Autonomous Ground Vehicle Path Tracking

Jeff Wit
Wintec, Inc.
104 Research Road, Building 9738
Tyndall Air Force Base, Florida 32403

Carl D. Crane III* and David Armstrong
Center for Intelligent Machines and Robotics
Department of Mechanical
and Aerospace Engineering
University of Florida
Gainesville, Florida 32611
e-mail: ccrane@ufl.edu

Autonomous ground vehicle navigation requires the integration of many technologies such as path planning, position and orientation sensing, vehicle control, and obstacle avoidance. The work presented here focuses on the control of a nonholonomic ground vehicle as it tracks a given path. A new path tracking technique called "vector pursuit" is presented. This new technique is based on the theory of screws, which was developed by Sir Robert Ball in 1900. It generates a desired vehicle turning radius based on the vehicle's current position and orientation relative to the position of a point ahead on the planned path and the desired orientation along the path at that point. The vector pursuit algorithm is compared to other geometrical approaches, and it is shown to be more robust, resulting in more accurate path tracking.   © 2004 Wiley Periodicals, Inc.

1. INTRODUCTION

An autonomous vehicle is one that is capable of automatic navigation. It is self-acting and self-

regulating, therefore it is able to operate in and react to its environment without outside control. The process of automating vehicle navigation can be broken down into four steps: (1) perceiving and modeling the environment, (2) localizing the vehicle within the environment, (3) planning and deciding the vehicle's desired motion and (4) executing the vehicle's

desired motion.[1] Recently, there has been much interest and research done in each of these areas.

Kim[2] presents a method whereby longitudinal and lateral control are combined to control vehicle steering and speed as a vehicle follows a prescribed path. The paper also shows how range data from ultrasonic and laser range sensors can be combined to detect obstacles and how this data is incorporated with the lateral controller. Lenain[3] introduces a nonlinear adaptive control law in order to preserve guidance precision in the presence of sliding. Lizarralde[4] presents a method for mobile robot navigation where dead-reckoning data is combined with range sensor data via a Kalman filter to provide input to a nonlinear model predictive control (NMPC) algorithm. Scalzo[5] presents a new minimalist approach to the navigation problem called micronavigation. Here a mobile vehicle succeeds in finding a path to the goal with no symbolic planning. Yang[6] presents the design of a novel tracking controller for a mobile robot by integrating the neural dynamics model into a conventional feedback controller. Connor[7] shows how global control behaviors can be achieved through composition of low-level controls in a manner that guarantees performance. Loizou[8] achieves vehicle control via the smoothing of nonsmooth navigation functions. The approach is shown to be particularly applicable on systems that have limited computational resources. A multitude of other references could be listed, but the above list represents some of the most recently presented work.

The work presented here in this paper focuses on executing the vehicle's desired motion. A new path-tracking algorithm is developed based on the theory of screws,[9] and is therefore considered a geometric technique. Typically, geometric techniques use a look-ahead point, which is on the planned path at a distance D from the vehicle, to determine the desired motion of the vehicle. Unfortunately, there is a tradeoff in determining the distance D. Increasing D tends to dampen the system leading to a stable system with less oscillation. On the other hand, increasing D also tends to cause the vehicle to cut corners of a path. Therefore, it is desirable to have a small look-ahead distance in order to accurately navigate the path, but, out of necessity, a large value is typically used to achieve a stable system with little oscillation.

A factor that must be considered when choosing the look-ahead distance is the vehicle speed. As the vehicle speed increases, the look-ahead distance typically needs to be increased, too. Having a look-ahead distance greater than zero allows the vehicle to start turning before it actually reaches a curve in the path.

Starting the turn early is desirable due to the fact that a certain amount of time is required for the vehicle to execute a commanded turn. The faster the vehicle is going, the sooner the vehicle needs to start its turn.

Another factor that must be considered when choosing the look-ahead distance is the anticipated vehicle position and heading errors. These errors are obviously preferably small. Unfortunately, this is not always the case. One example where large position and heading errors may be expected is if an unexpected obstacle is encountered. Large errors may exist once the vehicle navigates around the obstacle and then continues to track the desired path.

Ideally then, a geometric path tracking technique would allow small look-ahead distances to accurately track the given path, and not be sensitive to small changes in vehicle speed. In addition, it would be able to handle large position and heading errors by quickly converging to the planned path, if possible without much oscillation.

Current geometric path tracking methods, such as pure pursuit[10–13] or follow-the-carrot,[14] only use a desired position in order to determine a desired vehicle motion. Another current path tracking method, proportional path tracking,[15] uses both a desired position and heading, but is geometrically meaningless by adding terms with different units. Vector pursuit is a new geometric path-tracking algorithm that takes advantage of a desired orientation as well as a desired position while remaining geometrically meaningful.

2. SCREW THEORY BASICS

Screw theory, as developed in the pioneering work of Sir Robert Stawell Ball,[9] can be used to describe the instantaneous motion of a moving rigid body relative to a given coordinate system. It therefore is natural and appropriate to use screw theory to represent the instantaneous desired motion of an autonomous ground vehicle (AGV), i.e., a rigid body, from its current position and orientation to a desired position and orientation that is on a given path. Before developing the new path tracking method, a brief overview of the screw theory used in this method is presented.

