Notes 15.0: Mathematical functions

COMP9021 Principles of Programming

School of Computer Science and Engineering
The University of New South Wales

2013 session 1

The math.h header file

Except for the functions `abs()`, `labs()` and `llabs()`, defined in `stdlib.h`, all functions described in this set of notes are defined in `math.h`. They all come in three variants, two of which add `f` or `l` to the base name to indicate that they take arguments of type `float` or `long double` rather than of type `double`, respectively, or arguments of type pointer to `float` or pointer to `long double` rather than of type pointer to `double`, respectively (they might also take arguments of another fixed type), and when the base function returns a value of type `double`, to indicate that they return values of type `float` or `long double`, respectively.

Some installations require to pass `-lm` as argument to the linker when compiling programs that make use of functions or macros defined in `math.h`.

Constants (1)

The following macros are provided for constants of type `double`.

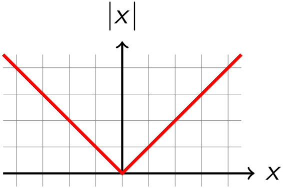
e	`M_E` (2.718282)
$\log_2(e)$	`M_LOG2E` (1.442695)
$\log_{10}(e)$	`M_LOG10E` (0.434294)
$\ln(2)$	`M_LN2` (0.693147)
$\ln(10)$	`M_LN10` (2.302585)

Constants (2)

π	`M_PI` (3.141593)
$\pi/2$	`M_PI_2` (1.570796)
$\pi/4$	`M_PI_4` (0.785398)
$1/\pi$	`M_1_PI` (0.318310)
$2/\pi$	`M_2_PI` (2.718282)
$2/\sqrt{\pi}$	`M_2_SQRTPI` (1.128379)
$\sqrt{2}$	`M_SQRT2` (1.414214)
$\sqrt{1/2}$	`M_SQRT1_2` (0.707107)

Absolute value

```
double fabs(double);
float fabsf(float);
long double fabsl(long double);
```

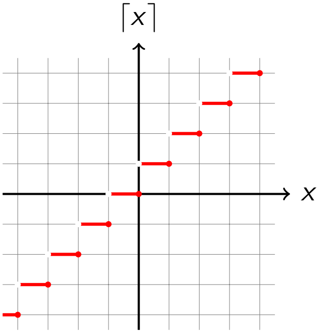


Defined in `stdlib.h`:

```
int abs(int);
long labs(long);
long long llabs(long long);
```

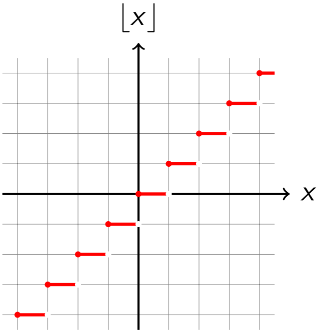
Ceiling

```
double ceil(double);
float ceilf(float);
long double ceill(long double);
```



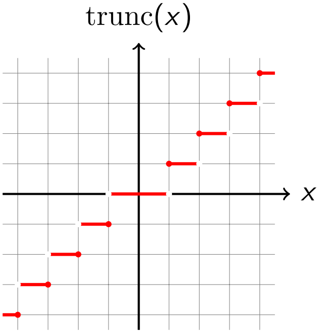
Floor

```
double floor(double);
float floorf(float);
long double floorl(long double);
```



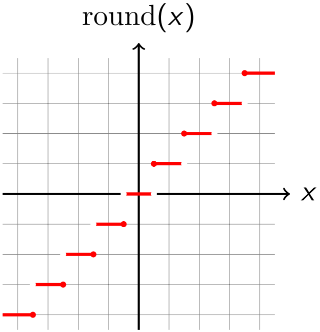
Truncation towards 0

```
double trunc(double);
float truncf(float);
long double truncl(long double);
```



Rounding towards integer larger in absolute value (1)

```
double round(double);
float roundf(float);
long double roundl(long double);
```



Rounding towards integer larger in absolute value (2)

```
long lround(double);
long lroundf(float);
long lroundl(long double);

long long llround(double);
long long llroundf(float);
long long llroundl(long double);
```

Rounding towards nearest integer (1)

```
double nearbyint(double);
float nearbyintf(float);
long double nearbyintl(long double);
```

The header file `fenv.h` defines

- the macros `FE_DOWNWARD`, `FE_UPWARD`, `FE_TONEAREST` and `FE_TOWARDZERO` that correspond to four rounding directions,
- the function `fegetround()` that returns the current rounding direction, and
- the function `fesetround()` that sets the rounding direction.

