ASTE 586 Computer Project, Supplement

Computing Precession, Nutation, and Spin

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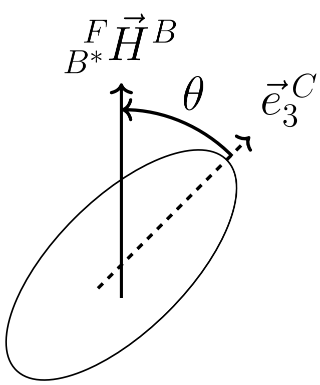

For axisymmetric bodies, we defined precession, nutation, and precession angles that all turned out to have simple behaviors. This diagram shows $\vec{e}_3^{\,C}$ as the axis of symmetry, but in other cases it could be $\vec{e}_2^{\,C}$ or $\vec{e}_1^{\,C}$.
As it is in this project, the particular axis of the C coordinate system that is your axis of symmetry (reference vector) may vary. Here we shall call it $\vec{e}_{\text{sym}}^{\,C}$. Likewise, an axis of the C coordinate system perpendicular to the axis of symmetry is some lateral axis, I'll call it $\vec{e}_{\text{node}}^{\,C}$. In particular if $\vec{e}_{\text{sym}}^{\,C}$ is $\vec{e}_3^{\,C}$, then $\vec{e}_{\text{node}}^{\,C}$ might be $\vec{e}_1^{\,C}$.

You are to compute precession, nutation, and spin angles from the simulation. You may do this with any method that you are comfortable with.

If you don't use one of the methods, below, then you should explain how your method works.

Many of the computations, below, involve the dot or cross product of two vectors. Remember that computing these products requires both vectors to be expressed in the same coordinate system for the result to be valid.

1 Method 1

You may solve for precession, nutation, and spin by deriving a rotation matrix symbolically as a function of θ, ψ, ϕ and setting it equal to a rotation matrix you compute from your simulation's Euler-Rodrigues parameters.

$$\left[{}^{F}C^{A}(\psi)\right]\left[{}^{A}C^{B}(\theta)\right]\left[{}^{B}C^{C}(\phi)\right] = {}^{F}C^{C}(\epsilon_1, \epsilon_2, \epsilon_3, \epsilon_4) \tag{1}$$

It is important to remember that this algorithm relies on the Euler-Rodrigues parameters having initial values, $\epsilon_i(0)$, that are consistent. For example, thinking of this as a 3-1-3 sequence, the initial values for the Euler-Rodrigues parameters must be such that the angular momentum vector ${}^{F}_{B^*}\vec{H}^{B}$ is aligned with $\vec{e}_3^{\,F}$ and the reference axis $\vec{e}^{\,C}_{\mathrm{ref}}$ must be $\vec{e}_3^{\,C}$.

Depending on your choice of $\epsilon_i(0)$, the precession angles computed with Equation 12 and Equation 1 may be different. If they only differ by a constant, that is perfectly acceptable.

In using this method, I recommend that you pursue expressions involving tangent of an Euler angle, like $\tan\phi = f(\epsilon_1, \epsilon_2, \epsilon_3, \epsilon_4)$. If possible, use of atan2 is generally preferred over atan. The right-hand side of this expression would involve more than one element of the rotation matrix, ${}^{F}C^{C}$, from the right-hand side of Equation 1. This way you avoid the ambiguities that come from relying solely on the sine or cosine function. For example, suppose that Equation 1 reveals that two of the elements of the rotation matrix, ${}_{i}C_{j}$ and ${}_{m}C_{n}$, are as follows:

$$\sin\psi\sin\theta = {}_{i}C_{j} \tag{2}$$

$$-\cos\psi\sin\theta = {}_{m}C_{n} \tag{3}$$

$$\boxed{\psi = \mathrm{atan2}\left({}_{i}C_{j}, -{}_{m}C_{n}\right)} \tag{4}$$

So, using the Euler-Rodrigues parameter values from the simulation at time t, you compute the rotation matrix ${}^{F}C^{C}(\epsilon_1, \epsilon_2, \epsilon_3, \epsilon_4)$. Then, you plug the i, j and m, n elements of that rotation matrix into Equation 4, giving you ψ for time t.

