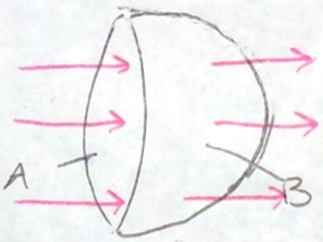
JACOB PIERSON | PHYSICS 206 - HW3 | 9/16/2022

4)



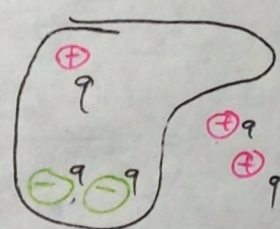
In this uniform electric field, is the magnitude of Φ_A <, >, or = Φ_B?

$\Phi_A = \Phi_B = 0$, the field is defined as uniform and as pictured, ~~as~~ is unchanged after passing through this object, suggesting the closed surface contains no net charge, so there can be no net flux through the surface.

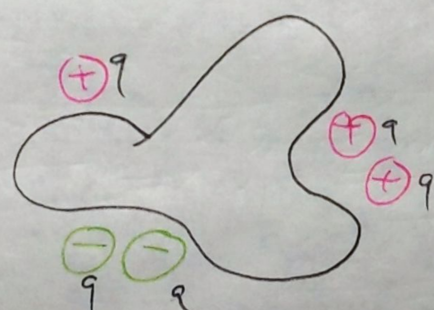
5) What is the electric flux (in terms of q/ε_0 through each of the surfaces?

a)

$\boxed{+q/\varepsilon_0}$

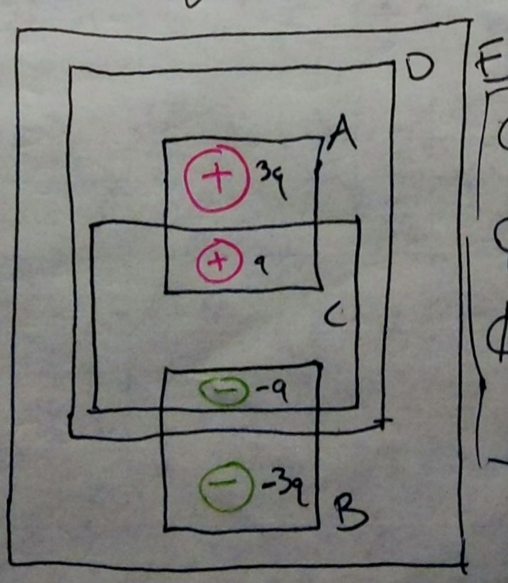


$\boxed{-q/\varepsilon_0}$



$\boxed{0}$

6) What is the flux through each of the surfaces A to E (in terms of q/ε_0)?



$$\Phi_A = \frac{4q}{\varepsilon_0} \qquad \Phi_D = \frac{3q}{\varepsilon_0}$$

$$\Phi_C = 0 \qquad \Phi_E = 0$$

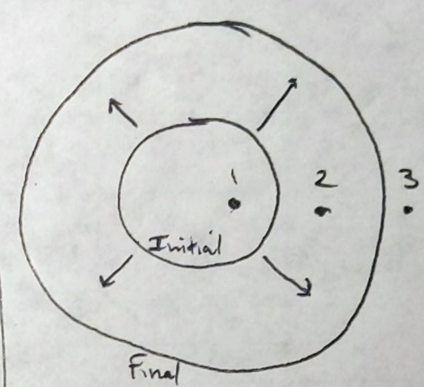
$$\Phi_B = -\frac{4q}{\varepsilon_0}$$

7) The charged balloon pictured expands as it blows up, increasing in size from the initial to final diameters shown. Consider how the electric field strength changes at points 1, 2, 3.

