



ℓ_k is the k^{th} layer of the network. There are $n + 1$ total layers. ($n = 3$ in the depicted network)
 $\vec{x}^{(\ell_k)}$ is the vector of inputs at layer k , the 0^{th} entry of which is always 1 (for the bias/intercept).
 $\dim(\vec{x}^{(\ell_k)}) = d_k$. In the depicted network, $\vec{d} = \langle 6, 7, 5, 2 \rangle$.

$W^{(\ell_k)}$ is the weight matrix connecting ℓ_k to ℓ_{k+1} .

$g_k = \lambda \vec{x} \cdot \langle 1 | \theta(W^{(\ell_k)T} \vec{x}) \rangle$ = Propagation function at layer k

$|$ denotes vector concatenation, and θ is the network's activation function (e.g. $\theta = \tanh$).

$\vec{x}^{(\ell_{k+1})} = g_k(\vec{x}^{(\ell_k)}) = \langle 1 | \theta(W^{(\ell_k)T} \vec{x}^{(\ell_k)}) \rangle$

$\dim(W^{(\ell_k)}) = d_k \times (d_{k+1} - 1)$

$f(\vec{x})$ = The function that the neural network approximates

$\tilde{f}(\vec{x})$ = The output of the neural network = $\vec{x}^{(\ell_n)}$

$\tilde{f} = \bigcirc_{k=0}^{n-1} g_k$, where \bigcirc represents iterated function composition.

$E(\vec{x}) = \frac{1}{2} ||f(\vec{x}) - \tilde{f}(\vec{x})||^2$

$$\frac{\partial E}{\partial W^{(\ell_{n-1})}}(\vec{x}) = ||f(\vec{x}) - \tilde{f}(\vec{x})||$$

$$\frac{\partial E}{\partial W_{ij}^{(\ell_k)}} = \frac{\partial E}{\partial \theta(W^{(\ell_k)T} \vec{x}^{(\ell_k)})_j} \frac{\partial \theta(W^{(\ell_k)T} \vec{x}^{(\ell_k)})_j}{\partial (W^{(\ell_k)T} \vec{x}^{(\ell_k)})_j} \frac{\partial (W^{(\ell_k)T} \vec{x}^{(\ell_k)})_j}{\partial W_{ij}^{(\ell_k)}}$$

$$\frac{\partial E}{\partial (W^{(\ell_k)T} \vec{x}^{(\ell_k)})_j} = \sum_{h=0}^{d_{k+1}-1} \left(\frac{\partial E}{\partial \theta(W^{(\ell_{k+1})T} \vec{x}^{(\ell_{k+1})})_h} \frac{\partial \theta(W^{(\ell_{k+1})T} \vec{x}^{(\ell_{k+1})})_h}{\partial (W^{(\ell_k)T} \vec{x}^{(\ell_k)})_h} W_{jh}^{(\ell_{k+1})} \right)$$

$$\frac{\partial \theta(W^{(\ell_k)T} \vec{x}^{(\ell_k)})_j}{\partial (W^{(\ell_k)T} \vec{x}^{(\ell_k)})_j} = \theta'(W^{(\ell_k)T} \vec{x}^{(\ell_k)})$$

$$\frac{\partial (W^{(\ell_k)T} \vec{x}^{(\ell_k)})_j}{\partial W_{ij}^{(\ell_k)}} = \frac{\partial}{\partial W_{ij}^{(\ell_k)}} \sum_{h=0}^{d_k-1} W_{hj}^{(\ell_k)} x_h^{(\ell_k)} = x_i^{(\ell_k)}$$