Characterization of multilayer mirrors by combination of complementary analytical methods using EUV and X-ray radiation

vorgelegt von
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der Technischen Universität Berlin
zur Erlangung des akademischen Grades
Doktor der Naturwissenschaften
Dr. rer. nat.

vorgelegte Dissertation

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Berlin 2017

Abstract

Multilayer mirrors for the extreme ultraviolet (EUV) spectral range are essential optical elements of next-generation lithography systems and in scientific applications, e.g. water window microscopes. Their lack in reaching theoretically predicted peak reflectivity values significantly hinders their applicability and raises the question of the reasons behind that limited performance. This thesis employs a combination of indirect characterization techniques using EUV and X-ray radiation to enable unambiguous judgments on the structural properties and interface morphologies of those multilayer systems.

This approach is used to study two sets of unpolished and interface polished Mo/Si/C multilayer systems designed to reflect EUV radiation with 13.5 nm wavelength. They have been fabricated with increasing molybdenum thickness from sample to sample. By examining the combination of EUV reflectivity and X-ray reflectivity (XRR) considering experimental uncertainties, structural parameters can be reconstructed and validated by deducting confidence intervals. By establishing a method for the analysis of EUV diffuse scattering, an observed dip in the peak reflectance for some samples can be related to variations in layer thickness and interface roughness associated with crystallization in the molybdenum layers. Increased roughness for samples at the crystallization threshold and intermixing can be identified impeding the measured reflectance.

Furthermore, this methodology is applied to Cr/Sc multilayer mirrors for the water window spectral range having individual layer thicknesses in the sub-nanometer regime. The combination of the analysis of EUV reflectivity and of XRR based on a binary layer model is shown to be insufficient to describe this system. The model is extended to explicitly take into account gradual interface profiles and strong intermixing. It is demonstrated by structural characterization and systematic validation of the extended model parameters, based on the analysis of EUV reflectivity, resonant extreme ultraviolet reflectivity (REUV), XRR and X-ray fluorescence (XRF) experiments, that only the combination of those analytic methods yields a consistent result. By augmenting the characterization through the EUV diffuse scattering analysis, the cause for the low reflectivity is determined.

Zusammenfassung

Mehrschichtspiegel für den EUV Wellenlängenbereich sind wichtige optische Komponenten für die nächste Halbleiterlithografiegeneration und kommen auch im wissenschaftlichen Bereich, beispielsweise in Mikroskopen für das Wasserfenster, zum Einsatz . Deren verminderte Reflektivität im Vergleich zu den theoretisch möglichen Werten, schränkt ihre Einsatzfähigkeit ein und wirft die Frage nach den Ursachen dafür auf. Die vorliegende Dissertation nutzt eine Kombination von indirekten Charakterisierungstechniken unter Anwendung von EUV und Röntgenstrahlung. So werden Rückschlüsse auf die Struktur und Grenzflächenmorphologie der Mehrschichtsysteme möglich.

Diese Methodik wird zur Untersuchung von Mo/Si/C Multilayersystemen mit polierten und unpolierten Grenzflächen, welche als Spiegel für EUV Strahlung mit 13.5 nm Wellenlänge dienen, eingesetzt. Die Mehrschichtsysteme wurden mit wachsender Molybdänschichtdicke von Probe zu Probe hergestellt. Die kombinierte Analyse von EUV Reflektivität und Röntgenreflektivität unter Berücksichtigung der experimentellen Unsicherheiten ermöglicht eine Bestimmung der strukturellen Modellparameter und deren Konfidenzintervalle. Die Einführung einer Methode zur Analyse diffuser EUV Streuung erlaubt ferner die Korrelation beobachteter Reflektivitätseinbrüche in bestimmten Proben mit Variationen der Schichtdicken und der Grenzflächenrauigkeit durch Kristallisation in den Molybdänschichten. Erhöhte Rauigkeit an der Kristallisationsschwelle und Durchmischung an den Grenzflächen können als Ursache der beinträchtigten Reflektivität identifiziert werden.

