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| Oregon State University |
| CS325 Project 3 |
| Linear Programming |

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Problem 1

1. i.

Xijk denotes the route a refrigerator travels from Plant Pi to Warehouse Wj to Retailer Rk

Minimize Shipping Costs = 15X111 + 16X112 + 17X113 + 20X114 + 27X123 + 23X124 + 25X125 + 29X126 + 16X211 + 17X212 + 18X213 + 21X214 + 20X223 + 16X224 + 18X225 + 22X226 + 18X311 + 19X312 + 20X313 + 23X314 + 20X323 + 16X324 + 18X325 + 14X326 + 23X334 + 21X335 + 21X336 + 15X337 + 26X423 + 22X424 + 24X425 + 28X426 + 22X434 + 20X435 + 20X436 + 14X437

X1jk <= 150

X2jk <= 450

X3jk <= 250

X4jk <= 150

Xij1 >= 100

Xij2 >= 150

Xij3 >= 100

Xij4 >= 200

Xij5 >= 200

Xij6 >= 150

Xij7 >= 100

Xijk >= 0 for all i, j, k

Optimal Solution for Shipping Routes with Minimal Cost

P1 to W1: 150 units

P2 to W1: 200 units

P2 to W2: 250 units

P3 to W2: 250 units

P4 to W3: 150 units

W1 to R1: 100 units

W1 to R2: 150 units

W1 to R3: 100 units

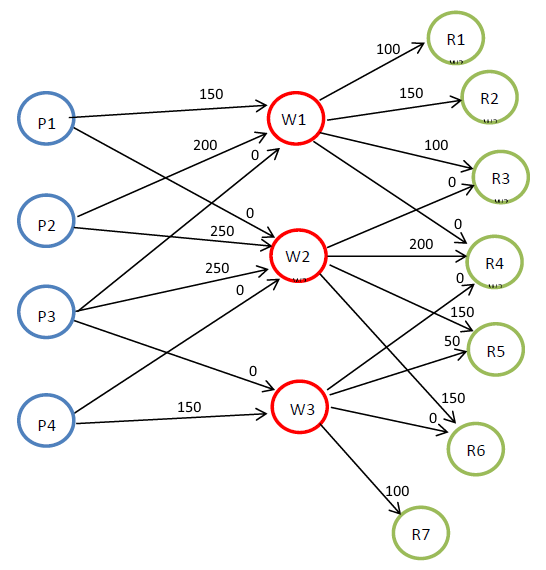
W2 to R4: 200 units

W2 to R5: 150 units

W2 to R6: 150 units

W3 to R5: 50 units

W3 to R7: 100 units



Code using Lindo7:

MIN

15 X111 + 16 X112 + 17 X113 + 20 X114 +

27 X123 + 23 X124 + 25 X125 + 29 X126 +

16 X211 + 17 X212 + 18 X213 + 21 X214 +

20 X223 + 16 X224 + 18 X225 + 22 X226 +

18 X311 + 19 X312 + 20 X313 + 23 X314 +

20 X323 + 16 X324 + 18 X325 + 14 X326 +

23 X334 + 21 X335 + 21 X336 + 15 X337 +

26 X423 + 22 X424 + 24 X425 + 28 X426 +

22 X434 + 20 X435 + 20 X436 + 14 X437

ST

X111 + X112 + X113 + X114 + X123 + X124 + X125 + X126 < 150

X211 + X212 + X213 + X214 + X223 + X224 + X225 + X226 < 450

X311 + X312 + X313 + X314 + X323 + X324 + X325 + X326 + X334 + X335 + X336 + X337 < 250

X423 + X424 + X425 + X426 + X434 + X435 + X436 + X437 < 150

X111 + X211 + X311 > 100

X112 + X212 + X312 > 150

X113 + X123 + X213 + X223 + X313 + X323 + X423 > 100

X114 + X124 + X214 + X224 + X314 + X324 + X334 + X424 + X434 > 200

X125 + X225 + X325 + X335 + X425 + X435 > 200

X126 + X226 + X326 + X336 + X426 + X436 > 150

X337 + X437 > 100

X111 > 0

X112 > 0

X113 > 0

X114 > 0

X123 > 0

X124 > 0

X125 > 0

X126 > 0

X211 > 0

X212 > 0

X213 > 0

X214 > 0

X223 > 0

X224 > 0

X225 > 0

X226 > 0

X311 > 0

X312 > 0

X313 > 0

X314 > 0

X323 > 0

X324 > 0

X325 > 0

X326 > 0

X334 > 0

X335 > 0

X336 > 0

X337 > 0

X423 > 0

X424 > 0

X425 > 0

X426 > 0

X434 > 0

X435 > 0

X436 > 0

X437 > 0

END

1. It is not feasible to close warehouse 3 and still meet the supply and demand constraints.

The simplest reason for this is that this would leave Warehouse 3 as the only warehouse to supply retailers R5, R6, R7. Combined the demand for those 3 Retailers is 450 units. However, Plants P3 and P4 can only produce 400 units, and they would be the only plants that can ship to W3.

1. Keeping W2 open at a limited capacity of 100 units would be feasible. This presents a new optimal solution:

P1 to W1: 150 units

P2 to W1: 400 units

P2 to W2: 50 units

P3 to W2: 50 units

P3 to W3: 200 units

P4 to W3: 150 units

W1 to R1: 100 units

W1 to R2: 150 units

W1 to R3: 100 units

W1 to R4: 200 units

W2 to R5: 50 units

W2 to R6: 50 units

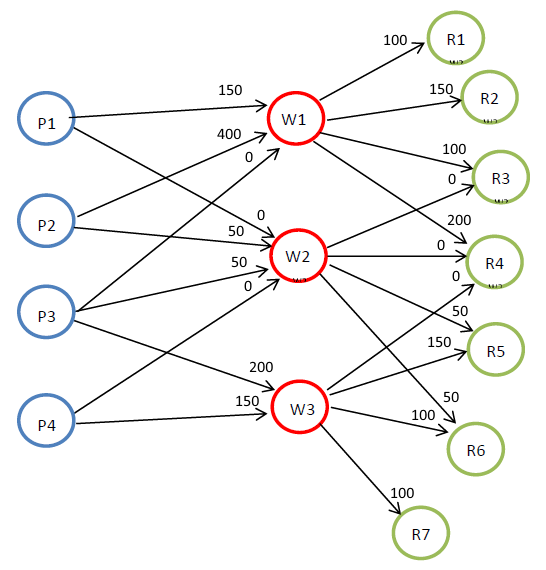
W3 to R5: 150 units

W3 to R6: 100 units

W3 to R7: 100 units

Code same as above but with added constraint:

X123 + X124 + X125 + X126 + X223 + X224 + X225 + X226 + X323 + X324 + X325 + X326 + X423 + X424 + X425 + X426 < 100



1. Linear programming model as mathematical formula

Define Variables

Pi: Plant that refrigerators are produced at

Wj: Warehouse that refrigerators are stored at

Rk: Retailer that sells refrigerators

Si: Associated supply of each Plant Pi

Dk: Associated demand of each Retailer Rk

cp(i,j): Cost of shipping 1 unit from Plant Pi to Warehouse Wj

cw(j,k): Cost of shipping 1 unit from Warehouse Wj to Retailer Rk

Objective

Minimize ∑ cp(i,j) \* Xij + ∑ cw(j,k) \* Xjk

Constraints

X ≥ 0 for all I, j, k

S1 ≤ 150

S2 ≤ 450

S3 ≤ 250

S4 ≤ 150

D1 ≥ 100

D2 ≥ 150

D3 ≥ 100

D4 ≥ 200

D5 ≥ 200

D6 ≥ 150

D7 ≥ 100

cp(1,1) = $10

cp(1,2) = $15

cp(2,1) = $11

cp(2,2) = $8

cp(3,1) = $13

cp(3,2) = $8

cp(3,3) = $9

cp(4,2) = $14

cp(4,3) = $8

cw(1,1) = $5

cw(1,2) = $6

cw(1,3) = $7

cw(1,4) = $10

cw(2,3) = $12

cw(2,4) = $8

cw(2,5) = $10

cw(2,6) = $14

cw(3,4) = $14

cw(3,5) = $12

cw(3,6) = $12

cw(3,7) = $6

Problem 2

1. Formulate as a linear problems:

Let: X1 = 100g of Tomatos

X2 = 100g of Lettuce

X3 = 100g of Spinach

X4 = 100g of Carrots

X5 = 100g of Sunflower Seeds

X6 = 100g of Smoked Tofu

X7 = 100g of Chickpeas

X8 = 100g of Oil

Minimize Z = 21X1 + 16X2 + 40X3 + 41X4 + 585X5 + 120X6 + 164X7 + 884X8

Where Z = number of calories (kcal)

