GENERAL ASSEMBLY

INTRO TO DATA SCIENCE
LECTURE 5: NAIVE BAYESIAN CLASSIFICATION

YUCHEN ZHAO / DAT-14

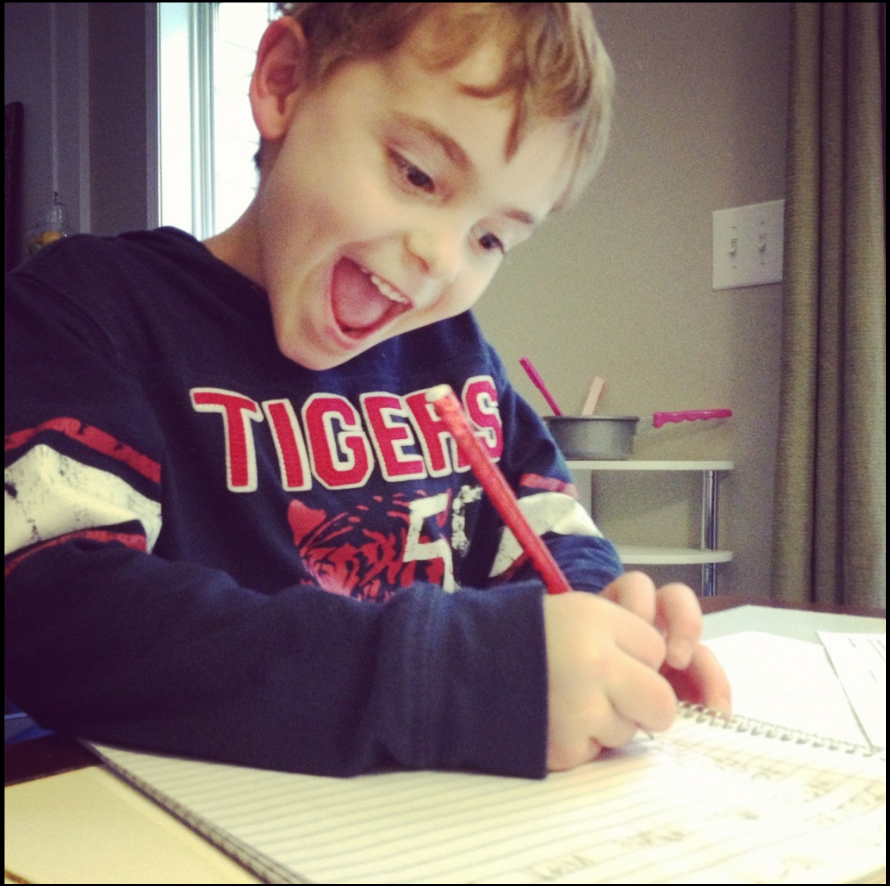
LAST TIME:

- CLASSIFICATION PROBLEMS
- TRAINING/TEST SETS & CROSS-VALIDATION
- KNN CLASSIFICATION

QUESTIONS?

GENERAL ASSEMBLY

HOW'S THE HOMEWORK GOING?


TIGERS


Where the magic happens
Your comfort zone

I. INTRO TO PROBABILITY
II. NAÏVE BAYESIAN CLASSIFICATION

EXERCISES:
III. NAÏVE BAYES CLASSIFICATION IN PYTHON

I. INTRO TO PROBABILITY

Q: What is a **probability***?*

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The probability of event A *is denoted* $P(A)$.

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A: This set is called the **sample space** Ω. *Event* A *is a member of the sample space, as is every other event.*

The probability of the sample space $P(\Omega)$ *is 1.*

Q: Consider two events A & B. *How can we characterize the* **intersection** *of these events?*

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A: With the **joint probability** *of* A *and* B, *written* $P(AB)$.

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NOTE

This information about B transforms the sample space.

Take a moment to convince yourself of this!

Q: Suppose event B *has occurred. What quantity represents the probability of* A **given** *this information about* B*?*

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This is called the **conditional probability** *of* A *given* B*, written* $P(A|B) = P(AB) / P(B)$.

