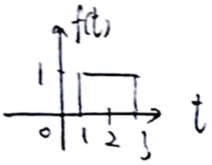
1. (1) $\int_{-\infty}^{+\infty}\delta(t)e^{-t}dt = \int_{-\infty}^{+\infty}\delta(t) = 1$

(2) $\int_{-\infty}^{t}\delta(\tau)e^{-\tau}d\tau = \int_{-\infty}^{t}\delta(\tau)d\tau = u(t)$

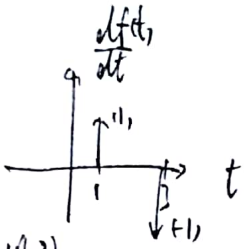
(3) $\delta[n]\cos w_0 n = \delta[n]\cos w_0\cdot 0 = \delta[n]$

(4) $\delta[n] * \cos w_0 n = \cos w_0 n$

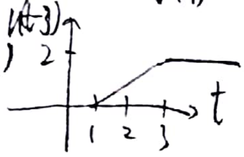
2. 

$f(t) = u(t-1) - u(t-3)$

$\frac{df(t)}{dt} = \delta(t-1) - \delta(t-3)$

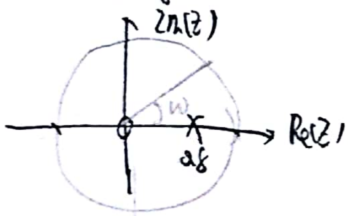


$\int_{-\infty}^{t} f(\tau)d\tau = (t-1)u(t-1) - (t-3)u(t-3)$



3. $x[n] = (0.8)^n u[n]$

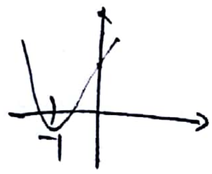
$X(z) = \frac{z}{z-0.8}$, $|z| > 0.8$ (这个就是记忆)　　$X(j\omega) = \frac{j\omega}{j\omega - 0.8}$



4. $A = \begin{bmatrix} 2 & -3 \\ a & -4 \end{bmatrix}$　　$\cancel{|A^*| = A^{-1} = \frac{\begin{bmatrix} -4 & 3 \\ -a & 2 \end{bmatrix}}{-8+3a}}$

$B = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$　　$(sI - A)^{-1} = \begin{pmatrix} s-2 & 3 \\ -a & s+4 \end{pmatrix}^{-1} = \frac{\begin{pmatrix} s+4 & -3 \\ a & s-2 \end{pmatrix}}{(s-2)(s+4)+3a}$

分母零极点全在 S 左半平面，$(s-2)(s+4)+3a = s^2+2s+3a-8$



对称轴：$s = -1$

只要，$\begin{cases} 3a-8 \geq 0 \\ \Delta \geq 0 \text{（这个不确定，我是觉得要有根点的）} \end{cases}$

$\Rightarrow \frac{8}{3} \leq a \leq 3$

5. (1) $\mathcal{F}[t f(-3t)] = +j\frac{d F(\frac{j\Omega}{3})}{3\, d\Omega}$

(2) $\mathcal{F}[e^{-2t}u(t) * \frac{df(t)}{dt}] = \mathcal{F}[e^{-2t}\delta(t) * f(t)] = \mathcal{F}[e^{-2t}f(t)] = F[j(\Omega+2)]$

6. $y(t)=T[x(t)]=x(t)\cos(3t+\frac{\pi}{4}+5)$

设 $y_1(t)=T[x_1(t)]$, $y_2(t)=T[x_2(t)]$

$$T[\alpha x_1(t)+\beta x_2(t)]=[\alpha x_1(t)+\beta x_2(t)]\cos(3t+\frac{\pi}{4}+5)$$
$$=\alpha x_1(t)\cos(3t+\frac{\pi}{4}+5)+\beta x_2(t)\cos(3t+\frac{\pi}{4}+5)$$
$$=\alpha y_1(t)+\beta y_2(t)$$

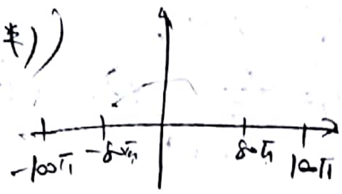
系统线性.

