BIOMECHANICS OF HUMAN MOVEMENT

ME 24-663 / BME 42-691 – FALL 2022
CARNEGIE MELLON UNIVERSITY
PROF. ENI HALILAJ

— HOMEWORK #4 —

Due: Thursday, November 3, 2022 at 11:30AM

1. A common problem encountered by the elderly is loss of balance, which sometimes results in falling. One biomechanical hypothesis suggests that falling in the elderly results from a lack of sufficient muscle strength. To study the issue of strength, suppose we asked a number of elderly women to rise from a chair in our motion lab. Our aim is to estimate the net joint torques for the major joints of the lower extremity during this task. Once we calculate the joint torques used to stand up from a chair, we can compare them to the maximum joint torques that our subjects can generate (maximum joint torques are a measure of muscle strength). This comparison will show what percentage of our subjects' maximum strength must be used to rise from a chair.

   Suppose we used high-speed video cameras, together with markers positioned judiciously over various body landmarks to estimate the time history of the absolute angular displacements of the shank, thigh, and trunk (i.e., $\theta 1$, $\theta 2$, $\theta 3$ in Figure 1). These data were subsequently filtered and numerically differentiated to obtain the absolute angular velocities and accelerations of each of these body segments over time. Assuming that the sit-to-stand activity can be characterized by predominantly planar (2-D) motion of all the body segments, a simple three-segment, planar linkage is used to model the human skeleton (Figure 1). Further assume that the body segments are connected by frictionless single degree-of-freedom revolute (i.e., hinge) joints.

   In terms of *kinematic data only*:

   (a) Please derive complete analytical expressions for the net joint torques exerted at the ankle (T1), knee (T2), and hip (T3) during sit-to-stand motion. Assume that the foot remains flat on the floor.

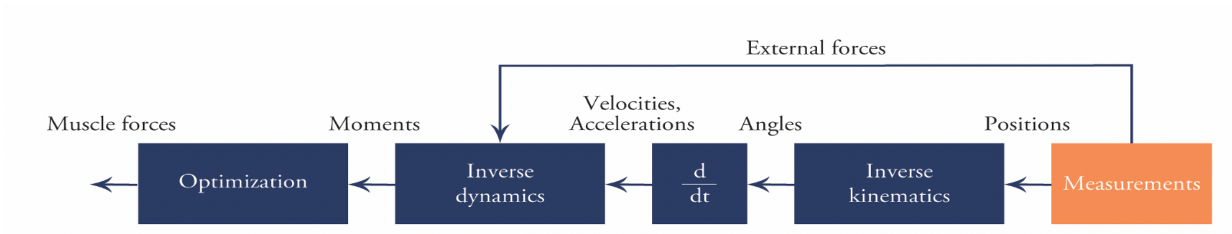
   (See attached pages)

   (b) Please derive complete analytical expressions for the ground reaction forces exerted in the horizontal (fore-aft) and vertical directions during the sit-to-stand motion. Again assume that the foot remains flat on the floor and that it is massless. Note that if you could measure Fgx and Fgy that you could calculate T1 without any kinematic data.

(See attached pages)

(c) Would you consider this problem to be inverse or direct dynamics? Why?

It is an inverse dynamics problem as we are looking to derive the moments of the joints which ultimately lead to motion. The problem follows this kind of flow which was explained in lecture:



(d) What is the physical significance of the inter-segmental forces (e.g., F1y)? Are the calculated inter-segmental forces equal to the true bone-on-bone forces? Why or why not?

The physical significance of the inter-segmental forces is to model the revolute joint as a pin which is known to have two force components (Fx and Fy).
The intersegmental forces are not equal to the true bone-on-bone loads. They could only be equal if our muscles applied pure joint torques like rotational motors. Our muscles do not generate such torques directly but rather generate pulling forces that are applied to the skeleton.

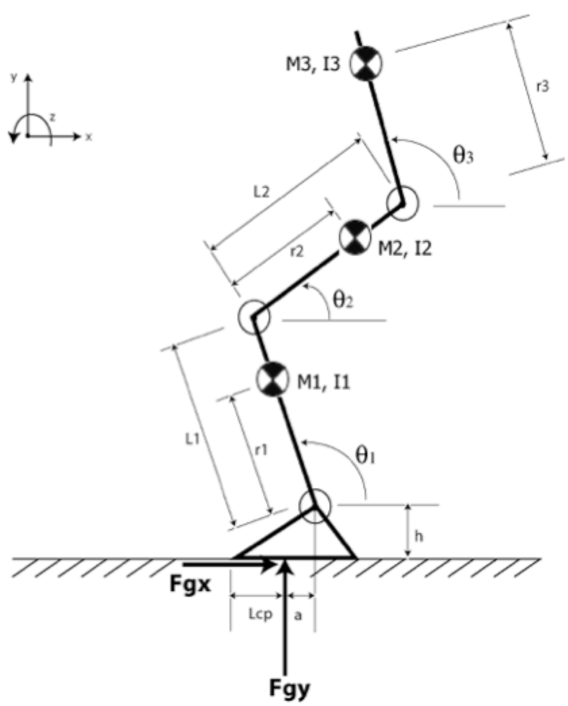


Figure 1. Planar 3-Segment Model.

2. Using our preliminary data from problem 1, we were able to land a small grant and have used the money to purchase a force plate. With our improved experimental set-up, we can now measure the horizontal (fore-aft) and vertical ground reaction forces during the sit-to-stand motion. Also, our video system remains operational, and we shall continue to record the time history of the angular displacements of the shank, thigh, and trunk.

   On the basis of the three segment, planar model given in Figure 1, use the kinematic *and force-plate data* now available to:

   (a) **Derive complete analytical expressions for the net joint torques exerted at the ankle (T1), knee (T2), and hip (T3) during the sit-to-stand motion. Again, assume that the foot remains flat on the floor.**

   (See attached pages)

   (b) **By comparing the terms in your equations for T1, T2, and T3 derived in problems 2a and 1a, discuss the advantages to being able to accurately estimate the ground reaction forces and relevant kinematic parameters when calculating joint torques.**

   As per the book, when calculating joint torques, it is important to estimate ground reactions forces and relevant kinematic parameters because when having access to both parameters, we can obtain an overdetermined system of equations. In this case, we can use the additional information to minimize the consequences of measurement error and use the "extra" information to improve the model performance. Therefore, the values of the joint torques become more accurate.