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Notice, with this we can also write $P(AB) = P(A|B) * P(B)$.

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A: Information about one does not affect the probability of the other.

This can be written as $P(A|B) = P(A)$.

Using the definition of the conditional probability, we can also write:

$$P(A|B) = P(AB) \,/\, P(B) = P(A) \rightarrow P(AB) = P(A) * P(B)$$

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$\rightarrow P(A|B) * P(B) = P(B|A) * P(A)$ *by combining the above*

$\rightarrow P(A|B) = P(B|A) * P(A) / P(B)$ *by rearranging last step*

This result is called **Bayes' theorem**. *Here it is again:*

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Some facts:

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- It's interesting because it's kind of a "wormhole" between two different "interpretations" of probability.

- It's a very powerful computational tool.

Briefly, the two interpretations can be described as follows:

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The Bayesian interpretation regards an event's probability as a "degree of belief," which can apply even to events that have not yet occurred.

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This a good direction to head if you like math and/or if you're interested in learning about cutting-edge data science techniques.

II. NAÏVE BAYESIAN CLASSIFICATION

Suppose we have a dataset with features $x_1, \ldots, x_n$ *and a class label* C.
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$$P(\text{class } C \mid \{x_i\}) = \frac{P(\{x_i\} \mid \text{class } C) \cdot P(\text{class } C)}{P(\{x_i\})}$$

Bayes' theorem can help us to determine the probability of a record belonging to a class, given the data we observe.

source: Data Analysis with Open Source Tools, by Philipp K. Janert. O'Reilly Media, 2011.

Each term in this relationship has a name, and each plays a distinct role in any Bayesian calculation (including ours).

$$P(\text{class } C \mid \{x_i\}) = \frac{P(\{x_i\} \mid \text{class } C) \cdot P(\text{class } C)}{P(\{x_i\})}$$

This term is the **likelihood function***. It represents the joint probability of observing features* $\{x_i\}$ *given that that record belongs to class* C.

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We can observe the value of the likelihood function from the training data.

This term is the **prior probability** *of* C. *It represents the probability of a record belonging to class* C *before the data is taken into account.*

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The value of the prior is also observed from the data.

This term is the **normalization constant**. *It doesn't depend on* C, *and is generally ignored until the end of the computation.*

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The normalization constant doesn't tell us much.

This term is the **posterior probability** *of* C*. It represents the probability of a record belonging to class* C *after the data is taken into account.*

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The goal of any Bayesian computation is to find ("learn") the posterior distribution of a particular variable.

The idea of Bayesian inference, then, is to **update** *our beliefs about the distribution of* C *using the data ("evidence") at our disposal.*

$$P(\text{class } C \mid \{x_i\}) = \frac{P(\{x_i\} \mid \text{class } C) \cdot P(\text{class } C)}{P(\{x_i\})}$$

Then we can use the posterior for prediction.

Q: What piece of the puzzle we've seen so far looks like it could intractably difficult in practice?

Remember the likelihood function?

$$P(\{x_i\}|C) = P(\{x_1, x_2, \ldots, x_n\})|C)$$

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Observing this exactly would require us to have enough data for every possible combination of features to make a reasonable estimate.

Q: What piece of the puzzle we’ve seen so far looks like it could intractably difficult in practice?

A: Estimating the full likelihood function.

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This "naïve" assumption simplifies the likelihood function to make it tractable.

$$P(\text{class } C \mid \{x_i\}) = \frac{P(\{x_i\} \mid \text{class } C) \cdot P(\text{class } C)}{P(\{x_i\})}$$

the training phase *of the model involves computing the* likelihood function, *which is the conditional probability of each feature given each class.*

the prediction phase *of the model involves computing the* posterior probability *of each class given the observed features, and choosing the class with the highest probability.*

Advantages:

– Fast to train (single scan). Fast to classify

– Not sensitive to irrelevant features

– Handles real and discrete data

– Handles streaming data well

Disadvantages:

– Assumes independence of features

LAB
III. NAIVE BAYESIAN CLASSIFICATION