Die hier etablierte Methodologie wird desweiteren auf Cr/Sc Mehrschichtspiegel für das Wasserfenster angewandt. Die Kombination von EUV- und Röntgenreflektivität basierend auf einem binären Schichtmodell stellt sich bei diesem System als unzureichende Beschreibung heraus. Daher wird das Modell erweitert, um graduelle Grenzflächenprofile und starke Vermischung explizit zu berücksichtigen. Auf Grundlage der Strukturanalyse mittels EUV Refklektivität, resonanter EUV Reflektivität, Röntgenreflektivität und Röntgenfluoreszenz und anschließender Validierung kann gezeigt werden, dass nur die Kombination all dieser analytischen Methoden ein konsistentes Ergebnis liefert. Die Erweiterung dieser Charakterisierung durch diffuse EUV Streuung erlaubt die Bestimmung der Gründe für die geringe Reflektivität.

1 Introduction

In 1959, Jack S. Kilby made an invention responsible for the technological revolution in the years to come. His development of the first integrated circuit was the realization of a logical element known as *flip-flop*, capable of storing a single bit, by implementing a layout that could host all required circuits on a single semiconductor wafer piece [75]. His achievement paved the way for the miniaturization of electronic circuits that enabled the technological advancements we experienced over the past 57 years, and was awarded as part of the Nobel prize in physics in 2000 [109]. Only two years after the original invention, Robert N. Noyce submitted a patent on the fabrication of integrated circuits in monolithic single crystals using photo lithography to create the necessary artificial structure [103]. This technique of using light to transfer a pattern from a photomask onto a semiconductor wafer has prevailed over the course of the technological development and is still the primary method for the fabrication of computer chips today [91]. As the technology improved over time, the progress roughly followed *Moore's law* of doubling the transistor count on a unit area of the wafer every two years [99]. In consequence, the structured feature sizes on the wafers shrank down to accommodate this large amount of circuits on a single chip. Today, structure sizes in the lower nanometer regime have been reached [69] but only through the implementation of additional methods appending the optical lithography. With the strong decrease in size, Moore's law is threatened to loose its validity [90, 115]. The technological requirements on the lithography systems used to fabricate those chips in mass production thus grew significantly.

A basic principle of optical resolution known as the *Rayleigh criterion* states, that the minimum structure size achievable with a purely optical system is proportional to the wavelength used [86]. Consequently, while the first lithography systems used in the semiconductor industry operated in the visible spectrum, wavelengths have been reduced down to the deep ultraviolet (DUV) at 193 nm used nowadays to keep pace with Moore's law. However, with feature sizes of only a few tenths of nanometer now necessary, a significant further reduction of the wavelength can not be avoided as the lithography at optical wavelengths has reached its physical limits. The next-generation lithography uses wavelengths in the extreme ultraviolet (EUV) spectral range of 13.5 nm. This radiation is strongly absorbed by all materials, including air, challenging the design of the optical lithography systems by effectively ruling out any optical design based on transmission lenses for focusing and imaging, as those are unavailable. With the semiconductor industry at the verge of a major technological change, the topic of reflective optical elements for EUV radiation has gained a large amount of attention and experienced extensive research efforts [10].

It was back in 1972, that Eberhart Spiller had proposed a new design for efficient mirror systems working at incidence angles near the surface normal for strongly absorbed radiation such as EUV. The idea was based on fabricating artificial layer systems reflecting portions of the incoming radiation at each interface that would interfere constructively at acceptable absorption levels overcoming the extremely low reflection otherwise seen from single surfaces [129]. The result is multilayer Bragg reflectors which fulfill the Bragg condition for constructive interference for specific pairs of wavelength and angle of incidence and thus require specific design. At angles close to the surface normal, layers with thicknesses in the order of half the wavelength are necessary, which requires fabrication methods capable of precisely depositing layers of only several nanometers in thickness. Since the original proposal, multilayer systems have been realized using evaporation and sputtering techniques and demonstrated to increase reflection [130, 139]. As the technology developed and more advanced sputtering techniques became available to fabricate at the necessary precision [134], the first important applications of focusing multilayer mirrors were space probes used for the observation of the sun in the EUV spectrum [32, 33, 131].

Theoretical models and calculations of candidate systems for large reflectivity close to normal incidence at a wavelength of 13.5 nm, show peak values of approximately 72 % by using multilayer systems based on molybdenum (Mo) and silicon (Si) [12, 13, 50]. State of the art systems fabricated nowadays reach values closely above 70 % [8, 29, 30, 49, 95], which is still a few percentage points below the theoretical limit. This is of particular concern for the usage in EUV lithography systems, where 11 near-normal incidence reflections from the source to the wafer are required to image a structure [71, 142]. Even at the theoretical threshold, with 11 reflections only 3 % of the radiation reaches the wafer. Therefore, the small difference to the theoretical reflection limit, additionally, has a large impact on the total radiant power at the wafer level. This is a very crucial point in the development of the next-generation lithography using EUV radiation.