A screw consists of a centerline that is defined in a given coordinate system and a pitch. The motion of a rigid body at any instant can be represented as if it was attached to a screw and rotating about that screw at some angular velocity. A screw that describes the instantaneous motion of a rigid body is called an instantaneous screw.
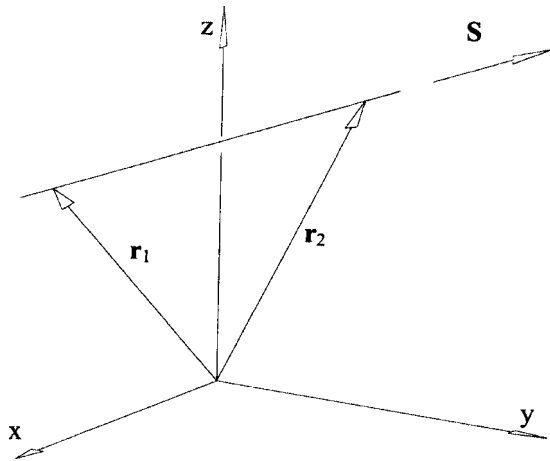
Figure 1. Line defined by two points.

One way to define the centerline of a screw is by using Plücker line coordinates. Two points given by the vectors $\mathbf{r}_1$ and $\mathbf{r}_2$ in a given coordinate system define a line as shown in Figure. This line can also be defined as a unit vector, $\mathbf{S}$, in the direction of the line and a moment vector, $\mathbf{S}_0$, of the line about the origin. From Figure 1 we see that

$$\mathbf{S} = \frac{\mathbf{r}_2 - \mathbf{r}_1}{|\mathbf{r}_2 - \mathbf{r}_1|} \tag{1}$$

and

$$\mathbf{S}_0 = \mathbf{r}_1 \times \underline{\mathbf{S}}. \tag{2}$$

It can be seen from (1) and (2) that the components of $\mathbf{S}$, the direction of the line, are dimensionless and the components for $\mathbf{S}_0$, the moment of the line, have units of length.

The vectors $(\mathbf{S};\mathbf{S}_0)$ are the Plücker line coordinates of this line. By defining $\mathbf{S}=[L,M,N]^{\mathrm{T}}$ and $\mathbf{S}_0 = [P,Q,R]^{\mathrm{T}}$, and noting that $\mathbf{r}_1=[x_1,y_1,z_1]^{\mathrm{T}}$ and $\mathbf{r}_2 = [x_2,y_2,z_2]^{\mathrm{T}}$, it is seen that

$$L = \frac{x_2 - x_1}{\sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}}, \tag{3}$$

$$M = \frac{y_2 - y_1}{\sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}}, \tag{4}$$

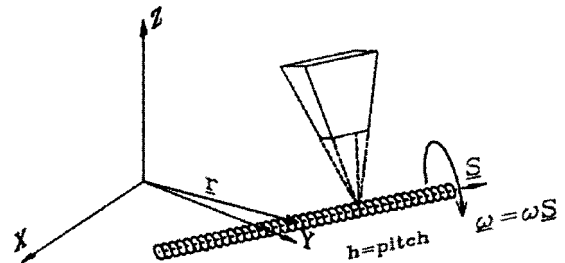$$N = \frac{z_2 - z_1}{\sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}}, \tag{5}$$

Figure 2. Instantaneous motion about a screw.

and

$$P = y_1 N - z_1 M, \tag{6}$$

$$Q = z_1 L - x_1 N, \tag{7}$$

$$R = x_1 M - y_1 L. \tag{8}$$

Figure 2 depicts the instantaneous motion of a rigid body rotating with an angular velocity, ω, about a screw, $\$$, that has a centerline defined by $(\mathbf{S};\mathbf{S}_0)$ and that has a pitch, h. The velocity of any point on the rigid body is equal to the velocity due to the rotation plus the translational velocity due to the pitch of the screw. The velocity of the rigid body can be quantified by

$$\omega\$ = (\omega\mathbf{S}; \omega\mathbf{S}_{0h}), \tag{9}$$

where

$$\mathbf{S}_{0h} = \mathbf{S}_0 + h\mathbf{S} = \mathbf{r} \times \mathbf{S} + h\mathbf{S}, \tag{10}$$

and $\mathbf{r}$ is any vector from the origin to the centerline of the screw. The instantaneous velocity of a point in the rigid body is coincident with the origin of the coordinate system, i.e., $\mathbf{v}_0$ is given by

$$\mathbf{v}_0 = \omega\mathbf{S}_{0h}. \tag{11}$$

Two specific screws are used in developing the path tracking algorithms, translation screws and rotation screws. The motion about a screw with an infinite pitch models pure translation of a rigid body at a velocity v along the direction $\mathbf{S}$. In the limit, as the pitch goes to infinity, (9) simplifies to

$$v\$ = (\mathbf{0}; v\mathbf{S}), \tag{12}$$

which is a screw that has a centerline at infinity.

On the other hand, the motion about a screw whose pitch is equal to zero models pure rotation of a rigid body. By substituting a pitch, h, equal to zero, (9) simplifies to

$$\omega\$ = (\omega\mathbf{S}; \omega\mathbf{S}_0). \tag{13}$$

In addition to using rotation and translation screws, a property of instantaneous screws that proves to be very useful is that they are additive. Note that the units of (12) and (13) are the same, i.e., the first three components have units of rad/time and the last three components have units of length/time, even though (12) is a translation screw and (13) is a rotation screw.

