A review of the molecular flow conductance for systems of tubes and components and the measurement of pumping speed

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The definition of molecular flow conductance for tubes and components is considered and the calculation of the conductance of long tubes with differently shaped cross-sections reviewed. The conductance and transmission probability for short tubes is discussed in relation to Clausing type integral equations and their approximate solutions. A review is given of theoretical and experimental investigations of molecular gas flow emission patterns for tubes and components. An estimation for the effective transmission probability and conductance of more complex systems, particularly of tubes, diaphragms, baffles etc connected in series, is considered as well as their experimental determination using scale models and statistical Monte Carlo type methods for their more accurate calculation. The definition of pumping speed and the development of different procedures which have been adopted in recent years for its measurement under molecular flow conditions are reviewed. It has been shown that the difficulty in the various methods was associated with the determination of the pressure at the mouth of the pump which depended on a number of inter-related factors. The most important of these concerned the design and dimensions of the test dome which it is necessary to attach to the pump, the position and distribution of gas entry to this dome as well as the method and place of pressure measurement. The physical interpretation of speeds measured with various arrangements is discussed in relation to recent proposals of a test dome arrangement which forms the basis for an internationally agreed standard for pump speed specification.

1. Introduction

Historians of vacuum technology have usually shown an interest in establishing the beginnings of vacuum by the early developments of pumps and the investigations linked with the names of Torricelli, Robert Boyle, Robert Hooke and Otto von Guericke of the Magdeburg hemisphere fame, ie the 17th century. I would suggest on the contrary that the theoretical and scientific foundations of vacuum physics and rarefied gas flow in particular are not much older than 100 years. If we consider the development of the kinetic theory of gases as a starting point, we can almost celebrate today its hundred years anniversary, with the invention of the vapour pump by Gaede and Langmuir and that of the ionisation gauge by Buckley thrown in as a half way mark about 50 years ago.

The recognition that gas pressure is entirely due to the effect of molecular impacts took a long time. An exception was some early publications of D Bernoulli[1] in his Hydrodynamics of 1738*. These were of course, purely philosophic speculations as he had no experimental evidence. It has also been pointed out by Tait[2](1885) that an even earlier, though less well known, anticipation was due to Robert Hooke[3] (1678), who incidentally is also accredited with the discovery of Boyle's law. The common notion in those days was to talk of the "elastic tension" of gases. This concept survives to this day in the current edition of the Shorter Oxford Dictionary, where a manometer is defined as an "instrument for ascertaining the elastic forces of gases or vapours".

*Figure (1) is an illustration from this work.

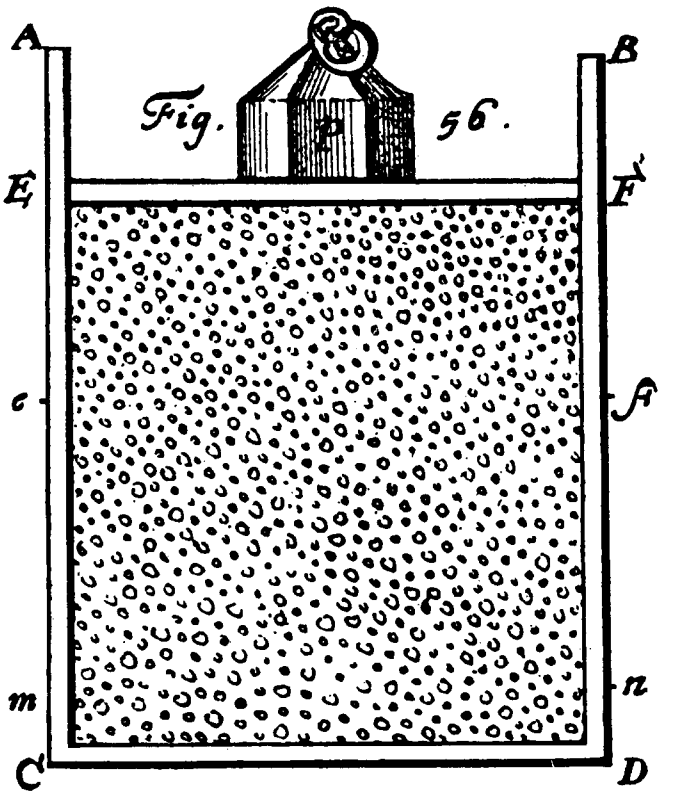


Figure 1. Illustration from book on hydrodynamics by D. Bernoulli (1738) indicating the bombardment of a cylindrical vessel and loaded piston by particles of a gas.

An interesting development occurred in 1845 when a man named Waterston[4] sent a paper to the Royal Society in which many later developments in the kinetic theory of gases were anticipated. However, this was not thought worthy of publication at the time and *not* published until 1892, when rediscovered by Lord Rayleigh. The present theory dates back to some suggestions made by Joule in 1851, which led to papers by A Kronig (1856), essentially restating an earlier account by Herapath (1823/4) and R Clausius[5](1857). It was then fully developed over the next 30 years by Clausius[6], Maxwell[7] and Boltzmann[8], which takes us to about 1890.

The real basis for flow of gases at low pressures and related phenomena was laid by the detailed investigations of M Knudsen[9,10] and M von Smoluchowski[11] around 1910, to be followed by Gaede[12,13] and Langmuir, from about 1913, with the invention and publication of the diffusion pump in 1915 by Gaede[14] and 1916 by Langmuir[15,16]. For early reviews on pumps the book by Dushman[17] (1922) and the review by Gaede[18] (1923) are of interest.

In the development of the kinetic theory, unsound derivations and errors have been quite common, especially so perhaps because theory was often well in advance of exact experimental verification. As a relatively recent and amusing example, may be mentioned a suggestion by Fairbourne[19] (1922) according to which a pressure difference would be set up by introducing a funnel shaped solid into a low pressure gas; he neglected however, the compensating effect of collisions on the outside of the funnel as shown by Witmer[20] (1924) and Fisher[21] (1924).

For the logical foundation of the kinetic and statistical theory and a discussion of many defects in the usual proofs, reference may be made to the exposition by P and T Ehrenfest[22] (1911). It is also instructive to read the discussions and occasional controversy between Knudsen and Smoluchowski (1910/1911) in the development of the theory for rarefied gases, and between Gaede and Langmuir concerning the operation of the diffusion pump (diffusion versus condensation) which had its counterpart more recently as indicated below.

We are not concerned here so much with tracing the history, but with developing the fundamental principles in so far as they affect the question of pump speed measurement. Before discussing these, however, let us briefly consider why one is interested in the clarification of pump speed measurement. One can, in fact, distinguish two entirely different motivations:

(1) An understanding of factors which determine the speed of a pump is undoubtedly of importance for the proper assessment of pumps of all types. This in turn leads to their improvement and further development. To this end such intrinsic pump parameters as Ho coefficient (for diffusion pumps), ionisation pumping as against getter pumping efficiency (for sputter-ion pumps and evapor-ion pumps), and say sticking coefficient (for cryopumps) are investigated. These parameters can then be related to the speed of the pump as defined relative to its inlet (or pump mouth).

(2) A proper specification of pump speed is required by the user of the pump (who can regard the pump for this purpose merely as a magical black box). It is of course important that this pump speed be soundly defined, for the user must be able to compare the performance of two apparently similar products, and furthermore be able to apply the quoted speed characteristics to practical pumping systems so that possible performances may be approximately predicted.

It will be important to recognise this difference in outlook and interest; the first of the above cannot be adequately discussed in this review.

As in other branches of applied science, we must be careful to distinguish between precision and approximation in the physical and mathematical solution to the various problems. The boundary between them can be almost indistinguishable or, in other instances, unbridgeable. Thus particularly, one needs to distinguish between those approximations which can approach a precise solution as closely as we please (as in a series solution, if a sufficient number of terms are taken, or in a statistical approach, if the sample size is taken sufficiently large), and what we may call a "rough" approximation derived from not strictly correct "simplified" assumptions which often give solutions sufficiently close for all practical purposes, but whose indiscriminate application can lead to gross errors or ridiculous answers. In the following discussion recourse will be had to both types of approximation.

Problems associated with establishing the molecular flow conductance and transmission probability of tubes will first be reviewed, followed by a consideration of flow through more complex systems of combined tubes and vessels of various shapes. The pumping speed of different pump types, the combined characteristics of pumps with tube systems, and the question of pump speed measurement with the aid of test domes will then be discussed.

2. Molecular flow conductance

A number of fundamental assumptions will be made; the first is that molecular flow conditions* prevail ie:

(1) Pressures are assumed to be sufficiently low for the mean free path for molecular collisions in the gas to be large compared with the largest cross-sectional diameter in any tube or component considered. This means in effect that the movement of molecules can be taken to be quite independent of each other.

We may note some results applicable to molecular flow conditions, which all follow from the kinetic theory of gases and will be referred to below:

The rate of impingement of molecules on a small surface of area dS assuming that they are all travelling in one and the same direction, ie in a molecular beam, and where this beam is making an angle θ with the normal to the surface, is given by

$$n\bar{v}\cos\theta dS \tag{1}$$

where n is the molecular density in the beam and $\bar{v}$ is the mean velocity of the molecules.

To obtain the rate of impingement on such a surface (on one side only) from a stationary gas, ie where molecules are in random motion it is necessary to integrate over all directions so that this rate is given by

$$N\mathrm{d}S=\frac{1}{4}n\bar{v}\mathrm{d}S \tag{2}$$

From the Maxwellian distribution of velocities it follows that the mean velocity

$$\bar{v}=4\left(\frac{\mathbf{k}T}{2\pi m}\right)^{\frac{1}{2}}=1.455\times10^{4}\sqrt{\frac{T}{M}}\ \mathrm{cm/sec} \tag{3}$$

where m is the mass of the molecule and M its molecular weight.

*In some recent literature this has also been termed "free-molecule flow".

We may also note that the pressure

$$p = nkT \tag{4}$$

so that the number of molecules per cm^3 in terms of p is given by

$$n = \frac{p}{kT} = 9.6 \times 10^{18}\, p/T \text{ (with } p \text{ in torr)} \tag{5}$$

The cosine law relates to the number of molecules emitted by a small area dS of a diffusely reflecting surface within the solid angle dw which makes an angle θ with the normal to the surface. This rate of emission is given by

$$N_{\theta,dw}\mathrm{d}S = \frac{\mathrm{d}w}{4\pi} n\bar{v} \cos\theta \mathrm{d}S \tag{6}$$

$$\text{ie } N_{\theta,dw} = \frac{\mathrm{d}w}{\pi} N \cos\theta \tag{7}$$

We now consider the special case of steady molecular flow in a tube. The following additional assumptions will be made:

(2) A single tube is assumed to be connected between two large vessels so that the distribution of the position and angle at which molecules enter the tube at its openings to the vessels is independent of flow in other parts of the system.

(3) The flow is steady, ie has been established for a sufficiently long time.

(4) It is further assumed that steady pressure p_1 and p_2 of the same gas are maintained in the two vessels respectively, where $p_2 > p_1$, and where these pressures are measured sufficiently far from the openings so as not to be affected by the gas flow Q which will be established through the tube.

Under these assumptions one may define a conductance of the tube U given by:

$$U = \frac{Q}{p_2 - p_1} \tag{8}$$

3. Interaction between molecules and surfaces

Knudsen[9] first calculated the quantity of gas streaming per second through a tube connecting two vessels filled with the same gas but at different pressures. He was particularly concerned with the case of "molecular flow" at very low pressures and used as a basic assumption the "diffuse" reflection of molecules from the walls of the tube following the "cosine law", see equations (6) and (7) above. This law was incidentally subject to much experimental work about this time and was amply justified. Reference may be made for example to the discussion by Gaede[13] and the later work by Clausing[27–31] indicating its derivation from the 2nd law of thermodynamics. The same law holds of course for the diffuse reflection of light where it is known as Lambert's law and is of importance in illumination problems and photometry. Many results worked out for diffuse light reflection therefore have their exact counterpart in problems of molecular flow. For such results, see for example, Moon[35]. Thus Clausing[25] discussing the amount of uniformly diffused light passing through two apertures in series, stressed the analogy with the stationary flow of gases and in his derivation of molecular flow through tubes Clausing [32, 33, 34] made use of results obtained by Walsh[36, 37] for radiation of light. In another application Steckelmacher and Holland[38] while calculating the distribution of deposits evaporated from surface sources under molecular flow conditions indicated the analogy with illumination from distributed diffuse sources of illumination. An analogy for flow in tubes was also suggested by Monk[39] (1952).

There is of course still the possibility, as first discussed by Maxwell, of preferred reflection or specular reflection, of molecules in their interaction with the surface. In effect, Maxwell[7] proposed that a fraction, say $(1-f)$ of incident molecules were specularly scattered, while the remainder were scattered in a completely diffuse fashion. The effect of this on molecular flow through tubes was already discussed in the early papers by Smoluchowski[11]. More recently de Marcus[40] considering flow through tubes of any length with partially reflecting walls showed that Smoluchowski's results would give a rather poor approximation. However, as indicated above the value of f is often taken as unity, ie completely diffuse reflection. Millikan[41] described experiments with a number of materials (particularly liquids) intereacting with gases to determine f. He also reviewed all available experimental data at that time concluding for example that

$f = 0.89$ for air-glass.

Much early work was also carried out by Stern, Esterman and others in their molecular beam experiments using cleaved crystals and other surfaces. Recently this subject has again received attention because it is obviously of such fundamental importance particularly with regard to rarefied gas flow problems. Hurblut[42] and in his review[43] gave results of careful experiments carried out with gas beams interacting with a number of practical technical surfaces including polished and unpolished glass, teflon, polished aluminium and steel. He concluded that by far the greatest number of molecules were scattered at random as from an ideally rough surface for all combinations of gases and surfaces tested. The measured flux distributions showing rather close agreement with the cosine or "diffuse" scattering pattern with *no* indications of any sharp perturbation of that pattern which would be attributable to "specular" lobes—with the possible exception of certain results for teflon and glass surfaces, especially at high angles of incidence for the latter, where a small fraction of the incident molecules were scattered into the specular direction in addition to the generally prevailing purely random distribution. Some of his results are shown in Figure 2. Evaluating these measurements showed, contrary to Millikan[41], that f was as high as 0.97 for nitrogen-glass and very near to 1 for many other practical material combinations. For more recent results, reference may be made to papers by Nocilla[44], Devienne and Forestier[45], and Jawutsch[46].

Diffuse reflections by cosine law also applies in certain cases of radiant heat transfer, and again methods developed here have direct application, see eg the book by Jakob[47]. For some of these problems an equivalent electrical network analysis may be applied as shown by Oppenheim[48]. A recent detailed discussion stressing the similarities between transport processes in rarefied gases and thermal radiation was given by Eckert[49] and specifically for flow in tubes by Sparrow, Jonsson and Lundgren[50].

