Swarm Aggregations Using Artificial Potentials and Sliding Mode Control

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***Abstract*— In this brief article we consider a control strategy of multi-agent systems or simply swarms based on artificial potential functions and the sliding mode control technique. First, we briefly discuss a "kinematic" swarm model in n-dimensional space introduced in an earlier work. In this model the inter-individual interactions are based on artificial potential functions and the motion of the individuals is along the negative gradient of the combined potential. After that we consider a general model for vehicle dynamics of each agent (swarm member) and use sliding mode control theory to force their motion to obey the dynamics of the kinematic model. In this context, the results for the initial model serve as a "proof of concept" for multi-agent coordination and control (swarm aggregation), whereas the present results serve as possible implementation method for engineering swarms with given vehicle dynamics. The presented control scheme is robust with respect to disturbances and system uncertainties.**

I. Introduction

In nature, coordinated and cooperative behavior or simply swarming behavior can be seen in a large range of beings starting from simple bacteria to mammals. Examples of swarms include flocks of birds, schools of fish, herds of animals, and colonies of bacteria. The evolution of such behavior could be due to its inherent advantages such as avoiding predators and increasing the chance of finding food. In [1] Grünbaum explains how social foragers as a group more successfully perform chemotaxis over noisy gradients than individually. In other words, individuals do much better collectively compared to the case when they forage on their own.

It is amazing to observe how many biological societies which consist of relatively simple members (creatures) can collectively perform complex, meaningful, and intelligent tasks. Developing systems consisting of multiple autonomous agents, such as mobile robots or unmanned undersea or areal vehicles, that (similarly to biological swarms) can perform coordinated motion or cooperatively perform tasks is of paramount importance for the engineering community. Such systems may allow tasks which are impossible to perform by traditional robotic systems to be easily performed. Moreover, they could be more flexible and robust. Furthermore, it may be possible to build such systems for cheaper than the traditional intelligent autonomous robotic systems.

One approach to develop engineering swarming systems is to study biological swarms and apply the derived principles (governing their motion) to the design of engineering swarms. In [2], [3] we considered a biologically inspired n-dimensional continuous time synchronous swarm model based on artificial potentials and obtained results on cohesive swarm aggregation. In [4] the model in [2], [3] was augmented with a term representing the environment and convergence to (divergence from) more favorable regions (unfavorable regions) was shown. Similar results based on artificial potentials and virtual leaders were independently obtained by Leonard and coworkers in [5], [6] for agents with point mass dynamics. In [6] they also considered sampling affects on the swarm motion and gradient descent. In [7] the same authors also extended their model in [5] to the multi-agent formation control problem.

Artificial potential functions have been used extensively for robot navigation and control. See for example [8], [9]. One of the first works applying artificial potentials to agent coordination is by Reif and Wang [10], where they consider distributed control approach of groups of robots, called *social potential fields* method. Similar work is by Yamaguchi [11], where he describes a cooperative control method for coordinating the motion of a group of holonomic mobile robots to capture/enclose a target by making group formations.

The above mentioned articles mostly consider continuous time synchronous models. In the literature there are also approaches working with discrete time models and involving asynchronism and imperfect information (due to sensing and/or communication delays). In [12] swarm stability under *total asynchronism* (i.e., asynchronism with time delays) was considered. The authors consider one dimensional discrete time totally asynchronous swam model for both stationary and mobile swarms and prove asymptotic convergence under total asynchronism conditions and finite time convergence under *partial asynchronism* conditions (i.e., total asynchronism with a bound on the maximum possible time delay). For the mobile swarm case they prove that cohesion will be preserved during motion under conditions, expressed as bounds on the maximum possible time delay. In [13] the work in [12] has been extended to the multi-dimensional case by imposing special constraints on the "leader" movements and using specific communication topology. In [14] we obtained similar results to those in [12] for a swarm with a different mathematical model for the inter-member interactions using some earlier results developed for parallel and distributed computation.

There is also a relevant literature on formation control of autonomous vehicles. Some of the work we would like to mention here include the work by Desai and his colleagues in [15], [16], where the authors use graph theory to model a formation of robots navigating a terrain with obstacles; Egerstedt and coworkers in [17], [18], where the authors develop a formation control strategy based on formation functions (which are in a sense potential functions) and use a kinematic model for the robot motions and control Lyapunov functions for controller development; Olfati-Saber and Murray [19], where the authors develop a formation control strategy by using concepts from graph theory and Lyapunov theory; Lawton and his colleagues [20], where the authors consider nonholonomic robots and develop a strategy based on decomposing complex formation maneuvers into a sequence of maneuvers between formation patterns. In [21] we also developed a formation control strategy based on output regulation (servomechanism) techniques.