It's OK to use a subroutine from a library or some such you find online, as long as you are able to use it properly.

2 Method 2

It is inevitable that problems arise in implementing computations such as these. I suggest that you implement them in the order they are presented. The nutation angle is arguably the simplest to compute, so it makes sense to approach it first. Somewhat surprisingly, the spin angle seems to be less complicated than precession, so I recommend it next. Leaving the precession for last.

Remember that you'll be computing these angles for each time node of your simulation results. Likewise, you'll need to compute intermediate quantities like ${}^{F}_{B^*}\vec{H}^{B}$ for every time node, as well.

2.1 Nutation Angle

Recall that θ is the nutation angle, a measure of how the body is nodded over - it's the angle between the body's axis of symmetry and the angular momentum vector.

$$\theta = \arccos\left(\vec{e}^{\,C}_{\mathrm{ref}} \cdot \frac{{}^{F}_{B^*}\vec{H}^{B}}{\left|{}^{F}_{B^*}\vec{H}^{B}\right|}\right) \tag{5}$$

That's equivalent to Equation 13.

2.2 Spin Angle

Now for the spin angle, ϕ, which describes how the body is rotating around it's own axis of symmetry. This is very similar to the precession angle, but you need to measure against vectors fixed in the body frame (express them in the body-fixed coordinate system). The vector $\vec{e}^{\,C}_{\mathrm{node}}$ is such a reference

$$\vec{e}^{\,C}_{\mathrm{node}} = \frac{{}^{F}_{B^*}\vec{H}^{B} \times \vec{e}^{\,C}_{\mathrm{ref}}}{\left|{}^{F}_{B^*}\vec{H}^{B} \times \vec{e}^{\,C}_{\mathrm{ref}}\right|} \tag{6}$$

For the purpose of the spin angle, compute $\vec{e}^{\,C}_{\mathrm{node}}$ at the initial time in body-fixed coordinates and keep it fixed. You may then define $\vec{e}^{\,C}_{\mathrm{x}}$ as

$$\vec{e}^{\,C}_{\mathrm{x}} \triangleq \vec{e}^{\,C}_{\mathrm{ref}} \times \vec{e}^{\,C}_{\mathrm{node}} \tag{7}$$

$\vec{e}^{\,C}_{\mathrm{x}}$ will also be body-fixed and not varying with time, like $\vec{e}^{\,C}_{\mathrm{node}}$.[1] Next, you need a vector in body-fixed coordinates that captures the spin motion, we'll

[1]Alternate choices for $\vec{e}^{\,C}_{\mathrm{x}}$ and $\vec{e}^{\,C}_{\mathrm{node}}$ are the axis vectors, like $\vec{e}^{\,C}_{\mathrm{node}} = \vec{e}^{\,C}_{1}$, $\vec{e}^{\,C}_{\mathrm{x}} = \vec{e}^{\,C}_{2}$.

call it $\vec{u}$.

$$\vec{u} = \frac{{}^{F}_{B^*}\vec{H}^{B} \times \vec{e}^{\,C}_{\mathrm{ref}}}{\left|{}^{F}_{B^*}\vec{H}^{B} \times \vec{e}^{\,C}_{\mathrm{ref}}\right|} \tag{8}$$

Compute $\vec{u}$ in the body-fixed coordinate system – Yes, this looks just like Equation 6, but you will compute it for each time node t. Then you may compute ϕ at time node t.

$$\phi = -\mathrm{atan2}\left(\left(\vec{u} \cdot \vec{e}^{\,C}_{\mathrm{x}}\right), \left(\vec{u} \cdot \vec{e}^{\,C}_{\mathrm{node}}\right)\right) \tag{9}$$

Note that $\mathrm{atan2}\left(\left(\vec{u} \cdot \vec{e}^{\,C}_{\mathrm{x}}\right), \left(\vec{u} \cdot \vec{e}^{\,C}_{\mathrm{node}}\right)\right)$ computes an angle measured with postive values from $\vec{e}^{\,C}_{\mathrm{node}}$ to $\vec{u}$. The spin angle, ϕ, is from the B coordinate system to the C coordinate system. Since that's the opposite, we negate the expression to get Equation 9.