`nearbyint()` and associated functions compute the nearest integer according to the current value of the rounding direction. The program rounding.c illustrates.

Rounding towards nearest integer (2)

```
double rint(double);
float rintf(float);
long double rintl(long double);

long lrint(double);
long lrintf(float);
long lrintl(long double);

long long llrint(double);
long long llrintf(float);
long long llrintl(long double);
```

`rint()` and associated functions are like `nearbyint()`, but raise an exception (`FE_INEXAC`) when their argument is not an integer. The `fenv.h` header file defines functions to show the value of exception status flags determined by which exceptions have been raised, and to clear exception status flags. This is illustrated in fe_inexact.c.

Floating-point remainder

```
double fmod(double, double);
float fmodf(float, float);
long double fmodl(long double, long double);

double remainder(double, double);
float remainderf(float, float);
long double remainderl(long double, long double);
```

`fmod()` and associated functions, given `x` and `y` as arguments, return `x - trunc(x/y)`, which always has the same sign as `x`.

`remainder()` and associated functions, given `x` and `y` as arguments, return `x - nearbyint(x/y)` with `FE_TONEAREST` as rounding direction, which does not always has the same sign as `x`. (For all integers `n`, `nearbyint(2*n + 0.5)` is equal to `2*n` and `nearbyint(2*n + 1.5)` to `2*n + 2`.) For instance, `remainder(9, 2)` evaluates to `1` while `remainder(11, 2)` evaluates to `-1`.

Decomposition of floating-point numbers (1)

```
double frexp(double, int *);
float frexpf(float, int *);
long double frexpl(long double, int *);
```

`frexp()` and associated functions,

- given `0` and `&n` as arguments, return `0` and store `0` at location `&n`;
- given nonzero `x` and `&n` as arguments, compute the unique f and n such that $0.5 \leq |f| < 1$ and $f \times 2^n$ is equal to `x`, return f and store n at location `&n`.

For instance, `frexp(2, &n)` returns `0.5` and stores `2` at location `&n`, `frexp(3, &n)` returns `0.75` and stores `2` at location `&n`, and `frexp(4, &n)` returns `0.5` and stores `3` at location `&n`.

Decomposition of floating-point numbers (2)

```
double ldexp(double, int);
float ldexpf(float, int);
long double ldexpl(long double, int);
```

`ldexp()` and associated functions are the inverse of `frexp()` and associated functions, in that given a real f and an integer n as arguments, they return $f \times 2^n$.

Decomposition of floating-point numbers (3)

```
double modf(double, double *);
float modff(float, float *);
long double modfl(long double, long double *);
```

`modf()` and associated functions, given `x` and `&n` as arguments, compute the unique f and n such that $|f| < 1$, f and n have the same sign as `x` and $f + n$ is equal to `x`, return f and store n at location `&n`. For instance, `modf(2.6, &n)` returns `0.6` and stores `2` at location `&n` and `modf(-2.6, &n)` returns `-0.6` and stores `-2` at location `&n`.

Scaling of floating-point numbers

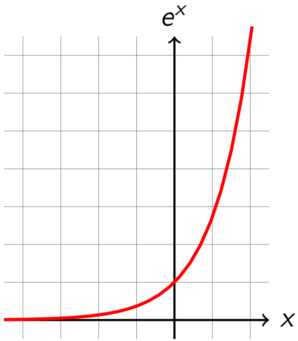
```
double scalbn(double, int);
float scalbnf(float, int);
long double scalbnl(long double, int);

double scalbln(double, long);
float scalblnf(float, long);
long double scalblnl(long double, long);
```

`scalbn()` and associated functions, given `x` and `n` as arguments, return the product of `x` with `FLT_RADIX` raised to the power `n`. The macro `FLT_RADIX` is defined in the header file `float.h` and typically has the value `2`.

