GENERAL ASSEMBLY

INTRO TO DATA SCIENCE
LECTURE 5: NAIVE BAYESIAN CLASSIFICATION

YUCHEN ZHAO / DAT-14

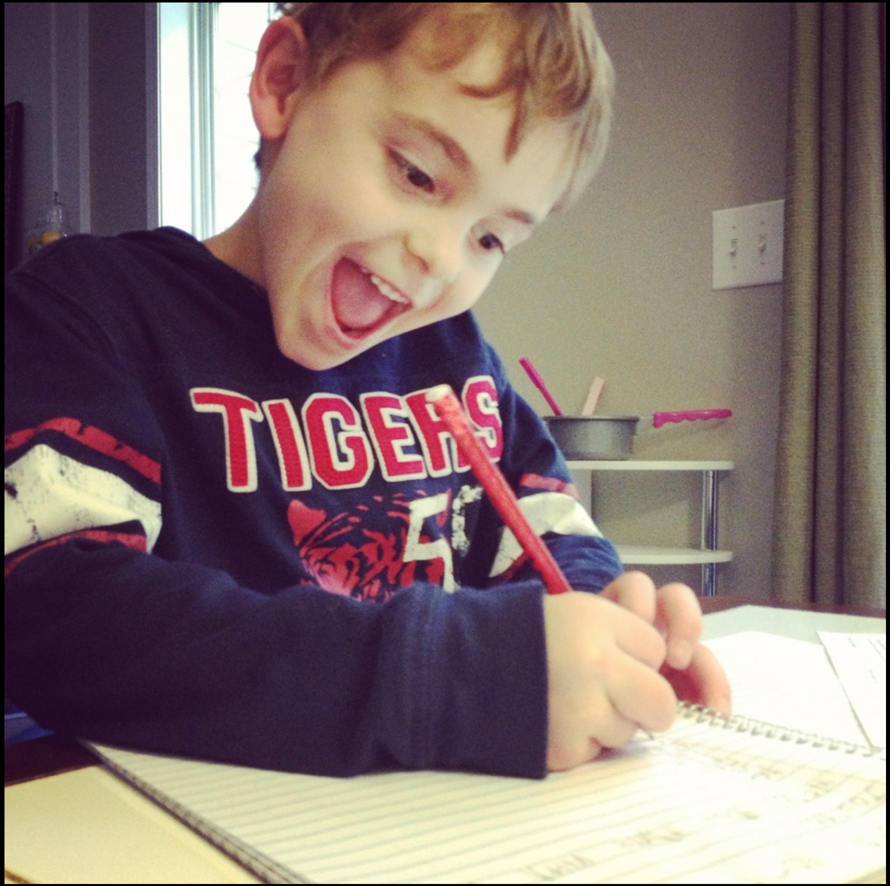
LAST TIME:

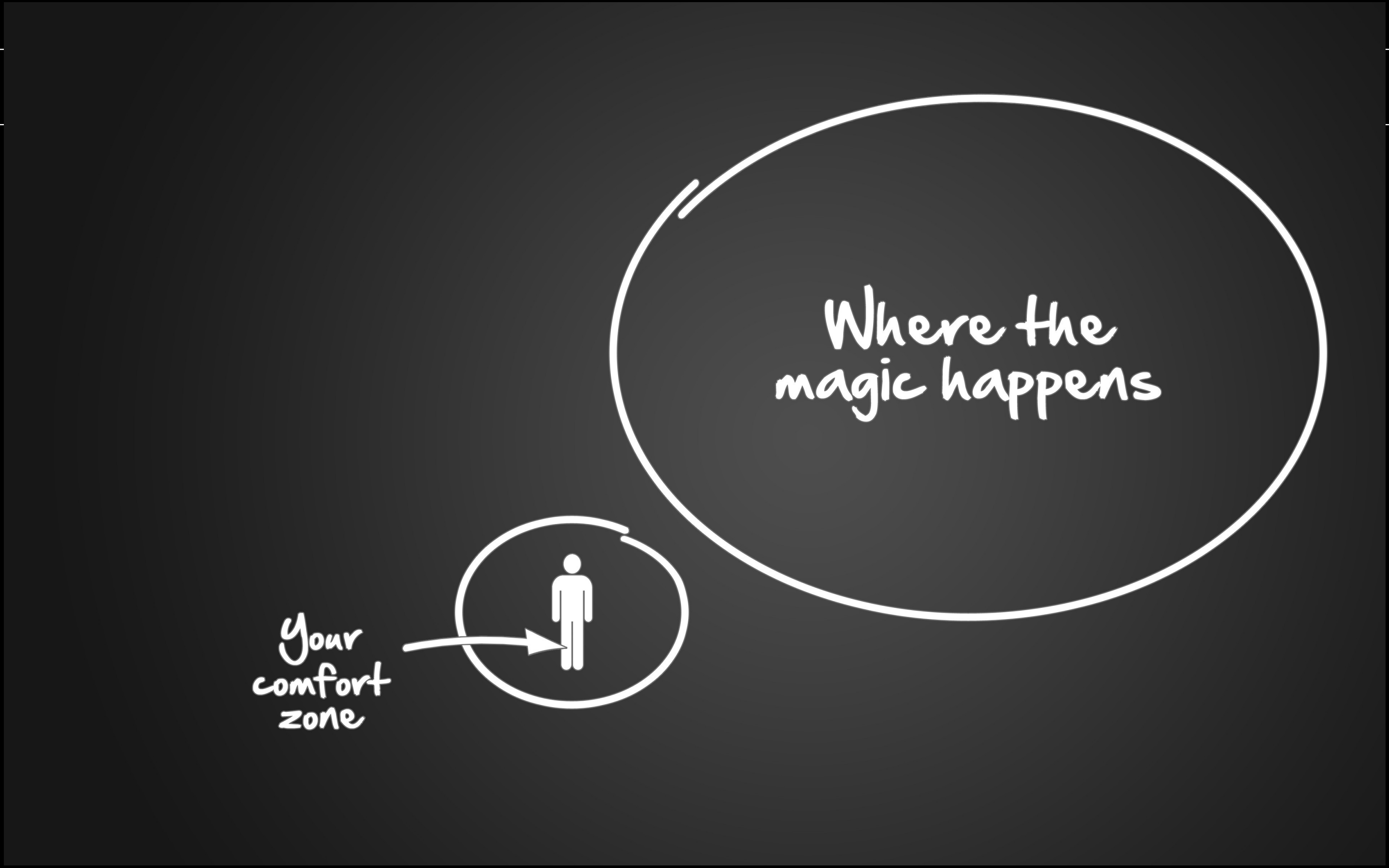
- CLASSIFICATION PROBLEMS
- TRAINING/TEST SETS & CROSS-VALIDATION
- KNN CLASSIFICATION

QUESTIONS?

GENERAL ASSEMBLY

HOW'S THE HOMEWORK GOING?


TIGERS


Where the magic happens
Your comfort zone

I. INTRO TO PROBABILITY
II. NAÏVE BAYESIAN CLASSIFICATION

EXERCISES:
III. NAÏVE BAYES CLASSIFICATION IN PYTHON

I. INTRO TO PROBABILITY

Q: What is a **probability***?*

Q: What is a **probability***?*

A: A number between ? and ? that characterizes the likelihood that some event will occur.

Q: What is a **probability***?*

A: A number between 0 and 1 that characterizes the likelihood that some event will occur.

Q: What is a **probability**?

A: A number between 0 and 1 that characterizes the likelihood that some event will occur.

The probability of event A *is denoted* $P(A)$.

Q: What is the set of all possible events called?

Q: What is the set of all possible events called?

A: This set is called the **sample space** Ω. *Event* A *is a member of the sample space, as is every other event.*

Q: What is the set of all possible events called?

A: This set is called the **sample space** Ω*. Event* A *is a member of the sample space, as is every other event.*

The probability of the sample space $P(\Omega)$ *is 1.*

Q: Consider two events A & B. *How can we characterize the* **intersection** *of these events?*

Q: Consider two events A & B*. How can we characterize the* **intersection** *of these events?*

A: With the **joint probability** *of* A *and* B*, written* $P(AB)$.

