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Lennard-Jones Potential Field-based Swarm Systems for Aggregation and Obstacle Avoidance

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Abstract: Main concern of this paper is constructing a swarm model using Lennard-Jones potential for N agents. The Lennard-Jones potential [1] is one of basic models to interpret motion of atoms or molecules in molecular dynamics. The swarm model generates interaction forces composed of attractive force and repulsive force as a function of initial positions of every agent on the basis of Lennard-Jones potential model with velocity matching. In that case, motion of the agents in the swarm shows aggregation behavior. Also, one of main issues of swarm system is how to avoid obstacles. In our case, we apply additional repulsive potential function into our swarm system for obstacle avoidance. After conducting a simulation, we confirm that a group of agents appears swarm behavior. We expect that this behavior can be applied to control movement of multiple robots or UAVs.

Keywords: Swarm, Lennard-Jones Potential, Obstacle Avoidance

1. INTRODUCTION

Swarm intelligence inspired by the nature becomes one of fascinating research areas and has a lot of attentions due to its various applications. Representative research results of swarm intelligence are ant colony optimization (ACO) [2] and particle swarm optimization (PSO) [3]. ACO is one of optimization algorithms to search a goal location governed by basic algorithm of numerous agents. PSO is an optimization method of continuous nonlinear function by a huge population of particles. Based on ACO and PSO, many papers suggest various applications, such as wireless-sensor networks [4], electric power systems [5], path-planning of mobile robots [6] and so on. These ideas come from observed behavior of creatures in the nature, such as foraging foods in a colony of ants and making social behavior of a school of fishes and a flock of birds.

The initial researches on swarm intelligence presents mathematical models based grouping behaviors [7], [8], [9]. Similarly, there is another point of view in the swarm intelligence. It is called potential-field based swarm systems. The potential-field can generate attractive force and positive force, and swarm behavior can be achieved using the forces governed by single integrator [10]. To achieve stability analysis, they show that trajectory of all involved agents of swarm is bounded under continuous case [11] and discrete case [12]. Flocking is one of swarm behaviors, double integrator dynamics based flocking behavior is introduced in [13] and [14]. In addition, consensus algorithm can be applied into the swarm with velocity matching [16], and a group of agents can be divided into two different groups using different range of potential function [17].

Our paper also considers potential model that contains attractive force and repulsive force by distance range between agents using Lennard-Jones potential. The Lennard-Jones potential is one of potential models in molecular dynamics that help investigate motion of atoms and molecules. As a part of process to design swarm structure, the main aim of this paper is constructing a swarm model for aggregation behavior and obstacle avoidance using Lennard-Jones potential [1] and repulsive potential function. We expect that applying those potential functions with cut-off range into swarm model makes agents appear actual phenomenon of the swarm in nature. Especially, we adapt a moving obstacle as an enemy. Sometimes, the swarm may divide into several parts due to other changing environments. For example, Some of flocks or schools may break into several parts due to attack from enemies. Also, this phenomenon is required to control movement of a herd of cattle for classification from owner. These behaviors are helpful to operate numerous robots or Unmanned Aerial Vehicles (UAVs).

In our paper, we found aggregating behavior of swarm under a simulation result. As a basic step, we present the phenomenon using several types of swarm cases. This paper is constructed as follows. In section 2.we briefly introduce Lennard-Jones potential and repulsive potential between two agents. In section 3extended Lennard-Jones potential and force for aggregation and repulsive potential and force for obstacle avoidance among N agents will be given. In section 4.we present several types of simulation results using the interactions. Finally, in section 5. , conclusion will be given.

