CS215 Assignment 1 Solutions

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Question 1. There are two friends playing a dice-roll game. Friend A has (n + 1) fair dice and Friend B has n fair dice (a fair die has equal probability of every face). On every roll, a win is achieved if we get a prime number on the top. What is the probability that A will have more wins than B if both roll all of their dice?

Solution 1. So A or B wins when they get either 2, 3 or 5, whose probability is $\frac{1}{2}$. So the probability that A wins i number of times when all of the $(n+1)$ dice are rolled is

$$\begin{aligned} p(A_i) &= \binom{n+1}{i}\left(\frac{1}{2}\right)^{i}\left(\frac{1}{2}\right)^{n+1-i} && (1) \\ &= \binom{n+1}{i}\left(\frac{1}{2}\right)^{n+1} && (2) \end{aligned}$$

Similarly, probability that B gets i wins when all of the n are rolled is

$$\begin{aligned} p(B_i) &= \binom{n}{i}\left(\frac{1}{2}\right)^{i}\left(\frac{1}{2}\right)^{n-i} && (3) \\ &= \binom{n}{i}\left(\frac{1}{2}\right)^{n} && (4) \end{aligned}$$

Now the probability that A has more wins than B is

$$p(i > j) = \sum_{i=0}^{n+1}\sum_{\substack{j=0 \\ i>j}}^{n} p(A_i)p(B_j) \tag{5}$$

Since number of wins of A is independent from that of B. By writing each possible term $\binom{n+1}{i}$ and $\binom{n}{j}$ in a grid manner. We observe that our required terms cover exactly half of the grid. Thus equation (5) simplifies to

$$p(i > j) = \frac{\sum_{i=0}^{n+1}\binom{n+1}{i}\left(\frac{1}{2}\right)^{n+1}\cdot\sum_{j=0}^{n}\binom{n}{j}\left(\frac{1}{2}\right)^{n}}{2} = \frac{1}{2} \tag{6}$$

Thus, the probability of A winning more times than B is $\frac{1}{2}$.