In many instances, researchers are interested in three-dimensional kinematics and dynamics and use homogeneous transformation matrices to get the three-dimensional translations and rotations of body segments. In other instances, researchers are able to use just a planar model to answer clinical questions. Part I provided you with an example of a simple inverse dynamics problem involving a planar model.

The following will give you some "real-world" experience in gait analysis and will build upon the planar model. We *strongly recommended* that you use a computer program to solve them. We suggest Matlab, but you are free to use a program with which you are most comfortable.

We will conduct a planar analysis and use the "6 marker set" that is frequently used in biomechanics labs. With this marker convention, a reflective marker is placed on the iliac crest, the greater trochanter (the hip), the lateral epicondyle (the knee), the lateral malleolus (the ankle), the calcaneus (the heel), and the 5th metatarsal (bone by the little toe). The location of these markers can be seen in Figure 2.

The associated Excel file contains real gait data, and the calculations that you will be performing are the same calculations that biomechanists perform in a gait analysis laboratory. (Actually, these are the calculations that they have a computer perform for them). The Excel file contains the following worksheets:

- **Demographics:** Information about the height (in meters) and mass (kg) of the subject.
- **Raw Marker Data:** The x and y positions of various markers expressed in the global coordinate system. All dimensions are in meters.
- **Force Plate Data:** The x and y components of the ground reaction force, the free torque (you can ignore it for this assignment), and the x location of the center of pressure (COP) (i.e., the point that represents the location of the GRF). All dimensions are in metric units (meters, Newtons, etc.).

Kinematics

1) Please calculate and plot the hip, knee, and ankle flexion angles as a function of time. (Note that you have more than 1 gait cycle in the data file.) To do so, follow these steps:

   a) Establish the vectors that represent the HAT (head, arms, torso), thigh (femur), shank (tibia and fibula), and foot. Please have all vectors originating at the distal point and being directed proximally. (For example, the thigh (femur) is defined by hip - knee).

   b) Normalize the vectors that represent the HAT, thigh, shank, and foot.

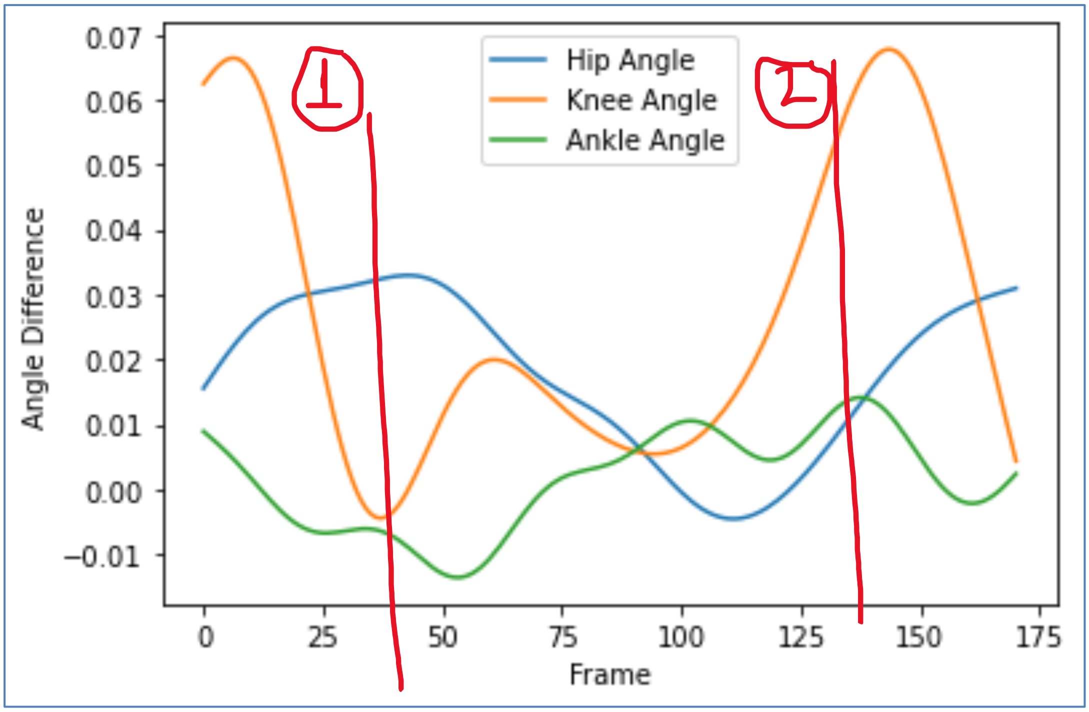
   c) Calculate the angle between the x-axis in the global coordinate system and vectors representing the HAT (θ_4), thigh (θ_3), and shank (θ_2). (Figure 2). (Hint: use the dot product)

   d) Calculate the angle between the y-axis and vector representing the foot (θ_1).

   e) Calculate the hip, knee, and ankle angles with the following formulas
      i) Hip angle = thigh angle - HAT angle
      ii) Knee angle = thigh angle - shank angle
      iii) Ankle angle = foot angle - shank angle

   f) Plot your results.

2) Label when heel strike and toe off occur on the three plots you just created.



1: Heel Strike
2: Toe off

3) Differentiate your angles for the foot, shank, thigh, and HAT to obtain the appropriate angular velocities and accelerations. Use the following formulas to conduct this numerical differentiation. The camera samples at 120 Hz. You don't need to find the velocities and accelerations for the first and last data points.

$$f'(t_i) \approx \frac{f(t_{i+1}) - f(t_{i-1})}{2\Delta t}$$

$$f''(t_i) \approx \frac{f(t_{i-1}) - 2f(t_i) + f(t_{i+1})}{\Delta t^2}$$

4) Numerically differentiate your knee and ankle position data to obtain the appropriate linear velocities and accelerations for these markers. You don't need to find the velocities and accelerations for the first and last data points.

Kinetics

5) The moment of inertia and center of mass of each segment is needed to perform inverse dynamic analysis. Anthropometric information from Winter (1990) can be used to calculate this information. Using the data in the table below, calculate the mass of the segment, the length of the segment, the position of the center of mass (COM) in the segment, and the moment of inertia about the center of mass for the foot, shank (tibia and fibula), thigh (femur), and HAT. Assume linear segments as shown in the figure.