2. MODEL OF LENNARD-JONES POTENTIAL FIELD BASED SWARM SYSTEMS AND REPULSIVE POTENTIAL FIELD FOR OBSTACLE AVOIDANCE

Consider a swarm system that consists of N agents moving along the 2-dimensional Euclidean spaces. Let x_i, v_i, and m_i denote position, velocity, and mass of i-th agent in the swarm respectively, where $x_i \in \mathbb{R}^2$, $v_i \in \mathbb{R}^2$, $m_i \in \mathbb{R}$, $\alpha_{ij} \in \mathbb{R}$, and $\beta_{ij} \in \mathbb{R}$. Potential field that is generated by distance values among agents offers interaction forces to them governed by the following double integrator.

$$\begin{cases} \dot{x}_i = v_i \\ \dot{v}_i = -\nabla_{x_i} U(x_i) \end{cases} \tag{1}$$

2.1 Lennard-Jones potential field based swarm systems between two agents

Lennard-Jones potential [1] is used as a potential model of atoms or molecules to interpret motion of them in molecular dynamics. By a distance range between atoms or molecules, the potential function generates repulsive force or attractive force. The following equations (2) and (3) and figures (Fig. 1 and 2) are potential and force of our systems inspired by the Lennard-Jones potential model.

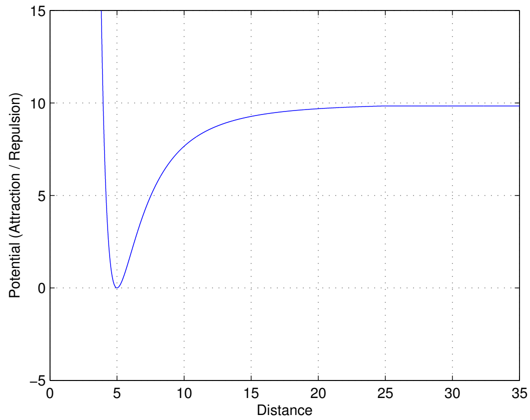

Fig. 1: Lennard-Jones Potential where $\alpha_{ij} = 5$, $\varepsilon = 10$, $l = 6$, $k = 3$, and $\gamma = 25$

Let $x_{ij} = (x_i - x_j)$, where $x_{ij} \in \mathbb{R}^2$, then the potential and force are represented by

$$U_{int}(x_{ij}) = \begin{cases} \varepsilon([(\frac{\alpha_{ij}}{\|x_{ij}\|})^l - \frac{l}{k}(\frac{\alpha_{ij}}{\|x_{ij}\|}^k] + (\frac{l-k}{k})), & 0 < x_{ij} \le \gamma \\ \varepsilon([(\frac{\alpha_{ij}}{\|\gamma\|})^l - \frac{l}{k}(\frac{\alpha_{ij}}{\|\gamma\|}^k] + (\frac{l-k}{k})), & x_{ij} > \gamma \end{cases} \tag{2}$$

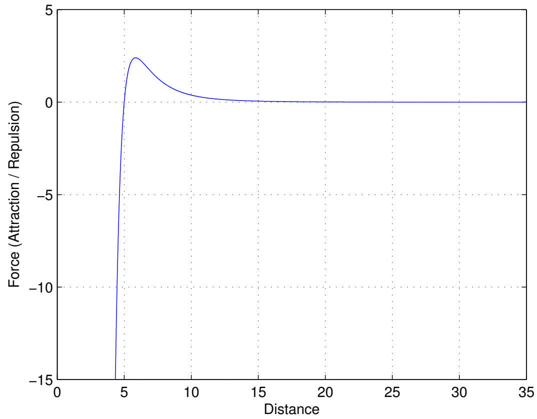

Fig. 2: Lennard-Jones Force where $\alpha_{ij} = 5$, $\varepsilon = 10$, $l = 6$, $k = 3$, and , $\gamma = 25$

$$\nabla_{x_i} U_{int}(x_{ij}) = \begin{cases} -\frac{\varepsilon l}{\alpha_{ij}}[(\frac{\alpha_{ij}}{\|x_{ij}\|})^{l+1} - (\frac{\alpha_{ij}}{\|x_{ij}\|})^{k+1}] \cdot \hat{x}_{ij}, & 0 < x_{ij} \le \gamma \\ 0, & x_{ij} > \gamma \end{cases} \tag{3}$$

where $\lim_{x_{ij} \to 0} U_{int}(x_{ij}) = \infty$, $\lim_{x_{ij} \to \infty} U_{int}(x_{ij}) = U_{int}(x_\infty) = U_{int}(\gamma)$, $U_{int}(\alpha_{ij}) = 0$, $l > k > 0$, $\hat{x}_{ij} = \frac{(x_i - x_j)}{\|x_i - x_j\|}$, and $\|\cdot\|$ is Euclidean norm.

