Chapter 13
Magnetically Coupled Circuits

13.1 What is a transformer?

13.2 Mutual Inductance

13.3 Energy in a Coupled Circuit

13.4 Linear Transformers

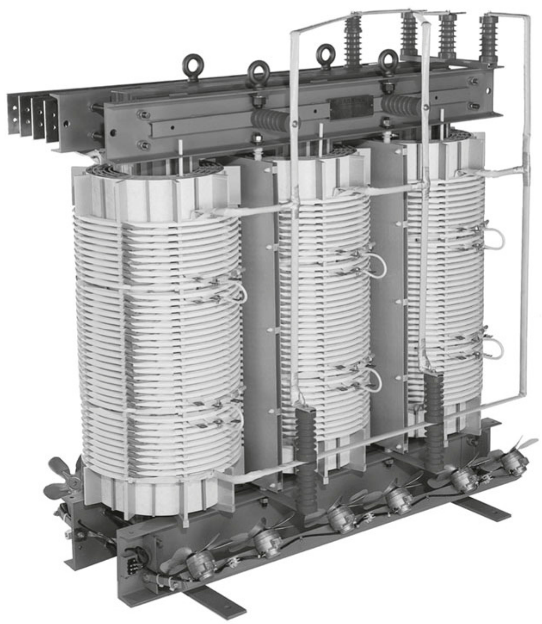
13.5 Ideal Transformers

13.6 Ideal Autotransformers

13.9 Applications

13.1 What is a transformer? (1)

- It is an electrical device designed on the basis of the concept of magnetic coupling
- It uses magnetically coupled coils to transfer energy from one circuit to another
- It is the key circuit elements for stepping up or stepping down ac voltages or currents, impedance matching, isolation, etc.



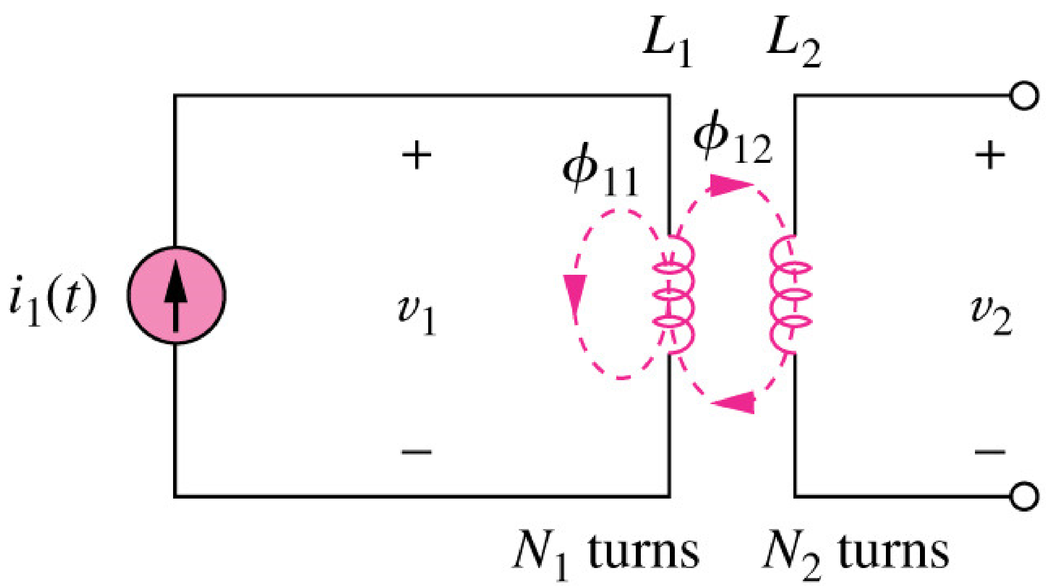
(a)

(b)

Courtesy of: (a) Electric Service Co., (b) Jensen Transformers

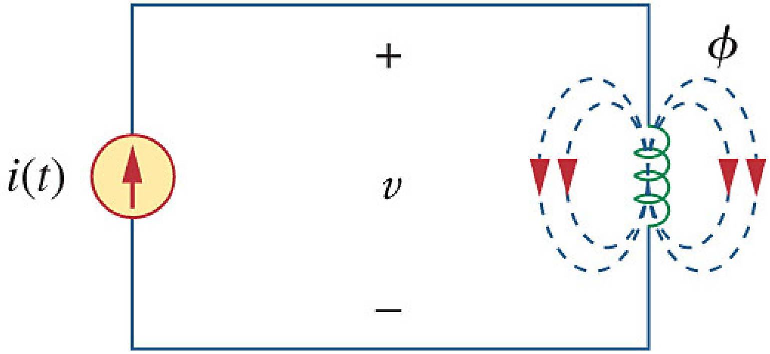
13.2 Mutual Inductance (1)

- Mutual Inductance
  - When two inductors (or coils) are in close proximity of each other, the magnetic flux caused by current in one coil links with the other coil, thereby inducing voltage in the latter.



13.2 Mutual Inductance (2)

- First consider a single inductor, a coil with N turns:



When current i flows through the coil, a magnetic flux Φ is produced around it.

- According to Faraday's law, the voltage v induced in the coil is proportional to the number of turns N and the time rate of change of magnetic flux Φ:

$$v = N \frac{d\phi}{dt}$$

13.2 Mutual Inductance (3)

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- Voltage induced in the coil given by:

$$v = N\frac{d\phi}{dt}$$

- But the flux Φ is produced by current i so that any change in Φ is caused by a change in the current:

$$v = N\overbrace{\underbrace{\frac{d\phi}{di}}\frac{di}{dt}}$$

- Recall the voltage-current relationship for an inductor:

$$v = L\frac{di}{dt}$$

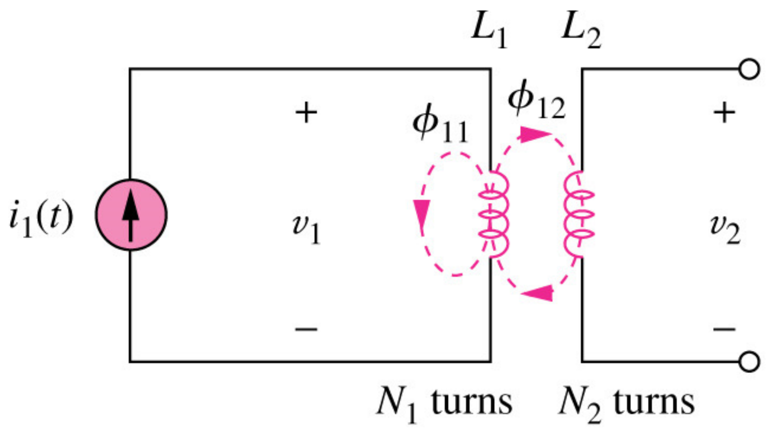
- The "Self" inductance L of the inductor is thus given by:

$$\boxed{L = N\frac{d\phi}{di}}$$

- Self-inductance L relates the voltage induced in a coil by a time-varying current in the same coil.

13.2 Mutual Inductance (4)

- Now consider two coils with self-inductances L_1 and L_2 that are in close proximity of each other:



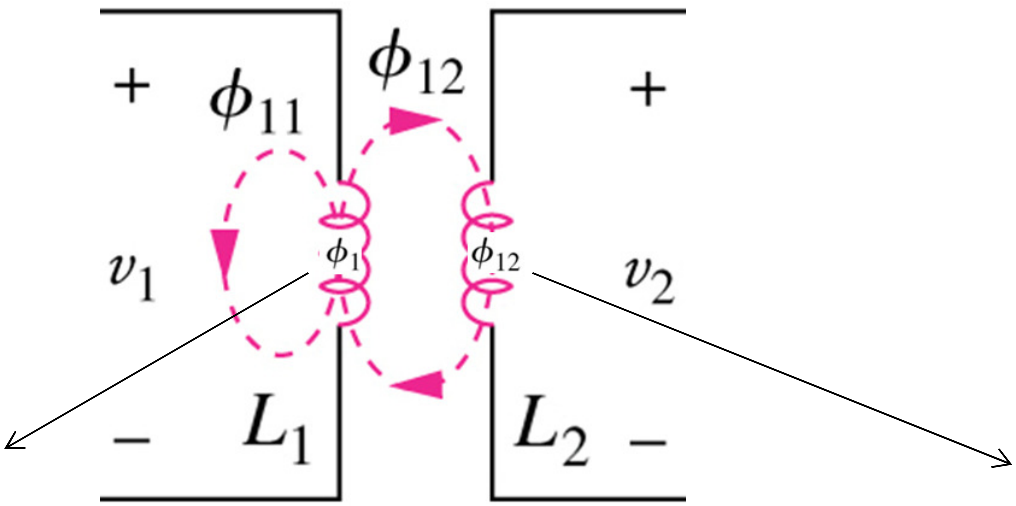
Coil 1 has N_1 turns
Coil 2 has N_2 turns.
Assume coil 2 carries no current.

- The total magnetic flux Φ_1 emanating from coil 1 has two components:
  - Φ_{11} links only coil 1
  - Φ_{12} links both coils

$$\phi_1 = \phi_{11} + \phi_{12}$$

13.2 Mutual Inductance (5)

- Although the two coils are physically separated, they are magnetically coupled.
- The voltage induced in each coil is proportional to the flux in each coil.

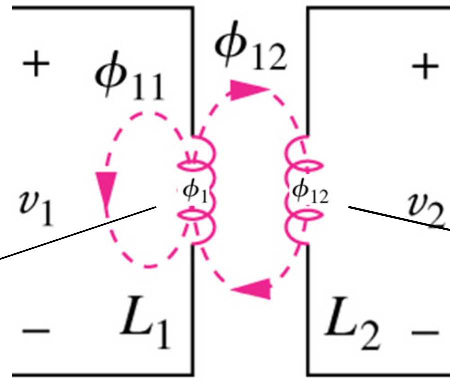


$$v_1 = N_1 \frac{d\phi_1}{dt}$$

$$v_2 = N_2 \frac{d\phi_{12}}{dt}$$

13.2 Mutual Inductance (6)

- The voltage equations can be rewritten as follows:



v_2 is the open-circuit mutual voltage (or induced voltage) across coil 2

$$v_1 = N_1 \frac{d\phi_1}{dt}$$

$$v_1 = \underbrace{N_1 \frac{d\phi_1}{di_1}}_{L_1} \frac{di_1}{dt}$$

$$v_1 = L_1 \frac{di_1}{dt}$$

$$v_2 = N_2 \frac{d\phi_{12}}{dt}$$

$$v_2 = \underbrace{N_2 \frac{d\phi_{12}}{di_1}}_{M_{21}} \frac{di_1}{dt}$$

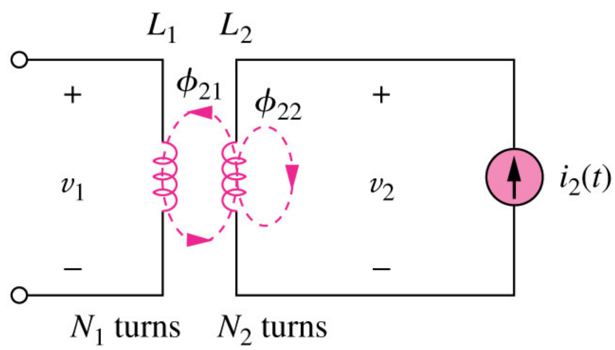
$$v_2 = M_{21} \frac{di_1}{dt}$$

L_1 is the self-inductance of coil 1

M_{21} is the mutual inductance of coil 2 with respect to coil 1

13.2 Mutual Inductance (7)

- Suppose now we let current i_2 flow in coil 2 while coil 1 carries no current:



- The magnetic flux Φ_2 emanating from coil 2 comprises flux Φ_{22} that links only coil 2 and flux Φ_{21} that links both coils:

$$\phi_2 = \phi_{21} + \phi_{22}$$

- The resulting symmetry is true:

$$v_2 = L_2 \frac{d\,i_2}{d\,t} \qquad v_1 = M_{12} \frac{d\,i_2}{d\,t} \qquad M_{12} = N_1 \frac{d\,\phi_{21}}{d\,i_2}$$

13.2 Mutual Inductance (8)

- $M_{12} = M_{21} = M$; The "Mutual inductance" between the coils
- Mutual inductance M is measured in Henrys (just like inductors)
- Mutual inductance only exists when inductors or coils are in close proximity and the circuits are driven by time-varying sources
- Although mutual inductance M is always a positive quantity, the mutual voltage M di/dt may be negative or positive, just like the self-induced voltage L di/dt (determined by "Dot" convention)

13.2 Mutual Inductance (9)

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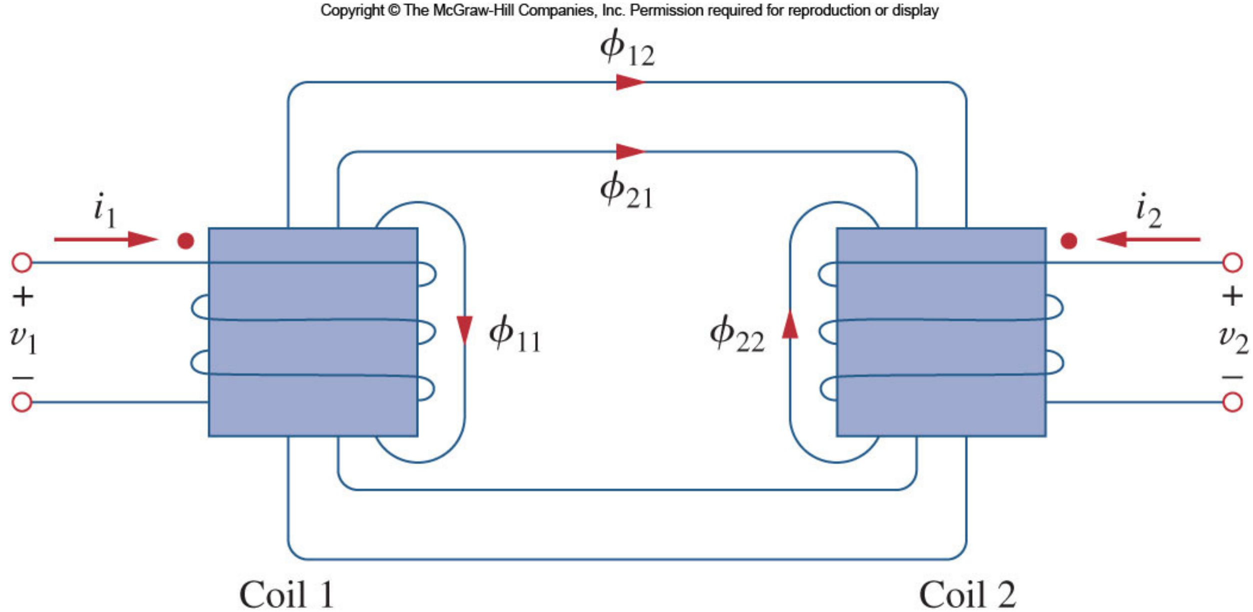
- Self-induced voltage polarity is determined by the reference direction of the current and the reference polarity of the voltage,
- The polarity of the mutual voltage is not as easy to determine (depends on winding direction of the coils).
- We use the dot convention to determine



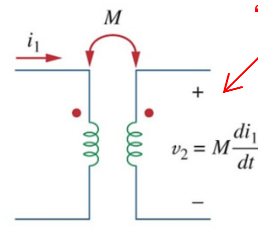
13.2 Mutual Inductance (10)

“Dot Convention”

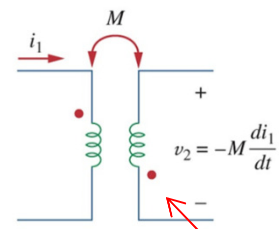
- If a current **enters** the dotted terminal of one coil, the reference polarity of the mutual voltage in the second coil is **positive** at the dotted terminal of the second coil.

Enters the “Dot”

Voltage is “Positive”



(a)

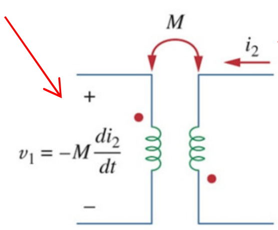


(b)

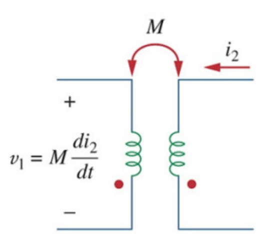
Voltage is “Positive” **At “Dotted” Terminal**

Voltage is “Negative”

Enters the “Un-Dotted” side



(c)



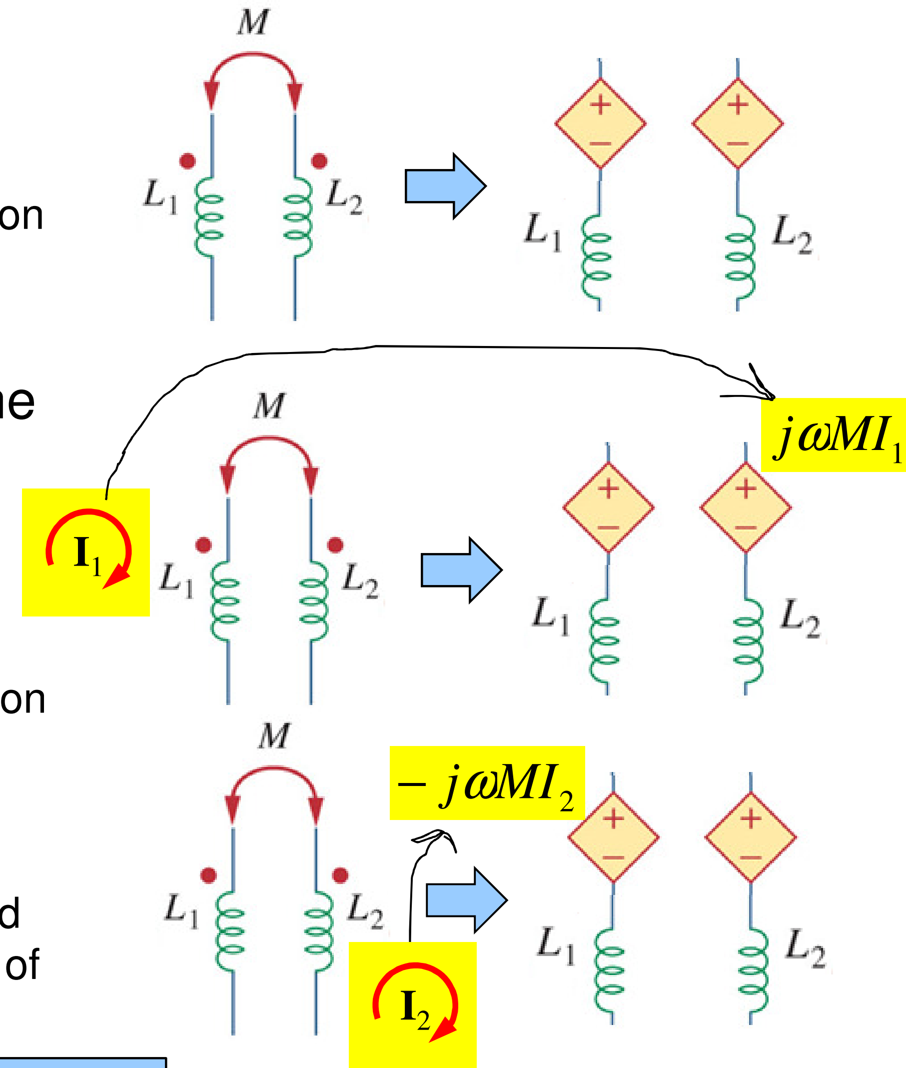
(d)

- Alternatively, if a current **leaves** the dotted terminal of one coil, the reference polarity of the mutual voltage in the second coil is **negative** at the dotted terminal of the second coil.

13.2 Mutual Inductance (11)

“The Model” (Phasor domain)

- Replace the “Dot” with a controlled voltage source.
  - Place on same side as the Dot
  - Positive terminal in same direction as the Dot
- Now look at the current entering each dot to determine the magnitude of the voltage source
  - If the current enters the dotted terminal, it will induce a positive voltage in the controlled source on the opposite terminal of jωM(I)
  - If the current enters the “un-dotted” terminal, it will induce a negative voltage in the controlled source on the opposite terminal of –jωM(I)

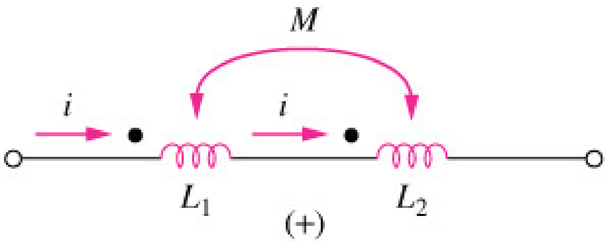


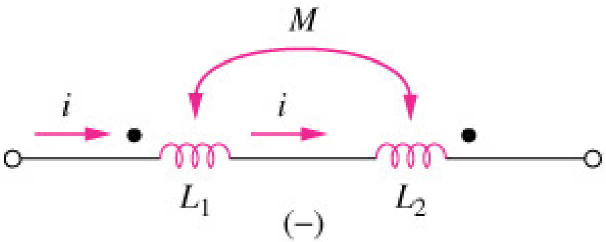
Note: This method differs slightly from the text

KEY CONCEPT !

13.2 Mutual Inductance (12)

Dot convention for coils in series; the sign indicates the polarity of the mutual voltage





$$L = L_1 + L_2 + 2M$$

(series - aiding connection)

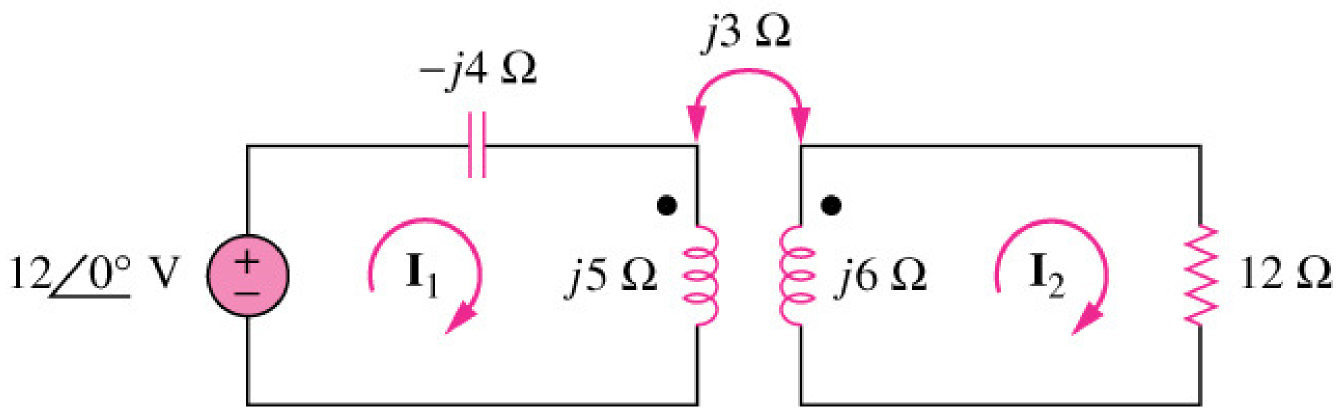
$$L = L_1 + L_2 - 2M$$

(series - opposing connection)

We can show this using the model!

Example 13.1 (Textbook)

Calculate the phasor currents I_1 and I_2 in the circuit shown below.

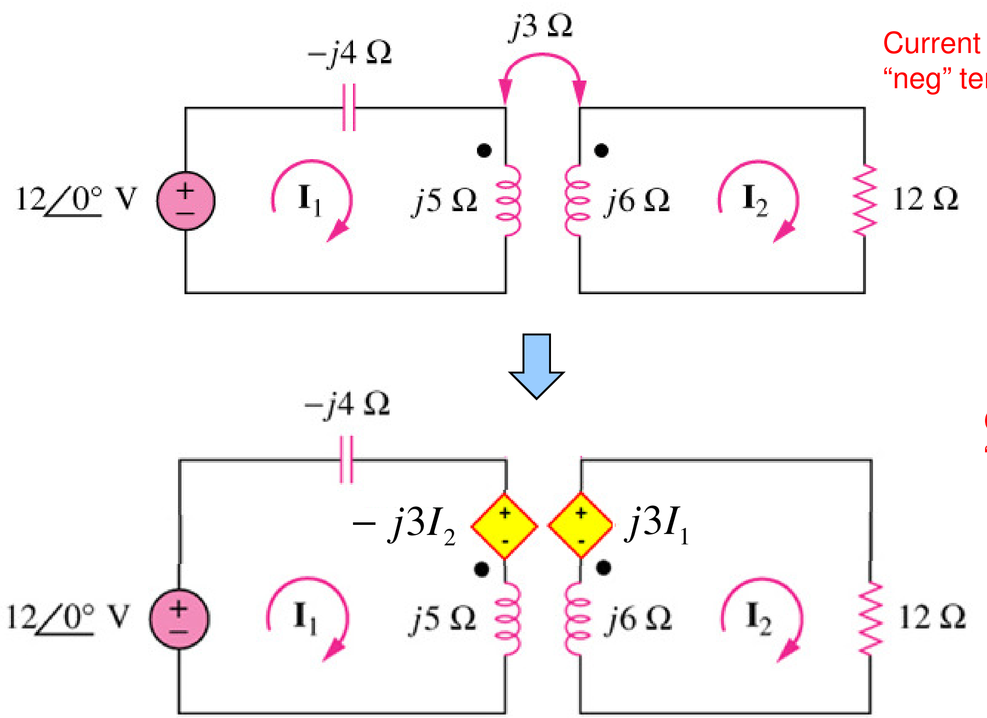


Ans: $I_1 = 13.02\angle -49.40°\text{A};\quad I_2 = 2.91\angle 14.04°\text{A}$

13.2 Mutual Inductance (14)

Example Problem 13.1

First, replace with the model:



Then, solve Loop Equations:

Current enters "neg" terminal

Current enters "+" terminal but value is – j3I$_2$

Loop I$_1$

$$-12 - j4I_1 + (-j3I_2) + j5I_1 = 0$$
$$jI_1 - j3I_2 = 12$$

Current enters "neg" terminal

Loop I$_2$

$$j6I_2 - j3I_1 + 12I_2 = 0$$
$$-j3I_1 + (12 + j6)I_2 = 0$$

NOTE:

- I$_1$ goes "into" the dot → Induced voltage on the second coil is "Positive"
- I$_2$ goes "into" the un-dotted side → Induced voltage on the first coil is "Negative"

Pay Attention to Sign Conventions !

13.2 Mutual Inductance (15)

Example Problem 13.1

Lastly, solve 2 equations, 2 unknowns (expected you know this):

Loop I_1 $\longrightarrow$ $jI_1 - j3I_2 = 12$ $\longrightarrow$ $jI_1 = 12 + j3I_2 \Rightarrow I_1 = 3I_2 - j12$

Loop I_2 $\longrightarrow$ $-j3I_1 + (12 + j6)I_2 = 0$

Substitution: $-3(12 + j3I_2) + (12 + j6)I_2 = 0$

÷ by 3 $-12 - j3I_2 + (4 + j2)I_2 = 0$

Collect Terms: $(4 - j)I_2 = 12$

Solve for I_2 $I_2 = \dfrac{12}{(4 - j)} = 2.824 + j0.706 = \boxed{2.91\angle 14.04°\ \text{A}}$

Find I_1 (Substitute back to Eq1)

$$I_1 = 3(2.824 + j0.706) - j12$$

$$I_1 = 8.471 + j2.118 - j12$$

$$I_1 = 8.471 - j9.882 = \boxed{13.02\angle -49.40°\ \text{A}}$$

Think about where we could have made mistakes!

- Applying the model incorrectly (wrong sign convention)
- Incorrect Phasor notation (Understand: jωM ↔ j3 ; jωL ↔ j5 ; 1/ jωC ↔ -j4)
- Incorrect sign convention for loop equations
  - Current entering negative terminal of a voltage source
- Understanding of complex numbers!
  - Familiarity with your calculators handling of complex numbers
  - Converting from rectangular to polar

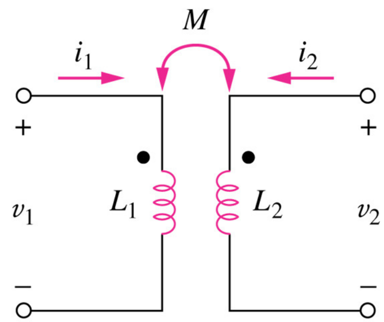
Review / Understand these concepts to avoid these mistakes

13.2 Recommended Viewing:

- Watch these videos illustrating solving mutual inductance problems:
  - http://www.youtube.com/watch?v=tD35a-uzd34
  - http://www.youtube.com/watch?v=hzU4XKQYTWw
  - http://www.youtube.com/watch?v=OqSvesTtnUo
- Uses the model described previously to solve mutual inductance problems.

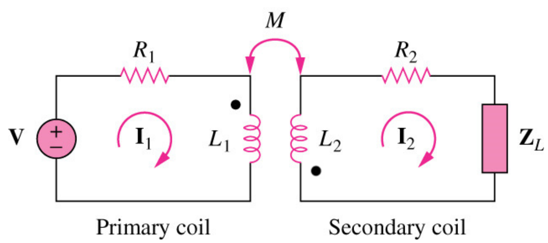
13.3 Energy in a Coupled Circuit (1)

- The instantaneous energy w stored in the circuit is:



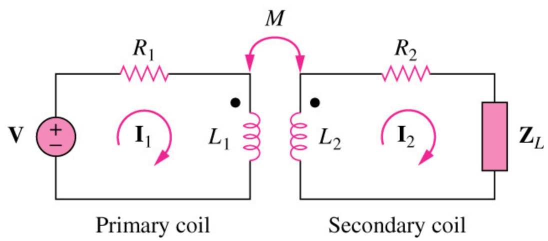
$$\boxed{w = \frac{1}{2}L_1 i_1^2 + \frac{1}{2}L_2 i_2^2 \pm M i_1 i_2}$$

- If **both** currents enter (or **both** leave) the dotted terminal the mutual term is (positive):



$$w = \frac{1}{2}L_1 i_1^2 + \frac{1}{2}L_2 i_2^2 + M i_1 i_2$$

- Otherwise the mutual term is (negative):



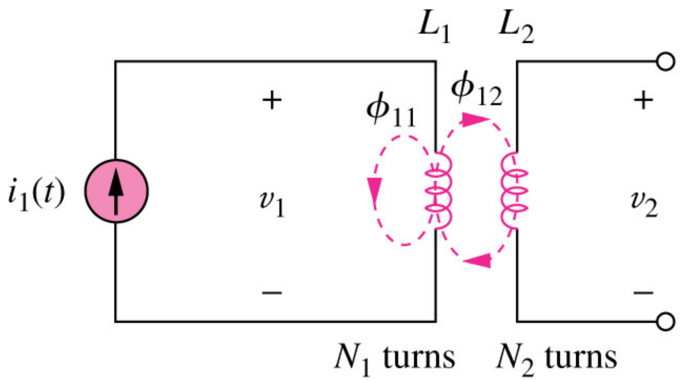
$$w = \frac{1}{2}L_1 i_1^2 + \frac{1}{2}L_2 i_2^2 - M i_1 i_2$$

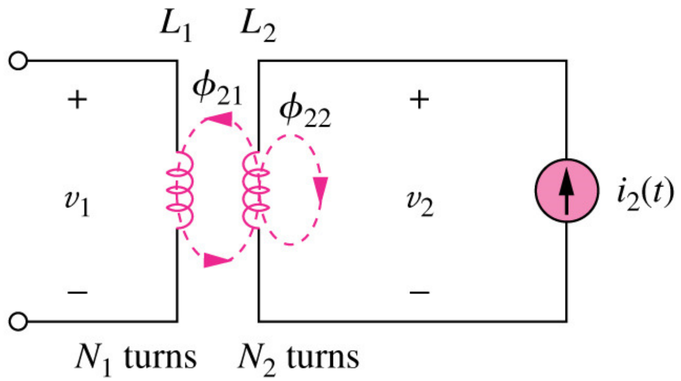
13.3 Energy in a Coupled Circuit (2)

- The coupling coefficient, k, is a measure of the magnetic coupling between two coils; $0 \leq k \leq 1$.

$$\boxed{M = k\sqrt{L_1 L_2}}$$

K = 1 coils perfectly coupled
K < 0.5 coils loosely coupled
K > 0.5 coils tightly coupled





$$K = \frac{\phi_{12}}{\phi_{11} + \phi_{12}}$$

$$K = \frac{\phi_{21}}{\phi_{21} + \phi_{22}}$$

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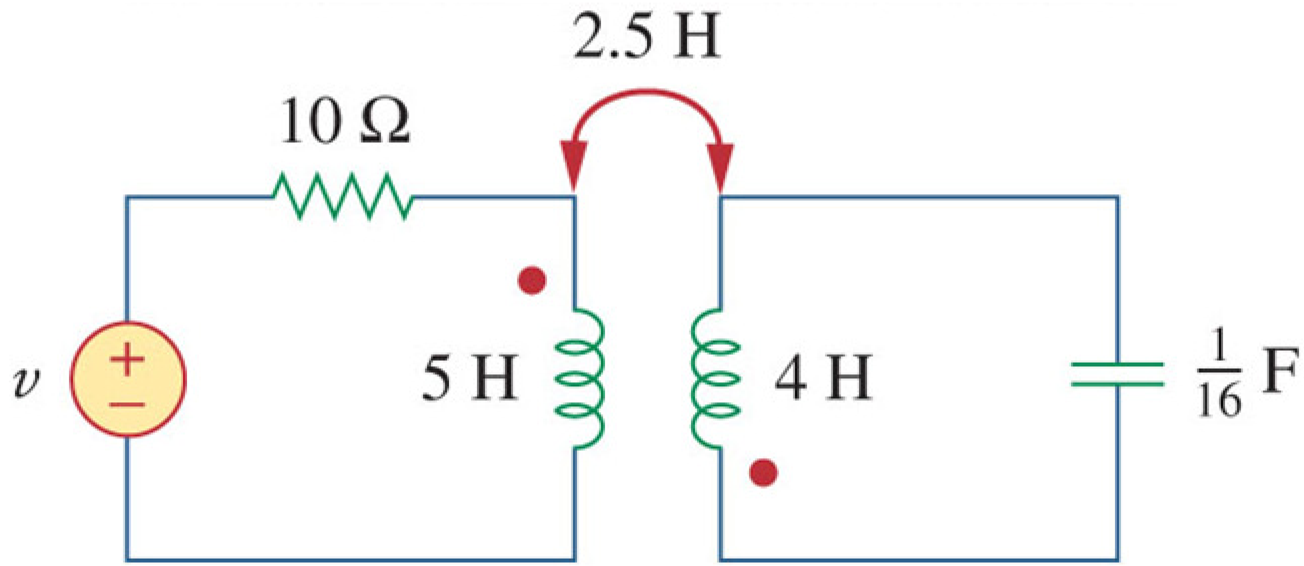
Example 13.3 (Textbook)

Consider the circuit below. Determine the coupling coefficient. Calculate the energy stored in the coupled inductors at time t = 1s if *v=60cos(4t +30°)* V.



Ans: *k*=0.56; *w(1)*=20.73J

13.3 Energy in a Coupled Circuit (4)

Example 13.3

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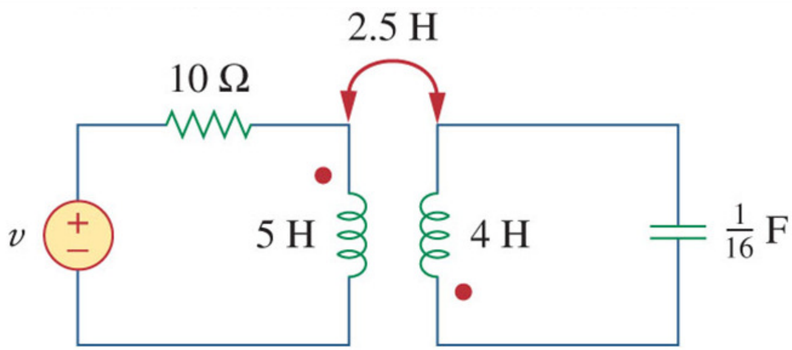
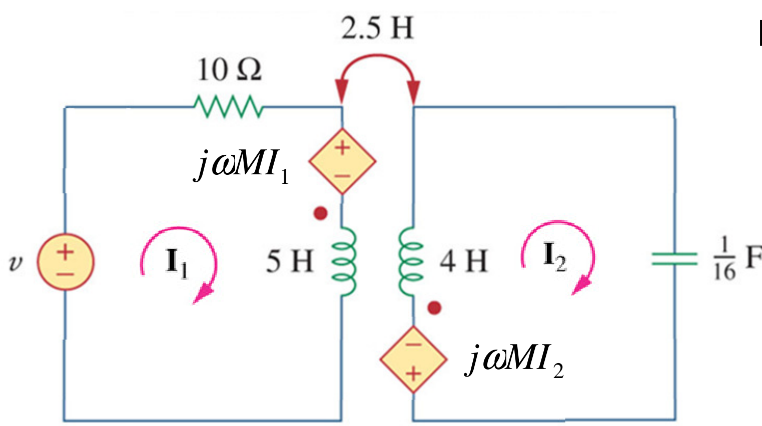


To find k, use the relation between L_1, L_2, and M:

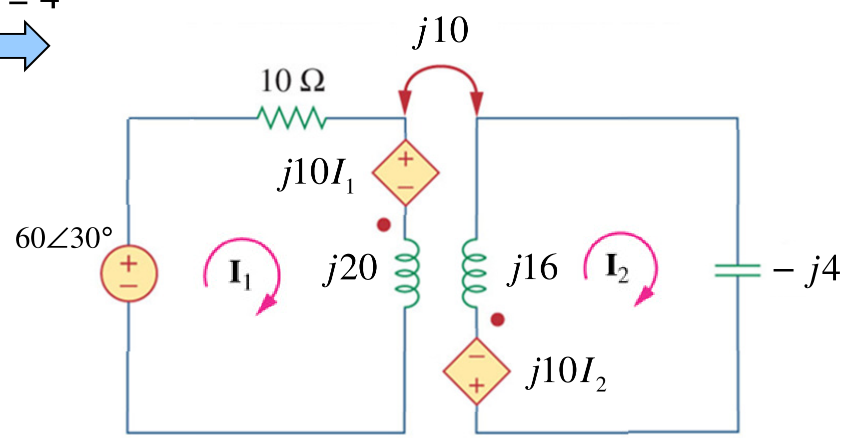
$$L_1 = 5 \;;\; L_2 = 4 \;;\; M = 2.5$$

$$k = \frac{M}{\sqrt{L_1 L_2}} = \frac{2.5}{\sqrt{(5)(4)}} = 0.56$$

Apply the "Model"



Convert to Phasor
ω = 4



Solve Mesh equations for I_1 and I_2

13.3 Energy in a Coupled Circuit (5)

Example 13.3

From Mesh analysis:

$$I_1 = 3.905\angle -19.4° \text{ A}$$
$$I_2 = 3.254\angle 160.6° \text{ A}$$

At t = 1, the value of ωt = (4)(1) = 4 radians = 229.2º

To find the energy at t = 1:

$$i_1 = 3.905\cos(\omega t - 19.4°)$$
$$i_2 = 3.254\cos(\omega t + 160.6°)$$

$$i_1 = 3.905\cos(229.2° - 19.4°) = -3.389$$
$$i_2 = 3.254\cos(229.2° + 160.6°) = 2.824$$

Positive since current (as defined) enters **both** dots

$$w = \frac{1}{2}L_1 i_1^2 + \frac{1}{2}L_2 i_2^2 + Mi_1 i_2$$

$$w = \frac{1}{2}(5)(-3.389)^2 + \frac{1}{2}(4)(2.824)^2 + 2.5(-3.389)(2.824) = 20.73 \text{ J}$$

Homework #2

Due in class Monday, January 26, 2015

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- 13.1
- 13.7
- 13.9
- 13.24

13.1 What is a transformer?

13.2 Mutual Inductance

13.3 Energy in a Coupled Circuit

13.4 Linear Transformers

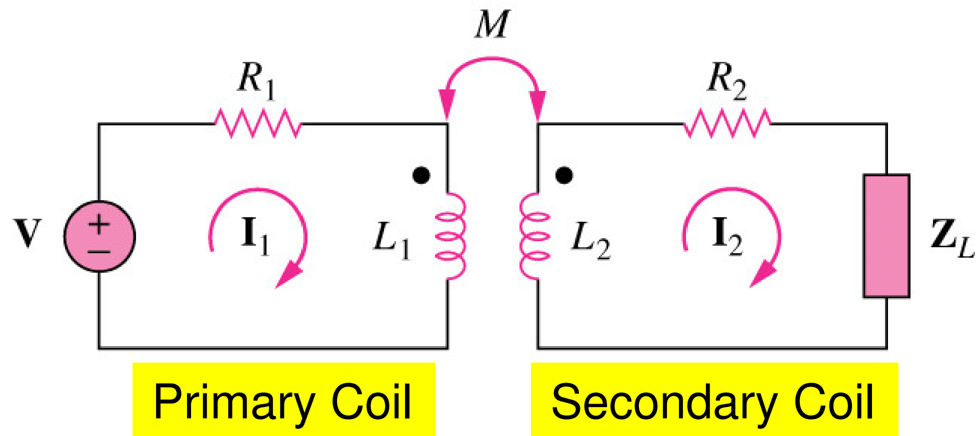
13.5 Ideal Transformers

13.6 Ideal Autotransformers

13.9 Applications

13.4 Linear Transformers (1)

- A transformer is generally a four-terminal device comprising two (or more) magnetically coupled coils. Below is a "simple" model:

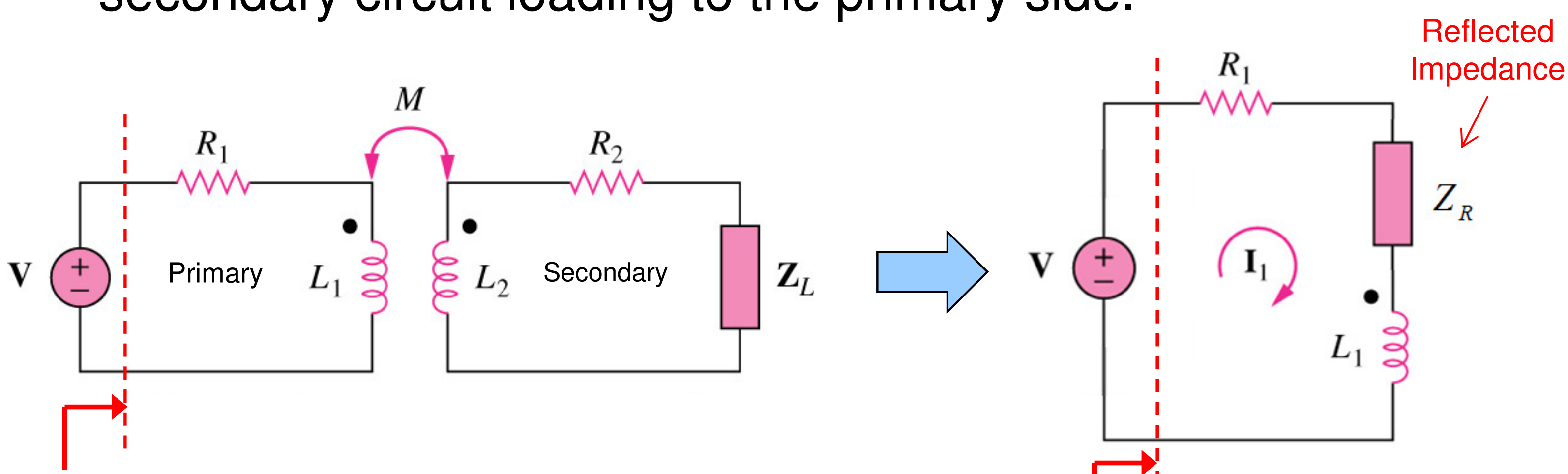


- The coil connected to the voltage source is called the primary winding.
- The coil connected to the load is called the secondary winding.
- Resistances R_1 and R_2 are included to account for the losses in the coils.
- A transformer is said to be linear if the coils are wound on a magnetically linear material for which the permeability is constant.
  - Air, plastic, Bakelite, wood, etc.
  - Most materials are magnetically linear.

13.4 Linear Transformers (2)

Reflected Impedance

- Often we are interested in the input impedance Z_{in} seen by the source.
  - For example, may want to match Z_{in} to the source impedance for maximum power transfer!
- To simplify our analysis we can break Z_{in} up into:
  - $Z_{primary}$ -- The impedance of the "primary" circuit $\left(Z_{primary} = R_1 + j\omega L_1\right)$
  - Z_R -- The "reflected" impedance back to the "primary".
- The "reflected" impedance Z_R is the contribution of the secondary circuit loading to the primary side.



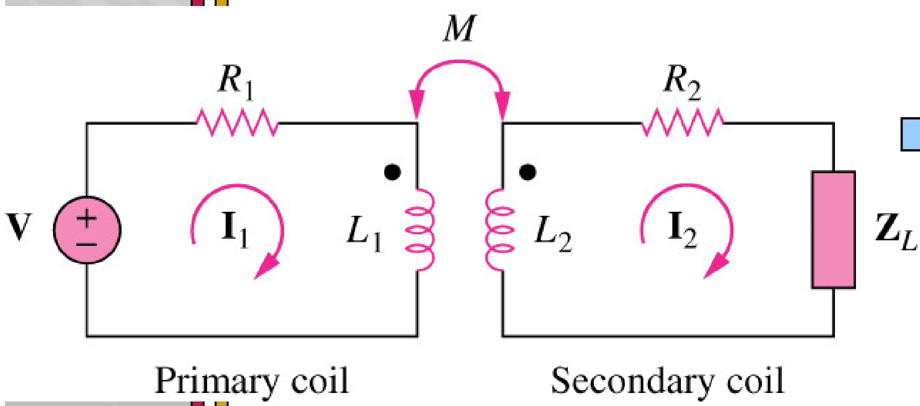
$$Z_{in} = Z_{primary} + Z_R$$

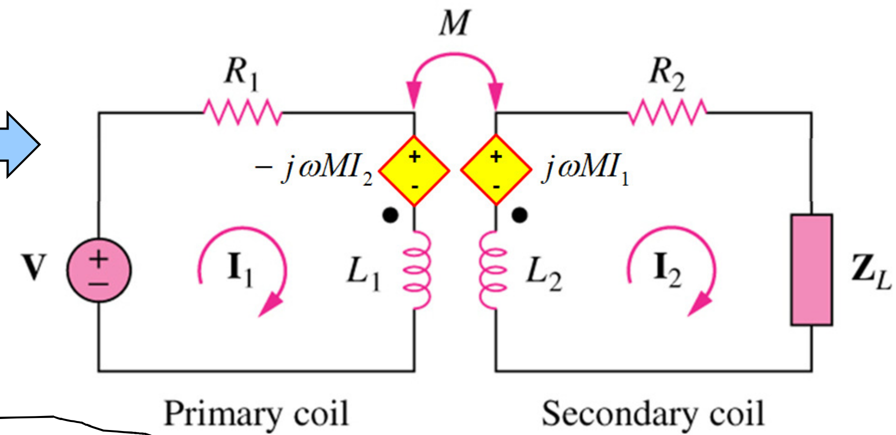
$$Z_{in} = Z_{primary} + Z_R$$

13.4 Linear Transformers (2)

Reflected Impedance

To obtain the input impedance Z_{in} as seen from the source:





Mesh 1: $V=(R_1+j\omega L_1)I_1 - j\omega M I_2$

Current enters positive terminal so it is **+** ($-j\omega MI_2$)

Mesh 2: $0=-j\omega M I_1 + (R_2+j\omega L_2+Z_L)I_2$

Current enters negative terminal so it is **-** ($j\omega MI_2$)

From Mesh 2:

$$I_2=\frac{j\omega M}{R_2+j\omega L_2+Z_L}I_1$$

Substituting into Mesh 1 gives:

$$V=\underbrace{(R_1+j\omega L_1)}I_1+\underbrace{\frac{\omega^2M^2}{R_2+j\omega L_2+Z_L}}I_1$$

$$V=Z_{primary}\,I_1+Z_{reflected}\,I_1$$

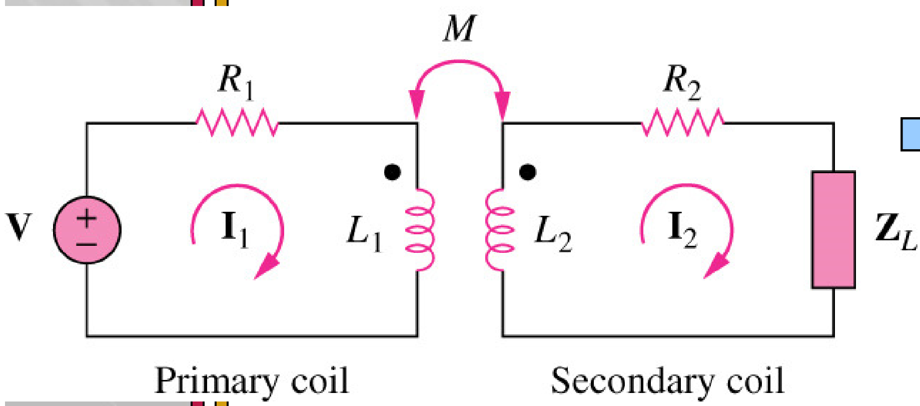
Reflected Impedance

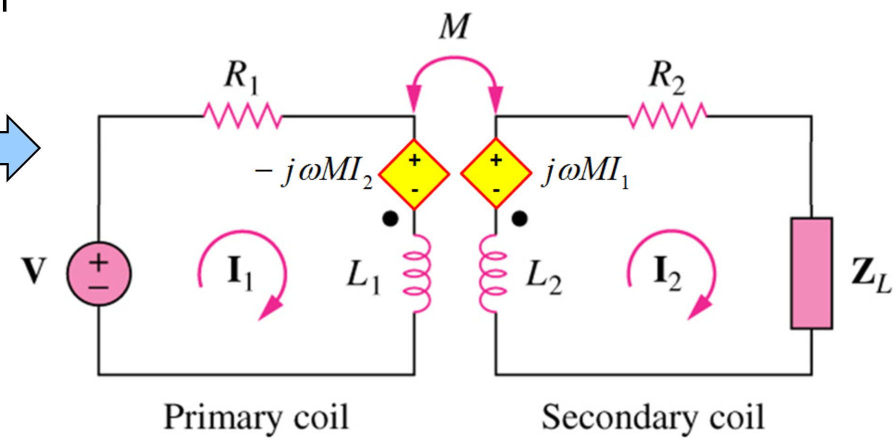
$$\boxed{Z_R=\frac{\omega^2M^2}{R_2+j\omega L_2+Z_L}}$$

13.4 Linear Transformers (3)

Reflected Impedance (Another way of looking at it !)

Obtain input impedance Z_{in} as seen from the source:





Mesh 1: $V = (Z_{primary})I_1 - j\omega M I_2$

Mesh 2: $0 = -j\omega M I_1 + (Z_{Secondary})I_2$

$Z_{Secondary}$ = the total series impedance in the secondary loop

From Mesh 2:

$$\mathrm{I}_2 = \frac{j\omega M}{Z_{Secondary}}\mathrm{I}_1$$

Substituting into Mesh 1 gives:

$$V = \underbrace{Z_{primary}}\, I_1 + \underbrace{\frac{\omega^2 M^2}{Z_{Secondary}}}\, I_1$$

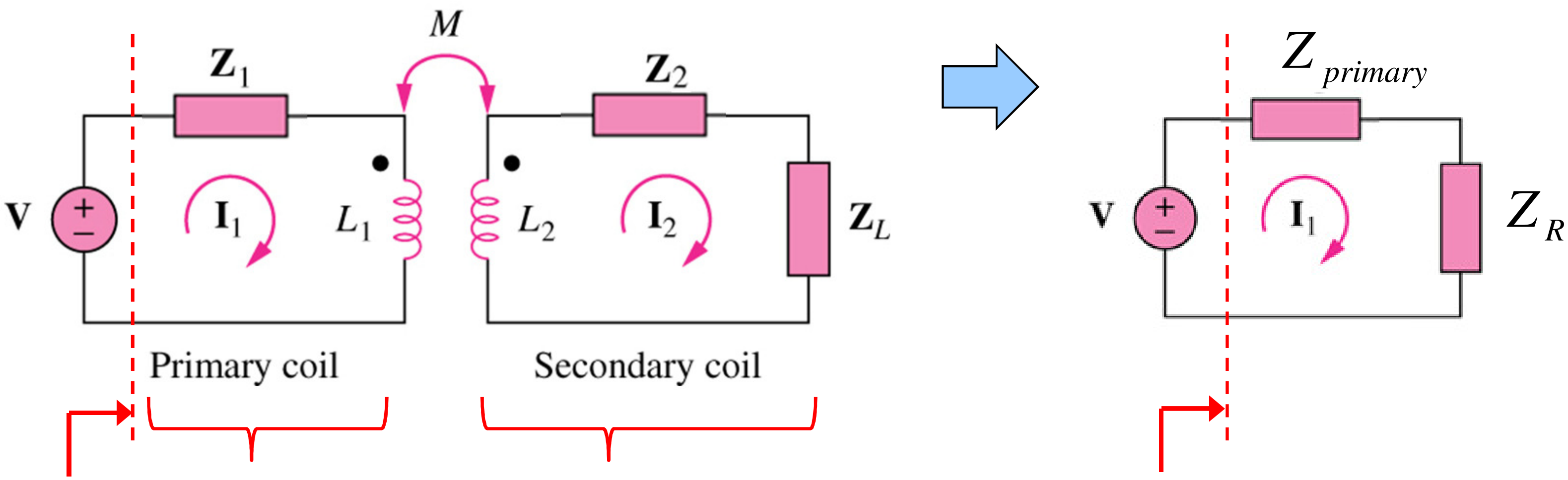
$$V = Z_{primary}\, I_1 + Z_{reflected}\, I_1$$

Reflected Impedance

$$\boxed{Z_R = \frac{\omega^2 M^2}{Z_{Secondary}}}$$

13.4 Linear Transformers (4)
Reflected Impedance

- The input impedance can be broken into two parts as follows:



$$Z_{in} = \underbrace{Z_{primary}}_{\text{Primary coil}} + \underbrace{Z_R}_{\text{Secondary coil}}$$

- Series Impedance in Primary Coil: $Z_{primary} = Z_1 + j\omega L_1$

- Series Impedance in Secondary Coil: $Z_{secondary} = Z_2 + Z_L + j\omega L_2$

- Input Impedance: $\boxed{Z_{in} = Z_{primary} + \dfrac{\omega^2 M^2}{Z_{Secondary}}}$

Note: Z_{in} will be the same if the dot on L2 is switched

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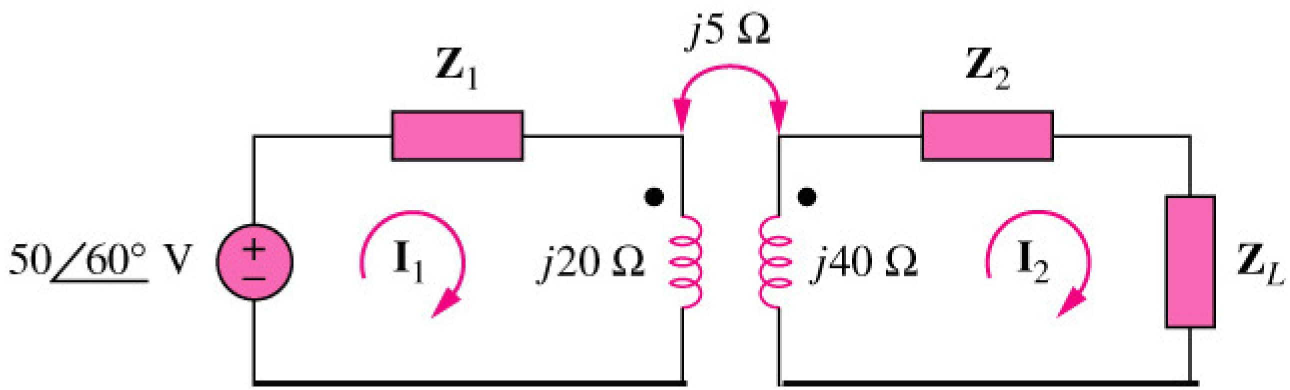
Example 13.4 (textbook)

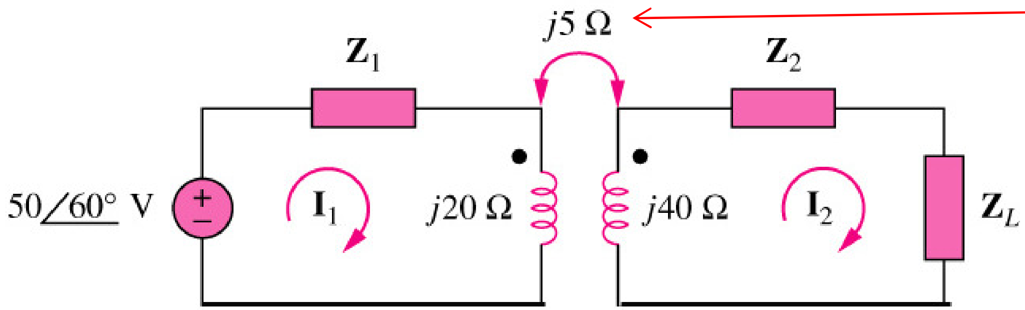
In the circuit below, calculate the input impedance and current I_1. Take Z_1=60-j100Ω, Z_2=30+j40Ω, and Z_L=80+j60Ω.