3. VECTOR PURSUIT PATH TRACKING

Vector pursuit is a new geometric path tracking method that uses the theory of screws. It is similar to other geometric methods in that a look-ahead distance is used to define a current goal point, and then geometry is used to determine the desired motion of the vehicle. On the other hand, it is different from current geometric path tracking methods, such as follow-the-carrot or pure pursuit, which do not use the desired orientation of the vehicle at the look-ahead point (the desired orientation at the look-ahead point is assumed to be tangent to the path at the look-ahead point). Proportional path tracking is a geometric method that does use the desired orientation at the look-ahead point. This method adds the current position error multiplied by some gain to the current orientation error multiplied by some gain, and therefore becomes geometrically meaningless since terms with different units are added. Vector pursuit uses both the location and orientation of the look-ahead point while remaining geometrically meaningful.

The first step in vector pursuit calculates two instantaneous screws. The first instantaneous screw, $\$_t$, accounts for the translation from the current vehicle position to the location of the look-ahead point while the second instantaneous screw, $\$_r$, accounts for the rotation from the current vehicle orientation to the desired orientation at the look-ahead point. Again, the desired orientation at the look-ahead point is defined as the direction tangent to the path at the look-
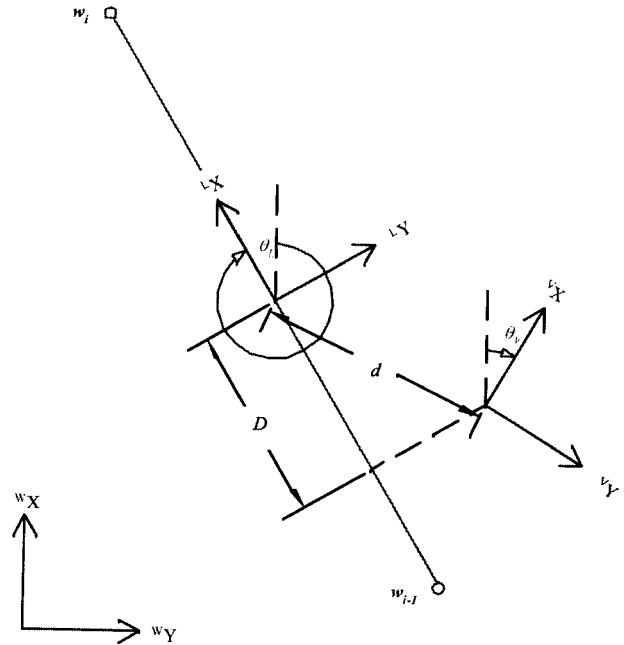

Figure 3. Defined coordinate systems.

ahead point. The second step uses the additive property of instantaneous screws to calculate $\$_d$, the sum of $\$_t$ and $\$_r$, which defines the desired instantaneous motion of the vehicle.

Before developing the screw theory based path tracking methods, a few coordinate systems must first be defined. First, the world coordinate system is defined where the x-axis points north, the z-axis points down and the y-axis points east to form a right hand coordinate system. The origin of the world coordinate system defined here is determined by the conversion from a geodetic coordinate system to a UTM coordinate system. It is assumed that the desired path is given, or can be converted to, the world coordinate system. The world coordinate system can be seen in Figure 3.

In addition to the world coordinate system, both a moving and the vehicle coordinate systems are also shown in Figure 3. A moving coordinate system is defined where the origin is a point on the planned path, the look-ahead point, which is a given distance called the look-ahead distance, D, in front of the orthogonal projection of the vehicle's position onto the planned path. It should be noted that in this work paths were modeled as a series of line segments. If the end of the current path segment is closer than the distance D, then the look-ahead point is determined by intersecting a circle of radius D with the next path line segment. If there is no additional path line segment, then

the look-ahead point is set as the end point of the current path line segment. The x-axis of the moving coordinate system is oriented in the direction of the planned path at that point, i.e., the direction from the previous waypoint w_{i-1} to the current waypoint w_i, the z-axis is down and the y-axis is defined to form a right hand coordinate system. Since the moving coordinate system's origin is located at the look-ahead point, this coordinate system will be referred to as the look-ahead coordinate system. The selection of the distance D will be discussed later.

Finally, the vehicle coordinate system is defined where the x-axis is in the forward direction of the vehicle, the z-axis is down and the y-axis forms a right hand coordinate system. The origin of the vehicle coordinate system depends on the type of vehicle. For vehicles with Ackerman or differential steering, it is defined in a way that decouples the control of the linear and angular velocities. For example, on a car-like vehicle with rear wheel drive, the origin is defined to be the center of the rear axle.

A method is required to indicate the coordinate system in which a vector is referenced since more than one coordinate system was defined here. Therefore, vectors are written with a leading superscript indicating the coordinate system to which they are referenced. With these three coordinate systems and the notation defined, the development of vector pursuit path tracking is now presented.