4. Flow through 'long' tubes and 'thin' apertures

In effect, Knudsen derived what is today known as the "long tube" formula for flow through an infinitely long (ie length $\gg$ radius) tube;

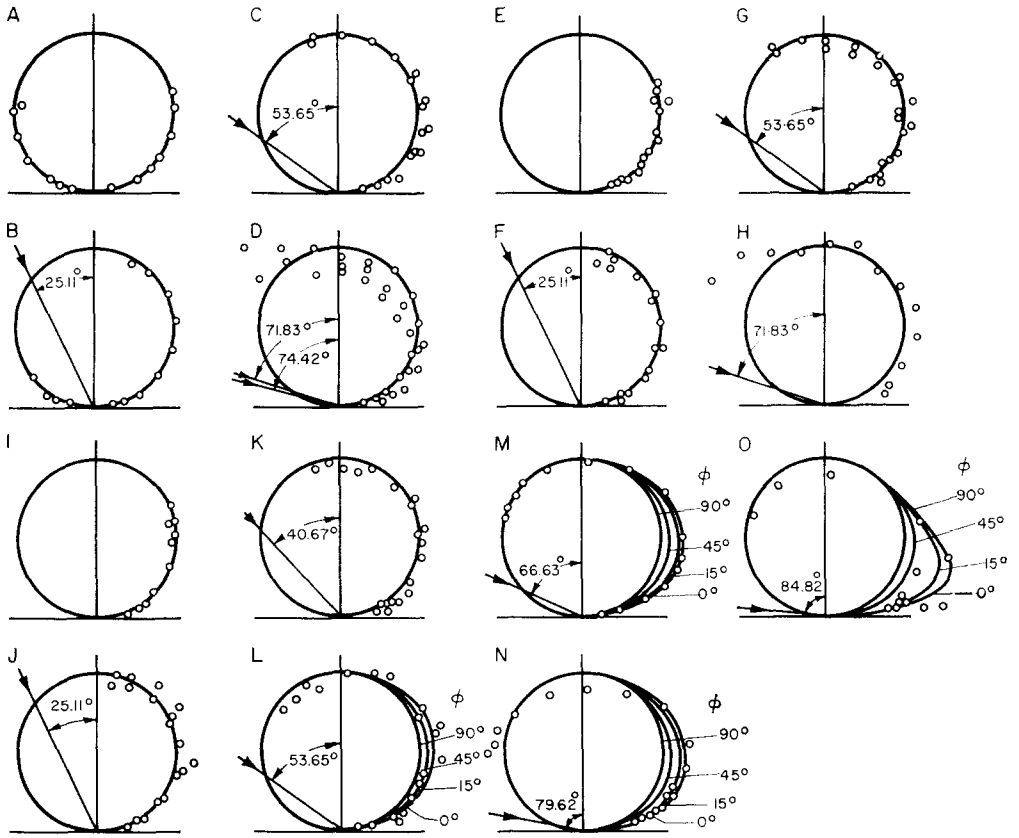


Figure 2. Polar plots of experimental results for the interaction of molecular beams with surfaces of polished steel *A—D*, polished aluminium *E—H*, and unpolished glass *I—O* (following Hurblut, 1957, 1959).

With a cylindrical tube of length L and circular cross section of radius R connected between two large vessels, and taking L to be very large compared to R, the conductance as established by Knudsen is given by

$$U=\frac{8}{3}\pi\frac{R^3}{L}\sqrt{\frac{\mathbf{k}T}{2\pi m}} \tag{9}$$

Other derivations were given by Smoluchowski[11] and Lorentz[51]. See also the books by Loeb[52] and Kennard[53].

It is interesting to note that only fairly recently, in the fundamental work of De Marcus (though leaning heavily on the Clausing approach to molecular flow problems) the first rigorous derivation of the Knudsen long tube formula is given. In this report expressions were first vigorously developed for tubes of arbitrary length and then by an asymptotic expansion an expression was derived valid for $L \gg R$. This incidentally yielded the Knudsen result together with additional higher order terms.

Clausing who basically took the correct approach came up against an integral equation, and in order to find approximations to this, he assumed that as L became large compared with R, the Knudsen long tube formula should be true, but this of course already involved approximations in its derivation.

In the same original paper Knudsen[10] also deduced the equation for gas effusing through an opening in a very thin walled vessel under molecular flow conditions.

The conductance of such an opening is given by

$$U_o=\sqrt{\frac{\mathbf{k}T}{2\pi m}}\cdot A \tag{10}$$

where A is the cross sectional area of the opening.

From (3) it is seen that

$$U_o=\frac{1}{4}\bar{v}A \tag{11}$$

ie in analogy with (2), we may consider the rate of molecular bombardment of this opening. For molecules moving randomly in a gas of molecular density n their rate of impingement on to one side of an opening of area A is

$$F_o=N_oA=\frac{1}{4}n\bar{v}A=nU_o \tag{12}$$

This is of course exactly the rate at which molecules will pass through such an opening into a large vessel on the other side assumed to be completely evacuated. There is in fact no loss of generality if one assumes that the pressure in one vessel is p_1 and the pressure in the other is negligibly low, ie $p_2=$o. In the more general case, there is merely a counter stream from the second vessel at pressure p_2 with $p_1=$o. Since intermolecular collisions may be neglected, these two streams are quite independent of one another so that we have eg for the flow through an opening

$F_{01}=n_1U_0$ for the first stream
and $F_{02}=n_2U_0$ for the counter stream
and the net flow rate is simply given by

$$F_{02}-F_{01}=(n_2-n_1)U_0 \tag{13}$$

where n_2 and n_1 are now the molecular densities in the two vessels corresponding to the pressures p_2 and p_1 respectively.

Actually, Knudsen[10] also derived a more general formula intended for flow through long cylindrical tubes of arbitrary

cross-section. This may be written in the form

$$U=\frac{8}{3}\pi\frac{A^2}{BL}\sqrt{\frac{\mathbf{k}T}{2\pi m}}$$

where A=cross sectional area of the tube
and B=perimeter of the tube

It is perhaps fortuitous that it gives the correct result when the tube is of circular cross section, and it is surprising to find this formula still widely quoted in relatively recent books on vacuum techniques. However, this was shown to be in error and a correct general expression was derived by Smoluchowski[11] (see also Lorentz[51], Kennard[53], Loeb[52]).

In this the conductance for a long tube of arbitrary cross-section may be put

$$U=\frac{1}{2}\frac{I}{L}\sqrt{\frac{\mathbf{k}T}{2\pi m}} \tag{14}$$

where $I=\int_s\int_{-\pi/2}^{\pi/2}\frac{1}{2}\rho\,.\cos\theta\,\mathrm{d}\theta\mathrm{d}s$

where ρ is a chord of the cross-section making the angle θ with the normal to ds (of the perimeter).

This formula, or modifications of it involving similar double integrals, were used to obtain expressions for long tubes of rectangular cross-section by Smoluchowski[11], tubes with off axis and coaxial inserts (ie the annulus) by Clausing[26] and Barrett and Bosanquet[54] (see also Guthrie and Wakerling[55]), and tubes with elliptical cross-section by Turnbull[56].

5. Flow through tubes of any length

Dushman[17,57] first gave a formula which was intended also for short tubes. This had the virtue that in the limit $L=0$, it gave the correct conductance corresponding to a thin orifice and not infinity as would be obtained with the application of the Knudsen long tube formula. These results were obtained for tubes with circular cross-section.

In effect Dushman argued (see also Dushman[58]), that if the Knudsen tube conductance (9) is compared with the conductance through the opening of the tube (ie (10) with $A=\pi R^2$), the ratio

$$r=\frac{U_o}{U}=\frac{3}{8}\frac{L}{R} \tag{15}$$

indicates that small values of $\frac{L}{R}$, gives tube conductances which are much too high. One way of rectifying this situation is to assume that conductances are added inversely as in the electrical analogy. For two conductances U_1 and U_2 in series, we then have the combined conductance U given by

$$\frac{1}{U}=\frac{1}{U_1}+\frac{1}{U_2} \tag{16}$$

Secondly, he suggested that for a short tube one should consider separately the conductance of the opening U_0 and that of the tube U so that for the combined conductance U^1 of a short tube

$$\frac{1}{U^1}=\frac{1}{U_o}+\frac{1}{U} \tag{17}$$

Hence, from (15) we have

$$\frac{1}{U^1}=\frac{1}{U_o}(1+r) \tag{18}$$

with (10) the Dushman formula for a short tube gives:

$$U^1=U_o\left(\frac{1}{1+r}\right)=U_o\,.\,\frac{1}{1+\frac{3}{8}\frac{L}{R}}=\sqrt{\frac{\mathbf{k}T}{2\pi m}}\,.\,\frac{\pi R^2}{1+\frac{3}{8}\frac{L}{R}} \tag{19}$$

Clausing[23,24,32,33,34] derived a more accurate result for tubes of any length.

For $\frac{L}{R}=0$ and $\frac{L}{R}\approx\infty$

this gave the same results as were obtained by Dushman (namely the Knudsen formulae) but not in the intervening region, where he showed considerable deviations between his and Dushman's simple result. These differences were discussed particularly by Clausing[32] where he showed that the Dushman formula was really a very rough approximation, while his own derivation was a strictly accurate solution of the problem even though it involved an approximation when obtaining actual numerical values from the resulting integral equation. In this connection it is interesting to note that the situation was later again confused by an incorrect analysis published by Florescu[60] suggesting that Clausing's results were in reality only an approximation (for details of this point—see discussion below).

6. The transmission probability

In Clausing's approach he considered the transmission probability for molecules which entered the tube from say, the first vessel to emerge at the exit in the other vessel without having been back in the first. With this concept, the net flow rate of molecules through a pipe with two openings can be written as:

$$F=A_1N_1P_{1\to2}-A_2N_2P_{2\to1} \tag{20}$$

Where F is the net number of molecules per unit time that flows through the opening 1, and $N_1\ A_1$ is the rate of entry of molecules at the opening 1 of area A_1 and $P_{1\to2}$ is the probability that a molecule entering the opening 1 will leave by opening 2. This is of course a generalisation of (13) above for flow through an orifice. Davis[61] indicated that (20) could be generalised for a pipe with several openings:

$$F_i=\sum_{j=1}^{M}(A_iN_iP_{i\to j}-A_jN_jP_{j\to i}),\quad i,j=1,2\,..\,M \tag{21}$$

In this expression the effect of pressure and temperature on flow rate are contained in the factor N, while the geometrical dependence on the flow rate is contained in the factors $P_{i\to j}$, ie the transmission probabilities.

When the pressures and temperatures are equal in the two vessels, there is no net flow through the pipe, and then

$$A_1N_1P_{1\to2}=A_2N_2P_{2\to1}$$

also then $N_1=N_2$

so that $A_1P_{1\to2}=A_2P_{2\to1}$

It follows also in the more general case of several openings

$$\sum_{i=1}^{M}A_iP_{i\to j}=A_j$$

where M is the number of openings as discussed by Davis[61].

A generalisation of the above, considering the probability of passage of molecules through cavities with arbitrarily located inlet and outlet apertures was considered by Lozgachev[62,63]. The calculation of transmission probabilities for simple systems, since the pioneering work of Clausing, will now be briefly reviewed. As was pointed out in the case of flow through an orifice, each of the molecular streams $A_1N_1P_{1\to 2}$ and $A_2N_2P_{2\to 1}$ can be considered separately without loss of generality, so that one may consider the simplified case of zero pressure in the second vessel, ie $N_2=0$ and then (20) reduces to

$$F=AN_0W=F_0W \tag{22}$$

where we have put the transmission probability or probability of passage from vessel 1 to vessel 2 : $P_{1\to 2}=W$. Provided that the tube is connected to a sufficiently large vessel for random molecular impacts at the opening (see also assumption (2) above) the rate at which molecules enter this opening is given by N_0 as in equation (12) above. The rate of molecular transmission and hence the rate of entry of molecules into the second vessel is then given by F.

We should note here the connection between the quantities in equation (22) (ie F, and W) and those in our earlier definition (8) of conductance. The rate of impingement on the opening $F_0=N_0A$ was given in (12) above. All we need to consider here is that Q the flow rate in pressure volume per unit time units in (8) may be written in terms of F of the rate of molecular flow by the relation

$$Q=F.\mathbf{k}T \tag{23}$$

and since in (8) $p_2=0$ and $p_1=n\mathbf{k}T$, we have

$$U=\frac{F}{n} \tag{24}$$

or with (22) and (12):

$$U=\frac{AN_0}{n}.W=U_0.W \tag{25}$$

ie the transmission probability as defined above also expresses the factor by which the entrance conductance of a tube needs to be multiplied in order to get the tube conductance. In terms of the transmission probability W, the flow rate Q in pressure volume units may be written as

$$Q=U(p_2-p_1)=\frac{1}{4}\bar{v}AW(p_2-p_1)$$

or

$$Q=\left[\frac{\mathbf{k}T}{2\pi m}\right]^{\frac{1}{2}}AW(p_2-p_1) \tag{26}$$

7. Clausing's investigation of the transmission probability of a tube

Clausing[25–34] calculated the probability W for a tube of circular cross-section

$$W=\int_0^L W_{SR}(x).w(x)dx+W_{ss}(L) \tag{27}$$

where $w(x)$ is given by an integral equation

$$w(x)=\int_0^L W_{RR}(\xi-x)d\xi.w(\xi)+W_{RS}(L-x) \tag{28}$$

explicitly the complexity of the problem can be seen by writing down $w(x)$ in the case of a circular tube:

$$w(x)=\frac{1}{4R}\int_0^L\left\{2+\frac{(\xi-x)^3}{[(\xi-x)^2+4R^2]^{\frac{3}{2}}}-\frac{3(\xi-x)}{[(\xi-x)^2+4R^2]^{\frac{1}{2}}}\right\}$$
$$w(\xi)\,d\xi+\frac{1}{4R}\left\{[(\overline{L-x)^2+4R^2}]^{\frac{1}{2}}+\frac{(L-x)^2}{[(\overline{L-x)^2+4R^2}]^{\frac{1}{2}}}-\right.$$
$$\left.-2(L-x)\right\} \tag{29}$$

Where W_{SR}, W_{SS}, W_{RR} and W_{RS} are appropriate functions of R and relate to probabilities of the molecular passage and emittance of molecules (assuming a cosine law of emission) from different parts of the tube wall. Clausing also showed that the function $w(x)$ was related to the impact density $g(x)$ for molecules impinging on the walls of the tube, where x is measured along the tube length. Defining the relative impact density $h(x)=\frac{g(x),}{N_0}$ he proved the identity

$$h(x)\equiv w(L-x) \tag{30}$$

This proof depends on the principle of detailed balancing according to which for each direction and velocity the number of emitted molecules is equal to the number adsorbed (see also Clausing[27]).

In trying to solve the integral equation Clausing[32–34] assumes that for $\frac{R}{L}$ large ($\geqslant 1$) a good solution is given by

$$w(x)=\alpha+\frac{1-2\alpha}{L}.x \tag{31}$$

with α =const. Substitution of this in the integral actually gave an expression for α which may be written in the form:

$$\alpha=\frac{[u(u^2+1)^{\frac{1}{2}}-u^2]-[v(v^2+1)^{\frac{1}{2}}-v^2]}{\frac{u(2v^2+1)-v}{(v^2+1)^{\frac{1}{2}}}-\frac{v(2u^2+1)-u}{(u^2+1)^{\frac{1}{2}}}}=\alpha\left(\frac{R}{L},\frac{x}{L}\right) \tag{32}$$

where $u=(L-x)/2R$ and $v=x/2R$, ie $u=(L/R)-v$ (34)

He then selected α such that $W=\frac{8R}{3L}$ for long tubes, ie assuming the Knudsen formula for long tubes. He showed that a good approximation was obtained for short tubes when $L\leqslant 4R$ by taking

$$\alpha=\frac{\sqrt{L^2+4R^2}-L}{2R+\frac{4R^2}{\sqrt{L^2+4R^2}}} \tag{35}$$

, and when $L>4R$, $\alpha=\alpha\left(\frac{R}{L},\frac{x}{L}\right)$ (35)

given by the above formula for $\alpha\left(\frac{R}{L},\frac{x}{L}\right)$ but with

$$x/L=2R\sqrt{7}/(3L+2R\sqrt{7}) \text{ ie } u=\frac{L\sqrt{7}}{3L+2R\sqrt{7}} \tag{36}$$

With this choice (he points out, it is one of many), for very small R/L

$$\alpha\to\frac{4R}{3L} \text{ so that } W\to\frac{8R}{3L}.$$

With these approximations Clausing then calculated W for a range of values of L/R, which he tabulated, and these Clausing probability factors formed the basis for flow calculations in tubes for more than 20 years.