Ans: $\quad Z_{\text{in}} = 100.14\angle -53.1°\Omega;\ I_1 = 0.5\angle 113.1°\text{A}$

13.4 Linear Transformers (5)

Example 13.4



Note:

$$j\omega M = j5$$
$$\omega M = 5$$
$$(\omega M)^2 = 25$$

- The series impedance in the primary coil:

$$Z_{primary} = (60 - j100) + j20 = 60 - j80$$

- The series impedance in the primary coil:

$$Z_{secondary} = (30 + j40) + (80 + j60) + j40 = 110 + j140$$

- Input Impedance given by:

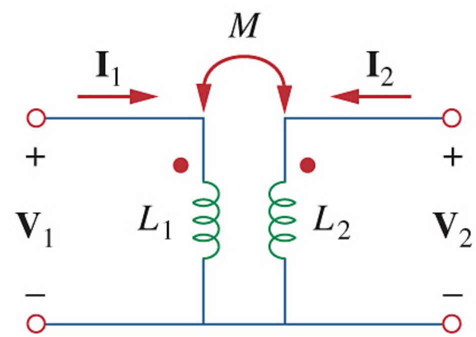
$$Z_{in} = Z_{primary} + \frac{\omega^2 M^2}{Z_{Secondary}} = (60 - 80j) + \frac{25}{(110 + j140)} = 60.09 - j80.11$$

- Current I_1 given by: $I_1 = \frac{V_s}{Z_{in}} = \frac{50\angle 60°}{60.09 - j80.11} = \frac{50\angle 60°}{100.14\angle -53.1°}$

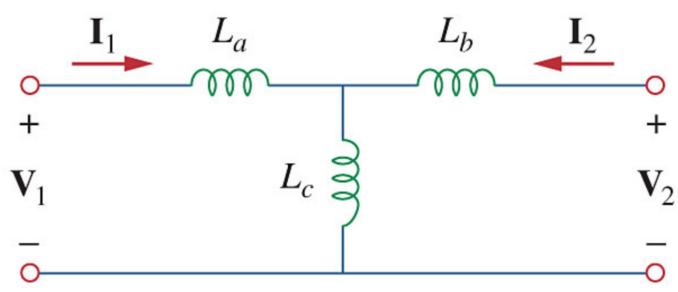
13.4 Linear Transformers (6)

Equivalent T and π Circuits:

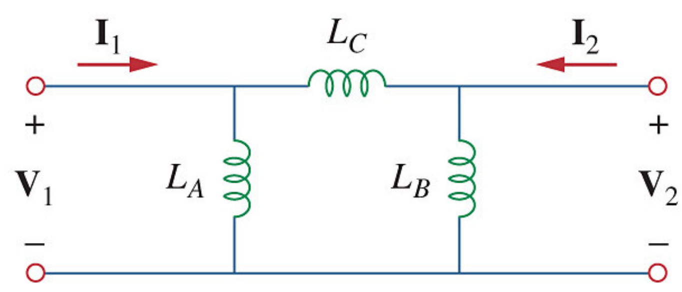
- It is sometimes convenient to replace a magnetically couple circuit with an equivalent circuit with no magnetic coupling.
- We can replace the linear transformer with an equivalent T or π circuit that has no mutual inductance.



T Circuit



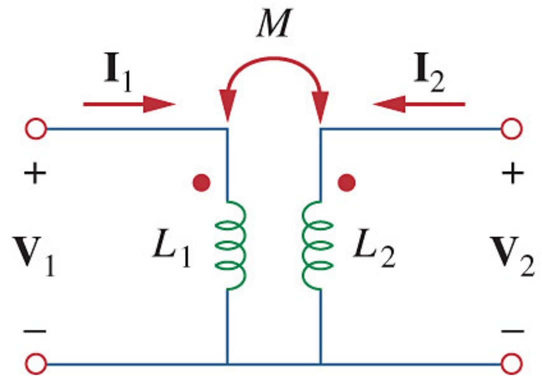
π Circuit



13.4 Linear Transformers (7)

Equivalent T Circuit:

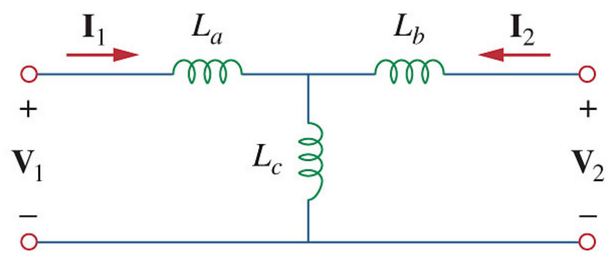
Linear Transformer Circuit



Mesh Analysis of this transformer circuit results in the following:

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} j\omega L_1 & j\omega M \\ j\omega M & jwL_2 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

Equivalent T Network



Mesh Analysis of the equivalent T network results in the following:

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} j\omega(L_a + L_c) & jwL_c \\ jwL_c & jw(L_b + L_c) \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

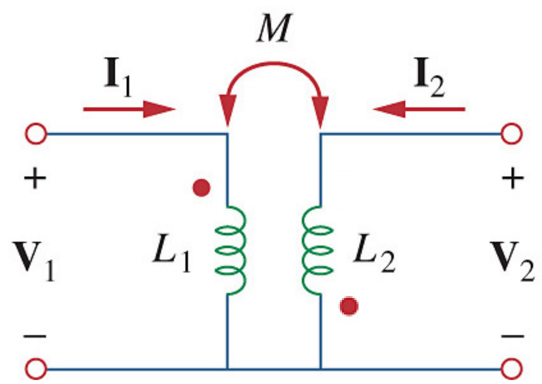
Equating the terms gives the following relationships:

$L_a = L_1 - M$ $\quad$ $L_b = L_2 - M$ $\quad$ $L_c = M$

13.4 Linear Transformers (8)

Equivalent T Circuits (Swapped Dots):

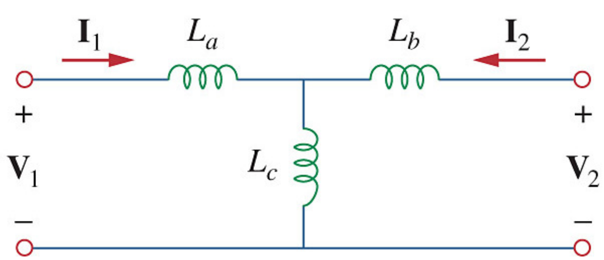
Linear Transformer Circuit



Mesh Analysis of this transformer circuit results in the following:

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} j\omega L_1 & -j\omega M \\ -j\omega M & jwL_2 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

Equivalent T Network



Mesh Analysis of the equivalent T network results in the following:

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} j\omega(L_a + L_c) & jwL_c \\ jwL_c & jw(L_b + L_c) \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

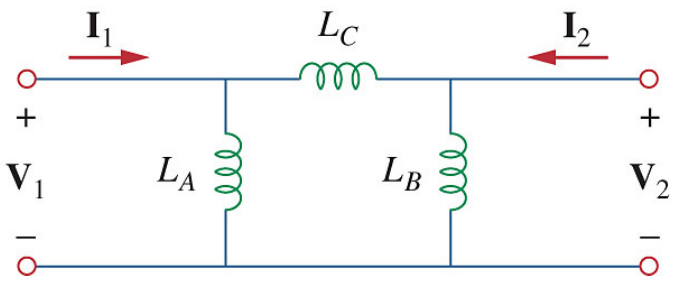
Equating the terms gives the following relationships:

$L_a = L_1 + M \qquad L_b = L_2 + M \qquad L_c = -M$

13.4 Linear Transformers (9)

Equivalent π Circuit:

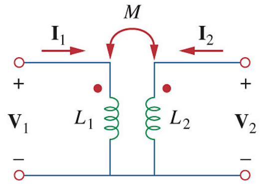
Similarly, for the π network nodal analysis provides:



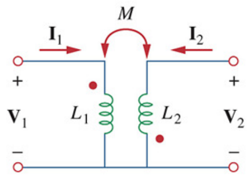
Equivalent π Network

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} \frac{1}{j\omega L_A} + \frac{1}{j\omega L_C} & -\frac{1}{j\omega L_C} \\ -\frac{1}{j\omega L_C} & \frac{1}{j\omega L_B} + \frac{1}{j\omega L_C} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

By equating terms in admittance matrices, for the π equivalent network we obtain (note if dots are different, replace with –M):



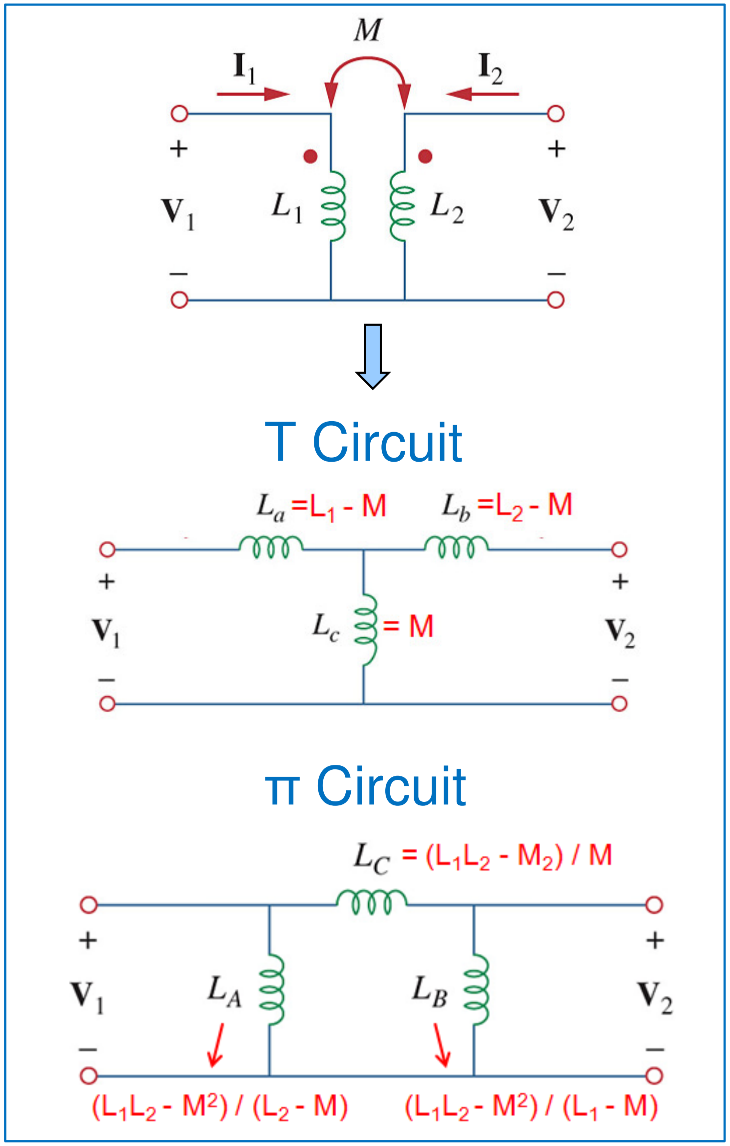
$$L_A = \frac{L_1 L_2 - M^2}{L_2 - M} \;;\; L_B = \frac{L_1 L_2 - M^2}{L_1 - M} ;\; L_C = \frac{L_1 L_2 - M^2}{M}$$

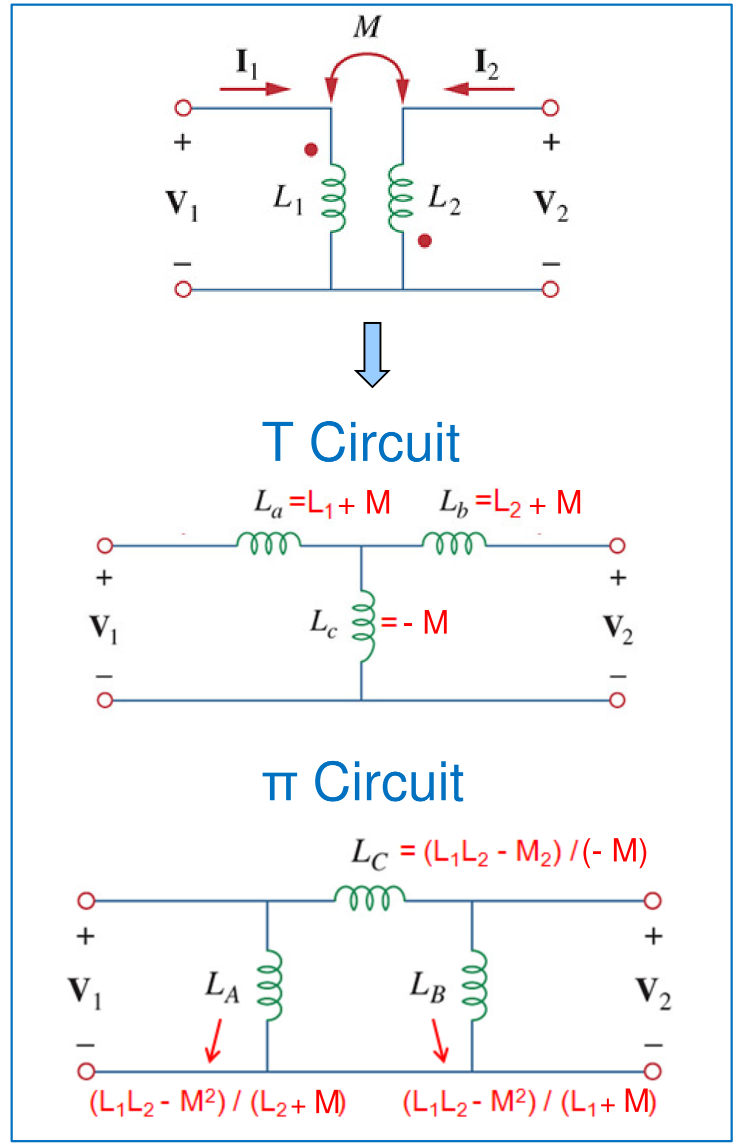


$$L_A = \frac{L_1 L_2 - M^2}{L_2 + M} \;;\; L_B = \frac{L_1 L_2 - M^2}{L_1 + M} ;\; L_C = \frac{L_1 L_2 - M^2}{-M}$$

13.4 Linear Transformers (10)

Equivalent T or π Circuits Summary

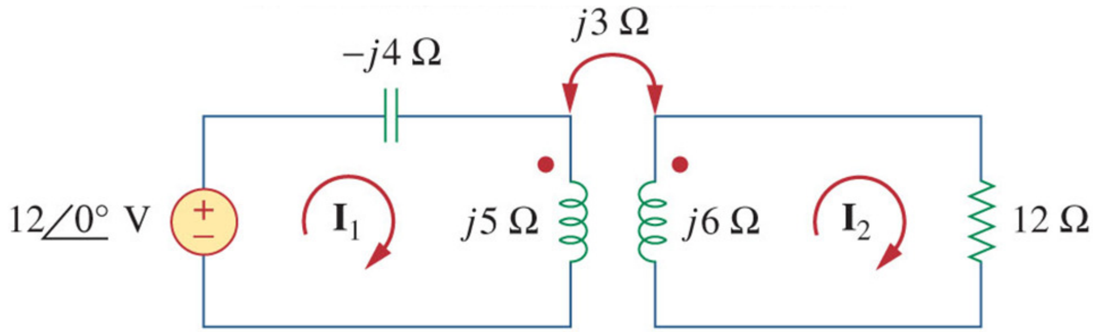


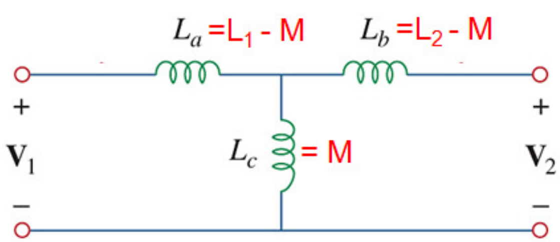


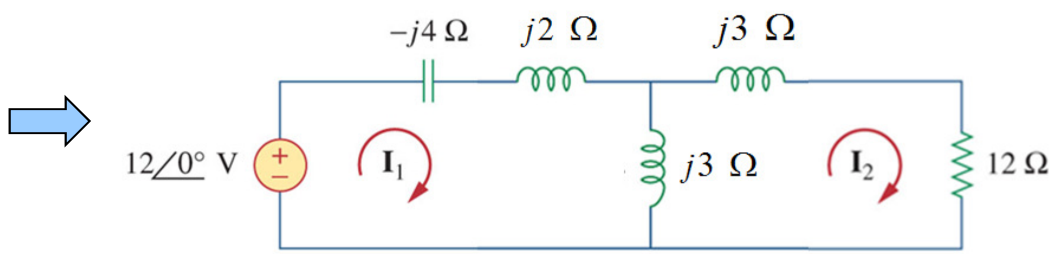
13.4 Linear Transformers (11)

Practice Problem 13.6

Find I_1 and I_2 using the T equivalent circuit





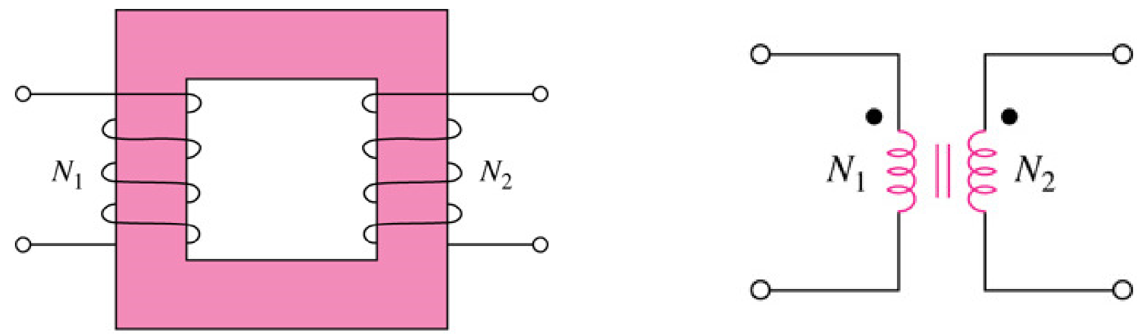


Mesh I_1: $-12\angle 0° - j4I_1 + j2I_1 + j3(I_1 - I_2) = 0$ $\qquad jI_1 - j3I_2 = 12\angle 0°$

Mesh I_2: $j3(I_2 - I_1) + j3I_2 + 12I_2 = 0$ $\qquad -j3I_1 + (12 + j6)I_2 = 0$

13.5 Ideal Transformers (1)

- An ideal transformer has perfect coupling (k=1).
- It consists of two or more coils with a large number of turns wound on a common core of high permeability.

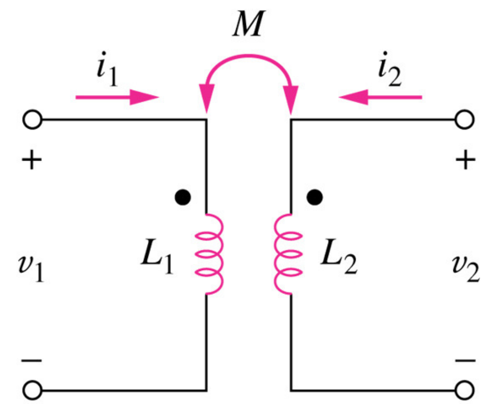


- Because of the high permeability of the core, the flux links all the turns of both coils, thereby resulting in a perfect coupling.

13.5 Ideal Transformers (2)

“Dot’s the same polarity”

Recall the coupled circuit:



$$V_1 = j\omega L_1 I_1 + j\omega M I_2 \qquad (1)$$

$$V_2 = j\omega M I_1 + j\omega L_2 I_2 \qquad (2)$$

Solving for I_1 in (1): $\mathrm{I}_1 = \dfrac{V_1 - j\omega M I_2}{j\omega L_1}$ (3)

Substituting (3) into (2): $V_2 = j\omega L_2 I_2 + \dfrac{MV_1}{L_1} - \dfrac{j\omega M^2 I_2}{L_1}$

$M = \sqrt{L_1 L_2}$ For perfect coupling (k=1):

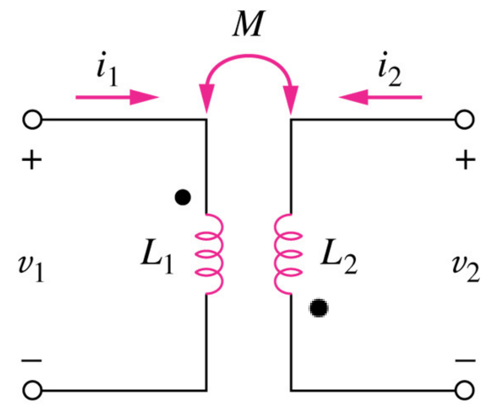
$$V_2 = j\omega L_2 I_2 + \frac{\sqrt{L_1 L_2}V_1}{L_1} - \frac{j\omega L_1 L_2 I_2}{L_1} = \sqrt{\frac{L_2}{L_1}}V_1 = nV_1$$

Therefore: $V_2 = nV_1$ where $n = \sqrt{L_2 / L_1}$ = turns ratio

13.5 Ideal Transformers (3)

"Dot's opposite each other"

Mesh Equations give the following:



$$V_1 = j\omega L_1 I_1 - j\omega M I_2 \qquad (1)$$

$$V_2 = -j\omega M I_1 + j\omega L_2 I_2 \qquad (2)$$

Solving for I_1 in (1): $\mathrm{I}_1 = \dfrac{V_1 + j\omega M I_2}{j\omega L_1}$ (3)

Substituting (3) into (2): $V_2 = j\omega L_2 I_2 - \dfrac{MV_1}{L_1} - \dfrac{j\omega M^2 I_2}{L_1}$

$M = \sqrt{L_1 L_2}$ For perfect coupling (k=1):

$$V_2 = j\omega L_2 I_2 - \frac{\sqrt{L_1 L_2} V_1}{L_1} - \frac{j\omega L_1 L_2 I_2}{L_1} = -\sqrt{\frac{L_2}{L_1}} V_1 = -nV_1$$

Therefore: $V_2 = -nV_1$ If dot is swapped at output

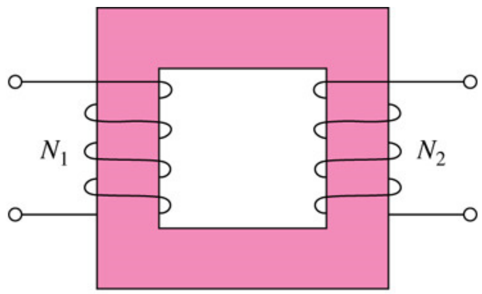
13.5 Ideal Transformers (4)

Properties

- A transformer is said to be ideal if it has the following properties:
  1. Coils have very large reactances (L_1, L_2, $M \rightarrow \infty$)
  2. Coupling coefficient is equal to unity (k=1)
  3. Primary and secondary coils are lossless ($R_1 = R_2 = 0$)
- An ideal transformer is a unity-coupled (k=1) lossless transformer in which the primary and secondary coils have infinite self-inductances (L_1 & $L_2 \rightarrow \infty$).
- Iron core transformers are close approximations to ideal transformers and are used in power systems and electronics.

13.5 Ideal Transformers (5)

- When a sinusoidal voltage is applied to the primary winding, the same magnetic flux Φ goes through both windings.



$$v_1 = N_1 \frac{d\phi}{dt} \quad ; \quad v_2 = N_2 \frac{d\phi}{dt}$$

$$\frac{v_2}{v_1} = \frac{N_2}{N_1} = n =$$ Turns ratio or transformation ratio

- Using the phasor voltages rather than the instantaneous voltages:

$$\frac{V_2}{V_1} = \frac{N_2}{N_1} = n$$

13.5 Ideal Transformers (6)

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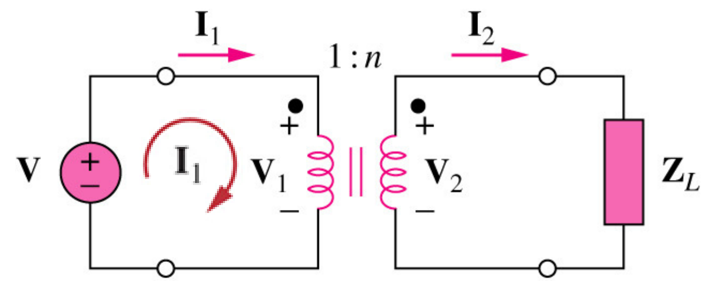
- Power conservation: $v_1 i_1 = v_2 i_2$
- The energy supplied to the primary must equal the energy absorbed by the secondary, since there are no losses in an ideal transformer.
- In phasor form: $\boxed{\dfrac{I_1}{I_2} = \dfrac{V_2}{V_1} = n}$



  - n=1 → isolation transformer ($V_2 = V_1$)
  - n>1 → step-up transformer ($V_2 > V_1$)
  - n<1 → step-down transformer ($V_2 < V_1$)

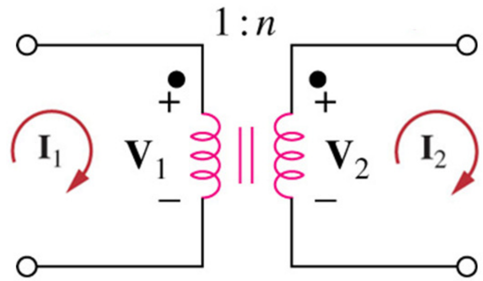
13.5 Ideal Transformers (7)

- Transformer ratings are usually specified as V_1 / V_2
- Power companies often generate at some convenient voltage and use the step-up transformer to increase the voltage so that the power can be transmitted at very high voltage and low current over transmission lines, resulting in significant cost savings. Near residential consumers, step-down transformers are used to bring the voltage down to 120 V.
- It is important to get the proper polarity of the voltages and the direction of the currents for the transformer.

13.5 Ideal transformers (8)

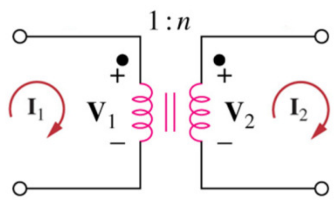
V and I relationships (simpler way to remember)

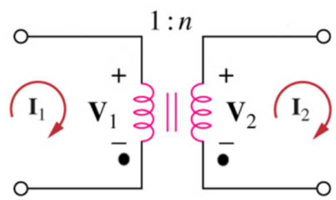
Given this standard definition for Voltages and currents



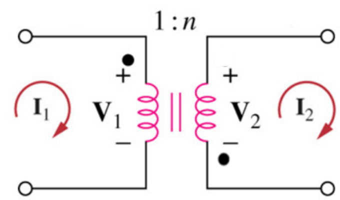
Note: This definition of I_2 differs from the text.

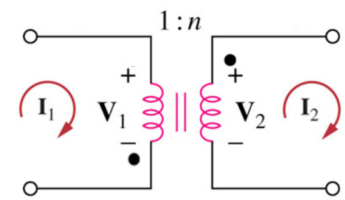
Dots "Same" → "+"





Dots "Different" → "-"





$$V_2 = nV_1$$

$$I_2 = \frac{I_1}{n}$$

Reflected Impedance →

$$Z_{in} = \frac{Z_L}{n^2}$$

Note: The larger the turns ratio
The larger "n"
The larger "N_2"
The larger V_2

Turns Ratio

$$n = \frac{N_2}{N_1}$$

$$V_2 = -nV_1$$

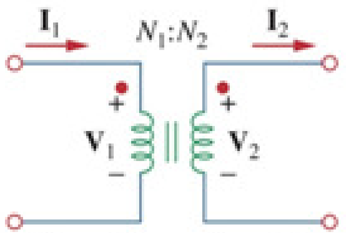
$$I_2 = -\frac{I_1}{n}$$

$$Z_{in} = \frac{Z_L}{n^2}$$

13.5 Ideal Transformers (9)

V and I relationships

- Expressing V_1 in terms of V_2 and I_1 in terms of I_2 or vice versa:



$$\frac{V_2}{V_1} = \frac{N_2}{N_1} \qquad \frac{I_2}{I_1} = \frac{N_1}{N_2}$$

$$V_1 = \frac{V_2}{n} \qquad V_2 = nV_1$$

Positive, if Voltage "**same**" polarity at dot

$$I_2 = \frac{I_1}{n} \qquad I_1 = nI_2$$

Positive, if current "**different**" polarity at dot

- Complex Power is:

Complex Conjugate of I_2

$$S_1 = V_1 I_1^* = \frac{V_2}{n}(nI_2)^* = V_2 I_2^* = S_2$$

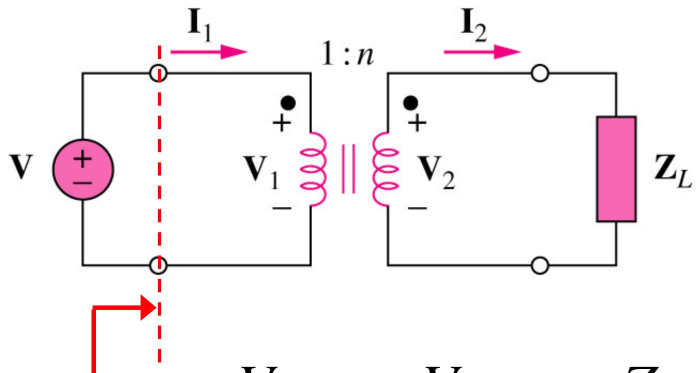
- Complex power supplied to the primary is delivered to the secondary without loss.

- The ideal transformer is **lossless** and absorbs **no power**.

13.5 Ideal Transformers (10)

Reflected impedance

- The input impedance as seen by the source is:



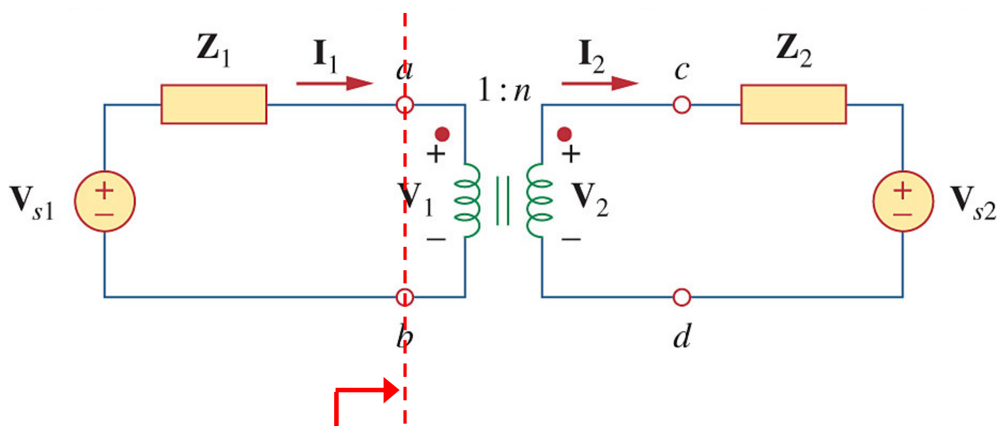
$$Z_{in} = \frac{V_1}{I_1} = \frac{V_2}{n^2 I_2} = \frac{Z_L}{n^2}$$

- The input impedance is also called the reflected impedance since it appears as if the load impedance is reflected to the primary side.
- The ability of the transformer to transform a given impedance to another allows impedance matching to ensure maximum power transfer.

13.5 Ideal Transformers (11)

Equivalent circuit analysis

- In analyzing a circuit containing an ideal transformer, it is common practice to eliminate the transformer by reflecting impedances and sources from one side of the transformer to the other.
- Suppose we want to reflect the secondary side of the circuit to the primary side:

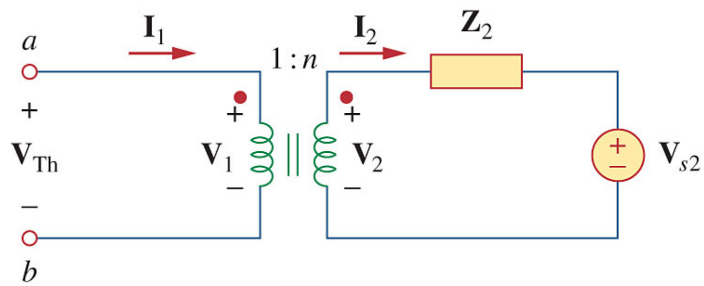


$$Z_R = \frac{Z_2}{n^2}$$

13.5 Ideal Transformers (12)

Equivalent circuit analysis

- We find the Thevenin equivalent of the circuit to the right of a-b:
- Obtaining V_{th} from "open circuit voltage":



(a)

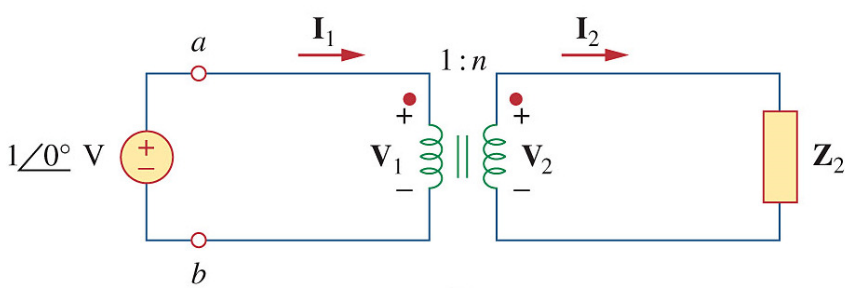
$I_1 = 0 = I_2$ Since a-b is open

$V_2 = V_{s2}$

$$V_{Th} = V_1 = \frac{V_2}{n} = \boxed{\frac{V_{s2}}{n}}$$

- Obtaining Z_{Th} (remove the voltage source in the secondary and insert a unit source at a-b terminals.)

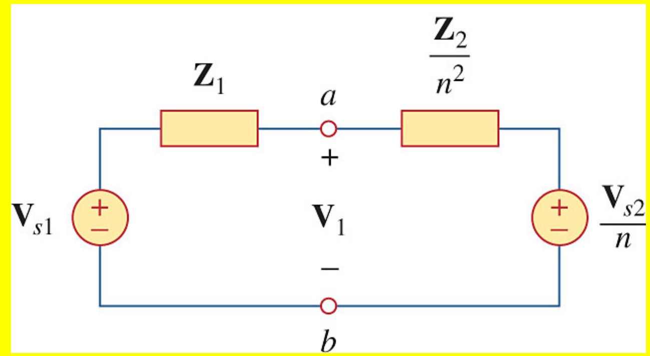
$$Z_{Th} = \frac{V_1}{I_1} = \frac{V_2}{n^2 I_2} = \boxed{\frac{Z_2}{n^2}}$$



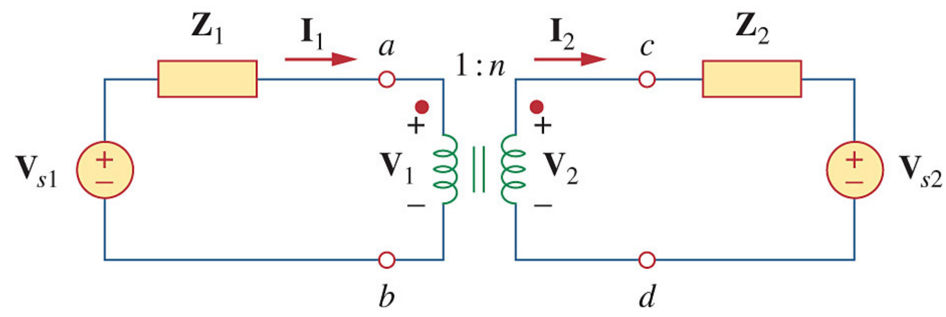
(b)

Equivalent Circuit

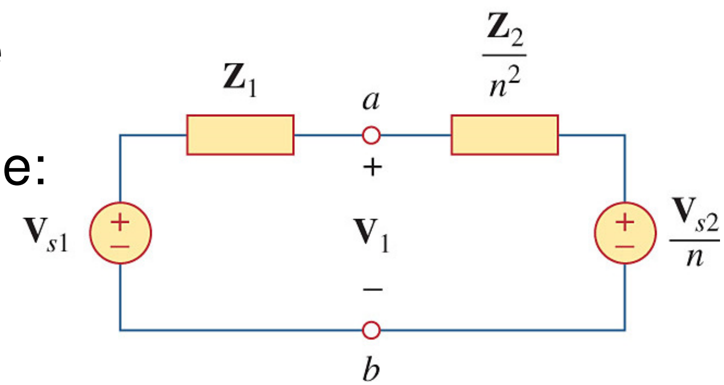
Reflecting the secondary to the primary



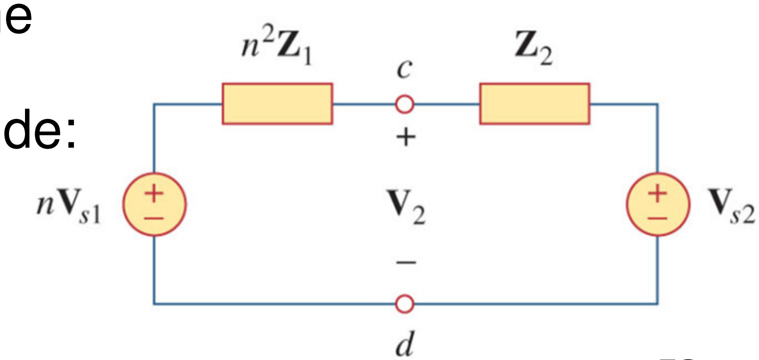
13.5 Ideal Transformers (13)



- The general rule for eliminating the transformer and reflecting the secondary circuit to the primary side:
  - Divide the secondary impedance by n^2
  - Divide the secondary voltage by n
  - Multiply the secondary current by n



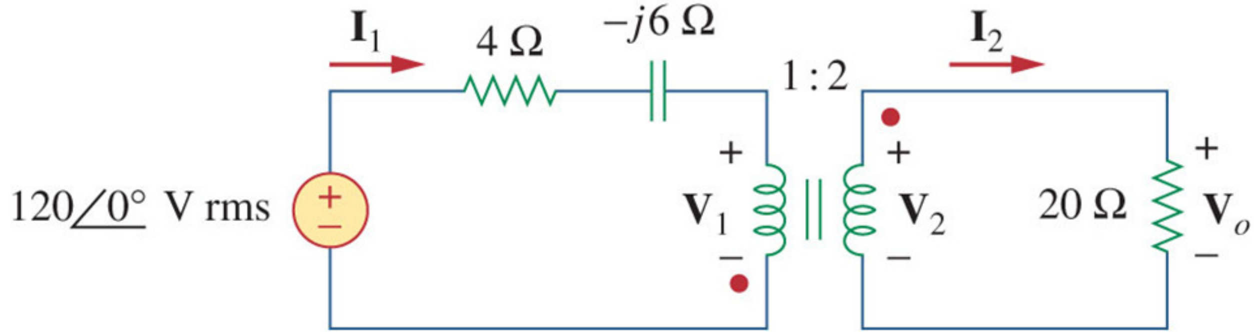
- The general rule for eliminating the transformer and reflecting the secondary circuit to the primary side:
  - Multiply the primary impedance by n^2
  - Multiply the primary voltage by n
  - Divide the primary current by n



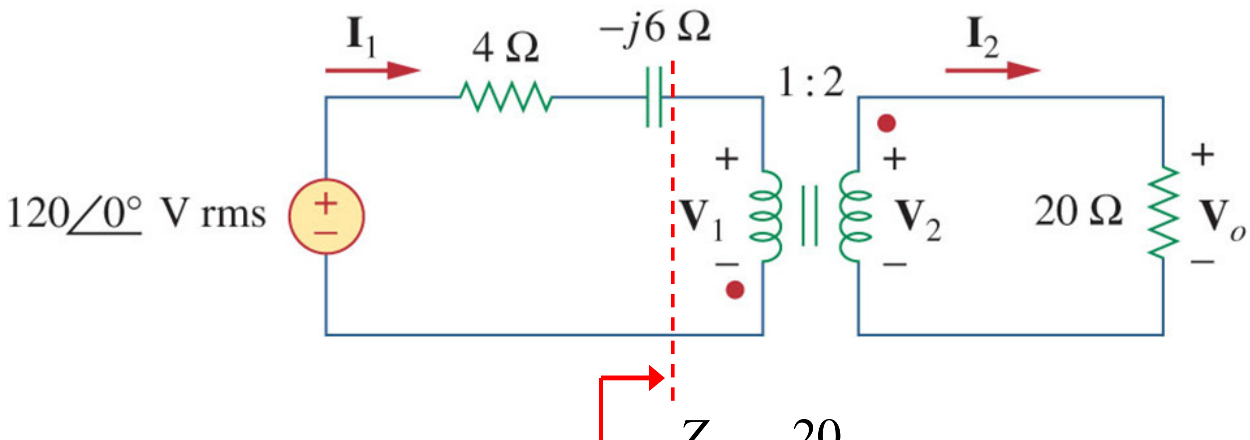
13.5 Ideal Transformer (14)

Example 13.8 (Textbook)

For the ideal transformer, find: (a) the source current I_1, (b) the output voltage V_0, and (c) the complex power supplied by the source



13.5 Ideal Transformer (14)



$$Z_R = \frac{Z_2}{n^2} = \frac{20}{2^2} = 5$$

Impedance seen by the Voltage source is:

$$Z_{in} = (4 - j6) + 5 = 9 - j6 = 10.82\angle -33.69^\circ \ \Omega$$

Input current I_1 is:

$$I_1 = \frac{V_s}{Z_{in}} = \frac{120\angle 0^\circ}{10.82\angle -33.69^\circ} = 11.09\angle 33.69^\circ \ \text{A}$$

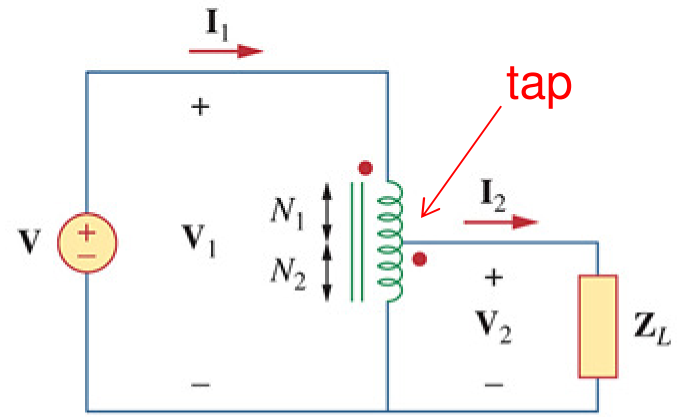
$$I_2 = -\frac{I_1}{n} = \frac{-11.09\angle 33.69^\circ}{2} = 5.55\angle -146.31^\circ \ \text{A}$$

$$V_o = 20 I_2 = 20(5.55\angle -146.31^\circ) = 110.9\angle -146.31^\circ \ \text{V}$$

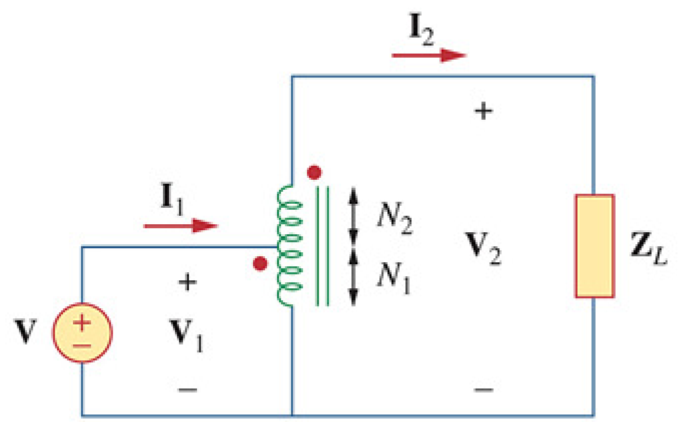
- 13.1 What is a transformer?
- 13.2 Mutual Inductance
- 13.3 Energy in a Coupled Circuit
- 13.4 Linear Transformers
- 13.5 Ideal Transformers
- 13.6 Ideal Autotransformers
- 13.9 Applications

13.6 Ideal Auto-Transformers (1)

- An **autotransformer** is a transformer in which both the primary and the secondary are in a single winding
- A connection point called a *tap* is separates the primary and secondary.
- The tap is often adjustable to provide a desired turns ratio.
- An adjustable tap provides a variable voltage to the load
- A disadvantage of the autotransformer is it provides *no electrical isolation*



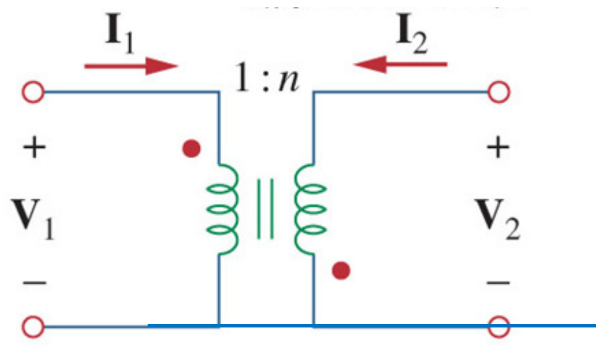
Step Down Auto-Transformer



Step Up Auto-Transformer

13.6 Ideal Auto-Transformers (2)

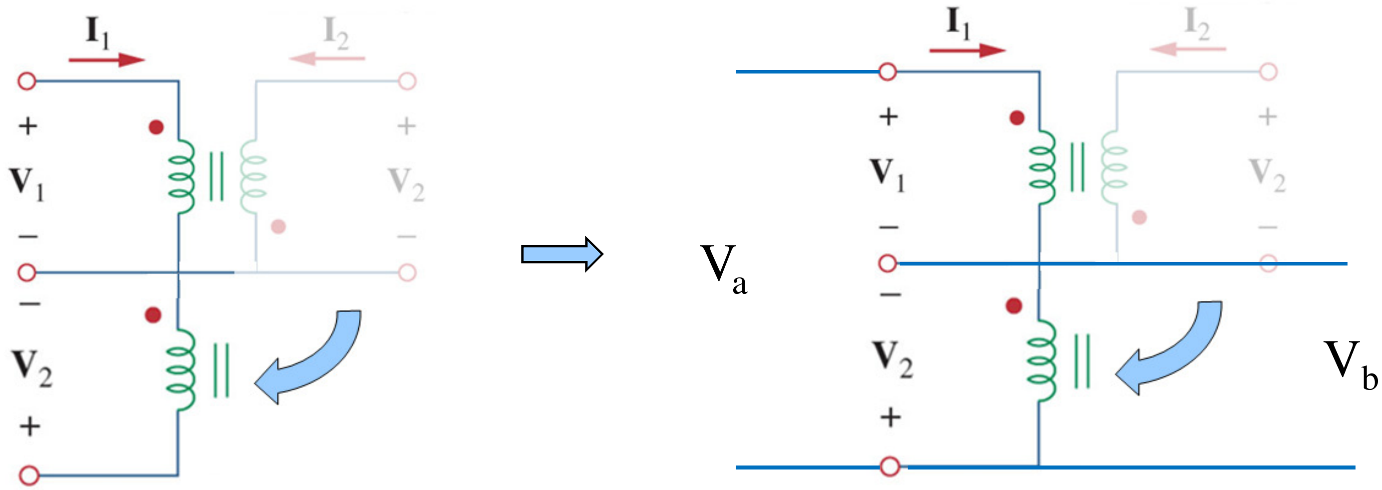
- In analyzing the autotransformer, consider the following circuit:



From earlier we know the following relationship

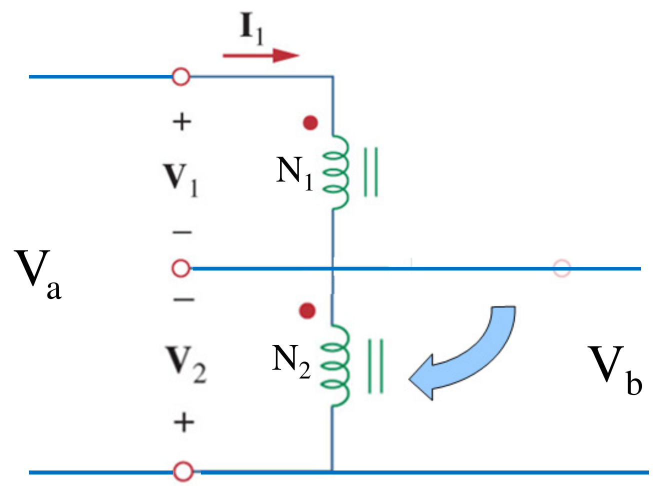
$$V_2 = -nV_1$$

- If we flip the secondary side underneath the primary, we can create an autotransformer as shown



13.6 Ideal Auto-Transformers (3)

- Analysis of this circuit provides the following results:



Primary Side

$$V_a = V_1 - V_2$$

$$V_a = V_1 + nV_1$$

$$V_a = (1+n)V_1$$

Secondary Side

$$V_b = -V_2 = nV_1$$

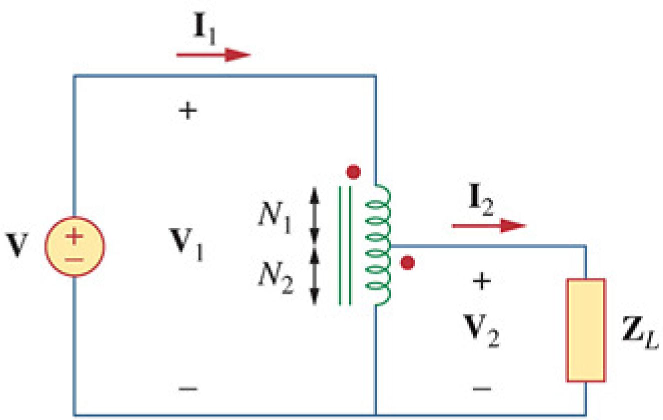
Ratio Primary/Secondary

$$\frac{V_a}{V_b} = \frac{1+n}{n} = \frac{N_1 + N_2}{N_2} \Rightarrow \boxed{V_b = \left(\frac{N_2}{N_1 + N_2}\right) V_a}$$

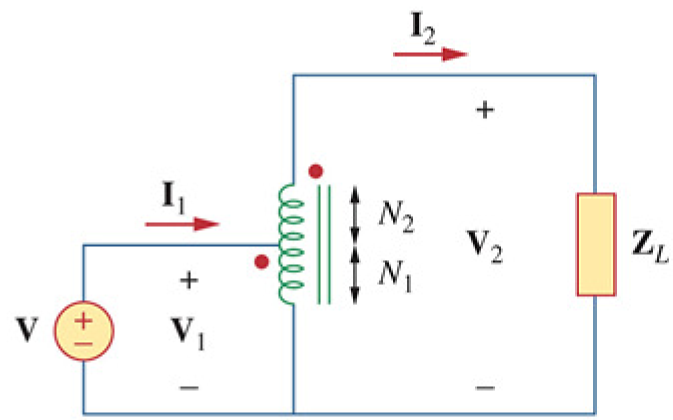
Notice, this looks like a Voltage Divider !

13.6 Ideal Auto-Transformers (4)

- The voltage / current relationships for the lossless ideal autotransformer are as follows:



Step Down Auto-Transformer



Step Up Auto-Transformer

Similar to Voltage Divider equation

$$V_2 = \frac{N_2}{N_1 + N_2} V_1 \qquad V_2 = \frac{N_1 + N_2}{N_1} V_1$$

Inverse Relation

$$I_2 = \frac{N_1 + N_2}{N_2} I_1 \qquad I_2 = \frac{N_1}{N_1 + N_2} I_1$$

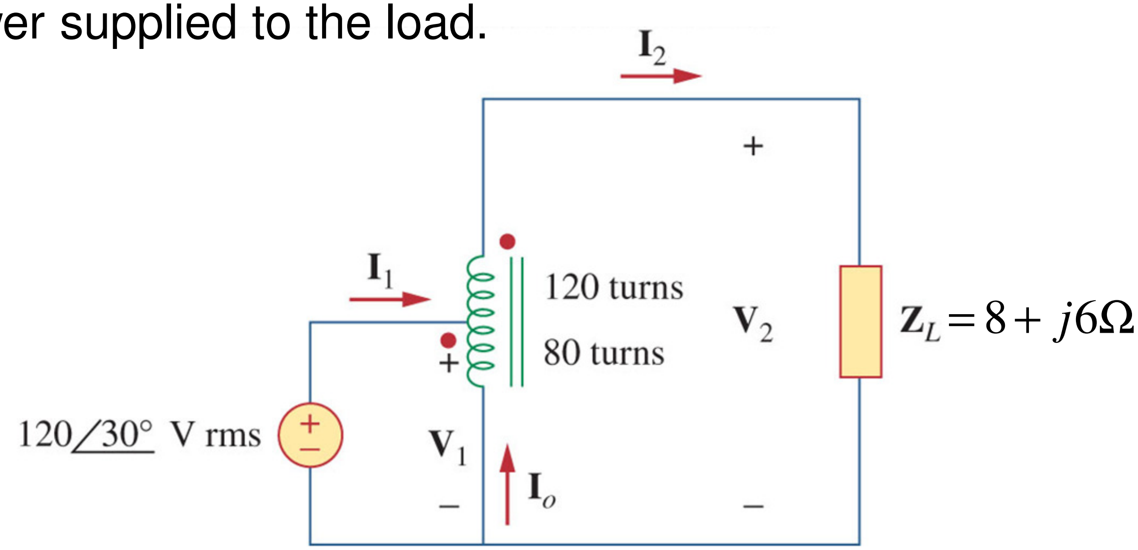
Derive from V/I

$$Z_{in} = \left(\frac{N_1 + N_2}{N_2}\right)^2 Z_L \qquad Z_{in} = \left(\frac{N_1}{N_1 + N_2}\right)^2 Z_L$$

13.6 Ideal Auto-Transformers (5)

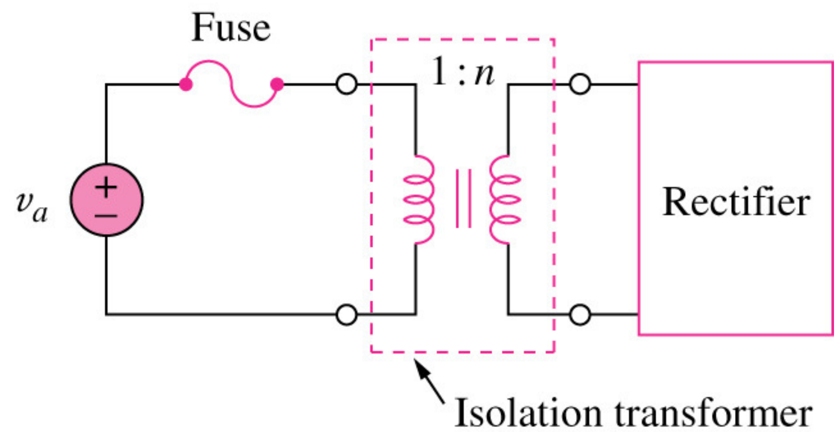
Example 13.11 (Textbook)

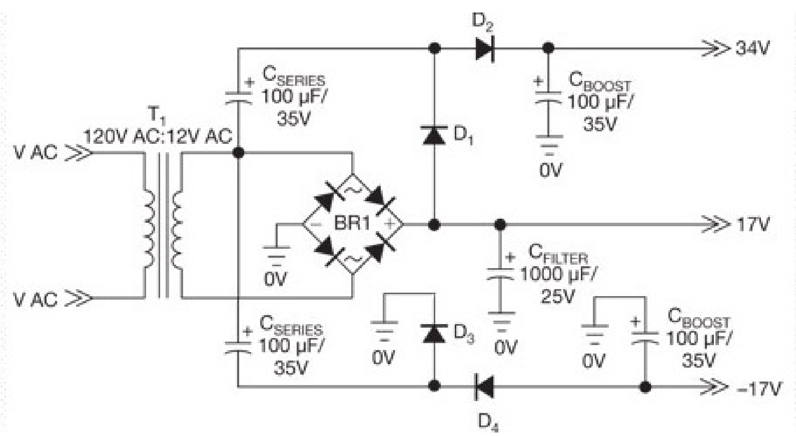
For the autotransformer below, find: (a) the currents I_1 , I_2 , I_0, (b) the complex power supplied to the load.

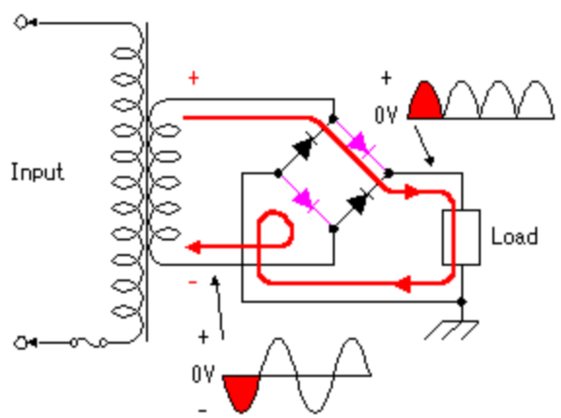


13.9 Applications (1)

- Transformer as an Isolation Device to isolate ac supply from a rectifier

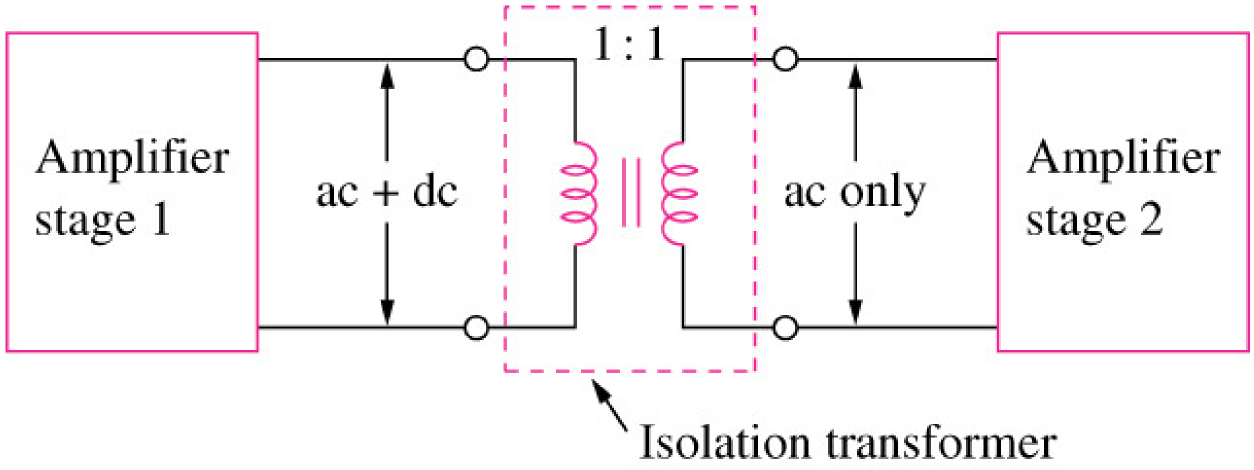






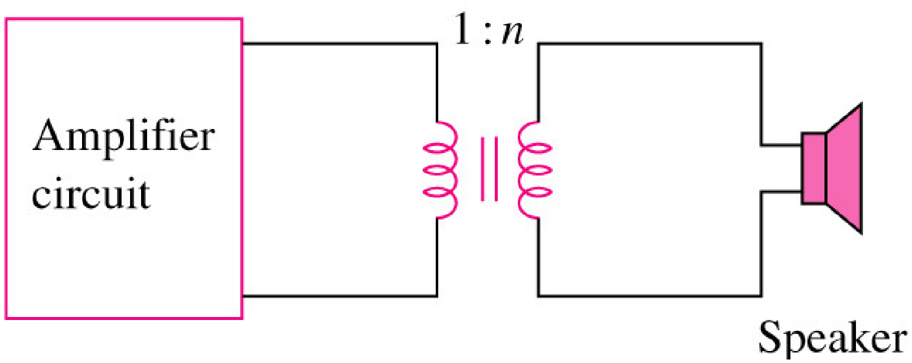
13.9 Applications (2)

- Transformer as an Isolation Device to isolate dc between two amplifier stages.