Q: Suppose event B *has occurred. What quantity represents the probability of* A **given** *this information about* B*?*

Q: Suppose event B *has occurred. What quantity represents the probability of* A **given** *this information about* B*?*

A: The intersection of A & B *divided by region* B.

Q: Suppose event B has occurred. What quantity represents the probability of A ***given*** *this information about B?*

A: The intersection of A & B divided by region B.

NOTE

This information about B transforms the sample space.

Take a moment to convince yourself of this!

Q: Suppose event B *has occurred. What quantity represents the probability of* A **given** *this information about* B*?*

A: The intersection of A & B *divided by region* B.

This is called the **conditional probability** *of* A *given* B*, written* $P(A|B) = P(AB) / P(B)$.

NOTE

This information about B transforms the sample space.

Take a moment to convince yourself of this!

Q: Suppose event B has occurred. What quantity represents the probability of A **given** *this information about B?*

A: The intersection of A & B divided by region B.

This is called the **conditional probability** *of A given B, written* $P(A|B) = P(AB) / P(B)$.

Notice, with this we can also write $P(AB) = P(A|B) * P(B)$.

NOTE

This information about B transforms the sample space.

Take a moment to convince yourself of this!

Q: What does it mean for two events to be **independent***?*

Q: What does it mean for two events to be **independent**?

A: Information about one does not affect the probability of the other.

Q: What does it mean for two events to be **independent***?*

A: Information about one does not affect the probability of the other.

This can be written as $\mathrm{P(A|B) = P(A)}$.

Q: What does it mean for two events to be **independent**?
A: Information about one does not affect the probability of the other.

This can be written as $P(A|B) = P(A)$.

Using the definition of the conditional probability, we can also write:

$$P(A|B) = P(AB) \,/\, P(B) = P(A) \rightarrow P(AB) = P(A) * P(B)$$

Probably the only calculation in the whole course:

Probably the only calculation in the whole course:

$$P(AB) = P(A|B) * P(B)$$

from last slide

Probably the only calculation in the whole course:

$P(AB) = P(A|B) * P(B)$ *from last slide*

$P(BA) = P(B|A) * P(A)$ *by substitution*

Probably the only calculation in the whole course:

$P(AB) = P(A|B) * P(B)$ *from last slide*

$P(BA) = P(B|A) * P(A)$ *by substitution*

But $P(AB) = P(BA)$ *since event* AB = *event* BA

Probably the only calculation in the whole course:

$P(AB) = P(A\|B) * P(B)$	*from last slide*
$P(BA) = P(B\|A) * P(A)$	*by substitution*
But $P(AB) = P(BA)$	*since event* AB = *event* BA
$\rightarrow P(A\|B) * P(B) = P(B\|A) * P(A)$	*by combining the above*

Probably the only calculation in the whole course:

$P(AB) = P(A|B) * P(B)$ *from last slide*

$P(BA) = P(B|A) * P(A)$ *by substitution*

But $P(AB) = P(BA)$ *since event* AB = *event* BA

→ $P(A|B) * P(B) = P(B|A) * P(A)$ *by combining the above*

→ $P(A|B) = P(B|A) * P(A) / P(B)$ *by rearranging last step*

This result is called **Bayes' theorem**. *Here it is again:*

$$P(A|B) = P(B|A) * P(A) \,/\, P(B)$$

This result is called **Bayes' theorem**. *Here it is again:*

$$P(A|B) = P(B|A) * P(A) / P(B)$$

Some facts:

- This is a simple algebraic relationship using elementary definitions.

This result is called **Bayes' theorem**. *Here it is again:*

$$P(A|B) = P(B|A) * P(A) / P(B)$$

Some facts:

- This is a simple algebraic relationship using elementary definitions.

- It's interesting because it's kind of a "wormhole" between two different "interpretations" of probability.

This result is called **Bayes' theorem**. *Here it is again:*

$$P(A|B) = P(B|A) * P(A) / P(B)$$

Some facts:

- This is a simple algebraic relationship using elementary definitions.