This matter was first taken up again by De Marcus and Hopper[64] and De Marcus[65] who originally were mainly interested in precise values of the Clausing probability factor for accurate vapour pressure measurements using a Knudsen cell. It will only be possible to give here a very brief sketch of the De Marcus approach.

8. Transmission probability of tubes using integral equations

De Marcus[65] gave a very generalised treatise for solving molecular flow problems (he uses the term "Knudsen flow") in terms of Clausing type integral equations. In his detailed analysis of these equations, he shows how various techniques may be used to obtain approximate solutions particularly by methods he so aptly describes as "squeezing" and "scissoring". In the first of these it is shown how solutions may be squeezed between two functions by an iteration process yielding both an upper and a lower bound. In the scissoring process, which is shown to be particularly suitable to the one dimensional Knudsen flow application, the integral equation is first reduced to one involving only the interval 0 to $L/2$ before a squeezing method is applied. Finally he showed that the problem could be solved by a variational method (always applicable when the Kernel is symmetrical). He then proceeded to show how these general results may be applied to specific systems and used the variational method to obtain results for Knudsen flow in circular tubes, systems of parallel semi-infinite plates and a bed of scattering spheres. By estimating a rigorous upper bound for the transmission probability he was able to tabulate approximate values of W with a greater accuracy than Clausing. Thus he showed that an exact solution of the Boltzmann equation could be obtained for Knudsen molecular flow with the aid of the ergodic theorem and principle of detailed balancing, provided that a subsidiary integral equation of the collision density was solved. For the case of "one-dimensional" systems, ie where a single space co-ordinate is sufficient to describe the collision density (note, a rectangular duct would not be in this category) he set up the generalised Clausing integral equation:

$$V(x)=V_1(x)+\int_o^L K(x,y)V(y)dy \tag{37}$$

where $V(x)$=rate of molecular collision with the walls, falling between x and $x+dx$.

$V_1(x)$=contribution to $V(x)$ due to molecules entering the flow system and making their first collision between x and $x+dx$.

$K(x,y)$=probability that molecules colliding with the walls at y will make their next collision between x and $x+dx$.

$$V_1(x)\propto P/(2\pi m\mathbf{k}T)^{\frac{1}{2}}$$

Further, if $N_1(x)$ is the probability that the first collision of a molecule entering at $x=o$ takes place beyond x, then $n_1(x)=\dfrac{dN_1(x)}{dx}$. $P(x;L)$, $P(x;o)$ are defined as probabilities that molecules emitted or scattered at x will proceed without intervening collisions out of the flow system at $x=L$ or $x=o$. The molecular current flowing is

$$W=N_1(L)+\int_o^L n(x)P(x;L)dx \tag{38}$$

where $n(x)$ is the solution to the integral equation

$$n(x)=n_1(x)+\int_o^L K(x;y)n(y)dy \tag{39}$$

The molecular current is also the probability that molecules entering the flow systems at $x=o$ will exit at $x=L$, ie W is the transmission probability.

The reflection coefficient R is numerically $1-W$ and may be expressed as

$$R=\int_o^L n(x)P(x;o)dx \tag{40}$$

adding expressions for W and R

$$1-N_1(L)=\int_o^L n(x)(P(x;o)+P(x;L))dx \tag{41}$$

In general, only one of these various functions need be found due to the relations between them. In the case of the circular tube, De Marcus solved for $N_1(x)$

$$N_1(x)=\frac{x^2+2R^2-x\sqrt{x^2+4R^2}}{2R^2} \tag{42}$$

and obtained the Clausing equation:

$$n(x)=\frac{1}{2R^2}\left[(x^2+4R^2)^{\frac{1}{2}}+\frac{x^2}{(x^2+4R^2)^{\frac{1}{2}}}-2x\right]+\frac{1}{4R}\int_o^L\left\{2-\frac{3|x-y|}{[(x-y)^2+4R^2]^{\frac{1}{2}}}+\frac{(|x-y|)^3}{[(x-y)^2+4R^2]^{\frac{3}{2}}}\right\}n(y)dy \tag{43}$$

He went on to show a method of obtaining a variational solution for W involving the adjustment of parameters to maximise a functional. From this, tables of values of W were prepared to an improved accuracy over those given by Clausing, De Marcus and Hopper[64] on the other hand gave a table applicable for short tubes using the squeezing method.

In this comprehensive paper, De Marcus[65] deduced an asymptotic value for the transmission probability. For sufficiently large values of L/R he obtained the Knudsen formula ie

$$W=\frac{8}{3}\frac{R}{L}$$

He pointed out that this was in fact the first rigorous proof of this formula, because in effect previous proofs assumed a strictly linear collision density (which is false) and thus in a sense assumed the truth of the theorem as a premise. In fact, he showed that the formula was only "just" correct, since the next term in the expansion was of the order

$$\frac{R^2}{L^2}\cdot ln\left(\frac{L}{R}\right)$$

rather than of the order $\dfrac{R^2}{L^2}$ as is usually the case for a term to be neglected. However, since $\dfrac{lnL}{L^2}$ does tend to 0 with increasing L, the Knudsen formula for long tubes is after all seen to be correct.

De Marcus[40] also considered variational techniques to compute the influence of specular reflection on molecular flow

and in De Marcus[66] studied the influence of molecular rough surfaces on molecular flow in circular tubes, though without being able to give explicit solutions to the resulting integral equations.

An analysis of molecular flow both through short cylindrical tubes and through conical ducts was given by German[67] in which Clausing type integral equations were set up. However, no suggestions were made to improve the methods of solution or approximation.

An interesting recent development was given in a note by Berman[68], in which the integrals indicated by De Marcus, have been evaluated to obtain equations for the direct calculation of the transmission probability for tubes, flat plates and beds of spheres. Only the result for tubes are quoted here:

Defining $W = W_1 - W_2$ (44)

and with $S = \frac{L}{R}$, $W_1 = 1 + S^2/4 - (S/4)(S^2+4)^{\frac{1}{2}}$

$$W_2 = \frac{((8-S^2)(S^2+4)^{\frac{1}{2}} + S^3 - 16)^2}{72S(S^2+4)^{\frac{1}{2}} - 288\ ln[S+(S^2+4)^{\frac{1}{2}}] + 288\ ln2}$$

Berman also gave expressions which may ease computation for certain values of L/R

For small S $(=L/R)$

$$W = 1 - \frac{S}{2} + \frac{S^2}{4} - \frac{5S^3}{48} + \frac{S^4}{32} - \frac{13S^5}{2560} - \frac{S^6}{3840} + \ldots . \quad (45)$$

which is stated to be in error by about 0.04% at $S=1$ and less than 10^{-4}% at $S=0.5$

For large S $(=L/R)$

$$W \sim \frac{8}{3S} - \frac{2(lnS)}{S^2} - \frac{91}{18S^2} + \frac{32\,lnS}{3\,S^3} + \frac{8}{3S^3} - 8\frac{(lnS)^2}{S^4} + O\left(\frac{1}{S^4}\right) \quad (46)$$

which is said to be in error by about 1% at $S=20$

9. Flow pattern for tubes and components

Clausing [27–31] investigated the angular distribution pattern of gas molecules emerging from cylindrical tubes of various lengths under molecular flow conditions. He derived expressions from which the flow pattern could be deduced, depending only on the ratio of tube radius to tube length (R/L) and gave numerical values for the flow intensity as a function of exit angle for the case when $L=2R$. The flow pattern through an orifice of negligible thickness corresponds to the cosine law distribution, ie in the form of a sphere tangentially to the orifice opening, where the probability of molecules issuing in any given direction are given by the length of the radius vector as shown in the polar diagram (Figure 3). Clausing showed that in the case of even a short tube considerable beaming occurs giving a modified flow pattern, having a relatively sharp apex as shown in the polar diagram. Such flow patterns were confirmed experimentally in molecular beam experiments by Ellett[69] using a Pirani detector, though he gave no experimental details. Dayton[102] discussed Clausing's earlier work and derived also the complementary gas flow pattern appearing at the tube entrance due to those molecules which return without passing through. The two angular distribution patterns at the exit and entrance were shown to be complementary in that they must add up to correspond to the cosine distribution for the original molecules entering the tube from the first chamber. Dayton gave computed results at 5° intervals for

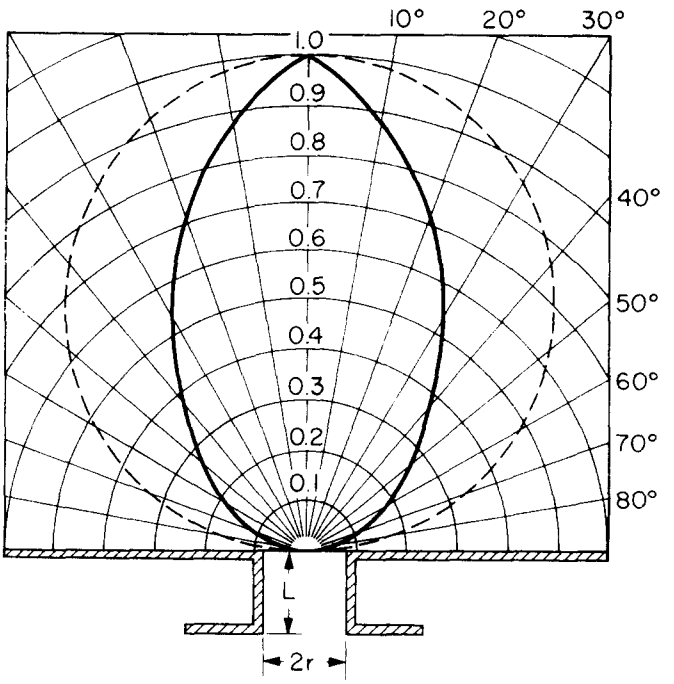


Figure 3. Polar diagram of gas flow issuing from a short cylindrical tube into a vacuum (for the spherical case $L=2r$) compared to flow through an orifice calculated by Clausing (1930).

the four values of $L/R=1$, 2, 4 and 10 respectively and these are shown plotted in Figure 4.

Although there is little doubt in the essentially correct approach of the Clausing and Dayton analysis reported here (within the various small approximations made), there have been some difficulties for various reasons, in establishing these results in every case experimentally. Thus Clausing[27–31] quotes some early work of Mayer[71,72] which apparently showed much less pronounced beaming. More recently Günther[73] carried out some careful interferometric measurements of deposits of silicon monoxide evaporated through a Knudsen source with a tubular exit of the Clausing dimensions $(L/R=2)$. These also indicated a weaker beaming effect than predicted by theory and led to an explanation in terms of a wall effect or "diffusion" stream. However, it should perhaps be noted that special difficulties would have arisen with the use of SiO which is not a very stable evaporant material.

Winterbottom and Hirth[74] suggested that the discrepancy could be explained by the adsorption of molecules on the walls followed by a diffusion (or creep) of the adsorbed molecules before they are re-evaporated. A similar mechanism had previously been proposed by Sears[75] in connection with flow through very fine tubes. This model was further discussed by Ruth, Winterbottom and Hirth[76] where they showed how their model could be fitted to Gunther's experimental results assuming a mean diffusion path length (ie the distance which it is assumed molecules need to traverse while being adsorbed) of only 0.07 cm. In recent work by Gokcen[77] the surface diffusion effect was further discussed in relation to the Winterbottom and Hirth[74] proposals.

The intensity distribution of metal evaporated deposits from conical and wire sources of different designs was investigated by Preuss[78,79] and Preuss and Begunis[80] using radio active evaporants. As expected the results for conical sources showed marked beaming effects and they indicated the effectiveness of this experimental technique (Figure 5).

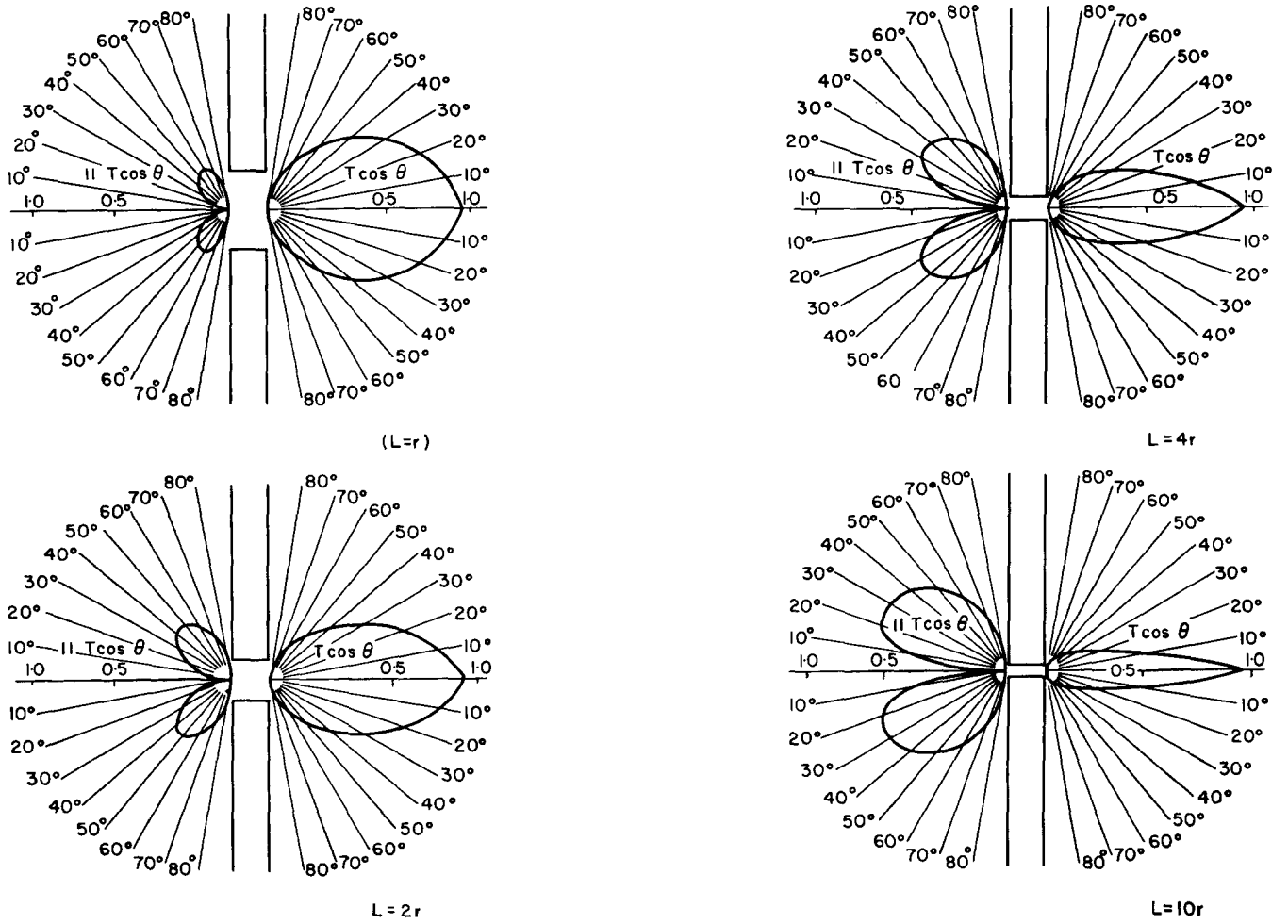


Figure 4. Polar diagrams of gas flow at entrance and exit of cylindrical tubes when $L=r$, $L=2r$, $L=4r$ and $L=10r$, (calculated by Dayton, 1957).