The method developed to calculate $\$_t$ and $\$_r$ first takes into account the vehicle's nonholonomic and minimum turning radius constraints. In order to satisfy the constraints, the centerlines of the instantaneous screws must be on the vehicle's y-axis and at a distance from the x-axis greater than or equal to the vehicle's minimum turning radius. The requirement that the instantaneous screws' centerlines be a distance greater than or equal to the vehicle's minimum turning radius, from the x-axis is initially ignored. It is ignored at first due to the fact that some vehicles, e.g., a differentially driven vehicle, with nonholonomic constraints have no minimum turning radius. Therefore, the only initial constraint placed on the location of the centerlines of the instantaneous screws is that they must be on the vehicle's y-axis. With this in mind, the point in the XY plane on the centerline of the screw to correct the translational error, $\$_t$ (which in this case represents a rotation and not a translation), was selected as the center of a circle that passes through the origins of the vehicle coordinate system and the look-ahead coordinate system and
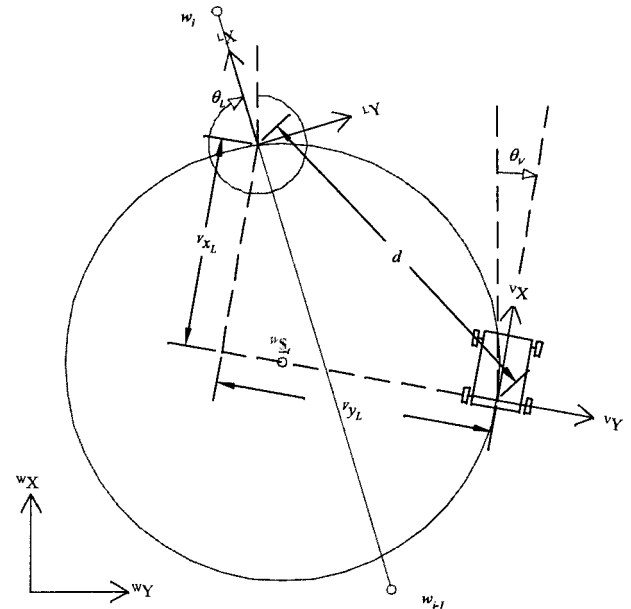

Figure 4. Instantaneous screw for translating to look-ahead point.

which is tangent to the vehicle's current orientation, i.e., the x-axis of the vehicle coordinate system (see Figure 4). Hence, $\$_t$ is defined to be

$$^W\$_t = k_t\left(0,0,1; \, ^Wy_v + \frac{d^2}{2\,^Vy_L}\cos(\theta_v), -\,^Wx_v\right.$$
$$\left. + \frac{d^2}{2\,^Vy_L}\sin(\theta_v),0\right), \qquad (14)$$

where d is the distance from the origin of the vehicle coordinate system to the origin of the look-ahead coordinate system, $(^Vx_L, \, ^Vy_L)$ are the coordinates of the look-ahead coordinate system's origin in the vehicle coordinate system, $(^Wx_V, \, ^Wy_V)$ are the coordinates of the vehicle position in the world coordinate system, and θ_V is the angle from the x-axis of the world coordinate system to the x-axis of the vehicle coordinate system. The term k_t is used as a weighting factor that will be dealt with later. Equation (14) is valid only if the term ^Vy_L is nonzero. Otherwise, $\$_t$ is determined by

$$^W\$_t = k_t\left(0,0,0; \, \frac{^Wx_L - \,^Wx_V}{d}, \frac{^Wy_L - \,^Wy_V}{d},0\right). \qquad (15)$$

The instantaneous screw $\$_r$ is defined to be

$$^{W}\$_r = k_r(0,0,1; {}^{W}y_V, -{}^{W}x_V, 0), \qquad (16)$$

where k_r is a weighting factor. Note that the axis of rotation is chosen to pass through the origin of the vehicle coordinate system so that no translation is associated with $\$_r$.

The desired instantaneous screw is now determined as either

$$
\begin{aligned}
^{W}\$_d &= {}^{W}\$_t + {}^{W}\$_r \\
&= \Bigg(0,0,k_t+k_r\,;k_r\,{}^{W}y_V \\
&\quad + k_t\Big({}^{W}y_V + \frac{d^2}{2\,{}^{V}y_L}\cos(\theta_V)\Big), -k_r\,{}^{W}x_V \\
&\quad + k_t\Big(-{}^{W}x_V + \frac{d^2}{2\,{}^{V}y_L}\sin(\theta_V)\Big),0\Bigg),
\end{aligned}
\qquad (17)
$$

if the term $^{V}y_L$ is nonzero, or

$$
\begin{aligned}
^{W}\$_d &= {}^{W}\$_t + {}^{W}\$_r \\
&= \Bigg(0,0,k_r\,;k_r\,{}^{W}y_V + k_t\Big(\frac{{}^{W}x_L - {}^{W}x_V}{d}\Big), -k_r\,{}^{W}x_V \\
&\quad + k_t\Big(\frac{{}^{W}y_L - {}^{W}y_V}{d}\Big),0\Bigg),
\end{aligned}
\qquad (18)
$$

if the term $^{V}y_L$ is zero.

