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Approximate equations have been derived for the molecular flow conductance of tubes with sharp bends or elbows, the conductance of two or more tubes in series with different cross sections or with diaphragms which involve junctions across which the gas molecules are scattered back and forth in complicated beaming patterns.

A formula for the transmission probably W_θ of a sharp bend generated by rotating a circular cross section through the angle θ about an axis tangent to the circle, where the bend is located between two large chambers, was derived by Balson[1] using the method of Clausing.[2] While Balson's final equation is acceptable, he gives some incorrect equations for certain of the probability relations used in the derivation. It can be shown that the following simple equation gives a good approximation to Balson's more complicated formula:

$$W_\theta = [(\theta^2+4)^{\frac{1}{2}}-\theta]/[1+2/(\theta^2+4)^{\frac{1}{2}}].$$

The transmission probability for cylindrical tubes of radius R and of various over-all lengths containing a right angle elbow, formed by cutting the tube at an angle of 45° and joining the parts to form a right angle, were computed by Davis[3] using the Monte Carlo technique. His curves show that the effect of the bend depends on the ratio $(A+B)/R$, or L/R, where A is the axial length of one arm, B is the axial length of the other arm, and $L=A+B$ is the total axial length of the elbow as located between two large chambers. We have found it convenient to express this effect as an equivalent length L_e which must be added to the axial length L in computing the transmission probability from various well-known formulas or graphs for the transmission probability of a straight cylindrical tube, where L_e is given by the empirical relation

$$L_e/R = 0.014(L/R)^2 + 0.04(L/R) - 0.38 \quad \text{for} \quad 2 \le L/R \le 6$$

and

$$L_e/R = 2.0 - 9.5(R/L) \quad \text{for} \quad 6 < L/R < \infty.$$

In 1956 the author[4] mentioned that the equations derived by Harries[5] for the transmission probability through two tubes in series were not exact because Harries assumed that the probability of passage for gas flowing from one tube through a second is the same as the probability when the second tube is located between two large chambers; whereas, the probability of passage through the second tube is in fact not the same but is modified by beaming effects across the junction. When these beaming effects are taken into consideration, it can be shown that the most general formula for the transmission probability W_{12} through two tubes in series located between two large chambers, where the first tube has a uniform cross section of area A_1 and a length L_1, while the second tube has a uniform cross section A_2 equal to or less than A_1 and length L_2, is

$$W_{12} = w_{11}f_1 + w_{11}\sum_{n=1}^{\infty} f_{(n+1)} \prod_{i=1}^{n} (1-f_i)(1-b_i)$$

where f_i represents the probability that a molecule which has crossed the entrance of the first tube and succeeded in crossing the junction opening (either directly or after diffuse reflections from the wall of the junction and the first tube), and has been scattered back and forth across the junction opening making a total of i crossings in the forward direction from the first tube to the second, will then pass through the second tube and escape through the exit; b_i represents the probability that a molecule which has crossed the entrance of the first tube and has been scattered back and forth across the junction opening, making i crossings in the backward direction from the second tube into the first tube, will then pass back through the first tube and escape through the entrance into the chamber from which it originated; and w_{11} is the probability that a molecule which has crossed the entrance to the first tube will then cross for the first time in the forward direction through the opening in the junction, either directly or after multiple scattering back and forth from the junction wall and the wall of the first tube. Each of the quantities f_i and b_i have different values for different values of the index i because of successive "beaming effects" across the junction. When the beaming effects are ignored and it is assumed that the molecules always cross the junction with a cosine law distribution pattern, $f_i = w_2$, where w_2 is the ordinary transmission probability for the second tube when located separately between two large chambers, and b_i is given by

$$1/b_i = (A_2/A_1)[(1/w_1) - 1] + 1,$$

where w_1 is the transmission probability for the first tube separately (without any junction diaphragm) when located between two large chambers. Neglecting beaming effects and assuming that the molecules scattered back from the junction wall into the first tube have approximately the same uniform cosine law flux pattern as molecules entering the tube from a large chamber, it can be shown that

$$1/w_{11} = 1/w_1 + A_1/A_2 - 1.$$

The above equation for W_{12} then reduces to the formula of Harries which was also derived by Oatley[6]:

$$1/W_{12} = 1/w_1 + (A_1/A_2 w_2) - 1.$$

Following the procedure used in the article by Stickney and Dayton[7] for estimating the effect of beaming, we can assume as a first approximation that

$$f_1 = w_2(1 + \cot\theta_{12})/2,$$

where θ_{12} is the average angle with respect to the tube axis (not the most probable angle) made by the "average trajectory" for molecules crossing the junction aperture for the first time from the first tube to the second tube,

$$f_i = w_2(1 + \cot\theta_{212})/2 \quad (i > 1),$$

where θ_{212} is the average angle with respect to the tube axis made by the average trajectory for molecules scattered back across the junction aperture from the first tube into the second after having been returned from the second tube to the walls of the first tube, and

$$b_i = \frac{(1 + \cot\theta_{121})/2}{(A_2/A_1)[(1/w_1) - 1] + 1} \quad (i > 0),$$

where θ_{121} is the average angle with respect to the tube axis made by the average trajectory for molecules scattered back across the aperture from the second tube to the first after having crossed the junction from the first tube and colliding with the walls of the second tube. In the special case that the second tube is a continuation of the first tube and both tubes have the same circular cross section ($A_2 = A_1 = \pi R^2$) the average angle θ_{12} may be computed from the formula

$$\theta_{12} = \frac{1}{w_1}\int_0^{\pi/2} 2T_L\theta \cos\theta \sin\theta \, d\theta$$