Here, as depicted in Fig. 1 and Fig. 2, the value α_{ij} divides interaction force into two parts, attractive force and repulsive force. On the value α_{ij}, if the distance x_{ij} is less than α, then the interaction force generates repulsive force. Inversely, if the distance x_{ij} is more than α, then the interaction force generates an attractive force between them. In this case, the system gets equilibrium point when $x_{ij} = \alpha_{ij}$ and $v_i = \mathbf{0}$ for all i.

2.2 Repulsive potential field based swarm systems between two agents

Additionally, we apply additional potential function for avoiding obstacles using repulsive potential field. The repulsive potential field and force are represented by

$$U_{obs}(x_{io_j}) = \begin{cases} \varepsilon(\frac{\beta_{ij}}{\|x_{io_j}\|}), & 0 < x_{ij} \le \gamma \\ \varepsilon(\frac{\beta_{ij}}{\|\gamma\|}), & x_{ij} > \gamma \end{cases} \tag{4}$$

$$\nabla_{x_i} U_{obs}(x_i) = \begin{cases} -\frac{\varepsilon}{\beta_{ij}}(\frac{\beta_{ij}}{\|x_{io_j}\|})^2 \cdot \hat{x}_{io_j}, & 0 < x_{ij} \le \gamma \\ 0, & x_{ij} > \gamma \end{cases} \tag{5}$$

where $x_{io_j} = (x_i - o_j)$, o_j is location of j-th obstacle ($o_j \in \mathbb{R}^2$), $\lim_{x_{io_j} \to 0} U_{obs}(x_{io_j}) = \infty$, $\lim_{x_{io_j} \to \infty} U_{obs}(x_{io_j}) = U_{obs}(x_\infty) = U_{obs}(\gamma)$. The potential and force are depicted in Fig. 3 and Fig. 4.

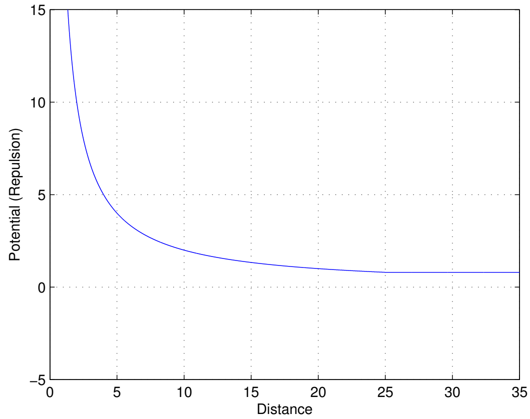

Fig. 3: Repulsive Potential for Obstacle Avoidance where $\epsilon = 20$, $\beta_{ij} = 1$, and $\gamma = 25$.

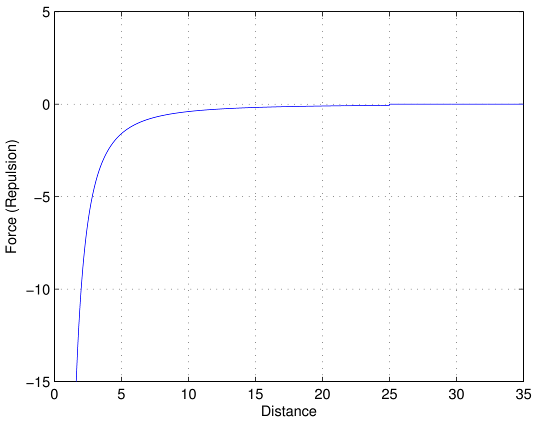

Fig. 4: Repulsive Force for Obstacle Avoidance where $\epsilon = 20$, $\beta_{ij} = 1$, and $\gamma = 25$.