In this article we consider a control strategy of swarms based on artificial potential functions and sliding mode control. The sliding mode control method is an important technique that has been used extensively for robot navigation and control (we will not mention these here). It has a variety of advantageous properties, which make it an attractive control technique. These properties include its robustness to system uncertainties and external disturbances and its ability to reduce the problem of controller design to a lower dimensional space with the choice of an appropriate switching surface. In [22], [23], [24] the sliding mode control technique was used for robot navigation and obstacle avoidance in an environment modeled with harmonic potentials. The strategy was based on forcing the motion of the robot along the gradient of the potential field representing the obstacles. In this article, we show that a similar procedure can be applied also for implementing engineering aggregating swarm models such as those considered in [2], [3] and foraging swarms considered in [4] as well as formation control. Therefore, the results presented here serve as an implementation method of the earlier results in [2], [3], [4]. However, the presented procedure is general and is not limited only to the models in [2], [3], [4], but can be applied also to other similar models or contexts. An initial version of this article appeared in [25]. Recently we used similar ideas to those considered here for

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tracking/intercepting moving targets [26].

II. A "Kinematic" Model for Aggregating Swarms

Consider a multi-agent system (i.e., a swarm) consisting of N individuals (members) in an n-dimensional Euclidean space. Let $x^i \in \mathbb{R}^n$ denote the position vector of individual i and define $x^\top = [x^{1\top}, \ldots, x^{N\top}]$ as the vector of the positions of all the agents in the swarm. Assume that the agents are required to move based on the equation

$$\dot{x}^i = -\nabla_{x^i} J(x), i = 1, \ldots, N, \tag{1}$$

where $J : \mathbb{R}^{nN} \to \mathbb{R}$ is a potential function. It is to be specified (i.e., chosen) by the multi-agent system designer based on the desired structure and/or behavior of the swarm. It can be a function that represents the (artificial) potential energy due to the relative configuration of the agents or some kind of an artificial potential function that is used to specify the inter-individual interactions (or simply the attraction/repulsion relationship between the swarm members). Such potential functions are being used for swarm aggregations, formation control, and multi-agent coordination and control under different names. For example, in [2], [3], [4] we used attraction/repulsion functions for swarm aggregations, in [5], [6] Leonard and coworkers used similar potentials for control of a group of point-mass agents. Similarly, in [18] Egerstedt and Hu used formation functions, whereas in [19] Olfati-Saber and Murray used structural potentials for formation control. Although the motions in all of the above articles may not necessarily be of the form of (1), their definition of the desired "structure" of the swarm of agents is in terms of the system (or environment) potential. In this article we consider a multi-agent system consisting of agents with general vehicle dynamics. It is assumed that the inter-agent interactions (or system structure) is defined in terms of a potential function $J(x)$ with desired motion as in (1). We show that using sliding mode control it is possible to force the agents to obey the motion in (1) and therefore achieve the desired behavior or structure (or configuration). This leads to recovering the existing results for the model in (1).

In this article we will keep the discussion in the context of swarm aggregations and use potential functions of type considered in [2], [3], [4]. However, we would like also to emphasize that the procedure can be used in other contexts as well. The potential $J(x)$ may represent only the inter-individual interactions as in [2], [3] or may include also environmental effects as in [4] or may be defined for some other purpose. For simplicity consider the case in which it does not include any environmental terms. In particular consider the potential functions of the form

$$J(x) = \sum_{i=1}^{N-1} \sum_{j=i+1}^{N} J_{ij}(\|x^i - x^j\|), \tag{2}$$

where $J_{ij}(\|x^i - x^j\|)$ is the potential between i and j and can be different for different pairs. Moreover, we assume that $J_{ij}(\|x^i - x^j\|)$ satisfy

(A1) The potentials $J_{ij}(\|x^i - x^j\|)$ are symmetric and satisfy
$$\nabla_{x^i} J_{ij}(\|x^i - x^j\|) = -\nabla_{x^j} J_{ij}(\|x^i - x^j\|). \tag{3}$$

(A2) There exist corresponding function $g_{ar}^{ij} : \mathbb{R}^+ \to \mathbb{R}^+$ such that
$$\nabla_y J_{ij}(\|y\|) = y g_{ar}^{ij}(\|y\|)$$

(A3) There exist *unique distances* δ_{ij} at which we have $g_{ar}^{ij}(\delta_{ij}) = 0$ and $g_{ar}^{ij}(\|y\|) > 0$ for $\|y\| > \delta_{ij}$ and $g_{ar}^{ij}(\|y\|) < 0$ for $\|y\| < \delta_{ij}$.

Potential functions satisfying these are *odd* functions which are attractive on distances $\|y\| > \delta_{ij}$ and repulsive on distances $\|y\| < \delta_{ij}$. The term $g_{ar}^{ij}(\|y\|)$ determines the attraction-repulsion relationship between the individuals and usually is of the form $g_{ar}^{ij}(\|y\|) = g_a^{ij}(\|y\|) + g_r^{ij}(\|y\|)$, where $g_a^{ij}(\|y\|)$ represents the attraction and $g_r^{ij}(\|y\|)$ represents the repulsion. The distance δ_{ij} is the *equilibrium distance* at which the attraction and the repulsion balance. It can be shown that the functions satisfying the conditions (A1)-(A3) result in aggregating warm behavior [2], [3]. In particular, assuming that the motion of the individuals is given by (1), it can be shown that the following hold

(R1) The center $\bar{x} = \frac{1}{N}\sum_{i=1}^{N} x^i$ of the swarm is stationary for all time.