A	B	C	D	E
Segment	**Segment mass/ total mass**	**Segment length/ height**	**COM position from proximal end of segment /segment length**	**Radius of gyration about COM/ segment length**
Foot	0.0145	0.055	0.5	0.475
Shank	0.0465	0.246	0.433	0.302
Thigh	0.1	0.245	0.433	0.323
HAT	0.678	0.47	0.626	0.496

The height of the subject is 1.7526 m, and the mass of the subject is 66.7 kg.

Columns B-D are fractions that should be scaled by total mass, height, or segment length as listed.

Mom ent of inertia of a segment is defined as:
$I = mass * (radius\_of\_gyration)^2$

Radius of gyration should be determined from the information given.

6) Please plot the vertical and horizontal ground reaction forces during the stance phase as a percentage of body weight.

7) Draw a free body diagram for the foot, shank, thigh, and HAT segments. You may ignore the mass and moment of inertia of the foot, as it is sometimes assumed that the mass of the foot is negligible when compared to the mass of shank.

8) Using problem 2 as a primer, please derive the equations necessary to solve for the torques about the hip, knee, and ankle joints *during the stance phase of gait* for this model. Again, for simplicity, you may ignore the mass of the foot. Some things to consider:

    a) The location of the origin of the coordinate system for this problem is different than the location of the coordinate system in Part I. Therefore, we are measuring positions, velocities, and accelerations with respect to a fixed laboratory reference frame. Think about how this changes your equations from the ones you had in Part I.

    b) You have markers at the hip, knee, and ankle joints. Think about how you can use this information to simplify your equations from those used in Part I.

9) For the given data, please use the equations you just derived in step 8 to calculate the torques for the hip, knee, and ankle joints *during the stance phase of gait*. (Note: when solving the equations, remember that all angular velocities and accelerations need to be in radians).Plot your results and normalize them with respect to body mass (i.e., units are Nm/kg).

Discussion

10) We used anthropometric data to determine the lengths of the segments in this assignment. Suppose Ted (of homework 2 fame) wants to work with you and suggests that you could just use motion data to determine segment length. Ted says that the magnitude of the distance between any 2 markers in the 6-marker convention could give you the appropriate segment length.

    To test Ted's idea, plot the magnitude of the vector between the hip and knee for all of the time points in this study. Comment on the behavior of the curve you just plotted and compare this magnitude to the length of the thigh determined in question 5. Comment on some possible explanations for the observed differences.

    Length of thigh: 0.429

    The trend of the curve mainly demonstrates an upward trend, with certain peaks and valleys which correspond to the length of the thigh in the respective moments of the swing and stance phase.
    The magnitude of the curve is always lower than the length of the thigh with a maximum of 0.4228. The reason behind this difference is that we are measuring the relative distances between the hip and knee. It is also known that with movement, the markers do not stay put due to movements of the muscles. These factors, contribute to the inaccuracies seen in the plotted curve.

11) In question 8, we asked you to normalize the joint torques by body mass. Why is it important to perform this normalization instead of just reporting a raw value in Newton-meters?

It is important to perform the normalization as this properly process scales the torques. The mass does not change, and scaling will give us a better understanding of how the torques are applied on the body and this way, we can compare the torques to one another.

12) Which joint moment has the greatest magnitude (hip, knee, or ankle)? Could you suggest an explanation as to why this is the case?

Looking at the plots of the book in chapter 8:
Of all lower-limb joint moments, the peak ankle plantarflexion moment is greatest. The reason for that is because the ankle is directly affected by the ground reaction forces. Another reason could be the fact that the ankle is heavily involved with pushing the foot off the ground at toe-off.

13) Look at the shapes of the curves for the hip and knee torques in early stance. Please suggest a few explanations for what you observe and propose some possible solutions to correct the problem. (Hints: consider your results for questions 6 and 10. Also, what type of marker and force plate data are you working with?).

Looking at the plots of the book in chapter 8:
During early stance, heel strike occurs which puts a significant stress on the body and therefore, makes the markers offset. This is also confirmed from our results in part 6 and 10. A jerk is initially observed and the ways to improve that is by performing signal processing and filtering to remove this observed noise.

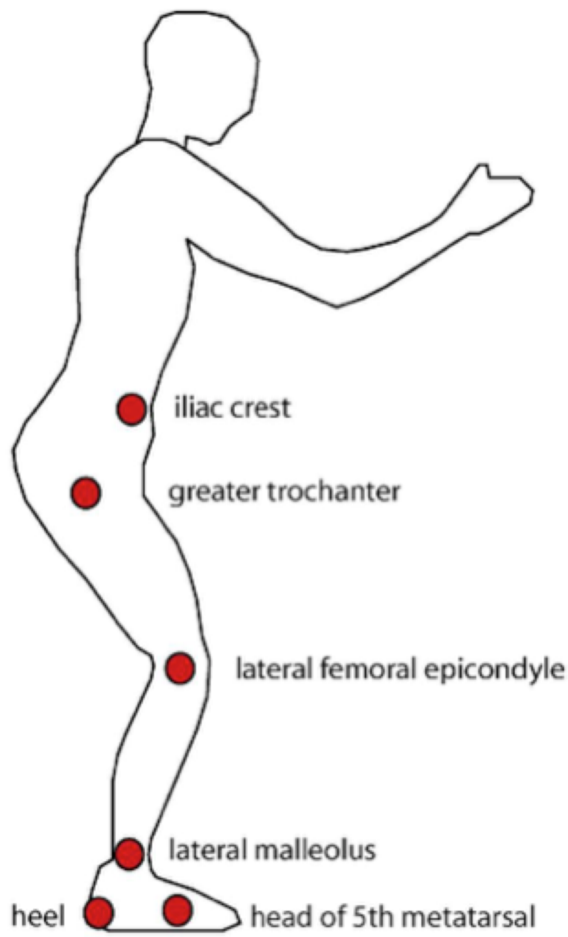


Figure 2. Marker Convention.