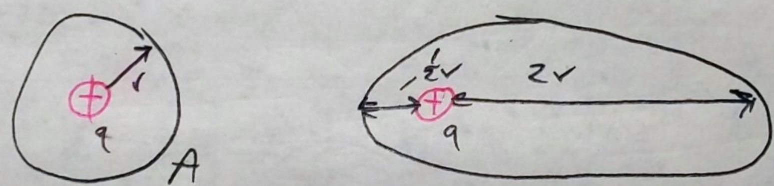
Point 1: zero, no surface will ~~enclosed~~ enclose any charge so field strength of zero won't change

point 2: decrease, as balloon expands beyond point 2, the surface will no longer enclose any charge (goes to zero) but initially does ~~enclose~~ enclose $q_{balloon}$

point 3: constant, surfaces at point three always enclose total charge of balloon, field strength doesn't change

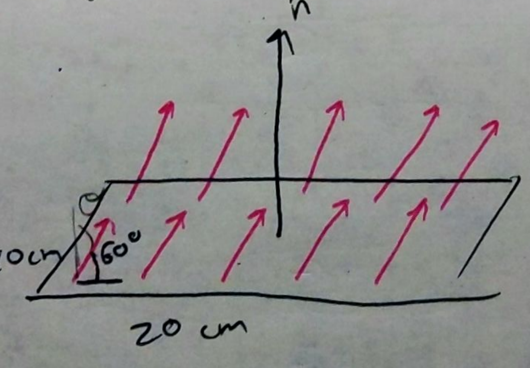


9) The ~~spheres~~ sphere + ellipsoid below surround equal charges, what can be said about the flux through A + B?



They are equal, based on the same charge (q) → or, both surfaces enclose the same charge, A makes for easier math. [student 2, paraphrased]

10) The electric flux through the surface shown is 25 N m²/C, what is the field strength?



$\theta = 30^\circ$

$$\phi_e = \vec{E} \cdot \vec{A}$$

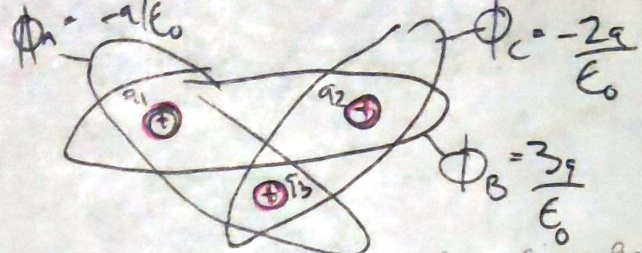
$$25\ Nm^2/C = \vec{E}(0.10m)(0.20m)\cos(30)$$

$$\vec{E} = \frac{25\ N\frac{m^2}{C}}{(0.10m)(0.20m)\cos(30)}$$

$$\vec{E} \approx \frac{25\ N\frac{m^2}{C}}{(0.02m^2)\cos(30)}$$

$$\boxed{\vec{E} \simeq 1443 N/C}$$

19) Below are three Gaussian surfaces and the electric flux through each. What are the three charges; q_1, q_2, + q_3?



$\phi_a = \frac{-q}{\epsilon_0} = q_1 + q_3$

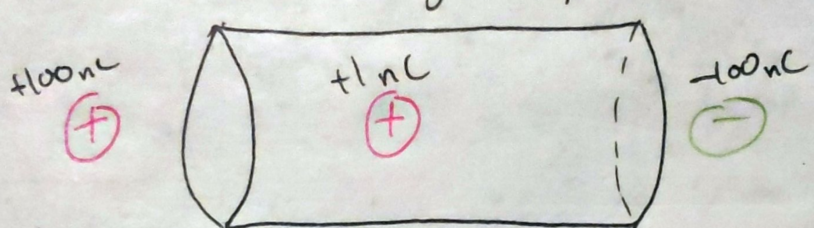
$\phi_b = \frac{3q}{\epsilon_0} = q_1 + q_2$

$\phi_c = \frac{-2q}{\epsilon_0} = q_2 + q_3$

$$\begin{array}{c|ccc|c} & q_1 & q_2 & q_3 & \phi \\ A & 1 & 0 & 1 & -1 \\ B & 1 & 1 & 0 & 3 \\ C & 0 & 1 & 1 & -2 \end{array} = A \quad rref(A) \text{ gives: } \Rightarrow \boxed{\begin{array}{l} q_1 = 2 \\ q_2 = 1 \\ q_3 = -3 \end{array}}$$

check $2 + (-3) = -\frac{q}{\epsilon_0}$ ✓

21) what is the net electric flux through the cylinder shown below?