- It's interesting because it's kind of a "wormhole" between two different "interpretations" of probability.

- It's a very powerful computational tool.

Briefly, the two interpretations can be described as follows:

Briefly, the two interpretations can be described as follows:

The frequentist interpretation regards an event's probability as its limiting frequency across a very large number of trials.

Briefly, the two interpretations can be described as follows:

The frequentist interpretation regards an event's probability as its limiting frequency across a very large number of trials.

The Bayesian interpretation regards an event's probability as a "degree of belief," which can apply even to events that have not yet occurred.

If this sounds crazy to you, don't worry…we won't dwell on the theoretical details.

If this sounds crazy to you, don't worry…we won't dwell on the theoretical details.

If this sounds interesting, there are plenty of resources available to learn more about Bayesian inference.

If this sounds crazy to you, don't worry…we won't dwell on the theoretical details.

If this sounds interesting, there are plenty of resources available to learn more about Bayesian inference.

This a good direction to head if you like math and/or if you're interested in learning about cutting-edge data science techniques.

II. NAÏVE BAYESIAN CLASSIFICATION

Suppose we have a dataset with features $x_1, \ldots, x_n$ *and a class label* c.
What can we say about classification using Bayes' theorem?

Suppose we have a dataset with features $x_1, \ldots, x_n$ *and a class label* C.
What can we say about classification using Bayes' theorem?

$$P(\text{class } C \mid \{x_i\}) = \frac{P(\{x_i\} \mid \text{class } C) \cdot P(\text{class } C)}{P(\{x_i\})}$$

Bayes' theorem can help us to determine the probability of a record belonging to a class, given the data we observe.

source: Data Analysis with Open Source Tools, by Philipp K. Janert. O'Reilly Media, 2011.

Each term in this relationship has a name, and each plays a distinct role in any Bayesian calculation (including ours).

$$P(\text{class } C \mid \{x_i\}) = \frac{P(\{x_i\} \mid \text{class } C) \cdot P(\text{class } C)}{P(\{x_i\})}$$

This term is the **likelihood function***. It represents the joint probability of observing features* $\{x_i\}$ *given that that record belongs to class* C.

$$P(\text{class } C \mid \{x_i\}) = \frac{P(\{x_i\} \mid \text{class } C) \cdot P(\text{class } C)}{P(\{x_i\})}$$

This term is the **likelihood function**. *It represents the joint probability of observing features* $\{x_i\}$ *given that that record belongs to class* C.

$$P(\text{class } C \mid \{x_i\}) = \frac{P(\{x_i\} \mid \text{class } C) \cdot P(\text{class } C)}{P(\{x_i\})}$$

We can observe the value of the likelihood function from the training data.

This term is the **prior probability** *of* C. *It represents the probability of a record belonging to class* C *before the data is taken into account.*

$$P(\text{class } C \mid \{x_i\}) = \frac{P(\{x_i\} \mid \text{class } C) \cdot P(\text{class } C)}{P(\{x_i\})}$$

This term is the **prior probability** *of* C. *It represents the probability of a record belonging to class* C *before the data is taken into account.*

$$P(\text{class } C \mid \{x_i\}) = \frac{P(\{x_i\} \mid \text{class } C) \cdot P(\text{class } C)}{P(\{x_i\})}$$

The value of the prior is also observed from the data.

This term is the **normalization constant**. *It doesn't depend on* C, *and is generally ignored until the end of the computation.*

$$P(\text{class } C \mid \{x_i\}) = \frac{P(\{x_i\} \mid \text{class } C) \cdot P(\text{class } C)}{P(\{x_i\})}$$

This term is the **normalization constant**. *It doesn't depend on* C, *and is generally ignored until the end of the computation.*

$$P(\text{class } C \mid \{x_i\}) = \frac{P(\{x_i\} \mid \text{class } C) \cdot P(\text{class } C)}{P(\{x_i\})}$$

The normalization constant doesn't tell us much.