HW 4

1\. a) Foot:

$\Sigma F_x = 0 \Rightarrow F_{xg} - F_{x_1} = 0$

$\Sigma F_y = 0 \Rightarrow F_{yg} - F_{y_1} = 0$

$\Sigma M = 0 \Rightarrow F_{xg} h - F_{yg} l - T_1 = 0 \Rightarrow T_1 = F_{xg} \cdot h - F_{yg} \cdot d$

Shank:

$x_1 = r_1 c\theta_1$

$\dot{x}_1 = -r_1 s\theta_1 \dot{\theta}$

$\ddot{x}_1 = -r_1 (s\theta_1 \ddot{\theta}_1 + c\theta \dot{\theta}_1^2)$

$y_1 = r_1 s\theta_1$

$\dot{y}_1 = r_1 c\theta_1 \dot{\theta}_1$

$\ddot{y}_1 = r_1 (c\theta_1 \ddot{\theta}_1 - s\theta_1 \dot{\theta}_1^2)$

$\Sigma F_x = m_1 \ddot{x}_1 \Rightarrow F_{x_1} - F_{x_2} = + m_1 \ddot{x}_1$

$\Sigma F_y = m_1 \ddot{y}_1 \Rightarrow F_{y_1} - F_{y_2} - m_1 g = m_1 \ddot{y}_1$

$\Sigma M = I_1 \ddot{\theta}_1 \Rightarrow T_1 - T_2 + F_{x_1} r_1 s\theta_1 - F_{y_1} r_1 c\theta_1 + F_{x_2} (l_1 - r_1) s\theta_1 - F_{y2} (l_1 - r_1) c\theta_1 = I_1 \ddot{\theta}_1$

Thigh

$x_2 = l_1 c\theta_1 + r_2 c\theta_2$

$\dot{x}_2 = -l_1 s\theta_1 \dot{\theta}_1 - r_2 s\theta_2 \dot{\theta}_2$

$\ddot{x}_2 = -l_1 (s\theta_1 \ddot{\theta}_1 + c\theta_1 \dot{\theta}_1^2) - r_2 (s\theta_2 \ddot{\theta}_2 + c\theta_2 \dot{\theta}_2^2)$

$y_2 = l_1 s\theta_1 + r_2 s\theta_2$

$\dot{y}_2 = l_1 c\theta_1 \dot{\theta}_1 + r_2 c\theta_2 \dot{\theta}_2$

$\ddot{y}_2 = l_1 (c\theta_1 \ddot{\theta}_1 - s\theta_1 \dot{\theta}_1^2) + r_2 (c\theta_2 \ddot{\theta}_2 - s\theta_2 \dot{\theta}_2^2)$

$\Sigma F_x = m_2 \ddot{x}_2 \Rightarrow F_{x_2} - F_{x_3} = m_2 \ddot{x}_2$

$\Sigma F_y = m_2 \ddot{y}_2 \Rightarrow F_{y2} - F_{y3} - m_2 g = m_2 \ddot{y}_2$

$\Sigma M = I_2 \ddot{\theta}_2 \Rightarrow T_2 - T_3 + F_{x_2} r_2 s\theta_2 - F_{y_2} r_2 c\theta_2 + F_{x_3} (l_2 - r_2) s\theta_2 - F_{y_3} (l_2 - r_2) c\theta_2 = I_2 \ddot{\theta}_2$

HAT

$x_3 = l_1 c\theta_1 + l_2 c\theta_2 + r_3 c\theta_3$

$\dot{x}_3 = -l_1 s\theta_1 \dot{\theta}_1 - l_2 s\theta_2 \dot{\theta}_2 - r_3 s\theta_3 \dot{\theta}_3$

$\ddot{x}_3 = l_1 (s\theta_1 \ddot{\theta}_1 + c\theta_1 \dot{\theta}_1^2) - l_2 (s\theta_2 \ddot{\theta}_2 + c\theta_2 \dot{\theta}_2^2) - r_3 (s\theta_3 \ddot{\theta}_3 + c\theta_3 \dot{\theta}_3^2)$

$y_3 = l_1 s\theta_1 + l_2 s\theta_2 + r_3 s\theta_3$

$\dot{y}_3 = l_1 c\theta_1 \dot{\theta}_1 + l_2 c\theta_2 \dot{\theta}_2 + r_3 c\theta_3 \dot{\theta}_3$

$\ddot{y}_3 = l_1 (c\theta_1 \ddot{\theta}_1 - s\theta_1 \dot{\theta}_1^2) + l_2 (c\theta_2 \ddot{\theta}_2 - s\theta_2 \dot{\theta}_2^2) + r_3 (c\theta_3 \ddot{\theta}_3 - s\theta_3 \dot{\theta}_3^2)$

$\Sigma F_x = m_3 \ddot{x}_3 \Rightarrow F_{x_3} = +m_3 \ddot{x}_3$

$\Sigma F_y = m_3 \ddot{y}_3 \Rightarrow F_{y_3} = m_3 \ddot{y}_3 + m_3 \cdot g$

Because ground forces are unknown

$\Sigma M = I_3 \ddot{\theta}_3 \Rightarrow F_{x_3} r_3 s\theta_3 - F_{y_3} r_3 c\theta_3 + T_3 = I_3 \ddot{\theta}_3$

$\Rightarrow \boxed{T_3 = -F_{x_3} r_3 s\theta_3 + F_{y_3} r_3 c\theta_3 + I_3 \ddot{\theta}_3}$

$$\boxed{\begin{aligned} T_2 = {} & I_2 \ddot{\theta}_2 + I_3 \ddot{\theta}_3 - F_{x_3} r_3 s\theta_3 - F_{x_3} (l_2 - r_2) s\theta_2 + F_{y_3} r_3 c\theta_3 + F_{y_3} (l_2 - r_2) c\theta_2 \\ & - F_{x_2} r_2 s\theta_2 + F_{y_2} r_2 c\theta_2 \end{aligned}}$$

$$\boxed{\begin{aligned} T_1 = {} & I_1 \ddot{\theta}_1 + I_2 \ddot{\theta}_2 + I_3 \ddot{\theta}_3 - F_{x_3} r_3 s\theta_3 - F_{x_3} (l_2 - r_2) s\theta_2 + F_{y_3} r_3 c\theta_3 + F_{y_3} (l_2 - r_2) c\theta_2 \\ & - F_{x_2} r_2 s\theta_2 + F_{y_2} r_2 c\theta_2 - F_{x_1} r_1 s\theta_1 + F_{y_1} r_1 c\theta_1 - F_{x_2} (l_1 - r_1) s\theta_1 + F_{y_2} (l_1 - r_1) c\theta_1 \end{aligned}}$$

b) $\Sigma F_x = 0 \Rightarrow F_{gx} = m_1 \ddot{x}_1 + m_2 \ddot{x}_2 + m_3 \ddot{x}_3$