$$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{in}}{\epsilon_0} = \boxed{\frac{1.0 \times 10^{-9} C}{\epsilon_0}}$$

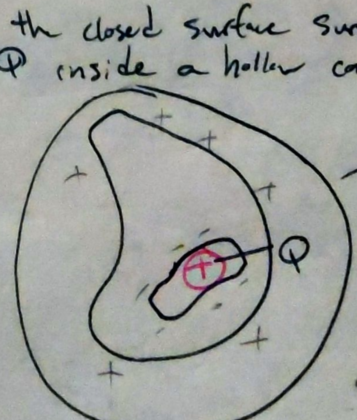
external charges do not contribute to flux through cylinder surface

28) What is the net electric ~~through~~ flux through the closed surface surrounding a neutral conductor w/ point charge Q inside a hollow cavity?

net flux $= \phi_e = \frac{Q_{in}}{\epsilon_0}$

Since the total conductor in enclosed by the Gaussian surface the interior +Q is distributed along the conductor's surface,

$\boxed{\text{so } \phi = \frac{Q}{\epsilon_0}}$



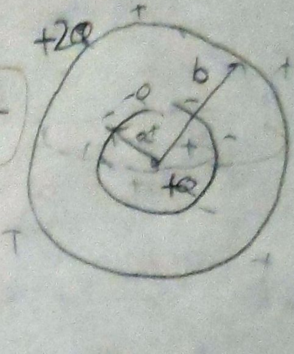
+Q manifests on surface of conductor

35) A hollow metal sphere has inner radius a and outer radius b. The hollow sphere has charge $+2Q$. A point charge $+Q$ sits at the center of the hollow sphere.

a) determine the electric fields in the three regions:

$\Phi_E = \oint \vec{E} \cdot d\vec{A} = \frac{Q_{in}}{\epsilon_0}$

$r \leq a$: $E \cdot \frac{Q}{A\epsilon_0} \cdot \frac{Q}{4\pi r^2 \epsilon_0}$ $Q_{in} = Q$ $\boxed{E = \frac{1}{4\pi r^2} \cdot \frac{Q}{\epsilon_0}}$



$a < r < b$:

$Q_{in} = +Q - Q = 0$, $\boxed{\vec{E} = 0}$

$r \geq b$:

$Q_{in} = 2Q + Q = 3Q$

$E = \frac{1}{4\pi r^2} \cdot \frac{3Q}{\epsilon_0} = \boxed{\frac{3Q}{4\pi r^2 \epsilon_0}}$

b) How much charge is on the inside surface of the hollow sphere? On the exterior?

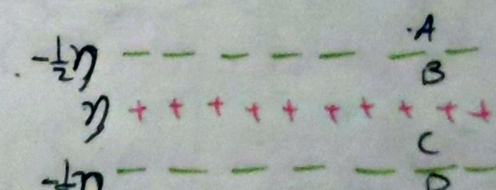
$\boxed{-Q \text{ to balance the } +Q \text{ at center so } \vec{E} \text{ inside conductor} = 0}$

44) Three parallel planes of charge shown below have surface charge densities of $-\frac{1}{2}\eta$, η, & $-\frac{1}{2}\eta$. Find the electric fields in each region (A-D)

$\boxed{A = D = 0}$

$\boxed{B = \frac{\frac{1}{2}\eta}{\epsilon_0} \uparrow \quad C = \frac{\frac{1}{2}\eta}{\epsilon_0} \downarrow}$

$B = -C$



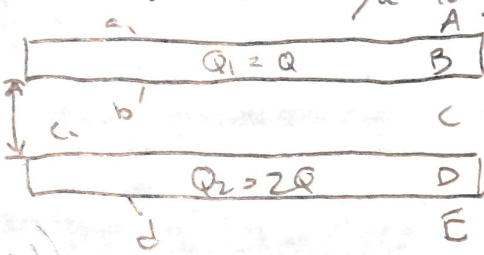
$E_{plane} = \frac{\eta}{2\epsilon_0}$

applying same logic from lecture example

47) Two large slabs of metal are parallel & distance l apart. The top & bottom of each slab has surface area A. Metal 1 has total charge $Q_1 = Q$ & metal 2 $Q_2 = 2Q$. Assume Q is positive. In terms of Q & A, determine:

a) the electric field strengths in regions A to E

b) the surface charge densities η_a to η_d on the four surfaces a to d



$E_{A\,tot} = \frac{Q}{A \cdot \epsilon_0} + \frac{2Q}{A\epsilon_0} = \frac{3Q}{A\epsilon_0}$

$$\boxed{E_B = E_D = 0} \quad \text{metals } \vec{E} = 0 \text{ inside}$$

$E_{plane} = \frac{\eta}{2\epsilon_0}$ $\qquad E_A = \frac{\eta_A}{\epsilon_0}$

Superposition $\vec{E}_A = \frac{1}{2}(E_{total})$

$$\boxed{\vec{E}_A = \frac{3}{2A}\frac{Q}{\epsilon_0}} \qquad \boxed{\eta_A = \frac{3}{2}\frac{Q}{A}}$$

$\eta_A = \frac{Q}{A}$

$\eta_D = \frac{2Q}{A}$

$$\boxed{\eta_B = -\frac{1}{2}\frac{Q}{A}}$$

$E_A = \frac{\eta_A}{2\epsilon_0} + \frac{\eta_B}{2\epsilon_0} + \frac{\eta_C}{2\epsilon_0} + \frac{\eta_D}{2\epsilon_0}$

~~54) A sphere~~

$$\vec{E}_C = E_A - \frac{Q}{A\epsilon_0} = \frac{3Q}{2A\epsilon_0} - \frac{Q}{A\epsilon_0} \Rightarrow \boxed{\vec{E}_C = \frac{Q}{2A\epsilon_0}}$$

$2\epsilon_0 \left[\frac{\eta_A}{\epsilon_0} = \frac{\eta_A}{2\epsilon_0} + \frac{\eta_B}{2\epsilon_0} + \frac{\eta_C}{2\epsilon_0} + \frac{\eta_D}{2\epsilon_0}\right]$

$\eta_A = \eta_B + \eta_C + \eta_D$

$$\vec{E}_E = -\vec{E}_A = \boxed{-\frac{3Q}{2A\epsilon_0}} \qquad \boxed{\eta_C = \frac{1}{2}\frac{Q}{A}} \quad \boxed{\eta_D = \eta_A = \frac{3}{2}\frac{Q}{A}} \quad \eta_B = -\eta_C$$

check $\frac{3}{2}\frac{Q}{A} = -\frac{1}{2}\frac{Q}{A} + \frac{1}{2}\frac{Q}{A} + \frac{3}{2}\frac{Q}{A}$ ✓

54) A spherical shell has inner radius R_{in} & outer radius R_{out}. The shell contains total charge Q (uniformly distributed), the interior is empty of charge & matter.

a) Find the electric ~~field strength~~ strength outside the shell, $r \geq R_{out}$

$\Phi_E = \oint \vec{E} \cdot d\vec{A} = \frac{Q_{in}}{\epsilon_0}$

$$\boxed{\vec{E} = \frac{Q}{4\pi r^2 \epsilon_0}} \quad \text{for } r \geq R_{out} \qquad Q_{in} = Q$$

$\vec{E} = \frac{Q}{(A \cdot \epsilon_0)} = \frac{Q_{in}}{4\pi r^2 \epsilon_0}$

b) Find the field strength interior of shell $r \leq R_{in}$

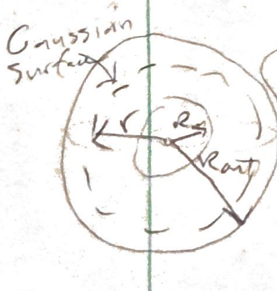
$$\boxed{\vec{E} = 0} \qquad Q_{in} = \emptyset$$