Constraints:

0.85X1 + 1.62X2 + 2.86X3 + 0.93X4 + 23.4X5 + 16X6 + 9X7 ≥ 15g Protein

2g Fat ≤ 0.33X1 + 0.2X2 + 0.39X3 + 0.24X4 + 48.7X5 + 5X6 + 2.6X7 + 100X8 ≤ 8g Fat

4.64X1 + 2.37X2 + 3.63X3 + 9.58X4 + 15X5 + 3X6 + 27X7 ≥ 4g Carbohydrates

9X1 + 28X2 + 65X3 + 69X4 + 3.8X5 + 120X6 + 78X7 ≤ 200mg Sodium

(X2 + X3) / (X1 + X2 + X3 + X4 + X5 + X6 + X7 + X8) ≥ 0.4

Xi ≥ 0 for all i

Solution to the salad with minimum calories while still meeting nutritional requirements:

Tomato 0g

Lettuce 58.548g

Spinach 0g

Carrot 0g

Sunflower Seeds 0g

Smoked Tofu 87.822g

Chickpeas 0g

Oil 0g

Code used in Lindo 7:

MIN 21 X1 + 16 X2 + 40 X3 + 41 X4 + 585 X5 + 120 X6 + 164 X7 + 884 X8

ST

0.85 X1 + 1.62 X2 + 2.86 X3 + 0.93 X4 + 23.4 X5 + 16 X6 + 9 X7 > 15

0.33 X1 + 0.2 X2 + 0.39 X3 + 0.24 X4 + 48.7 X5 + 5 X6 + 2.6 X7 + 100 X8 > 2

0.33 X1 + 0.2 X2 + 0.39 X3 + 0.24 X4 + 48.7 X5 + 5 X6 + 2.6 X7 + 100 X8 < 8

4.64 X1 + 2.37 X2 + 3.63 X3 + 9.58 X4 + 15 X5 + 3 X6 + 27 X7 > 4

9 X1 + 28 X2 + 65 X3 + 69 X4 + 3.8 X5 + 120 X6 + 78 X7 < 200

X2 + X3 - 0.4 X1 - 0.4 X2 - 0.4 X3 - 0.4 X4 - 0.4 X5 - 0.4 X6 - 0.4 X7 - 0.4 X8 > 0

X1 > 0

X2 > 0

X3 > 0

X4 > 0

X5 > 0

X6 > 0

X7 > 0

X8 > 0

END

The cost of the Low Calorie Salad is $2.33 ($2.327283 to be exact)

1. Formulate as a linear problems:

Let: X1 = 100g of Tomatos

X2 = 100g of Lettuce

X3 = 100g of Spinach

X4 = 100g of Carrots

X5 = 100g of Sunflower Seeds

X6 = 100g of Smoked Tofu

X7 = 100g of Chickpeas

X8 = 100g of Oil

Minimize Z = X1 + 0.75X2 + 0.5X3 + 0.5X4 + 0.45X5 + 2.15X6 + 0.95X7 + 2X8

Where Z = cost in dollars

Constraints:

0.85X1 + 1.62X2 + 2.86X3 + 0.93X4 + 23.4X5 + 16X6 + 9X7 ≥ 15g Protein

2g Fat ≤ 0.33X1 + 0.2X2 + 0.39X3 + 0.24X4 + 48.7X5 + 5X6 + 2.6X7 + 100X8 ≤ 8g Fat

4.64X1 + 2.37X2 + 3.63X3 + 9.58X4 + 15X5 + 3X6 + 27X7 ≥ 4g Carbohydrates

9X1 + 28X2 + 65X3 + 69X4 + 3.8X5 + 120X6 + 78X7 ≤ 200mg Sodium

(X2 + X3) / (X1 + X2 + X3 + X4 + X5 + X6 + X7 + X8) ≥ 0.4

Xi ≥ 0 for all i

Solution to the salad with minimum calories while still meeting nutritional requirements:

Tomato 0g

Lettuce 0g

Spinach 83.2298g

Carrot 0g

Sunflower Seeds 9.6083g

Smoked Tofu 0g

Chickpeas 115.2365g \*see note below

Oil 0g

\*note – Lindo 7 report gave Chickpeas answer as 115.2364, but on double checking values this caused Protein = 14.99999g and Fat = 7.999987. This appears to be a rounding displace issue with numbers with long decimal values. Increasing either Sunflower Seeds or Chickpeas by 0.0001 would cause Protein to reach 15g, but Sunflower Seeds would also cause Fat to go over 8g, so I have incremented Chickpeas in the solution above. The change to price ($0.000001) and calories (0.0002 kCal) is very small.