$$T[x_1(t+t_0)]=x_1(t+t_0)\cos(3t+t_0+\frac{\pi}{4}+5)\neq$$
$$y(t+t_0)=x_1(t+t_0)\cos(3t+3t_0+\frac{\pi}{4}+5)$$

系统时变

7. 记 $f(t)=Sa(50\pi t)+Sa^2(50\pi t)$ $\quad F[f(t)]=\frac{1}{50}[u(\omega+50\pi)-u(\omega-50\pi)]+\frac{1}{2500}[u(\omega+50\pi)-u(\omega-50\pi)]*[u(\omega+50\pi)-u(\omega-50\pi)]$

(重点是判断~~[illegible]~~不为0值的函数的存在区间([illegible]))



对 $F[f(t)]$, $\Omega_m=100\pi$

$\Omega_{smin}=2\Omega_m=200\pi$

$T_{smin}=\frac{2\pi}{\Omega_{smin}}=\frac{1}{100}\,s$

$f_{smin}=\frac{1}{T_{smin}}=100\,Hz$

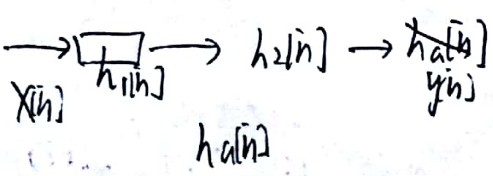
8. $X(s)=\frac{2s^3+s^2+2s+1}{s^2+3s+2}=2s-5+\frac{13s+11}{s^2+3s+2}$ $\quad s^2+3s+2=(s+1)(s+2)$

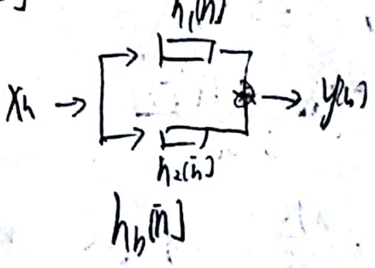
初值 ~~$\lim_{s\to\infty} f_0$~~ $\lim_{s\to\infty}\frac{s(13s+11)}{s^2+3s+2}=13$

终值 $\lim_{s\to 0}\frac{s(13s+11)}{s^2+3s+2}=0$

二、 $h_1[n]=\delta[n]+2\delta[n-1]+3\delta[n-2]+2\delta[n-3]+\delta(n-4)$

$h_2[n]=\delta[n]+2\delta[n-1]-2\delta[n-2]-\delta[n-3]$





$h_{(a)}[n] = h_1[n] * h_2[n] = (\qquad) * (\qquad)$

$= \delta[n] + 2\delta[n-1] - 2\delta[n-2] - \delta[n-3]$

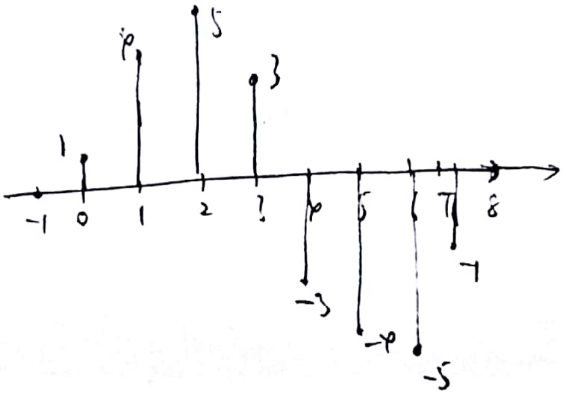
$+2(\delta(n-1) + 2\delta(n-2) - 2\delta(n-3) - \delta(n-4))$

$+3(\delta(n-2) + 2\delta(n-3) - 2\delta[n-4] - \delta[n-5])$

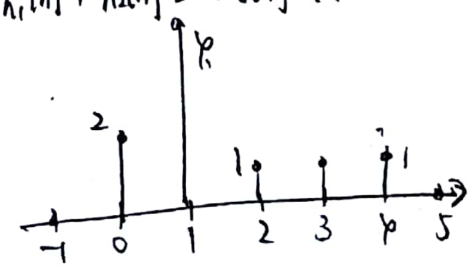
$+2(\delta(n-3) + 2\delta[n-4] - 2\delta[n-5] - \delta[n-6])$