$\rho = \frac{Q}{V}$

$V_{total} = \frac{4}{3}\pi(R_{out}^3 - R_{in}^3)$

$\rightarrow \rho = \frac{3Q}{4\pi(R_{out}^3 - R_{in}^3)}$

c) Find field strength within the shell $R_{in} \leq r \leq R_{out}$



$$\boxed{\vec{E} = \frac{1}{4\pi\epsilon_0}\frac{Q}{r^2}\left(\frac{r^3 - R_{in}^3}{R_{out}^3 - R_{in}^3}\right)} \qquad Q_{in} = Q\rho V_{in} \qquad V_{in} = \frac{4}{3}\pi(r^3 - R_{in}^3)$$

$Q_{in} = Q\left(\frac{3Q}{4\pi(R_{out}^3 - R_{in}^3)}\right)\left(\frac{4}{3}\pi(r^3 - R_{in}^3)\right) \Rightarrow Q_{in} = Q\left(\frac{r^3 - R_{in}^3}{R_{out}^3 - R_{in}^3}\right)$

d) Show solutions match at inner & outer boundaries

when $r = R_{in}$ ✓ $\qquad$ when $r = R_{out}$

$$\vec{E} = \frac{1}{4\pi\epsilon_0}\frac{Q}{R_{in}^2}\left(\frac{\cancel{R_{in}^3 - R_{in}^3}}{R_{out}^3 - R_{in}^3}\right)\hat{r} = 0 \qquad \vec{E} = \frac{1}{4\pi\epsilon_0}\frac{Q}{R_{out}^2}\left(\frac{\cancel{R_{out}^3 - R_{in}^3}}{\cancel{R_{out}^3 - R_{in}^3}}\right)$$

$$\boxed{\text{both match parts a \& b}} \qquad \boxed{\vec{E} = \frac{1}{4\pi\epsilon_0}\frac{Q}{R_{out}^2}\hat{r}} \ ✓$$

60) A sphere of radius R has total charge Q. The volume charge density (C/m^3) w/in the sphere is $\rho = \rho_0\left(1 - \frac{r}{R}\right)$ which decreases linearly from ρ_0 at the center to zero at the edge of the sphere.

$Q = \int dQ = \int \rho\, dV$

$dV = 4\pi r^2 dr$

a) Show that $\rho_0 = \frac{3Q}{\pi R^3}$

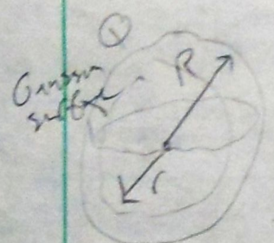
$$Q = \int \rho\, dV = 4\pi \rho_0 \int_0^R \left(1 - \frac{r}{R}\right) r^2\, dr$$

$$= 4\pi\rho_0 \left[\frac{r^3}{3} - \frac{r^4}{4R}\right]_0^R = \frac{\pi}{3}\rho_0 R^3$$

$$Q = \frac{\pi}{3}\rho_0 R^3 \Rightarrow \boxed{\frac{3Q}{\pi R^3} = \rho_0}$$

b) Show electric field inside the sphere points radially outward with magnitude

$$E = \frac{Qr}{4\pi\epsilon_0 R^3}\left(4 - 3\frac{r}{R}\right)$$



$$Q_{in} = \int \rho\, dV = 4\pi\rho_0 \int_0^r \left(1 - \frac{r}{R}\right) r^2\, dr = 4\pi\rho_0 \left[\frac{r^3}{3} - \frac{r^4}{4R}\right]$$

$$= \frac{4\pi\rho_0}{3}\left[r^3 - \frac{3r^4}{4R}\right]$$

$$Q_{in} = \frac{4\pi 3Q}{3\pi R^3}\left[r^3 - \frac{3r^4}{4R}\right] \Rightarrow Q\frac{r^3}{R^3}\left[4 - 3\frac{r}{R}\right]$$

$$\boxed{\hookrightarrow E(r) = \frac{Qr}{4\pi\epsilon_0 R^3}\left(4 - 3\frac{r}{R}\right)}$$

c) Show that result of part b has expected value at $r = R$

when $r = R$

$$E(R) = \frac{QR}{4\pi\epsilon_0 R^{\cancel{3}\,2}}\left(4 - 3\cancel{\frac{R}{R}}^{\,1}\right) \Rightarrow \boxed{\frac{Q}{4\pi\epsilon_0 R^2}}$$

✓