Code used in Lindo 7:

MIN X1 + 0.75 X2 + 0.5 X3 + 0.5 X4 + 0.45 X5 + 2.15 X6 + 0.95 X7 + 2 X8

ST

0.85 X1 + 1.62 X2 + 2.86 X3 + 0.93 X4 + 23.4 X5 + 16 X6 + 9 X7 > 15

0.33 X1 + 0.2 X2 + 0.39 X3 + 0.24 X4 + 48.7 X5 + 5 X6 + 2.6 X7 + 100 X8 > 2

0.33 X1 + 0.2 X2 + 0.39 X3 + 0.24 X4 + 48.7 X5 + 5 X6 + 2.6 X7 + 100 X8 < 8

4.64 X1 + 2.37 X2 + 3.63 X3 + 9.58 X4 + 15 X5 + 3 X6 + 27 X7 > 4

9 X1 + 28 X2 + 65 X3 + 69 X4 + 3.8 X5 + 120 X6 + 78 X7 < 200

X2 + X3 - 0.4 X1 - 0.4 X2 - 0.4 X3 - 0.4 X4 - 0.4 X5 - 0.4 X6 - 0.4 X7 - 0.4 X8 > 0

X1 > 0

X2 > 0

X3 > 0

X4 > 0

X5 > 0

X6 > 0

X7 > 0

X8 > 0

END

There are 278.4882 kCal in the salad given in the solution above.

There are two ways I could see approaching this problem:

* 1. Add a constraint of Cost being under $2.00 for the salad and solve for minimum calories;

OR

* 1. Add a constraint of Calories being under 250kCal and solve for minimum cost

In our world Veronica is greedy and wants to make the most profit possible on her low-calorie salad. So she took approach b) above and her ingredient list will end up like so:

Tomato 0g

Lettuce 0g

Spinach 76.1996g

Carrot 0g

Sunflower Seeds 9.383g

Smoked Tofu 16.8942g

Chickpeas 88.0222g

Oil 0g

This will give a salad with 249.9998kCal at a cost of $1.622658

Problem 3:

Least Absolute Deviations method to find from a set of points.

Part A:

* 1. Objective:

Constraints:

* 1. The LAD regression line appears to be. The sum of absolute deviations for the LAD regression is 12.5 compared to the 13.5 found with the LSR method.

LINDO Input:

|  |  |
| --- | --- |
| 1  2  3  4  5  6  7  8  9  10  11  12  13  14  15  16  17 | MIN U1 + U2 + U3 + U4 + U5 + U6  ST  U1 + A0 + A1> 5  U1 - A0 - A1 > -5  U1 + A0 + A1 > 3  U1 - A0 - A1 > -3  U2 + A0 + 2 A1 > 13  U2 - A0 - 2 A1 > -13  U3 + A0 + 3 A1 > 8  U3 - A0 - 3 A1 > -8  U4 + A0 + 4 A1 > 10  U4 - A0 - 4 A1 > -10  U5 + A0 + 5 A1 > 14  U5 - A0 - 5 A1 > -14  U6 + A0 + 6 A1 > 18  U6 - A0 - 6 A1 > -18  END |

Output:

|  |
| --- |
| LP OPTIMUM FOUND AT STEP 0  OBJECTIVE FUNCTION VALUE  1) 11.50000  VARIABLE VALUE REDUCED COST  U1 1.000000 0.000000  U2 6.500000 0.000000  U3 1.000000 0.000000  U4 1.500000 0.000000  U5 0.000000 0.750000  U6 1.500000 0.000000  A0 **1.500000** 0.000000  A1 **2.500000** 0.000000  ROW SLACK OR SURPLUS DUAL PRICES  2) 0.000000 -0.625000  3) 2.000000 0.000000  4) 2.000000 0.000000  5) 0.000000 -0.375000  6) 0.000000 -1.000000  7) 13.000000 0.000000  8) 2.000000 0.000000  9) 0.000000 -1.000000  10) 3.000000 0.000000  11) 0.000000 -1.000000  12) 0.000000 0.000000  13) 0.000000 -0.250000  14) 0.000000 -1.000000  15) 3.000000 0.000000  NO. ITERATIONS= 0 |

* 1. Visual inspection shows that both LAD and LSR provide good regression equations that fit the data set. As previously mentioned, the LAD process had less total absolute deviation from the data set than LSR.