While the semiconductor industry undoubtedly is a strong driving force in the development of EUV multilayer optics for 13.5 nm wavelength, mirrors for other spectral ranges suffer from the same issue. A relevant system to this work is a mirror designed to reflect radiation in the range of the so-called *water window* which is found between 2.3 nm and 4.4 nm. The water window is of special interest, because radiation in this spectral range shows low absorption in water, while it is absorbed by many elements, most importantly carbon and nitrogen, naturally occurring in organic molecules such as proteins [76]. This allows the study of biological systems in their native environment (water), where many proteins are biologically active. With the ability to produce radiation at those wavelengths at free-electron laser (FEL) sources [2, 118], more applications with strong and coherent pulses come in reach. High resolution imaging of protein samples, in addition to the required short wavelength, needs sufficient reflected radiation intensity and, more generally, optical elements capable of focusing and magnification. This can be achieved with high reflectance multilayer mirrors [63, 80]. A candidate system, relevant in this range, is made from chromium (Cr) and scandium (Sc) applying the very same principle as introduced above, however, having much thinner layer thicknesses due to the shorter wavelength. While at 3.1 nm wavelength, the theoretical reflectance limit is calculated to reach values above 50 % [114], state of the art mirrors only show reflectivities below 20 % [48, 145], less than half of the theoretically possible values.

The main reasons for radiation loss beyond unavoidable absorption inside the materials of both the Mo/Si and Cr/Sc multilayer systems are imperfections at the interfaces, such as compound formation, intermixing and roughness. As a result, the perfect multilayer system is distorted, since the interfaces are not chemically abrupt anymore. Thus, intermixing and compound formation lead to a diminished optical contrast and consequently to lower reflectance at the respective interface [101]. This is a known problem for multilayer optics, and measures taken to counteract this effect are the introduction of barrier layers hindering the formation of intermixing layers in some of the systems [29, 30]. In the case of roughness, the result of reduced optical contrast at the interfaces is the same on average for the impinging wavefield. However, in this case with the addition of diffuse scattering outside the specular beam direction [125], which is not present in the case of pure intermixing.

To minimize interface distortions and to ultimately increase the reflectivity of the respective systems, the research and industry groups concerned with fabricating multilayer mirrors require detailed information on the actual structural properties and the interface morphology of their samples. The characterization of those multilayer systems is thus a cornerstone in the effort for improvement and the fundamental understanding of the effects involved. There are several characterization techniques that exist and have been applied to assess and quantify the structure of the layer system, roughness and intermixing of materials at the interfaces of multilayer mirrors in the past. They can be roughly categorized as direct scanning methods and indirect ensemble methods.

Some widely used example of the first category is transmission electron microscopy (TEM), which establishes a microscopic approach to the problem of assessing the interface morphology with resolution at the nanoscale [9, 135]. By imaging the layer stack, interface imperfections can be made directly visible. In combination with high-resolution electron energy loss spectroscopy (HREELS), element specific interface profiles can be deducted giving insight into the intermixing behavior of two (or more) materials at the interfaces [43, 108]. A large downside of both methods, however, is the intrinsically local area of the image and thus the characterization of only very small local portions of the entire sample. Apart from that, the stack needs to be cut open to apply these techniques and thus leads to a destruction of the sample.

Another very popular method often used, before and after deposition of a multilayer stack, is atomic force microscopy (AFM) [19]. It is a scanning technique with nanometer resolution allowing to determine the morphology of a surface and thus to investigate their roughness. However, it faces the same locality obstacle as TEM or HREELS and can only operate on exposed areas. Thus, the morphology of buried structures remains hidden for this method. Nevertheless, it is applied to determine the initial substrate roughness and the condition of the final top surface as an important prerequisite for high-quality multilayer mirror fabrication [9, 88].