The weighting factors k_t and k_r are used to control how much the desired instantaneous screw is influenced by $\$_t$ and $\$_r$, respectively. These two weighting factors are related by the time required to translate to the look-ahead point and rotate to the desired orientation. Assuming that the term $^{V}y_L$ is nonzero, note that while the instantaneous screw defined in Eq. (14) describes a motion to translate the vehicle from its current location to the look-ahead point, it also describes a motion that rotates the vehicle. This can easily be seen in Figure 5. Therefore, from Eq. (13), the weighting factor k_t is an angular velocity. The amount of rotation, ϕ, can be determined by

$$\phi = \operatorname{atan2}((2\,{}^{V}y_L^2 - d^2),(2\,{}^{V}x_L\,{}^{V}y_L)) - \operatorname{atan2}\Big(\frac{d^2}{2\,{}^{V}y_L},0\Big).$$
$$(19)$$

where ϕ must be in the interval $(0,2\pi]$ rad. It is noted that the last part of Eq. (19) will always be $\pm\pi/2$ rad depending only on the sign of $^{V}y_L$.
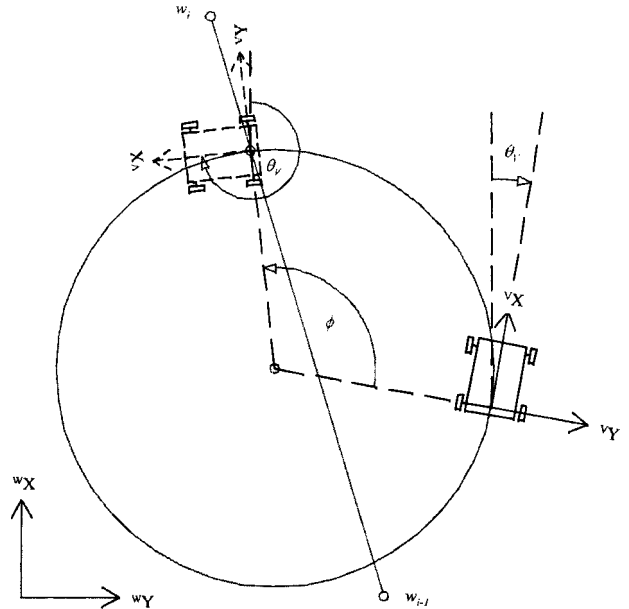
The time required to *translate* from the current ve-

Figure 5. Rotation defined by $\$_t$ instantaneous screw.

hicle position to the look-ahead point, assuming that $k_t = \omega_t$, some angular velocity, is determined by

$$t_t = \frac{\phi}{\omega_t}. \qquad (20)$$

The time required to rotate from the current vehicle orientation to the orientation at the look-ahead point must also account for the rotation, ϕ, due to $\$_t$. This will either increase or decrease the time needed to rotate. Assuming $k_r = \omega_r$, some angular velocity, this time can be determined by

$$t_r = \frac{(\theta_L - \theta_V) - \phi}{\omega_r}. \qquad (21)$$

An assumption is made that the relationship between t_t and t_r can be defined by

$$t_r = k t_t, \qquad (22)$$

where k is some positive constant greater than zero. Therefore, the weighting factors can now be determined from

$$k_t = \omega_t \qquad (23)$$

Figure 6. Navigation test vehicle.

and

$$k_r = \omega_r = \frac{(\theta_L - \theta_V) - \phi}{t_r}$$

$$= \frac{(\theta_L - \theta_V) - \phi}{kt_t} = \frac{\omega_t((\theta_L - \theta_V) - \phi)}{k\phi}. \quad (24)$$

Using Eq. (17), the coordinates of the point in the XY plane on the centerline of the desired screw can be determined in the world coordinate system by

$$^Wx_{\$_d} = {}^Wx_V - \frac{k_t}{k_t + k_r}\left(\frac{d^2}{2\,^Vy_L}\sin(\theta_v)\right)$$

$$= {}^Wx_V - \frac{k\phi}{(k-1)\phi + (\theta_L - \theta_V)}\left(\frac{d^2}{2\,^Vy_L}\sin(\theta_v)\right) \quad (25)$$

and

$$^Wy_{\$_d} = {}^Wy_V + \frac{k_t}{k_t + k_r}\left(\frac{d^2}{2\,^Vy_L}\cos(\theta_v)\right)$$

$$= {}^Wy_V + \frac{k\phi}{(k-1)\phi + (\theta_L - \theta_V)}\left(\frac{d^2}{2\,^Vy_L}\cos(\theta_v)\right). \quad (26)$$

Note that the above calculations of the weighting factors assumed that ^Vy_L was nonzero. If, on the other hand, ^Vy_L is zero, then from Eq. (12), the weighting factor k_t is a linear velocity. The amount of time to translate from the current vehicle position to the look-ahead point at some velocity, $k_t = v$, can be determined by

$$t_t = \frac{d}{v}. \quad (27)$$

The time required to rotate from the current vehicle orientation to the orientation at the look-ahead point can be calculated using Eq. (21) where ϕ is now zero. Therefore, assuming $k_r = \omega$, some angular velocity, this time can be determined by