Part B:

1. Objective:

Constraints:

1. The MMAD regression line for this data set is. The minimum of the maximum absolute deviation for the MMAD method was 3.84 compared to the 5.5 found with LSR.

LINDO Input:

|  |  |
| --- | --- |
| 1  2  3  4  5  6  7  8  9  10  11  12  13  14  15  16  17 | MIN T  ST  T + A0 + A1 > 5  T - A0 - A1 > -5  T + A0 + A1 > 3  T - A0 - A1 > -3  T + A0 + 2 A1 > 13  T - A0 - 2 A1 > -13  T + A0 + 3 A1 > 8  T - A0 - 3 A1 > -8  T + A0 + 4 A1 > 10  T - A0 - 4 A1 > -10  T + A0 + 5 A1 > 14  T - A0 - 5 A1 > -14  T + A0 + 6 A1 > 18  T - A0 - 6 A1 > -18  END |

Output:

|  |
| --- |
| LP OPTIMUM FOUND AT STEP 2  OBJECTIVE FUNCTION VALUE  1) 7.500000  VARIABLE VALUE REDUCED COST  T 7.500000 0.000000  A0 10.500000 0.000000  A1 0.000000 0.000000  ROW SLACK OR SURPLUS DUAL PRICES  2) 13.000000 0.000000  3) 2.000000 0.000000  4) 15.000000 0.000000  5) 0.000000 -0.500000  6) 5.000000 0.000000  7) 10.000000 0.000000  8) 10.000000 0.000000  9) 5.000000 0.000000  10) 8.000000 0.000000  11) 7.000000 0.000000  12) 4.000000 0.000000  13) 11.000000 0.000000  14) 0.000000 -0.500000  15) 15.000000 0.000000  NO. ITERATIONS= 2  LP OPTIMUM FOUND AT STEP 2  OBJECTIVE FUNCTION VALUE  1) 3.833333  VARIABLE VALUE REDUCED COST  T 3.833333 0.000000  A0 4.500000 0.000000  A1 2.333333 0.000000  ROW SLACK OR SURPLUS DUAL PRICES  2) 5.666667 0.000000  3) 2.000000 0.000000  4) 7.666667 0.000000  5) 0.000000 -0.333333  6) 0.000000 -0.500000  7) 7.666667 0.000000  8) 7.333333 0.000000  9) 0.333333 0.000000  10) 7.666667 0.000000  11) 0.000000 -0.166667  12) 6.000000 0.000000  13) 1.666667 0.000000  14) 4.333333 0.000000  15) 3.333333 0.000000  NO. ITERATIONS= 2  LP OPTIMUM FOUND AT STEP 2  OBJECTIVE FUNCTION VALUE  1) 3.833333  VARIABLE VALUE REDUCED COST  T 3.833333 0.000000  A0 4.500000 0.000000  A1 2.333333 0.000000  ROW SLACK OR SURPLUS DUAL PRICES  2) 5.666667 0.000000  3) 2.000000 0.000000  4) 7.666667 0.000000  5) 0.000000 -0.333333  6) 0.000000 -0.500000  7) 7.666667 0.000000  8) 7.333333 0.000000  9) 0.333333 0.000000  10) 7.666667 0.000000  11) 0.000000 -0.166667  12) 6.000000 0.000000  13) 1.666667 0.000000  14) 4.333333 0.000000  15) 3.333333 0.000000  NO. ITERATIONS= 2  T 3.833333 0.000000  A0 4.500000 0.000000  A1 2.333333 0.000000  ROW SLACK OR SURPLUS DUAL PRICES  2) 5.666667 0.000000  3) 2.000000 0.000000  4) 7.666667 0.000000  5) 0.000000 -0.333333  6) 0.000000 -0.500000  7) 7.666667 0.000000  8) 7.333333 0.000000  9) 0.333333 0.000000  10) 7.666667 0.000000  11) 0.000000 -0.166667  12) 6.000000 0.000000  13) 1.666667 0.000000  14) 4.333333 0.000000  15) 3.333333 0.000000  NO. ITERATIONS= 2  LP OPTIMUM FOUND AT STEP 2  OBJECTIVE FUNCTION VALUE  1) 3.833333  VARIABLE VALUE REDUCED COST  T 3.833333 0.000000  A0 **4.500000** 0.000000  A1 **2.333333** 0.000000  ROW SLACK OR SURPLUS DUAL PRICES  2) 5.666667 0.000000  3) 2.000000 0.000000  4) 7.666667 0.000000  5) 0.000000 -0.333333  6) 0.000000 -0.500000  7) 7.666667 0.000000  8) 7.333333 0.000000  9) 0.333333 0.000000  10) 7.666667 0.000000  11) 0.000000 -0.166667  12) 6.000000 0.000000  13) 1.666667 0.000000  14) 4.333333 0.000000  15) 3.333333 0.000000  NO. ITERATIONS= 2 |