Apart from the direct and local scanning techniques, indirect ensemble methods based on the elastic scattering of radiation are accurate and extensively used in multilayer characterization. Examples include X-ray reflectivity (XRR) and EUV reflectivity with resonant extreme ultraviolet reflectivity (REUV) as a variation of the latter. They are employed as standard approaches in multilayer mirror fabrication and the subsequent characterization [9, 30, 84]. Other techniques, sensitive to structural properties, are spectroscopic ellipsometry and X-ray fluorescence (XRF). Ellipsometry delivers information on the optical constants and layer thicknesses by measuring the altering of the polarization state of the impinging radiation after reflection from the sample [7, 85]. With XRF, fluorescence radiation of the materials inside the multilayer stack is excited using X-rays energetically closely above the respective absorption edges of these materials and detected to analyze the structure [72, 77]. The major advantage of all these techniques is, that they are destruction-free and contact less, and deliver fast information on the buried structure as well as on the top surface condition. Furthermore, statistical information across a large area depending on the beam footprint of the impinging radiation is obtained in contrast to the aforementioned local methods. However, it is no longer possible to directly gain information on the multilayer stack as theoretical models are required in all above examples to calculate and compare the expected results from a certain model to the measurement outcome. This is known as the *inverse problem*. Reconstruction of the model parameters by fitting calculations to the experimental data raises the question of uniqueness and accuracy of the solution found. Even the applicability of the model itself and its limitations are of importance in these considerations. Several researches have shown, that the combination of EUV and XRR can lead to significant improvements in the accuracy compared to standalone measurements with each technique individually [144]. By XRF, similarly further complementary information can be added to assist in the solution of this problem [54].

While structural information on the layer stack can be obtained through reconstruction of a model by conducting these experiments, only limited information is gained on the roughness of the interfaces which can not be distinguished from intermixing. However, as only roughness causes diffuse scattering, the analysis of the off-specular intensity upon irradiation of a multilayer stack is a natural tool for characterization of the interface morphology. A lot of theoretical and experimental work has been conducted in relation to the study of diffuse scattering from multilayer samples in grazing incidence geometries using X-rays, e.g. by grazing-incidence small-angle X-ray scattering (GISAXS), at small incidence angles [23, 24, 83, 96, 113, 124, 125], but also in the optical and EUV regime [4, 5, 45, 46, 119, 120], to deduct the desired information on the interface roughness.

As the German national metrology institute (NMI), the Physikalisch-Technische Bundesanstalt (PTB) is dedicated to precise measurements related to all fields of physics and technology providing metrology as its core mission. In fact, the international metrology organization, the Bureau International des Poids et Mesures, defines* metrology as *"the science of measurement, embracing both experimental and theoretical determinations at any level of uncertainty in any field of science and technology."*

In this spirit, the scope of this thesis is to provide metrology for the important field of multilayer optics. It is dedicated to the accurate and complete characterization of the structural properties and the interface morphology of multilayer mirrors to fathom the origin of their limited performance. The uniqueness problem associated with any model-based indirect characterization approach is largely unanswered and requires to be addressed. In this thesis, the data from different indirect experiments was aimed to be analyzed with respect to answering this question for the aforementioned important multilayer mirror systems. Experimental uncertainties, inevitably associated with any measurement, and model uncertainties had to be investigated with respect to the effect on the results obtained from each method employed. Based on theoretical optimization algorithms, confidence intervals for each reconstructed parameter of the underlying

* Source: `http://www.bipm.org/en/worldwide-metrology/`

models can be deducted, which allow to validate the results of the established characterization techniques. Solving the task of improving the models and exploiting several experimental techniques such that univocal judgments on the causes of the reduced multilayer reflectance can be made are thus the major focus of this work.

This thesis is structured in the following way. Chapter 2 introduces the fundamental theoretical concepts underlying the interaction of multilayer systems with EUV and x-ray radiation. The theoretical basis of the analytic experiments (EUV reflectivity, REUV, XRR, XRF and EUV diffuse scattering) conduced in this thesis to characterize the various samples is given. In chapter 3, the different experimental setups at the two storage rings metrology light source (MLS) and electron storage ring for synchrotron radiation (BESSY II) employed in obtaining the data analyzed here are presented. In this work, samples fitting in two major categories of multilayer mirrors for two different spectral ranges were investigated. Their fabrication was conduced using a sputtering technique, which is briefly reviewed. Furthermore, the extensive software that was developed over the course of this thesis is summarized here. The first relevant sample systems designed to operate as mirror at 13.5 nm wavelength, are two sets of Mo/Si/C multilayers with an increase of the molybdenum layer thickness from sample to sample from nominally 1.7 nm to 3.05 nm crossing the threshold for crystallites forming in these layers. The second set was treated using an ion polishing technique during deposition with the goal to reduce roughness at the interfaces. The methods employed for this system were compared to the reconstruction of a state-of-the-art Mo/B_4C/Si/C multilayer mirror. The second major sample system investigated here are Cr/Sc multilayer mirrors for the water window with nominal layer thicknesses in the sub-nanometer regime. The structural reconstruction of the Mo/Si and Cr/Sc multilayer mirrors based on the combination of the different experiments is presented in chapter 4. Here, the validity of the models and the accuracy of the reconstructed parameters with their confidence intervals is discussed in depth. Chapter 5 addresses the evaluation of the interface morphology of these samples based on the EUV diffuse scattering measurements and the models reconstructed in the previous chapter. The summary and conclusion of this thesis can be found in chapter 6.