$\Sigma F_y = 0 \Rightarrow F_{gy} = m_1 \ddot{y}_1 + m_1 \cdot g + m_2 \ddot{y}_2 + m_2 g + m_3 \ddot{y}_3 + m_3 \cdot g$

2. d) From 1b): $F_{gx} = F_{1x} = m_1 \ddot{x}_1 + m_2 \ddot{x}_2 + m_3 \ddot{x}_3$

$$F_{gy} = F_{1y} = m_1 \ddot{y}_1 + m_1 \cdot g + m_2 \ddot{y}_2 + m_2 g + m_3 \ddot{y}_3 + m_3 \cdot g$$

$$T_1 = F_{xg} \cdot h - F_{yg} \cdot a$$

$$\boxed{T_1 = (m_1 \ddot{x}_1 + m_2 \ddot{x}_2 + m_3 \ddot{x}_3) \cdot h - (m_1 \ddot{y}_1 + m_1 g + m_2 \ddot{y}_2 + m_2 g + m_3 \ddot{y}_3 + m_3 \cdot g) \cdot a}$$

Substituting T_1

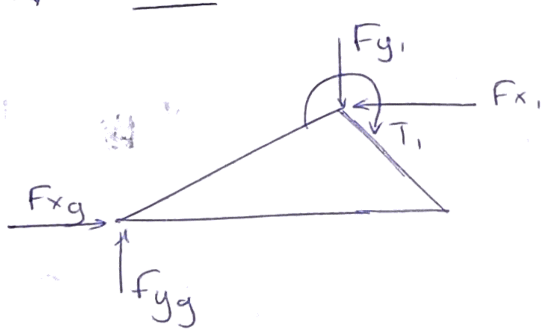
$$\boxed{T_2 = -I_1 \ddot{\theta}_1 + T_1 + F_{x1} r_1 s\theta_1 - F_{y1} r_1 c\theta_1 + F_{x2}(l_1 - r_1) s\theta_1 - F_{y2}(l_1 - r_1) c\theta_1}$$

$$\boxed{T_3 = T_2 - I_2 \ddot{\theta}_2 + F_{x2} r_2 s\theta_2 - F_{y2} r_2 c\theta_2 + F_{x3}(l_2 - r_2) s\theta_2 - F_{y3}(l_2 - r_2) c\theta_2}$$

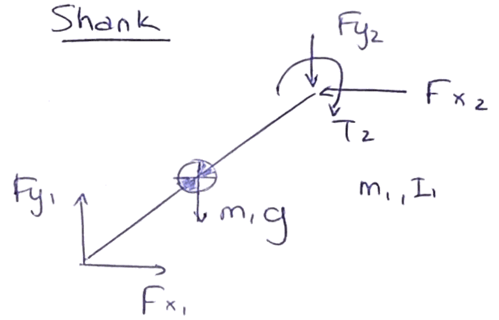
Substituting T_2

Kinematics

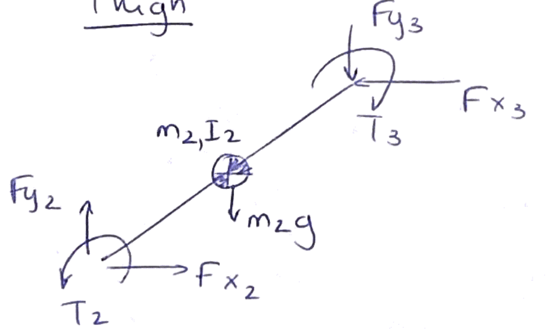
7) Foot



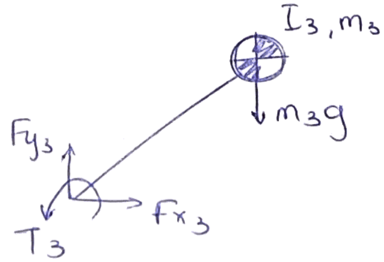
Shank



Thigh



HAT



In [177…
```
import pandas as pd
import numpy as np
import matplotlib.pyplot as plt
```

In [178…
```
xls = pd.ExcelFile("Homework 4 Gait Data (edits after class).xls")
data_2 = pd.read_excel(xls, "Raw Marker Data")
data_3 = pd.read_excel(xls, "Force Plate Data")
```

In [179…
```
data_2
```

Out[179]:

	Unnamed: 0	Iliac Crest	Iliac Crest.1	Hip	Hip.1	Knee	Knee.1	Ankl
0	PHASE	X	Y	X	Y	X	Y	)
1	SWING	-0.842302	1.017374	-0.85235	0.877174	-0.799229	0.486608	-1.05351
2	SWING	-0.832861	1.018299	-0.842193	0.878251	-0.781012	0.488419	-1.03278
3	SWING	-0.823422	1.0192	-0.83204	0.87931	-0.762884	0.490197	-1.01188
4	SWING	-0.813984	1.020067	-0.821896	0.880339	-0.744898	0.491922	-0.99067
...	...	...	...	...	...	...	...	..
167	SWING	0.914748	1.010509	0.909546	0.875817	1.055187	0.495521	1.10654
168	SWING	0.925093	1.008377	0.919371	0.873539	1.064641	0.492994	1.13058
169	SWING	0.935467	1.006165	0.929203	0.871169	1.073996	0.490342	1.15433
170	SWING	0.94586	1.003887	0.939038	0.868733	1.083282	0.487589	1.17784
171	SWING	0.956269	1.00156	0.948875	0.866247	1.092512	0.484762	1.20118

172 rows × 13 columns

In [180…
```
data_2 = data_2.drop(index = 0)
```

1)

In [181…
```
hat_v = np.array([data_2["Iliac Crest"] - data_2["Hip"], data_2["Iliac Crest
thigh_v = np.array([data_2["Hip"] - data_2["Knee"], data_2["Hip.1"] - data_2
shank_v = np.array([data_2["Knee"] - data_2["Ankle"], data_2["Knee.1"] - dat
foot_v = np.array([data_2["Heel"] - data_2["5th Met"], data_2["Heel.1"] - da
```