3. LENNARD-JONES POTENTIAL FIELD AND REPULSIVE POTENTIAL FIELD-BASED SWARM SYSTEMS AMONG N AGENTS WITH VELOCITY MATCHING

An expanded Lennard-Jones potential and force among N agents can be expressed as the following equations:

$$U(x_i) = U_{int}(x_i) + U_{obs}(x_i) \quad (6)$$

$$= \sum_{j=1,j\neq i}^{N} U_{int}(x_{ij}) + \sum_{j=1,j\neq i}^{N} U_{obs}(x_{ij}) \quad (7)$$

$$\nabla_{x_i} U(x_i) = \nabla_{x_i} U_{int}(x_{ij}) + \nabla_{x_i} U_{obs}(x_{ij}) \quad (8)$$

$$= \sum_{j=1,j\neq i}^{N} \nabla U_{int}(x_{ij}) + \sum_{j=1,j\neq i}^{N} \nabla U_{obs}(x_{ij}) \quad (9)$$

One of crucial issues of swarm systems is velocity matching. To apply the velocity matching [16] into our swarm systems, we use basic knowledge of graph theory. All agents and interactions between agents can be represented by nodes and edges. Especially, in our swarm framework, all agents are not connected each other due to existence of cut-off range on potential functions. Therefore, it is one of good ways to express the swarm systems as a graph structure. Let us define an adjacency matrix $A = [\rho_{ij}]$, which has following properties.

$$\rho_{ij} = \begin{cases} 1, & 0 < x_{ij} \le \gamma \\ 0, & \text{otherwise.} \end{cases} \quad (10)$$

Also, let us define a degree matrix $D = [d_{ij}]$, which has following properties.

$$d_{ij} = \begin{cases} \text{deg(i)}, & i = j \\ 0, & \text{otherwise.} \end{cases} \quad (11)$$

where $deg(i)$ is the number of connections. Here connection means that ρ_{ij}=1 between agent i and agent j.

Using the adjacency matrix A and the degree matrix G, we can calculate a laplacian matrix L as

$$L = D - A \quad (12)$$

The laplacian matrix $L = [l_{ij}]$ has following properties.

$$l_{ij} = \begin{cases} \text{deg(i)}, & i = j \\ -1, & \rho_{ij} = 1 \\ 0, & \text{otherwise.} \end{cases} \quad (13)$$

For calculating into two-dimensional spaces, the laplacian matrix is transfered using Kronecker product as below.

$$L_2 = L \otimes I_2 \quad (14)$$

Then, eventually, an agent i is effected by (15) as the velocity matching.

$$L_2(x_i)v_i = \sum_{j=1,j\neq i}^{N} \rho_{ij}(v_j - v_i) \quad (15)$$

Let a set of location terms $\mathbf{x} = [\mathbf{x_1}, \mathbf{x_2}, \cdots, \mathbf{x_n}]^\mathbf{T}$, a set of velocity terms $\mathbf{v} = [\mathbf{v_1}, \mathbf{v_2}, \cdots, \mathbf{v_n}]^\mathbf{T}$, a set of acceleration terms $\mathbf{a} = [\mathbf{a_1}, \mathbf{a_2}, \cdots, \mathbf{a_n}]^\mathbf{T}$, and a set of artificial mass terms $\mathbf{m} = [\mathbf{m_1}, \mathbf{m_2}, \cdots, \mathbf{m_n}]^\mathbf{T}$, where $x_i \in \mathbb{R}^2$, $v_i \in \mathbb{R}^2$, $a_i \in \mathbb{R}^2$, $m_i \in \mathbb{R}$. Then, by equations (7) and (9) and velocity matching term, all agents are governed by the following dynamics:

$$\begin{cases} \dot{\mathbf{x}} = \mathbf{v} \\ \dot{\mathbf{v}} = -\frac{1}{\mathbf{m}}(\mathbf{a} + \mathbf{L_2}(\mathbf{x})\mathbf{v}) \end{cases} \quad (16)$$

