State of the Cups

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- **Single Processor, online filler (i.e. deterministic):**
  Filler: $\Omega(\log n)$ (ignore the touched cup each time, equal water to all others, $\frac{1}{n}+\frac{1}{n-1}+\cdots+\frac{1}{1}$)
  Emptier: $O(\log n)$ (inductive proof)

- **Single Processor, offline filler (i.e. randomized):**
  Filler: $\Omega(\log\log n)$ (anchoring???)
  Emptier: $O(\log\log n)$ (??)

- **Multiprocessor, online filler (i.e. deterministic):**
  Filler: $\Omega(\log n)$ (For $p<n-\sqrt{n}$ see Bills paper that gets $\Omega(\log(n-p))$ which is tight for these small values of p by playing a single processor cup game on $n-p+1$ cups and anchoring the other $p-1$ cups. For $p>n-\sqrt{n}$ you build the anchor set, adding $n-p$ cups to it each time, to get $\log n-\log(n-p)$ backlog)
  Emptier: $O(\log n)$ (Bill's complicated paper, generalizes the inductive proof for single processor case using skewed averages)

- **MultiProcessor, offline filler (i.e. randomized):**
  Filler: $\Omega(\log\log n)$ (anchoring???, HYPOTHESIS: this is not tight! we should be able to get $\Omega(\log p+\log\log n)$ $Pr[\text{Hypothesis is correct}]\approx 0.5$)
  Emptier: $O(\log\log n+\log p)$

- **MultiProcessor, online filler with Δp power:**
  Filler: $\Omega(\log p+\log\log n)$ (using the superpower you can get $\Theta(p)$ cups with known constant fill in them. Recursing on these $\log p$ times gives $\log p$ backlog, and we already knew $\Omega(\log\log n)$)
  Emptier: $O(\log n)$ (inductive proof)

- **MultiProcessor, offline filler with Δp power:**
  Filler: $\Omega(p)$ (Recursive thing $f'(p)=0.9\cdot(f(p/2)+f(p/4)+\ldots)$, HYPOTHESIS: its actually unbounded)
  Emptier: ?? (HYPOTHESIS: unbounded!)

Current goals:

- Make upper bound and lower bound agree for Multiprocessor cup game with offline opponent (i.e. randomized)
- Discover the bounds on the Δp augmented filler in the offline and online multiprocessor cup games