In [182…
```
hat_v = np.transpose(hat_v)
thigh_v = np.transpose(thigh_v)
shank_v = np.transpose(shank_v)
foot_v = np.transpose(foot_v)
```

In [183…
```
N_hat_v = hat_v / np.linalg.norm(hat_v)
N_thigh_v = thigh_v / np.linalg.norm(thigh_v)
N_shank_v = shank_v / np.linalg.norm(shank_v)
N_foot_v = foot_v / np.linalg.norm(foot_v)
```

In [184…
```
x = np.zeros([171, 2])
x[:, 0] = 1
```

In [185…
```
hat = N_hat_v * x
```

```
hat = hat.astype(float)
hat_angle = np.arccos(hat[:,0])
```

In [186…

```
thigh = N_thigh_v * x
thigh = thigh.astype(float)
thigh_angle = np.arccos(thigh[:,0])
```

In [187…

```
shank = N_shank_v * x
shank = shank.astype(float)
shank_angle = np.arccos(shank[:,0])
```

In [188…

```
y = np.zeros([171, 2])
y[:, 1] = 1
foot = N_foot_v * y
foot = foot.astype(float)
foot_angle = np.arccos(foot[:,1])
```

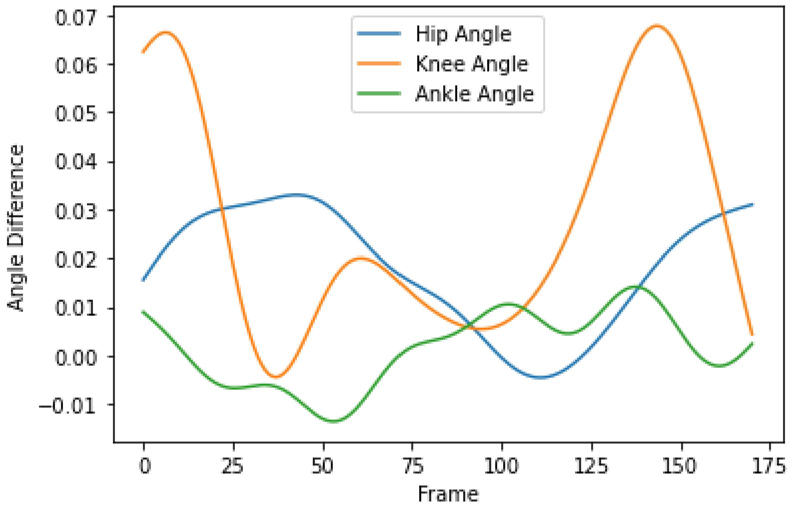
In [189…

```
hip_angle = thigh_angle - hat_angle
knee_angle = thigh_angle - shank_angle
ankle_angle = foot_angle - shank_angle
```

In [190…

```
plt.plot(hip_angle, label = "Hip Angle")
plt.plot(knee_angle, label = "Knee Angle")
plt.plot(ankle_angle, label = "Ankle Angle")
plt.xlabel("Frame")
plt.ylabel("Angle Difference")
plt.legend()
```

Out[190]: <matplotlib.legend.Legend at 0x14b076730>



3)

In [272…

```
Dt = 1/120
vel_foot = []
acc_foot = []
for i in range(1,170):
    vel = (foot_angle[i + 1] - foot_angle[i - 1])/(2*Dt)
    vel_foot.append(vel)
    acc = (foot_angle[i - 1] - 2*foot_angle[i] + foot_angle[i + 1])/(Dt**2)
    acc_foot.append(acc)

vel_foot = np.array(vel_foot)
acc_foot = np.array(acc_foot)
```

In [273…

```
vel_shank = []
acc_shank = []
for i in range(1,170):
    vel = (shank_angle[i + 1] - shank_angle[i - 1])/(2*Dt)
    vel_shank.append(vel)
    acc = (shank_angle[i - 1] - 2*shank_angle[i] + shank_angle[i + 1])/(Dt**
    acc_shank.append(acc)

vel_shank = np.array(vel_shank)
acc_shank = np.array(acc_shank)
```

In [274…

```
vel_thigh = []
acc_thigh = []
for i in range(1,170):
    vel = (thigh_angle[i + 1] - thigh_angle[i - 1])/(2*Dt)
    vel_thigh.append(vel)
    acc = (thigh_angle[i - 1] - 2*thigh_angle[i] + thigh_angle[i + 1])/(Dt**
    acc_thigh.append(acc)

vel_thigh = np.array(vel_thigh)
acc_thigh = np.array(acc_thigh)
```

In [275…

```
vel_hat = []
acc_hat = []
for i in range(1,170):
    vel = (hat_angle[i + 1] - hat_angle[i - 1])/(2*Dt)
    vel_hat.append(vel)
    acc = (hat_angle[i - 1] - 2*hat_angle[i] + hat_angle[i + 1])/(Dt**2)
    acc_hat.append(acc)

vel_hat = np.array(vel_hat)
acc_hat = np.array(acc_hat)
```

4)

In [279…

```
lvel_foot = []
lacc_foot = []
for i in range(1,170):
    vel = (foot_v[i + 1] - foot_v[i - 1])/(2*Dt)
    lvel_foot.append(vel)
    acc = (foot_v[i - 1] - 2*foot_v[i] + foot_v[i + 1])/(Dt**2)
    lacc_foot.append(acc)

lvel_foot = np.vstack(lvel_foot)
lacc_foot = np.vstack(lacc_foot)
```

In [280…

```
lvel_shank = []
lacc_shank = []
for i in range(1,170):
    vel = (shank_v[i + 1] - shank_v[i - 1])/(2*Dt)
    lvel_shank.append(vel)
    acc = (shank_v[i - 1] - 2*shank_v[i] + shank_v[i + 1])/(Dt**2)
    lacc_shank.append(acc)

lvel_shank = np.vstack(lvel_shank)
lacc_shank = np.vstack(lacc_shank)
```