(R2) If $J(x)$ is bounded from below, i.e., $J(x) > a$ for some finite $a \in \mathbb{R}$. Then, for any initial condition $x(0) \in \mathbb{R}^{nN}$, as $t \to \infty$ we have $x(t) \to \Omega_e$ where $\Omega_e = \{x : \dot{x} = 0\}$.

(R3) The swarm size will be bounded and the position x^i of all the agents will converge to a small region around its center $\bar{x}$, which is a hyperball of size ϵ, $B_\epsilon(\bar{x}) = \{x : \|x - \bar{x}\| \le \epsilon\}$. Moreover, convergence to $B_\epsilon(\bar{x})$ will occur in a finite time.

The result in (R1) implies that the relative motions of the individuals will balance each other and will result in a stationary center. This is expected due to the reciprocity in (3). The result in (R2) states that as time tends to infinity all the individuals will stop motion and the swarm will converge to a constant configuration. If $J(x)$ is chosen as a formation function which has a unique minimum at the desired formation (as in [18]), then the desired formation will be asymptotically achieved. If $J(x)$ is composed of inter-individual potentials which hold only on certain range (as in [5], [19]), then (R2) still holds but only locally. The size ϵ of the region in which the agents converge and the speed of convergence stated in (R3) depend on the attraction and repulsion parameters of the potential $J(x)$.

The above results are for the case in which $J(x)$ is composed of only inter-individual interaction terms and does not contain environmental affects. In foraging swarms [4], where agent motions are affected by the environment in which the swarm resides, (R1) does not hold. In other words, the center $\bar{x}$ is not stationary. However, under some conditions (R2) and (R3) still hold and it can be shown that the swarm will move to the favorable regions of the environment and away from the unfavorable regions [4]. In all the cases with proper choice of the potential functions $J(x)$ the model in (1) results in bounded motions and aggregating emergent behavior. However, we would like to also emphasize that the sliding mode control procedure discussed in the following sections will still be affective and applicable even if these properties do not hold, since it does not depend on them.

One shortcoming of the motion dynamics in (1) and therefore any results related to it is that (1) does not correspond to the dynamics of realistic agents. Therefore, the model in (1) is in a sense a *kinematic model* for swarm aggregation, formation control, or agent coordination dynamics. For this reason, the results derived for it serve as *proof of concept* for the behavior considered. However, they do not specify how that desired proven behavior could be achieved in engineering applications with given (i.e., predefined) agent (vehicle) dynamics. Nevertheless, they are still of practical interest and can serve as guidelines for designing swarming engineering multi-agent systems. In the next section, we will show how using the sliding mode control technique we can achieve the above type of motion for agents with general fully actuated vehicle dynamics even in the presence of disturbances and uncertainties.

III. Sliding Mode Control for Agents with Vehicle Dynamics

Consider a swarm of N agents the dynamics of each of which are described by the equation

$$M_i(x^i)\ddot{x}^i + f_i(x^i, \dot{x}^i) = u^i, 1 \le i \le N, \tag{4}$$

where $x^i \in \mathbb{R}^n$ is the position of agent i, $M_i(x^i) \in \mathbb{R}^{n \times n}$ is the mass or inertia matrix, $f_i(x^i, \dot{x}^i) \in \mathbb{R}^n$ represents the centripedal, Corriolis, gravitational effects and additive disturbances, and $u^i \in \mathbb{R}^n$ represents the control inputs. Assume that

$$f_i(x^i, \dot{x}^i) = f_i^k(x^i, \dot{x}^i) + f_i^u(x^i, \dot{x}^i), 1 \le i \le N,$$

where $f_i^k(\cdot,\cdot)$ represents the *known* part and $f_i^u(\cdot,\cdot)$ represents the *unknown* part. Also, assume that the unknown part is bounded. In other words, let

$$\|f_i^u(x^i, \dot{x}^i)\| \le \bar{f}_i(x^i, \dot{x}^i), 1 \le i \le N,$$

where $\bar{f}_i(x^i, \dot{x}^i)$ are known for all i. Moreover, it is assumed that for all i the mass/inertia matrix is nonsingular and lower and upper bounded by known bounds. In other words, the matrices $M_i(x^i)$ satisfy

$$\underline{M}_i \|y\|^2 \le y^\top M_i(x^i) y \le \bar{M}_i \|y\|^2,$$

for all i, where $\underline{M}_i > 0$ and $\bar{M}_i < \infty$ are known and $y \in \mathbb{R}^n$ is arbitrary.

Given the agent dynamics in (4), we would like to design each of the control inputs u^i such that to enforce satisfaction of (1). In other words, we want to design the control input such that to enforce the velocity of the agents along the negative gradient of the potential function $J(x)$. This will result in preserving (or recovering) the results derived for the model in (1) and will mean achieving stable swarming, formation stabilization, or agent coordination (based on the context) despite the uncertain vehicle dynamics in (4). To this end, we will use sliding mode control approach. The sliding mode control technique has the property of reducing the motion (and the analysis) of a system to a lower dimensional space, which makes it very suitable for this application (since we want to enforce the system dynamics to obey (1), which constitutes only a part of the agents state). Moreover, it is robust with respect to disturbances and system uncertainties. We will follow the procedure developed in [22], [23], [24] for robot navigation and obstacle avoidance.