$+ (\delta[n-4] + 2\delta[n-5] - 2\delta[n-6] - \delta[n-7]$

$= \delta[n] + 4\delta[n-1] + 5\delta[n-2] + 3\delta[n-3] - 3\delta[n-4] - 5\delta[n-5] - 4\delta[n-6] - \delta[n-7]$



$h_b[n] = h_1[n] + h_2[n] = 2\delta[n] + 4\delta[n-1] + \delta[n-2] + \delta[n-3] + \delta[n-4]$



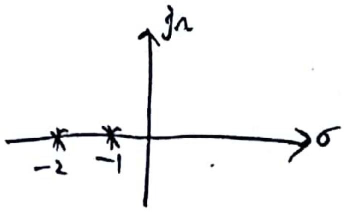
2. 当 $x(t) = u(t)$ 时, $y_{zs}(t) = (1 - 2e^{-t} + e^{-2t})u(t)$

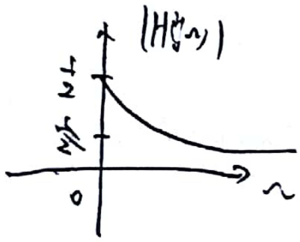
(1) $g(t) = (1 - 2e^{-t} + e^{-2t})u(t)$

$h(t) = g'(t) = \delta(t) + 2e^{-t}u(t) - 2\delta(t) - 2e^{-2t}u(t) + \delta(t)$

$= (2e^{-t} - 2e^{-2t})u(t)$

(2) $H(s) = \frac{2}{s+1} - \frac{2}{s+2} = \frac{2}{(s+1)(s+2)}$

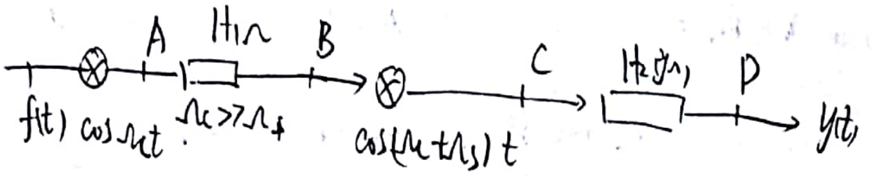




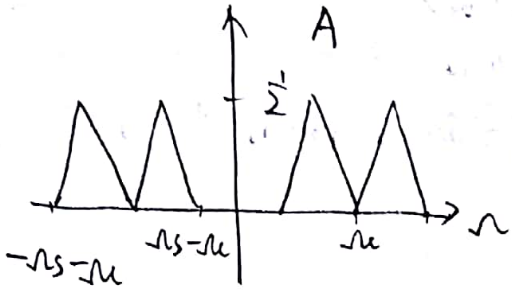
$H(j\omega) = \frac{2}{(j\omega+1)(j\omega+2)}$

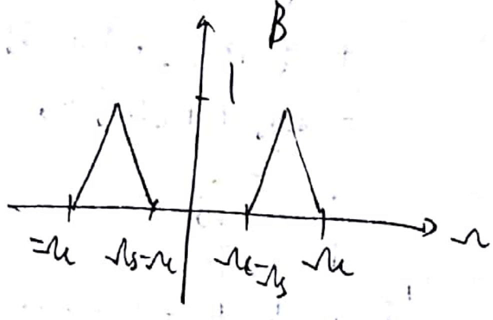
(3) $\frac{d^2y}{dt^2} + 3\frac{dy}{dt} + 2y(t) = 2x(t)$

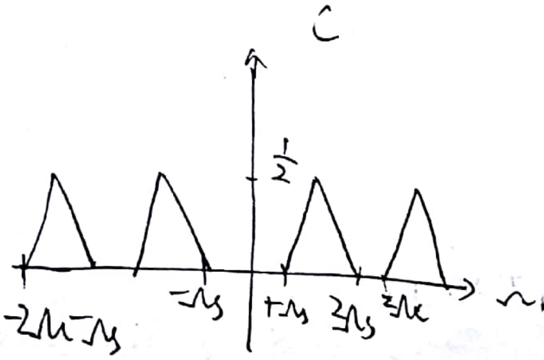
3.

