Advanced Probabilistic Machine Learning and Applications

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1 Tutorial 1: Introduction to probabilistic ML

Exercise 1: Multivariate Gaussian

Given a data set $\mathbf{X} = \{\mathbf{x}_1, \ldots, \mathbf{x}_N\}^\top$ in which the observations $\{\mathbf{x}_n\}$ are assumed to be drawn independently from a multivariate Gaussian distribution, i.e., $\mathbf{x}_1, \ldots, \mathbf{x}_N \sim \mathcal{N}(\mathbf{x}|\boldsymbol{\mu}_x, \boldsymbol{\Sigma}_x)$:

1. Estimate the mean and covariance parameters i. e., $\boldsymbol{\mu}_x$ and $\boldsymbol{\Sigma}_x$, by maximum likelihood.
2. Assume the covariance matrix $\boldsymbol{\Sigma}_x$ to be known and a Gaussian prior over the mean parameter $\boldsymbol{\mu}_x$ with mean $\boldsymbol{\mu}_0$ and identity covariance matrix, i.e., $\mathcal{N}(\boldsymbol{\mu}_x|\boldsymbol{\mu}_0, \mathbf{I})$. Compute the distribution a posteriori of the mean parameter μ_x given the observe data $\mathbf{X}$, i.e., $p(\boldsymbol{\mu}_x|\mathbf{X}, \boldsymbol{\mu}_0, \boldsymbol{\Sigma}_x)$, and its *Maximum a posteriori* (MAP) solution.

Exercise 2: Categorical distribution

Given a data set $\mathbf{X} = \{x_1, \ldots, x_N\}^\top$ in which the observations $x_n \in \{1, \ldots, k\}$ are assumed to be drawn independently from a Categorical distribution, i.e., $x_1, \ldots, x_N \sim Categorical(x|\pi_1, \ldots, \pi_k)$:

1. Estimate the parameters, i.e., the category probabilities $\{\pi_k\}$ by maximum likelihood.
2. Assume a Dirichlet prior over the category probabilities $\{\pi_k\}$ with hyperparameter α, i.e., $\pi_1, \ldots, \pi_k \sim Dirichlet(\pi_1, \ldots, \pi_k|\alpha)$. Compute the distribution a posteriori of the category probabilities $\{\pi_k\}$ given the observe data $\mathbf{X}$, i.e., $p(\pi_1, \ldots, \pi_k|\mathbf{X}, \alpha)$.

Exercise 3: Graphical models and corresponding joint distribution

1. Given the following generative model:

$$p(\{x_n, z_n\}_{n=1}^N, \{\pi_k, \mu_k\}_{k=1}^K) = \prod_n p(x_n|z_n, \{\mu_k\}_{k=1}^K, \sigma_x) p(z_n|\pi_1, \ldots \pi_k), p(\pi_1, \ldots \pi_k|\alpha),$$

   where $\{x_n\}$ are the observed variables, draw the corresponding graphical model.

2. Given the graphical model in Figure 1, write the generative model.

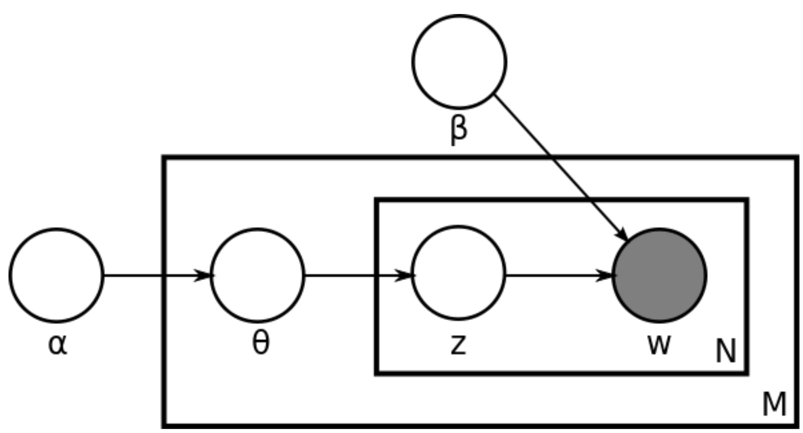

Figure 1: Graphical model for Exercise 3.1