First let us define the n-dimensional sliding manifold for agent i as

$$s^i = \dot{x}^i + \nabla_{x^i} J(x) = 0, i = 1, \ldots, N. \tag{5}$$

Our objective is to drive the agents to their corresponding sliding manifolds $s^i = 0$ since once all the agents reach their sliding manifolds we have

$$\dot{x}^i = -\nabla_{x^i} J(x), i = 1, \ldots, N,$$

which is exactly the motion equation in (1). Given the vehicle dynamics in (4), the problem becomes to design the control inputs u^i such that to enforce the occurrence of sliding mode. A sufficient condition for sliding mode to occur is given by [27]

$$s^{i\top} \dot{s}^i < 0, \tag{6}$$

for all $i = 1, \ldots, N$. This condition also guarantees that the sliding manifold is asymptotically reached, i.e., it guarantees that the *reaching conditions* are satisfied. Later we will show how to choose a controller which will actually guarantee finite time reaching of the sliding manifold. Differentiating the sliding manifold equation with respect to time we obtain

$$\dot{s}^i = \ddot{x}^i + \frac{d}{dt}\Big[\nabla_{x^i} J(x)\Big].$$

From the vehicle dynamics of the agents in (4) we have

$$\ddot{x}^i = M_i^{-1}(x^i)\Big[u^i - f_i(x^i, \dot{x}^i)\Big],$$

using which in the equation of the derivative of s^i and substituting in (6), the necessary conditions for occurrence of sliding mode become

$$s^{i\top}\left[M_i^{-1}(x^i)u^i - M_i^{-1}(x^i)f_i(x^i, \dot{x}^i) + \frac{d}{dt}\Big[\nabla_{x^i} J(x)\Big]\right] < 0.$$

One issue to note here is that the potential function $J(x)$ is not static. In other words, it depends on the relative positions of the individuals. Therefore, uncertainties and disturbances (including those acting on the system dynamics) as well as the agent motion can affect the time derivative of $J(x)$. Now, we have the following assumption about $J(x)$.

Assumption 1: The potential function $J(x)$ satisfies

$$\|\nabla_{x^i} J(x)\| \le \alpha(x)$$

for all x^i and

$$\|\nabla_{x^j}\left[\nabla_{x^i} J(x)\right]\| \le \beta(x)$$

for all x^i and x^j, where $\alpha(x)$ and $\beta(x)$ are known and finite and $\|\cdot\|$ denotes the Euclidean norm (the vector and the induced matrix norms, respectively).

The above assumption is in a sense a smoothness assumption since it requires bounds on the “first” and the “second” derivatives of $J(x)$. It is satisfied by many potential functions and certainly by those considered in [2], [3], [4]. For some $J(x)$ it is even possible to find constants $\bar{\alpha}$ and $\bar{\beta}$ such that $\alpha(x) \le \bar{\alpha}$ and $\beta(x) \le \bar{\beta}$ for the range of operating conditions. Later we will show an example of such a function.

Initially it may seem as if Assumption 1 is a restrictive assumption since $\alpha(x)$ and $\beta(x)$ must be known. However, note that the model in (1) requires the knowledge of $J(x)$ and once it is known computing $\alpha(x)$ and $\beta(x)$ is straightforward. Therefore, Assumption 1 does not bring much extra restriction to the system. Below we have one more assumption which states that all the agents are at rest initially and is also a reasonable assumption.

Assumption 2: At time $t = 0$ we have $\dot{x}^i(0) = 0$ for all $i = 1, \ldots, N$.

Using Assumption 1 and Assumption 2 one can establish a bound on $\left\|\frac{d}{dt}\left[\nabla_{x^i} J(x)\right]\right\|$ as

$$\begin{aligned}
\left\|\frac{d}{dt}\left[\nabla_{x^i} J(x)\right]\right\| &= \left\|\left[\sum_{j=1}^{N} \nabla_{x^j}\left[\nabla_{x^i} J(x)\right]\right]\dot{x}^j\right\| \\
&= \left\|\left[\sum_{j=1}^{N} \nabla_{x^j}\left[\nabla_{x^i} J(x)\right]\right]\Big[s^j - \nabla_{x^j} J(x)\Big]\right\| \\
&\le N\beta(x)\left[\alpha(x(0)) + \alpha(x)\right] \triangleq \bar{J}_i(x),
\end{aligned} \tag{7}$$

where the last inequality was established using

$$\|s^j(t)\| \le \|s^j(0)\| = \|\nabla_{x^j} J(x(0))\| \le \alpha(x(0)). \tag{8}$$

The second inequality in this equation follows from Assumption 1, whereas the equality in the middle follows from Assumption 2. For now assume that the first inequality holds; below we will show that it really does hold.