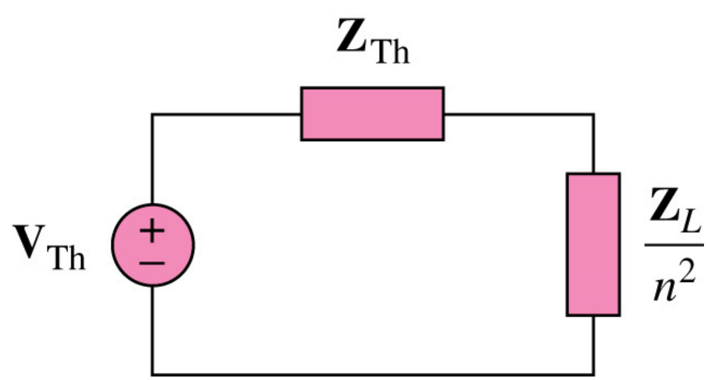
- Biasing is the application of a DC voltage to a transistor amplifier to produce a desired mode of operation.
- Each amplifier stage can be biased separately to operate in a particular mode.

- Transformer as a Matching Device



Using an ideal transformer to match the speaker to the amplifier

Equivalent circuit



13.9 Applications (4)

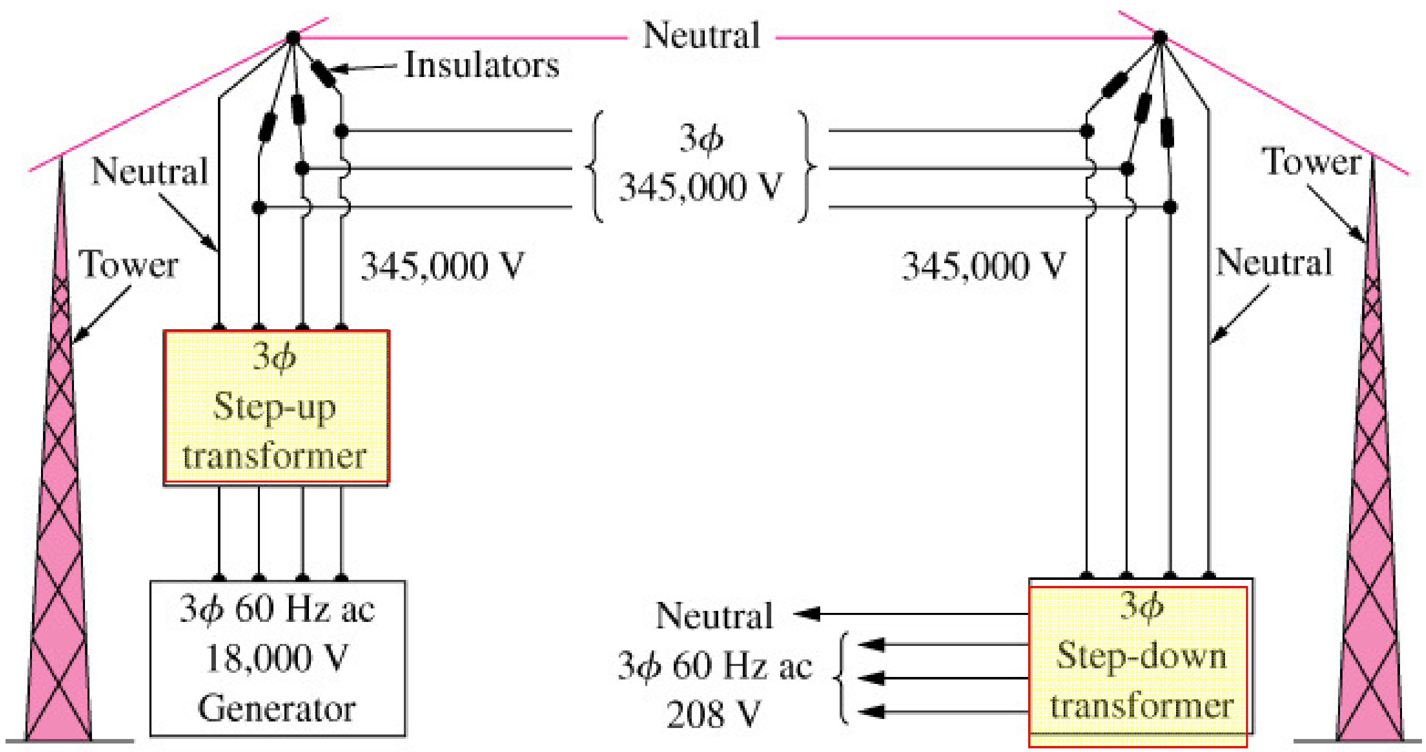
Practice Problem 13.16 (Textbook)

Calculate the turns ratio of an ideal transformer required to match a 400Ω load to a source with internal impedance of 2.5kΩ. Find the load voltage when the source voltage is 30V.

Ans: $n = 0.4$; $V_L = 6V$

13.9 Applications (5)

- A typical power distribution system



Homework #3

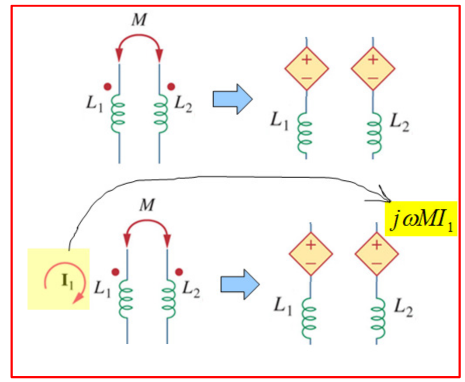
Due beginning of class Wednesday Feb 4, 2015

- 13.30
- 13.35
- 13.42
- 13.50
- 13.53 (modified)
- Autotransformer (See handout)

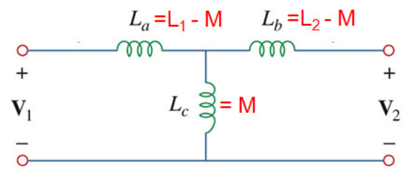
Exam over Chapter 13 on Monday Feb 9

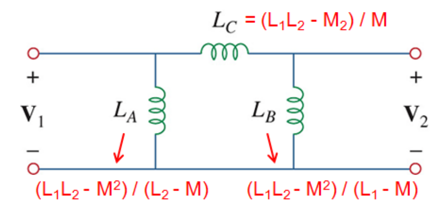
Equation / Analysis Summary

- Series Aiding $L = L_1 + L_2 + 2M$ / Opposing $L = L_1 + L_2 - 2M$
- Dot Convention Model



- Coupling coefficient "k" $M = k\sqrt{L_1 L_2}$
- Linear Transformer
  - Input Impedance: $Z_{in} = Z_{primary} + \dfrac{\omega^2 M^2}{Z_{Secondary}}$
  - Reflected Impedance: $Z_{reflected} = \dfrac{\omega^2 M^2}{Z_{Secondary}}$
  - Equivalent T or π Circuits:

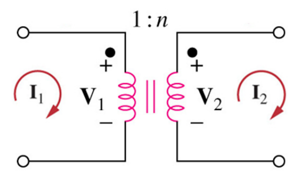




- Ideal Transformer

Turns Ratio $n = \dfrac{N_2}{N_1}$

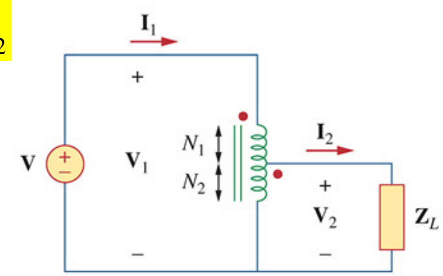
  - K = 1, L_1, L_2, → ∞
  - Lossless ($S_1 = S_2$)
  - Voltage / Current Relationship:
    - Dots "same" = +n, Dots "diff" = -n
  - Complex Power: $S_1 = V_1 I_1^* = V_2 I_2^* = S_2$



$V_2 = nV_1$ $\quad I_2 = \dfrac{I_1}{n}$ $\quad Z_{in} = \dfrac{Z_L}{n^2}$

$P_{ave} = |I_2|^2 R_L$

- Autotransformer
  - Adjustable "tap"
  - No electrical Isolation
  - Voltage Divider like relationship

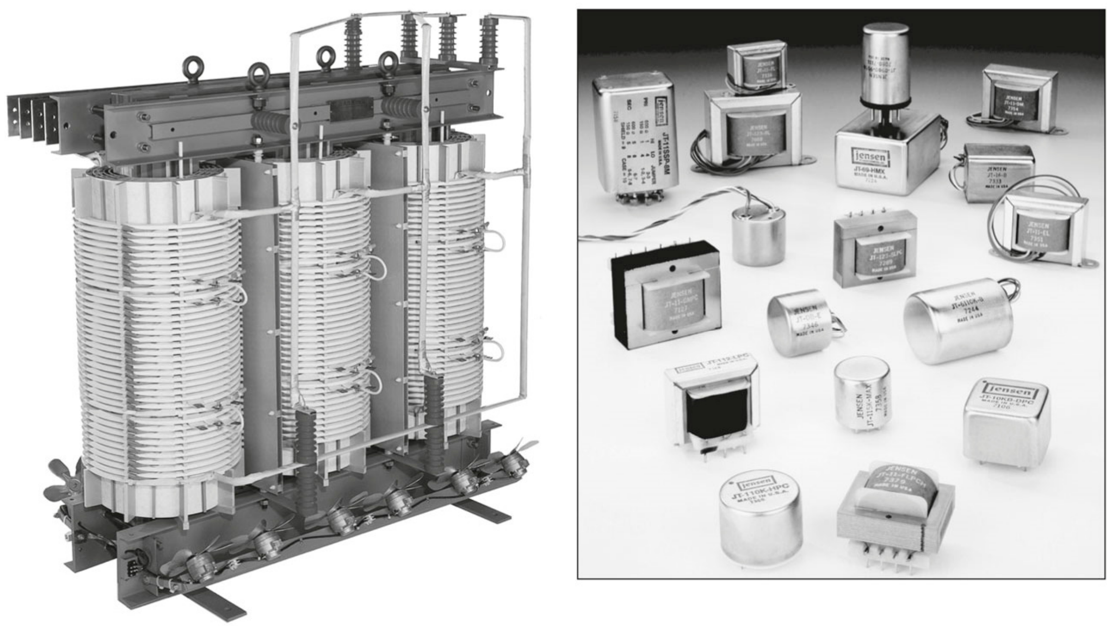


$V_2 = \dfrac{N_2}{N_1 + N_2} V_1$

$I_2 = \dfrac{N_1 + N_2}{N_2} I_1$

13.1 What is a transformer? (1)

- It is an electrical device designed on the basis of the concept of magnetic coupling
- It uses magnetically coupled coils to transfer energy from one circuit to another
- It is the key circuit elements for stepping up or stepping down ac voltages or currents, impedance matching, isolation, etc.

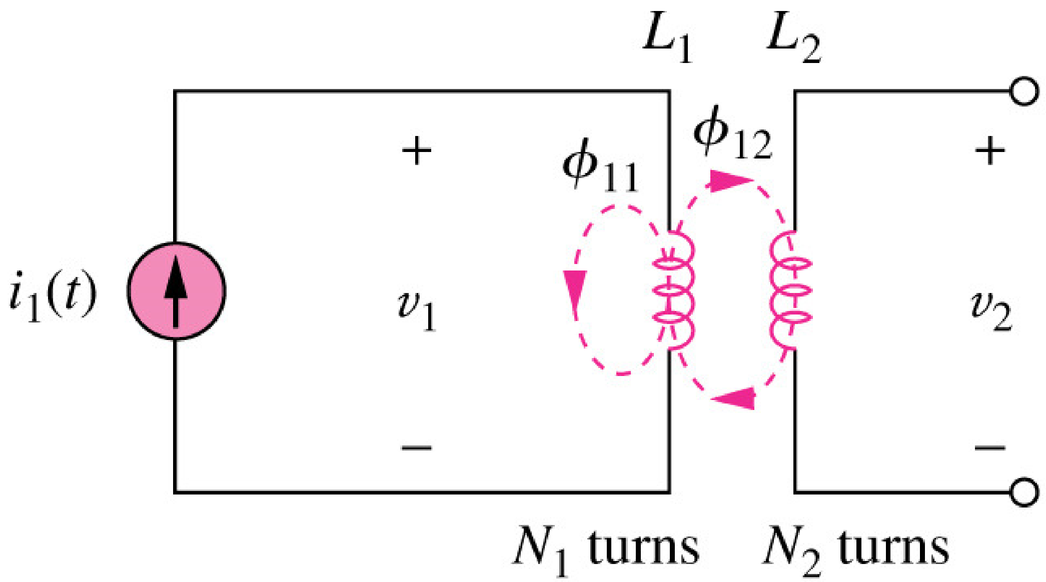


(a) (b)

Courtesy of: (a) Electric Service Co., (b) Jensen Transformers

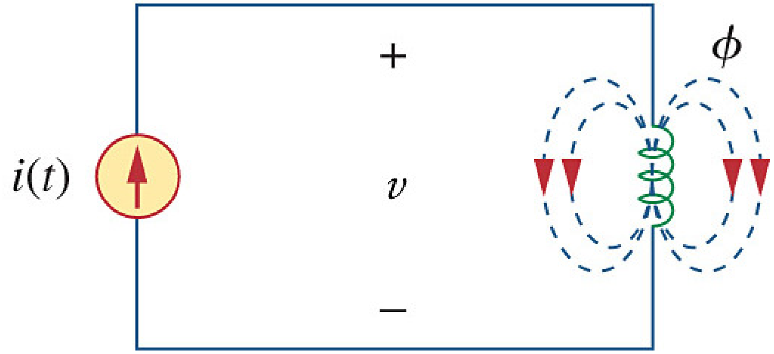
13.2 Mutual Inductance (1)

- Mutual Inductance
  - When two inductors (or coils) are in close proximity of each other, the magnetic flux caused by current in one coil links with the other coil, thereby inducing voltage in the latter.



13.2 Mutual Inductance (2)

- First consider a single inductor, a coil with N turns:



When current i flows through the coil, a magnetic flux Φ is produced around it.

- According to Faraday's law, the voltage v induced in the coil is proportional to the number of turns N and the time rate of change of magnetic flux Φ:

$$v = N \frac{d\phi}{dt}$$

13.2 Mutual Inductance (3)

- Voltage induced in the coil given by:

$$v = N\frac{d\phi}{dt}$$

- But the flux Φ is produced by current i so that any change in Φ is caused by a change in the current:

$$v = N\frac{d\phi}{di}\frac{di}{dt}$$

- Recall the voltage-current relationship for an inductor:

$$v = L\frac{di}{dt}$$

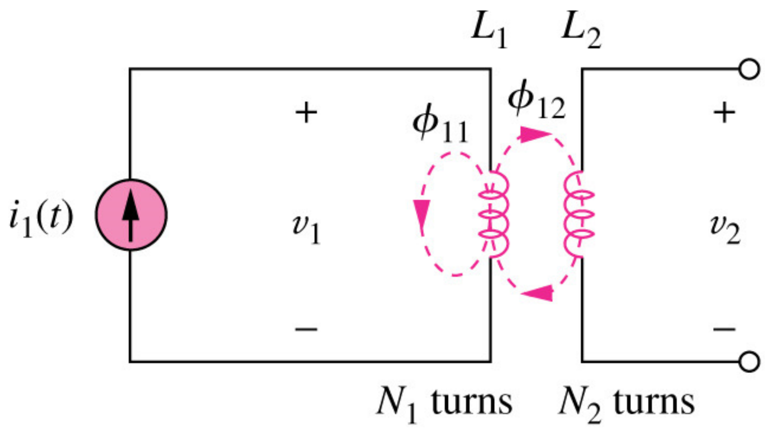
- The "Self" inductance L of the inductor is thus given by:

$$\boxed{L = N\frac{d\phi}{di}}$$

- Self-inductance L relates the voltage induced in a coil by a time-varying current in the same coil.

13.2 Mutual Inductance (4)

- Now consider two coils with self-inductances L_1 and L_2 that are in close proximity of each other:



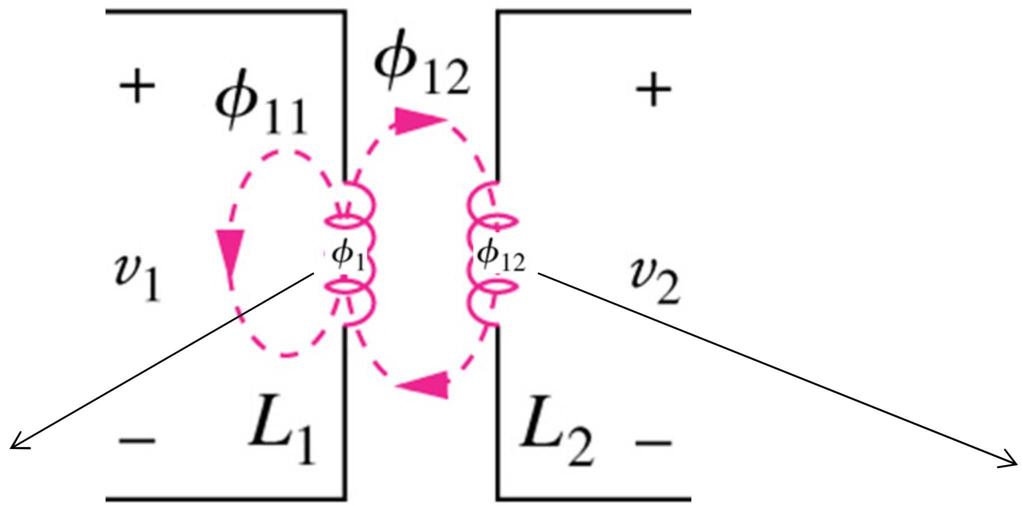
Coil 1 has N_1 turns
Coil 2 has N_2 turns.
Assume coil 2 carries no current.

- The total magnetic flux Φ_1 emanating from coil 1 has two components:
  - Φ_{11} links only coil 1
  - Φ_{12} links both coils

$$\phi_1 = \phi_{11} + \phi_{12}$$

13.2 Mutual Inductance (5)

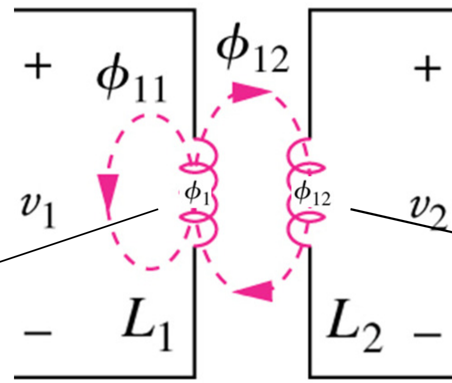
- Although the two coils are physically separated, they are magnetically coupled.
- The voltage induced in each coil is proportional to the flux in each coil.



$$v_1 = N_1 \frac{d\phi_1}{dt} \qquad v_2 = N_2 \frac{d\phi_{12}}{dt}$$

13.2 Mutual Inductance (6)

- The voltage equations can be rewritten as follows:



v_2 is the open-circuit mutual voltage (or induced voltage) across coil 2

$$v_1 = N_1 \frac{d\phi_1}{dt}$$

$$v_1 = \underbrace{N_1 \frac{d\phi_1}{di_1}} \frac{di_1}{dt}$$

$$v_1 = L_1 \frac{di_1}{dt}$$

$$v_2 = N_2 \frac{d\phi_{12}}{dt}$$

$$v_2 = \underbrace{N_2 \frac{d\phi_{12}}{di_1}} \frac{di_1}{dt}$$

$$v_2 = M_{21} \frac{di_1}{dt}$$

L_1 is the self-inductance of coil 1

M_{21} is the mutual inductance of coil 2 with respect to coil 1

13.2 Mutual Inductance (7)

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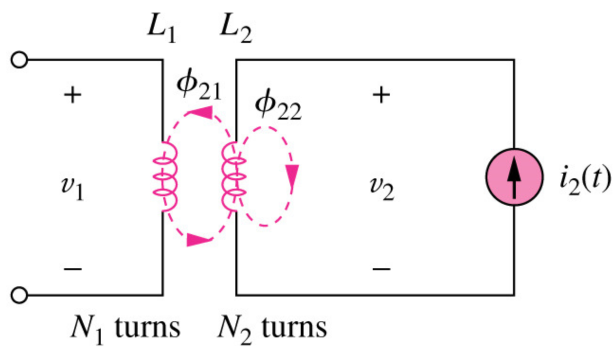
- Suppose now we let current i_2 flow in coil 2 while coil 1 carries no current:



- The magnetic flux Φ_2 emanating from coil 2 comprises flux Φ_{22} that links only coil 2 and flux Φ_{21} that links both coils:

$$\phi_2 = \phi_{21} + \phi_{22}$$

- The resulting symmetry is true:

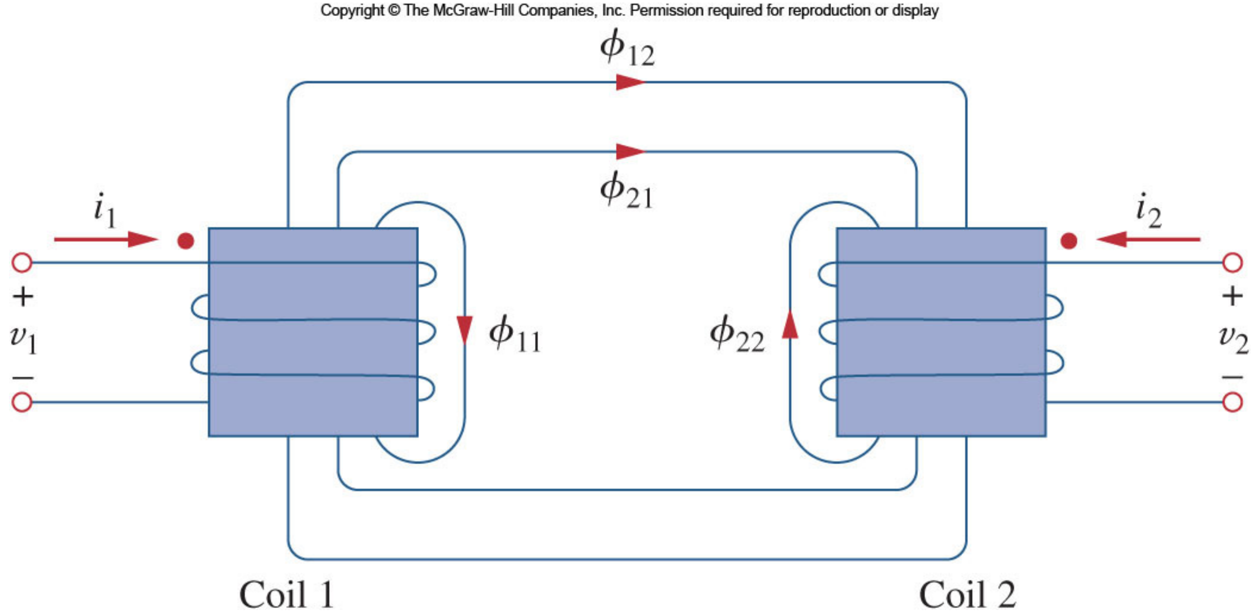
$$v_2 = L_2 \frac{d\, i_2}{d\, t} \qquad v_1 = M_{12} \frac{d\, i_2}{d\, t} \qquad M_{12} = N_1 \frac{d\, \phi_{21}}{d\, i_2}$$

13.2 Mutual Inductance (8)

- $M_{12} = M_{21} = M$; The "Mutual inductance" between the coils
- Mutual inductance M is measured in Henrys (just like inductors)
- Mutual inductance only exists when inductors or coils are in close proximity and the circuits are driven by time-varying sources
- Although mutual inductance M is always a positive quantity, the mutual voltage M di/dt may be negative or positive, just like the self-induced voltage L di/dt (determined by "Dot" convention)

13.2 Mutual Inductance (9)

- Self-induced voltage polarity is determined by the reference direction of the current and the reference polarity of the voltage,
- The polarity of the mutual voltage is not as easy to determine (depends on winding direction of the coils).
- We use the dot convention to determine



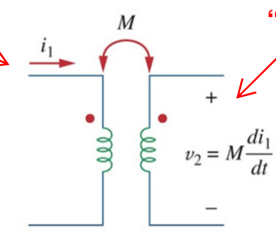
13.2 Mutual Inductance (10)

"Dot Convention"

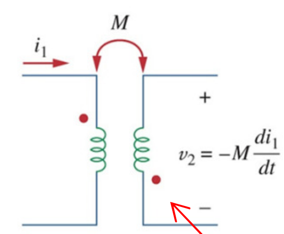
- If a current **enters** the dotted terminal of one coil, the reference polarity of the mutual voltage in the second coil is **positive** at the dotted terminal of the second coil.

Enters the "Dot"

Voltage is "Positive"



(a)

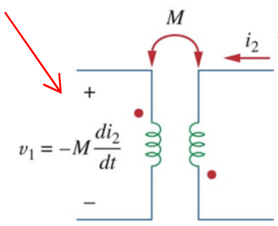


(b)

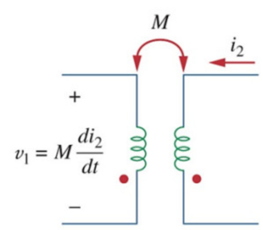
Voltage is "Positive" **At "Dotted" Terminal**

Voltage is "Negative"

Enters the "Un-Dotted" side



(c)



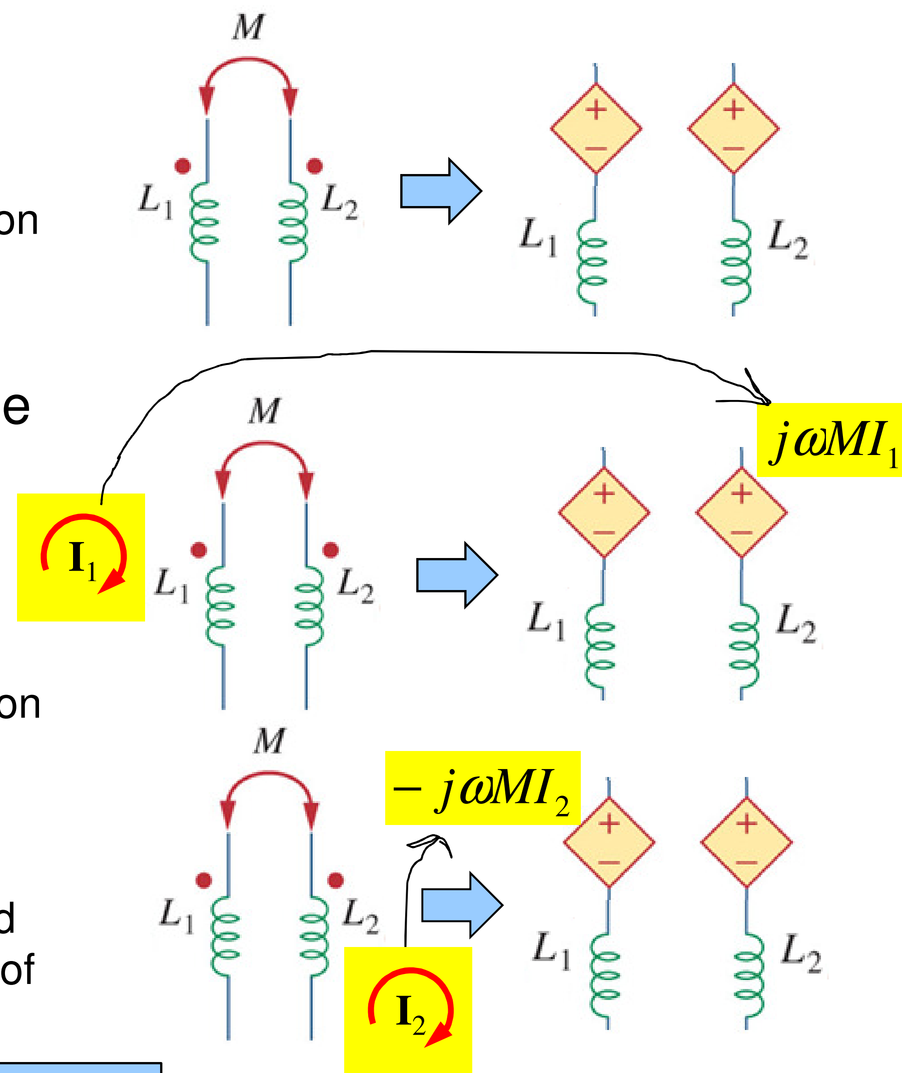
(d)

- Alternatively, if a current **leaves** the dotted terminal of one coil, the reference polarity of the mutual voltage in the second coil is **negative** at the dotted terminal of the second coil.

13.2 Mutual Inductance (11)

"The Model" (Phasor domain)

- Replace the "Dot" with a controlled voltage source.
  - Place on same side as the Dot
  - Positive terminal in same direction as the Dot
- Now look at the current entering each dot to determine the magnitude of the voltage source
  - If the current enters the dotted terminal, it will induce a positive voltage in the controlled source on the opposite terminal of jωM(I)
  - If the current enters the "un-dotted" terminal, it will induce a negative voltage in the controlled source on the opposite terminal of –jωM(I)

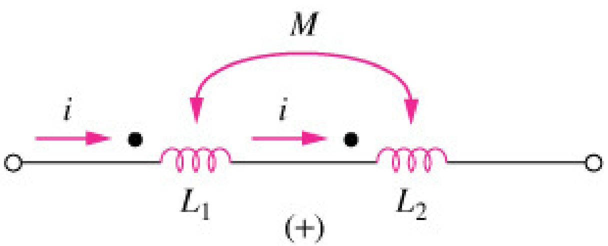


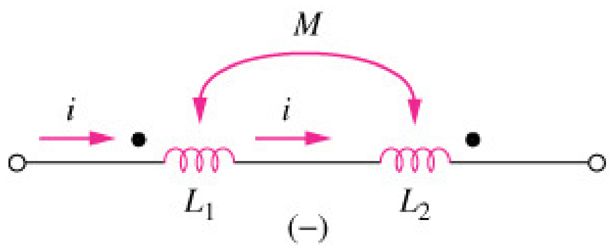
Note: This method differs slightly from the text

KEY CONCEPT !

13.2 Mutual Inductance (12)

Dot convention for coils in series; the sign indicates the polarity of the mutual voltage





$$L = L_1 + L_2 + 2M$$
(series - aiding connection)

$$L = L_1 + L_2 - 2M$$
(series - opposing connection)

We can show this using the model!

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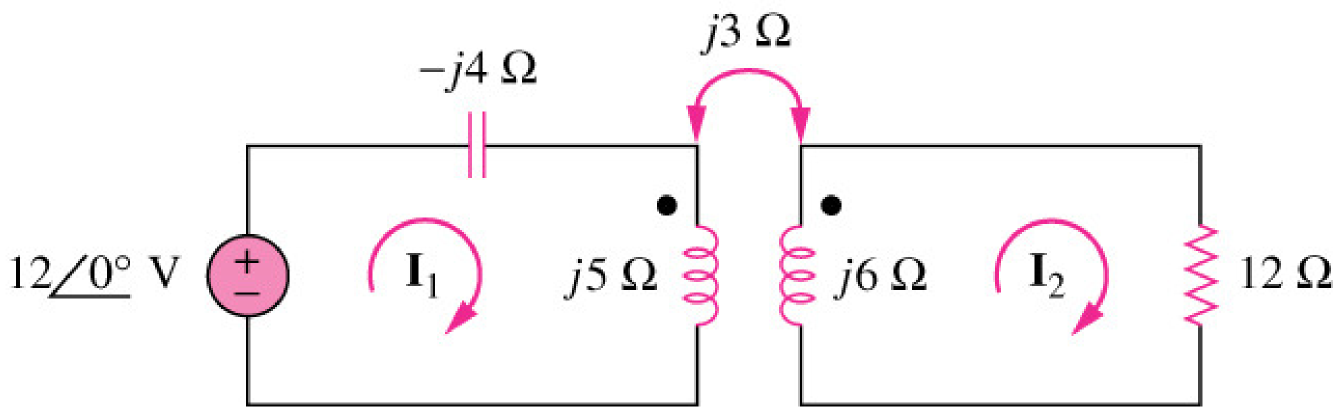
Example 13.1 (Textbook)

Calculate the phasor currents I_1 and I_2 in the circuit shown below.



Ans: $I_1 = 13.02\angle -49.40°\text{A};\quad I_2 = 2.91\angle 14.04°\text{A}$

Example Problem 13.1

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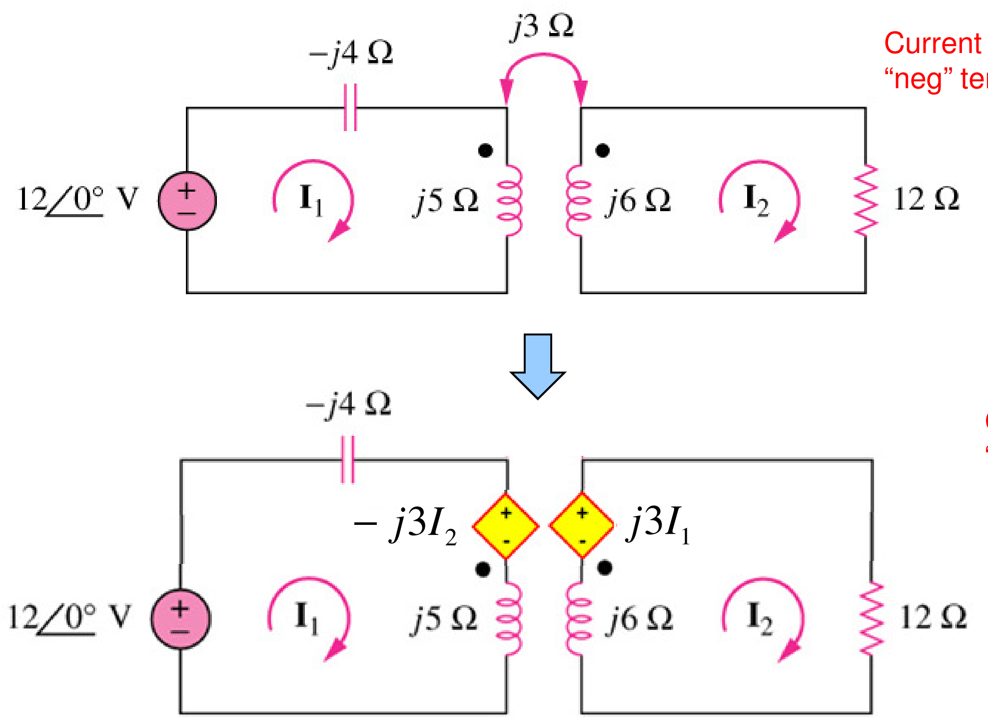
First, replace with the model:



Then, solve Loop Equations:

Current enters "neg" terminal

Current enters "+" terminal but value is $-j3I_2$

Loop I_1

$$-12 - j4I_1 + (-j3I_2) + j5I_1 = 0$$

$$jI_1 - j3I_2 = 12$$

Current enters "neg" terminal

Loop I_2

$$j6I_2 - j3I_1 + 12I_2 = 0$$

$$-j3I_1 + (12 + j6)I_2 = 0$$

NOTE:

- I_1 goes "into" the dot → Induced voltage on the second coil is "Positive"
- I_2 goes "into" the un-dotted side → Induced voltage on the first coil is "Negative"

Pay Attention to Sign Conventions !

Lastly, solve 2 equations, 2 unknowns (expected you know this):

Loop I_1 ⟶ $jI_1 - j3I_2 = 12$ ⟶ $jI_1 = 12 + j3I_2 \Rightarrow I_1 = 3I_2 - j12$

Loop I_2 ⟶ $-j3I_1 + (12 + j6)I_2 = 0$

Substitution: $-3(12 + j3I_2) + (12 + j6)I_2 = 0$

÷ by 3 $-12 - j3I_2 + (4 + j2)I_2 = 0$

Collect Terms: $(4 - j)I_2 = 12$

Solve for I_2 $$I_2 = \frac{12}{(4 - j)} = 2.824 + j0.706 = \boxed{2.91\angle 14.04^\circ \text{ A}}$$

Find I_1 (Substitute back to Eq1)

$$I_1 = 3(2.824 + j0.706) - j12$$
$$I_1 = 8.471 + j2.118 - j12$$
$$I_1 = 8.471 - j9.882 = \boxed{13.02\angle -49.40^\circ \text{ A}}$$

Think about where we could have made mistakes!

- Applying the model incorrectly (wrong sign convention)
- Incorrect Phasor notation (Understand: jωM ↔ j3 ; jωL ↔ j5 ; 1/ jωC ↔ -j4)
- Incorrect sign convention for loop equations
  - Current entering negative terminal of a voltage source
- Understanding of complex numbers!
  - Familiarity with your calculators handling of complex numbers
  - Converting from rectangular to polar

Review / Understand these concepts to avoid these mistakes

13.2 Recommended Viewing:

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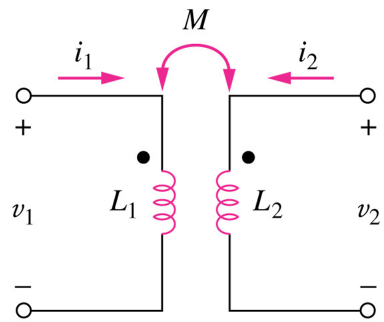
- Watch these videos illustrating solving mutual inductance problems:
  - http://www.youtube.com/watch?v=tD35a-uzd34
  - http://www.youtube.com/watch?v=hzU4XKQYTWw
  - http://www.youtube.com/watch?v=OqSvesTtnUo

- Uses the model described previously to solve mutual inductance problems.

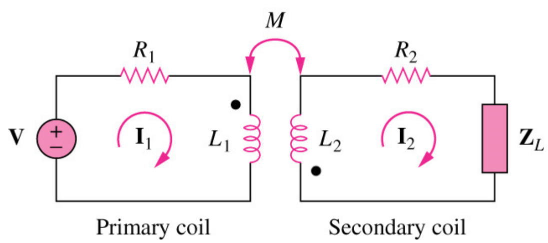
13.3 Energy in a Coupled Circuit (1)

- The instantaneous energy w stored in the circuit is:



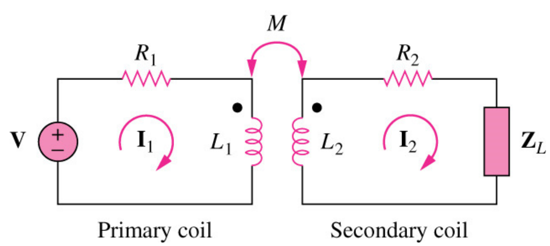
$$\boxed{w = \frac{1}{2}L_1 i_1^2 + \frac{1}{2}L_2 i_2^2 \pm M i_1 i_2}$$

- If **both** currents enter (or **both** leave) the dotted terminal the mutual term is (positive):



$$w = \frac{1}{2}L_1 i_1^2 + \frac{1}{2}L_2 i_2^2 + M i_1 i_2$$

- Otherwise the mutual term is (negative):



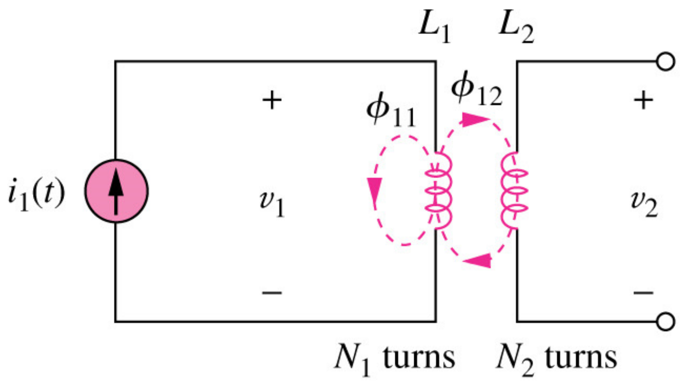
$$w = \frac{1}{2}L_1 i_1^2 + \frac{1}{2}L_2 i_2^2 - M i_1 i_2$$

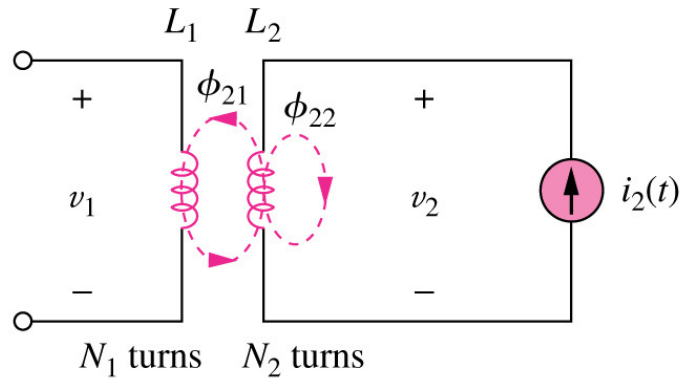
13.3 Energy in a Coupled Circuit (2)

- The coupling coefficient, k, is a measure of the magnetic coupling between two coils; $0 \leq k \leq 1$.

$$M = k\sqrt{L_1 L_2}$$

K = 1 coils perfectly coupled
K < 0.5 coils loosely coupled
K > 0.5 coils tightly coupled

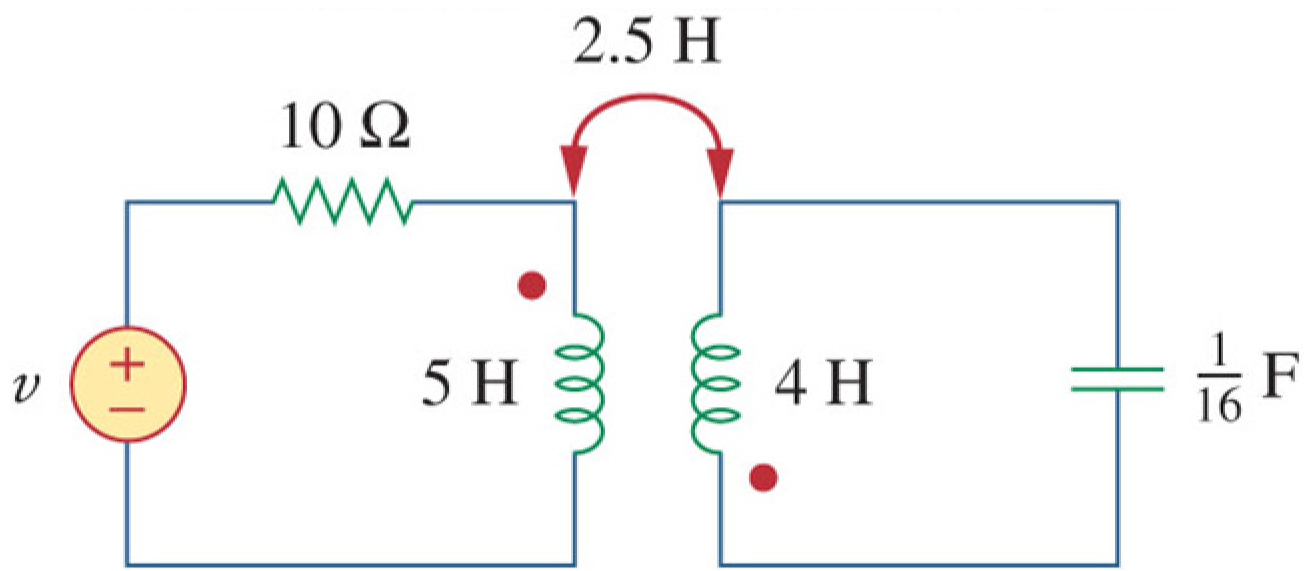




$$K = \frac{\phi_{12}}{\phi_{11} + \phi_{12}} \qquad K = \frac{\phi_{21}}{\phi_{21} + \phi_{22}}$$

Example 13.3 (Textbook)

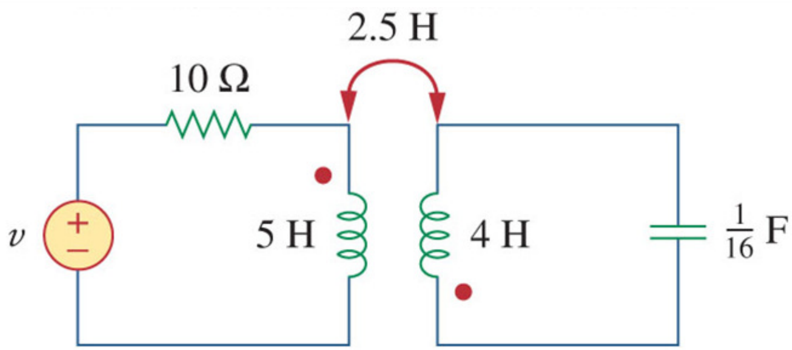
Consider the circuit below. Determine the coupling coefficient. Calculate the energy stored in the coupled inductors at time t = 1s if $v = 60\cos(4t + 30°)$ V.



Ans: k=0.56; $w(1)$=20.73J

13.3 Energy in a Coupled Circuit (4)

Example 13.3

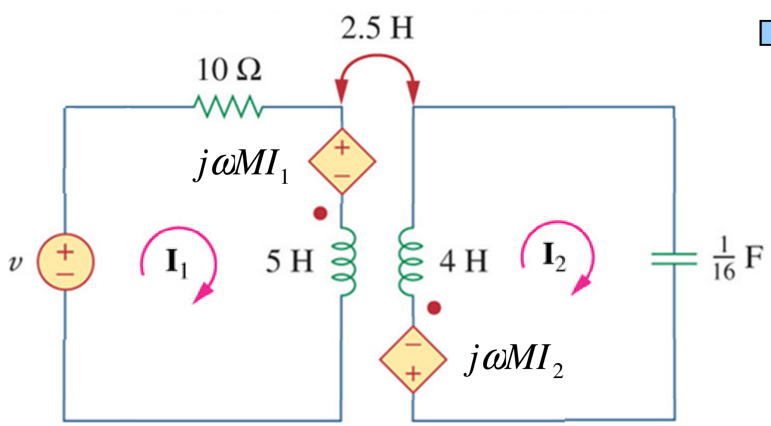


To find k, use the relation between L_1, L_2, and M:

$$L_1 = 5 \;;\; L_2 = 4 \;;\; M = 2.5$$

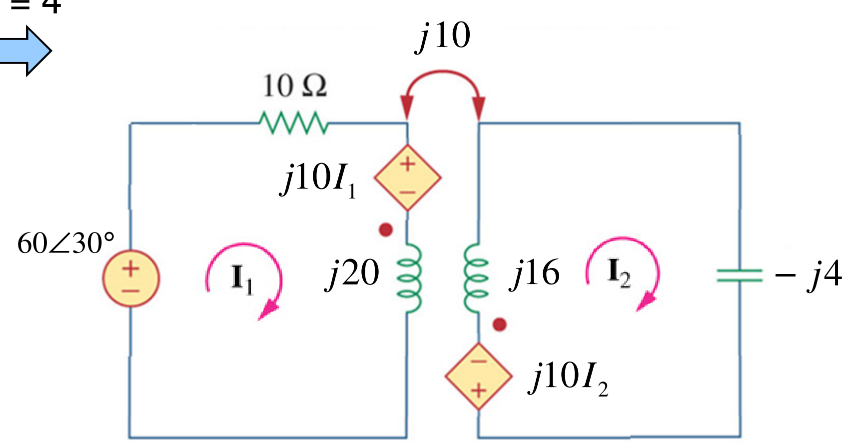
$$k = \frac{M}{\sqrt{L_1 L_2}} = \frac{2.5}{\sqrt{(5)(4)}} = 0.56$$

Apply the "Model"



Convert to Phasor

ω = 4



Solve Mesh equations for I_1 and I_2

13.3 Energy in a Coupled Circuit (5)

Example 13.3

From Mesh analysis:

$$I_1 = 3.905\angle -19.4° \text{ A}$$
$$I_2 = 3.254\angle 160.6° \text{ A}$$

At t = 1, the value of ωt = (4)(1) = 4 radians = 229.2º

To find the energy at t = 1:

$$i_1 = 3.905\cos(\omega t - 19.4°)$$
$$i_2 = 3.254\cos(\omega t + 160.6°)$$

$$i_1 = 3.905\cos(229.2° - 19.4°) = -3.389$$
$$i_2 = 3.254\cos(229.2° + 160.6°) = 2.824$$

Positive since current (as defined) enters **both** dots

$$w = \frac{1}{2}L_1 i_1^2 + \frac{1}{2}L_2 i_2^2 + M i_1 i_2$$

$$w = \frac{1}{2}(5)(-3.389)^2 + \frac{1}{2}(4)(2.824)^2 + 2.5(-3.389)(2.824) = 20.73 \text{ J}$$

Homework #2

Due in class Monday, January 26, 2015

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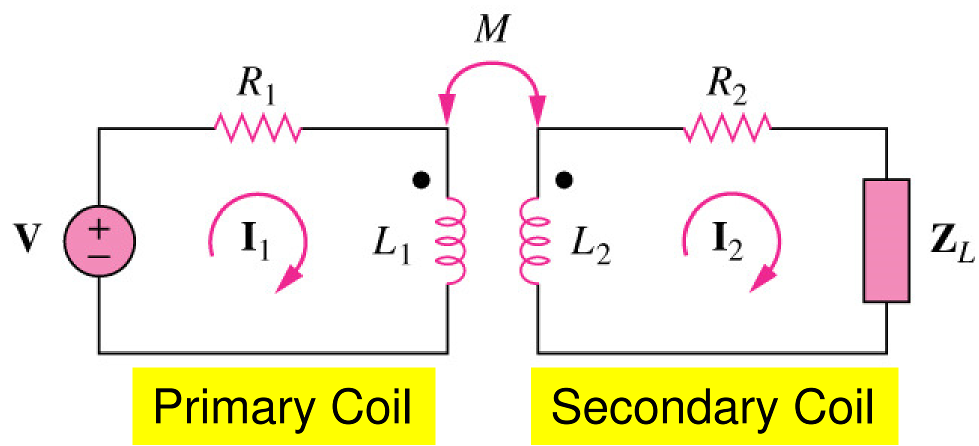
- 13.1
- 13.7
- 13.9
- 13.24

13.4 Linear Transformers (1)

- A transformer is generally a four-terminal device comprising two (or more) magnetically coupled coils. Below is a "simple" model:

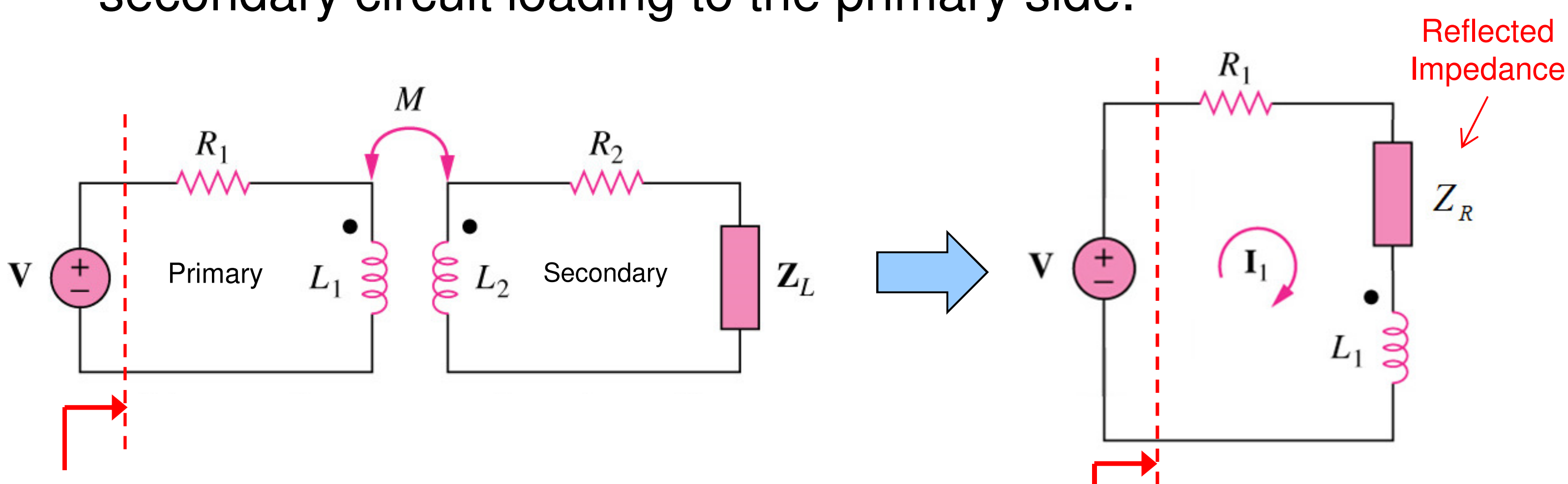


- The coil connected to the voltage source is called the primary winding.
- The coil connected to the load is called the secondary winding.
- Resistances R_1 and R_2 are included to account for the losses in the coils.
- A transformer is said to be linear if the coils are wound on a magnetically linear material for which the permeability is constant.
  - Air, plastic, Bakelite, wood, etc.
  - Most materials are magnetically linear.

13.4 Linear Transformers (2)

Reflected Impedance

- Often we are interested in the input impedance Z_{in} seen by the source.
  - For example, may want to match Z_{in} to the source impedance for maximum power transfer!
- To simplify our analysis we can break Z_{in} up into:
  - $Z_{primary}$ -- The impedance of the "primary" circuit $\left(Z_{primary} = R_1 + j\omega L_1\right)$
  - Z_R -- The "reflected" impedance back to the "primary".
- The "reflected" impedance Z_R is the contribution of the secondary circuit loading to the primary side.



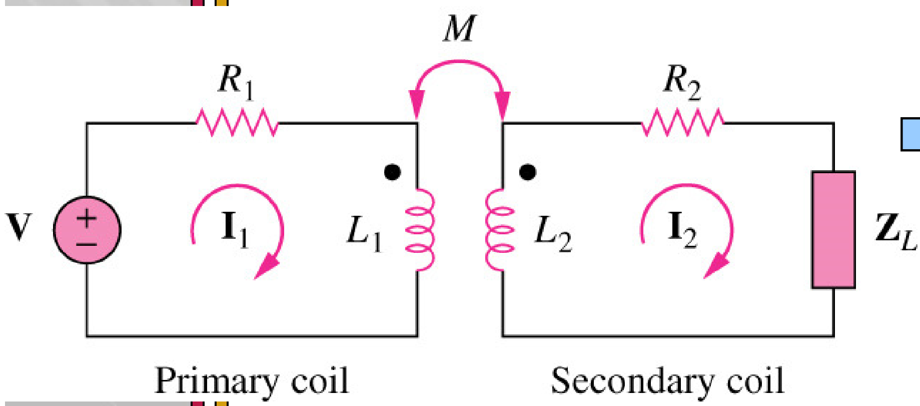
$$Z_{in} = Z_{primary} + Z_R$$

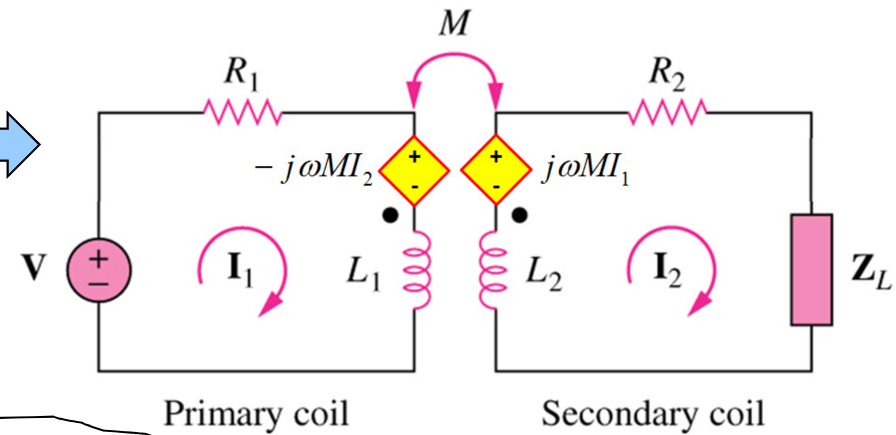
$$Z_{in} = Z_{primary} + Z_R$$

13.4 Linear Transformers (2)

Reflected Impedance

To obtain the input impedance Z_{in} as seen from the source:





Mesh 1: $V = (R_1 + j\omega L_1)I_1 - j\omega M I_2$

Current enters positive terminal so it is **+** (-jωMI_2)

Mesh 2: $0 = -j\omega M I_1 + (R_2 + j\omega L_2 + Z_L)I_2$

Current enters negative terminal so it is **-** (jωMI_2)

From Mesh 2:

$$I_2 = \frac{j\omega M}{R_2 + j\omega L_2 + Z_L} I_1$$

Substituting into Mesh 1 gives:

$$V = \underbrace{(R_1 + j\omega L_1)}I_1 + \underbrace{\boxed{\frac{\omega^2 M^2}{R_2 + j\omega L_2 + Z_L}}}I_1$$

$$V = Z_{primary}\, I_1 + Z_{reflected}\, I_1$$

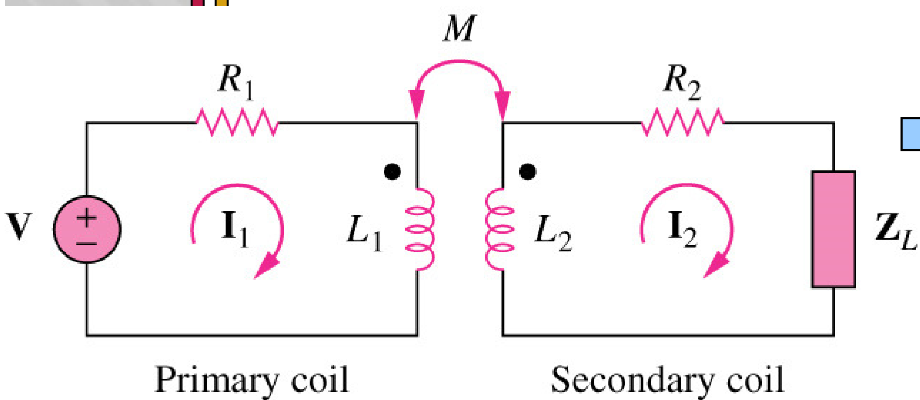
Reflected Impedance

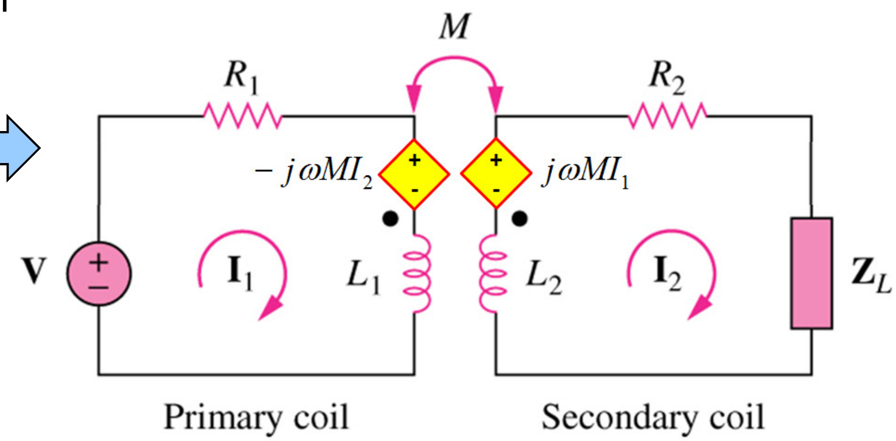
$$\boxed{Z_R = \frac{\omega^2 M^2}{R_2 + j\omega L_2 + Z_L}}$$

13.4 Linear Transformers (3)

Reflected Impedance (Another way of looking at it !)

Obtain input impedance Z_{in} as seen from the source:





Mesh 1: $V = \left(Z_{primary}\right)I_1 - j\omega M I_2$

Mesh 2: $0 = -j\omega M I_1 + (Z_{Secondary})I_2$

$Z_{Secondary}$ = the total series impedance in the secondary loop

From Mesh 2:

$$\mathrm{I}_2 = \frac{j\omega M}{Z_{Secondary}}\mathrm{I}_1$$

Substituting into Mesh 1 gives:

$$V = Z_{primary} I_1 + \frac{\omega^2 M^2}{Z_{Secondary}} I_1$$

$$V = Z_{primary} I_1 + Z_{reflected} I_1$$

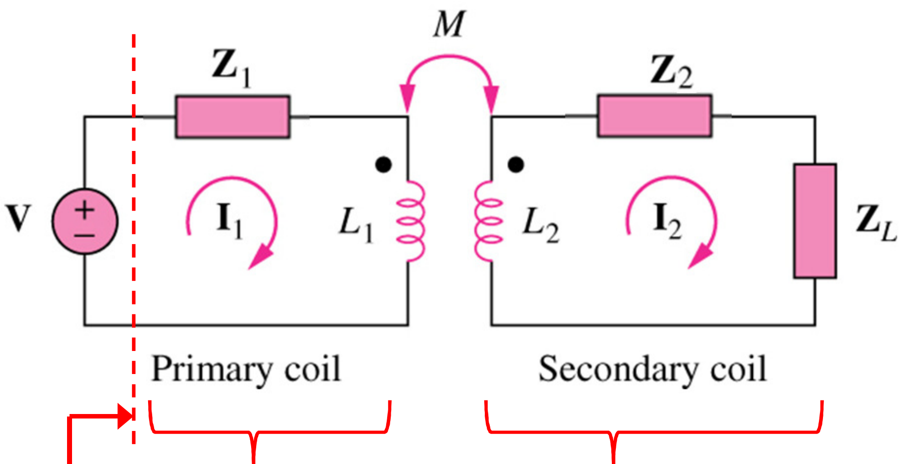
Reflected Impedance

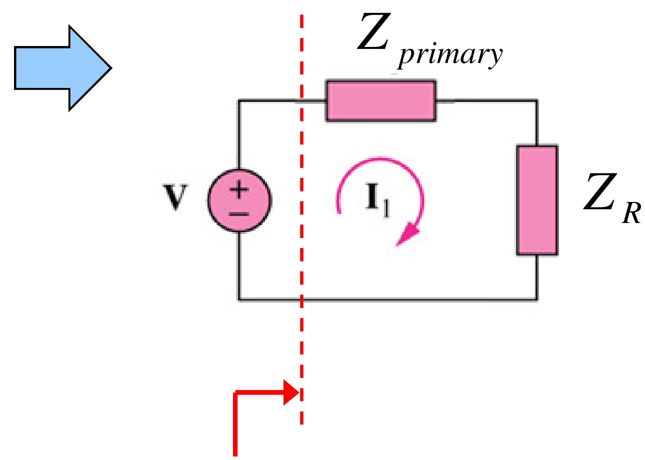
$$Z_R = \frac{\omega^2 M^2}{Z_{Secondary}}$$

13.4 Linear Transformers (4)

Reflected Impedance

- The input impedance can be broken into two parts as follows:





$$Z_{in} = Z_{primary} + Z_R$$

- Series Impedance in Primary Coil: $Z_{primary} = Z_1 + j\omega L_1$
- Series Impedance in Secondary Coil: $Z_{secondary} = Z_2 + Z_L + j\omega L_2$
- Input Impedance: $Z_{in} = Z_{primary} + \dfrac{\omega^2 M^2}{Z_{Secondary}}$

Note: Z_{in} will be the same if the dot on L2 is switched

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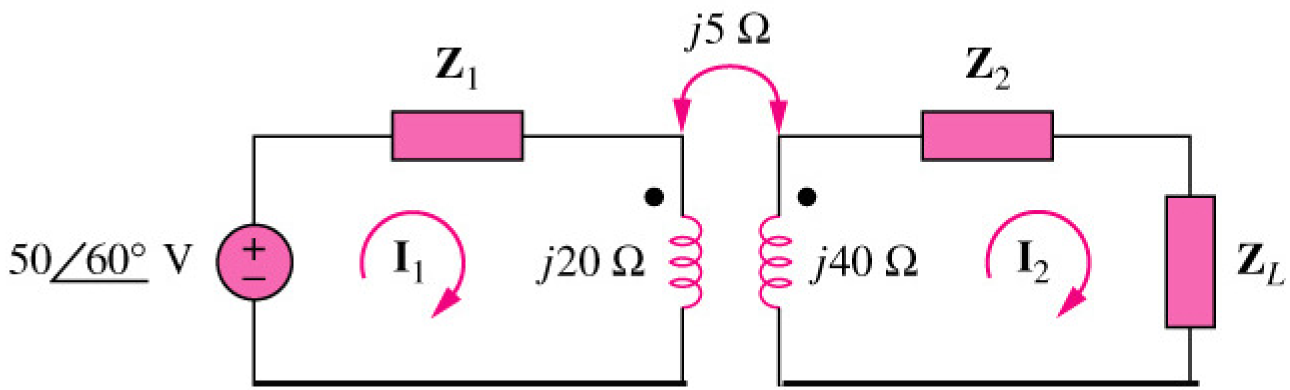
Example 13.4 (textbook)

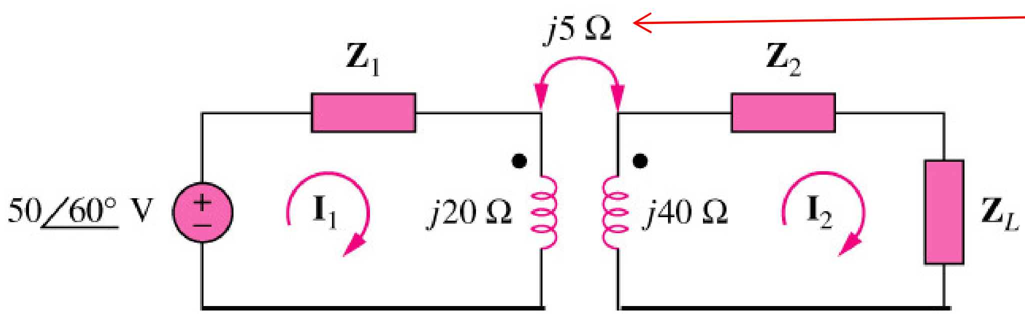
In the circuit below, calculate the input impedance and current I_1. Take $Z_1=60-j100\Omega$, $Z_2=30+j40\Omega$, and $Z_L=80+j60\Omega$.



Ans: $Z_{\text{in}} = 100.14\angle -53.1°\Omega;\ I_1 = 0.5\angle 113.1°\text{A}$

13.4 Linear Transformers (5)

Example 13.4



Note:

$$j\omega M = j5$$
$$\omega M = 5$$
$$(\omega M)^2 = 25$$

- The series impedance in the primary coil:

$$Z_{primary} = (60 - j100) + j20 = 60 - j80$$

- The series impedance in the primary coil:

$$Z_{secondary} = (30 + j40) + (80 + j60) + j40 = 110 + j140$$

- Input Impedance given by:

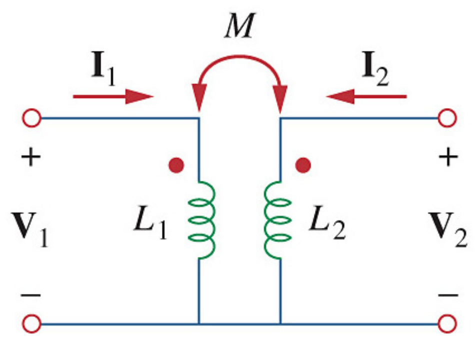
$$Z_{in} = Z_{primary} + \frac{\omega^2 M^2}{Z_{Secondary}} = (60 - 80j) + \frac{25}{(110 + j140)} = 60.09 - j80.11$$

- Current I_1 given by: $I_1 = \frac{V_s}{Z_{in}} = \frac{50\angle 60°}{60.09 - j80.11} = \frac{50\angle 60°}{100.14\angle -53.1°}$

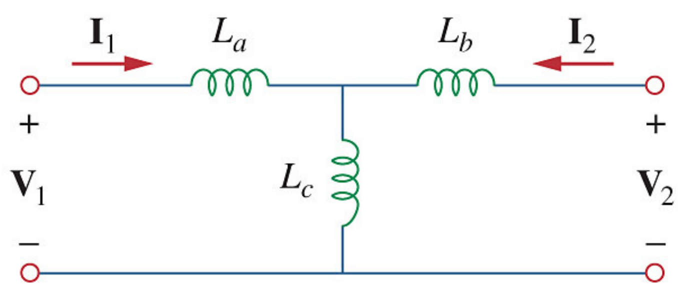
13.4 Linear Transformers (6)

Equivalent T and π Circuits:

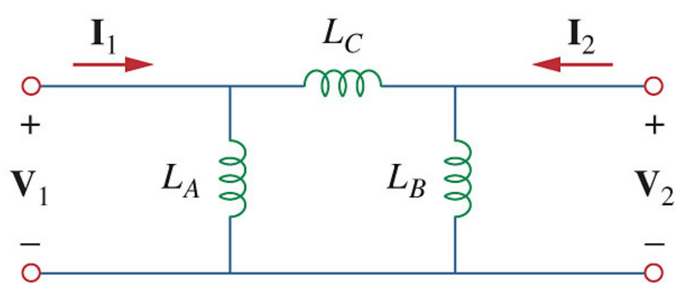
- It is sometimes convenient to replace a magnetically couple circuit with an equivalent circuit with no magnetic coupling.
- We can replace the linear transformer with an equivalent T or π circuit that has no mutual inductance.



T Circuit



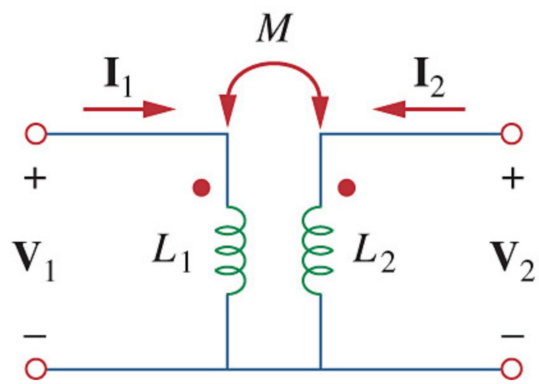
π Circuit



13.4 Linear Transformers (7)

Equivalent T Circuit:

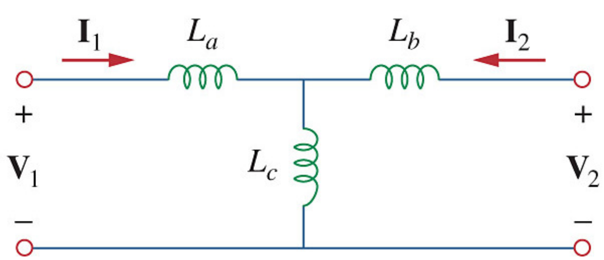
Linear Transformer Circuit



Mesh Analysis of this transformer circuit results in the following:

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} j\omega L_1 & j\omega M \\ j\omega M & jwL_2 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

Equivalent T Network



Mesh Analysis of the equivalent T network results in the following:

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} j\omega(L_a + L_c) & jwL_c \\ jwL_c & jw(L_b + L_c) \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

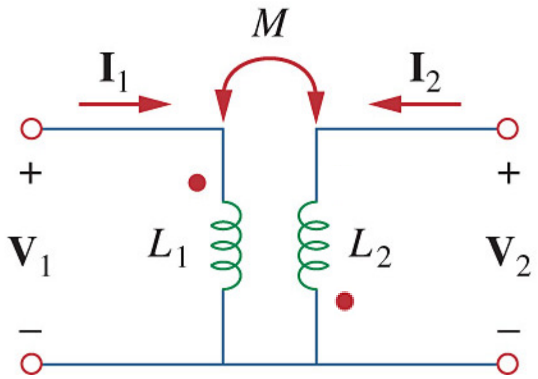
Equating the terms gives the following relationships:

$L_a = L_1 - M$ $\quad$ $L_b = L_2 - M$ $\quad$ $L_c = M$

13.4 Linear Transformers (8)

Equivalent T Circuits (Swapped Dots):

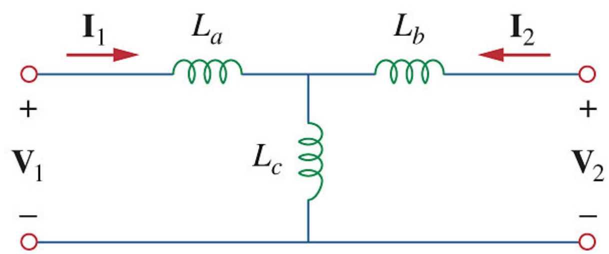
Linear Transformer Circuit



Mesh Analysis of this transformer circuit results in the following:

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} j\omega L_1 & -j\omega M \\ -j\omega M & jwL_2 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

Equivalent T Network



Mesh Analysis of the equivalent T network results in the following:

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} j\omega(L_a + L_c) & jwL_c \\ jwL_c & jw(L_b + L_c) \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

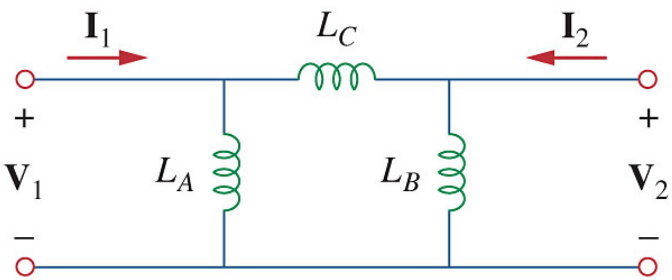
Equating the terms gives the following relationships:

$L_a = L_1 + M$ $\qquad$ $L_b = L_2 + M$ $\qquad$ $L_c = -M$

13.4 Linear Transformers (9)

Equivalent π Circuit:

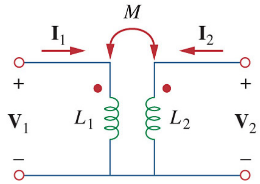
Similarly, for the π network nodal analysis provides:



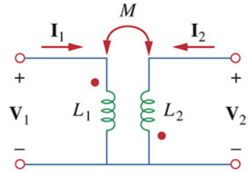
$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} \frac{1}{j\omega L_A} + \frac{1}{j\omega L_C} & -\frac{1}{j\omega L_C} \\ -\frac{1}{j\omega L_C} & \frac{1}{j\omega L_B} + \frac{1}{j\omega L_C} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

Equivalent π Network

By equating terms in admittance matrices, for the π equivalent network we obtain (note if dots are different, replace with –M):



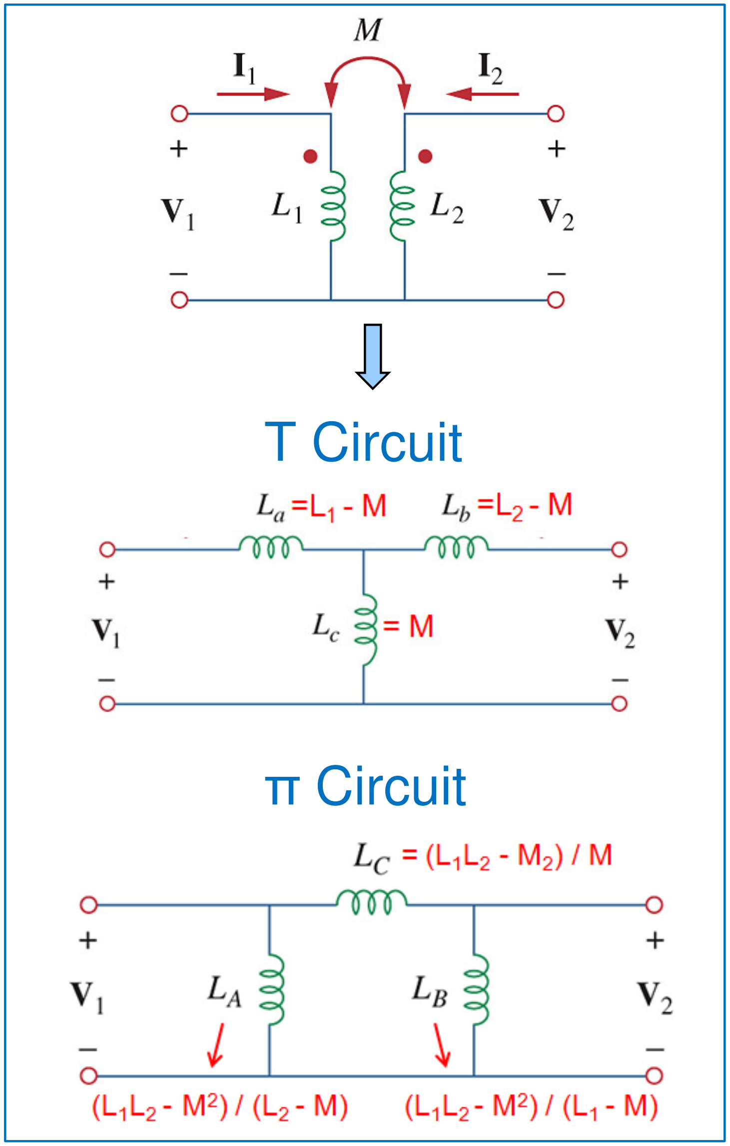
$$L_A = \frac{L_1 L_2 - M^2}{L_2 - M}\ ;\ L_B = \frac{L_1 L_2 - M^2}{L_1 - M};\ L_C = \frac{L_1 L_2 - M^2}{M}$$

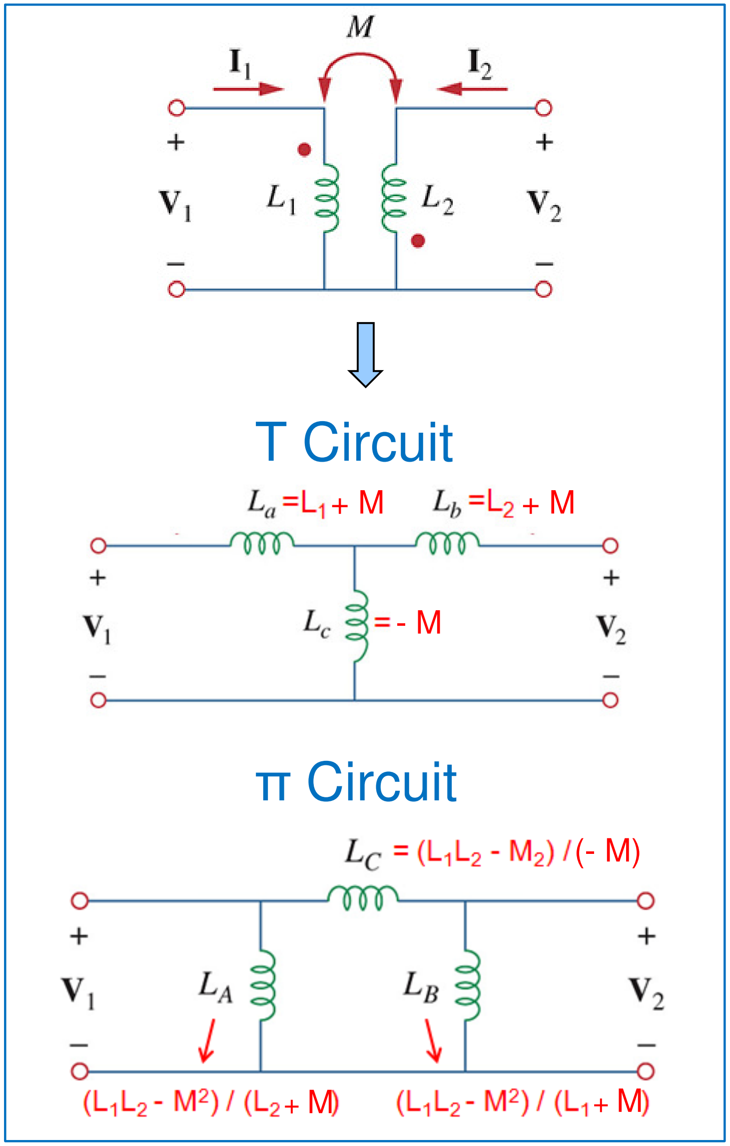


$$L_A = \frac{L_1 L_2 - M^2}{L_2 + M}\ ;\ L_B = \frac{L_1 L_2 - M^2}{L_1 + M};\ L_C = \frac{L_1 L_2 - M^2}{-M}$$

13.4 Linear Transformers (10)

Equivalent T or π Circuits Summary

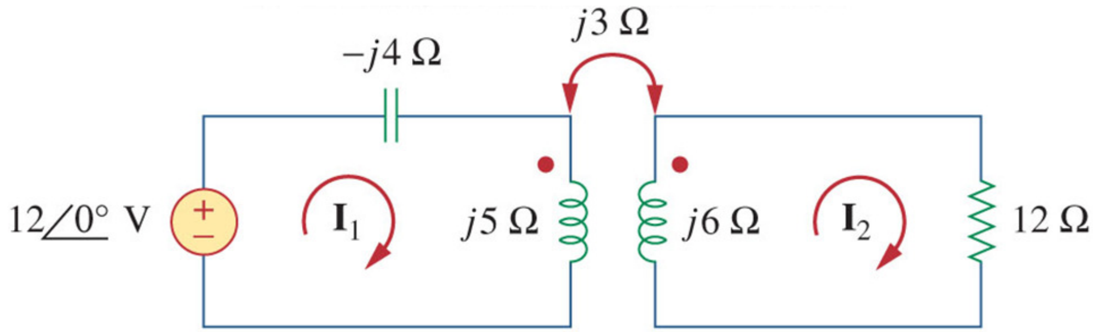


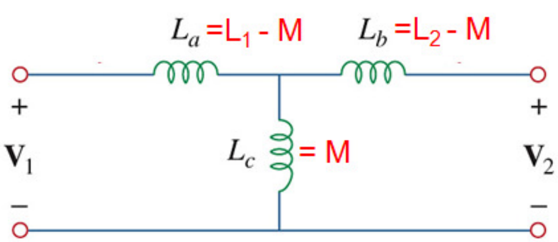


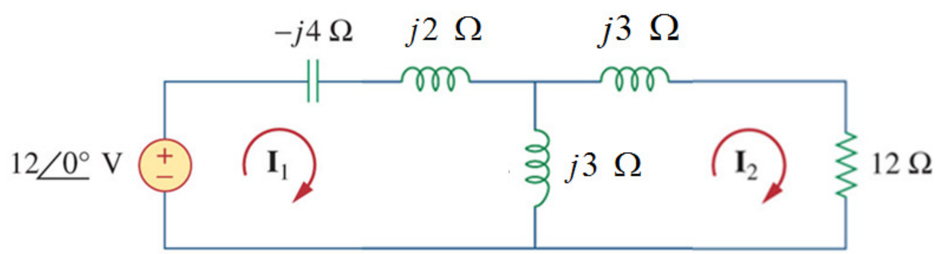
13.4 Linear Transformers (11)

Practice Problem 13.6

Find I_1 and I_2 using the T equivalent circuit





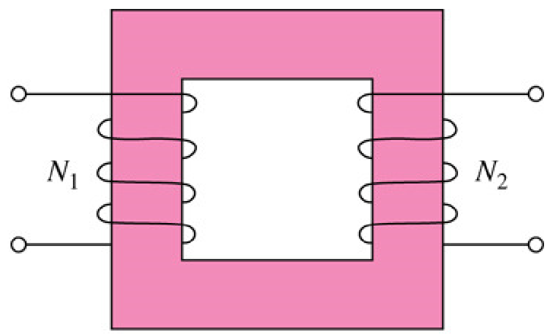


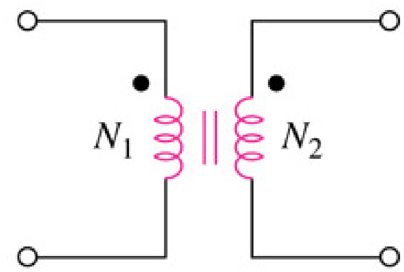
Mesh I_1: $-12\angle 0° - j4I_1 + j2I_1 + j3(I_1 - I_2) = 0$ $\qquad jI_1 - j3I_2 = 12\angle 0°$

Mesh I_2: $j3(I_2 - I_1) + j3I_2 + 12I_2 = 0$ $\qquad -j3I_1 + (12 + j6)I_2 = 0$

13.5 Ideal Transformers (1)

- An ideal transformer has perfect coupling (k=1).
- It consists of two or more coils with a large number of turns wound on a common core of high permeability.



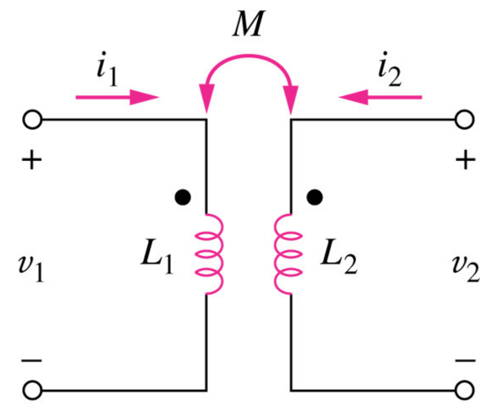


- Because of the high permeability of the core, the flux links all the turns of both coils, thereby resulting in a perfect coupling.

13.5 Ideal Transformers (2)

"Dot's the same polarity"

Recall the coupled circuit:



$$V_1 = j\omega L_1 I_1 + j\omega M I_2 \qquad (1)$$

$$V_2 = j\omega M I_1 + j\omega L_2 I_2 \qquad (2)$$

Solving for I_1 in (1): $\mathrm{I}_1 = \dfrac{V_1 - j\omega M I_2}{j\omega L_1}$ (3)

Substituting (3) into (2): $V_2 = j\omega L_2 I_2 + \dfrac{M V_1}{L_1} - \dfrac{j\omega M^2 I_2}{L_1}$

$M = \sqrt{L_1 L_2}$ For perfect coupling (k=1):

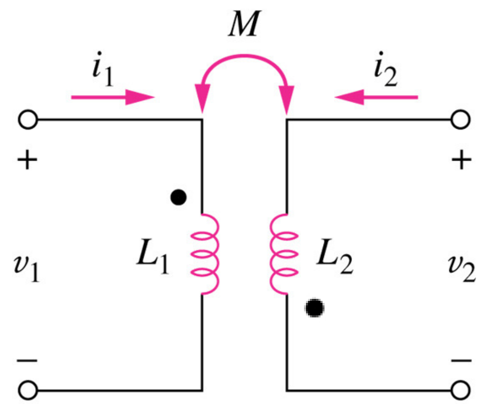
$$V_2 = j\omega L_2 I_2 + \frac{\sqrt{L_1 L_2} V_1}{L_1} - \frac{j\omega L_1 L_2 I_2}{L_1} = \sqrt{\frac{L_2}{L_1}} V_1 = nV_1$$

Therefore: $V_2 = nV_1$ where $n = \sqrt{L_2 / L_1}$ = turns ratio

13.5 Ideal Transformers (3)

"Dot's opposite each other"

Mesh Equations give the following:



$$V_1 = j\omega L_1 I_1 - j\omega M I_2 \qquad (1)$$

$$V_2 = -j\omega M I_1 + j\omega L_2 I_2 \qquad (2)$$

Solving for I_1 in (1): $\mathrm{I}_1 = \dfrac{V_1 + j\omega M I_2}{j\omega L_1}$ (3)

Substituting (3) into (2): $V_2 = j\omega L_2 I_2 - \dfrac{MV_1}{L_1} - \dfrac{j\omega M^2 I_2}{L_1}$

$M = \sqrt{L_1 L_2}$ For perfect coupling (k=1):

$$V_2 = j\omega L_2 I_2 - \frac{\sqrt{L_1 L_2}V_1}{L_1} - \frac{j\omega L_1 L_2 I_2}{L_1} = -\sqrt{\frac{L_2}{L_1}}V_1 = -nV_1$$

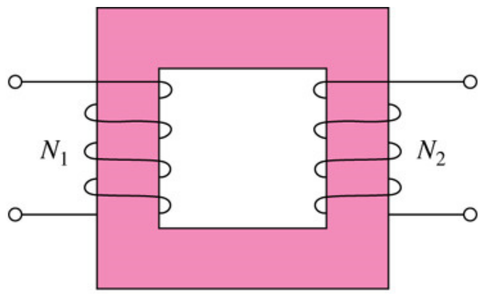
Therefore: $V_2 = -nV_1$ If dot is swapped at output

13.5 Ideal Transformers (4)
Properties

- A transformer is said to be ideal if it has the following properties:
  1. Coils have very large reactances (L_1, L_2, $M \rightarrow \infty$)
  2. Coupling coefficient is equal to unity ($k=1$)
  3. Primary and secondary coils are lossless ($R_1 = R_2 = 0$)
- An ideal transformer is a unity-coupled ($k=1$) lossless transformer in which the primary and secondary coils have infinite self-inductances (L_1 & $L_2 \rightarrow \infty$).
- Iron core transformers are close approximations to ideal transformers and are used in power systems and electronics.

13.5 Ideal Transformers (5)

- When a sinusoidal voltage is applied to the primary winding, the same magnetic flux Φ goes through both windings.



$$v_1 = N_1 \frac{d\phi}{dt} \quad ; \quad v_2 = N_2 \frac{d\phi}{dt}$$

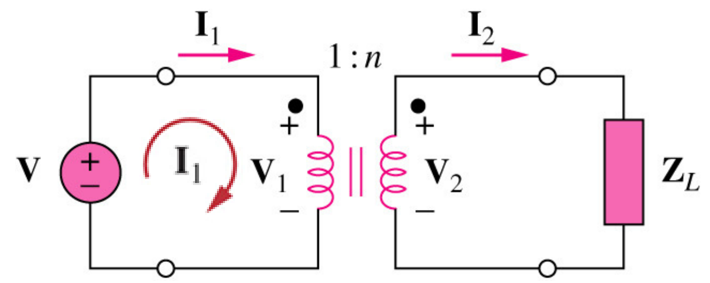
$$\frac{v_2}{v_1} = \frac{N_2}{N_1} = n =$$ Turns ratio or transformation ratio

- Using the phasor voltages rather than the instantaneous voltages:

$$\frac{V_2}{V_1} = \frac{N_2}{N_1} = n$$

13.5 Ideal Transformers (6)

- Power conservation: $v_1 i_1 = v_2 i_2$
- The energy supplied to the primary must equal the energy absorbed by the secondary, since there are no losses in an ideal transformer.
- In phasor form: $\boxed{\dfrac{I_1}{I_2} = \dfrac{V_2}{V_1} = n}$



  - n=1 → isolation transformer ($V_2 = V_1$)
  - n>1 → step-up transformer ($V_2 > V_1$)
  - n<1 → step-down transformer ($V_2 < V_1$)

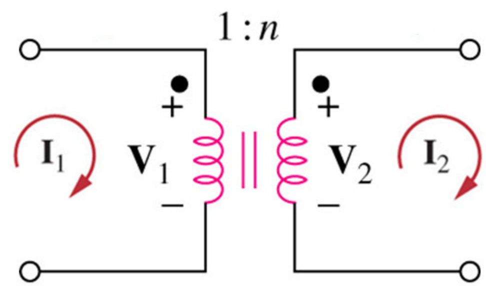
13.5 Ideal Transformers (7)

- Transformer ratings are usually specified as V_1 / V_2
- Power companies often generate at some convenient voltage and use the step-up transformer to increase the voltage so that the power can be transmitted at very high voltage and low current over transmission lines, resulting in significant cost savings. Near residential consumers, step-down transformers are used to bring the voltage down to 120 V.
- It is important to get the proper polarity of the voltages and the direction of the currents for the transformer.

13.5 Ideal transformers (8)

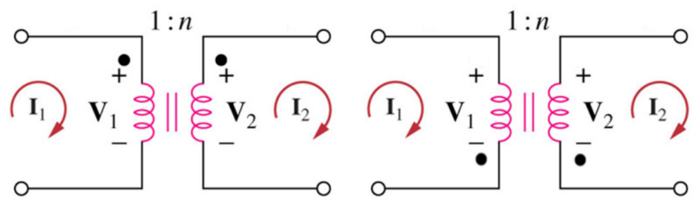
V and I relationships (simpler way to remember)

Given this standard definition for Voltages and currents



Note: This definition of I_2 differs from the text.

Dots "Same" → "+"



$$V_2 = nV_1$$

$$I_2 = \frac{I_1}{n}$$

Reflected Impedance

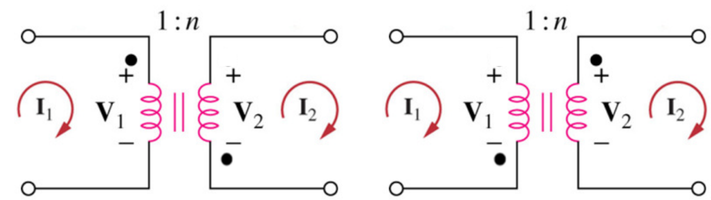
$$Z_{in} = \frac{Z_L}{n^2}$$

Note: The larger the turns ratio
The larger "n"
The larger "N_2"
The larger V_2

Turns Ratio

$$n = \frac{N_2}{N_1}$$

Dots "Different" → "-"



$$V_2 = -nV_1$$

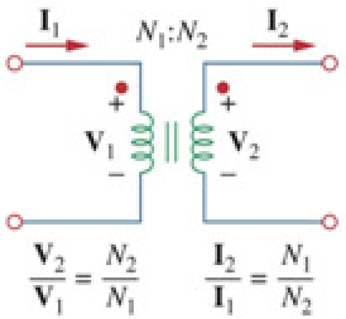
$$I_2 = -\frac{I_1}{n}$$

$$Z_{in} = \frac{Z_L}{n^2}$$

13.5 Ideal Transformers (9)

V and I relationships

- Expressing V_1 in terms of V_2 and I_1 in terms of I_2 or vice versa:



$$V_1 = \frac{V_2}{n} \qquad V_2 = nV_1$$

Positive, if Voltage "**same**" polarity at dot

$$I_2 = \frac{I_1}{n} \qquad I_1 = nI_2$$

Positive, if current "**different**" polarity at dot

- Complex Power is:

Complex Conjugate of I_2

$$S_1 = V_1 I_1^* = \frac{V_2}{n}(nI_2)^* = V_2 I_2^* = S_2$$

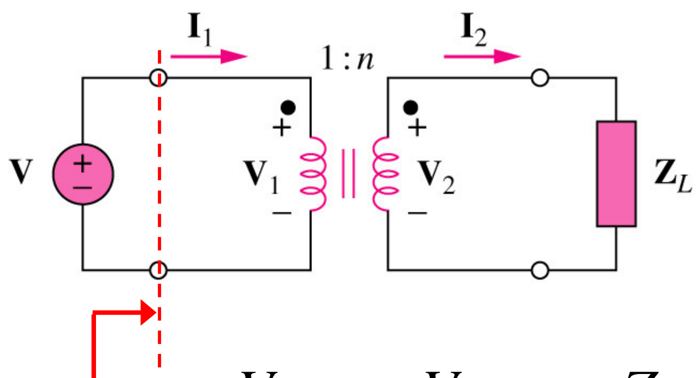
- Complex power supplied to the primary is delivered to the secondary without loss.

- The ideal transformer is **lossless** and absorbs **no power**.

13.5 Ideal Transformers (10)

Reflected impedance

- The input impedance as seen by the source is:



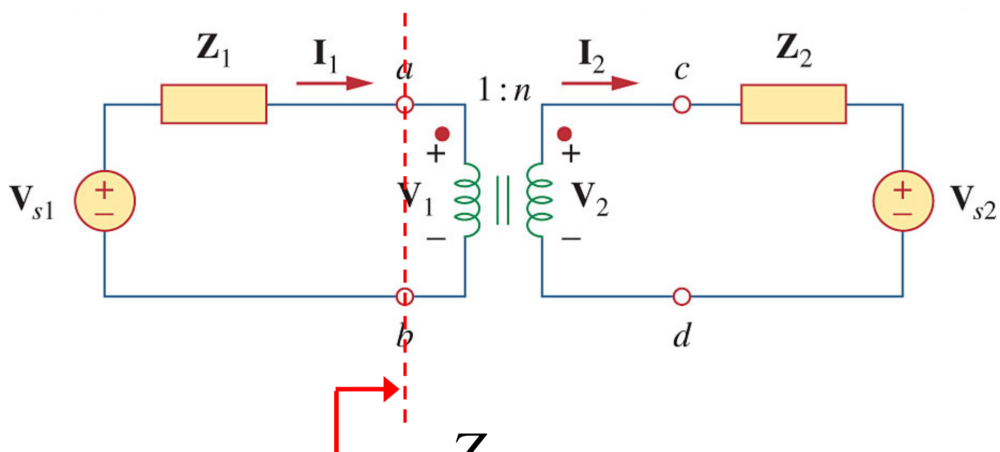
$$Z_{in} = \frac{V_1}{I_1} = \frac{V_2}{n^2 I_2} = \frac{Z_L}{n^2}$$

- The input impedance is also called the reflected impedance since it appears as if the load impedance is reflected to the primary side.
- The ability of the transformer to transform a given impedance to another allows impedance matching to ensure maximum power transfer.

13.5 Ideal Transformers (11)

Equivalent circuit analysis

- In analyzing a circuit containing an ideal transformer, it is common practice to eliminate the transformer by reflecting impedances and sources from one side of the transformer to the other.
- Suppose we want to reflect the secondary side of the circuit to the primary side:

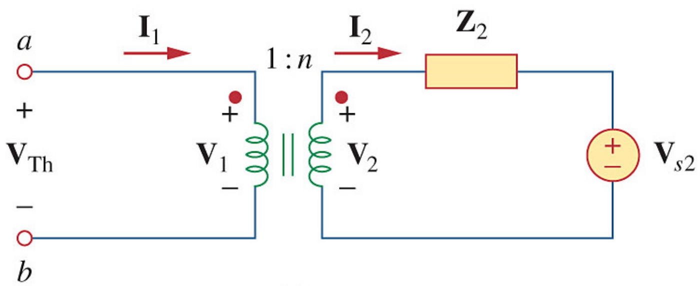


$$Z_R = \frac{Z_2}{n^2}$$

13.5 Ideal Transformers (12)

Equivalent circuit analysis

- We find the Thevenin equivalent of the circuit to the right of a-b:
- Obtaining V_{th} from "open circuit voltage":



(a)

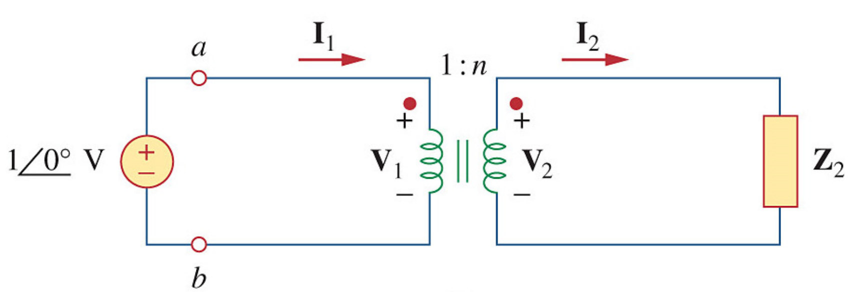
$$I_1 = 0 = I_2$$ Since a-b is open

$$V_2 = V_{s2}$$

$$V_{Th} = V_1 = \frac{V_2}{n} = \frac{V_{s2}}{n}$$

- Obtaining Z_{Th} (remove the voltage source in the secondary and insert a unit source at a-b terminals.)

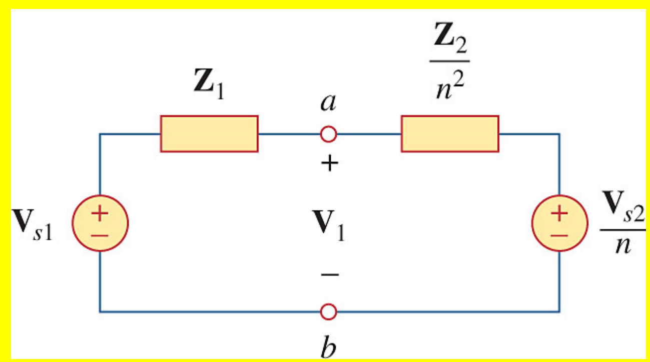
$$Z_{Th} = \frac{V_1}{I_1} = \frac{V_2}{n^2 I_2} = \frac{Z_2}{n^2}$$



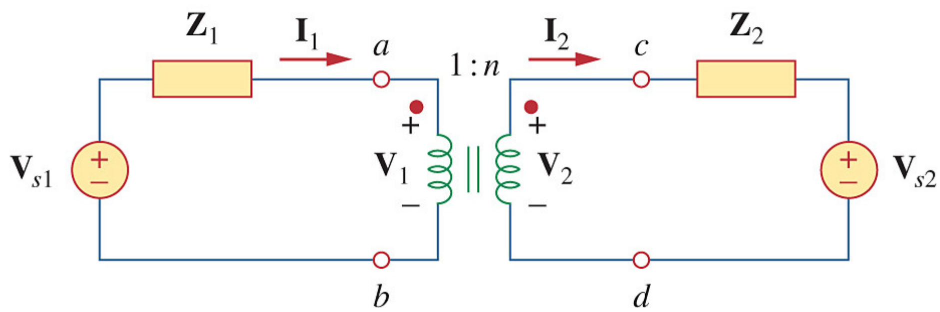
(b)

Equivalent Circuit

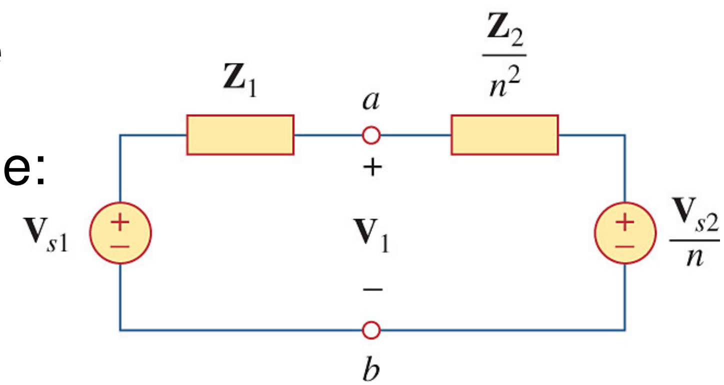
Reflecting the secondary to the primary



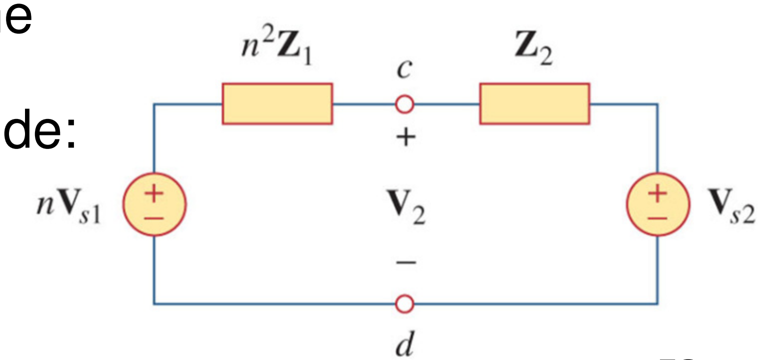
13.5 Ideal Transformers (13)



- The general rule for eliminating the transformer and reflecting the secondary circuit to the primary side:
  - Divide the secondary impedance by n^2
  - Divide the secondary voltage by n
  - Multiply the secondary current by n



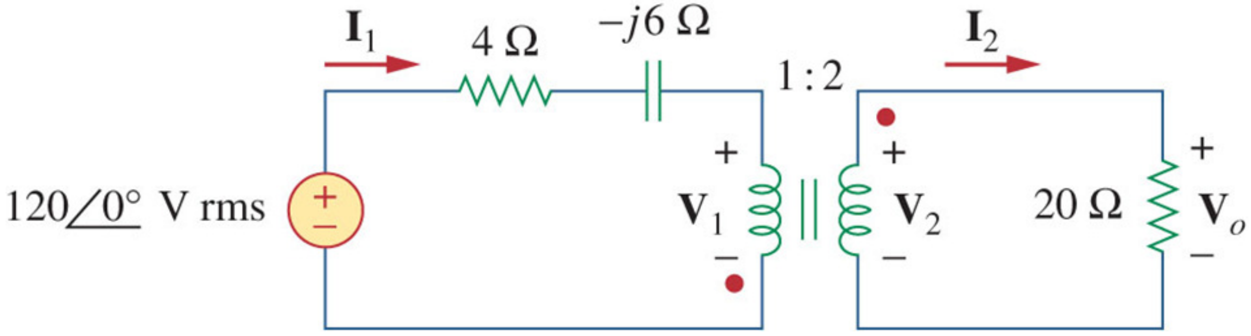
- The general rule for eliminating the transformer and reflecting the secondary circuit to the primary side:
  - Multiply the primary impedance by n^2
  - Multiply the primary voltage by n
  - Divide the primary current by n



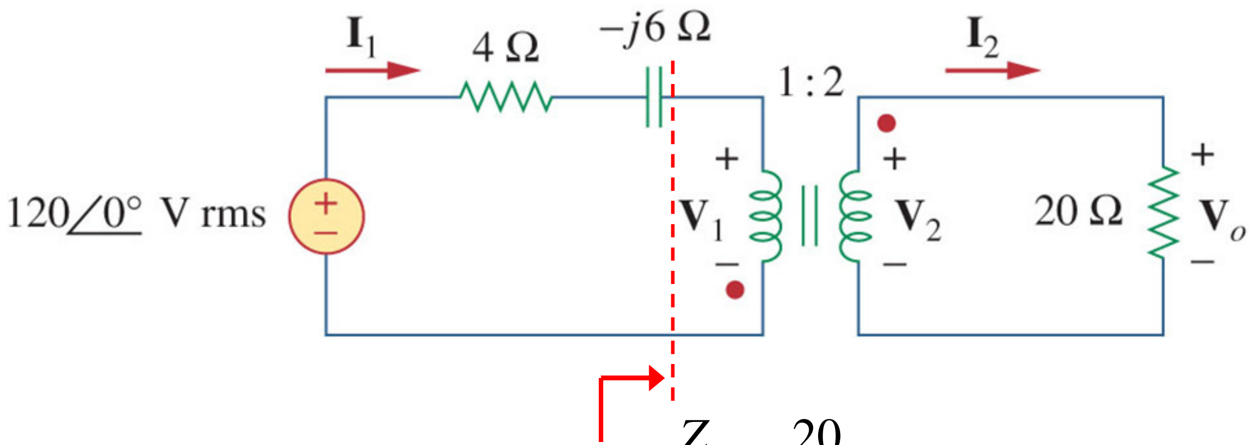
13.5 Ideal Transformer (14)

Example 13.8 (Textbook)

For the ideal transformer, find: (a) the source current I_1, (b) the output voltage V_0, and (c) the complex power supplied by the source



13.5 Ideal Transformer (14)



$$Z_R = \frac{Z_2}{n^2} = \frac{20}{2^2} = 5$$

Impedance seen by the Voltage source is:

$$Z_{in} = (4 - j6) + 5 = 9 - j6 = 10.82\angle -33.69° \ \Omega$$

Input current I_1 is:

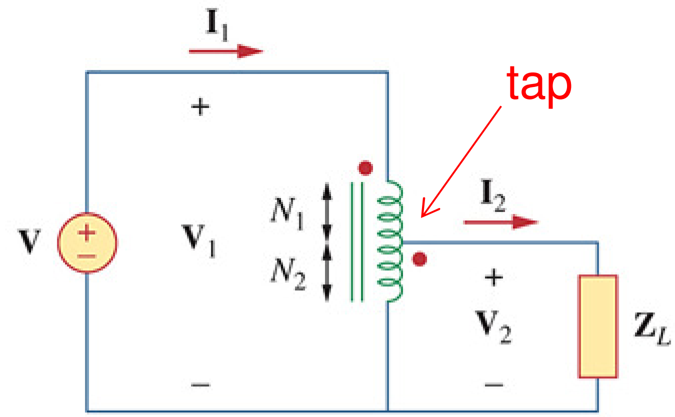
$$I_1 = \frac{V_s}{Z_{in}} = \frac{120\angle 0°}{10.82\angle -33.69°} = 11.09\angle 33.69° \ \text{A}$$

$$I_2 = -\frac{I_1}{n} = \frac{-11.09\angle 33.69°}{2} = 5.55\angle -146.31° \ \text{A}$$

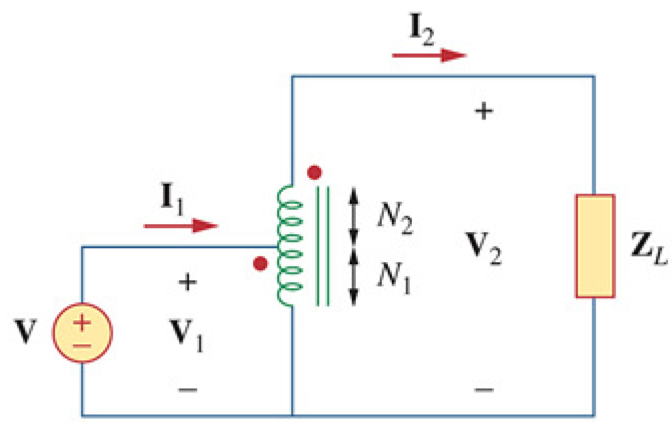
$$V_o = 20 I_2 = 20(5.55\angle -146.31°) = 110.9\angle -146.31° \ \text{V}$$

13.6 Ideal Auto-Transformers (1)

- An **autotransformer** is a transformer in which both the primary and the secondary are in a single winding
- A connection point called a *tap* is separates the primary and secondary.
- The tap is often adjustable to provide a desired turns ratio.
- An adjustable tap provides a variable voltage to the load
- A disadvantage of the autotransformer is it provides *no electrical isolation*



Step Down Auto-Transformer



Step Up Auto-Transformer

13.6 Ideal Auto-Transformers (2)

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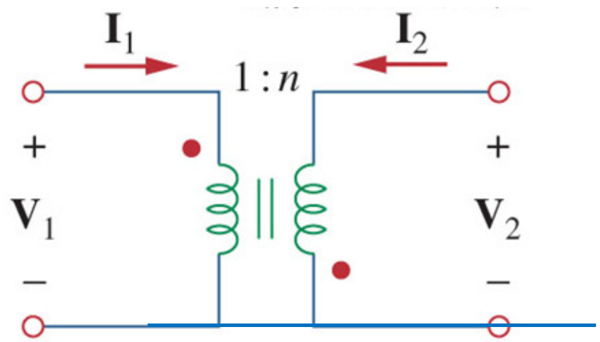
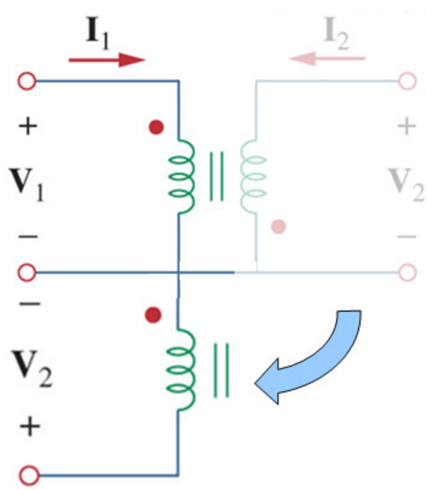
- In analyzing the autotransformer, consider the following circuit:

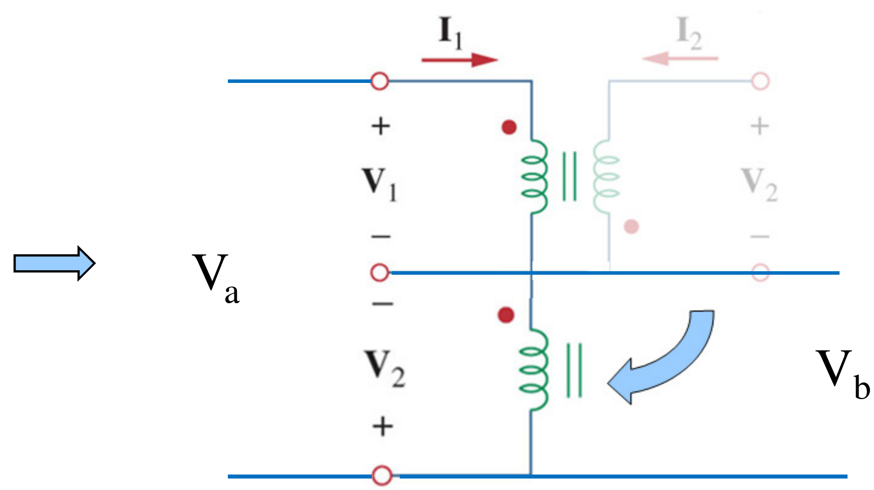


From earlier we know the following relationship

$$V_2 = -nV_1$$

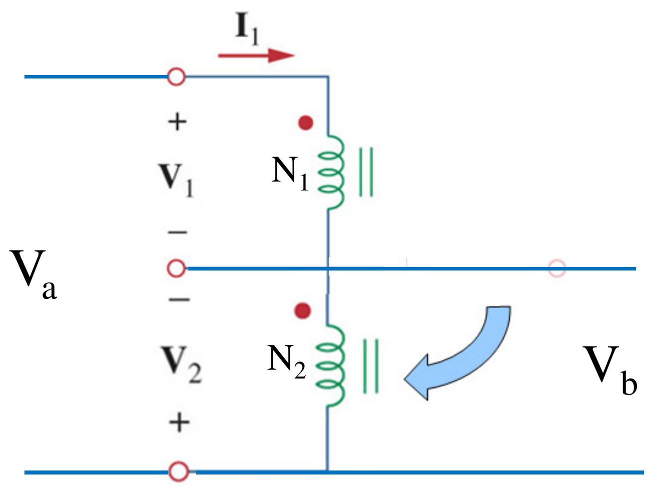
- If we flip the secondary side underneath the primary, we can create an autotransformer as shown





13.6 Ideal Auto-Transformers (3)

- Analysis of this circuit provides the following results:



Primary Side

$$V_a = V_1 - V_2$$

$$V_a = V_1 + nV_1$$

$$V_a = (1+n)V_1$$

Secondary Side

$$V_b = -V_2 = nV_1$$

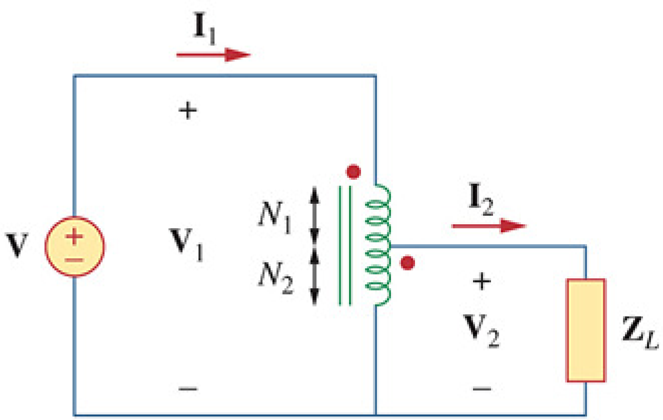
Notice, this looks like a Voltage Divider !

Ratio Primary/Secondary

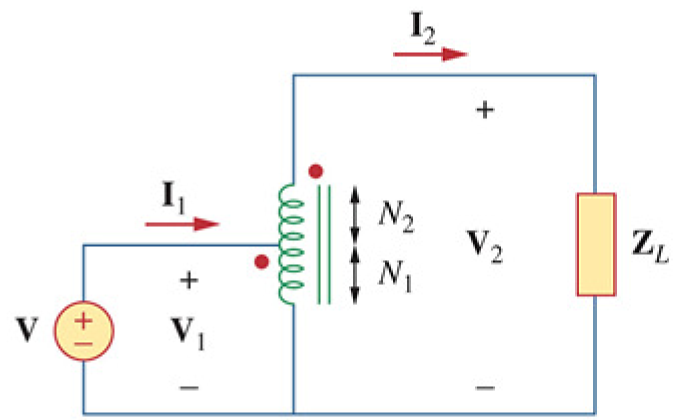
$$\frac{V_a}{V_b} = \frac{1+n}{n} = \frac{N_1+N_2}{N_2} \Rightarrow \boxed{V_b = \left(\frac{N_2}{N_1+N_2}\right)V_a}$$

13.6 Ideal Auto-Transformers (4)

- The voltage / current relationships for the lossless ideal autotransformer are as follows:



Step Down Auto-Transformer



Step Up Auto-Transformer

Similar to Voltage Divider equation

$$V_2 = \frac{N_2}{N_1 + N_2} V_1 \qquad V_2 = \frac{N_1 + N_2}{N_1} V_1$$

Inverse Relation

$$I_2 = \frac{N_1 + N_2}{N_2} I_1 \qquad I_2 = \frac{N_1}{N_1 + N_2} I_1$$

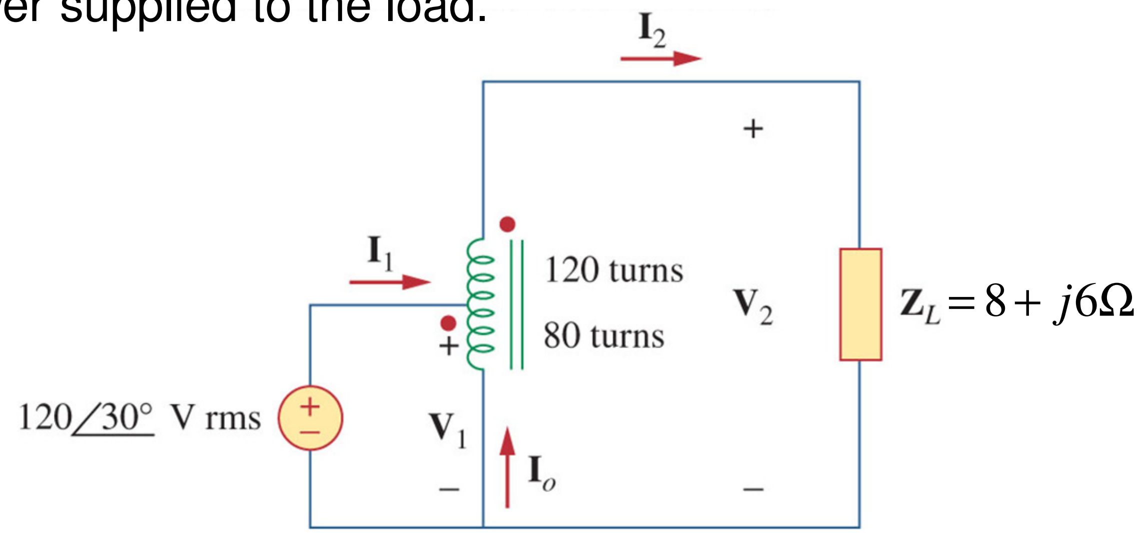
Derive from V/I

$$Z_{in} = \left(\frac{N_1 + N_2}{N_2}\right)^2 Z_L \qquad Z_{in} = \left(\frac{N_1}{N_1 + N_2}\right)^2 Z_L$$

13.6 Ideal Auto-Transformers (5)

Example 13.11 (Textbook)

For the autotransformer below, find: (a) the currents I_1 , I_2 , I_0, (b) the complex power supplied to the load.



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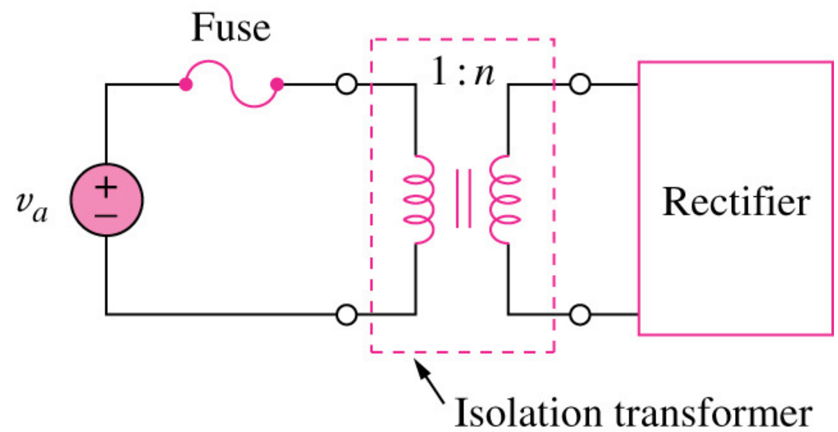
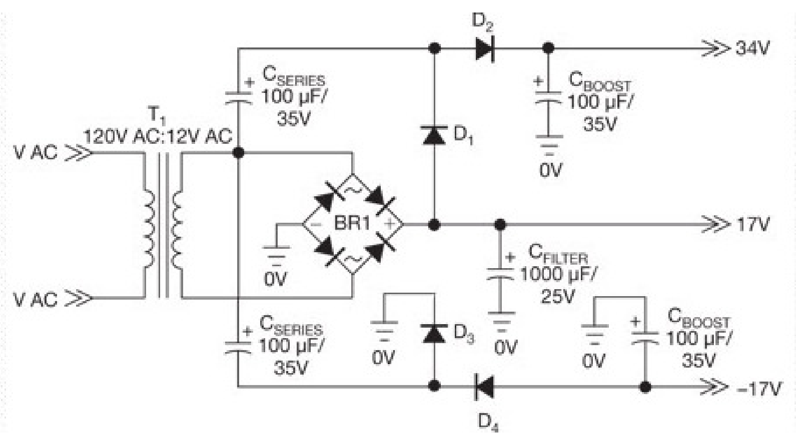
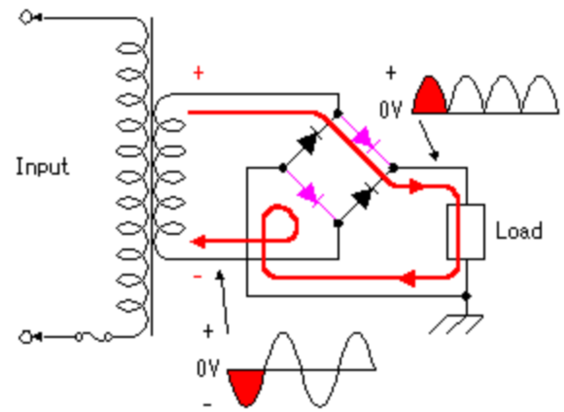
- Transformer as an Isolation Device to isolate ac supply from a rectifier

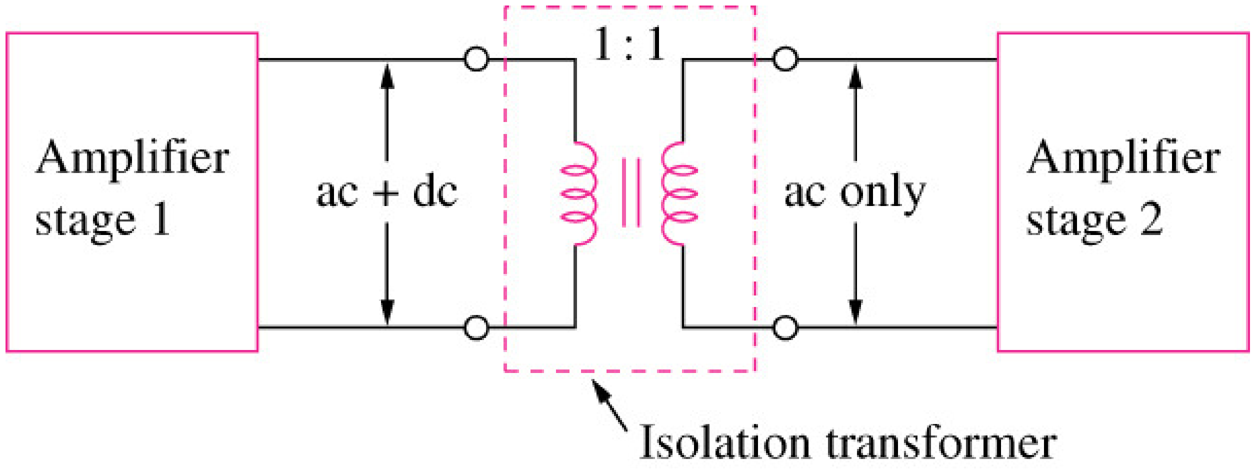






13.9 Applications (2)

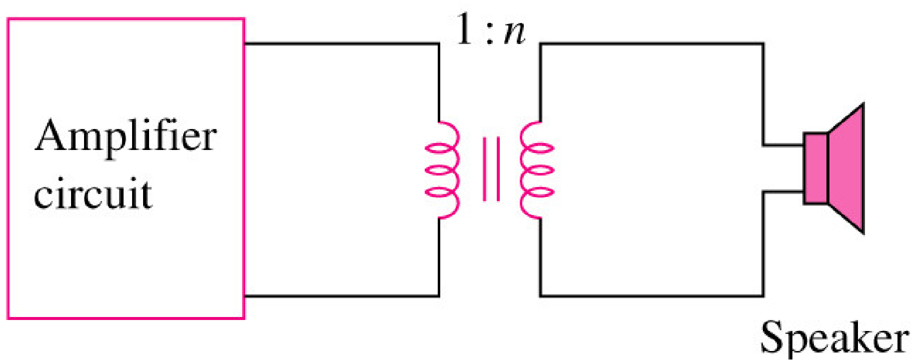
- Transformer as an Isolation Device to isolate dc between two amplifier stages.



- Biasing is the application of a DC voltage to a transistor amplifier to produce a desired mode of operation.
- Each amplifier stage can be biased separately to operate in a particular mode.

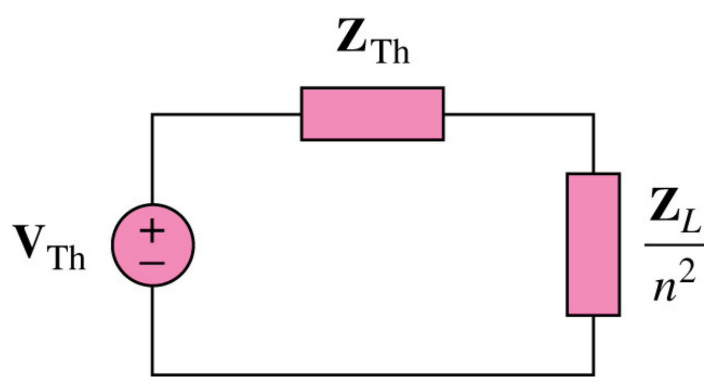
13.9 Applications (3)

- Transformer as a Matching Device



Using an ideal transformer to match the speaker to the amplifier

Equivalent circuit



13.9 Applications (4)

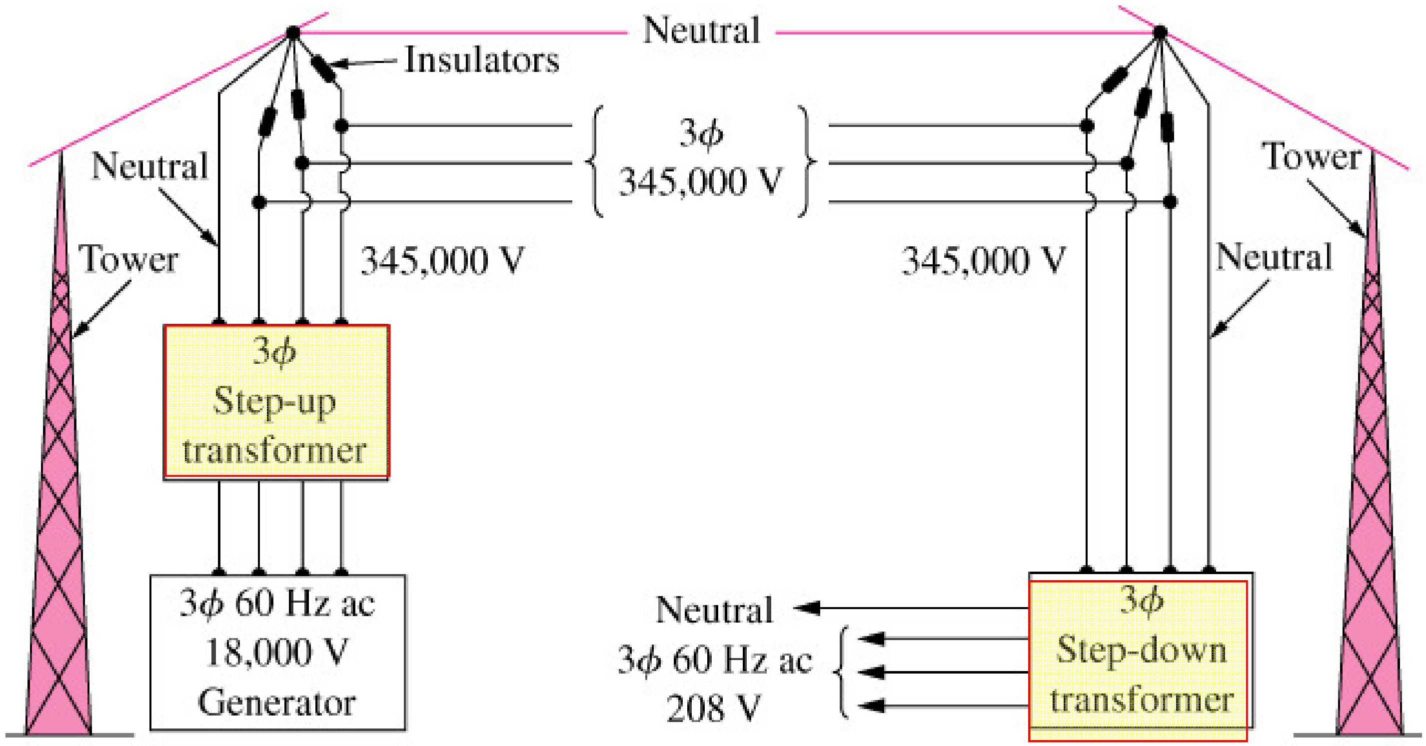
Practice Problem 13.16 (Textbook)

Calculate the turns ratio of an ideal transformer required to match a 400Ω load to a source with internal impedance of 2.5kΩ. Find the load voltage when the source voltage is 30V.

Ans: $n = 0.4$; $V_L = 6V$

13.9 Applications (5)

- A typical power distribution system



Homework #3

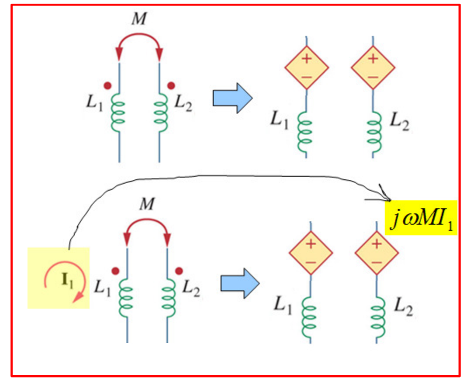
Due beginning of class Wednesday Feb 4, 2015

- 13.30
- 13.35
- 13.42
- 13.50
- 13.53 (modified)
- Autotransformer (See handout)

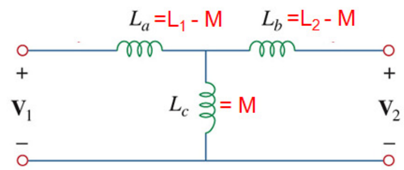
Exam over Chapter 13 on Monday Feb 9

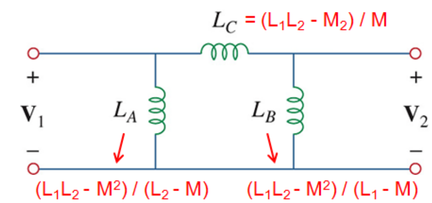
Chapter 13
Equation / Analysis Summary

- Series Aiding $L = L_1 + L_2 + 2M$ / Opposing $L = L_1 + L_2 - 2M$
- Dot Convention Model



- Coupling coefficient "k" $M = k\sqrt{L_1 L_2}$
- Linear Transformer
  - Input Impedance: $Z_{in} = Z_{primary} + \dfrac{\omega^2 M^2}{Z_{Secondary}}$
  - Reflected Impedance: $Z_{reflected} = \dfrac{\omega^2 M^2}{Z_{Secondary}}$
  - Equivalent T or π Circuits:

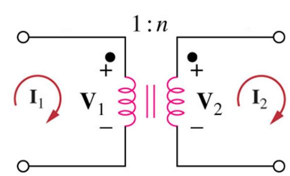




- Ideal Transformer

Turns Ratio
$n = \dfrac{N_2}{N_1}$

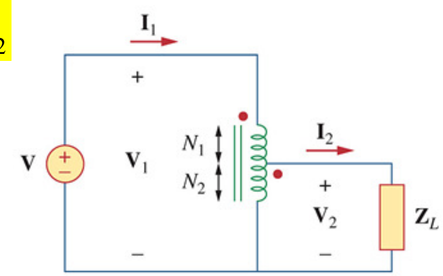
  - K = 1, L_1, L_2, → ∞
  - Lossless ($S_1 = S_2$)
  - Voltage / Current Relationship:
    - Dots "same" = +n, Dots "diff" = -n
  - Complex Power: $S_1 = V_1 I_1^* = V_2 I_2^* = S_2$



$V_2 = nV_1$ $\quad I_2 = \dfrac{I_1}{n}$ $\quad Z_{in} = \dfrac{Z_L}{n^2}$

$P_{ave} = |I_2|^2 R_L$

- Autotransformer
  - Adjustable "tap"
  - No electrical Isolation
  - Voltage Divider like relationship



$V_2 = \dfrac{N_2}{N_1 + N_2} V_1$

$I_2 = \dfrac{N_1 + N_2}{N_2} I_1$

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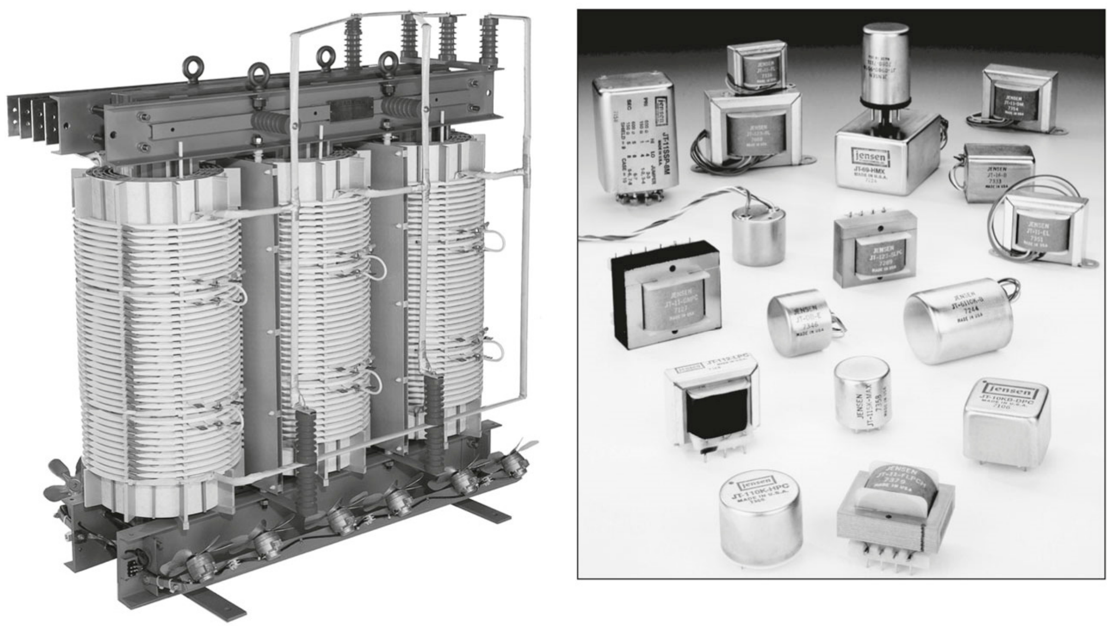
13.1 What is a transformer? (1)

- It is an electrical device designed on the basis of the concept of magnetic coupling
- It uses magnetically coupled coils to transfer energy from one circuit to another
- It is the key circuit elements for stepping up or stepping down ac voltages or currents, impedance matching, isolation, etc.

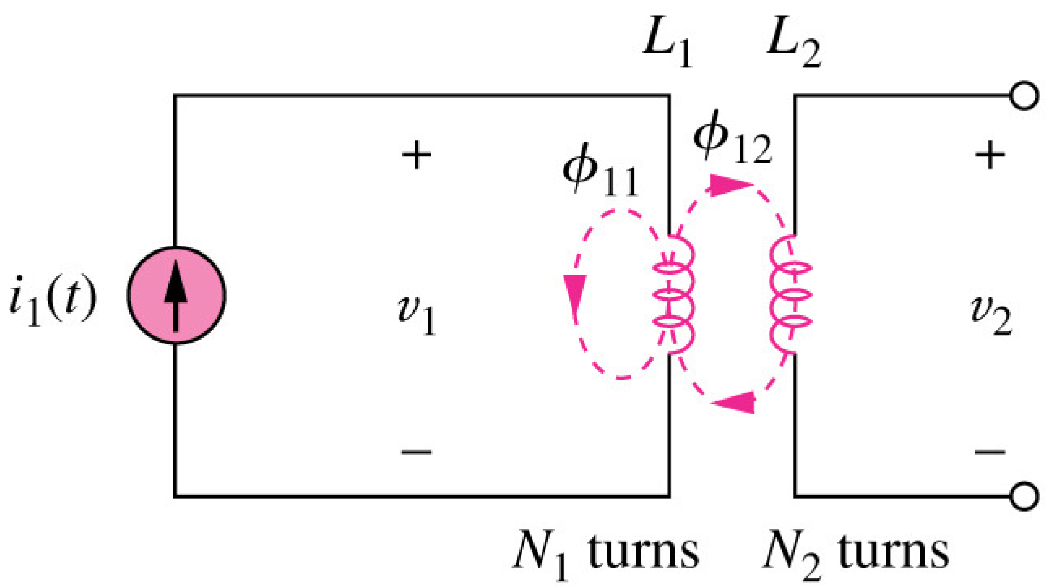


(a) (b)

Courtesy of: (a) Electric Service Co., (b) Jensen Transformers

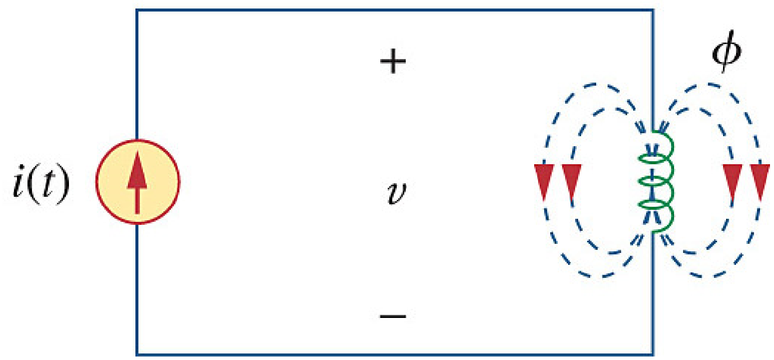
13.2 Mutual Inductance (1)

- Mutual Inductance
  - When two inductors (or coils) are in close proximity of each other, the magnetic flux caused by current in one coil links with the other coil, thereby inducing voltage in the latter.



13.2 Mutual Inductance (2)

- First consider a single inductor, a coil with N turns:



When current i flows through the coil, a magnetic flux Φ is produced around it.

- According to Faraday's law, the voltage v induced in the coil is proportional to the number of turns N and the time rate of change of magnetic flux Φ:

$$v = N\frac{d\phi}{dt}$$

13.2 Mutual Inductance (3)

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- Voltage induced in the coil given by:

$$v = N\frac{d\phi}{dt}$$

- But the flux Φ is produced by current i so that any change in Φ is caused by a change in the current:

$$v = N\frac{d\phi}{di}\frac{di}{dt}$$

- Recall the voltage-current relationship for an inductor:

$$v = L\frac{di}{dt}$$

- The "Self" inductance L of the inductor is thus given by:

$$\boxed{L = N\frac{d\phi}{di}}$$

- Self-inductance L relates the voltage induced in a coil by a time-varying current in the same coil.

13.2 Mutual Inductance (4)

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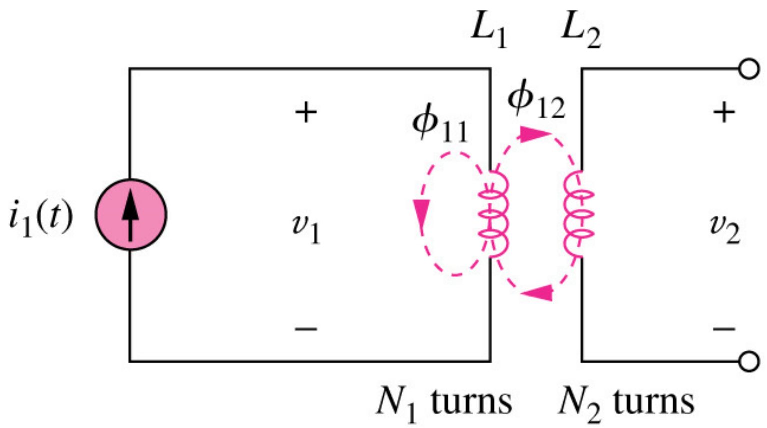
- Now consider two coils with self-inductances L_1 and L_2 that are in close proximity of each other:



Coil 1 has N_1 turns
Coil 2 has N_2 turns.

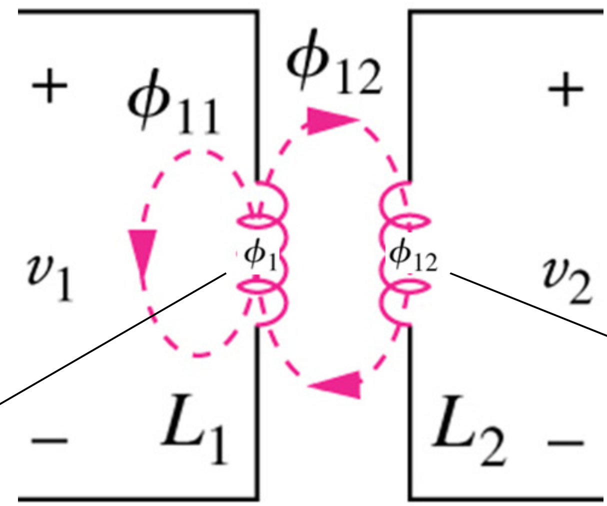
Assume coil 2 carries no current.

- The total magnetic flux Φ_1 emanating from coil 1 has two components:
  - Φ_{11} links only coil 1
  - Φ_{12} links both coils

$$\phi_1 = \phi_{11} + \phi_{12}$$

13.2 Mutual Inductance (5)

- Although the two coils are physically separated, they are magnetically coupled.
- The voltage induced in each coil is proportional to the flux in each coil.

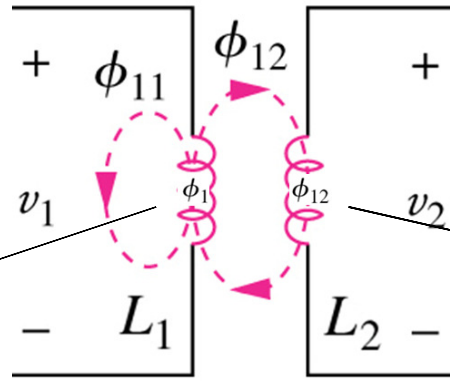


$$v_1 = N_1 \frac{d\phi_1}{dt}$$

$$v_2 = N_2 \frac{d\phi_{12}}{dt}$$

13.2 Mutual Inductance (6)

- The voltage equations can be rewritten as follows:



v_2 is the open-circuit mutual voltage (or induced voltage) across coil 2

$$v_1 = N_1 \frac{d\phi_1}{dt}$$

$$v_2 = N_2 \frac{d\phi_{12}}{dt}$$

$$v_1 = \underbrace{N_1 \frac{d\phi_1}{di_1}}_{L_1} \frac{di_1}{dt}$$

$$v_2 = \underbrace{N_2 \frac{d\phi_{12}}{di_1}}_{M_{21}} \frac{di_1}{dt}$$

$$v_1 = L_1 \frac{di_1}{dt}$$

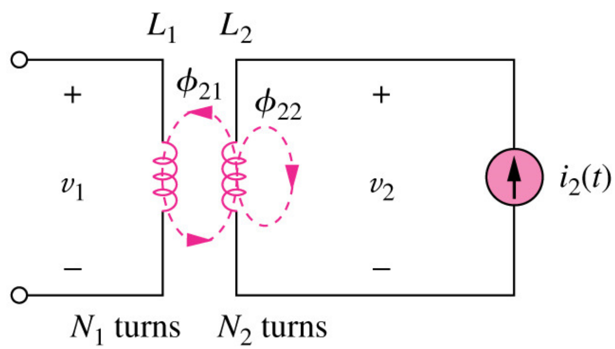
$$v_2 = M_{21} \frac{di_1}{dt}$$

L_1 is the <u>self-inductance</u> of coil 1

M_{21} is the <u>mutual inductance</u> of coil 2 with respect to coil 1

13.2 Mutual Inductance (7)

- Suppose now we let current i_2 flow in coil 2 while coil 1 carries no current:



- The magnetic flux Φ_2 emanating from coil 2 comprises flux Φ_{22} that links only coil 2 and flux Φ_{21} that links both coils:

$$\phi_2 = \phi_{21} + \phi_{22}$$

- The resulting symmetry is true:

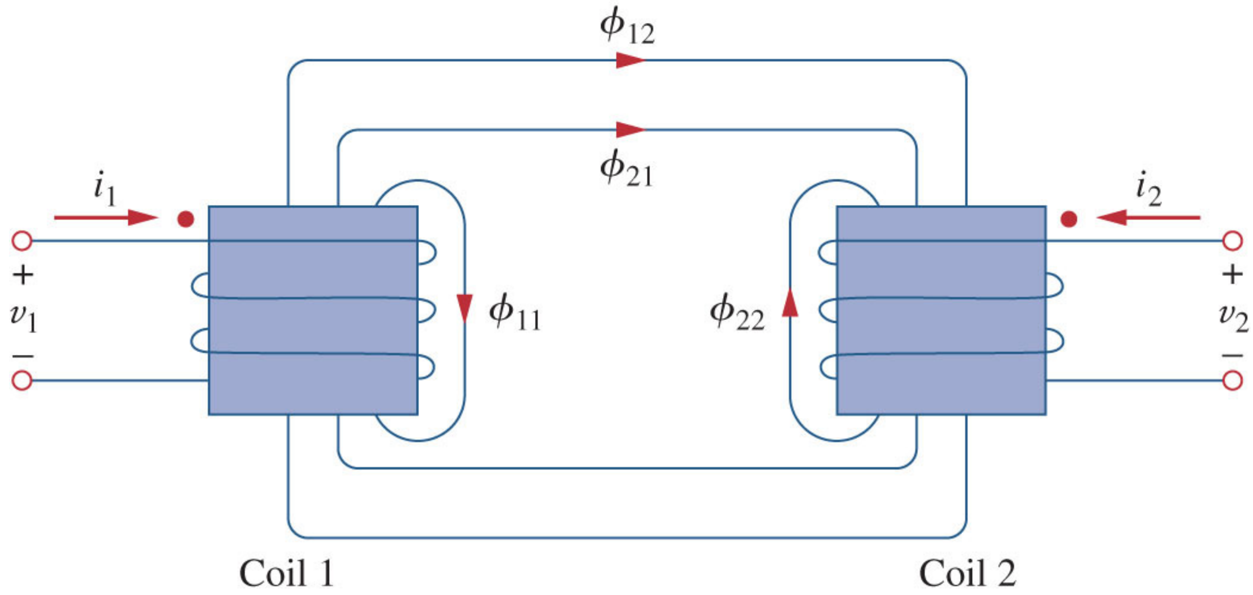
$$v_2 = L_2 \frac{d\,i_2}{d\,t} \qquad v_1 = M_{12} \frac{d\,i_2}{d\,t} \qquad M_{12} = N_1 \frac{d\,\phi_{21}}{d\,i_2}$$

13.2 Mutual Inductance (8)

- $M_{12} = M_{21} = M$; The “Mutual inductance” between the coils
- Mutual inductance M is measured in Henrys (just like inductors)
- Mutual inductance only exists when inductors or coils are in close proximity and the circuits are driven by time-varying sources
- Although mutual inductance M is always a positive quantity, the mutual voltage M di/dt may be negative or positive, just like the self-induced voltage L di/dt (determined by “Dot” convention)

13.2 Mutual Inductance (9)

- Self-induced voltage polarity is determined by the reference direction of the current and the reference polarity of the voltage,
- The polarity of the mutual voltage is not as easy to determine (depends on winding direction of the coils).
- We use the dot convention to determine



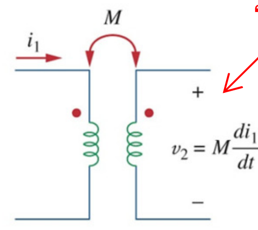
13.2 Mutual Inductance (10)

“Dot Convention”

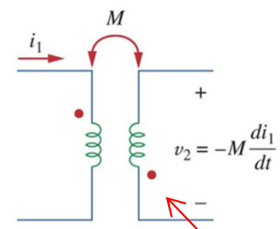
- If a current **enters** the dotted terminal of one coil, the reference polarity of the mutual voltage in the second coil is **positive** at the dotted terminal of the second coil.

Enters the “Dot”

Voltage is “Positive”



(a)

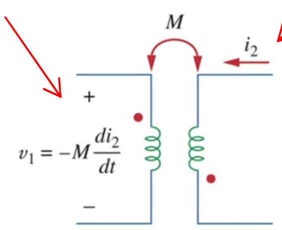


(b)

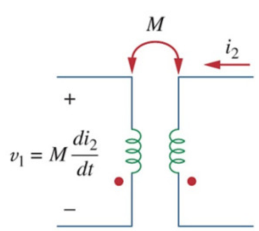
Voltage is “Positive” **At “Dotted” Terminal**

Voltage is “Negative”

Enters the “Un-Dotted” side



(c)



(d)

- Alternatively, if a current **leaves** the dotted terminal of one coil, the reference polarity of the mutual voltage in the second coil is **negative** at the dotted terminal of the second coil.

13.2 Mutual Inductance (11)

"The Model" (Phasor domain)

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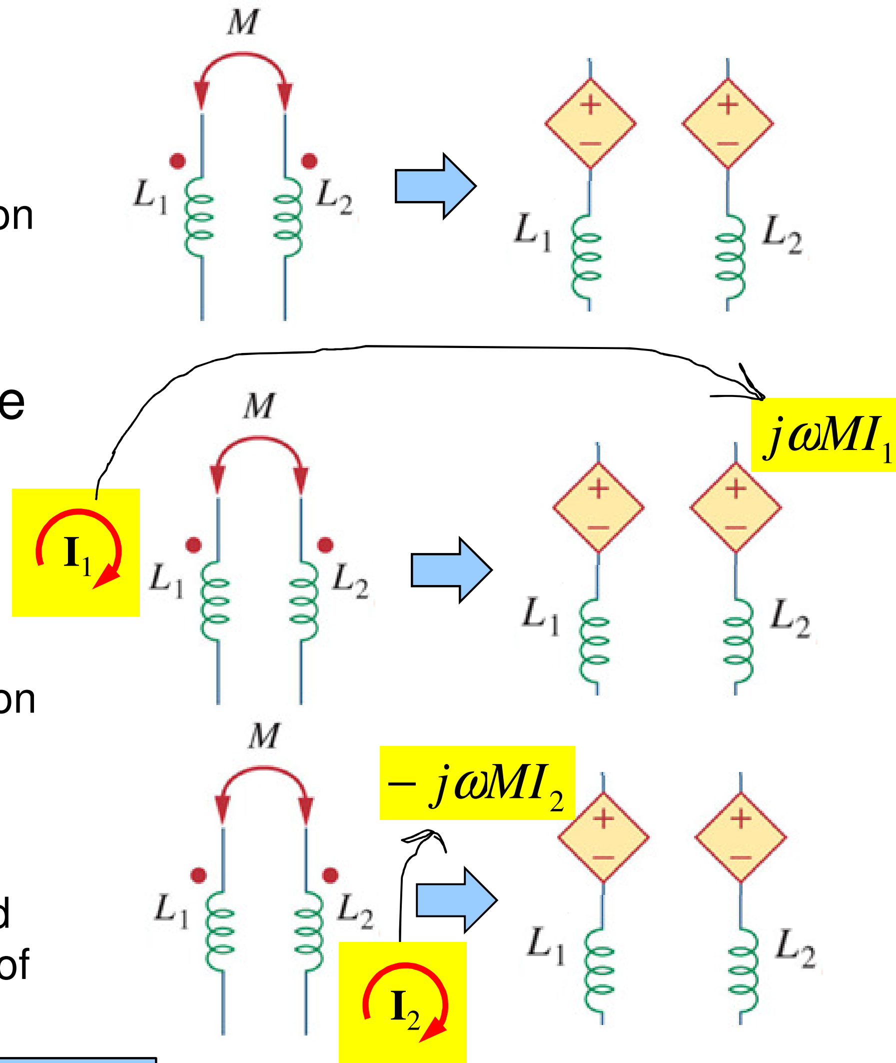
- Replace the "Dot" with a controlled voltage source.
  - Place on same side as the Dot
  - Positive terminal in same direction as the Dot
- Now look at the current entering each dot to determine the magnitude of the voltage source
  - If the current enters the dotted terminal, it will induce a positive voltage in the controlled source on the opposite terminal of jωM(I)
  - If the current enters the "un-dotted" terminal, it will induce a negative voltage in the controlled source on the opposite terminal of –jωM(I)

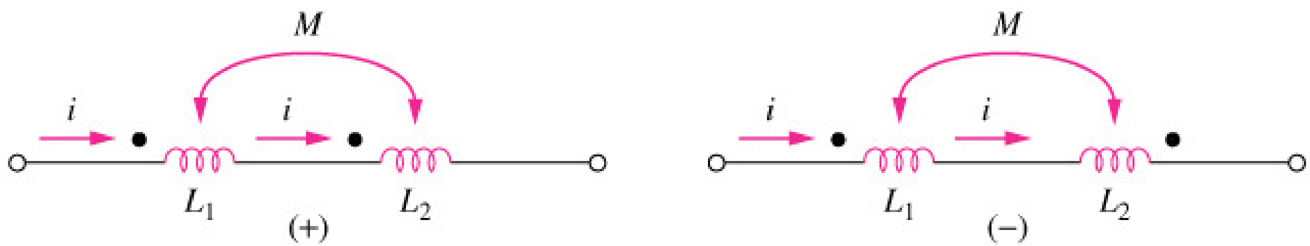


Note: This method differs slightly from the text

KEY CONCEPT !

13.2 Mutual Inductance (12)

Dot convention for coils in series; the sign indicates the polarity of the mutual voltage



$$L = L_1 + L_2 + 2M$$

(series - aiding connection)

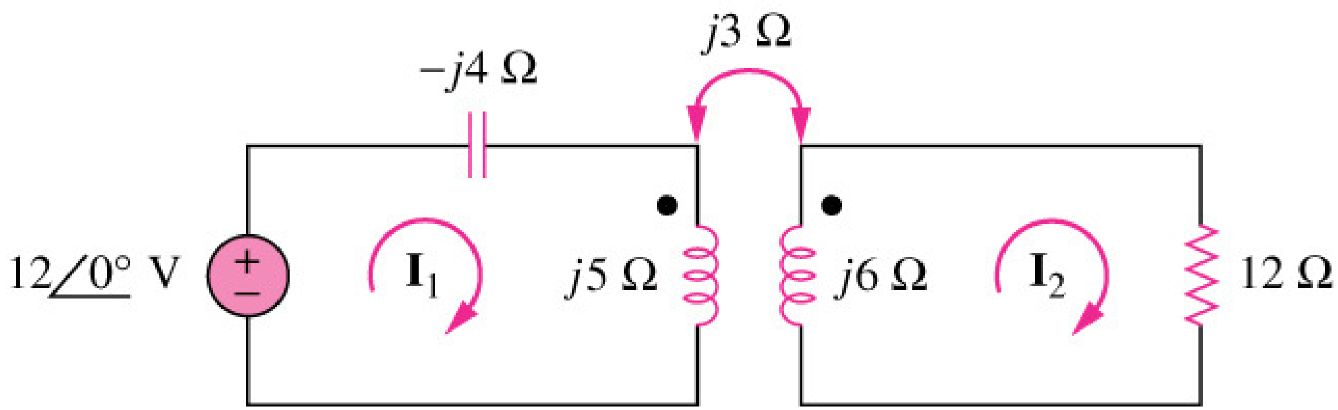
$$L = L_1 + L_2 - 2M$$

(series - opposing connection)

We can show this using the model!

Example 13.1 (Textbook)

Calculate the phasor currents I_1 and I_2 in the circuit shown below.



Ans: $I_1 = 13.02\angle -49.40°\text{A}; \quad I_2 = 2.91\angle 14.04°\text{A}$

13.2 Mutual Inductance (14)

Example Problem 13.1

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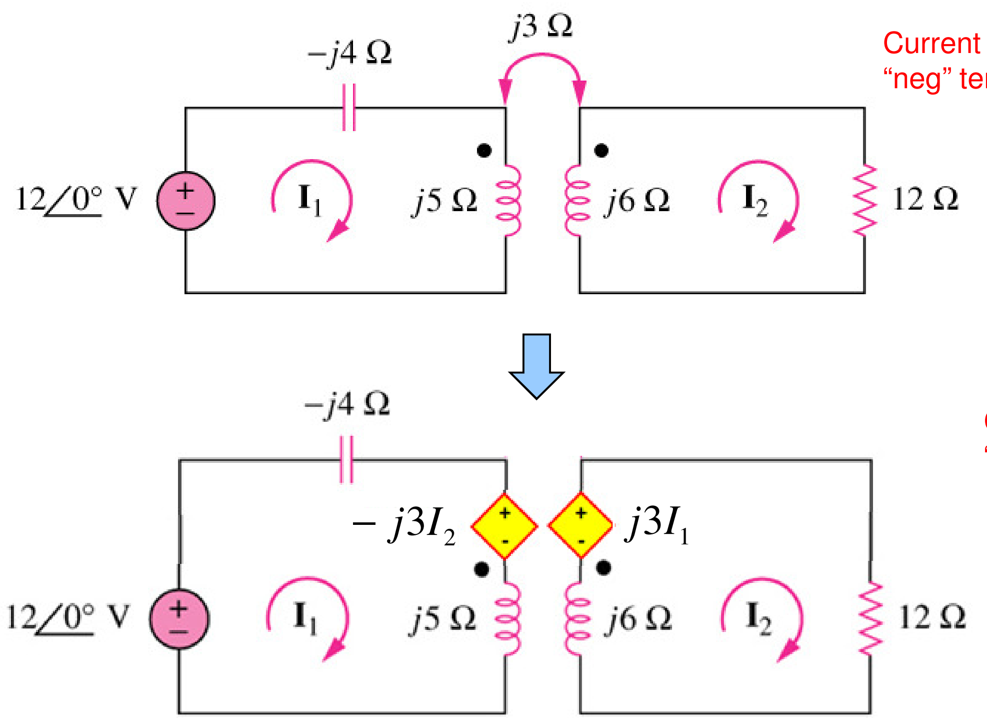
First, replace with the model:



Then, solve Loop Equations:

Current enters "neg" terminal

Current enters "+" terminal but value is $-j3I_2$

Loop I_1

$$-12 - j4I_1 + (-j3I_2) + j5I_1 = 0$$

$$jI_1 - j3I_2 = 12$$

Current enters "neg" terminal

Loop I_2

$$j6I_2 - j3I_1 + 12I_2 = 0$$

$$-j3I_1 + (12 + j6)I_2 = 0$$

NOTE:

I_1 goes "into" the dot → Induced voltage on the second coil is "Positive"

I_2 goes "into" the un-dotted side → Induced voltage on the first coil is "Negative"

Pay Attention to Sign Conventions !

13.2 Mutual Inductance (15)

Example Problem 13.1

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Lastly, solve 2 equations, 2 unknowns (expected you know this):

Loop I_1 $\longrightarrow$ $jI_1 - j3I_2 = 12$ $\longrightarrow$ $jI_1 = 12 + j3I_2 \Rightarrow I_1 = 3I_2 - j12$

Loop I_2 $\longrightarrow$ $-j3I_1 + (12 + j6)I_2 = 0$

Substitution: $-3(12 + j3I_2) + (12 + j6)I_2 = 0$

÷ by 3 $-12 - j3I_2 + (4 + j2)I_2 = 0$

Collect Terms: $(4 - j)I_2 = 12$

Solve for I_2 $$I_2 = \frac{12}{(4 - j)} = 2.824 + j0.706 = \boxed{2.91\angle 14.04° \text{ A}}$$

Find I_1 (Substitute back to Eq1)

$$I_1 = 3(2.824 + j0.706) - j12$$
$$I_1 = 8.471 + j2.118 - j12$$
$$I_1 = 8.471 - j9.882 = \boxed{13.02\angle -49.40° \text{ A}}$$

Think about where we could have made mistakes!

- Applying the model incorrectly (wrong sign convention)
- Incorrect Phasor notation (Understand: jωM ↔ j3 ; jωL ↔ j5 ; 1/ jωC ↔ -j4)
- Incorrect sign convention for loop equations
  - Current entering negative terminal of a voltage source
- Understanding of complex numbers!
  - Familiarity with your calculators handling of complex numbers
  - Converting from rectangular to polar

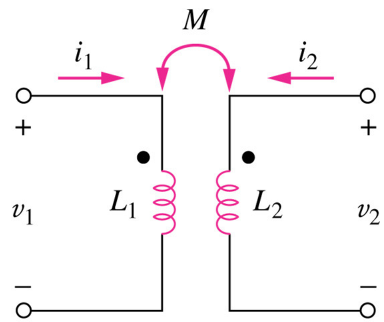
Review / Understand these concepts to avoid these mistakes

13.2 Recommended Viewing:

- Watch these videos illustrating solving mutual inductance problems:
  - http://www.youtube.com/watch?v=tD35a-uzd34
  - http://www.youtube.com/watch?v=hzU4XKQYTWw
  - http://www.youtube.com/watch?v=OqSvesTtnUo
- Uses the model described previously to solve mutual inductance problems.

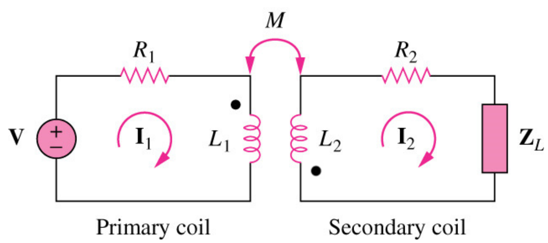
13.3 Energy in a Coupled Circuit (1)

- The instantaneous energy w stored in the circuit is:



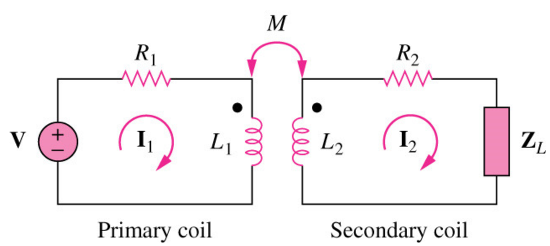
$$\boxed{w = \frac{1}{2}L_1 i_1^2 + \frac{1}{2}L_2 i_2^2 \pm M i_1 i_2}$$

- If **both** currents enter (or **both** leave) the dotted terminal the mutual term is (positive):



$$w = \frac{1}{2}L_1 i_1^2 + \frac{1}{2}L_2 i_2^2 + M i_1 i_2$$

- Otherwise the mutual term is (negative):



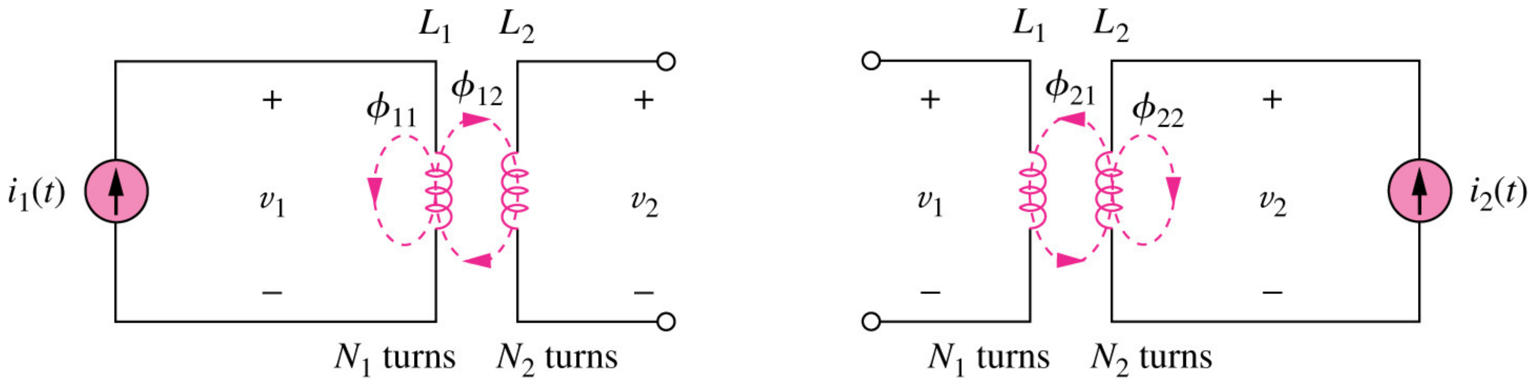
$$w = \frac{1}{2}L_1 i_1^2 + \frac{1}{2}L_2 i_2^2 - M i_1 i_2$$

13.3 Energy in a Coupled Circuit (2)

- The coupling coefficient, k, is a measure of the magnetic coupling between two coils; 0 ≤ k ≤ 1.

$$M = k\sqrt{L_1 L_2}$$

K = 1 coils perfectly coupled
K < 0.5 coils loosely coupled
K > 0.5 coils tightly coupled

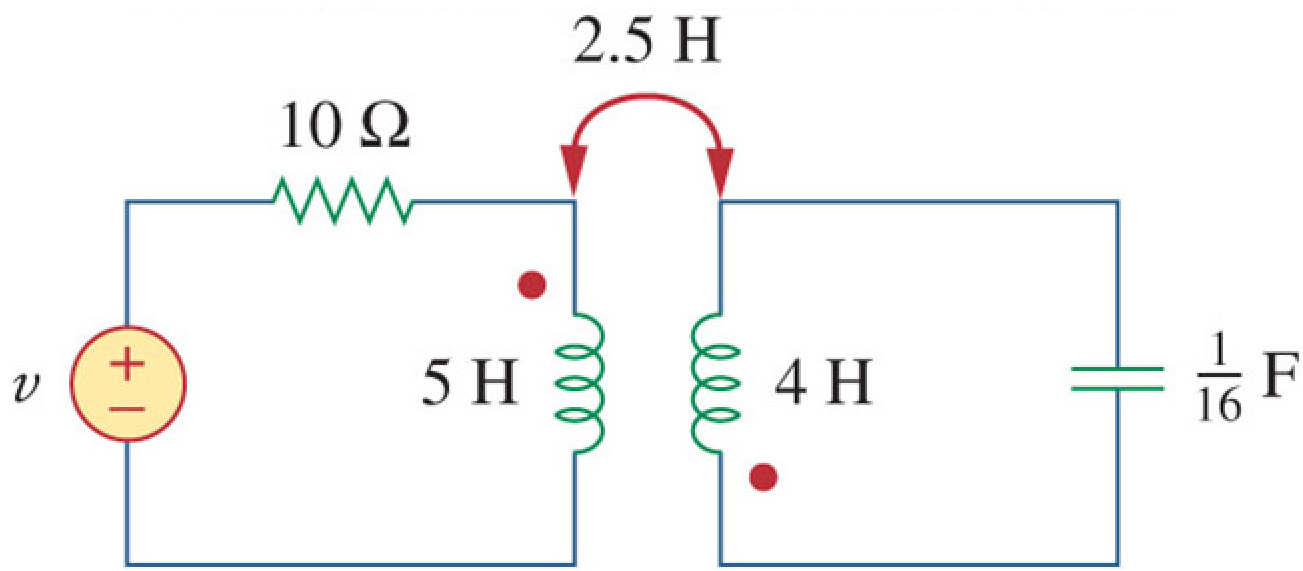


$$K = \frac{\phi_{12}}{\phi_{11} + \phi_{12}}$$

$$K = \frac{\phi_{21}}{\phi_{21} + \phi_{22}}$$

Example 13.3 (Textbook)

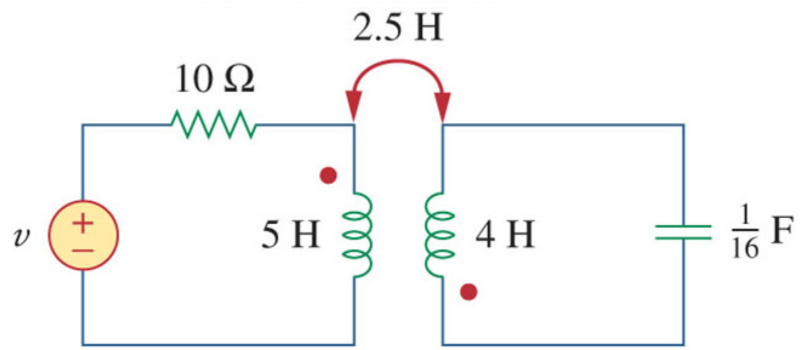
Consider the circuit below. Determine the coupling coefficient. Calculate the energy stored in the coupled inductors at time t = 1s if $v=60\cos(4t+30°)$ V.



Ans: k=0.56; $w(1)$=20.73J

13.3 Energy in a Coupled Circuit (4)

Example 13.3

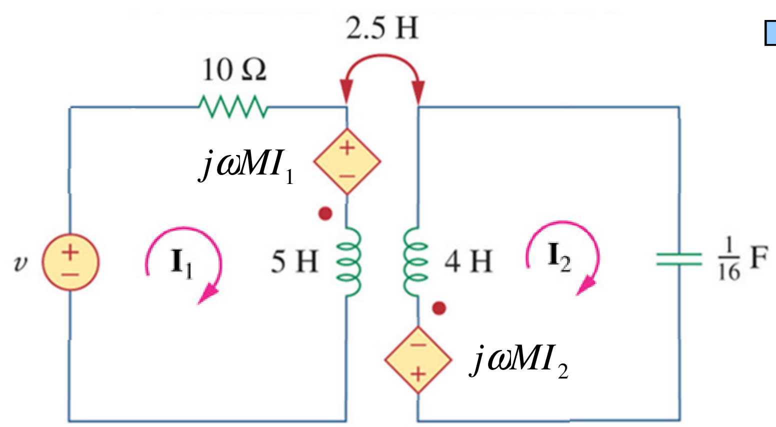


To find k, use the relation between L_1, L_2, and M:

$$L_1 = 5 \ ; \ L_2 = 4 \ ; \ M = 2.5$$

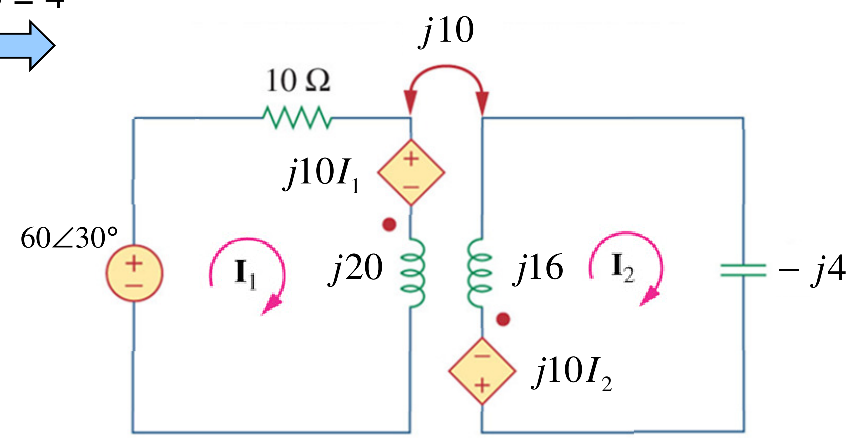
$$k = \frac{M}{\sqrt{L_1 L_2}} = \frac{2.5}{\sqrt{(5)(4)}} = 0.56$$

Apply the "Model"



Convert to Phasor

ω = 4



Solve Mesh equations for I_1 and I_2

13.3 Energy in a Coupled Circuit (5)

Example 13.3

From Mesh analysis:

$$I_1 = 3.905\angle -19.4^\circ \ \text{A}$$
$$I_2 = 3.254\angle 160.6^\circ \ \text{A}$$

At t = 1, the value of ωt = (4)(1) = 4 radians = 229.2º

To find the energy at t = 1:

$$i_1 = 3.905\cos(\omega t - 19.4^\circ)$$
$$i_2 = 3.254\cos(\omega t + 160.6^\circ)$$

$$i_1 = 3.905\cos(229.2^\circ - 19.4^\circ) = -3.389$$
$$i_2 = 3.254\cos(229.2^\circ + 160.6^\circ) = 2.824$$

Positive since current (as defined) enters **both** dots

$$w = \frac{1}{2}L_1 i_1^2 + \frac{1}{2}L_2 i_2^2 + M i_1 i_2$$

$$w = \frac{1}{2}(5)(-3.389)^2 + \frac{1}{2}(4)(2.824)^2 + 2.5(-3.389)(2.824) = 20.73 \ \text{J}$$

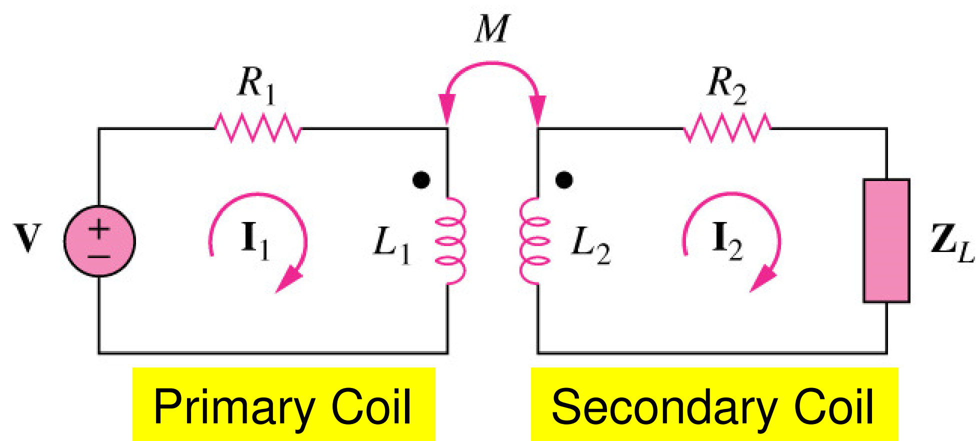
Homework #2

Due in class Monday, January 26, 2015

- 13.1
- 13.7
- 13.9
- 13.24

13.4 Linear Transformers (1)

- A transformer is generally a four-terminal device comprising two (or more) magnetically coupled coils. Below is a “simple” model:

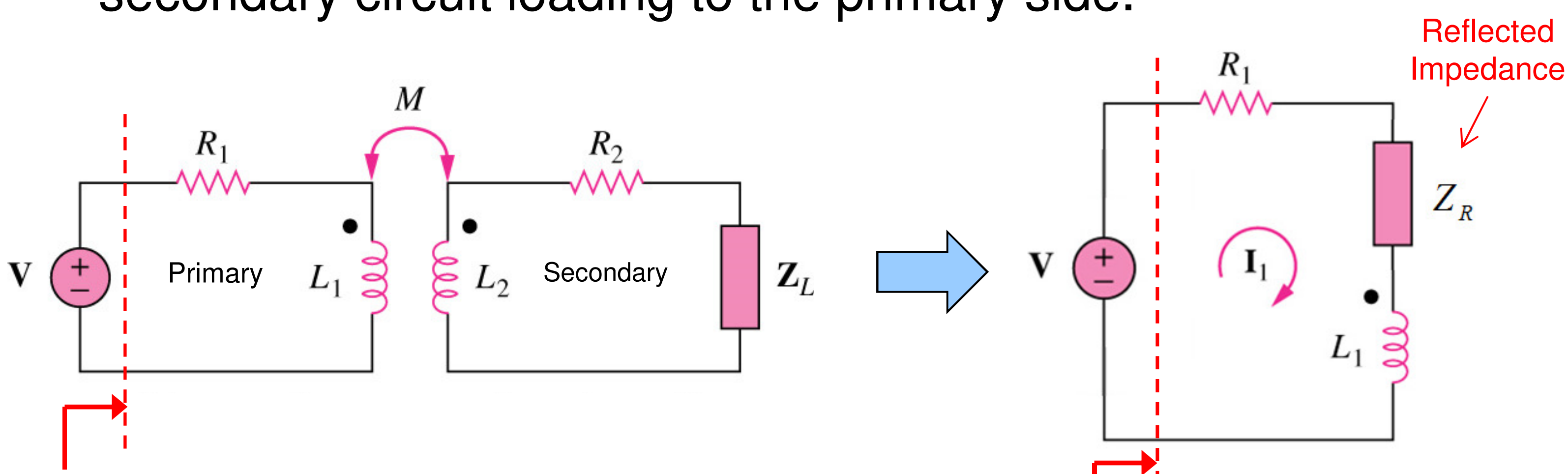


- The coil connected to the voltage source is called the primary winding.
- The coil connected to the load is called the secondary winding.
- Resistances R_1 and R_2 are included to account for the losses in the coils.
- A transformer is said to be linear if the coils are wound on a magnetically linear material for which the permeability is constant.
  - Air, plastic, Bakelite, wood, etc.
  - Most materials are magnetically linear.

13.4 Linear Transformers (2)

Reflected Impedance

- Often we are interested in the input impedance Z_{in} seen by the source.
  - For example, may want to match Z_{in} to the source impedance for maximum power transfer!
- To simplify our analysis we can break Z_{in} up into:
  - $Z_{primary}$ -- The impedance of the "primary" circuit $\left(Z_{primary} = R_1 + j\omega L_1\right)$
  - Z_R -- The "reflected" impedance back to the "primary".
- The "reflected" impedance Z_R is the contribution of the secondary circuit loading to the primary side.



$$Z_{in} = Z_{primary} + Z_R$$

$$Z_{in} = Z_{primary} + Z_R$$

13.4 Linear Transformers (2)

Reflected Impedance

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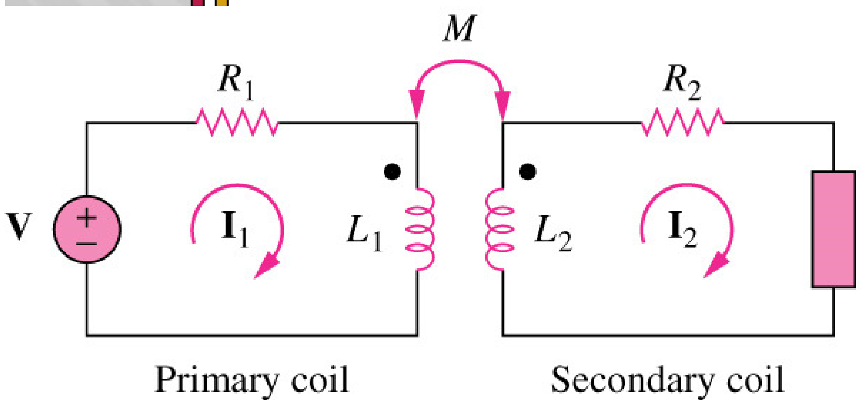
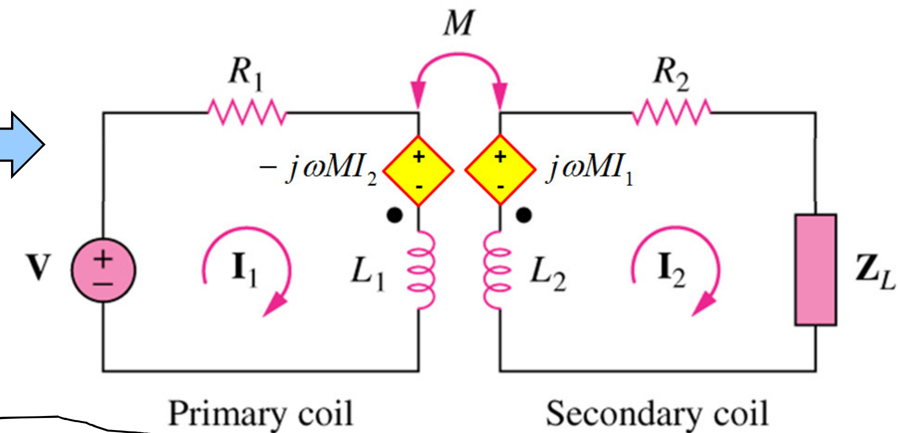
To obtain the input impedance Z_{in} as seen from the source:





Mesh 1: $V = (R_1 + j\omega L_1)I_1 - j\omega M I_2$

Current enters positive terminal so it is **+** ($-j\omega MI_2$)

Mesh 2: $0 = -j\omega M I_1 + (R_2 + j\omega L_2 + Z_L)I_2$

Current enters negative terminal so it is **-** ($j\omega MI_2$)

From Mesh 2:

$$I_2 = \frac{j\omega M}{R_2 + j\omega L_2 + Z_L} I_1$$

Substituting into Mesh 1 gives:

$$V = (R_1 + j\omega L_1)I_1 + \frac{\omega^2 M^2}{R_2 + j\omega L_2 + Z_L} I_1$$

$$V = Z_{primary}\, I_1 + Z_{reflected}\, I_1$$

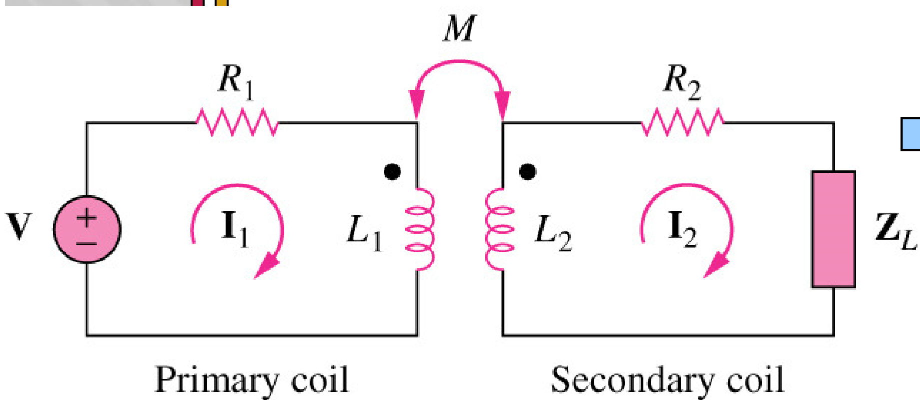
Reflected Impedance

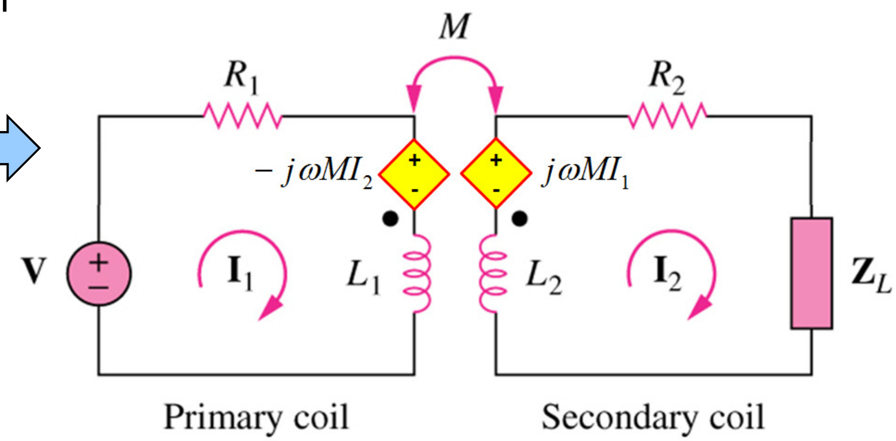
$$\boxed{Z_R = \frac{\omega^2 M^2}{R_2 + j\omega L_2 + Z_L}}$$

13.4 Linear Transformers (3)

Reflected Impedance (Another way of looking at it !)

Obtain input impedance Z_{in} as seen from the source:





Mesh 1: $V = \left(Z_{primary}\right)I_1 - j\omega M I_2$

Mesh 2: $0 = -j\omega M I_1 + (Z_{Secondary})I_2$

$Z_{Secondary}$ = the total series impedance in the secondary loop

From Mesh 2:

$$\mathrm{I}_2 = \frac{j\omega M}{Z_{Secondary}}\mathrm{I}_1$$

Substituting into Mesh 1 gives:

$$V = \underbrace{Z_{primary}} I_1 + \underbrace{\frac{\omega^2 M^2}{Z_{Secondary}}} I_1$$

$$V = Z_{primary} I_1 + Z_{reflected} I_1$$

Reflected Impedance

$$Z_R = \frac{\omega^2 M^2}{Z_{Secondary}}$$

13.4 Linear Transformers (4)
Reflected Impedance

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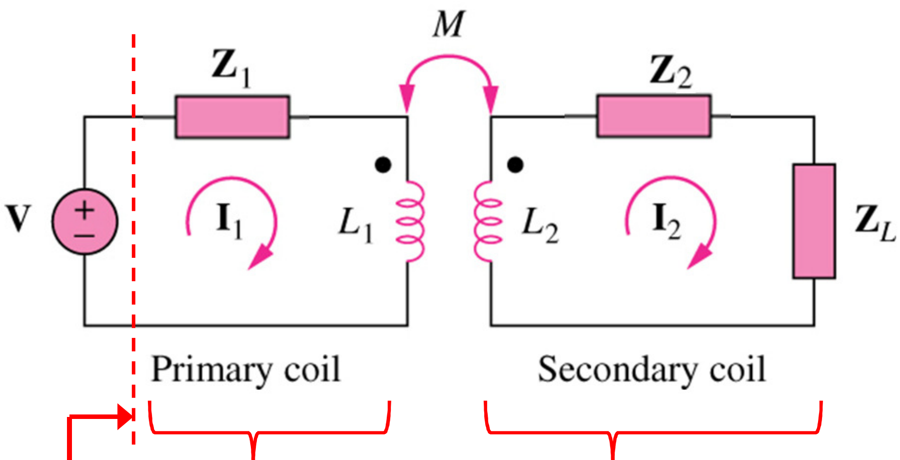
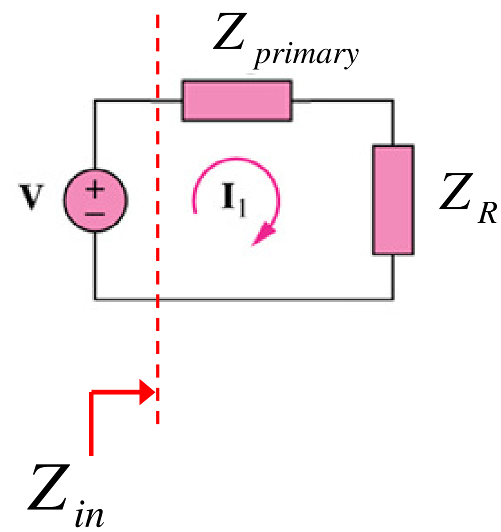
- The input impedance can be broken into two parts as follows:



$$Z_{in} = \underbrace{Z_{primary}}_{\text{Primary coil}} + \underbrace{Z_R}_{\text{Secondary coil}}$$



- Series Impedance in Primary Coil: $Z_{primary} = Z_1 + j\omega L_1$

- Series Impedance in Secondary Coil: $Z_{secondary} = Z_2 + Z_L + j\omega L_2$

- Input Impedance: $\boxed{Z_{in} = Z_{primary} + \dfrac{\omega^2 M^2}{Z_{Secondary}}}$

Note: Z_{in} will be the same if the dot on L2 is switched

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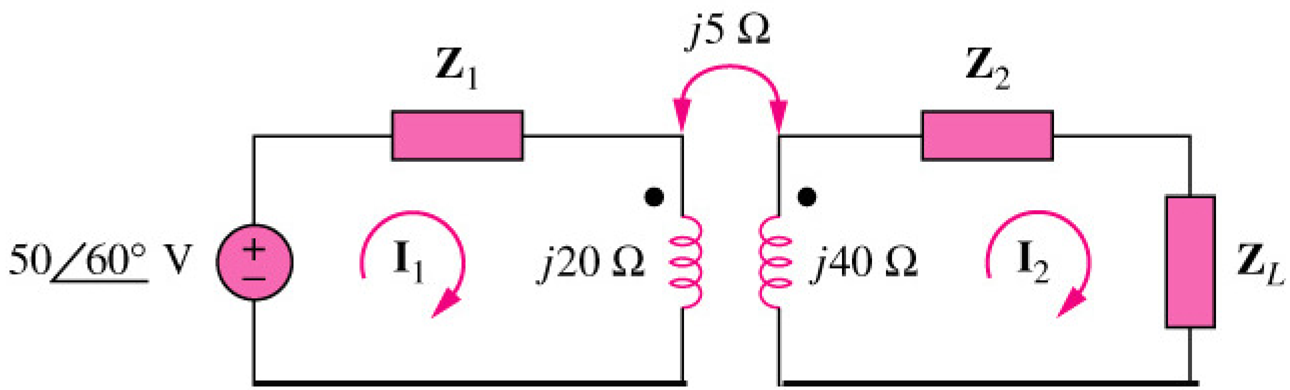
Example 13.4 (textbook)

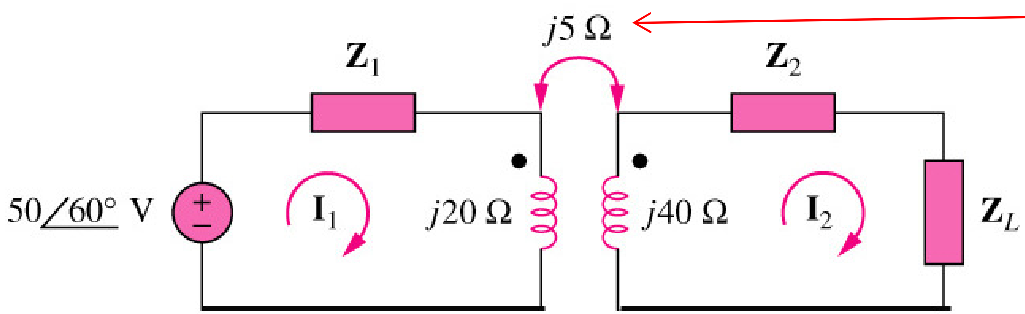
In the circuit below, calculate the input impedance and current I_1. Take Z_1=60-j100Ω, Z_2=30+j40Ω, and Z_L=80+j60Ω.



Ans: $Z_{in} = 100.14\angle -53.1°\Omega;\ I_1 = 0.5\angle 113.1°A$

13.4 Linear Transformers (5)

Example 13.4



Note:

$$j\omega M = j5$$
$$\omega M = 5$$
$$(\omega M)^2 = 25$$

- The series impedance in the primary coil:

$$Z_{primary} = (60 - j100) + j20 = 60 - j80$$

- The series impedance in the primary coil:

$$Z_{secondary} = (30 + j40) + (80 + j60) + j40 = 110 + j140$$

- Input Impedance given by:

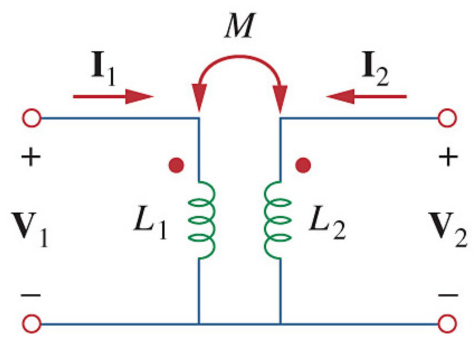
$$Z_{in} = Z_{primary} + \frac{\omega^2 M^2}{Z_{Secondary}} = (60 - 80j) + \frac{25}{(110 + j140)} = 60.09 - j80.11$$

- Current I_1 given by: $I_1 = \frac{V_s}{Z_{in}} = \frac{50\angle 60°}{60.09 - j80.11} = \frac{50\angle 60°}{100.14\angle -53.1°}$

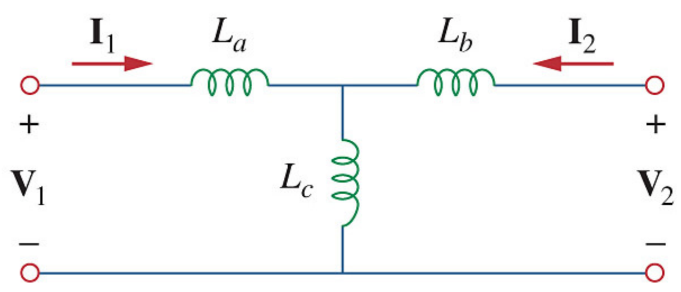
13.4 Linear Transformers (6)

Equivalent T and π Circuits:

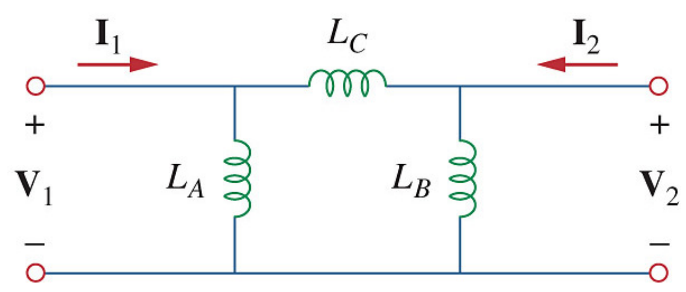
- It is sometimes convenient to replace a magnetically couple circuit with an equivalent circuit with no magnetic coupling.
- We can replace the linear transformer with an equivalent T or π circuit that has no mutual inductance.



T Circuit



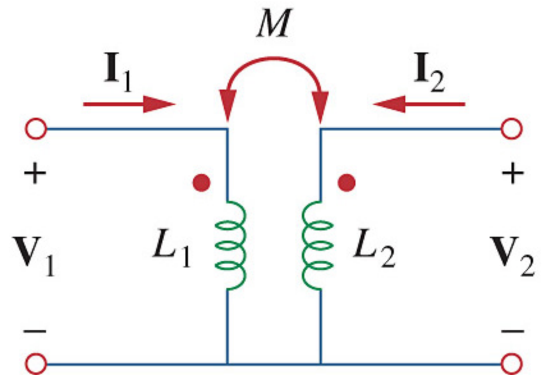
π Circuit



13.4 Linear Transformers (7)

Equivalent T Circuit:

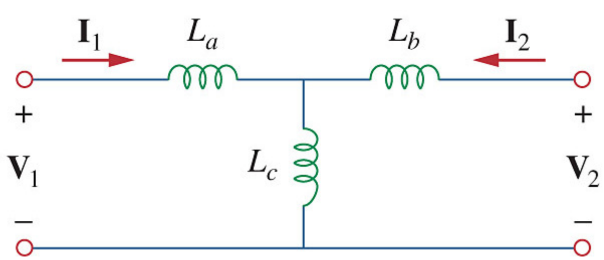
Linear Transformer Circuit



Mesh Analysis of this transformer circuit results in the following:

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} j\omega L_1 & j\omega M \\ j\omega M & jwL_2 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

Equivalent T Network



Mesh Analysis of the equivalent T network results in the following:

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} j\omega(L_a + L_c) & jwL_c \\ jwL_c & jw(L_b + L_c) \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

Equating the terms gives the following relationships:

$L_a = L_1 – M$ $\quad$ $L_b = L_2 – M$ $\quad$ $L_c = M$

13.4 Linear Transformers (8)

Equivalent T Circuits (Swapped Dots):

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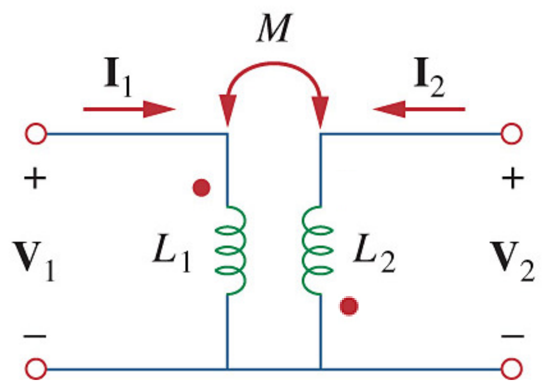
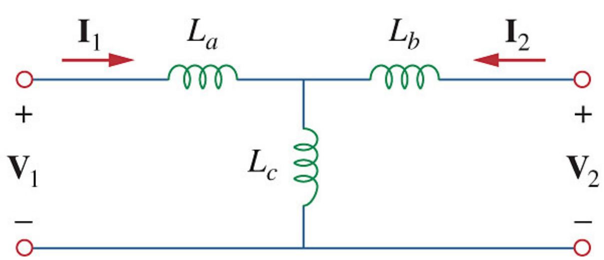
Linear Transformer Circuit



Mesh Analysis of this transformer circuit results in the following:

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} j\omega L_1 & -j\omega M \\ -j\omega M & jwL_2 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

Equivalent T Network



Mesh Analysis of the equivalent T network results in the following:

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} j\omega(L_a + L_c) & jwL_c \\ jwL_c & jw(L_b + L_c) \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

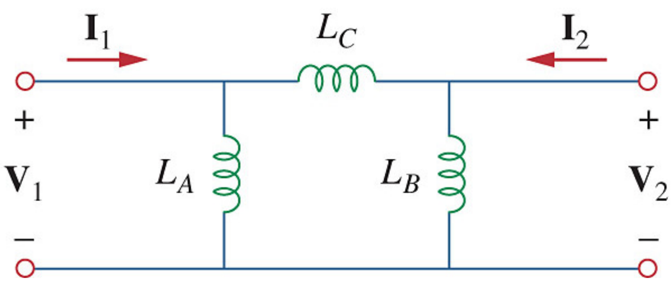
Equating the terms gives the following relationships:

$L_a = L_1 + M$ $\qquad$ $L_b = L_2 + M$ $\qquad$ $L_c = -M$

13.4 Linear Transformers (9)

Equivalent π Circuit:

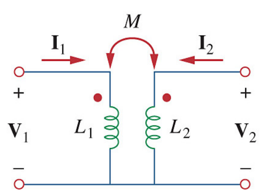
Similarly, for the π network nodal analysis provides:



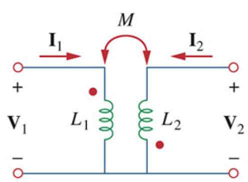
Equivalent π Network

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} \frac{1}{j\omega L_A} + \frac{1}{j\omega L_C} & -\frac{1}{j\omega L_C} \\ -\frac{1}{j\omega L_C} & \frac{1}{j\omega L_B} + \frac{1}{j\omega L_C} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

By equating terms in admittance matrices, for the π equivalent network we obtain (note if dots are different, replace with –M):



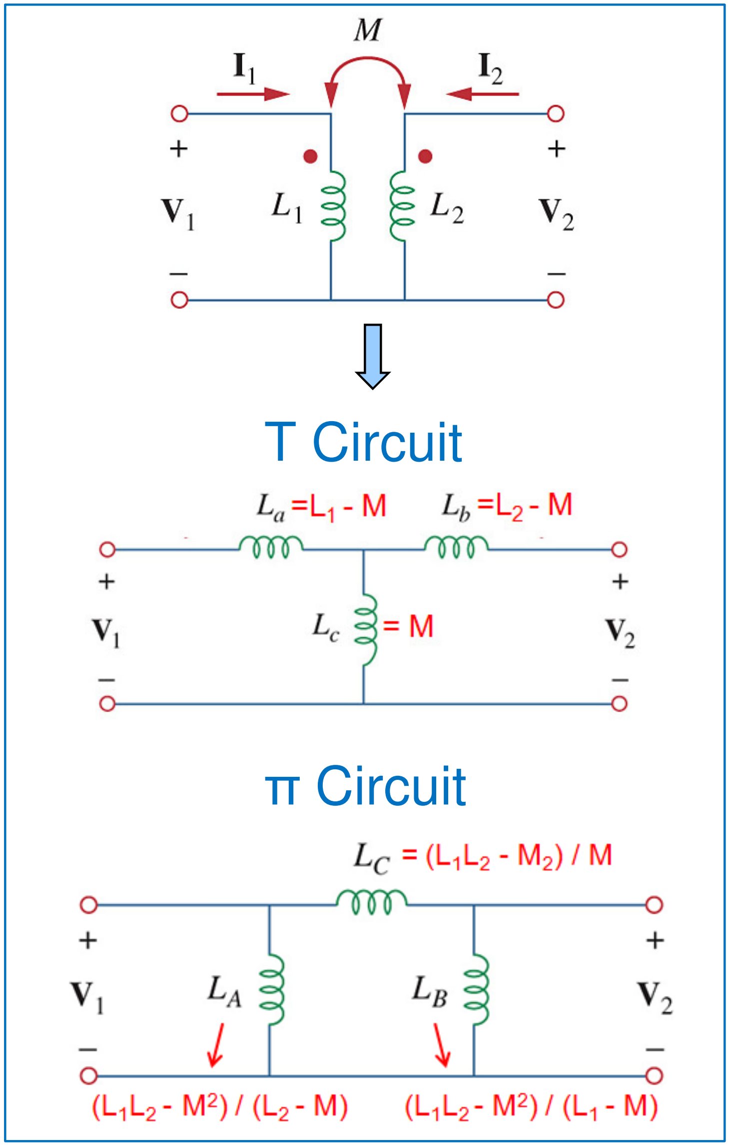
$$L_A = \frac{L_1 L_2 - M^2}{L_2 - M} \;;\; L_B = \frac{L_1 L_2 - M^2}{L_1 - M} \;;\; L_C = \frac{L_1 L_2 - M^2}{M}$$

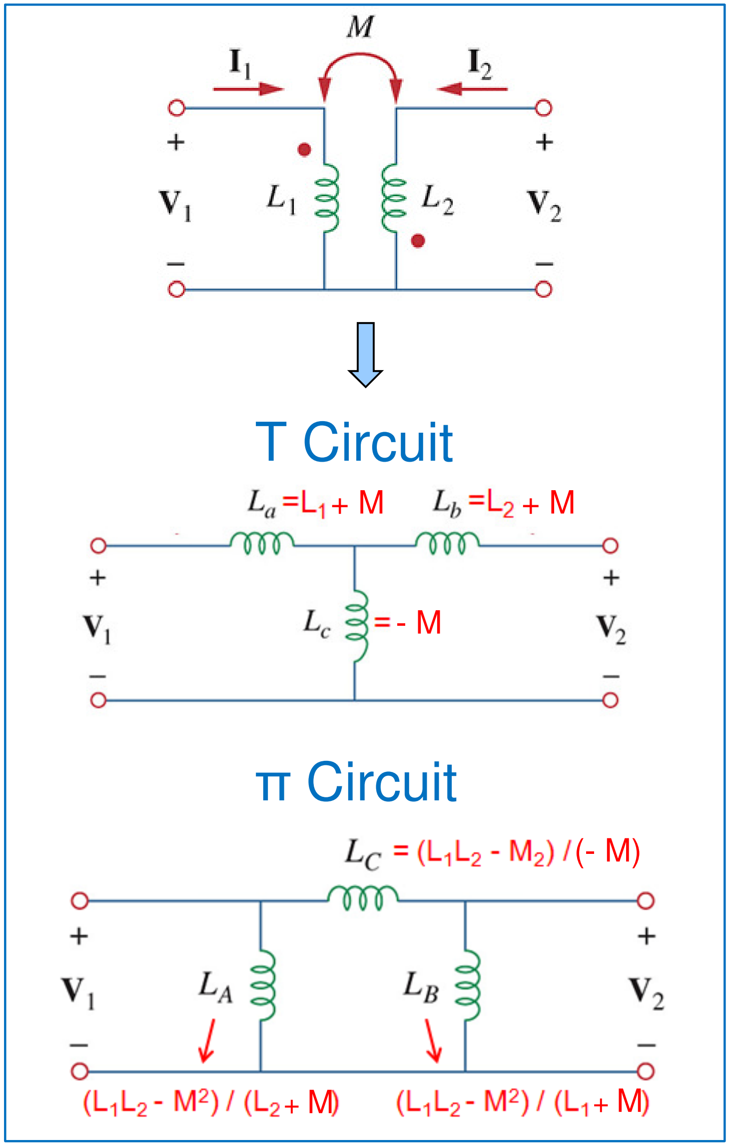


$$L_A = \frac{L_1 L_2 - M^2}{L_2 + M} \;;\; L_B = \frac{L_1 L_2 - M^2}{L_1 + M} \;;\; L_C = \frac{L_1 L_2 - M^2}{-M}$$

13.4 Linear Transformers (10)

Equivalent T or π Circuits Summary

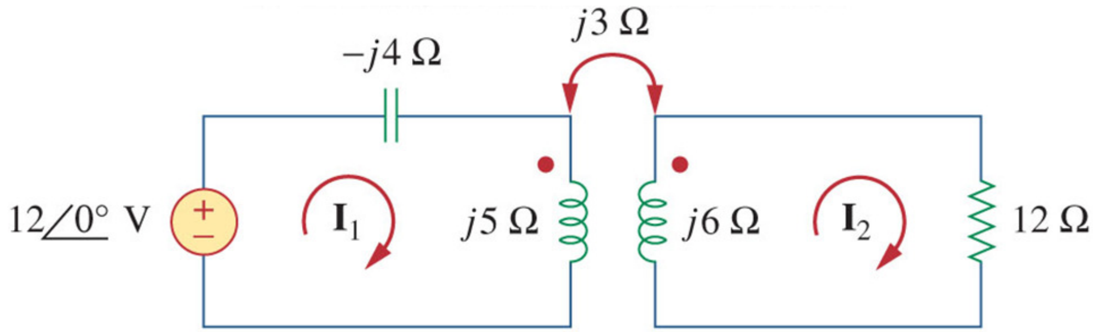


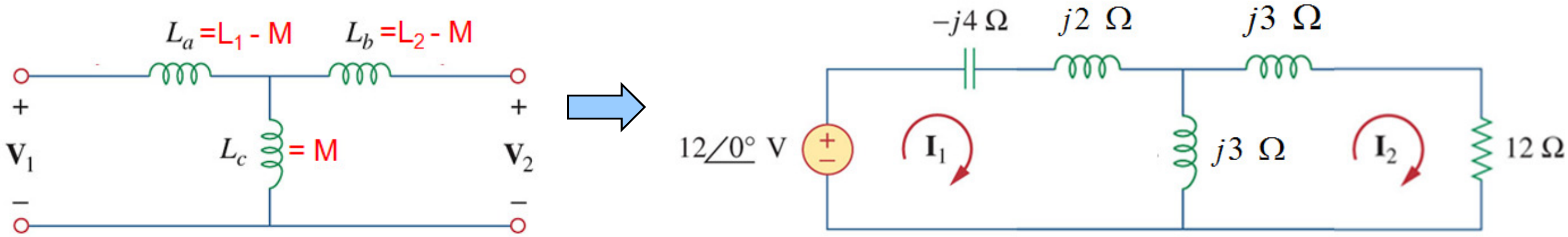


13.4 Linear Transformers (11)

Practice Problem 13.6

Find I_1 and I_2 using the T equivalent circuit





Mesh I_1: $-12\angle 0° - j4I_1 + j2I_1 + j3(I_1 - I_2) = 0$ $\qquad$ $jI_1 - j3I_2 = 12\angle 0°$

Mesh I_2: $j3(I_2 - I_1) + j3I_2 + 12I_2 = 0$ $\qquad$ $-j3I_1 + (12 + j6)I_2 = 0$

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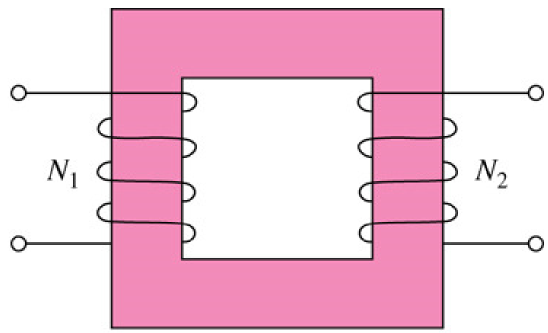
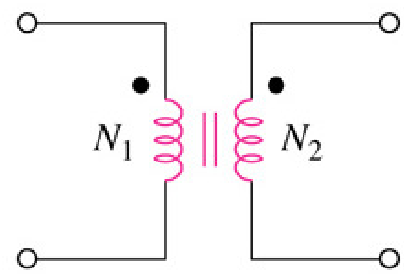
13.5 Ideal Transformers (1)

- An ideal transformer has perfect coupling (k=1).
- It consists of two or more coils with a large number of turns wound on a common core of high permeability.



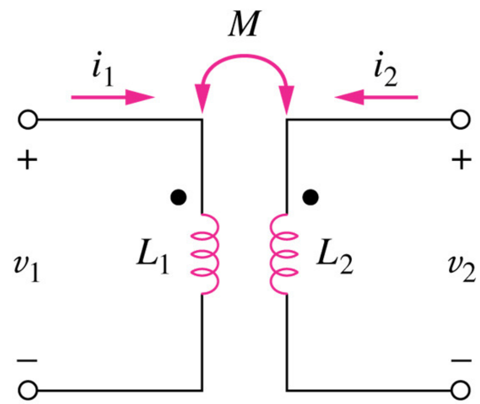


- Because of the high permeability of the core, the flux links all the turns of both coils, thereby resulting in a perfect coupling.

13.5 Ideal Transformers (2)

"Dot's the same polarity"

Recall the coupled circuit:



$$V_1 = j\omega L_1 I_1 + j\omega M I_2 \qquad (1)$$

$$V_2 = j\omega M I_1 + j\omega L_2 I_2 \qquad (2)$$

Solving for I_1 in (1): $\mathrm{I}_1 = \dfrac{V_1 - j\omega M I_2}{j\omega L_1}$ (3)

Substituting (3) into (2): $V_2 = j\omega L_2 I_2 + \dfrac{M V_1}{L_1} - \dfrac{j\omega M^2 I_2}{L_1}$

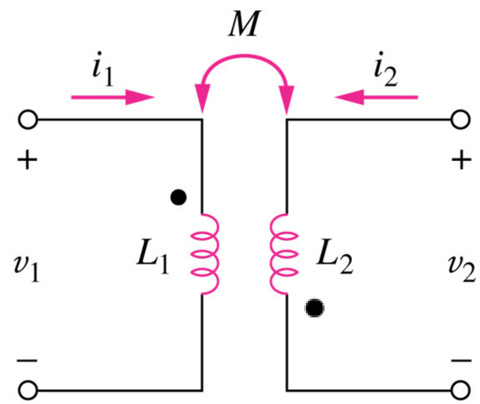
$M = \sqrt{L_1 L_2}$ For perfect coupling (k=1):

$$V_2 = j\omega L_2 I_2 + \frac{\sqrt{L_1 L_2}V_1}{L_1} - \frac{j\omega L_1 L_2 I_2}{L_1} = \sqrt{\frac{L_2}{L_1}}V_1 = nV_1$$

Therefore: $V_2 = nV_1$ where $n = \sqrt{L_2 / L_1}$ = turns ratio

13.5 Ideal Transformers (3)

"Dot's opposite each other"



Mesh Equations give the following:

$$V_1 = j\omega L_1 I_1 - j\omega M I_2 \qquad (1)$$

$$V_2 = -j\omega M I_1 + j\omega L_2 I_2 \qquad (2)$$

Solving for I_1 in (1): $\mathrm{I}_1 = \dfrac{V_1 + j\omega M I_2}{j\omega L_1}$ (3)

Substituting (3) into (2): $V_2 = j\omega L_2 I_2 - \dfrac{MV_1}{L_1} - \dfrac{j\omega M^2 I_2}{L_1}$

$M = \sqrt{L_1 L_2}$ For perfect coupling (k=1):

$$V_2 = j\omega L_2 I_2 - \frac{\sqrt{L_1 L_2}V_1}{L_1} - \frac{j\omega L_1 L_2 I_2}{L_1} = -\sqrt{\frac{L_2}{L_1}}V_1 = -nV_1$$

Therefore: $V_2 = -nV_1$ If dot is swapped at output

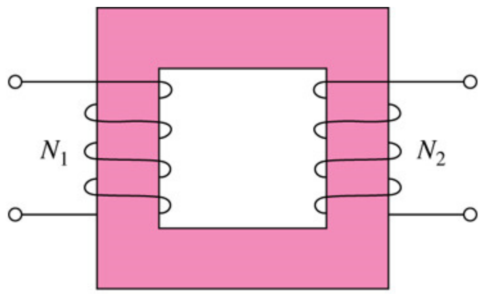
13.5 Ideal Transformers (4)

Properties

- A transformer is said to be ideal if it has the following properties:
  1. Coils have very large reactances (L_1, L_2, $M \rightarrow \infty$)
  2. Coupling coefficient is equal to unity (k=1)
  3. Primary and secondary coils are lossless ($R_1 = R_2 = 0$)
- An ideal transformer is a unity-coupled (k=1) lossless transformer in which the primary and secondary coils have infinite self-inductances (L_1 & $L_2 \rightarrow \infty$).
- Iron core transformers are close approximations to ideal transformers and are used in power systems and electronics.

13.5 Ideal Transformers (5)

- When a sinusoidal voltage is applied to the primary winding, the same magnetic flux Φ goes through both windings.



$$v_1 = N_1 \frac{d\,\phi}{d\,t} \quad ; \quad v_2 = N_2 \frac{d\,\phi}{d\,t}$$

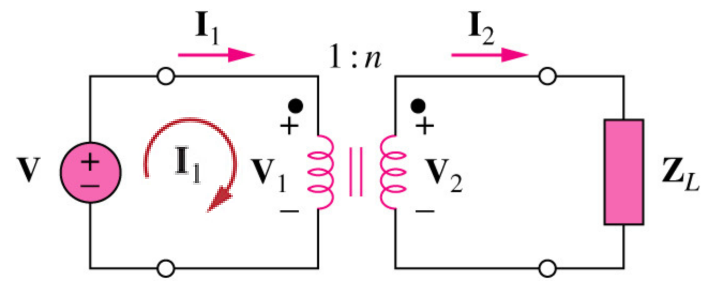
$$\frac{v_2}{v_1} = \frac{N_2}{N_1} = n =$$ Turns ratio or transformation ratio

- Using the phasor voltages rather than the instantaneous voltages:

$$\frac{V_2}{V_1} = \frac{N_2}{N_1} = n$$

13.5 Ideal Transformers (6)

- Power conservation: $v_1 i_1 = v_2 i_2$
- The energy supplied to the primary must equal the energy absorbed by the secondary, since there are no losses in an ideal transformer.
- In phasor form: $\boxed{\dfrac{I_1}{I_2} = \dfrac{V_2}{V_1} = n}$



  - n=1 → isolation transformer ($V_2 = V_1$)
  - n>1 → step-up transformer ($V_2 > V_1$)
  - n<1 → step-down transformer ($V_2 < V_1$)

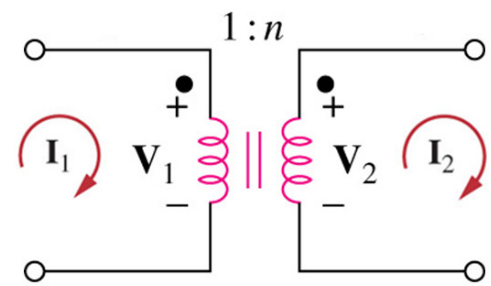
13.5 Ideal Transformers (7)

- Transformer ratings are usually specified as V_1 / V_2
- Power companies often generate at some convenient voltage and use the step-up transformer to increase the voltage so that the power can be transmitted at very high voltage and low current over transmission lines, resulting in significant cost savings. Near residential consumers, step-down transformers are used to bring the voltage down to 120 V.
- It is important to get the proper polarity of the voltages and the direction of the currents for the transformer.

13.5 Ideal transformers (8)

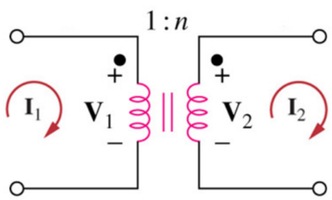
V and I relationships (simpler way to remember)

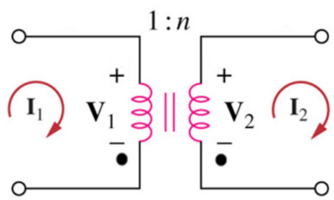
Given this standard definition for Voltages and currents



Note: This definition of I_2 differs from the text.

Dots "Same" → "+"





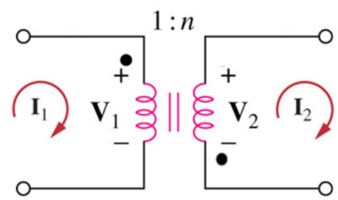
$$V_2 = nV_1$$

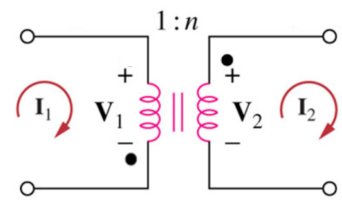
$$I_2 = \frac{I_1}{n}$$

Reflected Impedance →

$$Z_{in} = \frac{Z_L}{n^2}$$

Dots "Different" → "-"





$$V_2 = -nV_1$$

$$I_2 = -\frac{I_1}{n}$$

$$Z_{in} = \frac{Z_L}{n^2}$$

Note: The larger the turns ratio
The larger "n"
The larger "N_2"
The larger V_2

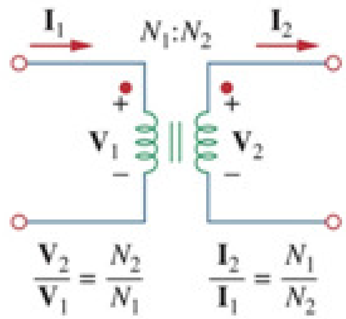
Turns Ratio

$$n = \frac{N_2}{N_1}$$

13.5 Ideal Transformers (9)

V and I relationships

- Expressing V_1 in terms of V_2 and I_1 in terms of I_2 or vice versa:



$$V_1 = \frac{V_2}{n} \qquad V_2 = nV_1$$

Positive, if Voltage "**same**" polarity at dot

$$I_2 = \frac{I_1}{n} \qquad I_1 = nI_2$$

Positive, if current "**different**" polarity at dot

- Complex Power is:

Complex Conjugate of I_2

$$S_1 = V_1 I_1^* = \frac{V_2}{n}(nI_2)^* = V_2 I_2^* = S_2$$

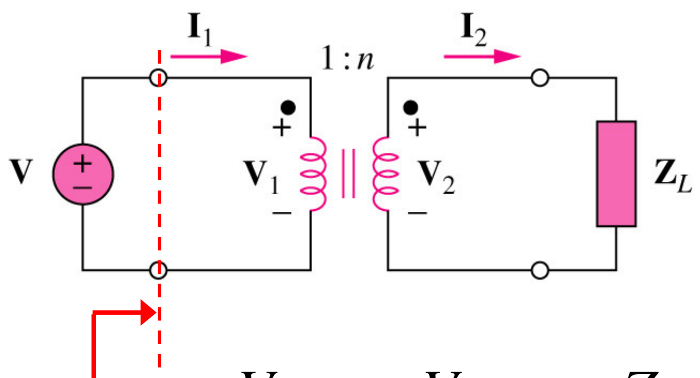
- Complex power supplied to the primary is delivered to the secondary without loss.

- The ideal transformer is **lossless** and absorbs **no power**.

13.5 Ideal Transformers (10)

Reflected impedance

- The input impedance as seen by the source is:



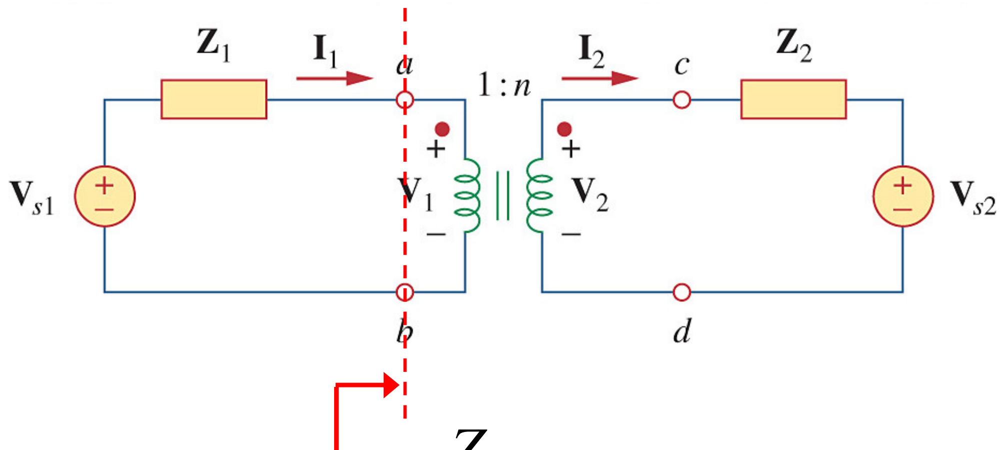
$$Z_{in} = \frac{V_1}{I_1} = \frac{V_2}{n^2 I_2} = \frac{Z_L}{n^2}$$

- The input impedance is also called the reflected impedance since it appears as if the load impedance is reflected to the primary side.
- The ability of the transformer to transform a given impedance to another allows impedance matching to ensure maximum power transfer.

13.5 Ideal Transformers (11)

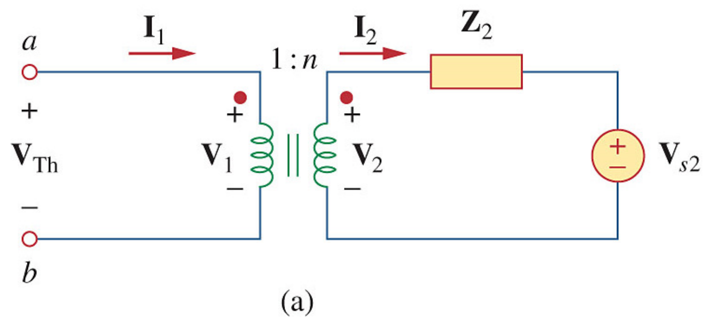
Equivalent circuit analysis

- In analyzing a circuit containing an ideal transformer, it is common practice to eliminate the transformer by reflecting impedances and sources from one side of the transformer to the other.
- Suppose we want to reflect the secondary side of the circuit to the primary side:



$$Z_R = \frac{Z_2}{n^2}$$

- We find the Thevenin equivalent of the circuit to the right of a-b:
- Obtaining V_{th} from "open circuit voltage":



(a)

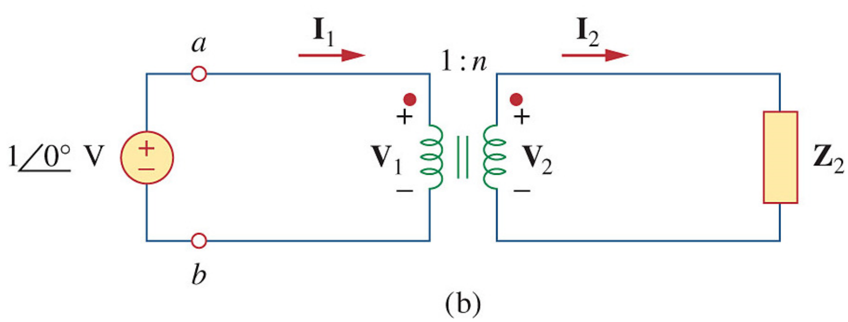
$$I_1 = 0 = I_2$$ Since a-b is open

$$V_2 = V_{s2}$$

$$V_{Th} = V_1 = \frac{V_2}{n} = \boxed{\frac{V_{s2}}{n}}$$

- Obtaining Z_{Th} (remove the voltage source in the secondary and insert a unit source at a-b terminals.)

