State of the Cups

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- **Single-Processor, online filler:**
  Filler: $\Omega(\log n)$ (ignore the touched cup each time, equal water to all others, $\frac{1}{n}+\frac{1}{n-1}+\cdots+\frac{1}{1}$)
  Emptier: $O(\log n)$ (inductive proof)

- **Single-Processor, offline filler:**
  Filler: $\Omega(\log\log n)$ (anchoring???)
  Emptier: $O(\log\log n)$ (??)

- **Multi-Processor, online filler:**
  Filler: $\Omega(\log n)$ (For $p<n-\sqrt{n}$ see Bills paper that gets $\Omega(\log(n-p))$ which is tight for these small values of p by playing a single processor cup game on $n-p+1$ cups and anchoring the other $p-1$ cups. For $p>n-\sqrt{n}$ you build the anchor set, adding $n-p$ cups to it each time, to get $\log n-\log(n-p)$ backlog)
  Emptier: $O(\log n)$ (Bill's complicated paper, generalizes the inductive proof for single processor case using skewed averages)

- **Multi-Processor, offline filler:**
  Filler: $\Omega(\log\log n)$ (anchoring???, HYPOTHESIS: this is not tight! we should be able to get $\Omega(\log p+\log\log n)$ $Pr[\text{Hypothesis is correct}]\approx 0.5$)
  Emptier: $O(\log\log n+\log p)$

- **Variable-Processor, online filler:**
  Filler: $\Omega(\text{poly}(p))$ (Amplification Lemma: $f'(p)=\frac{1}{2}\cdot(f(p/2)+f(p/4)+\ldots))$
  Emptier: $O(\text{poly}(p))$ (this is a conjecture, no legit thoughts on how to prove it yet)

- **Variable-Processor, offline filler:**
  Filler: $\Omega(\log p+\log\log n)$ (using the superpower you can get $\Theta(p)$ cups with known constant fill in them. Recursing on these $\log p$ times gives $\log p$ backlog, and we already knew $\Omega(\log\log n)$) CONJECTURE: we can probably do better: $\Omega(2^{\sqrt{\log n}/2})$ in $\text{poly}(n)$ moves (via careful use of Chernoff bounds)
  Emptier: Probably $O(2^{\sqrt{\log n}/2})$ is true (how to prove it? maybe not worth it...)

Current goals:

- Make upper bound and lower bound agree for multi-processor cup game with offline opponent (i.e. randomized)
- Prove bounds on variable-processor cup game against offline and online fillers