Given the bound in (7) we can choose u^i such that $s^{i\top}\dot{s}^i < 0$ hold for all i. In particular, by choosing

$$u^i = -u_0^i(x) sign(s^i) + f_i^k(x^i, \dot{x}^i), \tag{9}$$

where $sign(s^i) = [sign(s^i_1), \ldots, sign(s^i_N)]^\top$, we obtain

$$s^{i\top}\dot{s}^i < -\|s^i\| \left[\frac{1}{\bar{M}_i} u^i_0(x) - \frac{1}{\underline{M}_i} \bar{f}_i(x^i, \dot{x}^i) - \bar{J}_i(x) \right].$$

Then, by choosing the gain $u^i_0(x)$ of the control input as

$$u^i_0(x) > \bar{M}_i \left(\frac{1}{\underline{M}_i} \bar{f}_i(x^i, \dot{x}^i) + \bar{J}_i(x) + \epsilon^i \right), \tag{10}$$

for some $\epsilon^i > 0$, one can guarantee that

$$s^{i\top}\dot{s}^i < -\epsilon^i \|s^i\|$$

is satisfied and that sliding mode occurs. This equation also guarantees that the first inequality in (8) and therefore the bound in (7) hold for all $t \geq 0$. Note also that using the Lyapunov function $V_i = \frac{1}{2} s^{i\top} s^i$, the above inequality implies that $\dot{V}_i \leq -\epsilon^i \sqrt{V_i}$ and guarantees that the sliding manifold is reached in a *finite time* bounded by $t^i_{max} = \frac{2V_i(0)}{\epsilon^i}$. Then, sliding mode occurs on all the surfaces $s^i = 0$ in a *finite time* bounded by

$$\bar{t}_{sm} = \max_{i=1,\ldots,N} \left\{ \frac{2V_i(0)}{\epsilon^i} \right\}. \tag{11}$$

The above discussion can be formally summarized as follows.

Theorem 1: Consider a system of N agents with vehicle dynamics given by (4). Assume that the artificial potential function $J(x)$ satisfies Assumption 1 and that Assumption 2 holds. Let the controllers for the agents are given by (9) with gains as in (10). Then, sliding mode occurs in all the surfaces s^i and (1) is satisfied in a finite time bounded by the bound in (11).

In the above controller, we utilized the known part $f^k_i(x^i, \dot{x}^i)$ of the vehicle dynamics. If there are not known parts, then this portion of the controller can be set to zero. Note also that the controller does not need the exact mass/inertia matrix $M_i(x^i)$ of the robot. It just needs the bounds on them as is the case also with disturbances.

The model for the motion dynamics of the agents in (4) is more general than those in [5], [6], [7]. Moreover, it allows for possible additive disturbances (which are not included in [5], [6], [7]). This advantage is basically due to the robustness properties of the sliding mode control algorithm.

As mentioned before, the swarm models considered in [2], [3], [4] can be viewed essentially as *kinematic models* for swarm motions/aggregations. Therefore, they mostly serve as *proof of concept* for swarm behavior. In engineering swarm applications with agents with particular motion dynamics one can develop control algorithms taking into account the agent dynamics to achieve the required behavior. The control algorithm discussed in this section is one such method that could be applied if the agent dynamics are described by (4). Moreover, it can also be extended for agents with different vehicle dynamics. Furthermore, the procedure is not limited to the models/results in [2], [3], [4] and can be applied also for other similar systems as discussed before (provided that the stated assumptions are satisfied). Even further, the procedure is not limited to the motion in (1) and can easily be extended to cases when $\dot{x}^i$ contains other extra terms.

The design of the sliding mode surface we considered here is little different from conventional sliding mode control problems. In classical sliding mode control problems the surface $s = 0$ is chosen such that on it the tracking error asymptotically decays to zero. Here, the surfaces $s^i = 0$ are chosen so that the system motion equation obeys certain dynamics. Even though $\dot{x}^i$ can be viewed as the output of the system and s^i as the output error and stated that at $s^i = 0$ the output error becomes zero, there is still a difference since here s^i's are not constant surfaces and they can move as the agents move.

Theorem 1 suggests that the results on swarm stability (described earlier) will be recovered. However, in real applications usually it is not possible to achieve ideal sliding mode due to actuator non-idealities and other unmodeled dynamics which may lead to the so called *chattering phenomena*. Therefore, in practical implementations it may not be possible to ideally recover all the stability results that could be obtained for (1). For example, the statement that $\bar{x}$ is stationary for all time may not necessarily hold (due to chattering affects) and there may be small deviations of the center. Nevertheless, despite the non-idelities we still would expect that most of the results (such as swarm cohesiveness and finite time convergence) to be recovered with only small perturbations.

One issue which comes to mind is the issue of collisions. The sliding mode control procedure does not directly address the issue of collision avoidance. However, it can be handled by appropriately choosing the artificial potential $J(x)$. In particular, by having $J(x)$ with two parts, namely attraction and repulsion, such that attraction dominates on large distances and repulsion dominates on short distances (as in [2], [3], [4]), it is possible to avoid collisions between the agents.