This term is the **posterior probability** *of* C. *It represents the probability of a record belonging to class* C *after the data is taken into account.*

$$P(\text{class } C \mid \{x_i\}) = \frac{P(\{x_i\} \mid \text{class } C) \cdot P(\text{class } C)}{P(\{x_i\})}$$

This term is the ***posterior probability*** *of* C*. It represents the probability of a record belonging to class* C *after the data is taken into account.*

$$P(\text{class } C \mid \{x_i\}) = \frac{P(\{x_i\} \mid \text{class } C) \cdot P(\text{class } C)}{P(\{x_i\})}$$

The goal of any Bayesian computation is to find ("learn") the posterior distribution of a particular variable.

The idea of Bayesian inference, then, is to **update** *our beliefs about the distribution of* C *using the data ("evidence") at our disposal.*

$$P(\text{class } C \mid \{x_i\}) = \frac{P(\{x_i\} \mid \text{class } C) \cdot P(\text{class } C)}{P(\{x_i\})}$$

Then we can use the posterior for prediction.

Q: What piece of the puzzle we've seen so far looks like it could intractably difficult in practice?

Remember the likelihood function?

$$P(\{x_i\}|C) = P(\{x_1, x_2, \dots, x_n\})|C)$$

Remember the likelihood function?

$$P(\{x_i\}|C) = P(\{x_1, x_2, \ldots, x_n\})|C)$$

Observing this exactly would require us to have enough data for every possible combination of features to make a reasonable estimate.

Q: What piece of the puzzle we've seen so far looks like it could intractably difficult in practice?

A: Estimating the full likelihood function.

Q: So what can we do about it?

Q: So what can we do about it?

A: Make a simplifying assumption. In particular, we assume that the features x_i *are conditionally independent from each other:*

Q: So what can we do about it?

A: Make a simplifying assumption. In particular, we assume that the features x_i *are conditionally independent from each other:*

$$P(\{x_i\}|C) = P(x_1, x_2, \ldots, x_n|C) \approx P(x_1|C) * P(x_2|C) * \ldots * P(x_n|C)$$

Q: So what can we do about it?

A: Make a simplifying assumption. In particular, we assume that the features x_i *are conditionally independent from each other:*

$$P(\{x_i\}|C) = P(x_1, x_2, \ldots, x_n)|C) \approx P(x_1|C) * P(x_2|C) * \ldots * P(x_n|C)$$

This "naïve" assumption simplifies the likelihood function to make it tractable.

$$P(\text{class } C \mid \{x_i\}) = \frac{P(\{x_i\} \mid \text{class } C) \cdot P(\text{class } C)}{P(\{x_i\})}$$

the training phase *of the model involves computing the* likelihood function, *which is the conditional probability of each feature given each class.*

the prediction phase *of the model involves computing the* posterior probability *of each class given the observed features, and choosing the class with the highest probability.*

***Advantages*:**

– *Fast to train (single scan). Fast to classify*

– *Not sensitive to irrelevant features*

– *Handles real and discrete data*

– *Handles streaming data well*

***Disadvantages*:**

– *Assumes independence of features*

Classification in Practice: Resale on Ebay

Enough Equations!!!