$$t_r = \frac{\theta_L - \theta_V}{\omega}. \quad (28)$$

Using Eq. (22) for the relationship between the two times, the weighting factors can be determined from
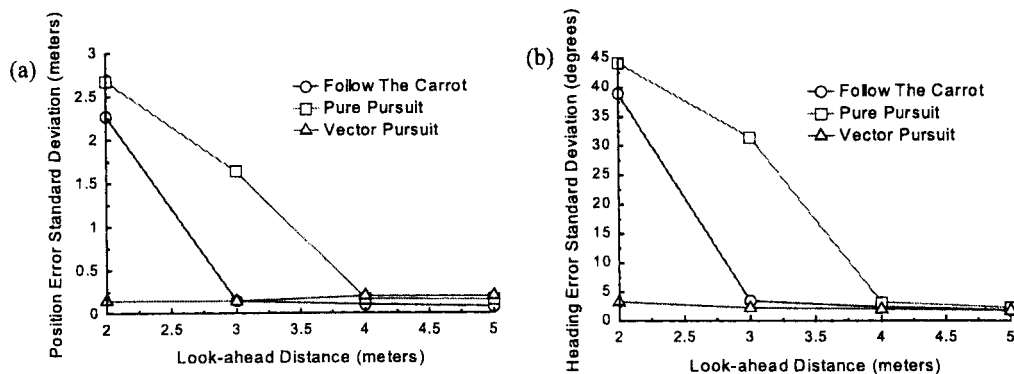

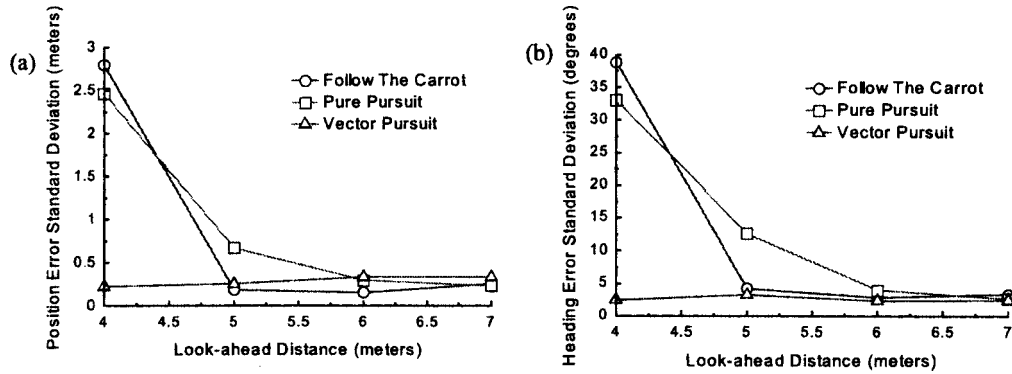**Figure 7.** Standard deviation of position and heading errors of a "U" shape path at 2 mps.

Figure 8. Standard deviation of position and heading errors of a "U" shape path at 3 mps.

$$k_t = v \qquad (29)$$

and

$$k_r = \omega = \frac{\theta_L - \theta_V}{t_r} = \frac{\theta_L - \theta_V}{k t_t} = \frac{v(\theta_L - \theta_v)}{kd}. \qquad (30)$$

Using Eq. (18), the coordinates of the point in the XY plane on the centerline of the desired screw can be determined in the world coordinate system by

$$^W x_{\$_d} = {}^W x_V - \frac{k_t}{k_r} \left(\frac{^W y_L - {}^W y_v}{d} \right) = {}^W x_V - k \left(\frac{^W y_L - {}^W y_v}{\theta_L - \theta_V} \right) \qquad (31)$$

and

$$^W y_{\$_d} = {}^W y_V + \frac{k_t}{k_r} \left(\frac{^W x_L - {}^W x_v}{d} \right) = {}^W y_V + k \left(\frac{^W x_L - {}^W x_v}{\theta_L - \theta_V} \right). \qquad (32)$$

Finally, the coordinates of the point in the XY plane on the centerline of the desired instantaneous screw can be determined in the vehicle coordinate system to determine the desired motion of the vehicle by

$$^V x_{\$_d} = {}^W x_V \cos(\theta_V) + {}^W y_V \sin(\theta_V) - ({}^W x_{\$_d} \cos(\theta_V) + {}^W y_{\$_d} \sin(\theta_V)) \qquad (33)$$

and

$$^V y_{\$_d} = -{}^W x_V \sin(\theta_V) + {}^W y_V \cos(\theta_V) - (-{}^W x_{\$_d} \sin(\theta_v) + {}^W y_{\$_d} \cos(\theta_v)). \qquad (34)$$

Recall that the vehicle's nonholonomic constraints were considered when calculating $\$_t$ and $\$_r$ but that the minimum turning radius was ignored. This has a nice result where $^V x_{\$_d}$ will always equal
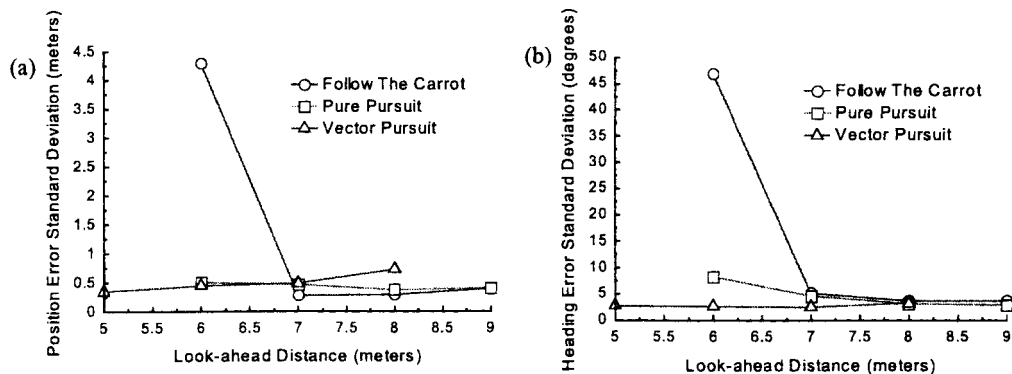