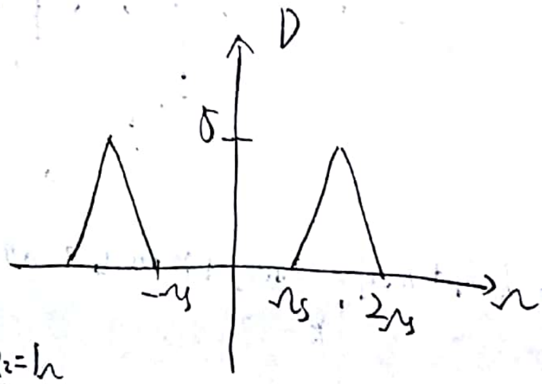


$H_1 = \begin{cases} 2, & |\Omega| < \Omega_c \end{cases}$ $\quad H_2 \begin{cases} 1, & |\Omega| \geq \Omega_c \end{cases}$



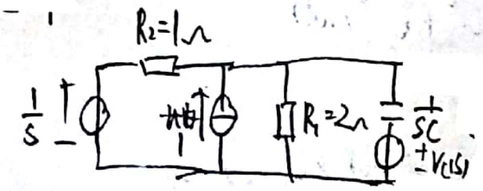






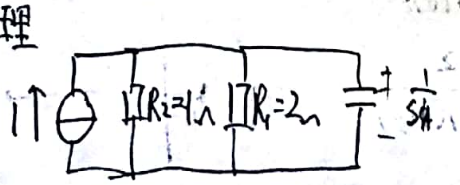
4.

(1) 作 s域模型.



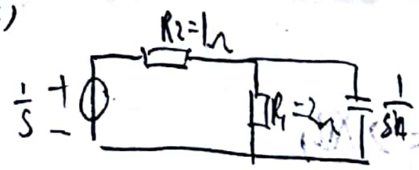
(2) 1. 零状态响应用叠加定理

① 只有电流源作用



$H_1(s) = 1, \quad y_{zs_1}(t) = \delta(t)$

② 只有电压源作用

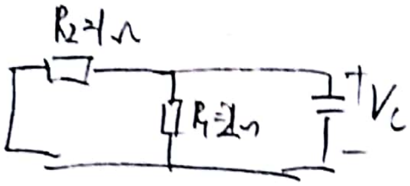


$$H_2(s) = \frac{\frac{1}{sc}}{1+\frac{\frac{2}{sc}}{2+\frac{1}{sc}}} =$$

$$y_{zs_2} = h_2(t) * u(t) = \frac{4}{9}u(t) + \frac{1}{3}t\,u(t) + -\frac{4}{9}u(t)e^{-\frac{3}{2}t}$$

得 $y_{zs} = \delta(t) + (\frac{4}{9} + \frac{1}{3}t + -\frac{4}{9}e^{-\frac{3}{2}t})u(t)$

2.零输入响应用三要素法(电路)



$V_c(0^+) = V_c(0^-) = 2V$

$V_c(\infty) = 0V$

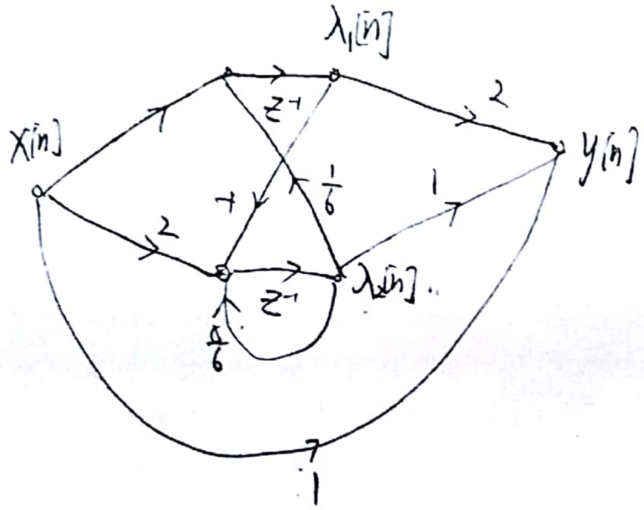
$R_{eq} = \frac{2}{3}\Omega$

$\tau = \frac{1}{R_{eq}C} = \frac{2}{3}$

$y_{zi} = V_c(\infty) + (V_c(0^-) - V_c(\infty))e^{-\frac{t}{\tau}}$

$= 2e^{-\frac{3}{2}t}$

5. (1) $A = \begin{bmatrix} 0 & \frac{1}{6} \\ -1 & \frac{5}{6} \end{bmatrix}$ $B = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$ $C = \begin{bmatrix} 2 & 1 \end{bmatrix}$ $D = 1$



