# State of the Cups

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- **Single-Processor, adaptive filler:**
  Filler: $\Omega(\log n)$ (ignore the touched cup each time, equal water to all others, $\frac{1}{n}+\frac{1}{n-1}+\cdots+\frac{1}{1}$)
  Emptier: $O(\log n)$ (inductive proof)

- **Single-Processor, oblivious filler:**
  Filler: $\Omega(\log\log n)$ (anchoring???)
  Emptier: $O(\log\log n)$ (no more than $\log n$ of the cups ever get fill above constant, play a cup game on the rest of the cups, get $O(\log\log n)$)

- **Multi-Processor, adaptive filler:**
  Filler: $\Omega(\log n)$ (For $p < n - \sqrt{n}$ see Bills paper that gets $\Omega(\log(n-p))$ which is tight for these small values of $p$ by playing a single processor cup game on $n-p+1$ cups and anchoring the other $p-1$ cups. For $p > n-\sqrt{n}$ you build the anchor set, adding $n-p$ cups to it each time, to get $\log n - \log(n-p)$ backlog)
  Emptier: $O(\log n)$ (Bill's complicated paper, generalizes the inductive proof for single processor case using skewed averages)

- **Multi-Processor, oblivious filler:**
  Filler: $\Omega(\log\log n)$ (anchoring???, HYPOTHESIS: this is not tight! we should be able to get $\Omega(\log p + \log\log n)$ $Pr[\text{Hypothesis is correct}] \approx 0.5$ )
  Emptier: $O(\log\log n + \log p)$

- **Variable-Processor, adaptive filler:**
  Filler: $\Omega(n^{1-\epsilon})$ for any constant $\epsilon > 0$ (Amplification Lemma: $f'(p) = \frac{1}{2}\cdot(f(p/2)+f(p/4)+\ldots)$)
  Emptier: $O(n)$ (this is a conjecture, no legit thoughts on how to prove it yet)

- **Variable-Processor, oblivious filler:**
  Filler: $\Omega(\log p + \log\log n)$ (using the superpower you can get $\Theta(p)$ cups with known constant fill in them. Recursing on these $\log p$ times gives $\log p$ backlog, and we already knew $\Omega(\log\log n)$) CONJECTURE: we can probably do better: $\Omega(2^{\sqrt{\log n}/2})$ in poly$(n)$ moves (via careful use of Chernoff bounds)
  Emptier: Probably $O(2^{\sqrt{\log n}/2})$ is true (how to prove it? maybe not worth it...)

**Current goals:**

- Make upper bound and lower bound agree for multi-processor cup game with oblivious opponent (i.e. randomized)
- Prove better bounds on variable-processor cup game against oblivious and adaptive fillers