2.3 Precession Angle

Finally, the precession angle. Loosely speaking, the precession angle describes how the body, particularly $\vec{e}^{\,C}_{\mathrm{ref}}$, marches around the angular momentum vector. One way to compute the precession angle begins with the cross product of the angular momentum vector direction and the reference axis at each time node of the simulation.

Refer back to Equation 8 – we need $\vec{u}$ again, this time expressed in the inertial coordinate system. Record $\vec{u}$ at the initial time as $\vec{u}_0$. So, $\vec{u}_0$ is at the initial time and $\vec{u}$ is at time node t. It is critical that $\vec{u}_0$ be inertially fixed - store it as components in the inertial coordinate system. $\vec{u}$ lies on the intersection of the plane perpendicular to $\vec{e}^{\,C}_{\mathrm{ref}}$ and the plane perpendicular to ${}^{F}_{B^*}\vec{H}^{B}$; that's called a nodal vector or node. A second helpful vector is defined perpendicular to ${}^{F}_{B^*}\vec{H}^{B}$ and $\vec{u}$:

$$\vec{v} = \frac{{}^{F}_{B^*}\vec{H}^{B}}{\left|{}^{F}_{B^*}\vec{H}^{B}\right|} \times \vec{u} \tag{10}$$

As for $\vec{u}$, record $\vec{v}$ at the initial time as $\vec{v}_0$. Here, too, it is critical that $\vec{v}_0$ be inertially fixed. Now you may compute the precession angle as

$$\psi = \mathrm{atan2}\left(\left(\vec{u} \cdot \vec{v}_0\right), \left(\vec{u} \cdot \vec{u}_0\right)\right) \tag{11}$$

Precession is the motion of the B coordinate system relative to the inertial reference frame. Vector $\vec{u}$ is fixed in the B coordinate system (It is expressed in coordinate system F). Vectors $\vec{u}_0$ and $\vec{v}_0$ represent the F (inertial) coordinate system.

3 Method 3

We may also compute our Euler angles with

$$\cos(\psi) = \left(\frac{\vec{e}_2^{\,F} \times {}^{F}_{B^*}\vec{H}^B}{\left| \vec{e}_2^{\,F} \times {}^{F}_{B^*}\vec{H}^B \right|} \right) \cdot \left(\frac{{}^{F}_{B^*}\vec{H}^B \times \vec{e}_{\text{ref}}^{\,C}}{\left| {}^{F}_{B^*}\vec{H}^B \times \vec{e}_{\text{ref}}^{\,C} \right|} \right) \tag{12}$$

$$\cos(\theta) = \vec{e}_{\text{ref}}^{\,C} \cdot \frac{{}^{F}_{B^*}\vec{H}^B}{\left| {}^{F}_{B^*}\vec{H}^B \right|} \tag{13}$$

$$\cos(\phi) = \vec{e}_{\text{node}}^{\,C} \cdot \left(\frac{{}^{F}_{B^*}\vec{H}^B \times \vec{e}_{\text{ref}}^{\,C}}{\left| {}^{F}_{B^*}\vec{H}^B \times \vec{e}_{\text{ref}}^{\,C} \right|} \right) \tag{14}$$

If the angular momentum is parallel to $\vec{e}_2^{\,F}$ instead of $\vec{e}_3^{\,F}$, meaning ${}^{F}_{B^*}\vec{H}^B \parallel \vec{e}_2^{\,F}$, then you'll need to replace $\vec{e}_2^{\,F}$ with $\vec{e}_1^{\,F}$ or $\vec{e}_3^{\,F}$ in Equation 12. Note, too, since these involve cosine, you'll only get positive angles. Having only positive angles means that all the rates will look positive. You may use this approach, but I'll still expect you to interpret the results properly and say if a rate is really positive or negative.