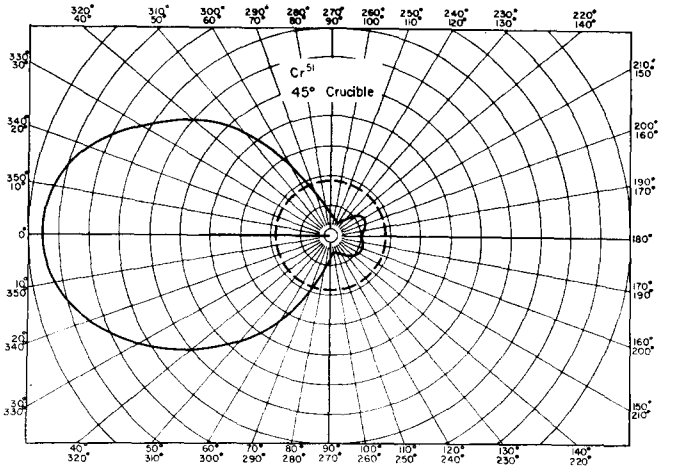
Some recent experimental work on the gas emission and flow patterns from tubes and components into a high vacuum using an ionisation gauge molecular beam detector was carried out in the author's laboratory. Holland and Priestland[81] described the performance of the detector and gave experimental results obtained for the flow through a thin aperture and a near parallel beam, while Holland and Priestland[82] described their experimental results for the gas flow pattern through tubes; the results being in general agreement with the calculated results of Clausing and Dayton though the resolution of the relatively large ion gauge detector was not sufficient to show the pointed peak in the flow pattern. (Figure 6).

Experimental results for cylindrical tubes of various sizes were reported by Schaetzle[83] also indicating the effect of gas scattering as the pressure was increased. These measurements showed a less pronounced beaming effect than that in the theoretical work of Dayton[70], but was in agreement with data of Cook *et al*[84]. It would appear that these results were much influenced by the detailed experimental conditions, particularly by the size and shape of the chamber behind the orifices and shows how practical difficulties can easily arise in the experimental verification of idealised theoretical data.

A general discussion of effusion studies and transmission probabilities for conical and cylindrical orifices was also given by Freeman and Edwards[85].

In an investigation limited to long tubes, both Giordmaine and Wang[86], and Becker[87] also considered the angular distribution of the emitted molecular beam. Their results are criticised and corrected in a recent paper by Zugenmaier[88].

Figure 5. Polar plot of dispersion patterns obtained by the evaporation of Cr^{51} from a 45° cone shaped crucible (as reported by Preuss, 1956).



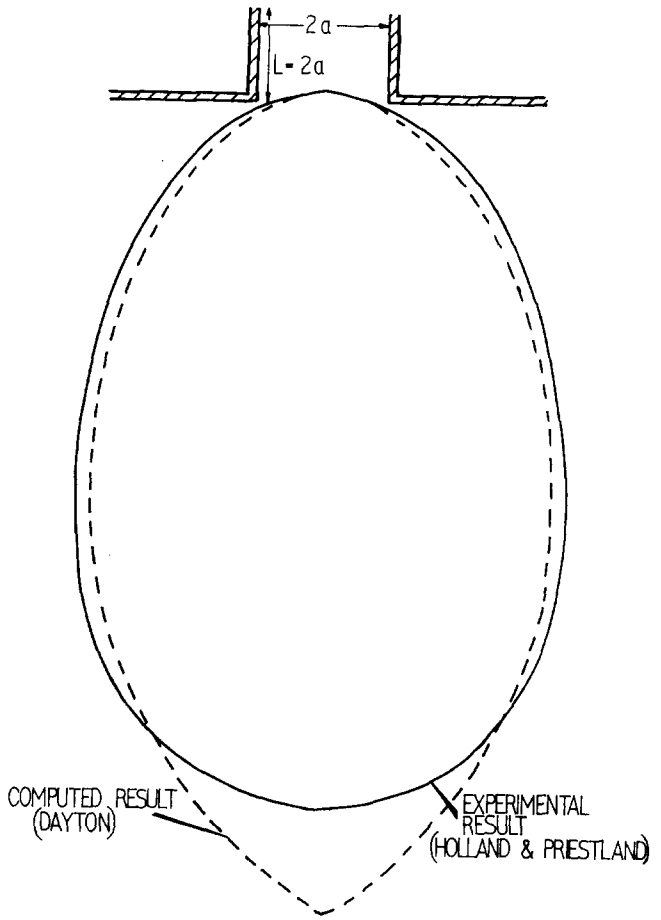


Figure 6. Polar diagram of experimental results for the gas flow issuing from a short cylindrical tube as measured with an ionisation gauge monitor by Holland and Priestland (1966).

This also includes a theoretical discussion for tubes of arbitrary length, which appears to make a number of simplifying assumptions leading to considerable deviations from Clausing's and Dayton's results.

The distribution function at various points in molecular flow through a circular tube was investigated recently by Sparrow and Haji-Sheikh[89]. They applied this to a calculation of the mass velocity profile development along the length of the tube. It is shown that this depends only on the R/L ratio ie tube dimensions, and *not* the temperature of particle fluxes. The most rapid changes are shown to occur near the tube inlet. For longer tubes, the velocity profile is shown to be fully developed and flat for an appreciable length of the tube. They further calculated the molecular or particle density distribution in the tube, which however is more complicated in that it is shown to depend on six parameters and co-ordinates.

For vapour pressure measurements, using a Knudsen cell in which the orifice is in the form of a short tube a Clausing correction is generally applied. It is occasionally useful to collect the evaporant material on a cooled plate and Clausing[32–34] in fact gave some approximate results for the amount collected by a cone, the sides of which made an angle θ with the centre line. A recent note by Miller[90] gives new four figure computations of the fraction of molecules collected by such conical surfaces based on Clausing's derivation. His results show good agreement in the limit when the cone becomes a plane with the more accurate De Marcus[65] results for the range of values 0.1 to 4 for L/R.

10. Molecular flow conductance of complex systems

In all our discussions so far we have limited our considerations to molecular flow in tubes or orifices assumed to be mounted between relatively large vessels so that gas molecules can be assumed to enter at random with a cosine distribution and exit in an unhindered fashion. We have seen that these problems are quite amenable to calculation. The question then arises how to treat flow through more complex structures, for example, tubes of different lengths and diameters joined in series, right angle bends, simple diaphragms and baffle plates, chevron baffles etc. as well as any combinations of these. Thus in the simplest case of say two tubes in series, it is clear that the conductance of each one as defined by (8) above can be determined with fair accuracy, but when these tubes are placed in series, the gas flow pattern from the one will be more or less beamed into the other as determined by their R/L ratio, and so upsetting all the calculations indicated above.

In the practical approach of Dushman[17], the tube impedance concept (the inverse of the conductance) was developed in analogy with electrical circuits so that impedance in series were additive. The electrical circuit analogy to molecular flow problems in vacuum systems has since then been discussed from time to time eg by Stops[91], Degras[92], Delafosse and Mongodin[93] and Guthrie[94]; though Oatley[95] pointed out some of the limitations in the concept of a molecular flow equivalent of Kirchhoff's laws. Dushman showed that the inverse of pump speeds could be treated as impedances, while for short tubes an entrance correction could be applied corresponding to the orifice impedance of the opening. This is, of course, equivalent to our equations (16) and (17) above for conductances. These ideas were also taken up by Clausing[32] at the end of his paper on the flow through short ducts, but he failed to give a warning that this method must be applied with caution for accurate results with short tubes. Clausing[32] showed how this impedance concept could be applied to estimate the time of evacuation of a vessel and related problems. For a recent summary of these procedures reference may be made to the usual texts on high vacuum techniques, particularly perhaps to the books by Guthrie and Wakerling[55] and Dushman[58]. A paper in the same vein, discussing particularly the flow of gas through tubes in series, by Harries[98] goes a little further. As before, he applies the Dushman short tube formula for each section of a tube and on finding that this is consistent with calculations of conductances for tubes in series he reaches the erroneous conclusion that the Dushman formula gives the law for molecular flow through a tube of any length with absolute accuracy. The same conclusions and approach were given in a paper by Florescu[99] in which he claims to give a physical justification for this procedure. By arbitrarily adding tube impedances calculated on the basis of the Clausing factor for short tubes he obtained discrepancies of up to 16.3% with his interpretation, ie based on the Dushman formula. As Dayton[102] already pointed out, all these arguments are of course fallacious, since the transmission probability "of any section is modified by the beaming effect across the junction and the exact solution should conform to Clausing's short tube formula rather than Dushman's."

Oatley[95, 96] also considered the flow of gas through composite systems, but from the point of view of transmission probabilities. Oatley first considered two tubes of equal radii connected in series as shown in Figure 7(a). If each one has transmission probabilities W_1 and W_2 respectively (defined as

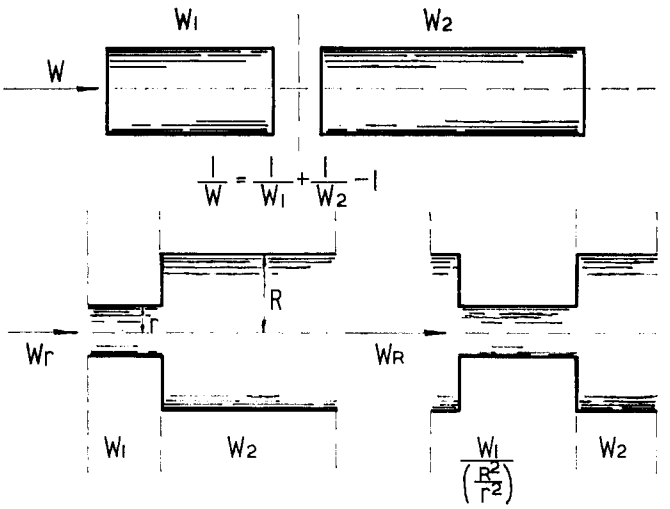


Figure 7. The flow of gases through simple composite tube systems (following Oatley, 1957):
(a) two tubes of equal radii connected in series.
(b) two tubes of different radii—considered as of radius *r*.
(c) two tubes of different radii—considered as of radius *R*.

in equation (22) above for each tube connected between large chambers), then he showed that a combined transmission probability could be derived leading to

$$\frac{1}{W}=\frac{1}{W_1}+\frac{1}{W_2}-1 \tag{47}$$

and similarly, for two tubes of different radii, as shown in Figure 7(b), the transmission probability W_R of the whole system, considered as of radius R as in 7(c):

$$\frac{1}{W_R}=\frac{R^2}{r^2}\cdot\frac{1}{W_1}+\frac{1}{W_2}-1 \tag{48}$$

or reverting to (b), and considering the inlet as of radius r:

$$\frac{1}{W_r}=\frac{1}{W_1}+\frac{r^2}{R^2}\cdot\frac{1}{W_2}-1 \tag{49}$$

In the same paper, Oatley showed that more complicated composite systems could be treated in this way, as well as pumps connected to tube systems, where the pumping speed could be taken just like the flow rate defined by F in equation (22) above.

In a note, Steckelmacher[107] indicated that formally at least, Oatley's results were equivalent to considering the conductance of the tube as made up of a tube conductance U_T in series with an entrance conductance U_A ie

$$\frac{1}{U}=\frac{1}{U_A}+\frac{1}{U_T} \tag{50}$$

Similarly, with an aperture of area A within a tube of area A_0 the conductance of the aperture was modified to U_A/A_0 given by

$$U_A/A_0=U_A(A_0/(A_0-A)) \tag{51}$$

It should be noted that the last mentioned relationship is equivalent to the expression quoted by Bureau, Lasslett and Keller[111] and was implicit in the paper of Oatley[96], who considered circular tubes so that the ratio $(A_0-A)/A=A_0/A-1=f-1$, where $f=R_0^2/R^2$, R_0 and R being the tube radius and aperture radius respectively so that for the transmission probability of the aperture in the tube W one can put

$$\frac{1}{W}=f-1.$$

For N tubes or N elements whose transmission probabilities are the same in either direction this Oatley expression (equation 47) generalises to:

$$\frac{1}{W_T}=\sum_{i=1}^{N}\frac{1}{W_i}+1-N$$

It was pointed out by Ballance[112] that these considerations were only valid for geometries in which the transmission probability was the same in each direction through the element (ie $P_{i\to j}=P_{j\to i}$ as discussed in section 6 above). He showed that in the general case for N elements the expression is modified to

$$\frac{1}{W_T}=\frac{1}{W_1}+\sum_{i=2}^{N}X_{i-1}\left(\frac{1}{W_i}-1\right)$$

Where $X_i=W_i^1/W_i$

and W_i is the transmission probability of the ith element and W_i^1 is the transmission probability in the opposite direction. He showed that this more general expression was exactly equivalent to the specific examples discussed by Oatley.

For a tube of radius R, the Clausing formula gives a conductance (see equation (25) above) which may be written as:

$$U=U_AW \tag{52}$$

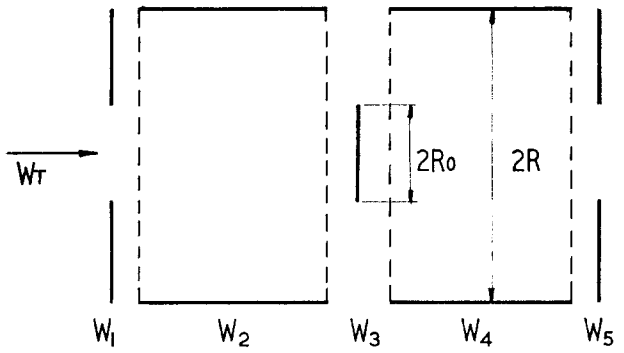
where $U_A=11.7A=11.7\pi R^2$ (for air at 25°C) which is of course, equivalent to the U_0 given in equations (10) and (11) above. From (50) it then follows that

$$U_T=U_AW/(1-W) \tag{53}$$

The application of these ideas to composite systems was given by Steckelmacher[107] and shown to give exactly equivalent answers to the method proposed by Oatley[96]. More recently Steckelmacher[109, 110] showed that the same principles could be applied to pumps used in conjunction with tube systems as discussed below.

The Oatley method (and hence also my own suggestions which give identical results) can be criticised on two main grounds:

Firstly, it does not give the correct limiting conditions for certain problems. This was demonstrated clearly in a recent paper by Ballance[112] in his example of a blocking plate in a tube shown in Figs. 8 & 9. Another case would be the example of a vapour trap given by Steckelmacher[107]. These formulae must be wrong for small lengths of tubing between the baffle. When the length of tubing shrinks to zero the trap becomes a solid wall; but instead of obtaining zero for U_T, Oatley obtained a finite value. If the method were more than just an



$$W_2 = W_4$$

$$\frac{1}{W_T} = 2 - 3\left(\frac{R_0}{R}\right)^2 + \frac{1}{W_2}\left(\frac{R_0}{R}\right)^2 + \frac{R_0^2}{(R^2 - R_0^2)} \cdot \frac{1}{W_2}$$

when $R_0 = R$, $W_T = 0$

but when $W_2 = 1$

$$\frac{1}{W_T} = 2 - \frac{2R_0^2}{R^2} + \frac{R_0^2}{R^2 - R_0^2}$$

hence, if $\frac{R_0^2}{R^2} = \frac{1}{2}$ $\quad \frac{1}{W_T} = 0.5$

instead of zero as it should be.