6
Summary

This thesis has treated the characterization Mo/Si and Cr/Sc multilayer mirror systems by means of the combination of several indirect methods based on reflection, fluorescence and scattering using EUV and X-ray radiation. Its focus was to validate and improve the applied theoretical models and determine the experimental techniques required to achieve an unambiguous solution to the inverse problem. For the reconstruction of the layer systems structure, a particle swarm optimization was applied to fit the model parameters to the measured data from extreme ultraviolet reflectivity, X-ray reflectivity, resonant extreme ultraviolet reflectivity and X-ray fluorescence experiments. A Markov-chain Monte Carlo algorithm was further employed to deduct the maximum likelihood distribution and thereby confidence intervals based on the measurement and model uncertainties. It was found, that different methods and models had to be applied depending on the system under investigation. The values and confidence intervals determined for each parameter of the respective model, allowed to draw conclusions on the actual structural layout of the samples.

The structural characterization methods could yield layer thicknesses, densities and even the distortion of the interfaces. However, they lack in the ability to identify these distortions as either roughness or intermixing. This could only be achieved by combining the results of the structural characterization with a method sensitive to roughness and again validate the accuracy of the result. This issue was approached by the analysis of EUV diffuse scattering with radiation impinging near-normal incidence as a suitable technique to deliver this distinction method. The method was introduced by analyzing the state-of-the-art Mo/B_4C/Si/C mirror reaching $(68.5 \pm 0.7)\,\%$ peak reflectance at its operation wavelengths of 13.5 nm. It was revealed that the high quality, and thus reflectivity, of the sample causes resonant enhancement of diffusely scattered radiation within the stack, which significantly contributes to the diffuse scattering intensities. These dynamic effects must be considered in the analysis by employing the theoretical framework of the distorted-wave Born approximation including multiple reflections at the interfaces of the multilayer. With this approach, the roughness properties for the samples could be extracted consistently. By comparing and combining the results of the structural characterization and the roughness analysis a consistent characterization of the multilayer mirrors could be achieved. Thereby, the analysis in this thesis could explain the lack of peak reflectivity compared to the theoretical expectation for an ideal system for both sample systems.

In the unpolished and polished set of the Mo/Si/C multilayer mirrors, it was revealed that the combination of EUV reflectivity and X-ray reflectivity yields an unambiguous result for the molybdenum layer thickness confirming the nominal trend in both sets. The confidence intervals for the molybdenum thickness could be determined ranging from 0.43 nm down to 0.24 nm, depending on the sample. In comparison, the analysis of EUV reflectivity for the Mo/B_4C/Si/C sample only yielded a confidence interval of approximately 1 nm. This demonstrated the need for combining multiple datasets, despite an excellent agreement of the calculated and measured curves, since multiple solutions exist. The sum of the thicknesses of all layers in a period shows a distinct increase for both sets at a certain molybdenum layer thickness, associated with a dip in peak reflectance with respect to the theoretical expectation. This effect, while observed in both sets, happens at significantly different molybdenum thicknesses, comparing the unpolished with the polished samples.

The analysis of the diffuse scattering intensity allowed an assessment of the interface morphology for these samples. The comparison with the structural analysis revealed an increase of roughness, associated with the jump in the period thickness and the dip in peak reflectance, which is compensated again at larger thicknesses in both sets. At this point, it may be concluded that these effects are caused by the onset of crystallization in the molybdenum layer, causing increased interface disturbances through roughness. In the analysis of the ion polished set, this threshold was shown to have moved towards lower molybdenum thicknesses. This is beneficial to the reflectance at the optimum molybdenum ratio with respect to the rest of the layers in a period, which in the polished set is now unaffected trough roughening due to crystallization. Nevertheless, comparing the roughness values found in the diffuse scattering analysis with the Névot-Croce factor from the optimized layer structure model, it became clear that while overall roughness was reduced significantly and led to a significant increase of the reflectivity in the polished set, the Nevót-Croce parameter has only reduced slightly, indicating that intermixing is still largely responsible for the remaining gap to the theoretically achievable reflectivity.