Hamiltonian function [18] that consists of potential and kinetic energy can be considered as the total amount of energy of the swarm at initial positions of all agents. Equation (17) is the Hamiltonian function we consider.

$$H(\mathbf{x},\mathbf{v}) = \frac{\mathbf{1}}{\mathbf{2}}\mathbf{m}\mathbf{v}^{\mathbf{T}}\mathbf{v} + \mathbf{U}(\mathbf{x}) \tag{17}$$

Time derivative of Hamiltonian function for N agents is

$$\begin{aligned} \dot{H}(\mathbf{x},\mathbf{v}) &= \mathbf{m}\mathbf{v}^{\mathbf{T}}\dot{\mathbf{v}} + \dot{\mathbf{x}}^{\mathbf{T}}\nabla\mathbf{U}(\mathbf{x}) \\ &= \mathbf{m}\mathbf{v}^{\mathbf{T}}(-\frac{\mathbf{1}}{\mathbf{m}}(\mathbf{a}+\mathbf{L_2}(\mathbf{x})\mathbf{v})) + \mathbf{v}^{\mathbf{T}}\mathbf{a} \\ &= -\mathbf{v}^{\mathbf{T}}\mathbf{L_2}(\mathbf{x})\mathbf{v} \\ &= -\frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{N}\rho_{ij}(v_j - v_i)^T(v_j - v_i) \\ &\leq 0 \end{aligned} \tag{18}$$

The above result means that the swarm model is energy consumptive system. Thus, the total energy of the swarm is decreasing for all time t. Thus, at the initial positions of agents, the swarm is aggregating continuously without any periodic vibration or escape if the agents initially located on some points which satisfy $U_{int}(x_{ij}) \leq \varepsilon([(\frac{\alpha_{ij}}{\|\gamma\|})^l - \frac{l}{k}(\frac{\alpha_{ij}}{\|\gamma\|}^k] + (\frac{l-k}{k}))$ between agent i and agent $j, \forall i, j$.

For calculating a center of positions and a center of mass, we use following equations given below. A center of positions of the swarm is

$$x_{cp} = \frac{1}{N}\sum_{i=1}^{N} x_i, \tag{19}$$

where N is the total number of agents in the swarm.

A center of mass of the swarm is

$$x_{cm} = \frac{1}{M}\sum_{i=1}^{N} m_i x_i, \tag{20}$$

where M is the total mass of agents in the swarm ($M = \sum_{i=1}^{N} m_i$).

4. SIMULATION RESULT

In order to confirm the swarm behavior, we conduct a simulation with nine agents as a group of swarm and three obstacles as enemies. Here, obstacles No. 1 and no. 2 are fixed on $(-30, 15)^T$ and $(-30, -5)^T$, and obstacle no. 3 is moving along the x-axis as -4 per second from $(70, 5)^T$. The initial points are depicted in Fig. 5. Initial position and artificial mass are as follows.

[Simulation] 9 agents are located on the 2-dimensional Euclidean spaces : [No.1]: $(-10, -5)^T$, $m_1 = 1$, [No.2]: $(0, -5)^T$, $m_2 = 1$, [No.3]: $(10, -5)^T$, $m_3 = 50$, [No.4]: $(-10, 5)^T$, $m_4 = 1$, [No.5]: $(0, 5)^T$, $m_5 = 1$, [No.6]: $(10, 5)^T$, $m_6 = 1$, [No.7]: $(-10, 15)^T$, $m_7 = 1$, [No.8]: $(0, 15)^T$, $m_8 = 1$, [No.9]: $(10, 15)^T$, $m_9 = 1$, where $\alpha_{ij} = 5$ and $\beta_{ij} = 1$ for all i and j, $\varepsilon = 5$, $l = 6$, and $k = 3$. Based on the initial condition, we have got a following simulation result.