In [281…

```
lvel_thigh = []
lacc_thigh = []
```

```
for i in range(1,170):
    vel = (thigh_v[i + 1] - thigh_v[i - 1])/(2*Dt)
    lvel_thigh.append(vel)
    acc = (thigh_v[i - 1] - 2*thigh_v[i] + thigh_v[i + 1])/(Dt**2)
    lacc_thigh.append(acc)

lvel_thigh = np.vstack(lvel_thigh)
lacc_thigh = np.vstack(lacc_thigh)
```

In [282…

```
lvel_hat = []
lacc_hat = []
for i in range(1,170):
    vel = (hat_v[i + 1] - hat_v[i - 1])/(2*Dt)
    lvel_hat.append(vel)
    acc = (hat_v[i - 1] - 2*hat_v[i] + hat_v[i + 1])/(Dt**2)
    lacc_hat.append(vel)

lvel_hat = np.vstack(lvel_hat)
lacc_hat = np.vstack(lacc_hat)
```

5)

Foot Segment

In [199…

```
height = 1.7526
mass = 66.7
```

In [200…

```
segment_length_foot = 0.055 * height
radius_foot = 0.475 * segment_length_foot
segment_mass_foot = 0.0145 * mass
I_foot = segment_mass_foot * radius_foot**2
print("Segment Length: ", segment_length_foot)
print("Radius: ", radius_foot)
print("Segment Mass: ", segment_mass_foot)
print("Intertia: ", I_foot)
```

```
Segment Length:  0.09639299999999999
Radius:  0.04578667499999999
Segment Mass:  0.9671500000000001
Intertia:  0.0020275522234474 22
```

Shank Segment

In [201…

```
segment_length_shank = 0.246 * height
radius_shank = 0.302 * segment_length_shank
segment_mass_shank = 0.0465 * mass
I_shank = segment_mass_shank * radius_shank**2
print("Segment Length: ", segment_length_shank)
print("Radius: ", radius_shank)
print("Segment Mass: ", segment_mass_shank)
print("Intertia: ", I_shank)
```

```
Segment Length:  0.43113959999999996
Radius:  0.13020415919999997
Segment Mass:  3.10155
Intertia:  0.05258095886699783
```

Thigh Segment

In [202…

```
segment_length_thigh = 0.245 * height
```

```
radius_thigh = 0.323 * segment_length_thigh
segment_mass_thigh = 0.1 * mass
I_thigh = segment_mass_thigh * radius_thigh**2
print("Segment Length: ", segment_length_thigh)
print("Radius: ", radius_thigh)
print("Segment Mass: ", segment_mass_thigh)
print("Intertia: ", I_thigh)
```

```
Segment Length:  0.42938699999999996
Radius:  0.13869200099999998
Segment Mass:  6.670000000000001
Intertia:  0.12830059251303128
```

HAT Segment

In [203…

```
segment_length_hat = 0.47 * height
radius_hat = 0.496 * segment_length_hat
segment_mass_hat = 0.678 * mass
I_hat = segment_mass_hat * radius_hat**2
print("Segment Length: ", segment_length_hat)
print("Radius: ", radius_hat)
print("Segment Mass: ", segment_mass_hat)
print("Intertia: ", I_hat)
```

```
Segment Length:  0.823722
Radius:  0.408566112
Segment Mass:  45.222600000000001
Intertia:  7.548839841594774
```

6)

In [204…

```
data_3
```

Out[204]:

	FX	FY	TY	POSX
0	14.816	85.273	0.204	-0.0935
1	-0.708	174.375	0.486	-0.0895
2	-9.544	215.167	0.686	-0.0861
3	-43.671	224.719	0.453	-0.0799
4	-77.349	273.885	0.344	-0.0742
...	...	...	...	...
81	36.190	146.510	0.482	0.1708
82	29.638	112.442	0.141	0.1762
83	23.841	86.878	0.050	0.1772
84	18.045	65.591	-0.233	0.1848
85	13.652	48.544	-0.305	0.1834

86 rows × 4 columns

In [205…

```
weight = mass * 9.81
```

In [206…

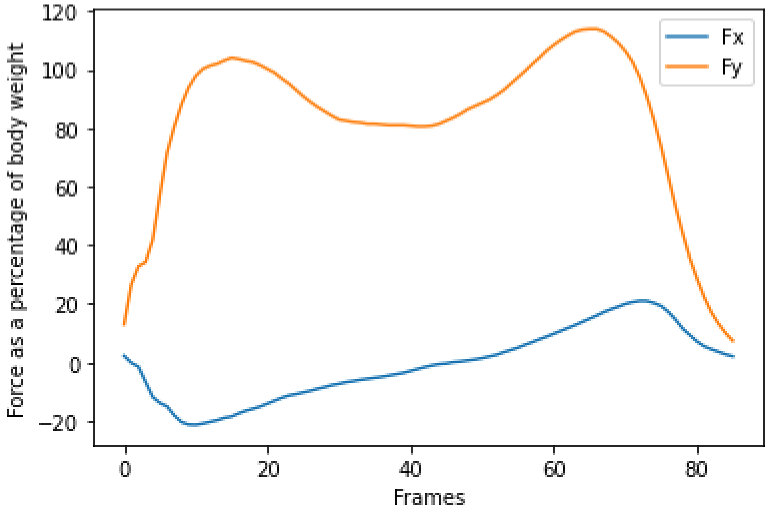
```
fx = (data_3["FX"] / weight) * 100
fy = (data_3["FY"] / weight) * 100
```

In [207…

```
plt.plot(fx, label = "Fx")
plt.plot(fy, label = "Fy")
plt.xlabel("Frames")
plt.ylabel("Force as a percentage of body weight")
plt.legend()
```

Out[207]:

```
<matplotlib.legend.Legend at 0x14b0fd160>
```



10)

In [208…

```
mag = []

for i in range(len(thigh_v)):
    m = np.sqrt(sum(thigh_v[i]**2))
    mag.append(m)
```

In [363…

```
np.max(mag)
```

Out[363]:

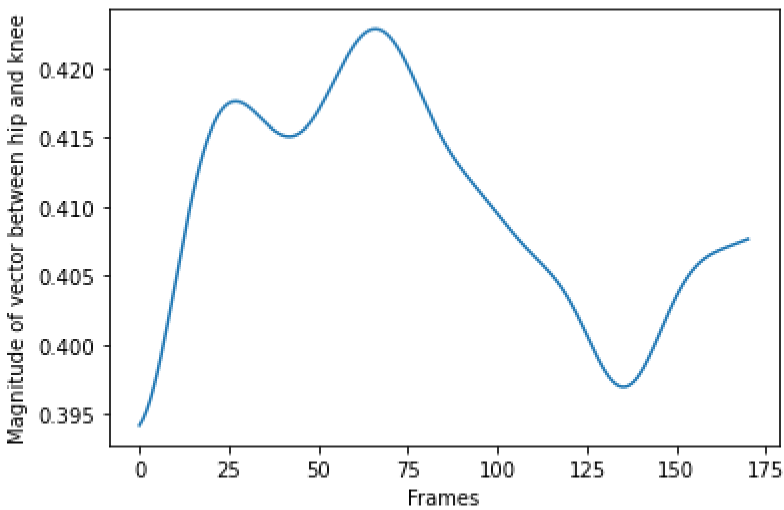
```
0.4228325341940565
```

In [355…

```
plt.plot(mag)
plt.ylabel("Magnitude of vector between hip and knee")
plt.xlabel("Frames")
```

Out[355]:

```
Text(0.5, 0, 'Frames')
```



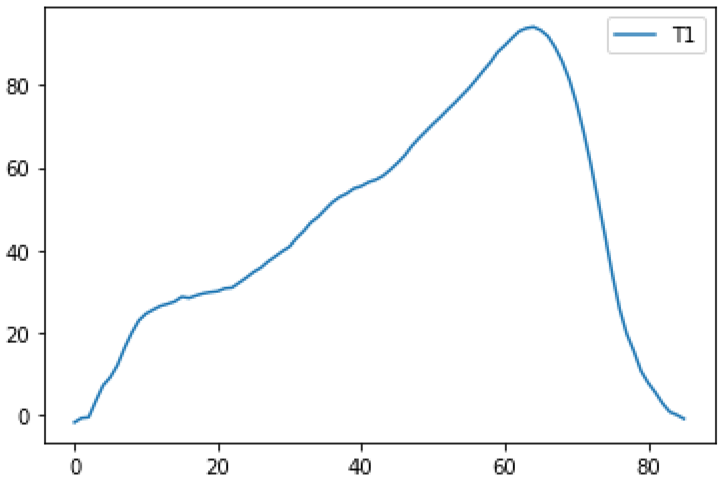
9)

In [295…
```
stance_x = np.array(data_2["Ankle"][41:127])
stance_y = np.array(data_2["Ankle.1"][41:127])
```

In [297…
```
T1 = (np.array(data_3["FY"]) * (np.array(data_3["POSX"]) - stance_x)) - (np.
```

In [298…
```
plt.plot(T1, label = "T1")
plt.legend()
```

Out[298]: <matplotlib.legend.Legend at 0x14b151fa0>



In [301…
```
Fx2 = np.array(data_3["FX"]) - segment_mass_shank * lacc_shank[40:126,0]
Fy2 = np.array(data_3["FY"]) - segment_mass_shank * lacc_shank[40:126,1] - s
```

In [302…
```
Fx1 = np.array(data_3["FX"])
Fy1 = np.array(data_3["FY"])
```

In [328…
```
com_shank = 0.433 * segment_length_shank
```

In [349…
```
T2 = -I_shank*acc_shank[40:126] + T1 + Fx1*com_shank*np.sin(shank_angle[41:1
        Fy1*com_shank*np.cos(shank_angle[41:127]) + \
        Fx2*(segment_length_shank - com_shank)*np.sin(shank_angle[41:127]) -
        Fy2*(segment_length_shank - com_shank)*np.cos(shank_angle[41:127])
```

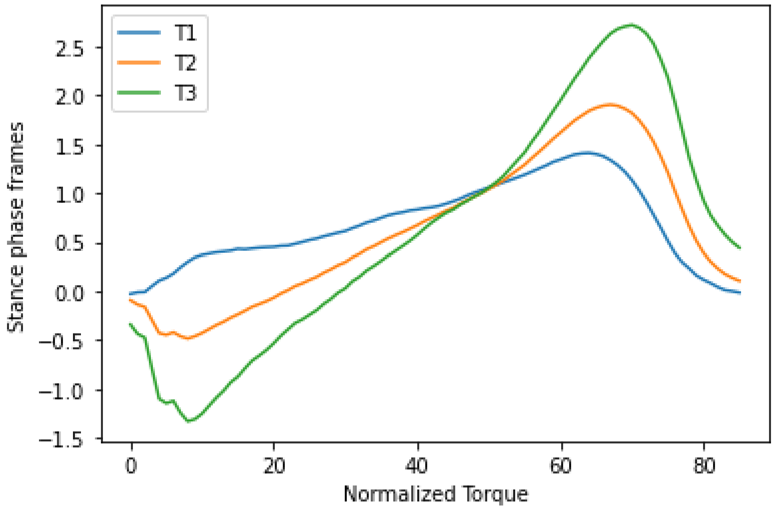
In [350…
```
Fx3 = Fx2 - segment_mass_thigh * lacc_thigh[40:126,0]
Fy3 = Fy2 - segment_mass_thigh * lacc_thigh[40:126,1] - segment_mass_thigh *
```

In [351…
```
com_thigh = com_shank
```

In [352…
```
T3 = -I_thigh*acc_thigh[40:126] + T2 + Fx2*com_thigh*np.sin(thigh_angle[41:1
        Fy2*com_thigh*np.cos(thigh_angle[41:127]) + \
        Fx3*(segment_length_thigh - com_thigh)*np.sin(thigh_angle[41:127]) -
        Fy3*(segment_length_thigh - com_thigh)*np.cos(thigh_angle[41:127])
```

In [365…
```
plt.plot(T1/mass, label = "T1")
plt.plot(T2/mass, label = "T2")
plt.plot(T3/mass, label = "T3")
plt.xlabel("Normalized Torque")
plt.ylabel("Stance phase frames")
plt.legend()
```

Out[365]: <matplotlib.legend.Legend at 0x14b9d5220>


T1
T2
T3
2.5
2.0
1.5
1.0
0.5
0.0
−0.5
−1.0
−1.5
Stance phase frames
0
20
40
60
80
Normalized Torque

In [ ]: