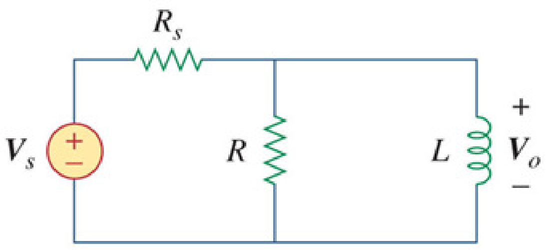
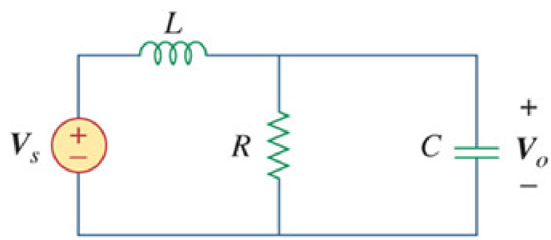
IUPUI ECE 202 Spring 2015:
Homework #4 (SOLUTIONS)   Name:

1. (Prob. 14.5 in text) For each of the circuits shown below, find the transfer function:

$$\mathbf{H}(s) = \mathbf{V}_o(s) / \mathbf{V}_s(s)$$





(a) Let $Z = R // sL = \dfrac{sRL}{R + sL}$

$$V_o = \frac{Z}{Z + R_s} V_s$$

$$H(s) = \frac{V_o}{V_s} = \frac{Z}{Z + R_s} = \frac{\dfrac{sRL}{R + sL}}{R_s + \dfrac{sRL}{R + sL}} = \frac{sRL}{RR_s + s(R + R_s)L}$$

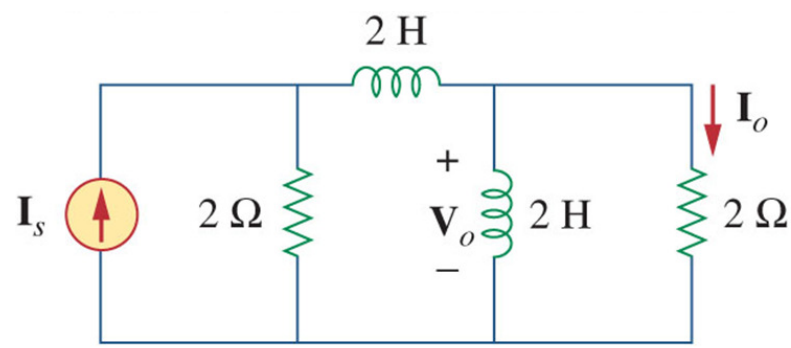
(b) Let $Z = R // \dfrac{1}{sC} = \dfrac{Rx\dfrac{1}{sC}}{R + \dfrac{1}{sC}} = \dfrac{R}{1 + sRC}$

$$V_o = \frac{Z}{Z + sL} V_s$$

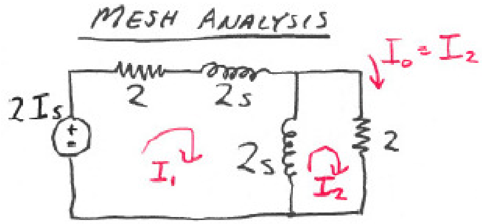
$$\mathrm{H(s)} = \frac{\mathrm{V_o}}{\mathrm{V_i}} = \frac{\mathrm{Z}}{\mathrm{Z + sL}} = \frac{\dfrac{\mathrm{R}}{1 + \mathrm{sRC}}}{\mathrm{sL} + \dfrac{\mathrm{R}}{1 + \mathrm{sRC}}} = \frac{\mathrm{R}}{\mathrm{s^2LRC + sL + R}}$$

2. (Prob. 14.6 from Text) For the circuit shown below, find the transfer function:

$$\mathbf{H}(s) = \mathbf{I}_o(s) / \mathbf{I}_s(s)$$



MESH ANALYSIS



MESH I_1: $2I_s = (2+2s)I_1 + 2sI_1 - 2sI_2$

$$I_s = (1+2s)I_1 - sI_2$$

MESH I_2: $0 = -2sI_1 + (2+2s)I_2$

$$I_1 = \frac{(1+s)}{s} I_2$$

SUB I_2 into I_1 MESH EQ^n

$$I_s = \frac{(1+2s)(1+s)}{s} I_2 - sI_2$$

$$sI_s = (1+3s+2s^2)I_2 - s^2 I_2$$

$$sI_s = (s^2+3s+1)I_2$$

SINCE $I_2 = I_o$

$$\boxed{\frac{I_o}{I_s} = \frac{s}{s^2+3s+1} = H(s)}$$

Node Voltage

NODE V_s:

$$I_s = \frac{V_s}{2} + \frac{V_s - V_o}{2s}$$

$$2sI_s = sV_s + V_s - V_o$$

$$2sI_s = (1+s)V_s - V_o$$

NODE V_o:

$$0 = \frac{V_o - V_s}{2s} + \frac{V_o}{2s} + \frac{V_o}{2}$$

$$0 = V_o - V_s + V_o + sV_o$$

$$V_s = (2+s)V_o$$

SUBSTITUTION V_o NODE INTO V_s NODE

$$2sI_s = (1+s)(2+s)V_o - V_o$$

$$= (2+3s+s^2-1)V_o$$

$$2sI_s = (s^2+3s+1)V_o$$

SINCE $V_o = 2I_o$

$$\cancel{2}sI_s = (s^2+3s+1)\cancel{2}I_o$$

SAME $\boxed{\frac{I_o}{I_s} = \frac{s}{s^2+3s+1} = H(s)}$

3. (Prob. 14.17 from Text) Sketch the magnitude and phase Bode plots for the following transfer function:

$$G(s) = \frac{s}{(s+2)^2(s+1)}$$

First put into proper (1+s/a) form:

Gain K = 0.25 → $20\log_{10}(0.25) = -12$ dB

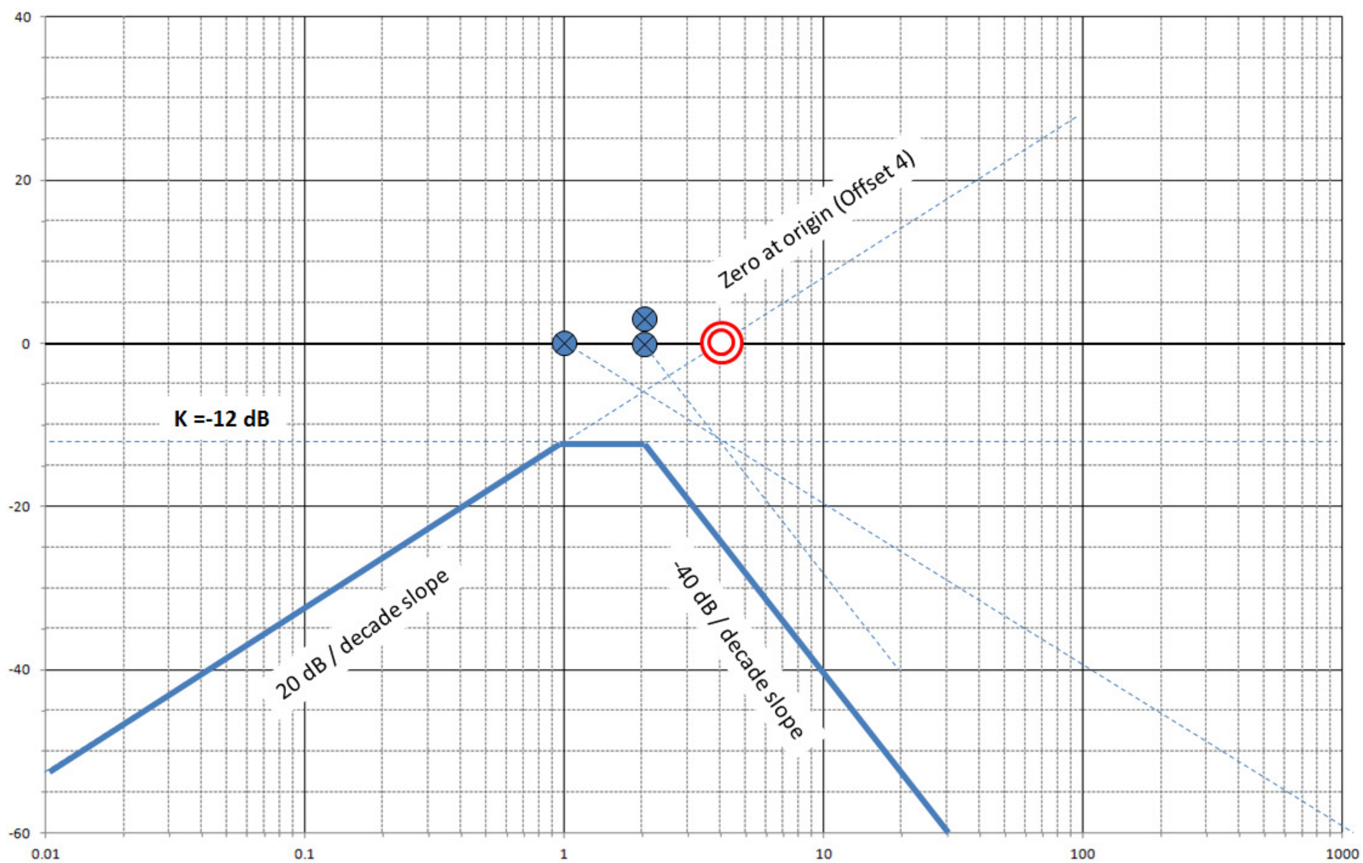
Zero at the origin (Offset 4)

$$G(s) = \frac{s}{(2)^2\left(1+\frac{s}{2}\right)^2\left(1+\frac{s}{1}\right)} = \frac{\left(\frac{1}{4}\right)s}{\left(1+\frac{s}{2}\right)^2\left(1+\frac{s}{1}\right)}$$

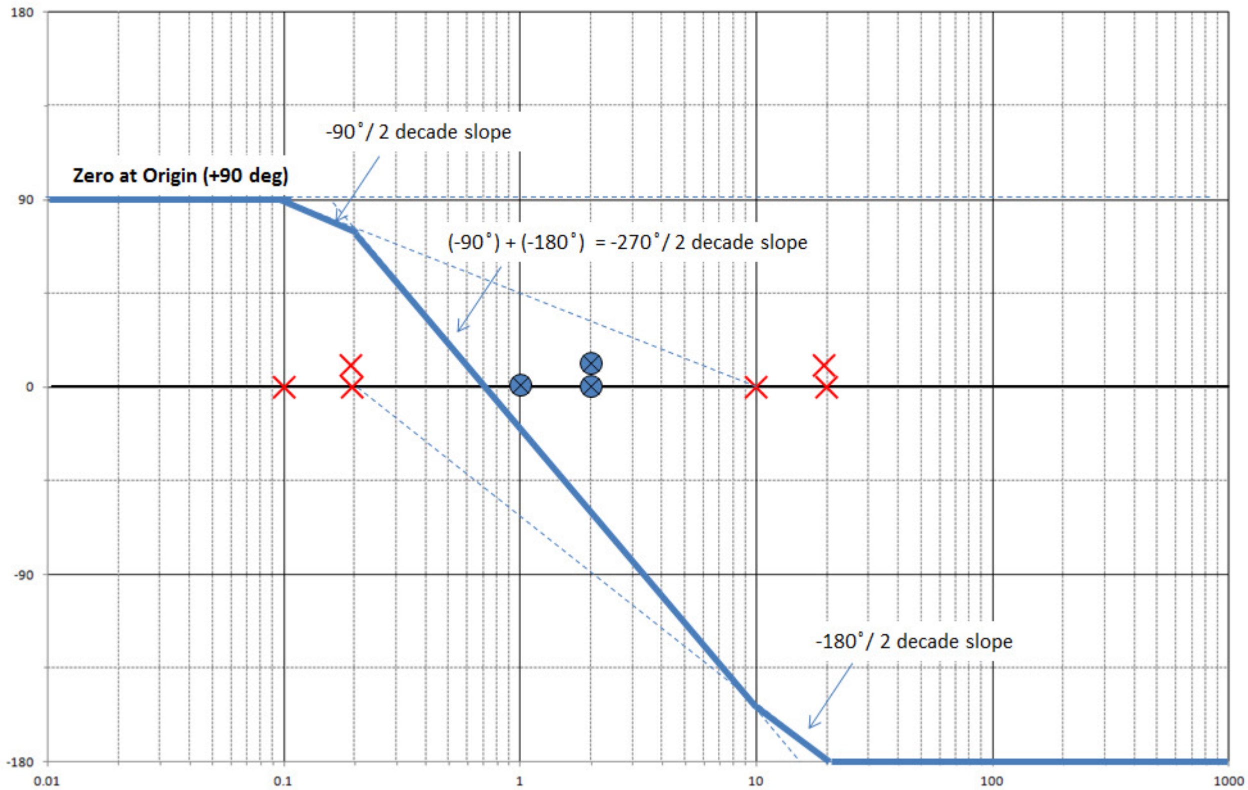
2 Poles at 2

1 Pole at 1

Magnitude Plot



Phase Plot



Notice:

$$G(s) = \frac{\left(\frac{1}{4}\right)s}{\left(1+\frac{s}{2}\right)^2\left(1+\frac{s}{1}\right)}$$

+90 One Zero

-90
-90 Three Poles
-90

-180 Phase as ω → ∞

4. (Prob. 14.19 from Text) Sketch the magnitude and phase Bode plots for the following transfer function:

$$H(s) = \frac{80s}{(s+10)(s+20)(s+40)}$$

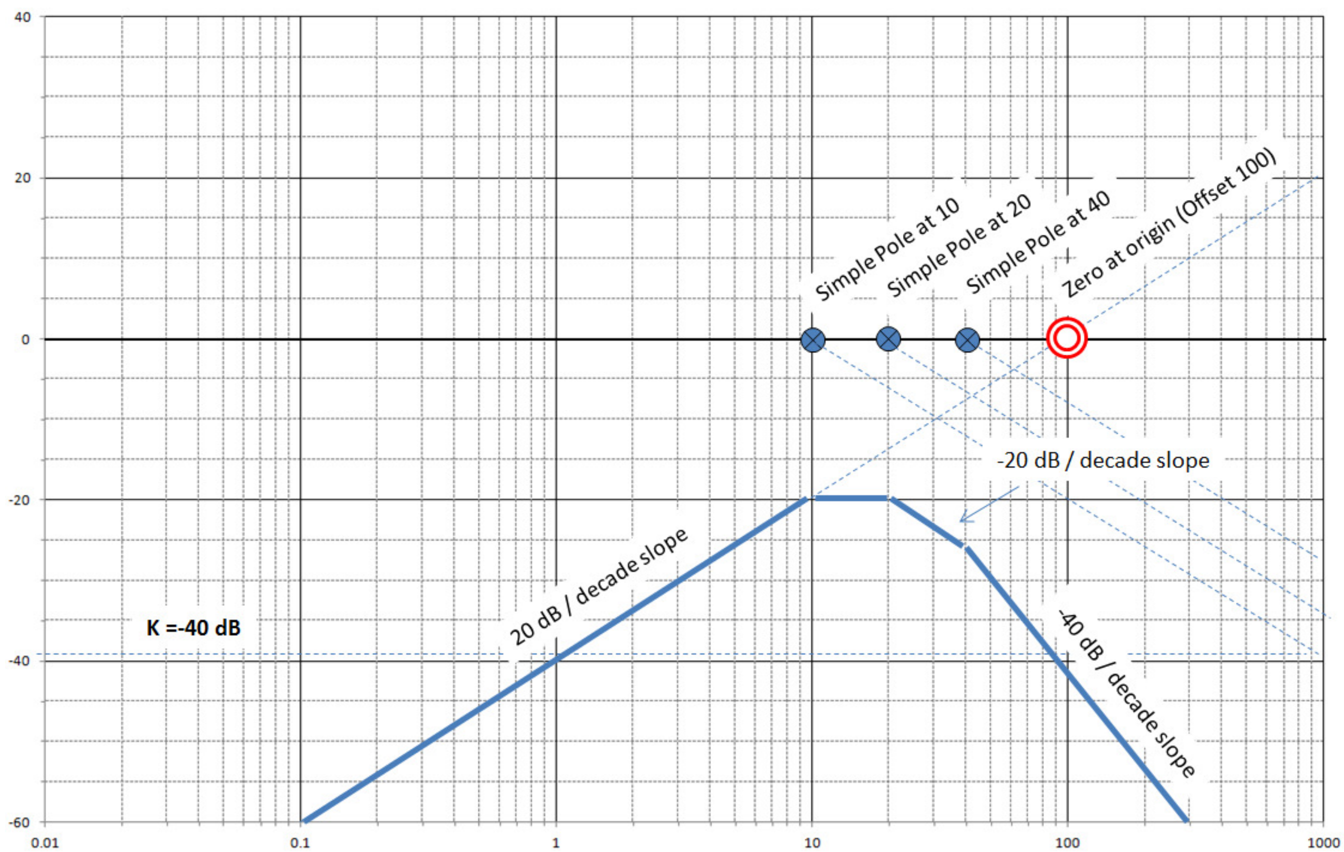
First put into proper (1+s/a) form:

Zero at the origin (Offset 100)

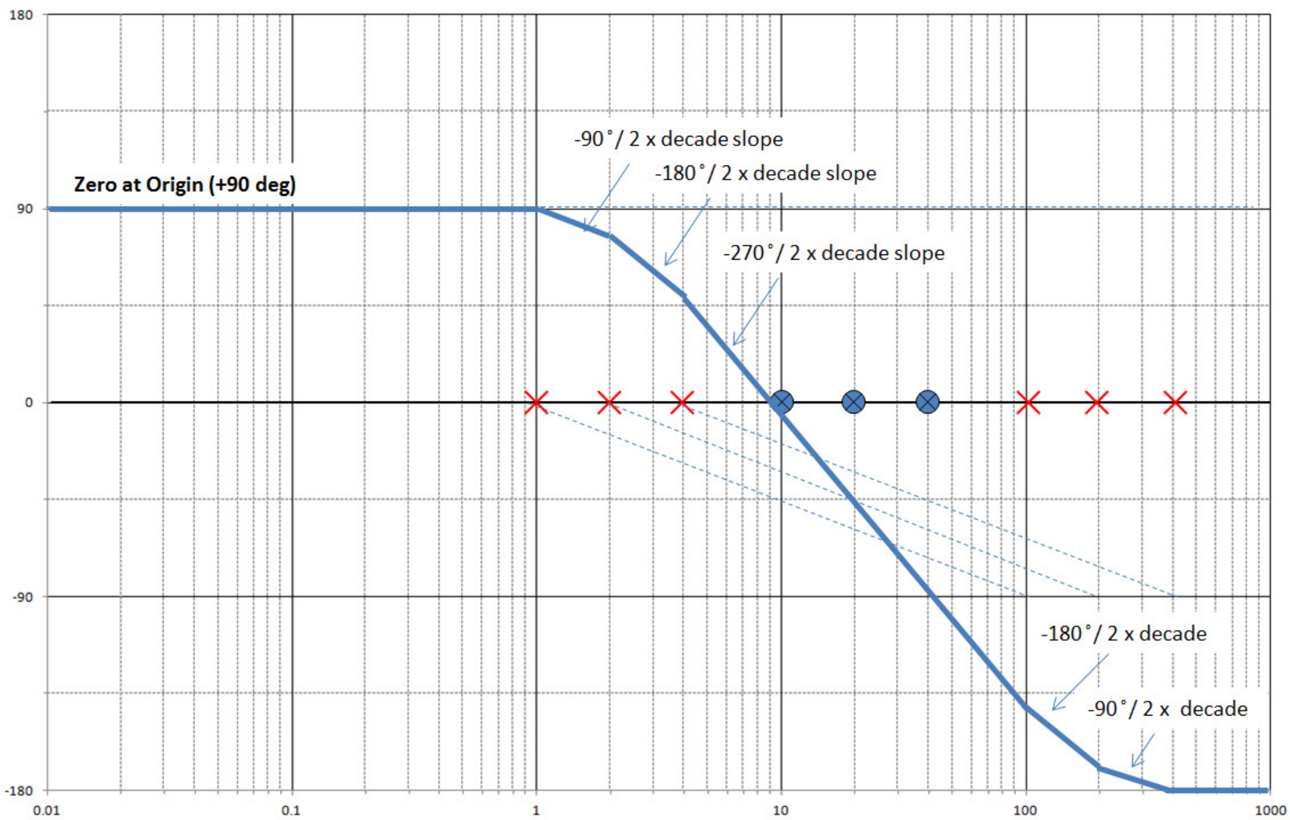
$$H(s) = \frac{80s}{10\left(1+\frac{s}{10}\right)20\left(1+\frac{s}{20}\right)40\left(1+\frac{s}{40}\right)} = \frac{\frac{1}{100}s}{\left(1+\frac{s}{10}\right)\left(1+\frac{s}{20}\right)\left(1+\frac{s}{40}\right)}$$

Poles at 10, 20, and 40

Magnitude Plot



Phase Plot



Notice:

$$H(s) = \frac{\frac{1}{100}s}{\left(1+\frac{s}{10}\right)\left(1+\frac{s}{20}\right)\left(1+\frac{s}{40}\right)}$$

→	+90	One Zero
→	-90	
	-90	Three Poles
	-90	
	-180	Phase as ω → ∞

5. (Prob. 14.22 from Text)
   a. **Part 1**: Find the transfer function H(ω) with the Bode magnitude plot as shown below:

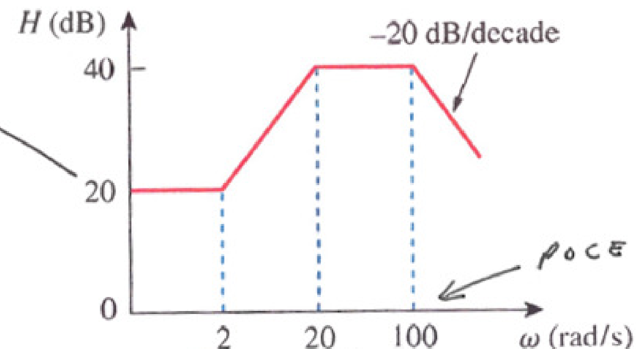
GAIN = 20

$20 \log_{10} k = 20$

$\Rightarrow K = 10$

$\log_{10} K = 1$

$K = 10^1 = 10$



POLE AT 100

zero @ 2

POLE AT 20

$$H(s) = \frac{10\left(1+\frac{s}{2}\right)}{\left(1+\frac{s}{20}\right)\left(1+\frac{s}{100}\right)} = \frac{10\left(\frac{1}{2}\right)(2+s)}{\left(\frac{1}{20}\right)(20+s)\left(\frac{1}{100}\right)(100+s)}$$

$$H(s) = \frac{10{,}000(2+s)}{(20+s)(100+s)}$$

sub $s = j\omega$ for $H(\omega)$

   b. **Part II:** Use MATLAB to generate the Bode plot from your derived transfer function

Command Window

New to MATLAB? Watch this Video, see Examples, or

```
>> num = [10000 20000];
>> den = [1 120 2000];
>> G = tf(num,den)

G =

   10000 s + 20000
  -----------------
  s^2 + 120 s + 2000

Continuous-time transfer function.

>> bode(G),grid
```