Calling `scalbn(x, n)` is meant to be more efficient than executing `x * pow(FLT_RADIX, n)`.

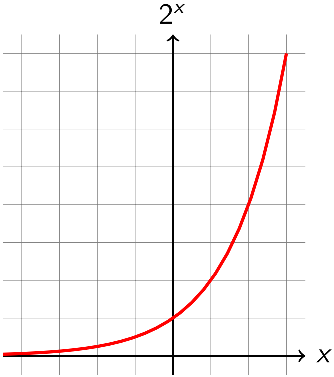
Exponential functions (1)

```
double exp(double x);
float expf(float);
long double expl(long double);
```



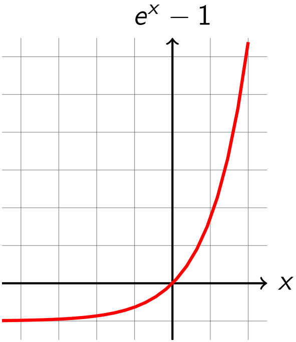
Exponential functions (2)

```
double exp2(double);
float exp2f(float);
long double exp2l(long double);
```



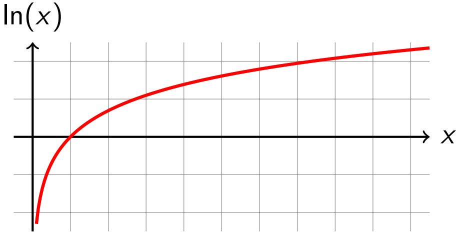
Exponential functions (3)

```
double expm1(double);
float expm1f(float);
long double expm1l(long double);
```



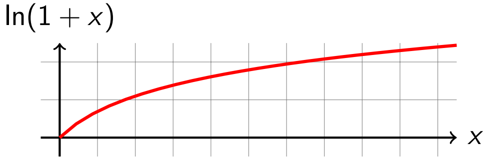
Logarithmic functions (1)

```
double log(double);
float logf(float);
long double logl(long double);
```



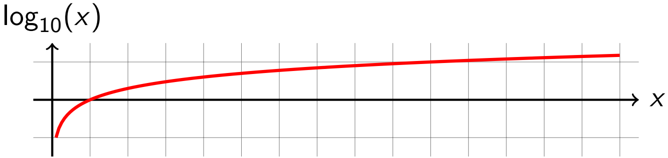
Logarithmic functions (2)

```
double log1p(double);
float log1pf(float);
long double log1pl(long double);
```



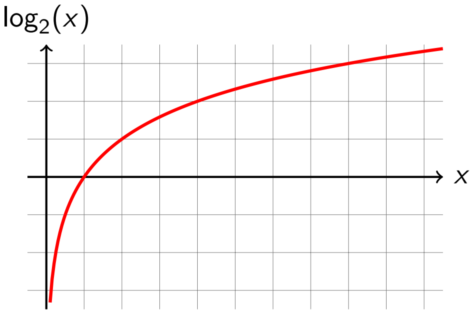
Logarithmic functions (3)

```
double log10(double);
float log10f(float);
long double log10l(long double);
```



Logarithmic functions (4)

```
double log2(double);
float log2f(float);
long double log2l(long double);
```



Logarithmic functions (5)

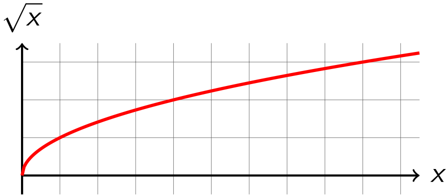
```
double logb(double);
float logbf(float);
long double logbl(long double);
int ilogb(double);
int ilogbf(float);
int ilogbl(long double);
```

`logb()`, `ilogb()` and associated functions, given nonzero `x` as argument, return the exponent of the representation of `x` as a floating-point number with `FLT_RADIX` used for the radix. The macro `FLT_RADIX` is defined in the header file `float.h` and typically has the value `2`, so for instance `logb(0.5)` and `ilogb(0.5)` return `-1`, and `logb(20)`, `ilogb(20)`, `logb(25)` and `ilogb(25)` return `4`.