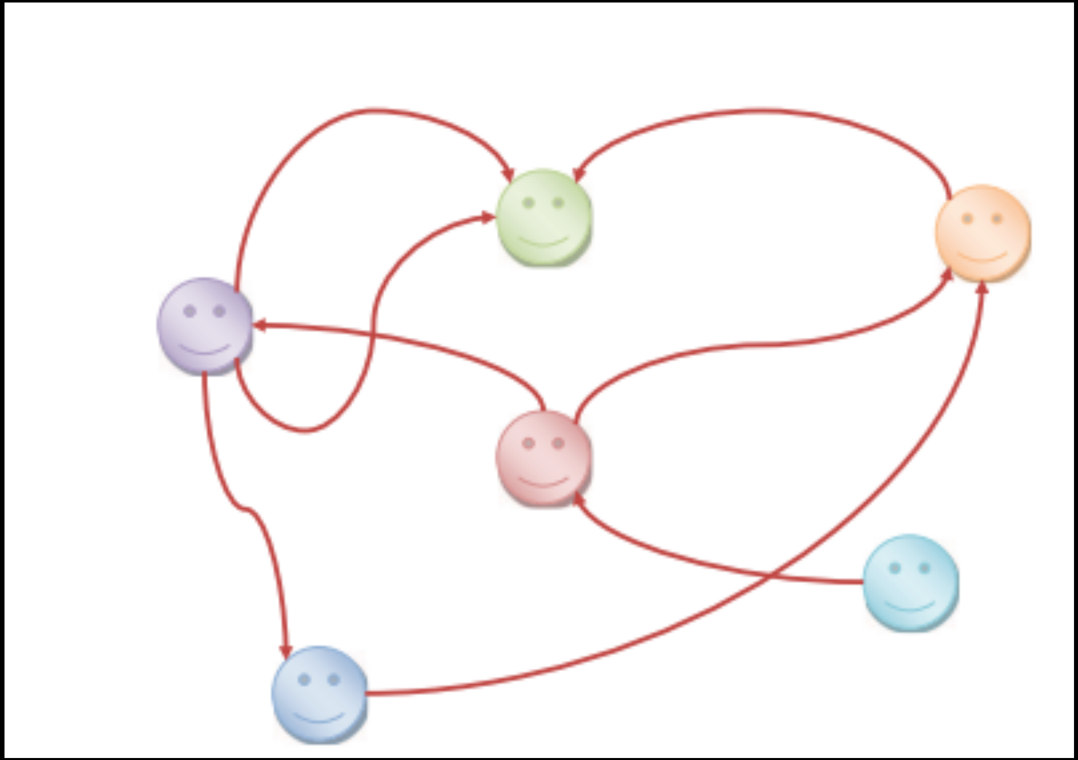
$$P(\text{class } C \mid \{x_i\}) = \frac{P(\{x_i\} \mid \text{class } C)}{P(\{x_i\})}$$

Show Me Something

Cool & Real!!!

Classification In Practice:
Resale On Ebay

Classification in Practice: Resale on Ebay



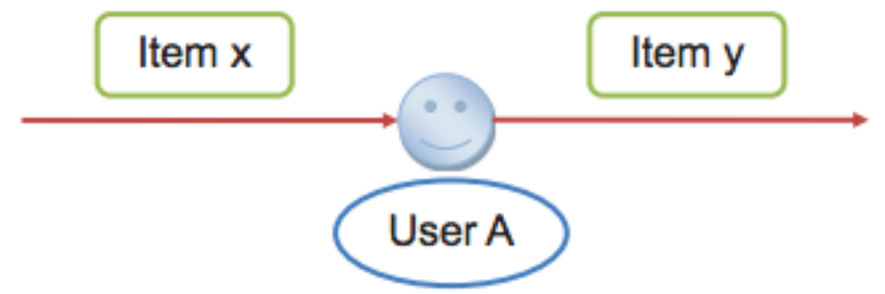
Study A Billion $$$ Market That No One Has Studied Before

Improving Reselling

Improving General Buying

Improving General Selling

Classification in Practice: Resale on Ebay



$(a) Exact\ Matching$

Challenge 1:

How To Develop Effective Criteria

To Accurately Identify Resale Activities?

Challenge 2:

How To Extract Resale Activities From Extremely Large-Scale Data Sets?

Challenge 3:

Can We Find Some Interesting Insights And Patterns From Resale Activities?

Classification Results

Table 7: Effectiveness of Various Models. The decision tree model achieves the best performance on four out of five measures.

Model	Acc.	Pre.	Rec.	F1	ROC
Naïve Bayes	64.6%	0.650	0.646	0.644	0.710
Log Regression	66.7%	0.669	0.667	0.666	0.710
Decision Tree	**75.4%**	**0.753**	**0.914**	**0.826**	0.718
Nearest Neighbor	70.9%	0.709	0.709	0.709	**0.721**

detail: http://www.cs.uic.edu/~yzhao/research/papers/pe012-zhao.pdf

LAB
III. NAIVE BAYESIAN CLASSIFICATION