in which T_L is a function of L_1/R as given by Eqs. (27 a,b,c) in the article by Stickney and Dayton.[7] Similarly, the average back scattering angle θ_{212} may be computed from

$$\theta_{212} = \int_0^{\pi/2} 2(1 - T_L)\theta \cos\theta \sin\theta \, d\theta \Big/ \int_0^{\pi/2} 2(1 - T_L) \cos\theta \sin\theta \, d\theta = \frac{(\pi/4) - w_1\theta_{12}}{1 - w_1},$$

where T_L is the same function of L_1/R. The average back scattering angle from the second tube into the first tube is computed from

$$\theta_{121} = \frac{(\pi/4) - w_2\theta_{21}}{1 - w_2},$$

where

$$\theta_{21} = \frac{1}{w_2}\int_0^{\pi/2} 2T_L\theta \cos\theta \sin\theta \, d\theta$$

in which T_L is now a function of L_2/R.

If we define

$$\beta_1 = \tfrac{1}{2}(1 + \cot\theta_{12}),$$
$$\gamma_1 = \tfrac{1}{2}(1 + \cot\theta_{212}),$$
$$\gamma_2 = \tfrac{1}{2}(1 + \cot\theta_{121}),$$

combining the above equations for the case $A_2 = A_1$ results in the approximate formula

$$W_{12} = w_1 w_2\left(\beta_1 + \frac{\gamma_1(1 - w_2\beta_1)(1 - w_1\gamma_2)}{1 - (1 - w_1\gamma_2)(1 - w_2\gamma_1)}\right)$$

which was obtained by Füstöss[8] using somewhat similar reasoning. From the fact that interchanging the sequence of tubes should not change the transmission probability when $A_1 = A_2$, Füstöss deduces from this equation that

$$w_1 - \frac{w_1\beta_1 - 1}{\gamma_1} = w_2 - \frac{w_2\beta_2 - 1}{\gamma_2} = k,$$

where k must be a constant. Füstöss arbitrarily chooses k to be zero. From our definitions of β_1, γ_1, and γ_2 above we find

$$w_1 \frac{w_1[(1 + \cot\theta_{12})/2] - 1}{(1 + \cot\theta_{212})/2} = k.$$

Values of k computed from this equation for various values of L_1/R are nearly constant and equal to 1.0 to within 2%, being exactly equal to 1.000 for $L_1/R = 0$

since then $\theta_{12}=\frac{1}{4}\pi$ and $\theta_{212}=\frac{1}{4}\pi$. Thus, $k=1$ would have been a better choice for the value of k. By choosing $k=0$ Füstöss obtains an equation for W_{12} in terms of w_1, w_2, β_1, and β_2. Applying this equation to the case of a compound tube of two equal sections he computes values of $\beta=\beta_1=\beta_2$ for various values of L/R using Clausing's values for W_{12}, w_1, and w_2. In this way he obtains values of β ranging from 0.9502 for $L/R=0.1$ to large negative values, such as -17.3991 for $L/R=50$. By using these physically meaningless values of β he shows that values of W_{12} agreeing very closely with the Clausing values can be computed from his equation by using the Clausing values of w_1 and w_2 for various combinations of two tubes with differing L/R ratios.

By setting $k=1$ we have obtained the equation

$$W_{12}=w_1w_2\left(\beta_1+\frac{(1-w_1\beta_1)(1-w_2\beta_1)(1-w_1-w_2+w_1w_2\beta_2)}{(1-w_1)(1-w_2)-(1-w_1-w_2+w_1w_2\beta_1)(1-w_1-w_2+w_1w_2\beta_2)}\right).$$

Following the procedure used by Füstöss we determine $\beta=\beta_1=\beta_2$ for the case $L_1/R=L_2/R$ and $w_1=w_2=w$. Then the following equation for computing β is obtained:

$$\beta=\frac{-b-\{b^2+w(1-W_{12})[2b+w(1-W_{12})]\}^{\frac{1}{2}}}{w^2(1-W_{12})},$$

where

$$b=2wW_{12}-w^2-W_{12}.$$

Using values of β_1 and β_2 computed in this way we also find very close agreement between the Clausing values and the values of W_{12} as computed from the above equation for various combinations of L_1/R and L_2/R.

When A_1 is not equal to A_2 we obtain

$$W_{12}=w_{11}w_2\left(\beta_1+\frac{\gamma_1(1-w_2\beta_1)[1-(A_1/A_2)w_{11}\gamma_2]}{1-(1-w_2\gamma_1)[1-(A_1/A_2)w_{11}\gamma_2]}\right),$$

where w_{11} is given by

$$1/w_{11}=1/w_1+(A_1/A_2)-1;$$

and assuming $k=1$,

$$\gamma_1=(1-w_1\beta_1)/(1-w_1)$$

and

$$\gamma_2=(1-w_2\beta_2)/(1-w_2).$$

These equations are useful for computing the transmission probability of a composite pipe line using tables or curves for w and β as functions of L/R. A more complete analysis of the gas flow in composite pipelines will be submitted for publication later.

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Problem of the Additivity of Transmission Probabilities for Composite Systems

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An investigation of the applicability of the concept of transmission probability is given. A method to calculate the transmission probabilities of composite systems is shown. The conditions of the applicability of Oatley's additive formula[1] and the end-correction introduced by Steckelmacher are examined. The efficiency of the applied method is compared with published Monte Carlo data and with our own ones as well. Interpolation formulas[2] are presented by which correct results can be derived in such ranges where the efficiency of any method is not satisfactory. The expectable error of the formulas giving transmission probabilities for composite systems is evaluated, as well as the conditions of their applicability and the possibilities of the choice of adequate formulas for different ranges.

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