When given `0` as argument, `logb()` and related functions return `-inf` while `ilogb()` and related functions return the value of the macro `FP_ILOGB0` (which on some machines is equal to -2^{32}).

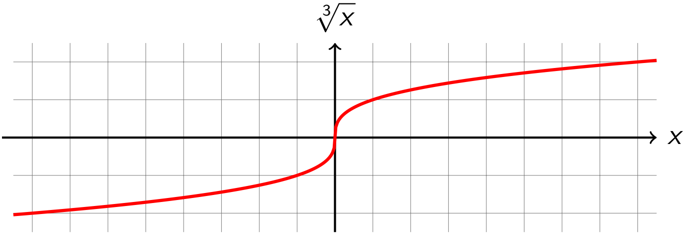
Square root

```
double sqrt(double);
float sqrtf(float);
long double sqrt2l(long double);
```



Cube root

```
double cbrt(double);
float cbrtf(float);
long double cbrt2l(long double);
```



Power and hypotenuse functions

```
double pow(double, double);
float powf(float, float);
long double powl(long double, long double);

double hypot(double, double);
float hypotf(float, float);
long double hypotl(long double, long double);
```

`pow()` and associated functions, given `x` and `y` as arguments, return the value of `x` raised to the power `y`. If `x` is equal to `0` then `y` has to be strictly positive; if `x` is negative then `y` has to be an integer.

`hypot()` and associated functions, given `x` and `y` as arguments, return the value of `sqrt(x*x + y*y)` without causing the undue overflow that the latter expression could cause.

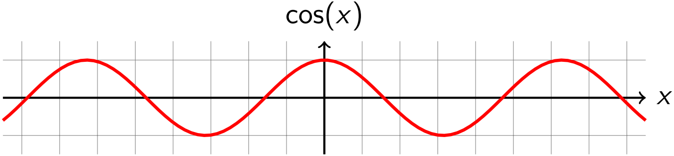
Multiply and add

```
double fma(double, double, double);
float fmaf(float, float, float);
long double fmal(long double,
                 long double,
                 long double);
```

`fma()` and associated functions, given `x`, `y` and `z` as arguments, return the value of `x * y + z`, with rounding to the precision of the return type done only to the final result, not to intermediate computations.

Cosine

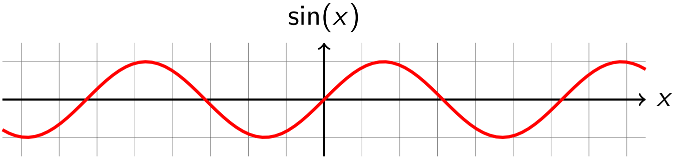
```
double cos(double);
float cosf(float);
long double cosl(long double);
```



The argument is taken in radian, and will yield a result of little significance if too large.

Sine

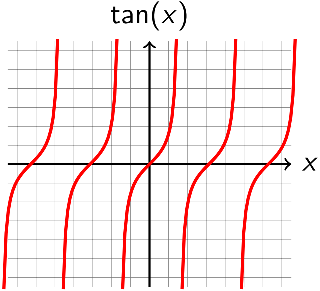
```
double sin(double);
float sinf(float);
long double sinl(long double);
```



The argument is taken in radian, and will yield a result of little significance if too large.

Tangent

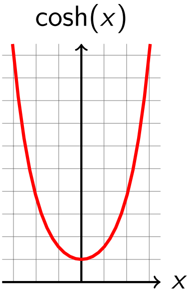
```
double tan(double);
float tanf(float);
long double tanl(long double);
```



The argument is taken in radian, and will yield a result of little significance if too large.

Hyperbolic cosine

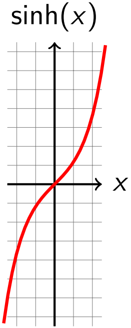
```
double cosh(double);
float coshf(float);
long double coshl(long double);
```



The argument is taken in radian.