1. The slopes of the LSR and MMAD regression lines were nearly identical (2.33 vs 2. 31). There just seems to be an offset in the direction to reduce the maximum standard deviation found on the third point.
2. All three methods of regression will create identical solutions if all elements of a data set lie on a line. For instance, produces for all regression methods.

Part C:

Objective:

Constraints:

LINDO Input:

|  |  |
| --- | --- |
| 1  2  3  4  5  6  7  8  9  10  11  12  13  14 | MIN U1 + U2 + U3 + U4 + U5 + U6  ST  U1 + A0 + A1 + A2 > 5  U1 - A0 - A1 - A2 > -5  U2 + A0 + A1 + 2 A2 > 9  U2 - A0 - A1 - 2 A2 > -9  U3 + A0 + 2 A1 + 2 A2 > 12  U3 - A0 - 2 A1 - 2 A2 > -12  U4 + A0 + A2 > 3  U4 - A0 - A2 > -3  U5 + A0 = 0  U6 + A0 + 1 A1 + 3 A2 > 11  U6 - A0 - 1 A1 - 3 A2 > -11  END |

LINDO Output:

|  |
| --- |
| LP OPTIMUM FOUND AT STEP 2  OBJECTIVE FUNCTION VALUE  1) 0.0000000E+00  VARIABLE VALUE REDUCED COST  U1 0.000000 1.000000  U2 0.000000 1.000000  U3 0.000000 1.000000  U4 0.000000 1.000000  U5 0.000000 1.000000  U6 0.000000 1.000000  T 5.333333 0.000000  A0 0.000000 0.000000  A1 3.833333 0.000000  ROW SLACK OR SURPLUS DUAL PRICES  2) 4.166667 0.000000  3) 6.500000 0.000000  4) 6.166667 0.000000  5) 4.500000 0.000000  6) 0.000000 0.000000  7) 10.666667 0.000000  8) 8.833333 0.000000  9) 1.833333 0.000000  10) 10.666667 0.000000  11) 0.000000 0.000000  12) 10.500000 0.000000  13) 0.166667 0.000000  14) 10.333333 0.000000  15) 0.333333 0.000000  NO. ITERATIONS= 2  LP OPTIMUM FOUND AT STEP 8  OBJECTIVE FUNCTION VALUE  1) 2.000000  VARIABLE VALUE REDUCED COST  U1 1.000000 0.000000  U2 0.000000 0.000000  U3 0.000000 0.500000  U4 0.000000 0.000000  U5 0.000000 1.500000  U6 1.000000 0.000000  A0 **0.000000** 0.000000  A1 **3.000000** 0.000000  A2 **3.000000** 0.000000  ROW SLACK OR SURPLUS DUAL PRICES  2) 2.000000 0.000000  3) 0.000000 -1.000000  4) 0.000000 -1.000000  5) 0.000000 0.000000  6) 0.000000 -0.500000  7) 0.000000 0.000000  8) 0.000000 -1.000000  9) 0.000000 0.000000  10) 0.000000 0.500000  11) 2.000000 0.000000  12) 0.000000 -1.000000  NO. ITERATIONS= 8 |

This algorithm results in.