IV. Illustrative Examples

In this section some numerical simulation examples will be presented in order to illustrate the effectiveness of the sliding mode control method discussed in the preceding section. For ease of plotting we use only $n = 2$ or $n = 3$; however, qualitatively the results will be the same for higher dimensions. We consider agents (robots) with point mass dynamics with unknown mass and additive sinosoidal disturbances. In other words, we consider the model

$$M_i \ddot{x}^i + f_i(x^i, \dot{x}^i) = u^i, 1 \leq i \leq N,$$

where $\underline{M}_i \leq M_i \leq \bar{M}_i$ is the unknown mass and $f_i(x^i, \dot{x}^i) = \sin(0.2t)$ is the unknown uncertainty in the system. Note that it satisfies the boundedness assumption $\|f(x^i, \dot{x}^i)\| = \|\sin(0.2t)\| \leq 1 \triangleq \bar{f}_i$. Without loss of generality we assume unity mass $M_i = 1$ for all the agents. Note that agents with point mass dynamics (but without uncertainties) are being considered in the literature (see for example [5], [6], [7]). As controller parameters in the simulations below we choose $\underline{M}_i = 0.5$ and $\bar{M}_i = 1.5$, $\bar{f}_i = 1$, and $\epsilon^i = 1$. Moreover, we approximated the signum function with a hyperbolic tangent. In other words, instead of the $sign(s^i)$ term in the controller we used $tanh(\gamma s^i)$ with $\gamma = 10$. This smooths the control action and reduces unwanted chattering due to the discontinuity in the controller. There are rigorous methods to prevent the chattering in the system (such as using a state observer [28]); however, they are outside of the scope of this article.

First, we will consider swarming behavior and will use the potential function

$$J(x) = \sum_{i=1}^{N-1} \sum_{j=i+1}^{N} \left[\frac{a}{2} \|x^i - x^j\|^2 + \frac{bc}{2} \exp\left(-\frac{\|x^i - x^j\|^2}{c} \right) \right], \tag{12}$$

considered in [2]. Here $J_a(\|x^i - x^j\|) = \frac{a}{2}\|x^i - x^j\|^2$ is an attraction potential, which dominates on long distances, and $J_r(\|x^i - x^j\|) = -\frac{bc}{2} \exp\left(-\frac{\|x^i - x^j\|^2}{c} \right)$ is a repulsion potential, which dominates on short distances. Both terms are always active and the eventual motion is an interplay between the two. This represents in its most basic form the attraction/repulsion forces in aggregating swarms in nature such as schools of fish. Note that for the potential in (12) it is straightforward to obtain the bounds

$$\alpha(x) = N \left[a \max_{1 \leq j \leq N} \|x^i - x^j\| + b\sqrt{\frac{c}{2}} \exp\left(-\frac{1}{2} \right) \right]$$

and

$$\beta(x) = a + 2b \exp\left(-\frac{3}{2}\right)$$

using which one can easily calculate the bound $\bar{J}_i(x)$. One issue to note, however, is that the above bounds $\alpha(x)$ and $\beta(x)$ are very conservative and result in large $\bar{J}_i(x)$ and the actual bounds can be much smaller. In fact, as we will see below the procedure works with much smaller $\bar{J}_i(x)$ in implementation.

Another issue to mention here is that for the considered potential $J(x)$ in (12) we have $\beta(x) = \bar{\beta}$, i.e., $\beta(x)$ is independent of x. Similarly, $\alpha(x)$ depends only on the relative distances $\|x^i - x^j\|$ between the individuals. From the results in the preceding sections we know that for $J(x)$ in (12) once on the sliding mode surface the system will be well behaved (i.e., its states will be bounded and the position of all the agents will converge to a small hyperball around its center) and also that the sliding mode surface will be reached in a finite time. This implies that for any given initial position $x(0)$ there is a constant $\bar{\alpha}(x(0))$ such that $\alpha(x) \leq \bar{\alpha}(x(0))$ for all $t \geq 0$ implying that both of the bounds in Assumption 1 can be set as constants for $J(x)$ in (12).

In the simulations below we used the parameters $a = 0.01$, $b = 20$, and $c = 1$ for the potential function in (12) and in the controller we used the bounds $\bar{J}_i(x) = 10N$ for the swarming case and $\bar{J}_i(x) = 20N$ for the formation control case, which are much smaller than the ones computed using the above $\alpha(x)$ and $\beta(x)$. Figure 1 shows in $\mathbb{R}^3$ the paths of the members of a swarm with $N = 10$ individuals. We chose the initial positions of the swarm members randomly and their initial velocity as zero (as is needed by Assumption 2). The initial positions of the swarm members are represented with circles and their paths with dots. Their positions after 40 seconds are also represented with circles. It is easily seen that all the members move toward each other and form a cohesive swarm cluster. Note that these results are very similar to those obtained in [2].

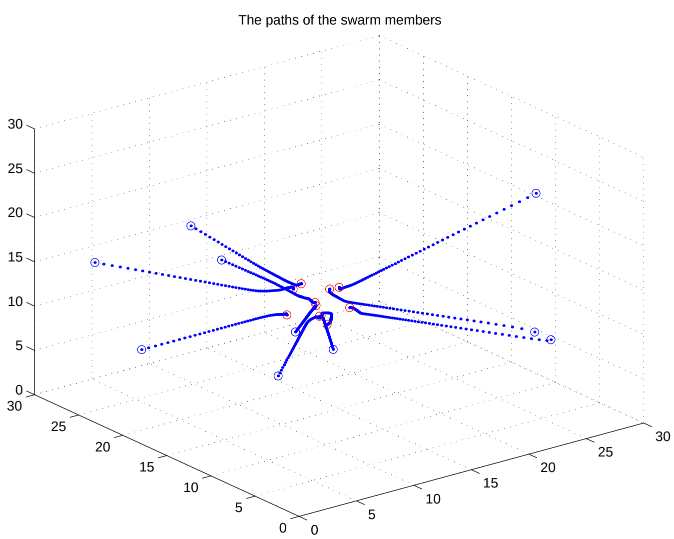

Fig. 1. The paths of the swarm members.