(2) $H(z) = C(zI-A)^{-1}B + D$

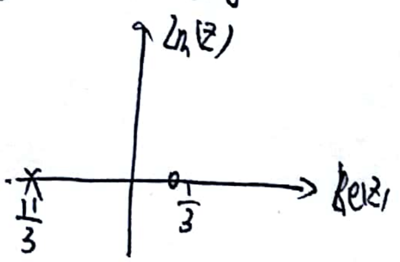
$= \begin{bmatrix} 2 & 1 \end{bmatrix} \begin{bmatrix} z & -\frac{1}{6} \\ 1 & z-\frac{5}{6} \end{bmatrix}^{-1} \begin{bmatrix} 1 \\ 2 \end{bmatrix} + 1$

$= \begin{bmatrix} 2 & 1 \end{bmatrix} \frac{1}{z(z-\frac{5}{6})+\frac{1}{6}} \cdot \begin{bmatrix} z-\frac{5}{6} & \frac{1}{6} \\ -1 & z \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix} + 1$

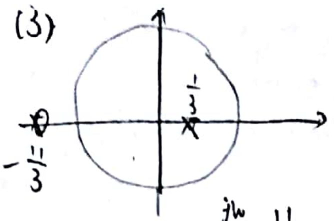
$= \frac{1}{(z-\frac{1}{2})(z-\frac{1}{3})} \cdot \begin{bmatrix} 2z-\frac{8}{3} & \frac{1}{3}+z \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix} + 1$

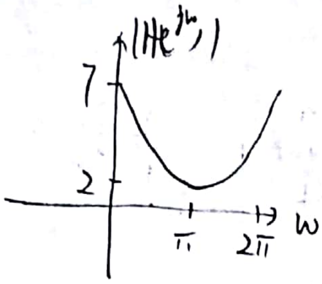
$= \frac{4(z-\frac{1}{2})}{(z-\frac{1}{2})(z-\frac{1}{3})} + 1$

$= \frac{z+\frac{11}{3}}{z-\frac{1}{3}}$



(3)





低通滤波特性

$$H(e^{jw}) = \frac{e^{jw} + \frac{11}{3}}{e^{jw} - \frac{1}{3}}$$

(4) 由(2)知 $H(z) = \frac{4}{z - \frac{1}{3}} + 1$

$$h[n] = \delta[n] + \left(\frac{1}{3}\right)^{n-1} u[n-1]$$

6. 已知 $\mathscr{F}[f(t)] = F(j\Omega)$，则由傅里叶变换定义

$$\int_{-\infty}^{+\infty} f(t) e^{-j\Omega t} dt = F(j\Omega)$$

对 $f(at)$，当 $a>0$ 时，令 $u=at$，$t=\frac{u}{a}$

$$\text{则} \int_{-\infty}^{\infty} f(at) e^{-j\Omega t} dt = \int_{-\infty}^{\infty} f(u) e^{-j\frac{\Omega u}{a}} \frac{1}{a} du$$

$$= \frac{1}{a} \int_{-\infty}^{\infty} f(u) e^{-\frac{j\Omega u}{a}} du$$

$$= \frac{1}{a} F(j\frac{\Omega}{a})$$

当 $a<0$ 时，令 $u=at$，$t=\frac{u}{a}$

$$\int_{-\infty}^{\infty} f(at) e^{-j\Omega t} dt = \int_{\infty}^{-\infty} f(u) e^{-j\frac{\Omega}{a}u} \cdot \frac{1}{a} du$$

$$= \frac{1}{a} \int_{\infty}^{-\infty} f(u) e^{-j\frac{\Omega}{a}u} du$$

$$= -\frac{1}{a} \int_{-\infty}^{\infty} f(u) e^{-j\frac{\Omega}{a}u} du$$

$$= -\frac{1}{a} F(j\frac{\Omega}{a})$$

于是，对全体 a，$\mathscr{F}[f(at)] = \frac{1}{|a|} F(j\frac{\Omega}{a})$