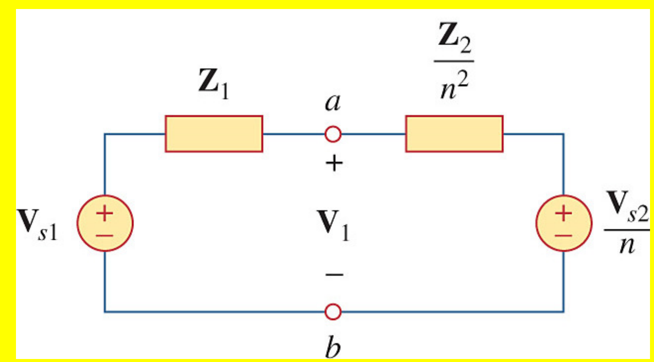
$$Z_{Th} = \frac{V_1}{I_1} = \frac{V_2}{n^2 I_2} = \boxed{\frac{Z_2}{n^2}}$$



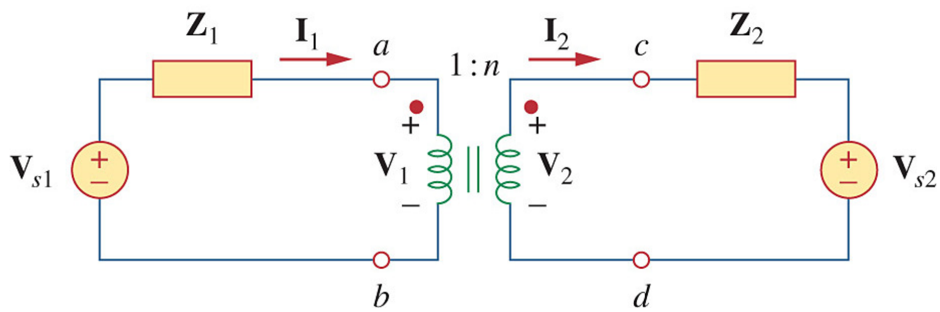
(b)

Equivalent Circuit

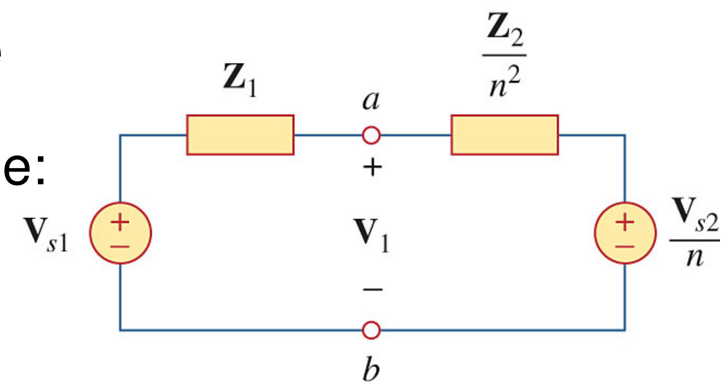
Reflecting the secondary to the primary



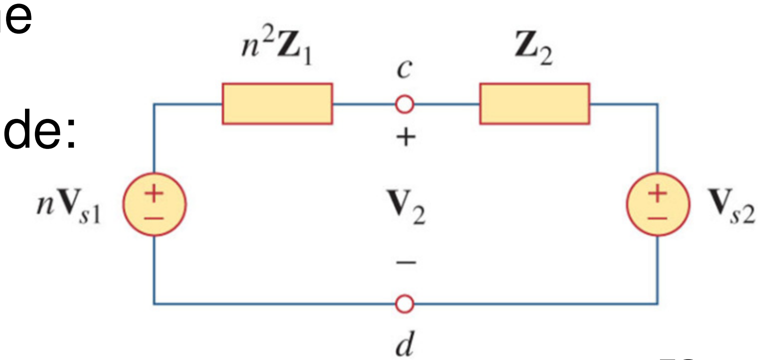
13.5 Ideal Transformers (13)



- The general rule for eliminating the transformer and reflecting the secondary circuit to the primary side:
  - Divide the secondary impedance by n^2
  - Divide the secondary voltage by n
  - Multiply the secondary current by n



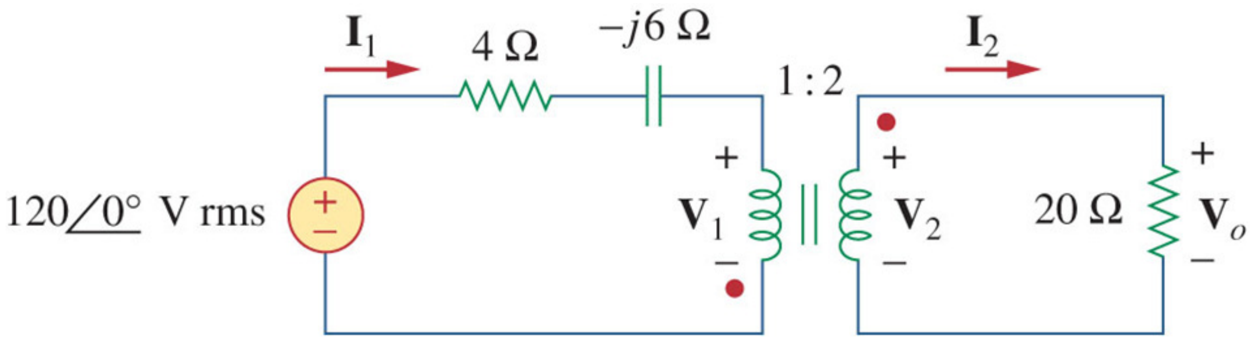
- The general rule for eliminating the transformer and reflecting the secondary circuit to the primary side:
  - Multiply the primary impedance by n^2
  - Multiply the primary voltage by n
  - Divide the primary current by n



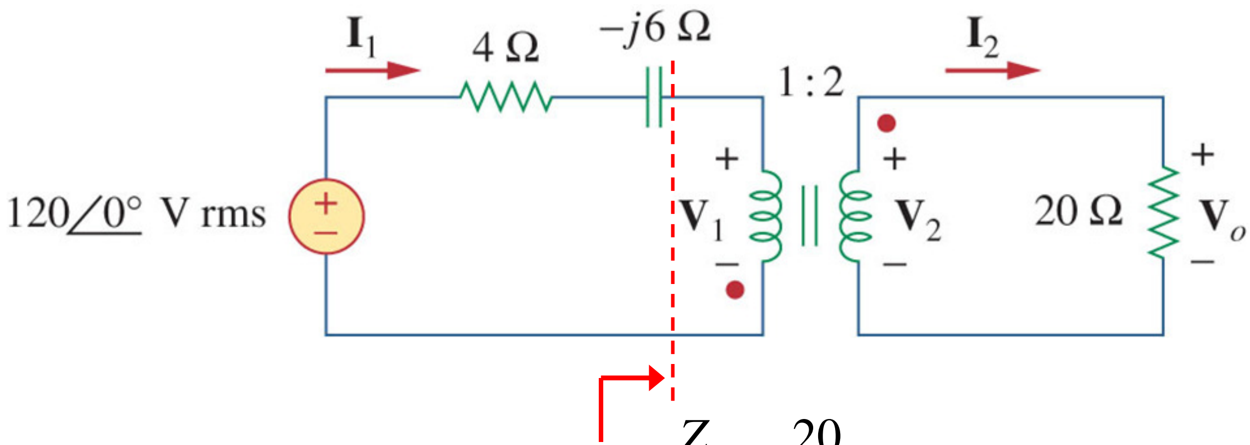
13.5 Ideal Transformer (14)

Example 13.8 (Textbook)

For the ideal transformer, find: (a) the source current I_1, (b) the output voltage V_0, and (c) the complex power supplied by the source



13.5 Ideal Transformer (14)



$$Z_R = \frac{Z_2}{n^2} = \frac{20}{2^2} = 5$$

Impedance seen by the Voltage source is:

$$Z_{in} = (4 - j6) + 5 = 9 - j6 = 10.82\angle -33.69^\circ \ \Omega$$

Input current I_1 is:

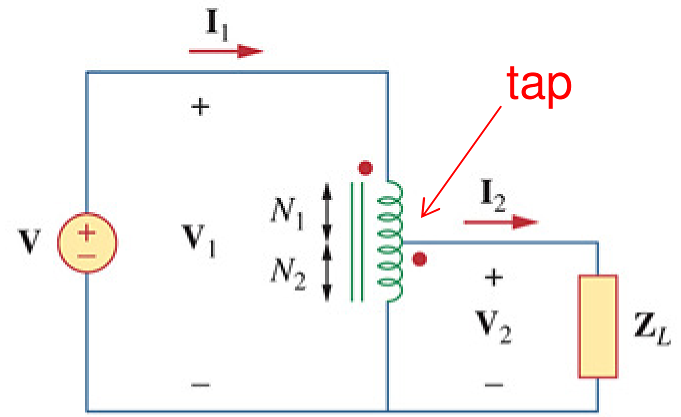
$$I_1 = \frac{V_s}{Z_{in}} = \frac{120\angle 0^\circ}{10.82\angle -33.69^\circ} = 11.09\angle 33.69^\circ \ \text{A}$$

$$I_2 = -\frac{I_1}{n} = \frac{-11.09\angle 33.69^\circ}{2} = 5.55\angle -146.31^\circ \ \text{A}$$

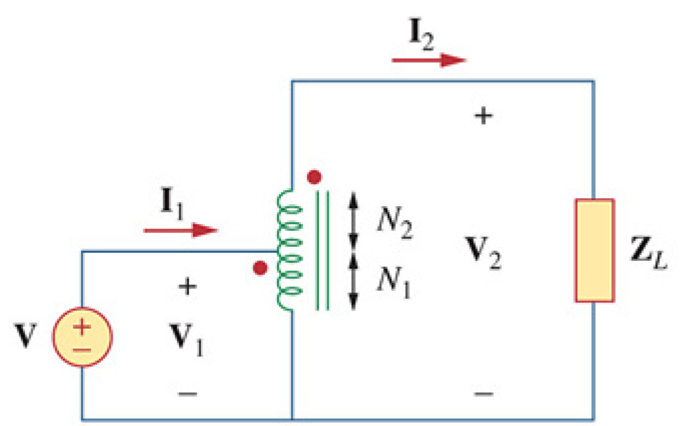
$$V_o = 20I_2 = 20(5.55\angle -146.31^\circ) = 110.9\angle -146.31^\circ \ \text{V}$$

13.6 Ideal Auto-Transformers (1)

- An **autotransformer** is a transformer in which both the primary and the secondary are in a single winding
- A connection point called a *tap* is separates the primary and secondary.
- The tap is often adjustable to provide a desired turns ratio.
- An adjustable tap provides a variable voltage to the load
- A disadvantage of the autotransformer is it provides *no electrical isolation*



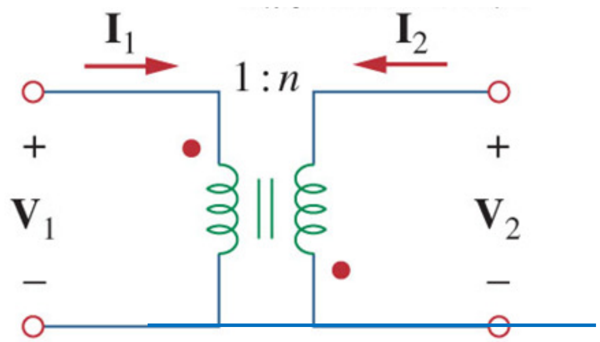
Step Down Auto-Transformer



Step Up Auto-Transformer

13.6 Ideal Auto-Transformers (2)

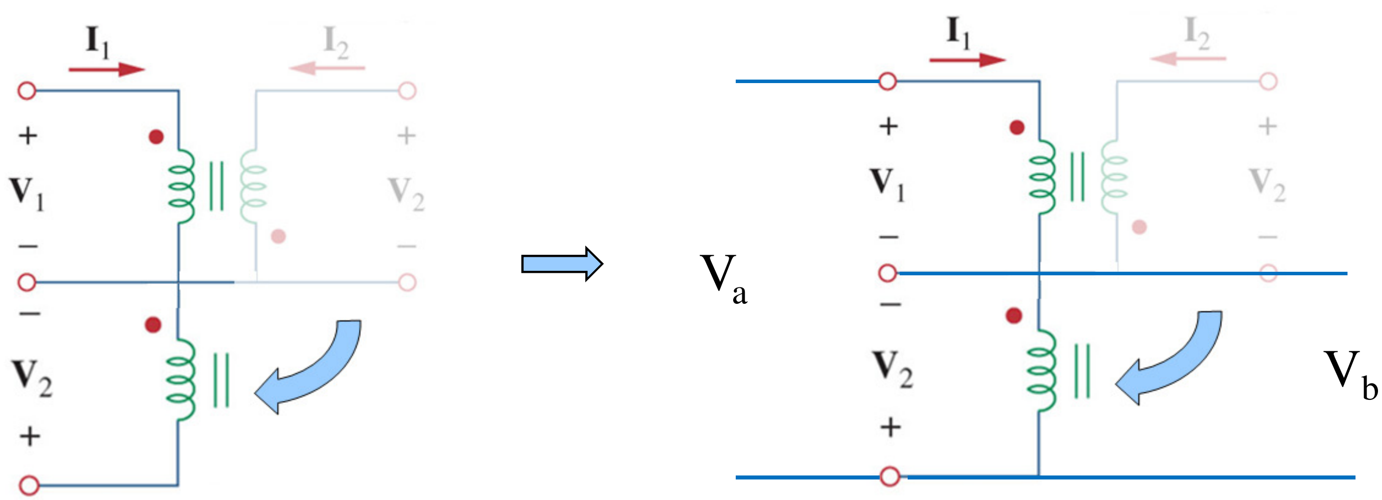
- In analyzing the autotransformer, consider the following circuit:



From earlier we know the following relationship

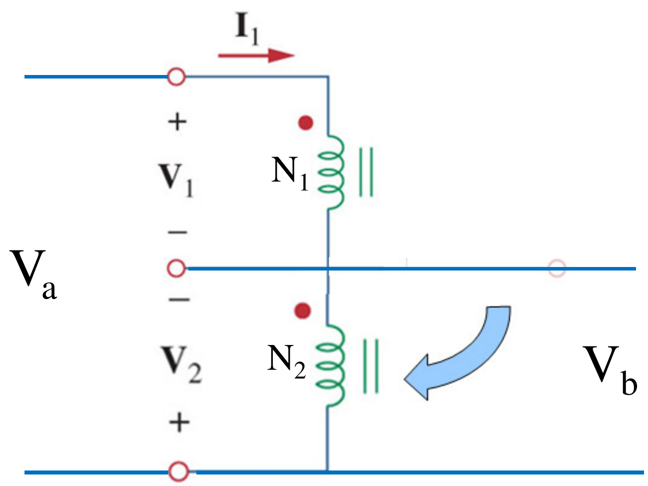
$$V_2 = -nV_1$$

- If we flip the secondary side underneath the primary, we can create an autotransformer as shown



13.6 Ideal Auto-Transformers (3)

- Analysis of this circuit provides the following results:



Primary Side

$$V_a = V_1 - V_2$$

$$V_a = V_1 + nV_1$$

$$V_a = (1+n)V_1$$

Secondary Side

$$V_b = -V_2 = nV_1$$

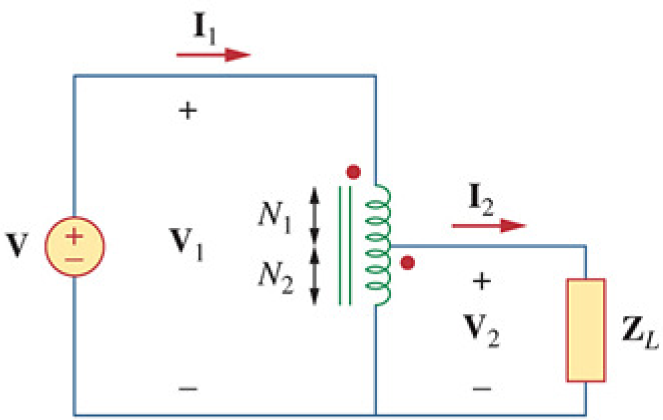
Notice, this looks like a Voltage Divider !

Ratio Primary/Secondary

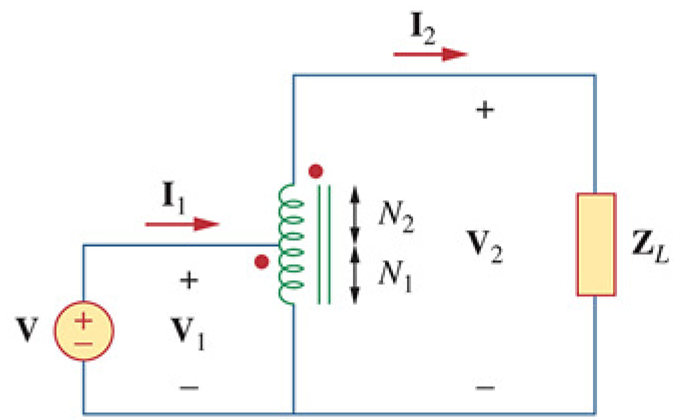
$$\frac{V_a}{V_b} = \frac{1+n}{n} = \frac{N_1+N_2}{N_2} \Rightarrow \boxed{V_b = \left(\frac{N_2}{N_1+N_2}\right)V_a}$$

13.6 Ideal Auto-Transformers (4)

- The voltage / current relationships for the lossless ideal autotransformer are as follows:



Step Down Auto-Transformer



Step Up Auto-Transformer

Similar to Voltage Divider equation

$$V_2 = \frac{N_2}{N_1 + N_2} V_1 \qquad V_2 = \frac{N_1 + N_2}{N_1} V_1$$

Inverse Relation

$$I_2 = \frac{N_1 + N_2}{N_2} I_1 \qquad I_2 = \frac{N_1}{N_1 + N_2} I_1$$

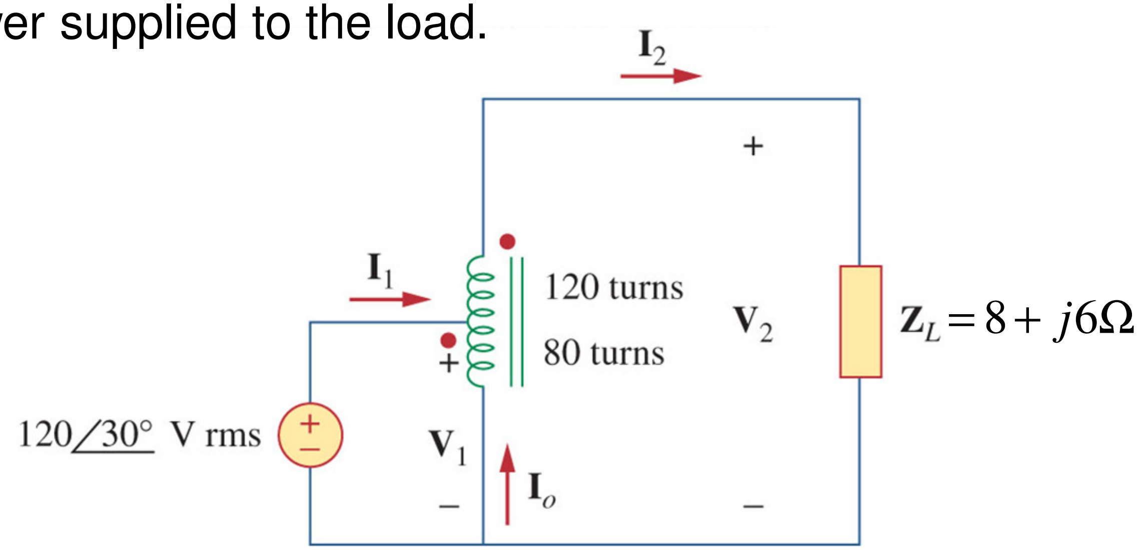
Derive from V/I

$$Z_{in} = \left(\frac{N_1 + N_2}{N_2}\right)^2 Z_L \qquad Z_{in} = \left(\frac{N_1}{N_1 + N_2}\right)^2 Z_L$$

13.6 Ideal Auto-Transformers (5)

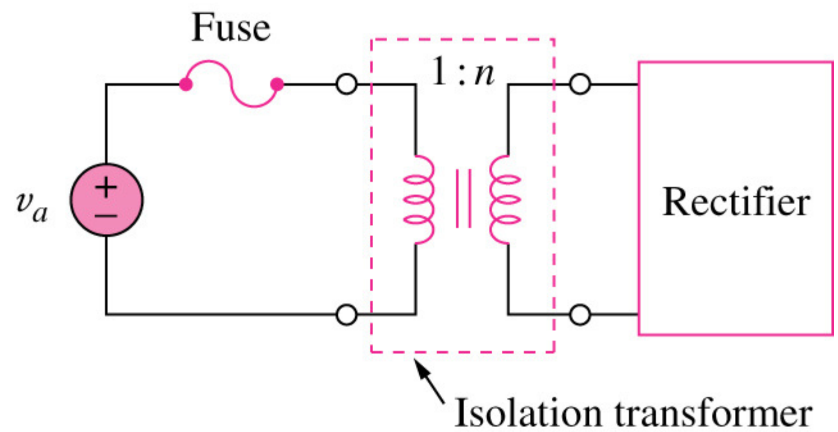
Example 13.11 (Textbook)

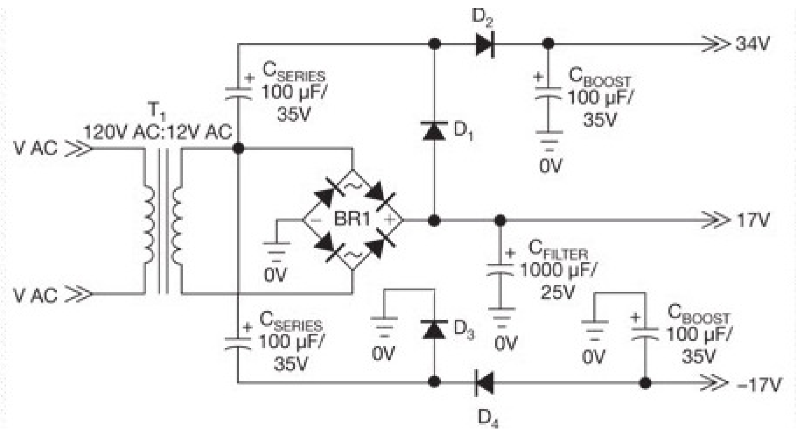
For the autotransformer below, find: (a) the currents I_1 , I_2 , I_0, (b) the complex power supplied to the load.

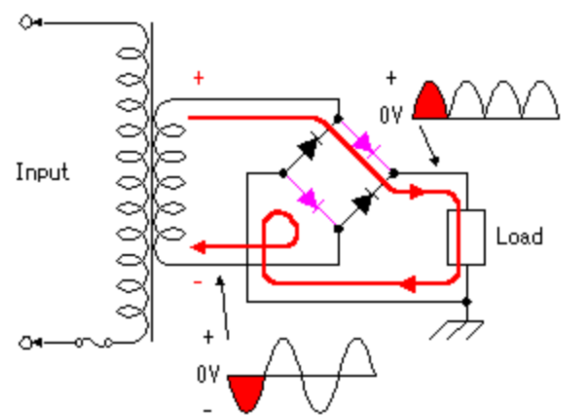


13.9 Applications (1)

- Transformer as an Isolation Device to isolate ac supply from a rectifier

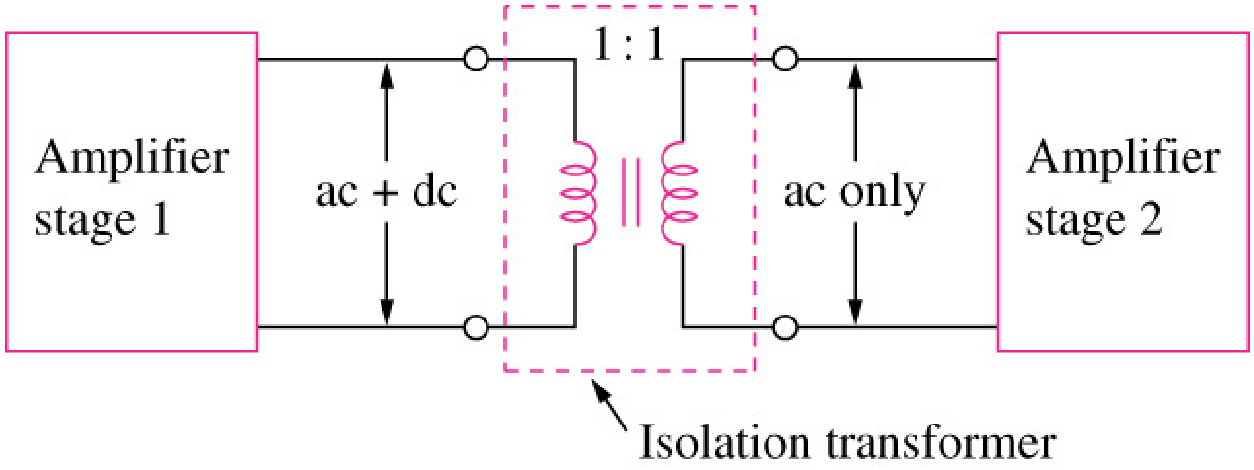






13.9 Applications (2)

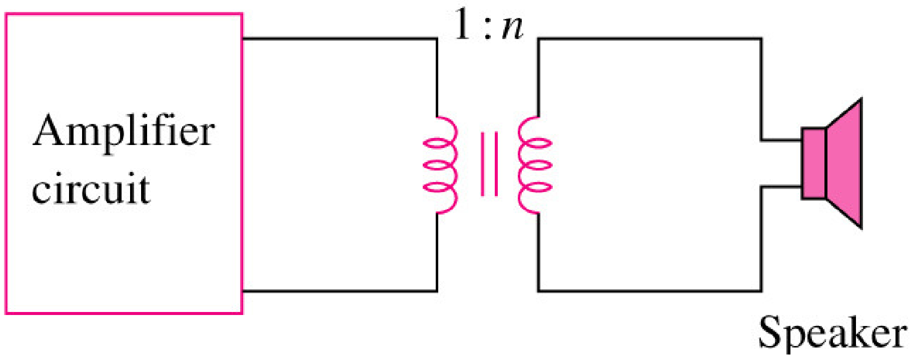
- Transformer as an Isolation Device to isolate dc between two amplifier stages.



- Biasing is the application of a DC voltage to a transistor amplifier to produce a desired mode of operation.
- Each amplifier stage can be biased separately to operate in a particular mode.

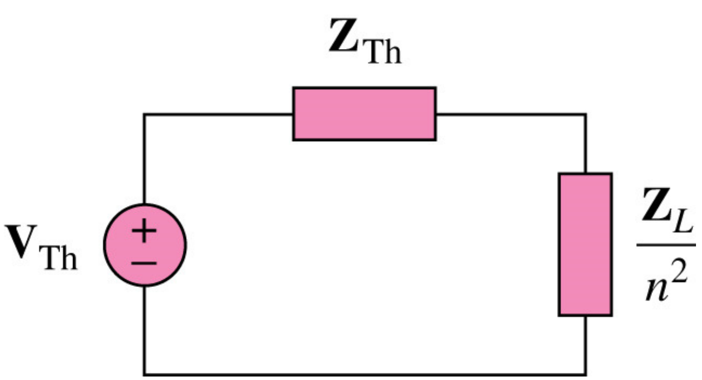
13.9 Applications (3)

- Transformer as a Matching Device



Using an ideal transformer to match the speaker to the amplifier

Equivalent circuit



13.9 Applications (4)

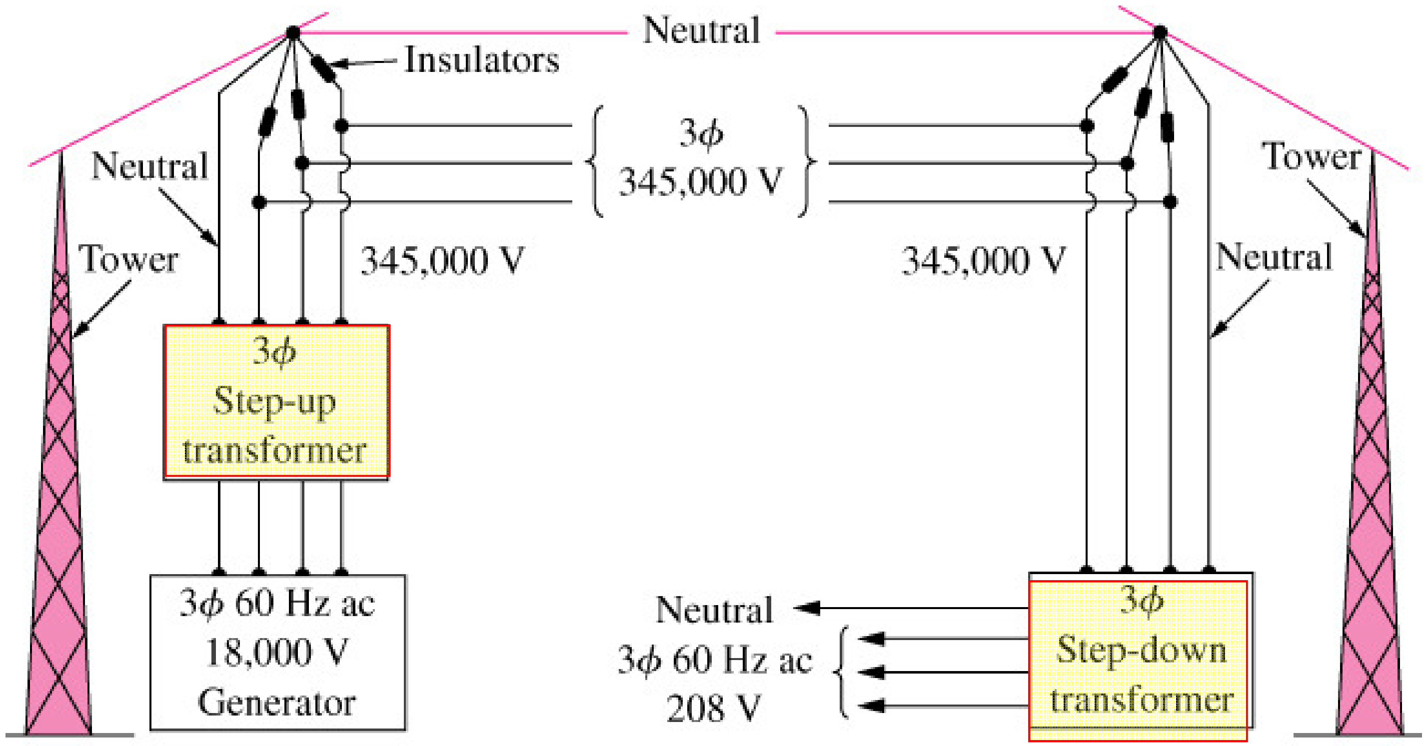
Practice Problem 13.16 (Textbook)

Calculate the turns ratio of an ideal transformer required to match a 400Ω load to a source with internal impedance of 2.5kΩ. Find the load voltage when the source voltage is 30V.

Ans: $n = 0.4$; $V_L = 6V$

13.9 Applications (5)

- A typical power distribution system



Homework #3

Due beginning of class Wednesday Feb 4, 2015

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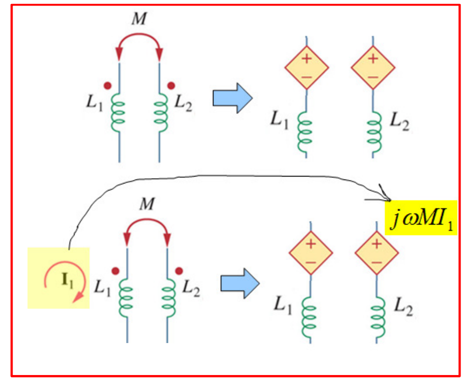
- 13.30
- 13.35
- 13.42
- 13.50
- 13.53 (modified)
- Autotransformer (See handout)

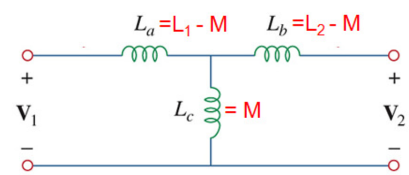
Exam over Chapter 13 on Monday Feb 9

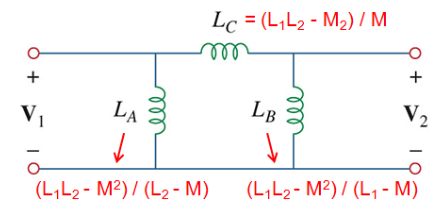
Chapter 13
Equation / Analysis Summary

- Series Aiding $L = L_1 + L_2 + 2M$ / Opposing $L = L_1 + L_2 - 2M$
- Dot Convention Model



- Coupling coefficient "k" $M = k\sqrt{L_1 L_2}$
- Linear Transformer
  - Input Impedance: $Z_{in} = Z_{primary} + \dfrac{\omega^2 M^2}{Z_{Secondary}}$
  - Reflected Impedance: $Z_{reflected} = \dfrac{\omega^2 M^2}{Z_{Secondary}}$
  - Equivalent T or π Circuits:

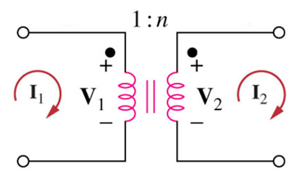




- Ideal Transformer

Turns Ratio $n = \dfrac{N_2}{N_1}$

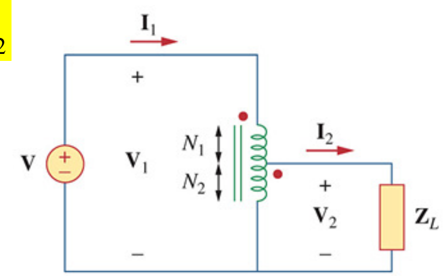
  - K = 1, L_1, L_2, → ∞
  - Lossless ($S_1 = S_2$)
  - Voltage / Current Relationship:
    - Dots "same" = +n, Dots "diff" = -n
  - Complex Power: $S_1 = V_1 I_1^* = V_2 I_2^* = S_2$



$V_2 = nV_1$ $\quad I_2 = \dfrac{I_1}{n}$ $\quad Z_{in} = \dfrac{Z_L}{n^2}$

$P_{ave} = |I_2|^2 R_L$

- Autotransformer
  - Adjustable "tap"
  - No electrical Isolation
  - Voltage Divider like relationship



$V_2 = \dfrac{N_2}{N_1 + N_2} V_1$

$I_2 = \dfrac{N_1 + N_2}{N_2} I_1$

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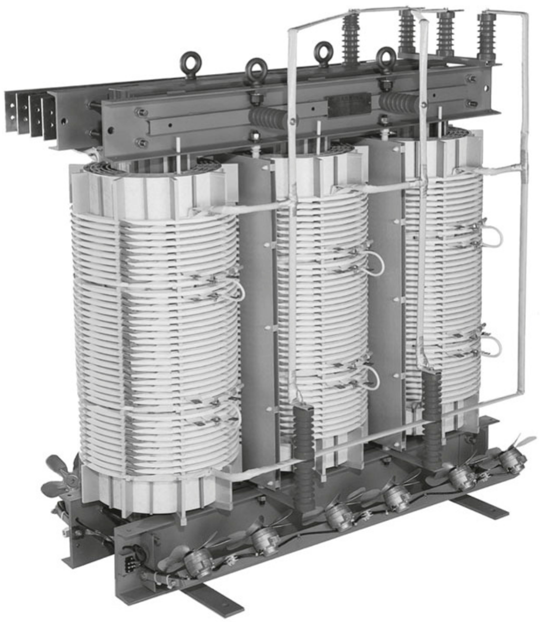
13.1 What is a transformer? (1)

- It is an electrical device designed on the basis of the concept of magnetic coupling
- It uses magnetically coupled coils to transfer energy from one circuit to another
- It is the key circuit elements for stepping up or stepping down ac voltages or currents, impedance matching, isolation, etc.



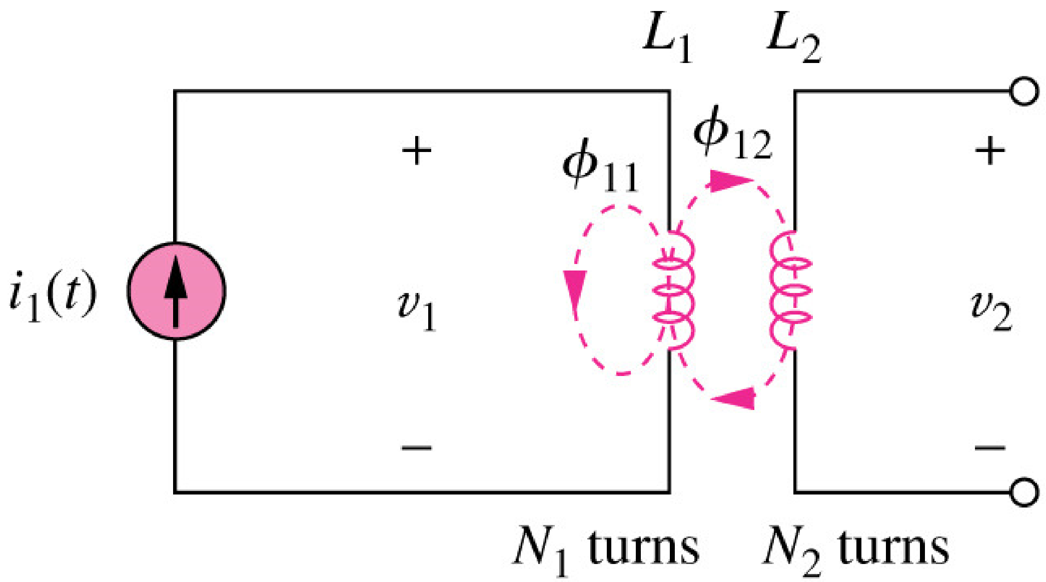
(a)

(b)

Courtesy of: (a) Electric Service Co., (b) Jensen Transformers

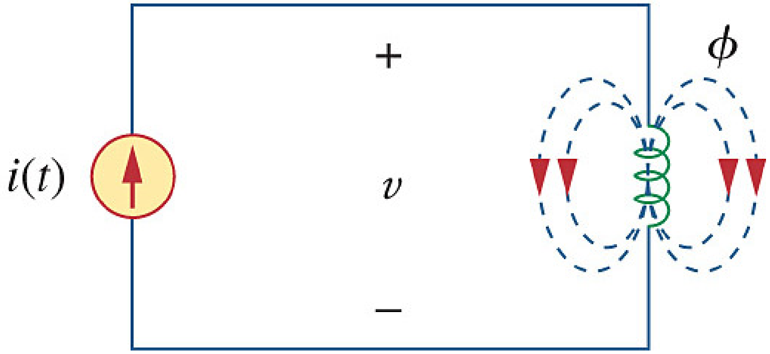
13.2 Mutual Inductance (1)

- Mutual Inductance
  - When two inductors (or coils) are in close proximity of each other, the magnetic flux caused by current in one coil links with the other coil, thereby inducing voltage in the latter.



13.2 Mutual Inductance (2)

- First consider a single inductor, a coil with N turns:



When current i flows through the coil, a magnetic flux Φ is produced around it.

- According to Faraday's law, the voltage v induced in the coil is proportional to the number of turns N and the time rate of change of magnetic flux Φ:

$$v = N \frac{d\phi}{dt}$$

13.2 Mutual Inductance (3)

- Voltage induced in the coil given by:

$$v = N\frac{d\,\phi}{d\,t}$$

- But the flux Φ is produced by current i so that any change in Φ is caused by a change in the current:

$$v = N\frac{d\,\phi}{d\,i}\frac{d\,i}{d\,t}$$

- Recall the voltage-current relationship for an inductor:

$$v = L\frac{d\,i}{d\,t}$$

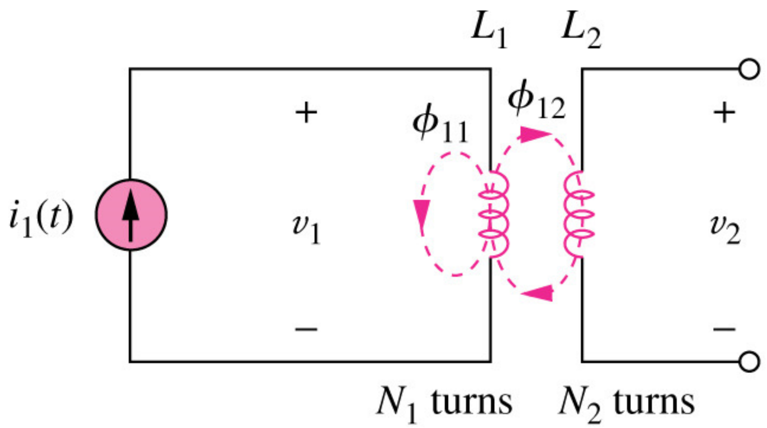
- The “Self” inductance L of the inductor is thus given by:

$$\boxed{L = N\frac{d\,\phi}{d\,i}}$$

- Self-inductance L relates the voltage induced in a coil by a time-varying current in the same coil.

13.2 Mutual Inductance (4)

- Now consider two coils with self-inductances L_1 and L_2 that are in close proximity of each other:



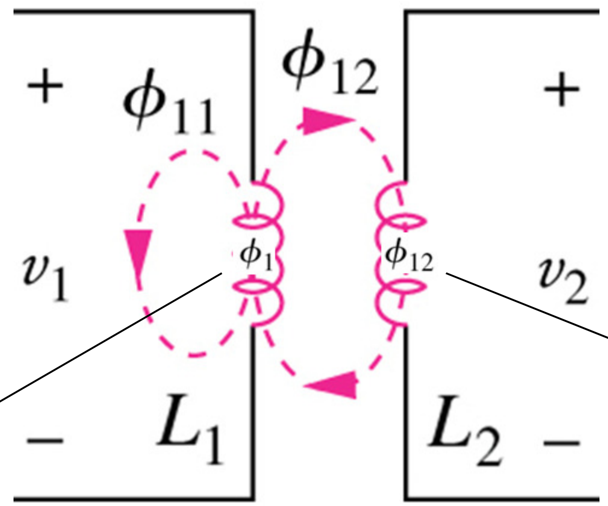
Coil 1 has N_1 turns
Coil 2 has N_2 turns.
Assume coil 2 carries no current.

- The total magnetic flux Φ_1 emanating from coil 1 has two components:
  - Φ_{11} links only coil 1
  - Φ_{12} links both coils

$$\phi_1 = \phi_{11} + \phi_{12}$$

13.2 Mutual Inductance (5)

- Although the two coils are physically separated, they are magnetically coupled.
- The voltage induced in each coil is proportional to the flux in each coil.

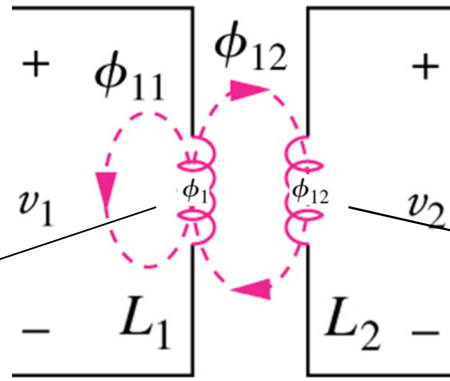


$$v_1 = N_1 \frac{d\,\phi_1}{d\,t}$$

$$v_2 = N_2 \frac{d\,\phi_{12}}{d\,t}$$

13.2 Mutual Inductance (6)

- The voltage equations can be rewritten as follows:



v_2 is the open-circuit mutual voltage (or induced voltage) across coil 2

$$v_1 = N_1 \frac{d\phi_1}{dt}$$

$$v_1 = \underbrace{N_1 \frac{d\phi_1}{di_1}}\frac{di_1}{dt}$$

$$v_1 = L_1 \frac{di_1}{dt}$$

L_1 is the self-inductance of coil 1

$$v_2 = N_2 \frac{d\phi_{12}}{dt}$$

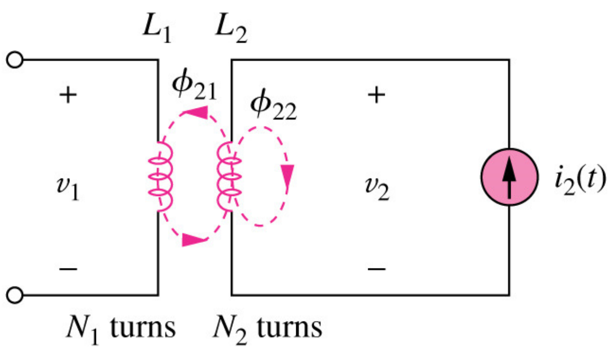
$$v_2 = \underbrace{N_2 \frac{d\phi_{12}}{di_1}}\frac{di_1}{dt}$$

$$v_2 = M_{21} \frac{di_1}{dt}$$

M_{21} is the mutual inductance of coil 2 with respect to coil 1

13.2 Mutual Inductance (7)

- Suppose now we let current i_2 flow in coil 2 while coil 1 carries no current:



- The magnetic flux Φ_2 emanating from coil 2 comprises flux Φ_{22} that links only coil 2 and flux Φ_{21} that links both coils:

$$\phi_2 = \phi_{21} + \phi_{22}$$

- The resulting symmetry is true:

$$v_2 = L_2 \frac{d\, i_2}{d\, t} \qquad v_1 = M_{12} \frac{d\, i_2}{d\, t} \qquad M_{12} = N_1 \frac{d\, \phi_{21}}{d\, i_2}$$

13.2 Mutual Inductance (8)

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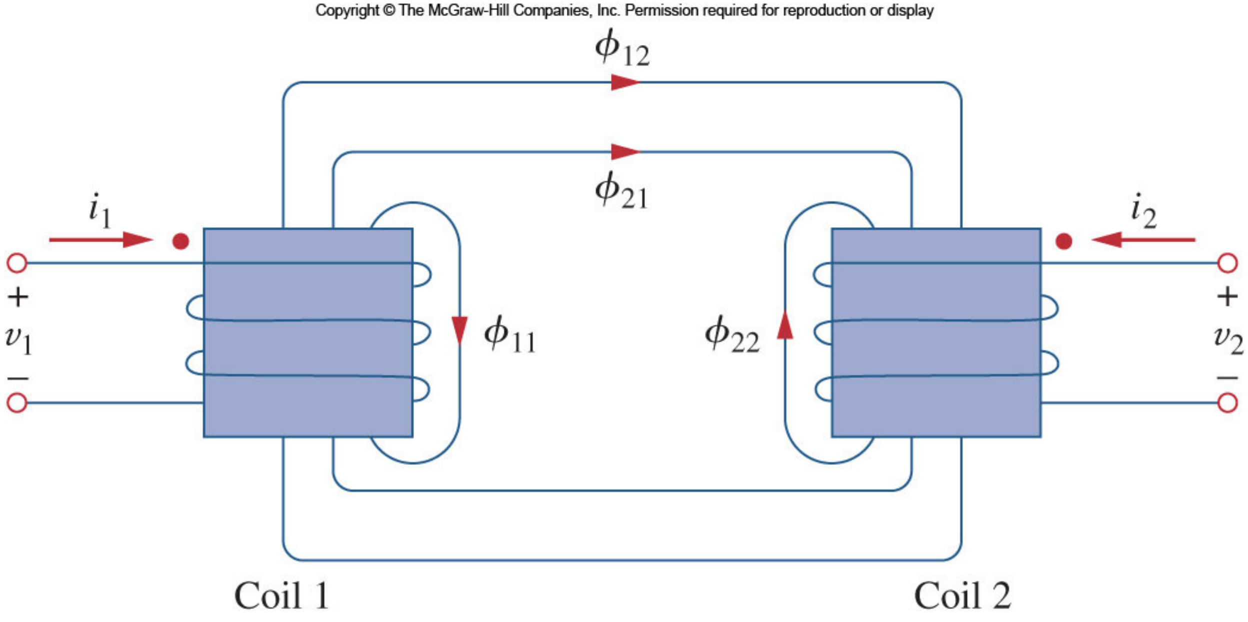
- $M_{12} = M_{21} = M$; The “Mutual inductance” between the coils
- Mutual inductance M is measured in Henrys (just like inductors)
- Mutual inductance only exists when inductors or coils are in close proximity and the circuits are driven by time-varying sources
- Although mutual inductance M is always a positive quantity, the mutual voltage M di/dt may be negative or positive, just like the self-induced voltage L di/dt (determined by “Dot” convention)

13.2 Mutual Inductance (9)

- Self-induced voltage polarity is determined by the reference direction of the current and the reference polarity of the voltage,
- The polarity of the mutual voltage is not as easy to determine (depends on winding direction of the coils).
- We use the dot convention to determine



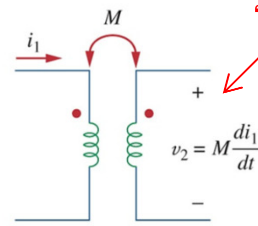
13.2 Mutual Inductance (10)

"Dot Convention"

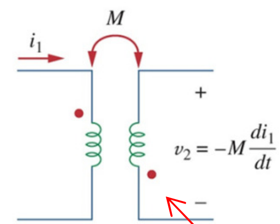
- If a current **enters** the dotted terminal of one coil, the reference polarity of the mutual voltage in the second coil is **positive** at the dotted terminal of the second coil.

Enters the "Dot"

Voltage is "Positive"



(a)

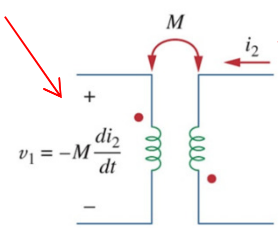


(b)

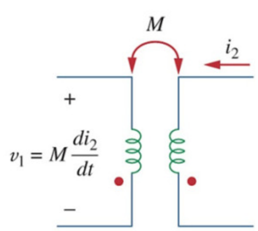
Voltage is "Positive" **At "Dotted" Terminal**

Voltage is "Negative"

Enters the "Un-Dotted" side



(c)



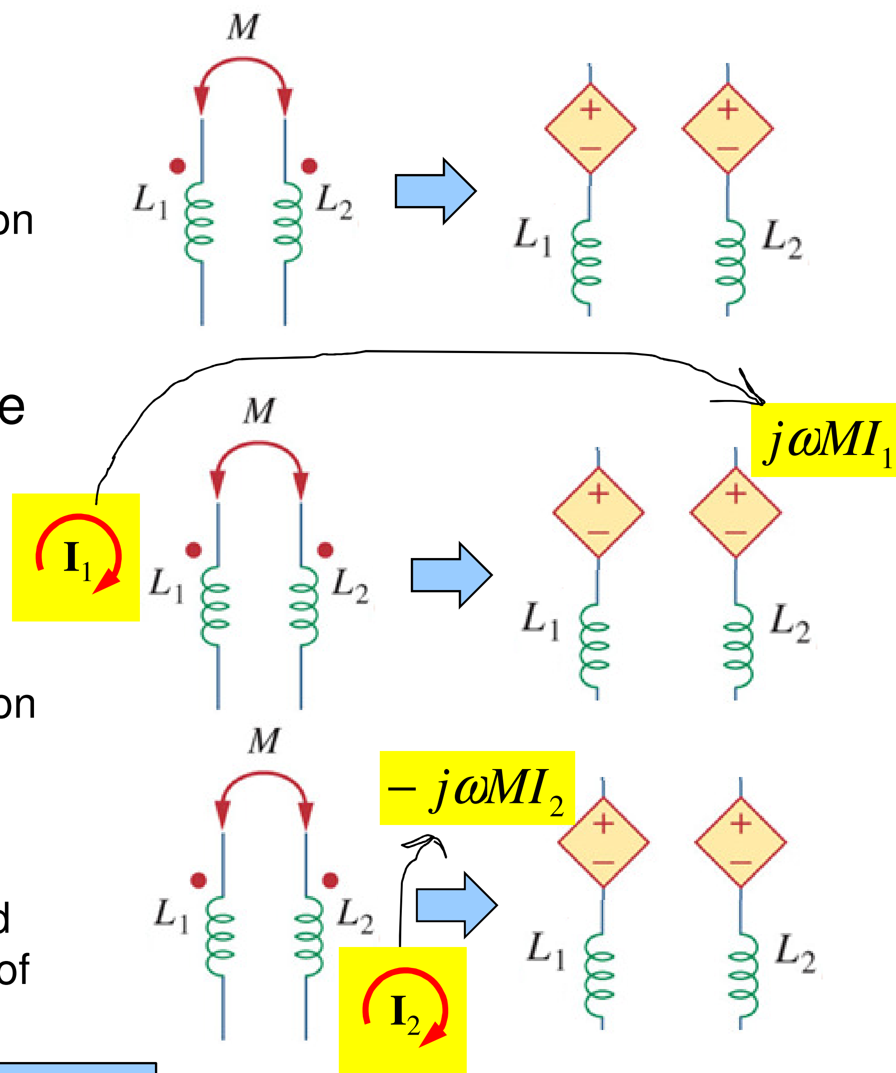
(d)

- Alternatively, if a current **leaves** the dotted terminal of one coil, the reference polarity of the mutual voltage in the second coil is **negative** at the dotted terminal of the second coil.

13.2 Mutual Inductance (11)

“The Model” (Phasor domain)

- Replace the “Dot” with a controlled voltage source.
  - Place on same side as the Dot
  - Positive terminal in same direction as the Dot
- Now look at the current entering each dot to determine the magnitude of the voltage source
  - If the current enters the dotted terminal, it will induce a positive voltage in the controlled source on the opposite terminal of jωM(I)
  - If the current enters the “un-dotted” terminal, it will induce a negative voltage in the controlled source on the opposite terminal of –jωM(I)

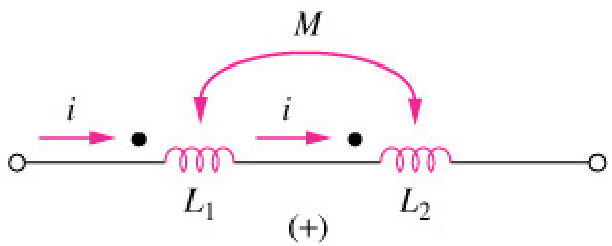


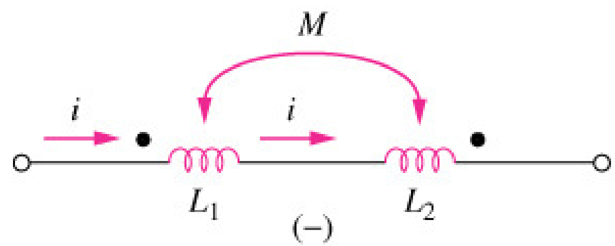
Note: This method differs slightly from the text

KEY CONCEPT !

13.2 Mutual Inductance (12)

Dot convention for coils in series; the sign indicates the polarity of the mutual voltage





$$L = L_1 + L_2 + 2M$$

(series - aiding connection)

$$L = L_1 + L_2 - 2M$$

(series - opposing connection)

We can show this using the model!

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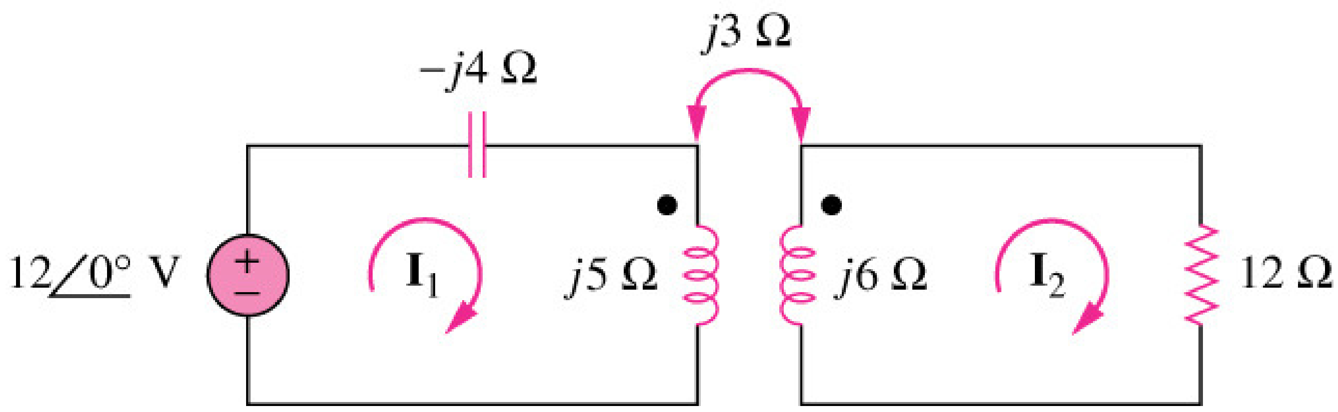
Example 13.1 (Textbook)

Calculate the phasor currents I_1 and I_2 in the circuit shown below.

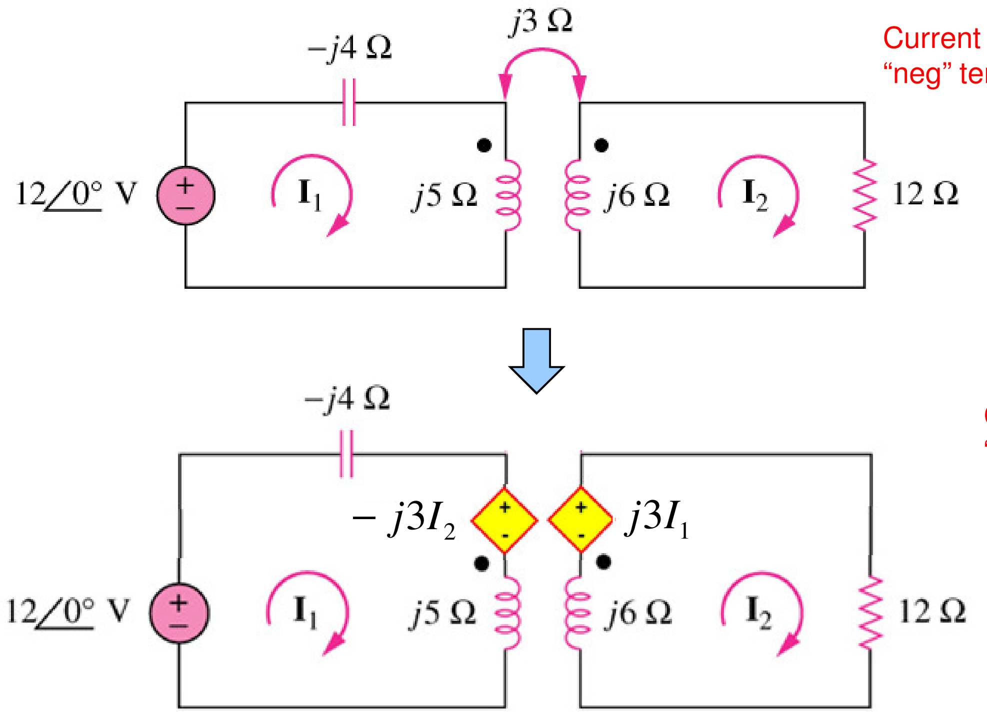


Ans: $I_1 = 13.02\angle -49.40°\text{A};\quad I_2 = 2.91\angle 14.04°\text{A}$

13.2 Mutual Inductance (14)

Example Problem 13.1

First, replace with the model:



Then, solve Loop Equations:

Current enters "neg" terminal

Current enters "+" terminal but value is $-j3I_2$

Loop I_1

$$-12 - j4I_1 + (-j3I_2) + j5I_1 = 0$$

$$jI_1 - j3I_2 = 12$$

Current enters "neg" terminal

Loop I_2

$$j6I_2 - j3I_1 + 12I_2 = 0$$

$$-j3I_1 + (12 + j6)I_2 = 0$$

NOTE:

I_1 goes "into" the dot → Induced voltage on the second coil is "Positive"

I_2 goes "into" the un-dotted side → Induced voltage on the first coil is "Negative"

Pay Attention to Sign Conventions !

13.2 Mutual Inductance (15)

Example Problem 13.1

Lastly, solve 2 equations, 2 unknowns (expected you know this):

Loop I_1 $\longrightarrow$ $jI_1 - j3I_2 = 12$ $\longrightarrow$ $jI_1 = 12 + j3I_2 \Rightarrow I_1 = 3I_2 - j12$

Loop I_2 $\longrightarrow$ $-j3I_1 + (12 + j6)I_2 = 0$

Substitution: $-3(12 + j3I_2) + (12 + j6)I_2 = 0$

÷ by 3 $-12 - j3I_2 + (4 + j2)I_2 = 0$

Collect Terms: $(4 - j)I_2 = 12$

Solve for I_2 $I_2 = \dfrac{12}{(4 - j)} = 2.824 + j0.706 = \boxed{2.91\angle 14.04° \text{ A}}$

Find I_1 (Substitute back to Eq1)

$$I_1 = 3(2.824 + j0.706) - j12$$
$$I_1 = 8.471 + j2.118 - j12$$
$$I_1 = 8.471 - j9.882 = \boxed{13.02\angle -49.40° \text{ A}}$$

Think about where we could have made mistakes!

- Applying the model incorrectly (wrong sign convention)
- Incorrect Phasor notation (Understand: jωM ↔ j3 ; jωL ↔ j5 ; 1/ jωC ↔ -j4)
- Incorrect sign convention for loop equations
  - Current entering negative terminal of a voltage source
- Understanding of complex numbers!
  - Familiarity with your calculators handling of complex numbers
  - Converting from rectangular to polar

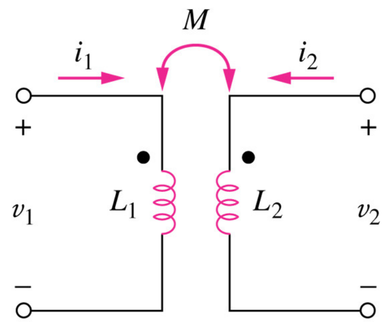
Review / Understand these concepts to avoid these mistakes

13.2 Recommended Viewing:

- Watch these videos illustrating solving mutual inductance problems:
  - http://www.youtube.com/watch?v=tD35a-uzd34
  - http://www.youtube.com/watch?v=hzU4XKQYTWw
  - http://www.youtube.com/watch?v=OqSvesTtnUo
- Uses the model described previously to solve mutual inductance problems.