Next, we consider the application of the procedure to the formation control (stabilization) problem. In particular, we consider $N = 6$ agents which are required to form an equilateral triangle formation (with any orientation) in $\mathbb{R}^2$ with three of the agents at the corners of the triangle and three of the agents at the middle point of the vertices. A potential function which has a minimum at the desired formation is

$$J(x) = \sum_{i=1}^{N-1} \sum_{j=i+1}^{N} \left[\frac{a_{ij}}{2} \|x^i - x^j\|^2 + \frac{b_{ij} c_{ij}}{2} \exp\left(-\frac{\|x^i - x^j\|^2}{c_{ij}} \right) \right]$$

where the parameters a_{ij}, b_{ij}, and c_{ij} depend on the desired relative distances of the individuals in the form of

$$\|x^i - x^j\| = d_{ij}, 1 \leq i, j \leq N.$$

In particular, we used $b_{ij} = 20, c_{ij} = 0.2$ for all i, j, and $a_{ij} \in \{0.1348, 6.1180 \times 10^{-6}, 4.1223 \times 10^{-8}\}$ which result in $d_{ij} \in \{1, \sqrt{3}, 2\}$ based on the desired relative positions of the agents. We would like to mention here that the above type potentials may have local minima leading to only local results. If global convergence to the desired formation is desired then potentials with unique minimum (at the desired formation) should be chosen.

Figure 2 illustrates the application of the procedure to the formation control problem. The system is once more initialized at rest with random initial positions. As expected, as the time progresses the swarm members move to their required inter-individual distances and form the desired formation. The trajectories shown illustrate their motion for 30 seconds. The plot of the positions of the swarm members after 30 seconds is shown in Figure 3. As one can see from the figure, the agents have formed the desired formation. The center of the agents is denoted by a star. During transient before occurrence of sliding mode it is possible for the center $\bar{x}$ to move. However, once all the agents reach their prospective sliding manifolds it is expected to be stationary. Close examination of the center position (not shown here) shows that this expectation is satisfied and $\bar{x}$ is stationary. However, on implementation on real hardware this may not necessarily be the case. Note that even though we considered an example of a formation stabilization, as mentioned before, the procedure can easily be applied to the case in which the formation has to move (as a whole entity) and track some desired trajectory.

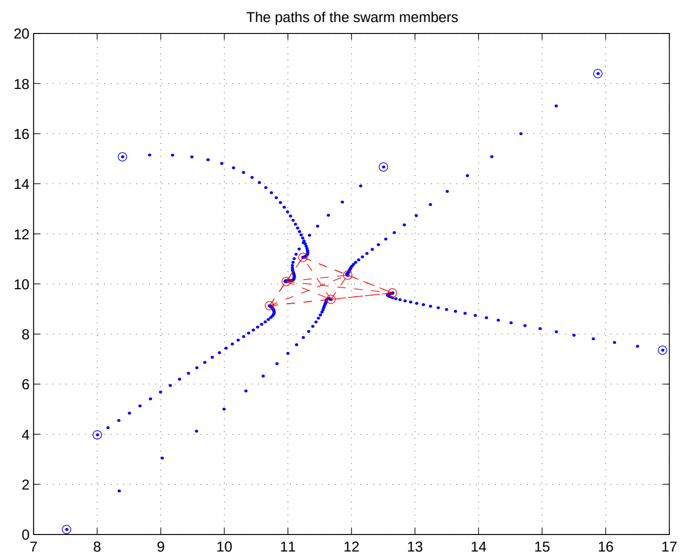

Fig. 2. Paths of the agents forming a triangle formation.

Finally, let us consider the case of foraging swarms, i.e., swarms moving in an environment modeled as a potential field (attraction/repulsion profile) as was considered in [4]. There, we considered a swarm model the motion of which was determined by a potential function of the form

$$\bar{J}(x) = J(x) + \sum_{i=1}^{N} \sigma(x^i)$$

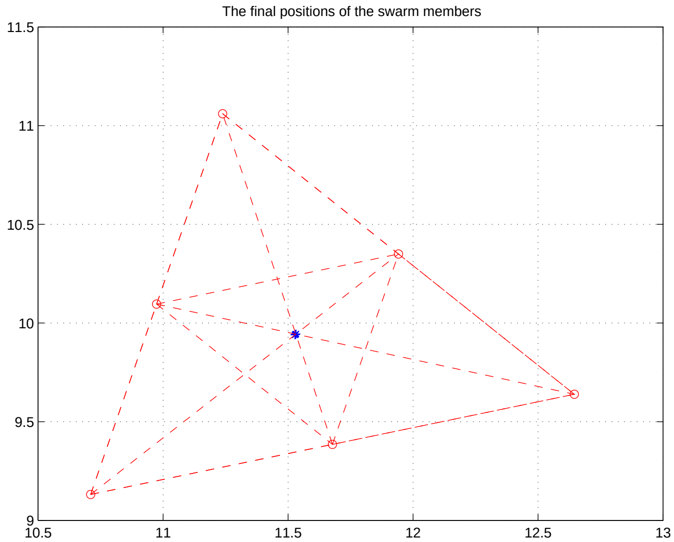

Fig. 3. Final positions of agents forming a triangle formation.