Figure 8. The flow of gases through a tube with blocking plate. Theoretical discussion by Oatley method.

approximation, it should satisfactorily handle such problems. As Ballance[112] pointed out, the Oatley method can give very valid and good approximations for molecular flow through complex systems, but in its application some obvious, and some not so obvious, errors are inherent.

The second criticism is one which I recently discussed, Steckelmacher[110], and which was hinted at already in Dayton[102] in his comment on Harries' paper. This refers to the streaming properties of flow through a tube which is more or less pronounced depending on tube length. Thus the method of Oatley, as well as the method discussed in my comments to his article, removed the major error of including the entrance correction twice, but still contained the minor error of assuming that the so-called "tube conductance" of the ith section of a composite system (analogous to (53) above),

$$U_{Ti} = U_A W_i/(1 - W_i) \tag{54}$$

depends only on the geometry of that section. Actually W_i really depends on the geometry of every element in a composite system.

This can be illustrated (following Oatley and Ballance) by calculating the transmission probability of two tubes of equal radius connected in series from equation (47) and comparing this with the actual transmission probability of the combined single tube with transmission probability W_3. One can then form the expression

$$\frac{W_3}{W} = \left(\frac{1}{W_1} + \frac{1}{W_2} - 1\right) W_3 \tag{55}$$

where W_1, W_2 and W_3 on the right hand side are the known transmission probabilities and the ratio W/W_3 should ideally be near unity for all L/R. The actual behaviour is shown in Figure 10 indicating maximum errors of about 4% for a tube in two sections and about 6.5% for three sections. (See Figure 11). The rather higher errors of up to 5.6% for a two section tube quoted by Oatley were due to his use of the Clausing values for the transmission probability, which is in error by a maximum

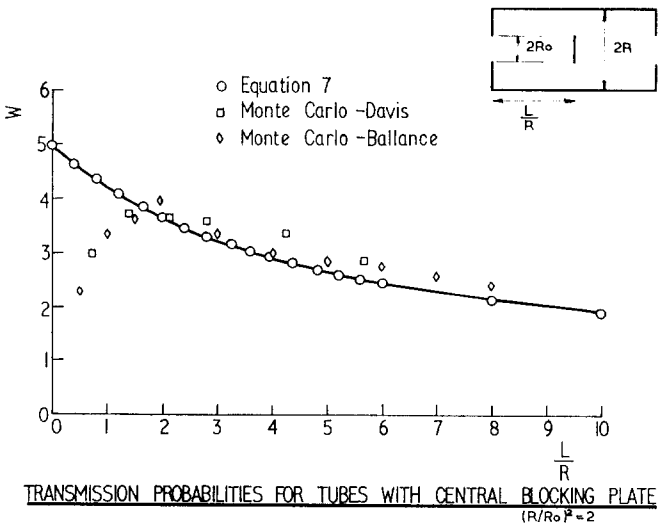


Figure 9. The flow of gases through a tube with a blocking plate. Comparison of Monte Carlo analysis of transmission probability with theoretical results—for different L/R with $(R/R_0)^2 = 2$ (following Ballance, 1966).

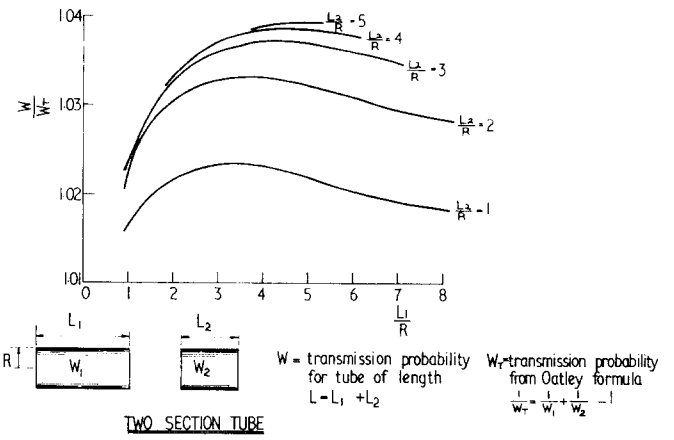


Figure 10. The transmission probability for a cylindrical tube made up of two sections compared with the combined transmission probability as calculated from the Oatley formula for various values of L/R.

of several per cent, whereas the improved De Marcus[65] values were used in the present calculation.

In the special case of tubes with diaphragm ends, Lozgachev[63] set up the corresponding Clausing integral equations, but he did not obtain numerical solutions. He also discusses briefly the transmission probability for two tubes connected in series. In a second paper Lozgachev[63] discussed the flow through ducts of arbitrary form in a rather general way by again setting up Clausing type integral equations. Special cases considered were that of a sphere and spherical segment for which solutions by successive approximation were derived, though he pointed out that a numerical integration was required to obtain transmission probabilities. Similarly he obtained expressions for the case of a cylinder with symmetrically located apertures in the base, but again the resulting integral equations would in general be difficult. An alternative procedure was therefore proposed for the case of a cavity with symmetrically located apertures which involved setting up a system of linear equations for the probabilities, the number of unknowns depending on the form of the cavities. This procedure was illustrated for the case of a rectangular cavity with two symmetrical apertures in its base.

A somewhat related problem to that of series connected complex tube systems is of gas beaming into a tube rather than entering it randomly. This has been considered in connection with the theory of impact tubes applied for example, to vacuum gauge measurements in satellites (Figure 12). In this application the effect of a superimposed mass velocity directed down the tube needs to be determined. Pond[13] considered the effect of such a superimposed entrance velocity on the flow of rarefied gas through a circular tube following very much on the lines of Clausing[32–34]. He determined a modified Clausing transmission probability as a function of the speed ratio s of the external gas (the speed ratio is defined as the ratio of mass flow to most probable speed of molecules) so that when $s=0$, the usual Clausing transmission probability is obtained. Numerical solutions for impact pressure probes were reported by de Leeuw and Rothe[114] and by Ballance[112, 114] including data for different angles of attack and orifice restricted tubes using Monte Carlo type calculations.

11. Molecular flow problems and the use of Monte Carlo methods

The discussion so far will have shown that immediately more complex systems need to be considered, the mathematical effort for obtaining solutions of the resulting integral equations can be considerable, while there are many cases which would appear intractable. An application of the Oatley method (or its equivalent) can of course be applied to obtain good approximations but there always remains some uncertainty as to the errors involved as well as in the suitability of its application to the particular problem in hand. An important breakthrough was therefore made by Levenson, Milleron and Davis[117, 118] and Davis[61] in their application of statistical Monte Carlo methods of calculation to the determination of molecular flow conductances. This method was applied to obtain the conductance of tubes and elbows of various shapes and also components including baffles and traps which would have been impossible to calculate by any other procedure.

Thus, Davis[61] indicated the procedure for such calculations: The entry and exit areas of the system were subdivided into equal area openings (eg concentric circles for circular tube systems or circular elbows). With these starting areas various

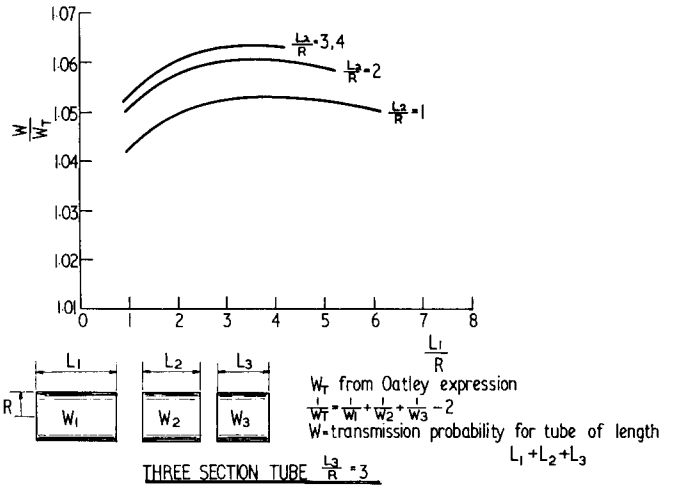


Figure 11. The transmission probability for a cylindrical tube made up of three sections compared with the combined transmission probability as calculated by the Oatley formula for various values of L/R for two sections, but with $L/R=3$ for the third section.

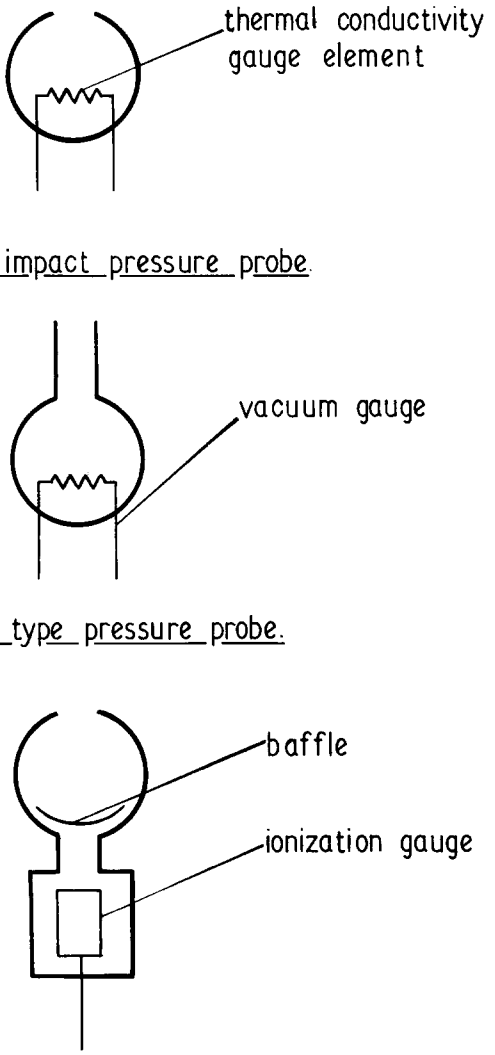


Figure 12. Diagram of different detector arrangements used in molecular flow studies. An orifice and tube type impact pressure probe has been used particularly with thermal conductivity type gauges as well as the molecular incidence rate pressure transducer proposed for measurements with an ionisation gauge.

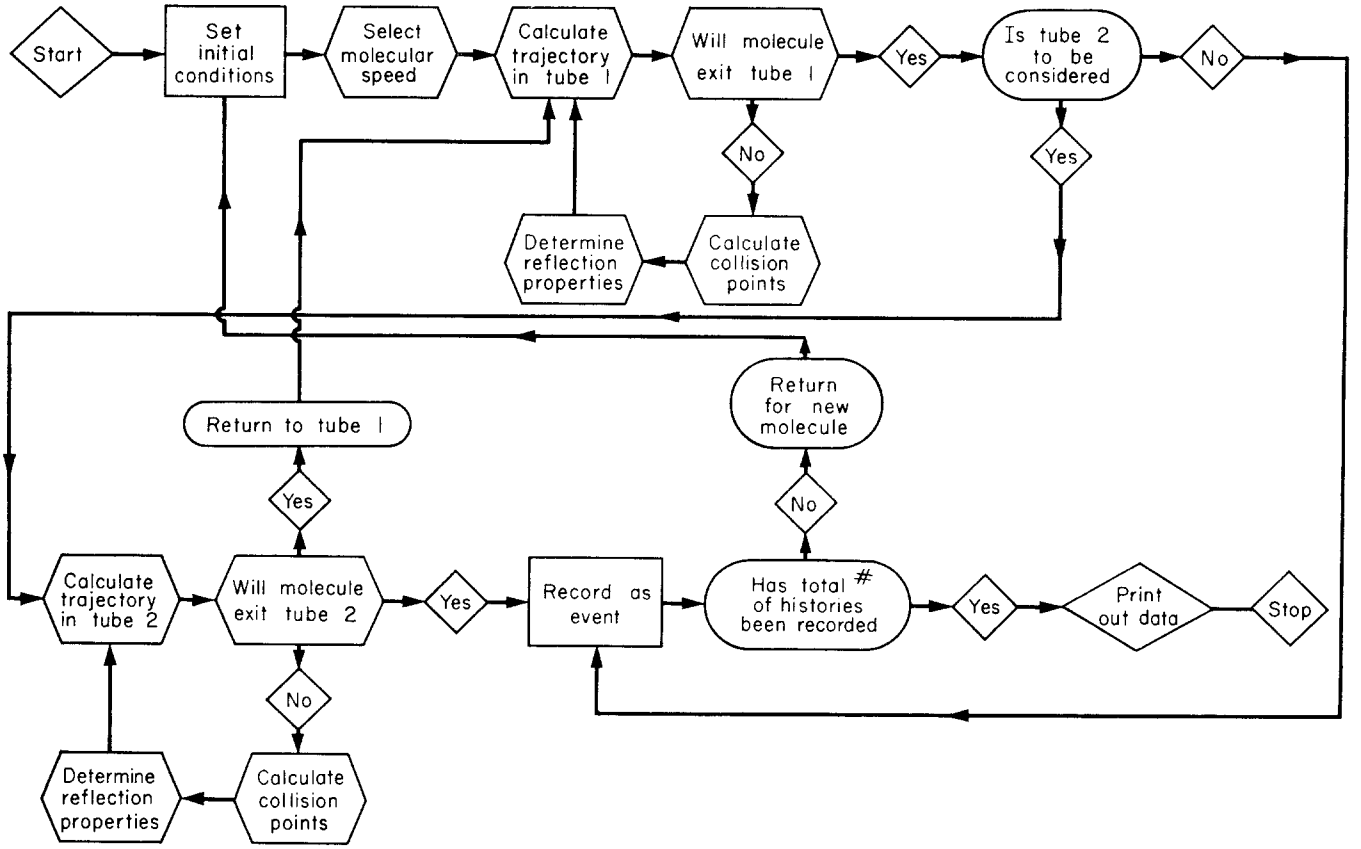


Figure 13. Computer flow diagram for Monte Carlo calculations of transmission probabilities (following Ballance, 1966).

possible molecular histories, assuming a cosine law entry and wall interaction were set up to work out the transmission probabilities. These were arrived at by using a sequence of random numbers to simulate the behaviour of individual molecules (see also Figure 13).

As indicated by Davis[61], and Davis, Levenson and Milleron[121] Monte Carlo calculations have an inherent error which depends on the number of molecular histories followed as well as on the transmission probability W. The standard deviation is given by:

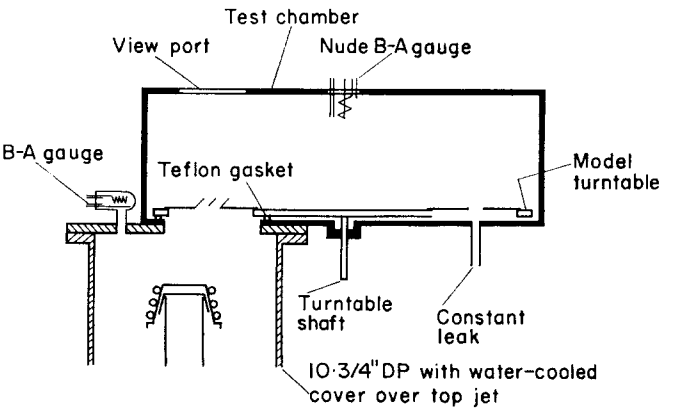
$$\sigma_W = (W(1-W)/N)^{\frac{1}{2}} \tag{56}$$

where N is the number of histories. Hence, with W of say 0.5, σ is of the order of 0.015 when $N=1000$, improving to 0.005 for $N=7000$. Davis[61] gave results for straight cylindrical tubes (showing good agreement with Clausing), cylindrical elbows (90°), the short cylindrical annulus, a straight tube with restricted openings, and a straight tube with restricted openings as well as with a central baffle plate.