Hyperbolic sine

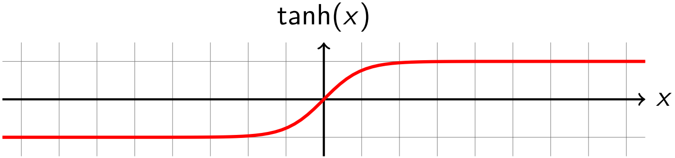
```
double sinh(double);
float sinhf(float);
long double sinhl(long double);
```



The argument is taken in radian.

Hyperbolic tangent

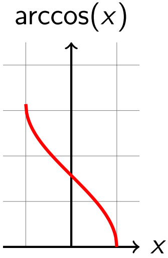
```
double tanh(double);
float tanhf(float);
long double tanhl(long double);
```



The argument is taken in radian.

Arc cosine

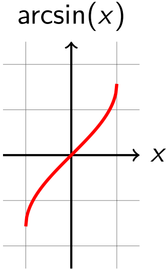
```
double acos(double);
float acosf(float);
long double acosl(long double);
```



The returned value is in radian.

Arc sine

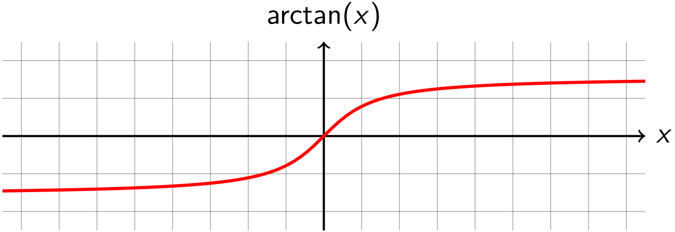
```
double asin(double);
float asinf(float);
long double asinl(long double);
```



The returned value is in radian.

Arc tangent

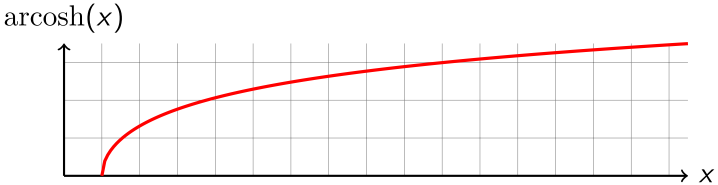
```
double atan(double);
float atanf(float);
long double atanl(long double);
```



The returned value is in radian.

Area hyperbolic cosine

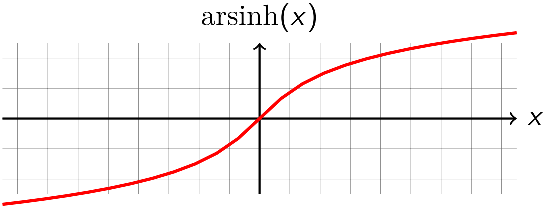
```
double acosh(double);
float acoshf(float);
long double acoshl(long double);
```



The returned value is in radian.

Area hyperbolic sine

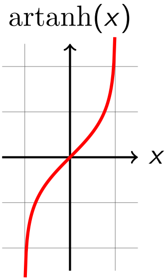
```
double asinh(double);
float asinhf(float);
long double asinhl(long double);
```



The returned value is in radian.

Area hyperbolic tangent

```
double atanh(double);
float atanhf(float);
long double atanhl(long double);
```



The returned value is in radian.

Difference, max and min

```
double fdim(double, double);
float fdimf(float, float);
long double fdiml(long double, long double);
double fmax(double, double);
float fmaxf(float, float);
long double fmaxl(long double, long double);
double fmin(double, double);
float fmif(float, float);
long double fminl(long double, long double);
```

`fdim()` and associated functions, given `x` and `y` as arguments, return `x - y` if `x` is greater than `y`, and `0` otherwise.

`fmax()` and associated functions, given `x` and `y` as arguments, return the maximum of `x` and `y`.

`fmin()` and associated functions, given `x` and `y` as arguments, return the minimum of `x` and `y`.