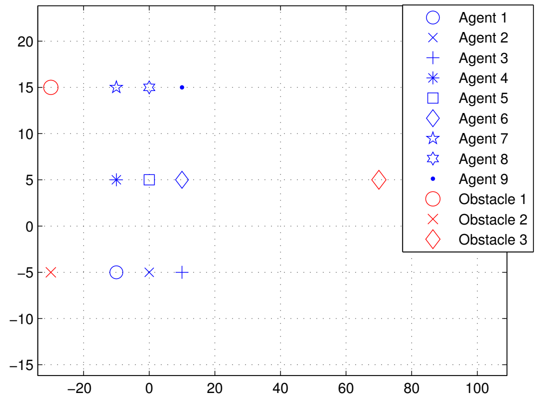

Fig. 5: Initial points of nine agents and three obstacles

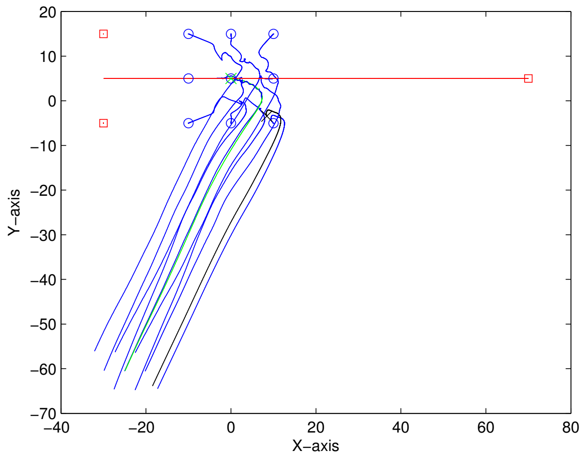

Fig. 6: Trajectories of 9 agents and 3 obstacles in the swarm for 25s (red □ points - given initial points of obstacles, red lines - trajectories of obstacle no.3, blue ◦ points - given initial points of agents, blue lines - trajectories of agents, black line - trajectories of center of mass, and green line - trajectories of center of position)

As can be seen in Fig.6, there is a leader agent no.3 in a swarm group. When the obstacle no.3 is approaching, A group of swarm has tried to move towards below direction for avoiding the obstacle.

5. CONCLUSION AND FUTURE WORK

In this paper, we have presented Lennard-Jones potential field and repulsive potential field-based swarm systems. The distance range among agents at the initial positions generate two types of interaction forces and at-

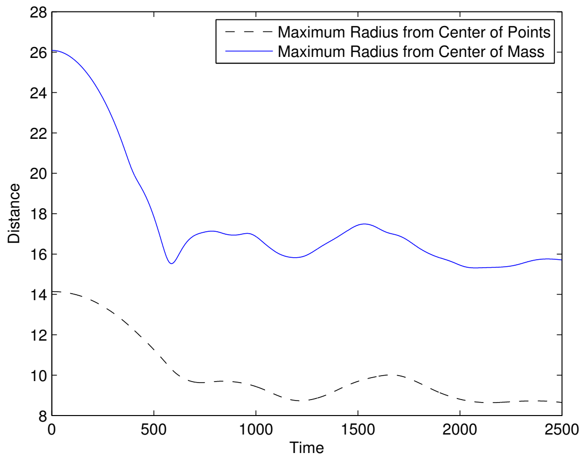

Fig. 7: Maximum radius of the swarm between center of points and center of mass under time varying.

tractive force for aggregating and one type of repulsive force for obstacle avoidance. Applying velocity matching into the swarm systems, the swarm showed stable aggregating behavior continuously. Also, due to existence of repulsive potential function, the swarm accomplished obstacles avoidance.

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