Figure 9. Standard deviation of position and heading errors of a "U" shape path at 4 mps.
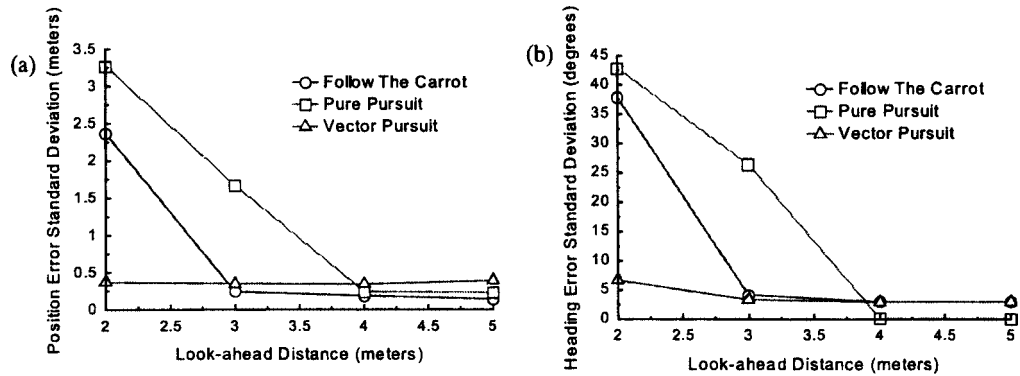
Figure 10. Standard deviation of position and heading errors of a figure-eight path at 2 mps.

zero, which does not break the nonholonomic constraints. In order to comply with the minimum turning radius constraint, the magnitude of $^{V}y_{\$_d}$ must be greater than or equal to the minimum turning radius.

An additional constraint on the vehicle's motion is imposed by requiring the vehicle not to change direction, i.e., forward to reverse or reverse to forward. Violation of this constraint is easily determined by the location of the look-ahead point relative to the vehicle coordinate system. When the vehicle is traveling forward, and the look-ahead point's x-value is negative, the vehicle direction would have to change from forward to reverse. Likewise, when the vehicle is traveling backward, and the look-ahead point's x-value is positive, the vehicle direction would have to change from reverse to forward. If either of these cases is violated, the vehicle is simply commanded to turn around.

Finally, it is important to note that the look-ahead distance, D, and the constant, k, are free choices in this method and as such represent parameters that must be selected in order to optimize or tune the vehicle's performance.

4. RESULTS

The Navigation Test Vehicle,[14] shown in Figure 6, is used to test the new path tracking technique. As mentioned earlier, two of the main factors for choosing the look-ahead distance are the vehicle speed and the anticipated position and heading errors. In order to test the new path tracking technique's sensitivity to the vehicle speed, two different paths are chosen. The first path is a "U" shape path with 60-m straight sections and a 15-m turning radius on the curved section. The second path is a figure eight path with a 15-m turning radius on each curved section. The look-ahead distance is initially chosen to be small and then slowly increased for successive runs. This is done for vehicle speeds of 2, 3, and 4 m/s. For comparison, the same tests are done using follow-the-carrot and pure pursuit path tracking techniques.

Results of these tests are given in Figures 7–9 for the "U" shape path and in Figures 10–12 for the figure-eight path. From these figures it is noted that for follow-the-carrot and pure pursuit a minimum look-ahead distance to achieve stability for a given
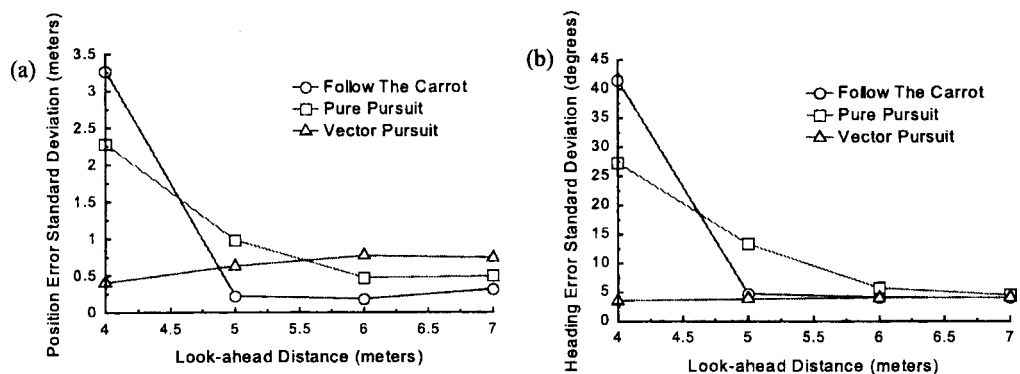

