ASTE 586 Computer Project, Part 1

1 Introduction

The first part of the computer project is to make sure that you have a proper setup to numerically integrate differential equations, with some control over accuracy, for an initial value problem. This initial work is really just getting your "tools" set-up for the project.

The assignment is to solve the given initial value problem analytically, on your own (not with a symbolic solver). Then solve it numerically, compare the two solutions, and show that your numerical solution is accurate.

2 The Initial Value Problem

The differential equations:

$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \\ \dot{x}_3(t) \\ \dot{x}_4(t) \end{bmatrix} = \begin{bmatrix} 0 & 2 & 0 & 0 \\ -2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 200 \\ 0 & 0 & -200 & 0 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \\ x_4(t) \end{bmatrix}$$

The initial values:

$$\begin{bmatrix} x_1(0) \\ x_2(0) \\ x_3(0) \\ x_4(0) \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix}$$

Be sure to compare your numerical and analytical results out to at least $t = 10$. You should be able to obtain enough accuracy such that, at $t = 10$ the maximum difference between your numerical and analytical solutions is about 10^{-3} or less for all four components of x. Show that this is satisfied. Plot $(x_1^2(t) + x_2^2(t) + x_3^2(t) + x_4^2(t) - 2)$ versus time to demonstrate that the result is very close to zero. Report your tolerance values.