Levenson, Milleron and Davis[117, 118] discussed the problem of optimisation of the molecular flow conductance for components connected to a pump—particularly a diffusion pump. They were therefore particularly concerned with baffle type geometries and discussed simple louvres, chevron systems as well as baffle plate systems also indicating the effect of the diffusion pump top cap which gives rise to uneven flow conditions at the pump mouth. In addition to the Monte Carlo calculation they also obtained experimental verification of some of their results based on a method suggested earlier by Oatley[97]. For a fixed flow rate of gas under molecular flow conditions in a large test chamber, the pressure difference across a scale model was compared with the pressure differences measured across several orifices of known area (with apparatus shown in Figure 14). The method was also discussed by Levenson Milleron and Davis[119] giving some additional results including flow through rectangular channels.

In all the investigations discussed so far, it was assumed that the walls bounding the flow system were smooth, though sufficiently rough (ie "molecularly" rough) to give rise to the diffuse re-emission of molecules (following the cosine law). De Marcus[66] considered a semi-infinite parallel plate flow channel with sawtooth pyramidal corrugations corresponding to rough surfaces. He obtained Clausing type integral equations

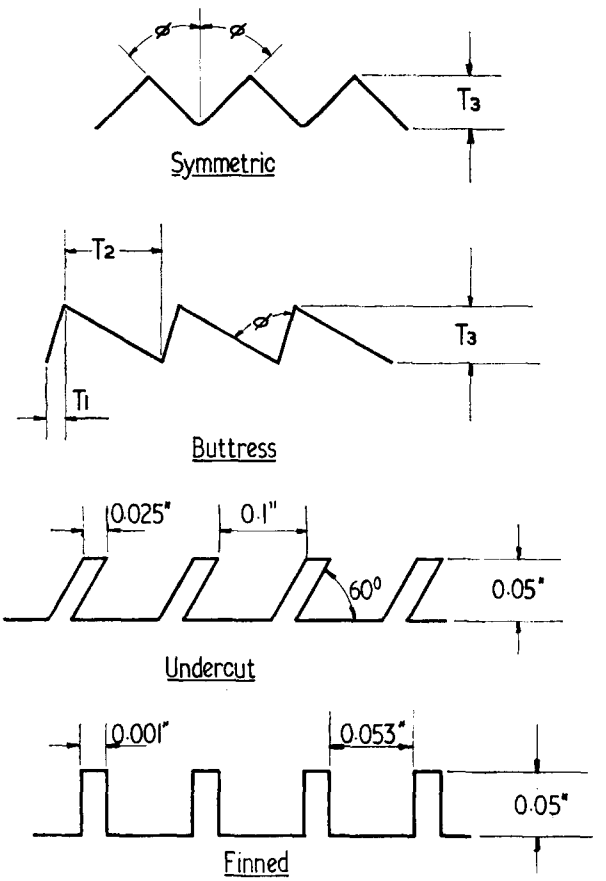
Figure 14. The experimental determination of transmission probabilities (and molecular flow conductances) by comparing the flow through scale models with the flow through known orifices (following Levenson, Milleron and Davis, 1961 and 1963).



for such a model to which approximate solutions were applied. These calculations indicated for some specific examples that the flow rate decreased (compared with a smooth wall) resulting in about a 1% change for a channel of length to height ratio of 0.1, and a 3% change for a channel 3 times as long.

The effect of surface roughness on molecular flow has proved to be another subject in which both Monte Carlo type calculations and careful experimental measurements on typical models can give useful information. In an extensive investigation Davis, Levenson and Milleron[121] described results obtained for short circular tubes with different engineering type wall finishes as well as for models of typically rough walls consisting of corrugations, fins and threads of various depths and angles as indicated in Figure 15. Their experiments were conducted in the apparatus mentioned above in which the flow route through the model could be compared with that through a known orifice. The engineering surfaces tested ranged from flame polished glass tubes with a surface roughness of 0.5μin to brass cylinders in which the roughness of different specimens varied from 3.5 to 63μin as measured with a Taly-surf machine. The results indicated flow rates from 5% above (for the longest smooth tube) to about 6% below the diffuse wall prediction (the higher figure for the roughest tube exposed). These experiments were checked for geometrical models of corrugated and finned wall finishes by Monte Carlo calculations. Conductance values from 15-20% lower than for smooth walls were found generally with a fair comparison between calculation and experiment (maximum deviation ±5%), but generally better agreement. It certainly appeared from this work that the roughness of engineering surfaces had a small effect (observable only by careful measurement) but that a gross surface roughness could have a considerable effect. The latter was predictable assuming diffuse reflecting properties from the surface elements.

Berman and Lund[122] showed by careful flow measurements of porous materials that the detailed nature of the gas-wall collision processes affect the flow behaviour. In control experiments with a nickel tube and orifice, on the other hand, good agreement with the Clausing transmission probability factor was confirmed.



PROFILES OF ROUGH SURFACES INVESTIGATED BY DAVIS, LEVENSON AND MILLERON (1964).

Figure 15. Profiles of rough surfaces of the type investigated in their molecular flow studies by Davis, Levenson and Milleron (1964).

12. The speed of pumps in the molecular flow region

Various definitions have been in use for the speed of pumps, and this has resulted in much confusion. It is therefore not surprising that some controversy should have arisen in the procedures for measurement.

If one considers pump speed in analogy with flow through tubes and considers a definition similar to the definition of conductance as discussed in section (1) above, then one needs to make assumptions on the same lines as those listed there:

(1) Assume molecular flow conditions.

(2) Consider a pump or pumping system, (ie a pump together with its associated components) connected to a large chamber so that the distribution and direction of molecules entering the pump mouth or system mouth is independent of the flow in other parts.

(3) The flow is steady.

(4) It is assumed that a gas flow Q is established in the chamber and into the pump such that a steady pressure p is maintained in the chamber, where p is assumed to be measured sufficiently far from the pump mouth so as not to be affected by the gas flow towards the pump.

Under these conditions one may define the pump speed:

$$S=\frac{Q}{p} \tag{57}$$

which is equivalent to equation (8) above. With the concept of molecular flow F developed in sections (4) and (6) above, and in analogy with equation (24)

$$S=\frac{F}{n} \tag{58}$$

where F is the net number of molecules which enter the pump (or pumping system) mouth per unit time and n is the molecular density (sufficiently far away from the pump mouth).

This can be expressed in yet another way by considering a probability of pumping σ (analogous to the transmission probability W) corresponding to the ratio of F to F_0 where F_0 is the number of molecules which would have passed through the pump mouth (or pumping system mouth) if this had been a thin orifice. Alternatively F_0 is simply the rate of molecular impingement on the pump mouth opening of area A. Hence in analogy with equation (22), we have:

$$F=F_0\sigma=AN_0\sigma=\frac{1}{4}n\bar{v}A\sigma \tag{59}$$

and so we can see that we have simply:

$$S=\frac{1}{4}\bar{v}A\sigma=\left(\frac{\mathbf{k}T}{2\pi M}\right)^{\frac{1}{2}}A\sigma \tag{60}$$

S can also be expressed in terms of the aperture conductance $U_0=\frac{1}{4}\bar{v}\,A$ by:

$$S=U_0\sigma \tag{61}$$

from which it is seen that an "ideal" pump is one which has the speed of just an aperture (with an infinite evacuated space behind it) for which $\sigma=1$ and

$$S=S_0=U_0.$$

To distinguish these definitions with others, Steckelmacher[109] suggested the term "intrinsic" pump speed and "intrinsic" conductance for the speed and conductance defined as in equation (57) and equation (8) above. In this paper it was shown that these definitions were consistent with carrying out Oatley type calculations (see section 10 above) for the combination of pumps and systems of tubes; though subject to the usual errors associated with this method due to the beaming effects, as indicated in Steckelmacher[110].

An advantage of this definition is that the pumping speed of an "ideal" pump as explained above comes out to be equivalent to the "aperture" speed (or conductance) as one would expect from physical considerations.

13. The theoretical pumping speed of a pump and its pumping efficiency

It may be suggested that with a good understanding of the theory of operation of a pump, its efficiency and speed of pumping should be capable of proper estimation and then all would be well. Let us consider this situation briefly in the case of the most common high vacuum pumping means—namely the diffusion pump. It is found that this pump was fated from the earliest days when Gaede and Langmuir had their controversy on its mode of operation—diffusion as against condensation, and in fact such controversies have had their counterpart in relatively recent literature in the search for a simple model to describe the action, performance and pumping speed of this type of pump. These theories have been discussed by Matricon[123], Ho[124, 125], Alexander[126], Avery and Witty[127], Jaeckel[128], Blears and Hill[129], Nöller[130], Jaeckel, Nöller and Kutscher[131], Florescu[59], Reichelt[132] as well as in numerous earlier papers. Thus Gaede[133, 18] (see also the discussion by Avery and Witty[127]) derived the following equation for the speed of a diffusion pump:

$$S=A\left(\frac{RT}{2\pi M}\right)^{\frac{1}{2}}(K\alpha-\beta P_2/P_1) \tag{62}$$

where A is the area of the jet annulus and P_2/P_1 is the ratio of fore-vacuum pressure to high vacuum pressure. K and α are "entrainment coefficients" and β a coefficient expressing the back diffusion of gas from the fore-vacuum.

A similar equation was derived by Matricon who suggested that the "constant" $k\alpha$ was an invariant for all pumps and gases (with a value of 9×10^{-2}) and that β was zero for all practical purposes.

These theories have in common an inverse square root dependence of pumping speed on the molecular weight of the gas which does not necessarily hold for practical pumps (see also Dayton[104], Blears and Hill[129] and Reichelt[132]). Ho[124, 125] recognising that the speed of practical pumps could be wrongly assessed depending on how the pressure in the vicinity of the pump mouth or diffusion pump jet was measured suggested a method of defining the efficiency which has become known as the Ho-coefficient, but this was still open to confusion. For a recent discussion concerning the Ho-coefficient and its interpretation reference may be made to Florescu[101] and Milleron and Levenson[135], the latter indicating that the Ho-coefficient was ambiguous with regard to pumping efficiency, but could be used as a measure of the gas entrainment by a jet with due consideration of conductance to the jet. The matter was further confused in that Ho specified an inexact method of pressure measurement in the plane of the pump mouth. One is therefore, naturally led in a full circle back to the practical question of measurement.

It is clear that similar remarks concerning the difficulties in attempts to derive the theoretical pumping speed and efficiency apply in the case of other methods of pumping whether they be mechanical (ie molecular), ion, getter or cryopumping even if due regard is taken of the various physical processes involved. The particular problems of non-uniform gas temperatures associated with cryopumping surfaces was considered by Mickelsen & Childs[136] and Barnes[137].

As Milleron and Levenson[134, 135] pointed out, the efficiency of any pumping system should be a measure of how well it utilises the wall area of the vacuum chamber to which it is connected. They suggest that the efficiency of any sort of free molecular flow pump system can be defined as the ratio of speed of the pump system to the theoretical speed associated with the area the pump system occupies on a chamber wall. In terms of our definition in Section 12 above, this efficiency is expressed by the quantity σ of equations (59-61).

14. Proposals for the measurement of pump speed

All definitions of pump speed put forward have in common the basic equation:

$$S=\frac{Q}{p} \tag{57}$$

The quantity of gas pumped, ie Q, may be well defined, though the manner of gas admission can profoundly influence the measurement. Provided this gas comes from a single source, and one assumes steady conditions, the quantity Q may be readily measured and is usually expressed in pressure-volume units per unit time (in the units usually used in vacuum technology this is expressed in torr-litres/second; incidentally, in SI units, where pressure is expressed in N/m^2, this turns out to be in watts). The measurement of pressure in the determination of pump speed, however, is not quite so straightforward and has been a subject of discussion over almost twenty years.

It has been shown above that irrespective of the pumping action, the speed should be defined relative to the mouth of the pump. This concept has led to the mistaken idea that the corresponding pressure in equation (57) should be measured there. The difficulties associated with attempts to measure this pressure were first properly outlined by Dayton[103], and discussed particularly by Venema[138, 139].

As discussed by Dayton[103], and Venema[138], some authors have introduced the ultimate pressure p_0 into the calculation of pump speed—so that they have defined the speed as $Q/(p-p_0)$. However, as considered in detail by

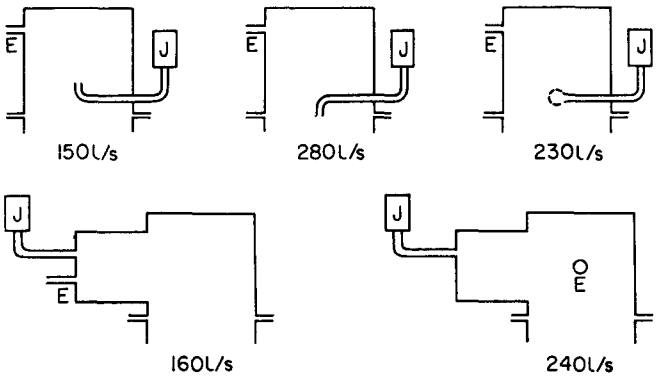


Figure 16. The effect of the gauge and gas inlet positions into a test dome on the measured speed of a diffusion pump (following Dayton, 1948).

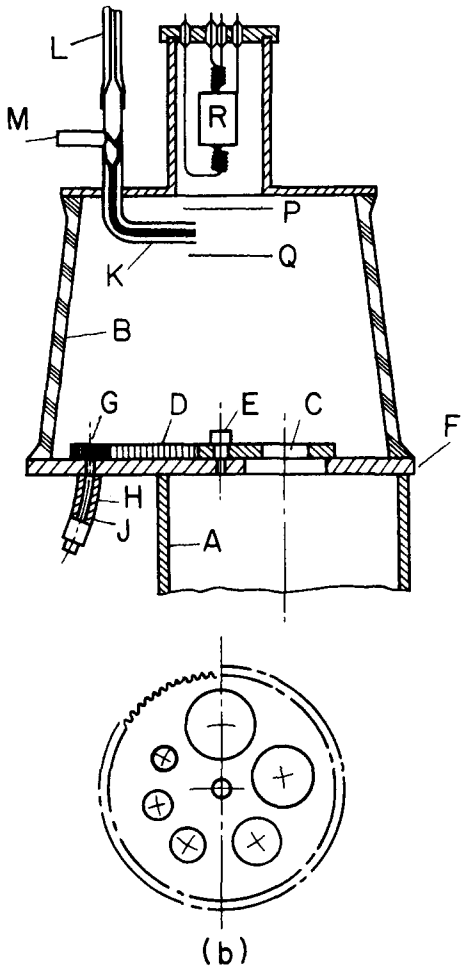


Figure 17. Test dome arrangement suggested by Oatley (1954). Six diaphragms of known conductance on a circular plate formed a division between two chambers on top of the pump.

Venema[138] this definition is not satisfactory for a number of reasons, and he points out that speed should preferably be measured under conditions such that p is much larger than p_0. This restriction could be relaxed if the gas flow established for speed measurement was that of a distinct and specific gas not already present as a residual gas in the vacuum system and the resulting partial pressure p measured with a mass spectrometer. Such an admittedly more complex procedure is advocated particularly for the evaluation of those pumps whose ultimate pressure may depend on their previous history, or where sorption effects come into play, ie at lower pressures.

Dayton[103], found that many methods for determining pump speeds in use up to that time would give completely misleading results. Thus in many cases simple "blank off" plates on top of the pump mouth were used. These contained a gas bleed directed straight at the pump nozzle with the gauge tubulation on the same plane also pointing towards the pump. Such an arrangement was shown to lead to a gross overestimation of the speed. He discussed the effect of the position and direction of the gauge opening relative to the pump mouth, as well as the effect of position and direction of the gas inlet, and methods of metering the gas flow. Some results reported in his investigation are shown in Figure 16.

