Compsoc 2023: Report for Modified Convergence Rule (v3)

Team Improbable: Alexander Whitefield

June 2023

1 Voting rule summary

We use ideas from Rank Centrality [3, 2] and Convergence voting [1] to contruct a voting rule. The voting rule uses voter preference orderings to build a Markov Chain between candidates. The score returned by the voting rule is the steady state probabilities for the candidates in the constructed Markov Chain. We refer to our voting rule as Modified Convergence voting.

2 Description of Modified Convergence voting

2.1 Construction of the Markov chain

Let $t(i, j)$ denote the transition probability from candidate i to j. Let n denote the number of candidates, and d denote a hyper-parameter which is analogous to the damping factor used in PageRank.

We compute $t(i, j)$ where $i \neq j$ as follows: First we compute p_{ji}: the proportion of voters who prefer j to i. Then we set:

$$t(i, j) := (1 - d) \cdot \frac{p_{ji}}{n - 1} + d \cdot \frac{1}{n}$$

Given the computations of $t(i, j)$ where $i \neq j$, $t(i, i)$ is determined by the constraints of the stochastic matrix (rows and columns must add to 1). We set $d = 0.1$ in our submission.

2.2 Scores assigned to candidates

Note that this Markov Chain is aperiodic and irreducible. Therefore, there exist steady state probabilities corresponding to each candidate. Moreover, we can quickly estimate the steady state probabilities using matrix multiplication. The score for candidate i is the steady state probability of candidate i.

2.3 Interpretation of Modified Convergence voting

Our Markov chain corresponds to the following thought experiment. Suppose we are at candidate i. With probability d we move to a random state. With probability $(1-d)$ we follow the following procedure: we randomly pick a state $j \neq i$ to compare to i. We then randomly pick a voter k. We move to j if voter k prefers j to i, and otherwise we remain at i.

Suppose we repeat this procedure, and record the amount of periods we spend on each candidate. As we increase the periods this procedure runs, the proportion of time we spend on each candidate will stabilise. These proportions correspond to the score given by the voting rule.

3 Expected properties of Modified Convergence voting

Satisfied properties: Anonymity, Neutrality, No Dictatorship, Non-imposition, Pareto Efficiency, Monotonicity and the Majority critereon

Unsatisfied properties: Condorcet critereon, IIA and Strategy-proofness.

3.1 Violation of IIA and Strategy-proofness

The conditions satisfied above, together with Arrow's impossibility theorem imply that Modified Convergence voting does not satisfy IIA.

As Modified Convergence voting is non-dictatorial, the Gibbard-Satterthwaite theorem implies that it is suspectible to tactical voting (i.e. it is not Strategy proof).

3.2 Violation of the Condorcet critereon

Our modification to the Markov Chain in [1] still allow us to use their example of violation of the Condorcet criterion. If the Condorcet criterion is desired, we could easily modify the voting rule by checking if there is a Condorcet winner, and if not, run the described algorithm.

3.3 Social welfare performance

The relative performance of Modified Converge voting depends on the model of voter utility. If the mass of voter utility is weighted towards their top candidate, we expect voting rules such as Plurality or Dowdall to do well. If the mass of voter utility is spread across alternatives, we expect Modified Convergence voting to do well.

References

[1] Gergei Bana, Wojciech Jamroga, David Naccache, and Peter YA Ryan. Convergence voting: From pairwise comparisons to consensus. *arXiv preprint arXiv:2102.01995*, 2021.

[2] Sahand Negahban, Sewoong Oh, and Devavrat Shah. Iterative ranking from pair-wise comparisons. *Advances in neural information processing systems*, 25, 2012.

[3] Lawrence Page, Sergey Brin, Rajeev Motwani, and Terry Winograd. The pagerank citation ranking: Bringing order to the web. Technical report, Stanford infolab, 1999.