Figure 11. Standard deviation of position and heading errors of a figure-eight path at 3 mps.
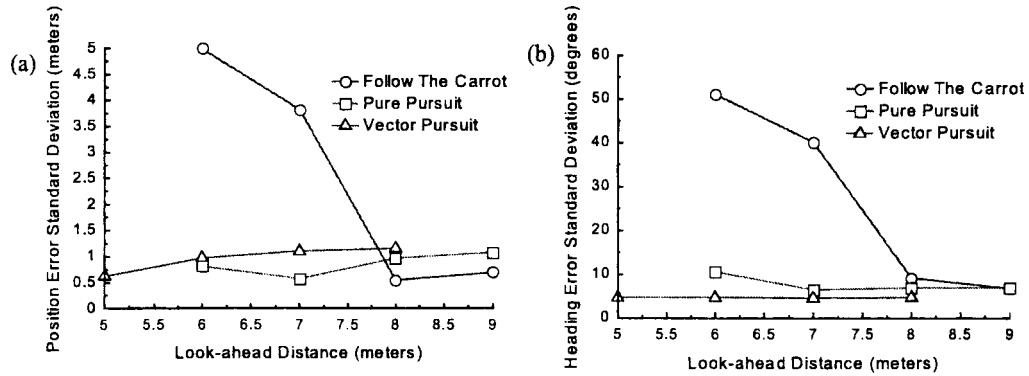
Figure 12. Standard deviation of position and heading errors of a figure-eight path at 4 mps.

velocity can be determined, while vector pursuit is stable over the entire range of look-ahead distances tested. Remaining stable over a range is definitely desirable. For example, suppose follow-the-carrot path tracking is utilized with a look-ahead distance of 3 m and a tracking speed of 2 m/s. Good results are obtained under normal circumstances. But, if the speed were to increase just slightly, e.g., going down a hill, stability would definitely be a concern. A similar example could be made of pure pursuit path tracking.

In addition to being concerned about the path tracking technique's sensitivity of the look-ahead distance to various speeds, the sensitivity of the path tracking technique to large position and heading errors is also a concern. In order to test this, a straight path is used with a jog in the middle, where this jog is varied from 2 to 6 m. This is again tested for follow-the-carrot and pure pursuit path tracking techniques in addition to vector pursuit, at tracking speeds of 2, 3, and 4 m/s. For each speed, a look-ahead distance is chosen so that all three path-tracking techniques would perform well with small position and heading

errors. The results of these tests are summarized in Figures 13–15 by looking at the overshoot of the vehicle position after the jog in the path. In each test, vector pursuit had a smaller overshoot than both follow-the-carrot and pure pursuit. In addition, follow-the-carrot resulted in large oscillations after the jog from which it could not recover for speeds of 3 and 4 m/s.

5. CONCLUSION

A new geometric path tracking technique based on the theory of screws was developed. This technique generates a desired vehicle turning radius based on the vehicle's current position and orientation relative to the position and orientation of a point ahead on the planned path. Previous techniques only considered the position of a point ahead on the path, or become geometrically meaningless when the orientation is considered in addition to the position of a point ahead on the path. The vector pursuit algorithm is
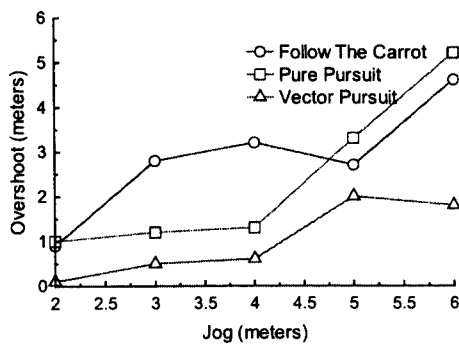

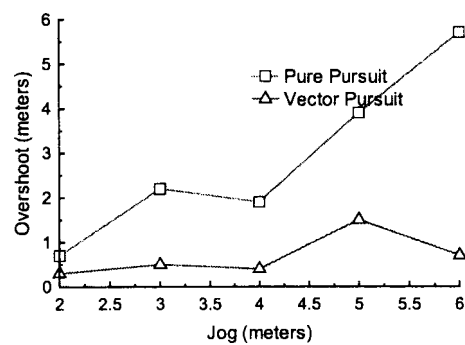**Figure 13.** Overshoot of vehicle position after jog at 2 mps.

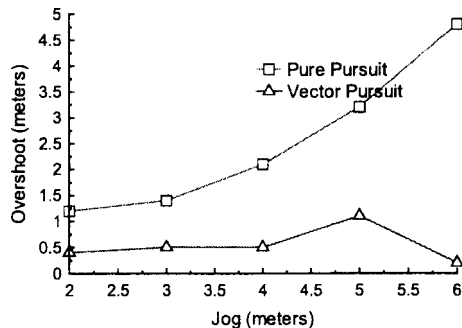**Figure 14.** Overshoot of vehicle position after jog at 3 mps.

Figure 15. Overshoot of vehicle position after jog at 4 mps.

compared to other approaches and it is shown that the new technique is more robust with respect to the sensitivity of the look-ahead distance to the current vehicle speed and the ability to handle sudden large position and heading errors.

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