A method in which the gas flow as well as the pressure may be determined by measuring the ratio of pressures recorded on two gauges was suggested by Blears and Hill[129]. This has the advantage that it can be used with different gases without having to know the response of the gauge. In this method a diaphragm of known conductance U was placed in the throat of the pump and the pump speed expressed as $S=U(p_1/p_2-1)$ where p_2 was the pressure above the diaphragm and p_1 the pressure "at the throat" or below the diaphragm. The method, as outlined, is invalidated by the impossibility of measuring p_1 in a meaningful way. Also, no specification was given of the chamber above the diaphragm, the gas inlet system used, or where p_2 was to be measured.

A somewhat related technique, already mentioned in section (11) above, was suggested by Oatley[97]. A series of six diaphragms on a circular plate as shown in Figure 17(b) were arranged to be brought in turn as a division between two large chambers. The lower chamber was connected to the pump under test and said to be sufficiently wide and short to have negligible effect on the speed of pumping. Pressure was measured by the gauge in the upper chamber. With the gas flow Q held constant the pressure p was plotted against the inverse conductances U of each diaphragm. Since p was proportional to $(1/U+1/S)$, this plot was a straight line which on extrapolation gave the speed of the pump. In effect the speed of the pump was compared with the conductance of an orifice. In principle this is a very sound method, and was used with success by Levenson and Milleron to determine the conductance of complex components. In his critical review on pump speed measurements, Venema[138] quite correctly questioned this method on two grounds:

Firstly, if the pipe connecting the chamber and diaphragms with the pump is not large enough, the assumption that the distribution of the molecules is homogeneous except for regions close to the pump inlet is not fulfilled.

Secondly as discussed above, the presence of the tube, if not sufficiently large would affect the assumption that the inverse effective pump speed, ie p/Q, could be equated to $1/U+1/S$. Both these criticisms can be overcome by making sure that both

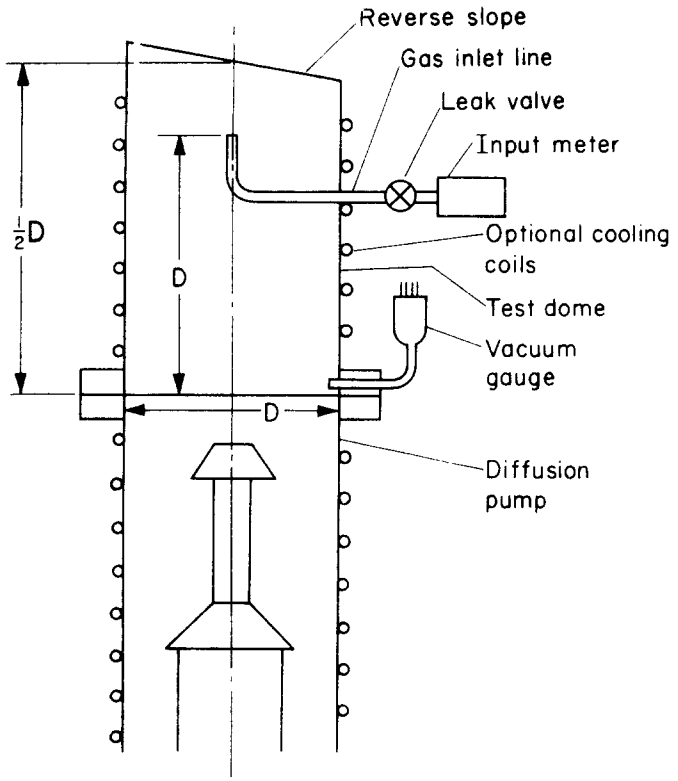


Figure 18. Test dome arrangement as proposed in the American Vacuum Society, 1963, tentative standard No. 4.1 for diffusion pump speed testing.

the chambers on either side of the diaphragm are made sufficiently large.

Dayton and Committee[70], ie the American Vacuum Society, Standards Committee, laid down pump speed measuring procedures with a test dome of the same internal diameter as the pump and a gauge connected near the pump mouth and gas inlet near the top of the dome. This formed the basis of the 1963 AVS recommendations for a tentative standard (No 4.1) (see Figure 18). This standard marked an advance in so far as it was an improvement on the previous situation where both the gauge position and gas inlet were left undefined and as Degras [158] pointed out in his review of test methods, it had the virtue of simplicity and gave consistent results. However, the pumping speed determined by this method did not correspond to the speed as defined above (see section 12), but gave a rather higher speed value due to the beaming effect of the test dome and the position of the gauge along it. These difficulties were in fact discussed in considerable detail by Venema[138, 139] (Figures 19 and 20). He came to the conclusion that with a spherical test dome a reasonably accurate measurement of the number of molecules incident on the wall of the dome was obtained if its diameter was 3× the diameter of the pump mouth, so that the gauge could be connected to a hole in this wall.

The effect of using a test dome corresponding to the Dayton[70] and AVS tentative standard on the assessment of diffusion pumps was clearly demonstrated in the experiments reported by Komiya and Ikeda[140] in which comparative tests were carried out with this as well as a dome of twice the pump diameter and with two gauge positions. They found consider-

Direction of Measurement	Reading of the McLeod gauge $\times 10^{-4}$ mm Hg	
	Throughput 9.2 mm.Hg cm³/sec	Throughput 102 mm.Hg cm³/sec
Tube-end points into the test dome ↑	0·61	5·53
→	0·52	4·80
Tube-end points into the pump ↓	0·44	4·25

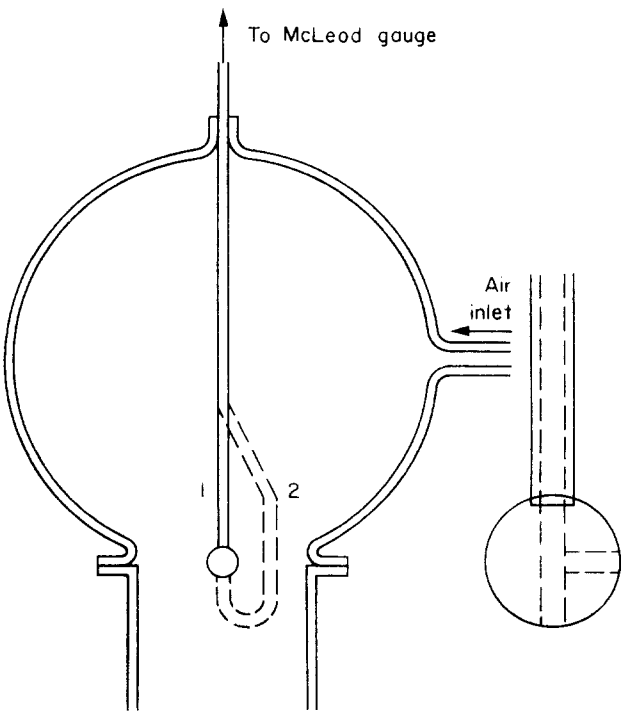


Figure 19. Experimental investigation of gauge orientation in the vicinity of the pump mouth using a spherical test dome (following Venema, 1954).

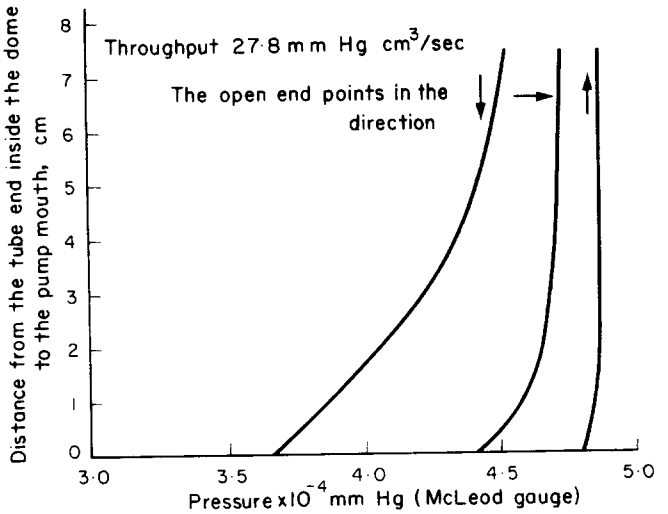


Figure 20. Experimental investigation of the influence of the pressure probe position relative to the pump mouth and the influence of the test dome diameter (following Venema, 1954).

Readings of the McLeod gauge : x10⁻⁴mm. Hg
Diameter of the pump mouth = 5·7cm.

Throughput	Diameter of the test dome(cm)				Mean free path cm.
mm Hg cm³/sec	6	10	15	24	
7·95	1·25	1·34	1·32	1·33	40
27·8	4 38	4·80	4·80	4·74	10
85·6	14·7	14·8	14·7	14·5	3

able differences in measured speed with test dome dimensions, the position and direction of the gauge connection to the dome as well as with variations of the gas inlet position.

In place of the diaphragm of Blears and Hill[129], a calibrated conductance tube has been widely used, particularly in the evaluation of getter films and in the study of the sorption properties of materials, when the gas flow to be measured was relatively small. As already pointed out, this has the advantage that gas flow may be measured as a pressure ratio. The method was adopted for diffusion pump speed evaluation by Landfors and Hablanian[141] though their arrangement would appear to give rise to gas beaming errors. This method has also been widely used for the evaluation of getter-ion pumps as indicated for example by Jepsen[142], Zaphiropoulis and Lloyd[143], and Brothers *et al*[144]. In the first two, the two gauges used to evaluate the gas flow were mounted into the calibrated conductance tube in a manner which would give gas beaming errors. The measured speed would also depend on an estimation of a test dome conductance of fairly complex geometry. In the latter case, three ionisation gauges were used—two mounted along a tube to measure the gas flow, the other near the mouth of the pump, using a test dome of the same diameter as the pump, as shown in Figure 21. Everyone of the three pressure measurements in this arrangement would appear to be subject to gas beaming errors leading to corresponding inaccuracies in the specification of the pump speed.

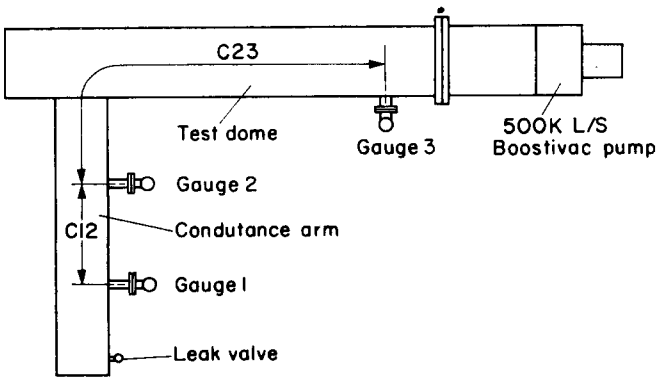


Figure 21. Test dome arrangement of the type used for the speed testing of getter-ion pumps in which gas flow is measured in terms of a tube conductance (following Brothers et al, 1963).

An AVS tentative standard type test dome has also been used in experiments with cryopumping surfaces. Thus Mullen and Jacobs[145] obtained pumping speeds, and hence sticking coefficients, much above the theoretical maximum, so that they had to apply a geometrical factor to take up all the uncertainties in their measurements including the effect of the test dome. It may be noted that the earlier paper by Moore[146], discussing cryopumping geometries as well as the effect of pressure measuring probe direction, gives experimental results obtained in a chamber which would also appear to be rather small and hence subject to gas beaming problems. Stickney and Dayton[147] gave a theoretical analysis of cryopumping speed measurements carried out with a cylindrical (AVS type) test dome and using a tubulated (or nude) gauge situated near the pumping surface. Their analysis was based on Clausing type calculations for the transmission probability of the test dome from which they arrived at a correction factor to be applied to the sticking coefficient. This factor was applied to a test with cryopumping of N_2 at 10°K indicated a corrected sticking coefficient σ very near unity. They also showed that the maximum theoretical pumping speed per unit area (with $\sigma=1$) should be about 11.8 1 sec^{-1} cm^{-2} for N_2 at 23°C corresponding to that measured with a large test dome, whereas for the worst case with a tubulated gauge this speed would be measured as 32.6 1 sec^{-1} cm^{-2} and for a nude gauge as 29.3 1 sec^{-1} cm^{-2}. They conclude that for accurate measurements of sticking coefficients, a large test dome (radius of at least ten times larger than the cryo panel radius) seemed preferable.

15. The simulation of a large test dome

It is seen that in the last few years the desirability of using a large test dome for example, on the lines suggested by Venema was being more generally accepted. There remained, however, two arguments against the immediate adoption of such a proposal.

(1) A practical difficulty arising from the increased size (and pumping speed) of diffusion pumps now generally available. In 1940 pumps of less than 10cm diameter were in general use,

now with pumps of several feet internal diameter, huge test domes of some 10ft diameter and 15ft high would need to be employed. It was therefore of interest to investigate alternative test procedures, based on smaller test domes, but simulating as far as possible results obtained with the large test dome.

(2) A perhaps not very valid but commercial point, was that the speed claimed for any pump would be lower by 20 or more per cent, so giving a commercial disadvantage to any manufacturer adopting the more correct test procedure.

Furthermore this situation can be and, in fact, has been exploited by changes in pump design, in which the top jet* is moved nearer the mouth of the pump to give a disproportionate increase in pump speed. This emphasises the urgent need for clarification together with agreements on national and international standardisation of sound test methods.

In the 1963 Gaede memorial issue of "Vacuum", (commemorating the 50 year anniversary of Gaede's application for a patent on the diffusion pump) Nöller[148] suggested an experimental method for calibrating the test domes used for measuring the speed of diffusion pumps, by which the measured speed, using a dome of diameter equal to the pump inlet could be converted to the "intrinsic speed" (in the sense defined above) that would be obtained with a test dome much larger than the diameter of the pump. His suggestion involved first placing the test dome on a much larger pump (corresponding to an "ideal" pump which would have the orifice speed $\bar{S}_0$ (of an opening the same size as the test dome diameter)). In this way a measured maximum speed value S_0 was obtained. By substituting this value into an expression derived by Nöller making certain simplifying assumptions, a correction could be made to the measured speed S with this test dome. The corrected speed was given by

$$\bar{S}=S/(1+S(1/\bar{S}_0-1/S_0)) \tag{63}$$

this correction is shown in the graph in Figure (22) where $\bar{S}/S$ is shown plotted against S/S_0 for various values of $\bar{S}_0/S_0$.

In a practical test quoted by Nöller, the correction factor amounted to between 18 and 24% assuming a "measured" Ho-coefficient $\bar{S}/S_0$ of .5 or .7 respectively. He showed that the correction factor increasingly deviated from unity, the nearer the pumping speed approached the maximum possible for the pump in question (ie the nearer it approached an "ideal" pump).

Commenting on Nöllers suggestion, Steckelmacher[108] mentioned that the effect of the dome geometry and gauge location could be calculated also from the equations given by Stickney and Dayton[147] in their cryopump investigation, and derived results for an "ideal" pump as shown in Figure (23). This indicated incidentally, that with a tubulated test dome (and an ideal pump), a gauge position could be used such that the measured speed was also the intrinsic speed.