In the case of the Cr/Sc multilayers for the water window spectral range, nominal layer thicknesses within a bilayer period are between 0.7 nm and 0.8 nm and thus noticeably thinner than for the Mo/Si systems. It was shown, that an approach to the structural characterization based on a discrete layer model for the chromium and scandium layers does not yield a solution valid for both, the EUV reflectivity and XRR experiments, with the same set of parameters. Thus, a solution fitting the EUV reflectivity experiment fails to describe the XRR curve and vice versa. The discrete layer model is not suitable to describe the physical structure of the sample. Any solution found for either one of the experiments can therefore not be related to the physical properties of the sample. Instead, a model describing a gradual interface profile and layers composed of a mixture of both materials was introduced. Based on this gradual model, the intermixing and roughness were parametrized separately and asymmetric interface profiles could be described explicitly.

It was found through the uniqueness and accuracy analysis, that the increased variability of the improved model requires more complementary information than the analysis of the Mo/Si samples. The goal of unambiguity of the solutions was achieved by performing EUV reflectivity, REUV, XRR and XRF experiments. Confidence intervals were determined, by evaluating each dataset individually and by combining all in a single analysis. The solutions and confidence intervals determined prove, that only the combination of all datasets could yield a consistent result. It was found, that none of the regions within the Cr/Sc stack are pure chromium or scandium. Furthermore, the interface regions show a strong asymmetry, which could not be determined with the required significance by any of the aforementioned standalone analytic experiments. Not even the combined analysis of these methods could distinguish between roughness and intermixing. Those two parameters were shown to have a strong correlation. To determine roughness and intermixing, the EUV diffuse scattering was measured and analyzed similarly as for the Mo/Si samples. The result shows a roughness value of $\sigma_r = 0.17(-0.01/+0.02)$ nm. Consequently, the intermixing could be determined to be $47(-4/+3)\,\%$, leaving any of the nominal chromium or scandium layers of the stack to contain large amounts of the other material on average. In conclusion, the roughness determined here is comparable to the values found for the polished Mo/Si/C samples. There, this roughness amplitude evidently allowed reflectivities much closer to the theoretical maximum value. Consequently, intermixing could be identified as the main cause for the small reflectivity achieved with Cr/Sc multilayer mirrors for the water window.

In summary, the work presented in this thesis proves the importance of assessing the uniqueness and accuracy of indirect characterization methods to deduct a meaningful result. As shown on several occasions in the analysis of the multilayer mirrors, reconstructions in very good agreement with the data curves were demonstrated to show ambiguities and inconsistencies. This was revealed by adding complementary information from other experiments or even when analyzing the data of a single experiment using global optimization algorithms. With the approach of combining multiple analytic techniques and determining confidence intervals of the reconstructed parameters, conclusions on the physical properties of the samples could be drawn reliably. This thesis augments the existing characterization methods for multilayer mirrors in that respect.

Finally, with the inclusion of EUV diffuse scattering, a technique to assess the interface morphology was established. It is suitable for characterization near-normal incidence offering an alternative to grazing-incidence methods such as grazing-incidence small-angle X-ray scattering (GISAXS). This has some unique advantages as any measurement using small incidence angles is inherently limited to flat or convex surfaces. Focusing mirrors, however, usually are concavely curved and thus characterization techniques with grazing angles of incidence are not applicable . Instead, with EUV diffuse scattering with radiation impinging near normal incidence, it is possible to extract the roughness information for those samples as well. In addition, radiation at the wavelengths of operation for these mirrors is suitable to conduct this experiment.

As an outlook extending the scope of this work, it would be interesting to evaluate the gain in accuracy and uniqueness of the solutions by applying the compilation of techniques used for the Cr/Sc system, also to the two Mo/Si/C sample sets. This may prove to be beneficial to further reduce the confidence intervals on the results, most prominently on the thickness of the barrier layers. In particular, as a straight forward approach, the improved model for the Cr/Sc mirrors could be carried over to these systems. Thereby, the role of the barrier and compound layers in the crystallization could be investigated based on validated reconstruction parameters. This could augment the analysis conducted on similar systems elsewhere [9]. In general, including further methods would deliver additional complementary information. Ellipsometry, for example, could yield results on the optical constants of the various materials in the layer stack.

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