Assignment 2: CS215

Satyam Sinoliya, 23B0958
Vaibhav Singh, 23B1068
Shaik Awez Mehtab, 23B1080

Solution 1

Task A

When $X \sim \text{Ber}(p)$, PGF of X is

$$G_{\text{Ber}}(z) = \mathbb{E}(z^X) \tag{1}$$
$$= \sum_{n=0}^{\infty} P[X = n]z^n \tag{2}$$

Since $P[X = 0] = (1 - p)$, $P[X = 1] = p$, $P[X = n] = 0$ when $n > 1$,

$$G_{\text{Ber}}(z) = P[X = 0]z^0 + P[X = 1]z^1 \tag{3}$$
$$= (1 - p) + pz \tag{4}$$

Task B

When $X \sim \text{Bin}(n, p)$, PMF of X, $P[X = k] = \binom{n}{k} p^k (1 - p)^{n-k}$ for $0 \leq k \leq n$. But for $k > n$, $P[X = k] = 0$.

$$G_{\text{Bin}}(z) = \sum_{k=0}^{\infty} P[X = k]z^k \tag{5}$$
$$= \sum_{k=0}^{n} \binom{n}{k} p^k (1 - p)^{n-k} z^k \tag{6}$$
$$= \sum_{k=0}^{n} \binom{n}{k} (pz)^k (1 - p)^{n-k} \tag{7}$$
$$= (1 - p + pz)^n. \tag{8}$$

By equation 4, $G_{\text{Bin}}(z) = (1 - p + pz)^n = (G_{\text{Ber}}(z))^n$. Hence proved.

Solution 2

Solution 3

Solution 4

Solution 5