where $J(x)$ is the potential in (12) and the term $\sigma : \mathbb{R}^n \to \mathbb{R}$ represents a "profile" of attractant/repellent substances (e.g., nutrients and/or toxic substances in biology and targets and/or threads in engineering) or simply is a model of the environment. In this model, it is assumed that the regions of lower values of $\sigma(\cdot)$ constitute more favorable regions. Then, note that the individuals try to move to more favorable regions along the negative gradient of the profile (due to the first term in the motion equation), while trying to stay cohesive (due to the second term in the motion equation).

For an environment potential we use the same potential (or profile) we used in [4] which is shown in Figure 4. The hills in the profile represent unfavorable regions, while the valleys represent favorable ones. We would expect the swarm to avoid hills and to move towards the valleys (provided the parameters are appropriately chosen). For the attraction/repulsion parameters a, b, and c we use the same parameters as for the swarming case above.

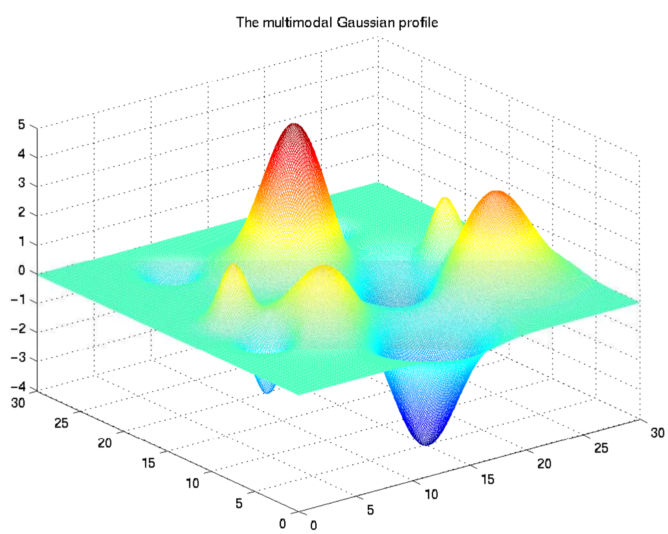

Fig. 4. The environment potential (profile).

Figure 5 shows a simulation run for the model with the above parameters and randomly chosen initial positions. The initial velocities are once again set to zero. It is easily seen from the figure that the expectations are satisfied and the agents move towards valleys while avoiding hills. One can easily see the similarity of the result with the results obtained in [4].

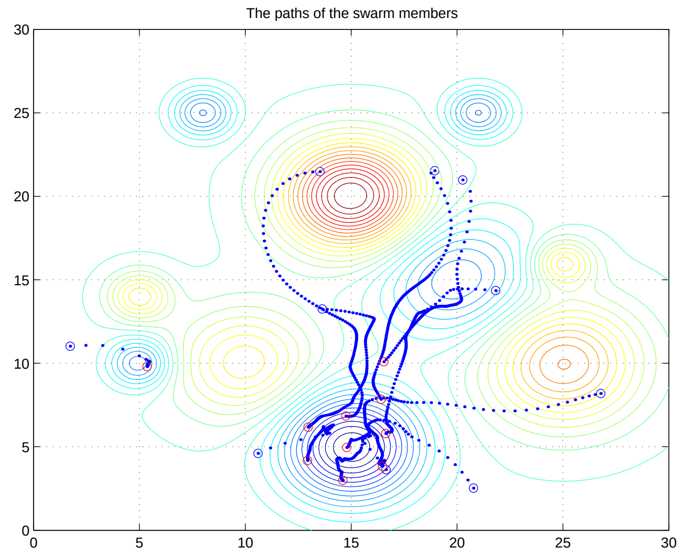

Fig. 5. Paths of the agents moving in the profile.

V. Concluding Remarks

Developing systems consisting of multiple autonomous agents, such as mobile robots or unmanned undersea or areal vehicles, that can cooperatively perform tasks or show some kind of emergent behavior is of paramount importance for the engineering community. Principles governing the dynamics of similar systems in nature such as schools of fish, flocks of birds, herds of animals or swarms of bacteria could be exploited for developing such automated systems. In our earlier work we had considered a swarm model based on artificial potentials and obtained results on cohesive swarm aggregation. In this article, we presented a procedure based on sliding mode control theory using which our previous results (i.e., stable aggregating behavior) can be implemented for engineering swarms with general fully actuated agent (vehicle) dynamics. We also showed that the procedure could be used for formation control and social foraging swarms (i.e., swarms moving in an environment of nutrients or toxic substances). Although, we introduced the procedure in the context of our swarm model, it is more general than it and can be used for other similar models using artificial potentials. Moreover, it has the advantage of being robust with respect to disturbances and system uncertainties.

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