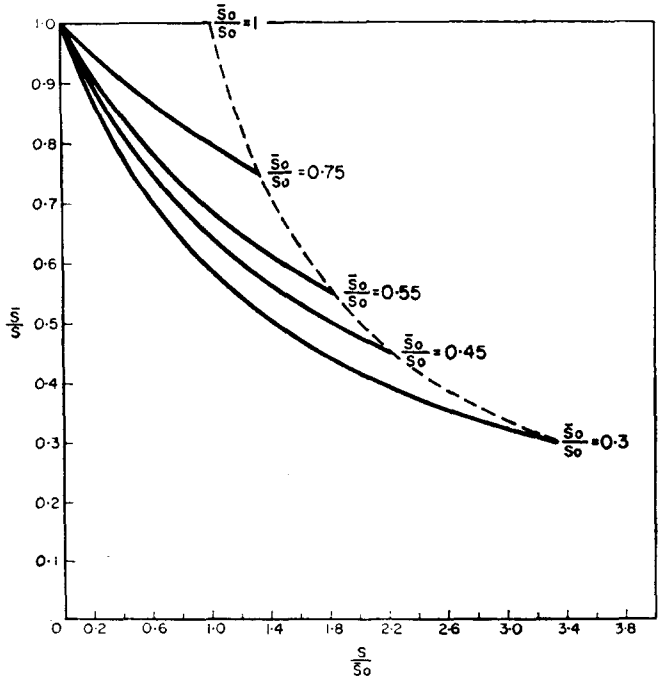


FIG. 3'. Replacing Fig. 3. in Nöller.

Figure 22. Correction of the pumping speed S measured with an imperfect test dome arrangement. Measured speed with dome on ideal pump is S_0 as against the orifice speed $\bar{S}_0$ and corrected speed is given by $\bar{S}$ (following Nöller, 1963, and commented on by Steckelmacher, 1964).

*of a diffusion pump.

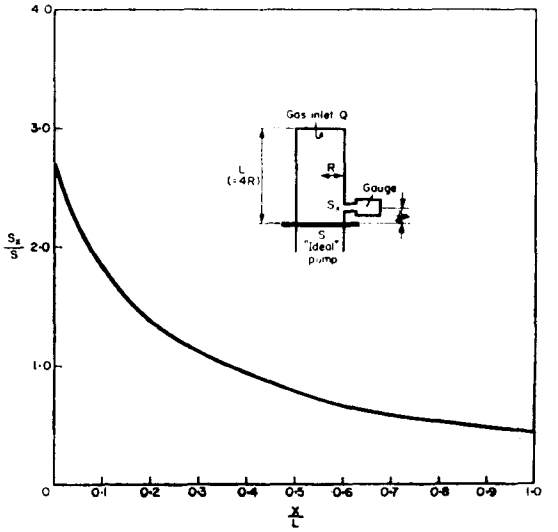


FIG. 2. Gas beaming effect on pump speed measurement as function of gauge position (following Dayton).

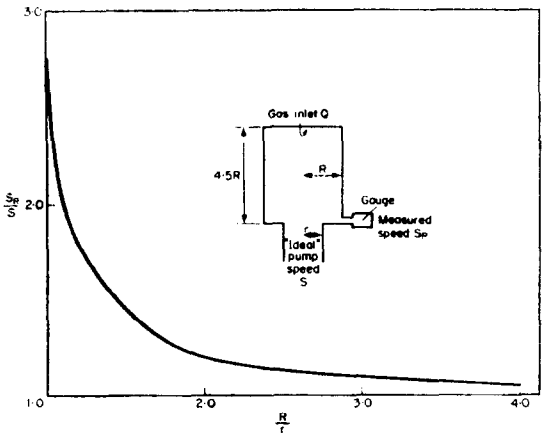


FIG. 1. Gas beaming effect on pump speed measurement as function of test dome diameter (following Dayton).

Figure 23. Effect on measured speed of an "ideal" pump of test dome diameter and gauge position calculated from equations given by Stickney and Dayton, 1963 (following Steckelmacher, 1964).

In a detailed review of the whole question Dayton[106] made some further comments on the measured speed of an ideal pump and in effect elaborated on the earlier procedures of Stickney and Dayton[147] suggesting that pump speeds should be specified as that measured with a test dome of diameter equal to that to the pump inlet and "static" pressure being measured at the plane of the pump inlet (ie corresponding to the Dayton and AVS test dome specification). His argument hinged on an analysis of the net speed to be expected in the interior of a vacuum chamber connected by a pipe of diameter equal to that of the pump inlet to a pump of speed S_t (ie measured speed with tubulated gauge). He suggested that correct results for this net speed was obtained by applying the correction not to the speed but to the conductance of such a tube. In this method the conductance should be taken as:

$$U_t = U_0 W/(1 - J_L W/2) \tag{64}$$

instead of $U = U_0 W$ as given by equation (25) above. J_L is a beaming coefficient for the gauge tube connection.

In a further comment Steckelmacher[109, 110] showed that this suggestion was basically equivalent to the procedure of taking the "intrinsic" speed of the pump and "intrinsic" conductance of the tube as combined by the approximate Oatley method (see section 10) for complex systems. Using the Stickney and Dayton analysis it was also shown that the intrinsic speed should be capable of simulation (with a tubulated test dome by moving the position of the gauge away from the pump mouth) in the more general case of a non-ideal pump. Figure 24 shows the recommended position of the gauge as a function of the effective length of the test dome. It is seen that for $L/R = 2$, y/R is approximately 1. Similar conclusions were arrived at independently by Buhl and Trendelenburg[151] in a paper giving also a number of other suggestions for the simulation of a large test dome.

A second order correction was discussed by Steckelmacher[110] pointing out that the take up of gas by the pump depends not only on pump design but also on the beaming pattern from the tube above the pump into the pump mouth. Hence, the intrinsic speed S as measured with a large test dome may differ from a "simulated" intrinsic speed S_t measured with a tubulated test dome, where we may write

$$S_t = \sigma_t U_0 \tag{65}$$

in place of equation (61) for the intrinsic speed S. In calculations for complex systems involving pumps and tubes, it may be argued that S_t should be used in preference to S. On the other hand in combinations of pumps and systems in which the flow is randomised before entering the pump mouth, the use of S may be more appropriate. No experimental data are as yet available to ascertain if the difference between S and S_t or σ and σ_t is important for actual pumps. Even in the case of apparently simple straightforward systems such as getter surfaces or cryopump panels, there may be a directional effect influencing the take up of gas and hence the sticking coefficient. However, a larger directional effect may well be expected in the case of a diffusion pump both with regard to the pumping action itself and possibly with regard to the geometry of the pump such as the distance of the top jet cap from the mouth of the pump.

The use of statistical Monte Carlo methods can be very useful in calculations on systems involving pumps, test domes and particularly the effect of combined pumps, traps and baffles, as was shown by Levenson, Milleron and Davis[118], and Pinson and Peck[149] (see also section 11 above). Chubb[150] has developed a Monte Carlo programme suitable for any axially symmetric systems of pumping and non-pumping surfaces. He applied this to an evaluation of a cryo-pumping surface using a tubulated test dome, and also to a problem of test dome simulation for pumping speed measurement. In this calculation, the impact density rather than the pressure (or rather molecular density) on the test dome wall was obtained.

In the general case of non-random (ie directional) impacts the reading of a tubulated gauge connected into a wall depends on both the molecular impact density and its directional

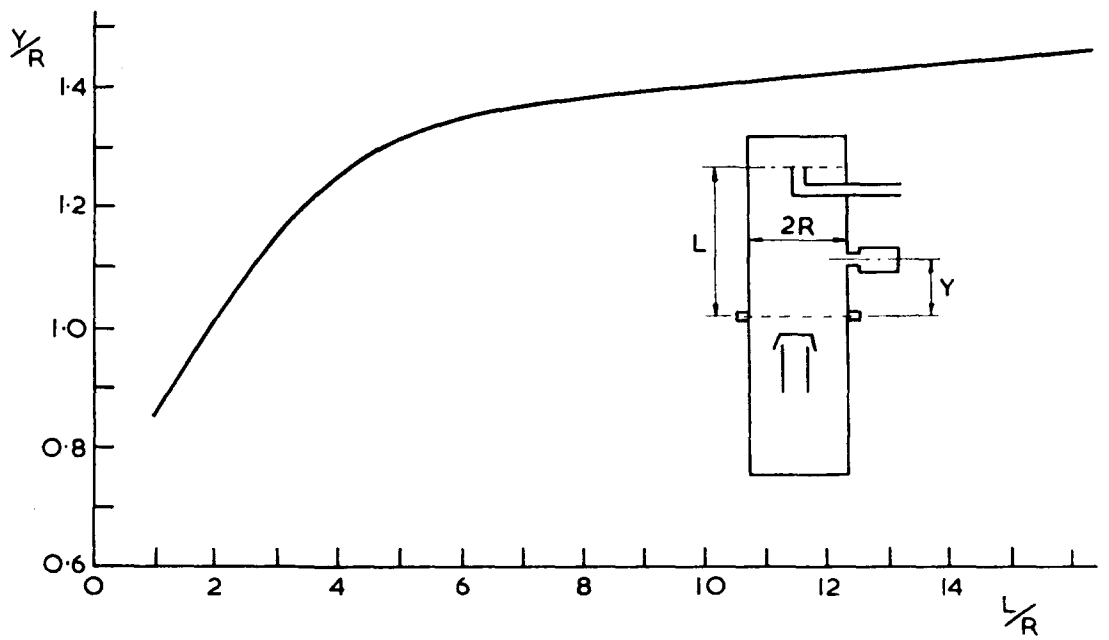


Figure 24. The gauge position y (measured from the pump flange) recommended for speed measurement with a test dome of length L and the same radius R as the pump mouth (following Steckelmacher, 1965).

distribution as was shown by Holland and Priestland[81, 82]. An interesting suggestion was made by Buhl and Trendelenburg[151] to use an impact "transducer" with which impact density could be measured irrespective of its directional distribution. This is shown schematically in Figure 12. It is seen that molecules impinging on the thin orifice of the transducer enter the spherical chamber where they are randomised before entering the gauge. Molecules leaving the spherical chamber in a random manner give rise to the impact density on the transducer orifice of $\frac{1}{4}n\bar{v}$ where n is the molecular density in the transducer. In the absence of net gas flow this is exactly equal to the external impact density onto the orifice no matter which way the molecules arrive. Although as yet only little experimental experience with such a transducer has been reported, it would appear to be a useful technique for molecular flow investigations*. Buhl and Trendelenburg[152] reported on the use of such a transducer to investigate a gas inlet system in order to obtain an improved gas distribution at the top of a tubulated test dome (ie more nearly corresponding to a cosine distribution at the gas inlet reference plane).**

An interesting recent application of the Monte Carlo method was reported by Smith and Lewin[172] giving data for the transmission probability through a cylindrical tube with wall sorption. With a sticking coefficient $s=0$, their results confirm closely the Clausing conductance as expected, but when $s\neq 0$, wall pumping was shown to greatly reduce the gas flow, even for small values of s. They also indicated the backscattering, the total flux retained at the wall and its distribution along the tube as well as the collimation of the emergent beam.

16. Conclusions

The discussion on molecular flow through tubes, components and systems will have shown that there are basically five methods by which such problems can be investigated:

(1) Calculations of the transmission probability as well as the gas flow pattern for simple systems leading to Clausing type integral equations which usually require approximations for their numerical solution, though these can generally be made to be as close as we please.

Slight complexities of the geometry or composite systems may be quite impossible to deal with by such methods. However, solution of, for example, the simple circular tube case has been important in establishing a criterion against which other methods can be tested.

(2) The method of Oatley or the use of the entrance aperture concept to deal with the transmission probability of complex systems has been shown to be deficient in that at best it gives only a rough approximation and in simple cases can be out by 4-6%. It is valuable in giving very valid and good approximations for flow through complex systems though it should be treated with care as it can lead to relatively large errors.

(3) Statistical Monte Carlo methods, in which the possible molecular trajectories are set up, can be applied to fairly complex problems and provided a sufficiently large number of molecular histories are followed (involving the use of a computer), a good accuracy can be achieved. There is at present a limit to the complexity of the geometries to which such computer programmes can be readily set up, but it is expected that this method can still be used to obtain solutions to many problems which have not yet been attempted.

(4) An experimental method in which gas flow through a scale model of the system under investigation is compared with flow through an orifice. This gives data on the conductance of the model and hence its transmission probability.

(5) An experimental method in which the angular distribution and density of gas emitted into a large vacuum chamber by a scale model is monitored by a molecular beam detector (eg an ionisation gauge detector) enabling the gas flow pattern to be recorded. An estimate of the transmission probability can be made by integration of the curve and comparison with the spherical cosine distribution pattern.

To elucidate the problem of pump speed measurement the first four methods have been used with some success. The speed of a pump under molecular flow conditions has been defined in analogy with the flow conductance of components and systems—for which the terms "intrinsic" speed and "intrinsic" conductance have been used. It is suggested that these definitions should be applied to any form of pump. It was shown that they were consistent with the speed of an "ideal" pump in giving just the "aperture" speed of conductance. In the review of the problems associated with an assessment of the theoretical pumping speed of vapour pumps a number of difficulties were indicated. The efficiency of any pumping system was best expressed as the ratio of the "intrinsic" speed to the "ideal" speed of the pump aperture.

A number of shortcomings in earlier proposals for pump speed measurements—particularly effects due to gas beaming and the measurement of pressure are discussed. A practical problem arises in the testing of large pumps which would make the provision of test domes much larger than the pump difficult. In order to measure speeds approximating to the "intrinsic" speed, methods of simulation using a test dome of the same size of the pump are indicated. It is shown that some remaining problems of molecular flow and gas beaming which can affect the accuracy of the simulation technique still need to be resolved, but this should not hold up the present endeavour to get international agreements for the standardisation of test procedures.

Historical notes (see introduction)

As an example of an early anticipation of the kinetic theory of gases the following extract by Robert Hooke published in 1678, may be of interest:

"In the next place for fluid bodies amongst which the greatest instance we have is air, though the same be in some proportion in all other fluid bodies".

"The air then is a body consisting of particles so small as to be almost equal to the particles of the Heterogeneous fluid medium incompassing the earth. It is bounded but on one side, namely, towards the earth, and is indefinitely extended upward being only hindered from flying away that way by its own gravity (the cause of which I shall some other time explain). It consists of the same particles single and separated, of which water and other fluids do, conjoyned and compounded, and being made of particles exceeding small, its motion (to make its balance with the rest of the earthy bodies) is exceeding swift, and its Vibrative Spaces exceeding large, comparative to the Vibrative Spaces of other terrestrial bodies. I suppose that of the Air next the Earth in its natural state may be 8000 times

*Orifice type spherical "impact pressure probes" with some similarities to this transducer have also been used for free-molecule flow studies. See e.g. Hughes[178] and Hughes and de Leeuw[179].

**For a recent review of pressure measurement problems in determining the speed of pumps, see also Buhl[185].

greater than that of Steel, and above a thousand times greater than that of common water, and proportionably I suppose that its motion must be eight thousand times swifter than the former, and above a thousand times swifter than the latter. If therefore a quantity of this body be inclosed by a solid body, and that be so contrived as to compress it into less room, the motion thereof (supposing the heat the same) will continue the same, and consequently the Vibrations and Occursions will be increased in reciprocal proportion, that is, if it be Condensed into half the space the Vibrations and Occursions will be double in number: if into a quarter the Vibrations and Occursions will be quadruple, etc.

"Again, if the containing Vessel be so contrived as to leave it more space, the length of the Vibrations will be proportionably enlarged, and the number of Vibrations and Occursions will be reciprocally diminished, that is, if it be suffered to extend to twice its former dimensions, its Vibrations will be twice as long, and the number of its Vibrations and Occursions will be fewer by half, and consequently its endeavours outward will be also weaker by half.

"These Explanations will serve *mutatis mutandis* for explaining the Spring of any other Body whatsoever".

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