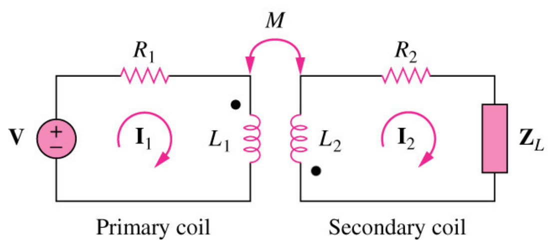
13.3 Energy in a Coupled Circuit (1)

- The instantaneous energy w stored in the circuit is:



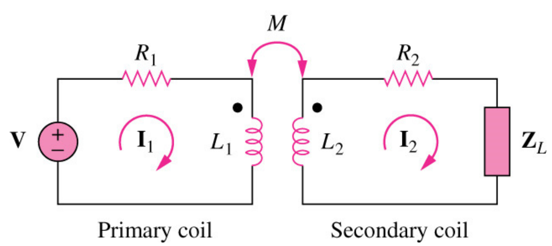
$$\boxed{w = \frac{1}{2}L_1 i_1^2 + \frac{1}{2}L_2 i_2^2 \pm M i_1 i_2}$$

- If **both** currents enter (or **both** leave) the dotted terminal the mutual term is (positive):



$$w = \frac{1}{2}L_1 i_1^2 + \frac{1}{2}L_2 i_2^2 + M i_1 i_2$$

- Otherwise the mutual term is (negative):



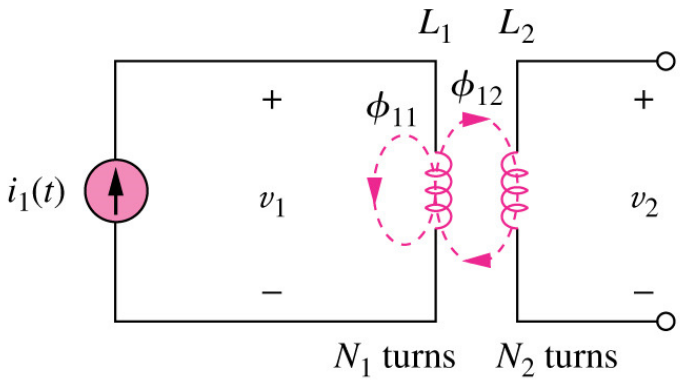
$$w = \frac{1}{2}L_1 i_1^2 + \frac{1}{2}L_2 i_2^2 - M i_1 i_2$$

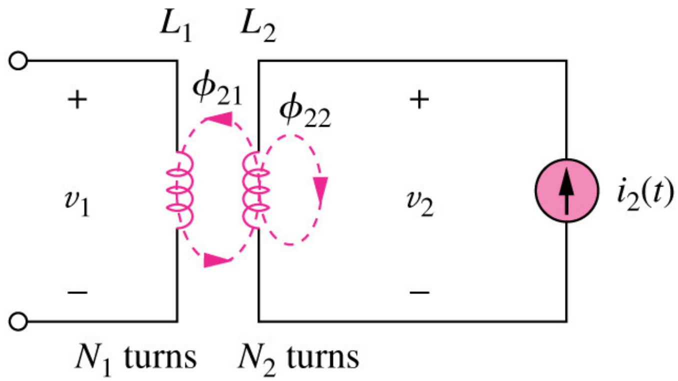
13.3 Energy in a Coupled Circuit (2)

- The coupling coefficient, k, is a measure of the magnetic coupling between two coils; $0 \le k \le 1$.

$$\boxed{M = k\sqrt{L_1 L_2}}$$

K = 1 coils perfectly coupled
K < 0.5 coils loosely coupled
K > 0.5 coils tightly coupled

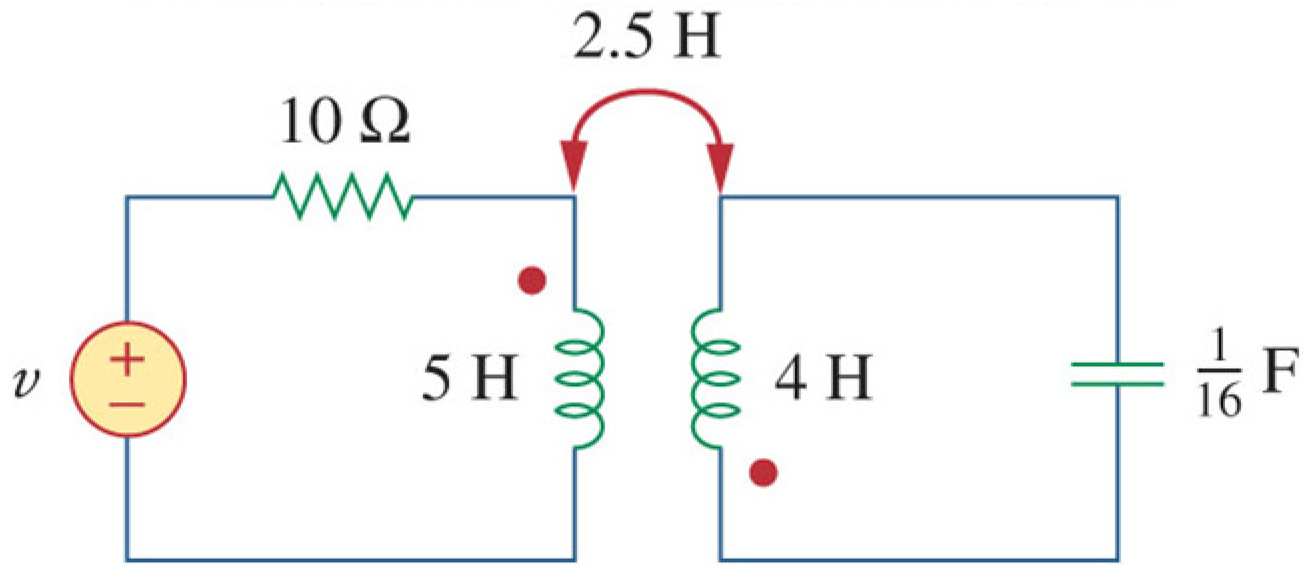




$$K = \frac{\phi_{12}}{\phi_{11} + \phi_{12}} \qquad K = \frac{\phi_{21}}{\phi_{21} + \phi_{22}}$$

Example 13.3 (Textbook)

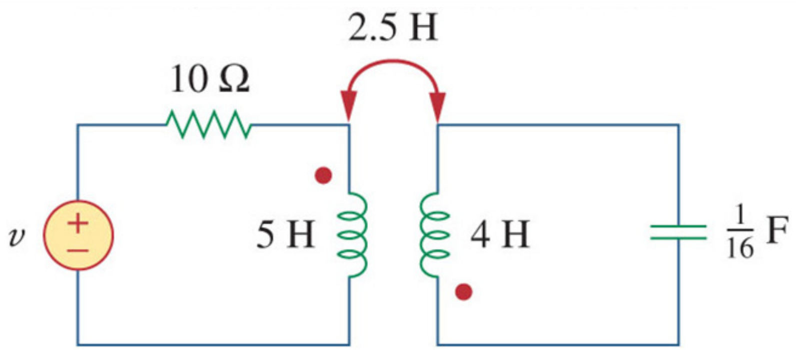
Consider the circuit below. Determine the coupling coefficient. Calculate the energy stored in the coupled inductors at time t = 1s if $v=60\cos(4t+30°)$ V.



Ans: k=0.56; $w(1)$=20.73J

13.3 Energy in a Coupled Circuit (4)

Example 13.3

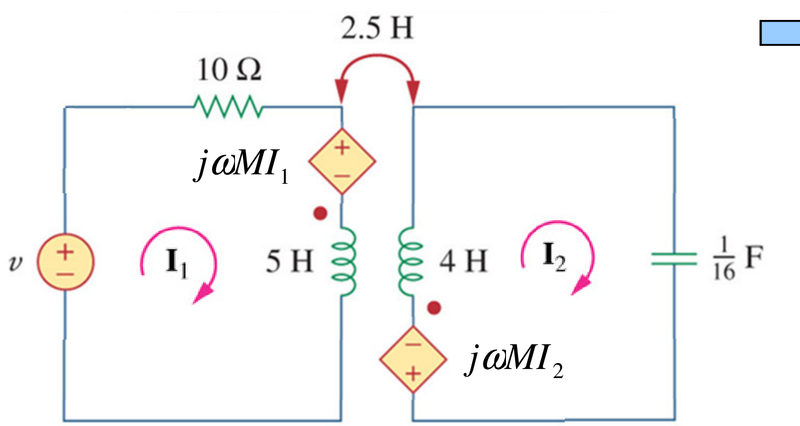


To find k, use the relation between L_1, L_2, and M:

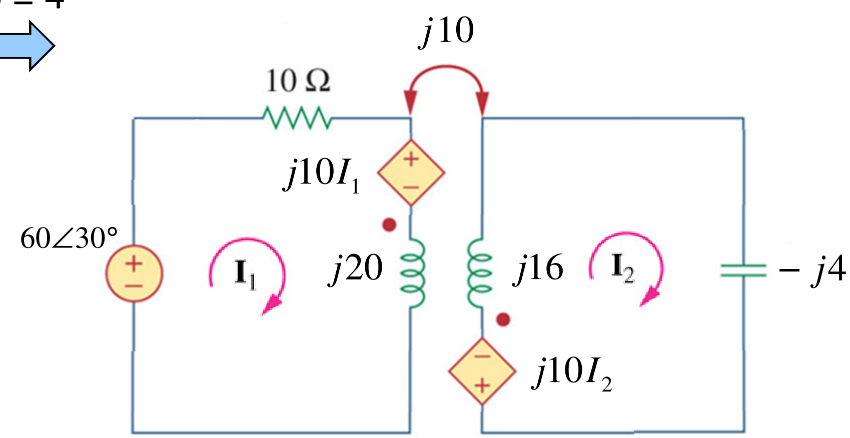
$$L_1 = 5 \ ; \ L_2 = 4 \ ; \ M = 2.5$$

$$k = \frac{M}{\sqrt{L_1 L_2}} = \frac{2.5}{\sqrt{(5)(4)}} = 0.56$$

Apply the "Model"



Convert to Phasor
ω = 4



Solve Mesh equations for I_1 and I_2

13.3 Energy in a Coupled Circuit (5)

Example 13.3

From Mesh analysis:

$$I_1 = 3.905\angle -19.4° \text{ A}$$
$$I_2 = 3.254\angle 160.6° \text{ A}$$

At t = 1, the value of ωt = (4)(1) = 4 radians = 229.2º

To find the energy at t = 1:

$$i_1 = 3.905\cos(\omega t - 19.4°)$$
$$i_2 = 3.254\cos(\omega t + 160.6°)$$

$$i_1 = 3.905\cos(229.2° - 19.4°) = -3.389$$
$$i_2 = 3.254\cos(229.2° + 160.6°) = 2.824$$

Positive since current (as defined) enters **both** dots

$$w = \frac{1}{2}L_1 i_1^2 + \frac{1}{2}L_2 i_2^2 + Mi_1 i_2$$

$$w = \frac{1}{2}(5)(-3.389)^2 + \frac{1}{2}(4)(2.824)^2 + 2.5(-3.389)(2.824) = 20.73 \text{ J}$$

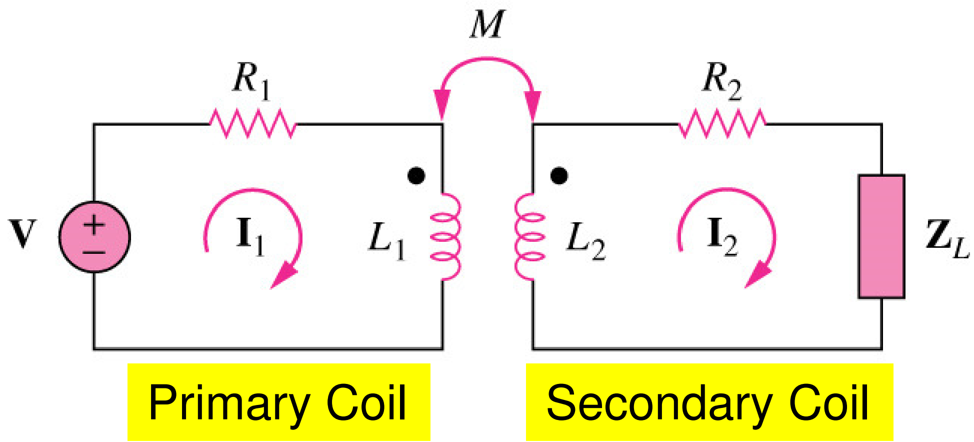
Homework #2

Due in class Monday, January 26, 2015

- 13.1
- 13.7
- 13.9
- 13.24

13.4 Linear Transformers (1)

- A transformer is generally a four-terminal device comprising two (or more) magnetically coupled coils. Below is a "simple" model:

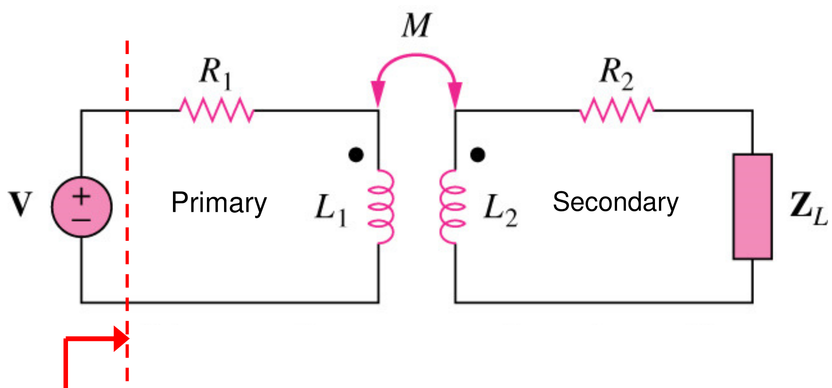


- The coil connected to the voltage source is called the primary winding.
- The coil connected to the load is called the secondary winding.
- Resistances R_1 and R_2 are included to account for the losses in the coils.
- A transformer is said to be linear if the coils are wound on a magnetically linear material for which the permeability is constant.
  - Air, plastic, Bakelite, wood, etc.
  - Most materials are magnetically linear.

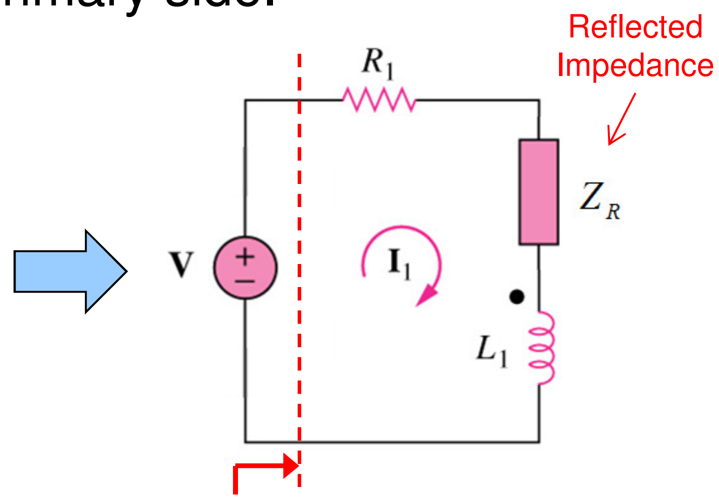
13.4 Linear Transformers (2)

Reflected Impedance

- Often we are interested in the input impedance Z_{in} seen by the source.
  - For example, may want to match Z_{in} to the source impedance for maximum power transfer!
- To simplify our analysis we can break Z_{in} up into:
  - $Z_{primary}$ -- The impedance of the "primary" circuit $(Z_{primary} = R_1 + j\omega L_1)$
  - Z_R -- The "reflected" impedance back to the "primary".
- The "reflected" impedance Z_R is the contribution of the secondary circuit loading to the primary side.



$$Z_{in} = Z_{primary} + Z_R$$

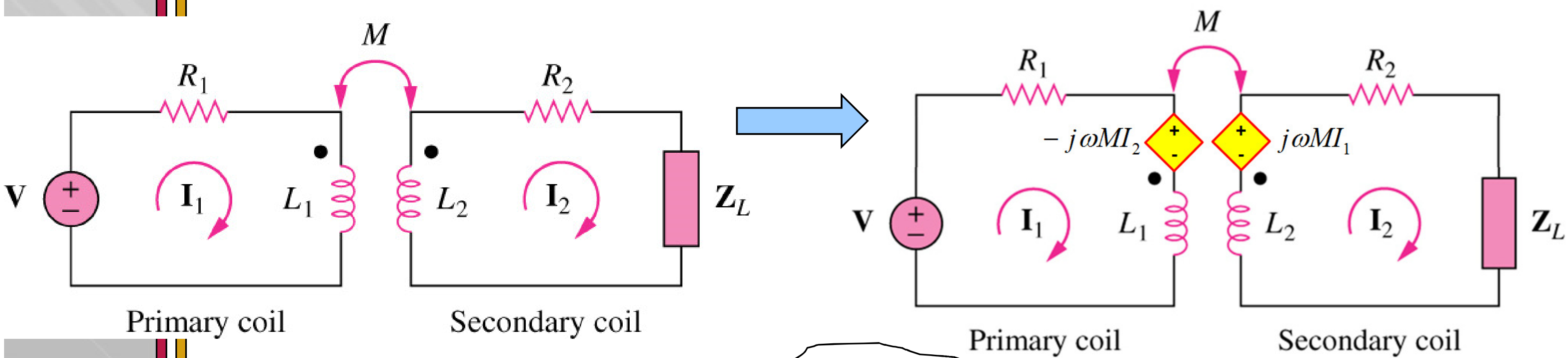


$$Z_{in} = Z_{primary} + Z_R$$

13.4 Linear Transformers (2)

Reflected Impedance

To obtain the input impedance Z_{in} as seen from the source:



Mesh 1: $V = (R_1 + j\omega L_1)I_1 - j\omega MI_2$

Current enters positive terminal so it is **+** ($-j\omega MI_2$)

Mesh 2: $0 = -j\omega MI_1 + (R_2 + j\omega L_2 + Z_L)I_2$

Current enters negative terminal so it is **-** ($j\omega MI_2$)

From Mesh 2:

$$I_2 = \frac{j\omega M}{R_2 + j\omega L_2 + Z_L} I_1$$

Substituting into Mesh 1 gives:

$$V = (R_1 + j\omega L_1)I_1 + \frac{\omega^2 M^2}{R_2 + j\omega L_2 + Z_L} I_1$$

$$V = Z_{primary}\, I_1 + Z_{reflected}\, I_1$$

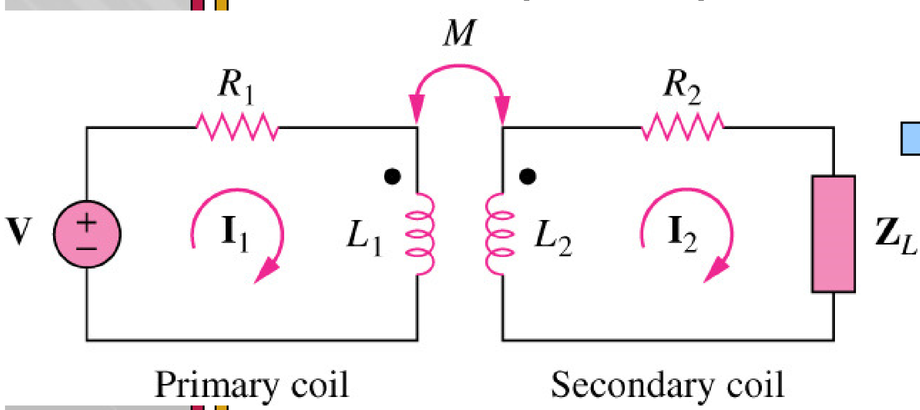
Reflected Impedance

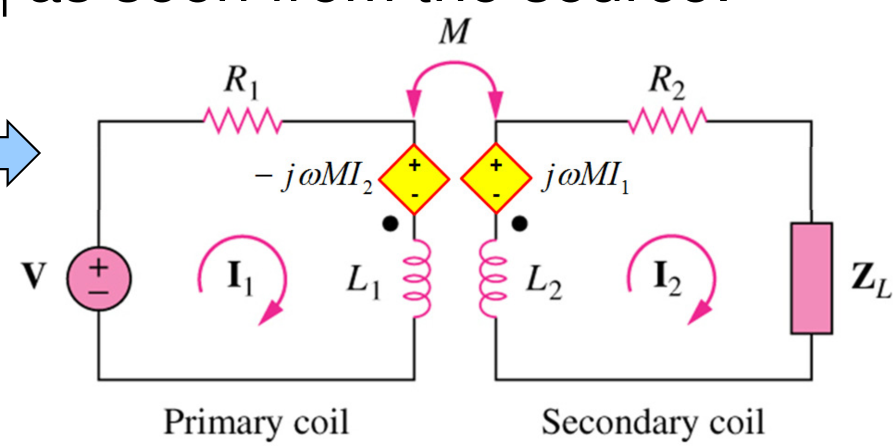
$$Z_R = \frac{\omega^2 M^2}{R_2 + j\omega L_2 + Z_L}$$

13.4 Linear Transformers (3)

Reflected Impedance (Another way of looking at it !)

Obtain input impedance Z_{in} as seen from the source:





Mesh 1: $V = \left(Z_{primary}\right)I_1 - j\omega M I_2$

Mesh 2: $0 = -j\omega M I_1 + (Z_{Secondary})I_2$

$Z_{Secondary}$ = the total series impedance in the secondary loop

From Mesh 2:

$$I_2 = \frac{j\omega M}{Z_{Secondary}} I_1$$

Substituting into Mesh 1 gives:

$$V = \underbrace{Z_{primary}}\, I_1 + \underbrace{\frac{\omega^2 M^2}{Z_{Secondary}}}\, I_1$$

$$V = Z_{primary} I_1 + Z_{reflected} I_1$$

Reflected Impedance

$$\boxed{Z_R = \frac{\omega^2 M^2}{Z_{Secondary}}}$$

13.4 Linear Transformers (4)

Reflected Impedance

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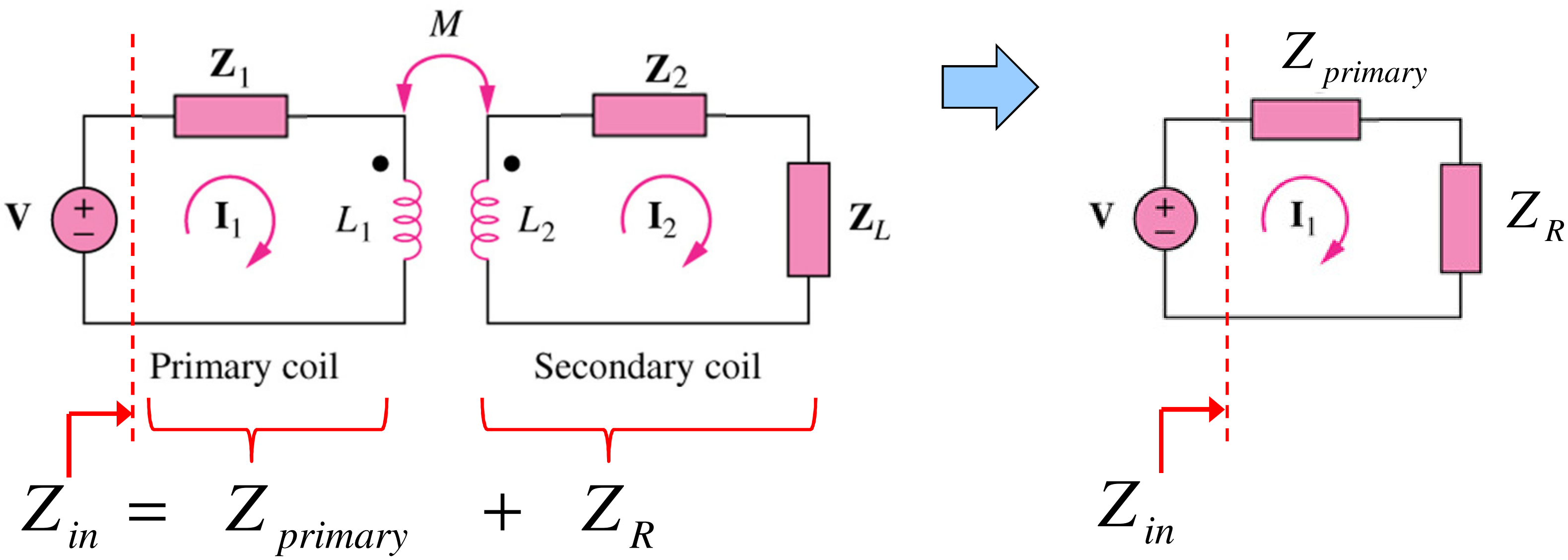
- The input impedance can be broken into two parts as follows:



$$Z_{in} = \underbrace{Z_{primary}}_{\text{Primary coil}} + \underbrace{Z_R}_{\text{Secondary coil}}$$

- Series Impedance in Primary Coil: $Z_{primary} = Z_1 + j\omega L_1$

- Series Impedance in Secondary Coil: $Z_{secondary} = Z_2 + Z_L + j\omega L_2$

- Input Impedance: $\boxed{Z_{in} = Z_{primary} + \dfrac{\omega^2 M^2}{Z_{Secondary}}}$

Note: Z_{in} will be the same if the dot on L2 is switched

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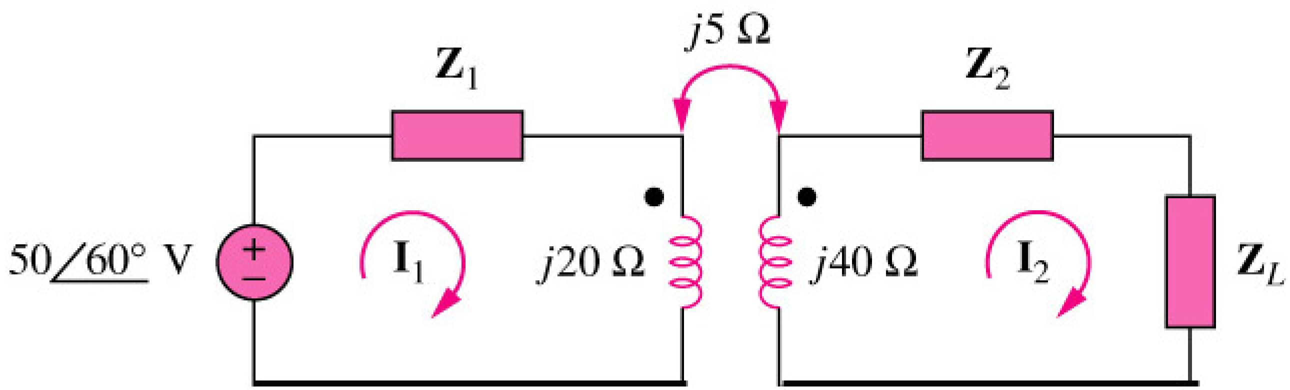
Example 13.4 (textbook)

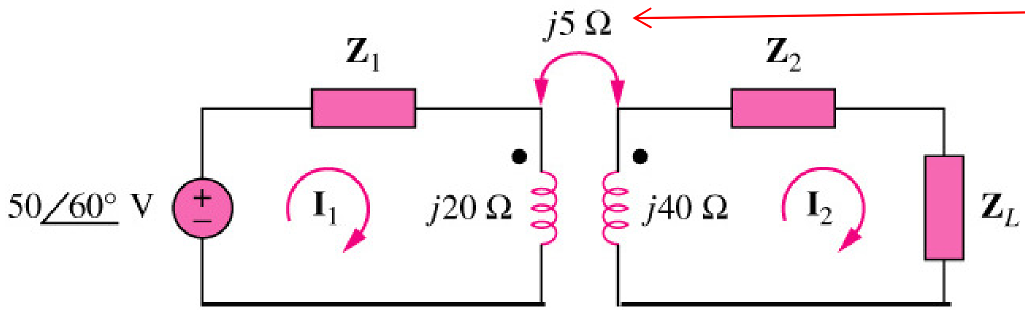
In the circuit below, calculate the input impedance and current I_1. Take Z_1=60-j100Ω, Z_2=30+j40Ω, and Z_L=80+j60Ω.



Ans: $$Z_{in} = 100.14\angle -53.1°\Omega;\ I_1 = 0.5\angle 113.1°\mathrm{A}$$

13.4 Linear Transformers (5)

Example 13.4



Note:

$$j\omega M = j5$$
$$\omega M = 5$$
$$(\omega M)^2 = 25$$

- The series impedance in the primary coil:

$$Z_{primary} = (60 - j100) + j20 = 60 - j80$$

- The series impedance in the primary coil:

$$Z_{secondary} = (30 + j40) + (80 + j60) + j40 = 110 + j140$$

- Input Impedance given by:

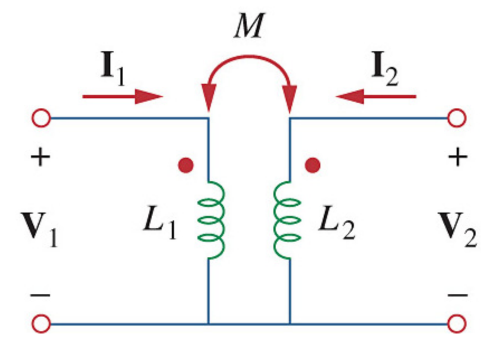
$$Z_{in} = Z_{primary} + \frac{\omega^2 M^2}{Z_{Secondary}} = (60 - 80j) + \frac{25}{(110 + j140)} = 60.09 - j80.11$$

- Current I_1 given by: $I_1 = \dfrac{V_s}{Z_{in}} = \dfrac{50\angle 60°}{60.09 - j80.11} = \dfrac{50\angle 60°}{100.14\angle -53.1°}$

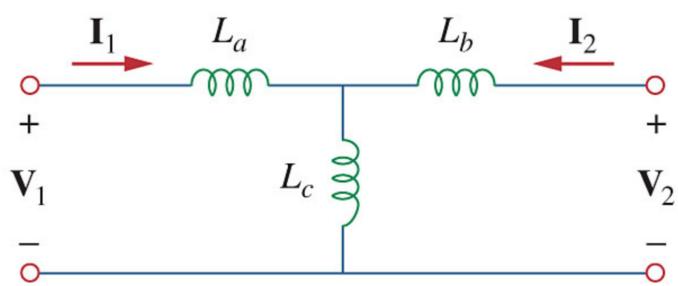
13.4 Linear Transformers (6)

Equivalent T and π Circuits:

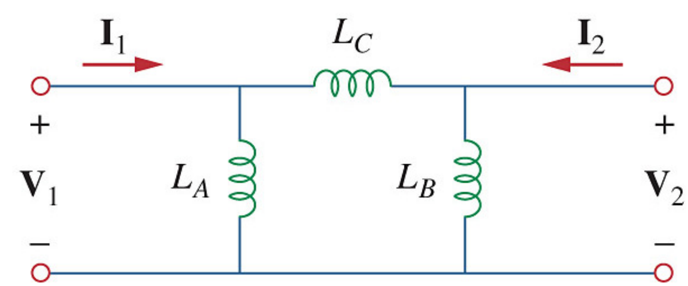
- It is sometimes convenient to replace a magnetically couple circuit with an equivalent circuit with no magnetic coupling.
- We can replace the linear transformer with an equivalent T or π circuit that has no mutual inductance.



T Circuit



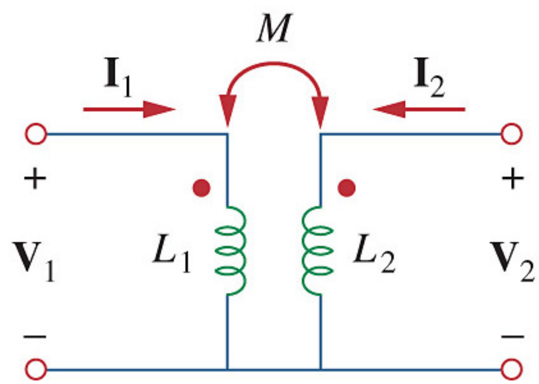
π Circuit



13.4 Linear Transformers (7)

Equivalent T Circuit:

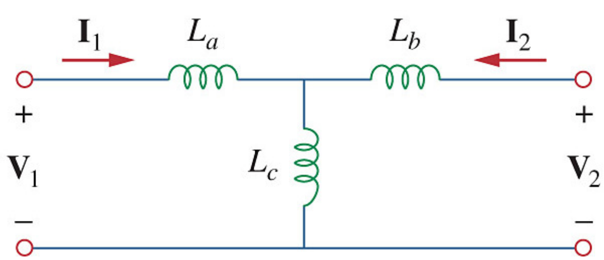
Linear Transformer Circuit



Mesh Analysis of this transformer circuit results in the following:

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} j\omega L_1 & j\omega M \\ j\omega M & jwL_2 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

Equivalent T Network



Mesh Analysis of the equivalent T network results in the following:

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} j\omega(L_a + L_c) & jwL_c \\ jwL_c & jw(L_b + L_c) \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

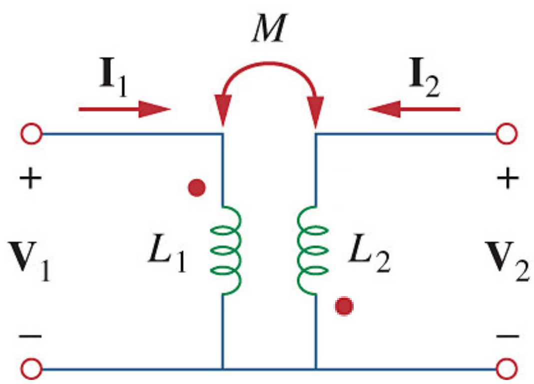
Equating the terms gives the following relationships:

$L_a = L_1 - M$ $\qquad$ $L_b = L_2 - M$ $\qquad$ $L_c = M$

13.4 Linear Transformers (8)

Equivalent T Circuits (Swapped Dots):

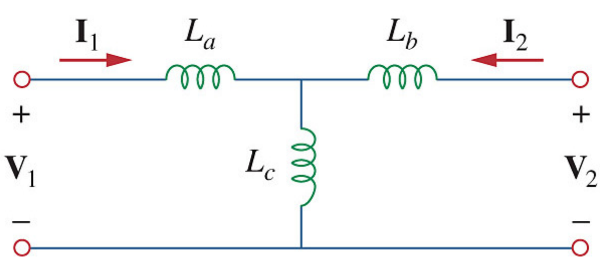
Linear Transformer Circuit



Mesh Analysis of this transformer circuit results in the following:

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} j\omega L_1 & -j\omega M \\ -j\omega M & jwL_2 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

Equivalent T Network



Mesh Analysis of the equivalent T network results in the following:

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} j\omega(L_a + L_c) & jwL_c \\ jwL_c & jw(L_b + L_c) \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

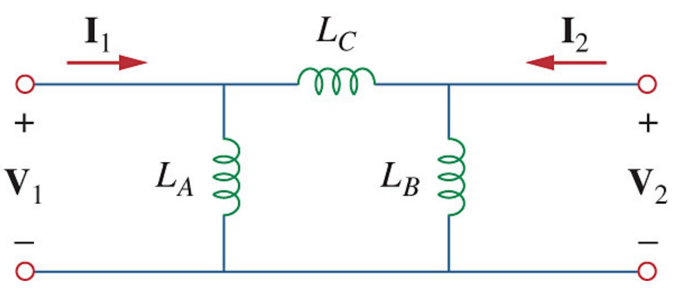
Equating the terms gives the following relationships:

$L_a = L_1 + M$ $\qquad$ $L_b = L_2 + M$ $\qquad$ $L_c = -M$

13.4 Linear Transformers (9)

Equivalent π Circuit:

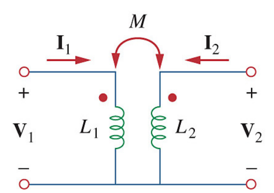
Similarly, for the π network nodal analysis provides:



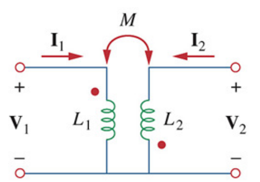
Equivalent π Network

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} \dfrac{1}{j\omega L_A} + \dfrac{1}{j\omega L_C} & -\dfrac{1}{j\omega L_C} \\ -\dfrac{1}{j\omega L_C} & \dfrac{1}{j\omega L_B} + \dfrac{1}{j\omega L_C} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

By equating terms in admittance matrices, for the π equivalent network we obtain (note if dots are different, replace with –M):



$$L_A = \frac{L_1 L_2 - M^2}{L_2 - M} \;;\; L_B = \frac{L_1 L_2 - M^2}{L_1 - M} \;;\; L_C = \frac{L_1 L_2 - M^2}{M}$$



$$L_A = \frac{L_1 L_2 - M^2}{L_2 + M} \;;\; L_B = \frac{L_1 L_2 - M^2}{L_1 + M} \;;\; L_C = \frac{L_1 L_2 - M^2}{-M}$$

13.4 Linear Transformers (10)

Equivalent T or π Circuits Summary

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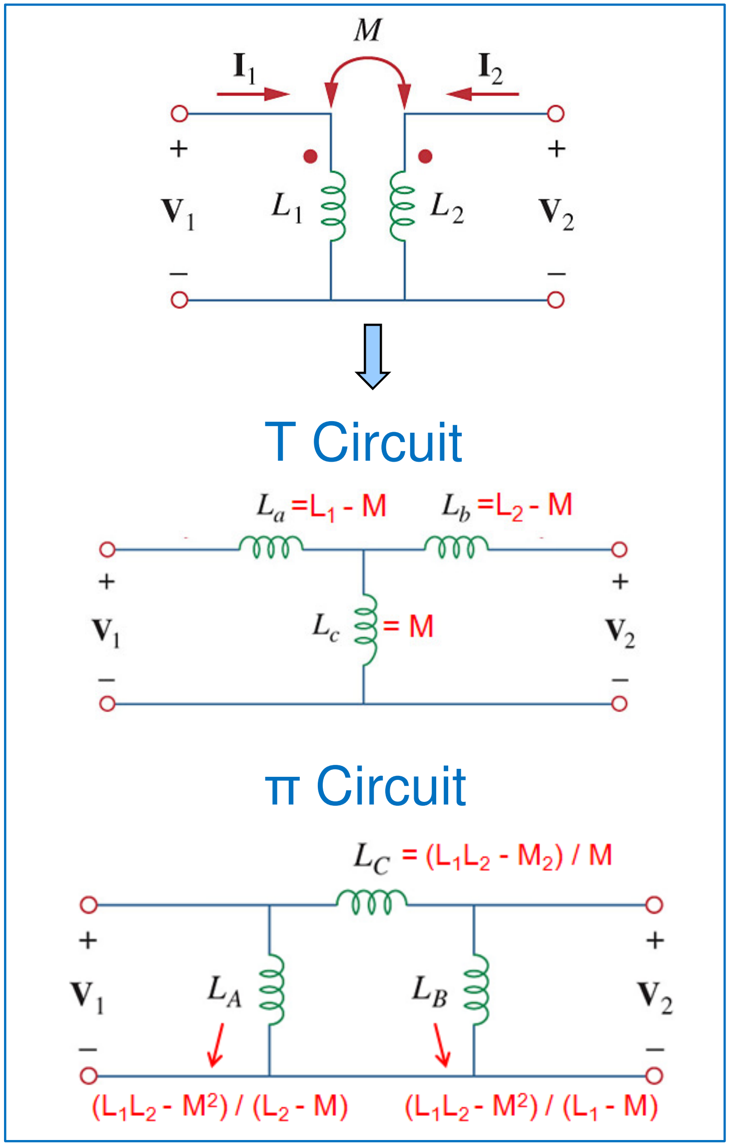
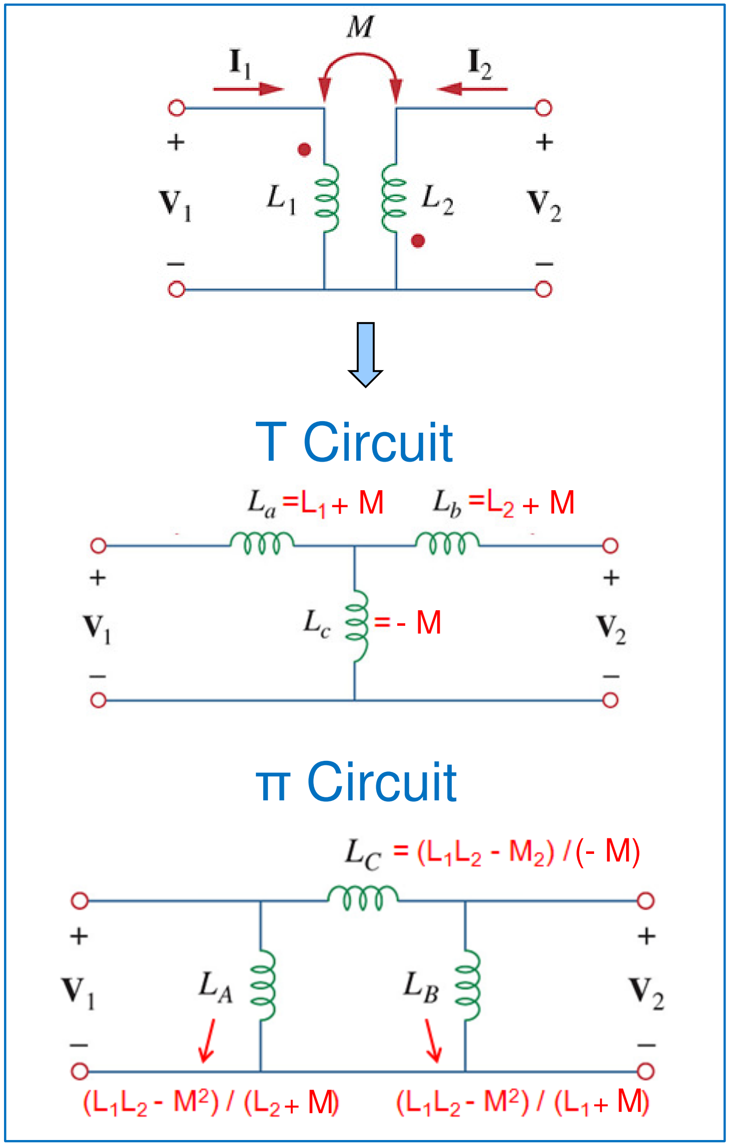
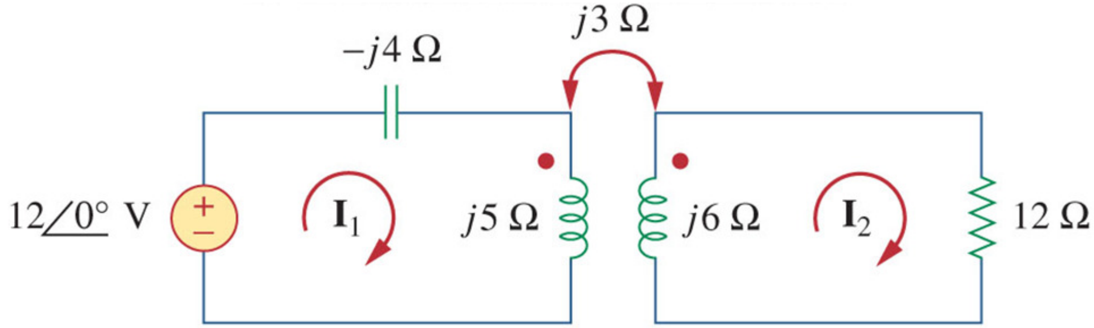


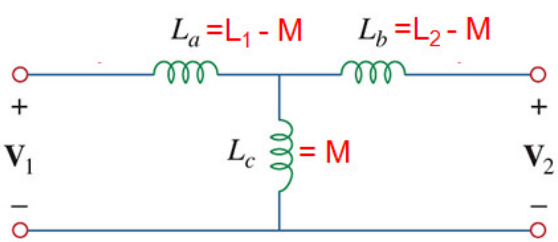


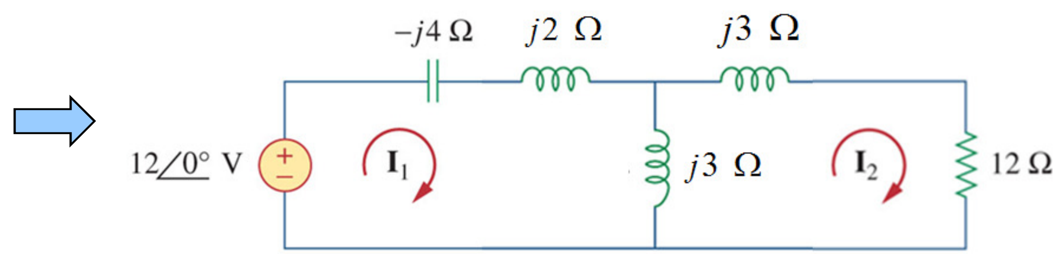
13.4 Linear Transformers (11)

Practice Problem 13.6

Find I_1 and I_2 using the T equivalent circuit





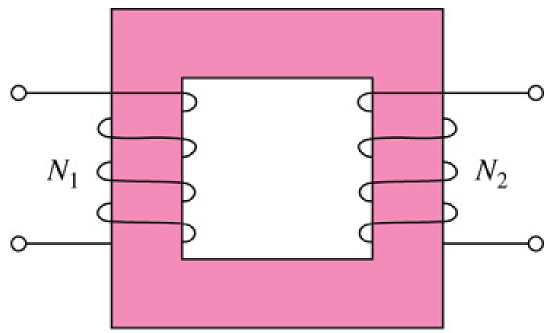


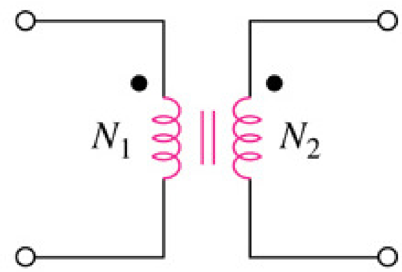
Mesh I_1: $-12\angle 0° - j4I_1 + j2I_1 + j3(I_1 - I_2) = 0$ $\qquad jI_1 - j3I_2 = 12\angle 0°$

Mesh I_2: $j3(I_2 - I_1) + j3I_2 + 12I_2 = 0$ $\qquad -j3I_1 + (12 + j6)I_2 = 0$

13.5 Ideal Transformers (1)

- An ideal transformer has perfect coupling (k=1).
- It consists of two or more coils with a large number of turns wound on a common core of high permeability.



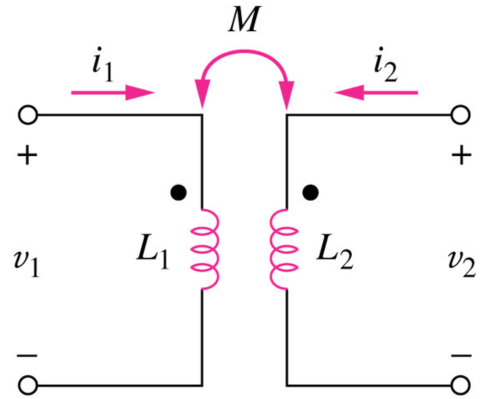


- Because of the high permeability of the core, the flux links all the turns of both coils, thereby resulting in a perfect coupling.

13.5 Ideal Transformers (2)

"Dot's the same polarity"

Recall the coupled circuit:



$$V_1 = j\omega L_1 I_1 + j\omega M I_2 \qquad (1)$$

$$V_2 = j\omega M I_1 + j\omega L_2 I_2 \qquad (2)$$

Solving for I_1 in (1): $\mathrm{I}_1 = \dfrac{V_1 - j\omega M I_2}{j\omega L_1}$ (3)

Substituting (3) into (2): $V_2 = j\omega L_2 I_2 + \dfrac{M V_1}{L_1} - \dfrac{j\omega M^2 I_2}{L_1}$

$M = \sqrt{L_1 L_2}$ For perfect coupling (k=1):

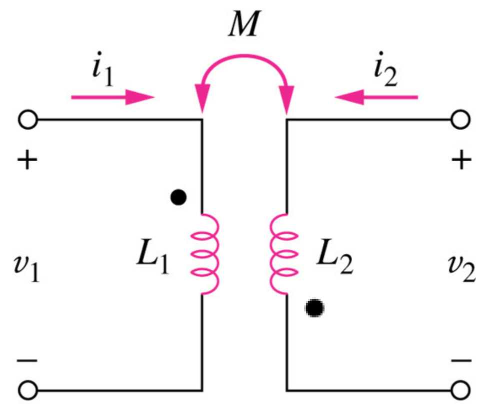
$$V_2 = j\omega L_2 I_2 + \frac{\sqrt{L_1 L_2}V_1}{L_1} - \frac{j\omega L_1 L_2 I_2}{L_1} = \sqrt{\frac{L_2}{L_1}}V_1 = nV_1$$

Therefore: $V_2 = nV_1$ where $n = \sqrt{L_2 / L_1}$ = turns ratio

13.5 Ideal Transformers (3)

“Dot’s opposite each other”

Mesh Equations give the following:



$$V_1 = j\omega L_1 I_1 - j\omega M I_2 \qquad (1)$$

$$V_2 = -j\omega M I_1 + j\omega L_2 I_2 \qquad (2)$$

Solving for I_1 in (1): $\mathrm{I}_1 = \dfrac{V_1 + j\omega M I_2}{j\omega L_1}$ (3)

Substituting (3) into (2): $V_2 = j\omega L_2 I_2 - \dfrac{MV_1}{L_1} - \dfrac{j\omega M^2 I_2}{L_1}$

$M = \sqrt{L_1 L_2}$ For perfect coupling (k=1):

$$V_2 = j\omega L_2 I_2 - \frac{\sqrt{L_1 L_2}V_1}{L_1} - \frac{j\omega L_1 L_2 I_2}{L_1} = -\sqrt{\frac{L_2}{L_1}}V_1 = -nV_1$$

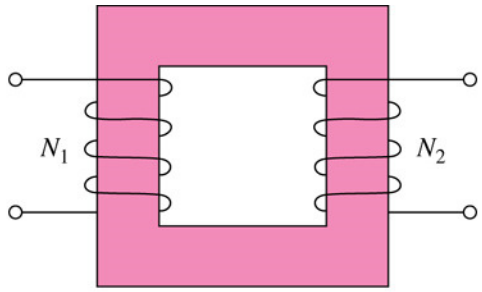
Therefore: $V_2 = -nV_1$ If dot is swapped at output

13.5 Ideal Transformers (4)
Properties

- A transformer is said to be ideal if it has the following properties:
  1. Coils have very large reactances (L_1, L_2, $M \rightarrow \infty$)
  2. Coupling coefficient is equal to unity (k=1)
  3. Primary and secondary coils are lossless ($R_1 = R_2 = 0$)
- An ideal transformer is a unity-coupled (k=1) lossless transformer in which the primary and secondary coils have infinite self-inductances (L_1 & $L_2 \rightarrow \infty$).
- Iron core transformers are close approximations to ideal transformers and are used in power systems and electronics.

13.5 Ideal Transformers (5)

- When a sinusoidal voltage is applied to the primary winding, the same magnetic flux Φ goes through both windings.



$$v_1 = N_1 \frac{d\phi}{dt} \quad ; \quad v_2 = N_2 \frac{d\phi}{dt}$$

$$\frac{v_2}{v_1} = \frac{N_2}{N_1} = n =$$ Turns ratio or transformation ratio

- Using the phasor voltages rather than the instantaneous voltages:

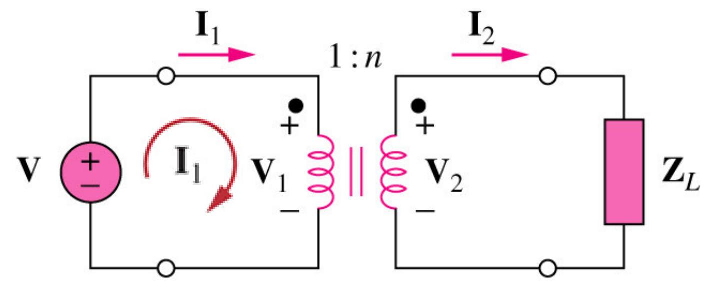
$$\frac{V_2}{V_1} = \frac{N_2}{N_1} = n$$

13.5 Ideal Transformers (6)

- Power conservation: $v_1 i_1 = v_2 i_2$

- The energy supplied to the primary must equal the energy absorbed by the secondary, since there are no losses in an ideal transformer.

- In phasor form: $\boxed{\dfrac{I_1}{I_2} = \dfrac{V_2}{V_1} = n}$



  - n=1 → isolation transformer ($V_2 = V_1$)
  - n>1 → step-up transformer ($V_2 > V_1$)
  - n<1 → step-down transformer ($V_2 < V_1$)

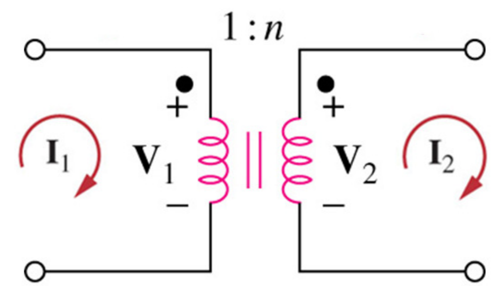
13.5 Ideal Transformers (7)

- Transformer ratings are usually specified as V_1 / V_2
- Power companies often generate at some convenient voltage and use the step-up transformer to increase the voltage so that the power can be transmitted at very high voltage and low current over transmission lines, resulting in significant cost savings. Near residential consumers, step-down transformers are used to bring the voltage down to 120 V.
- It is important to get the proper polarity of the voltages and the direction of the currents for the transformer.

13.5 Ideal transformers (8)

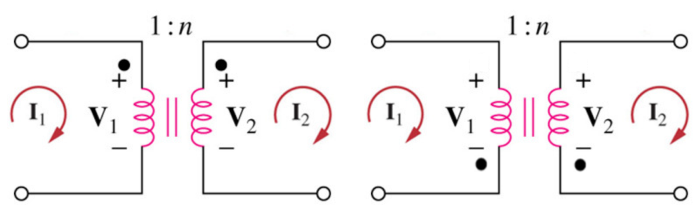
V and I relationships (simpler way to remember)

Given this standard definition for Voltages and currents



Note: This definition of I_2 differs from the text.

Dots "Same" → "+"



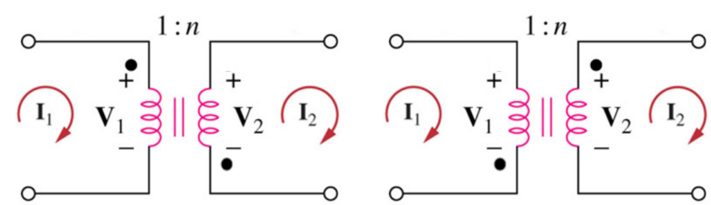
$$V_2 = nV_1$$

$$I_2 = \frac{I_1}{n}$$

$$Z_{in} = \frac{Z_L}{n^2}$$

Reflected Impedance

Dots "Different" → "-"



$$V_2 = -nV_1$$

$$I_2 = -\frac{I_1}{n}$$

$$Z_{in} = \frac{Z_L}{n^2}$$

Note: The larger the turns ratio
The larger "n"
The larger "N_2"
The larger V_2

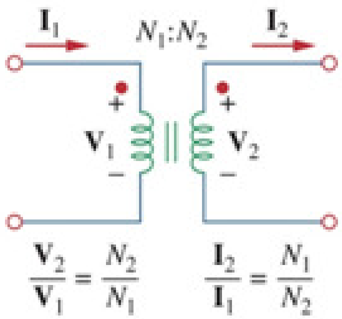
Turns Ratio

$$n = \frac{N_2}{N_1}$$

13.5 Ideal Transformers (9)

V and I relationships

- Expressing V_1 in terms of V_2 and I_1 in terms of I_2 or vice versa:



$$V_1 = \frac{V_2}{n} \qquad V_2 = nV_1$$

Positive, if Voltage "**same**" polarity at dot

$$I_2 = \frac{I_1}{n} \qquad I_1 = nI_2$$

Positive, if current "**different**" polarity at dot

- Complex Power is:

Complex Conjugate of I_2

$$S_1 = V_1 I_1^* = \frac{V_2}{n}(nI_2)^* = V_2 I_2^* = S_2$$

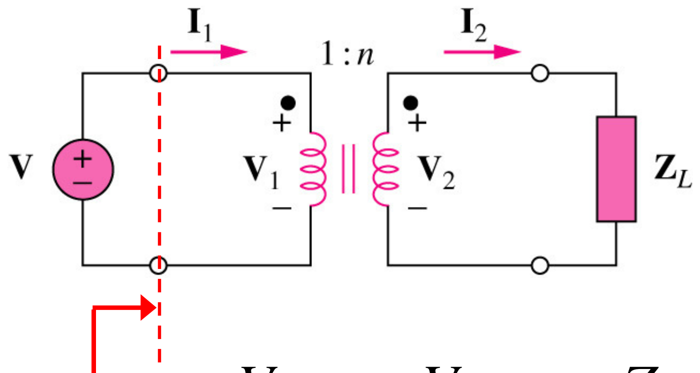
- Complex power supplied to the primary is delivered to the secondary without loss.

- The ideal transformer is **lossless** and absorbs **no power**.

13.5 Ideal Transformers (10)

Reflected impedance

- The input impedance as seen by the source is:



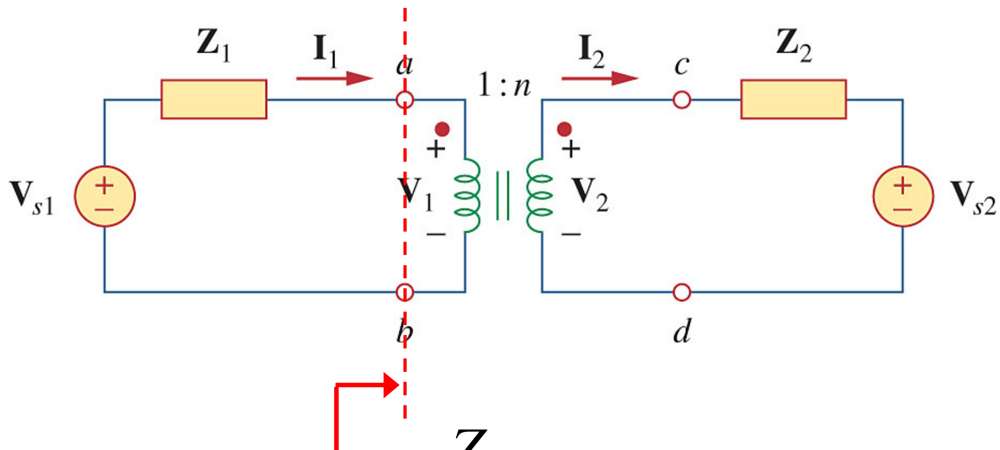
$$Z_{in} = \frac{V_1}{I_1} = \frac{V_2}{n^2 I_2} = \frac{Z_L}{n^2}$$

- The input impedance is also called the reflected impedance since it appears as if the load impedance is reflected to the primary side.
- The ability of the transformer to transform a given impedance to another allows impedance matching to ensure maximum power transfer.

13.5 Ideal Transformers (11)

Equivalent circuit analysis

- In analyzing a circuit containing an ideal transformer, it is common practice to eliminate the transformer by reflecting impedances and sources from one side of the transformer to the other.
- Suppose we want to reflect the secondary side of the circuit to the primary side:

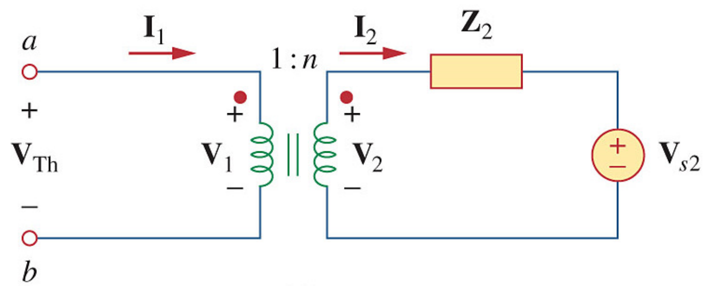


$$Z_R = \frac{Z_2}{n^2}$$

13.5 Ideal Transformers (12)

Equivalent circuit analysis

- We find the Thevenin equivalent of the circuit to the right of a-b:
- Obtaining V_{th} from "open circuit voltage":



(a)

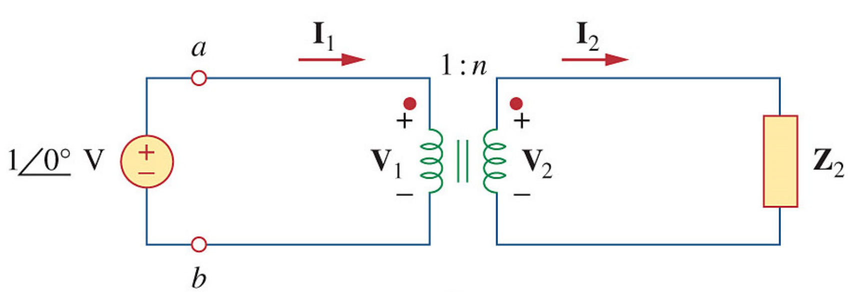
$I_1 = 0 = I_2$ Since a-b is open

$V_2 = V_{s2}$

$$V_{Th} = V_1 = \frac{V_2}{n} = \boxed{\frac{V_{s2}}{n}}$$

- Obtaining Z_{Th} (remove the voltage source in the secondary and insert a unit source at a-b terminals.)

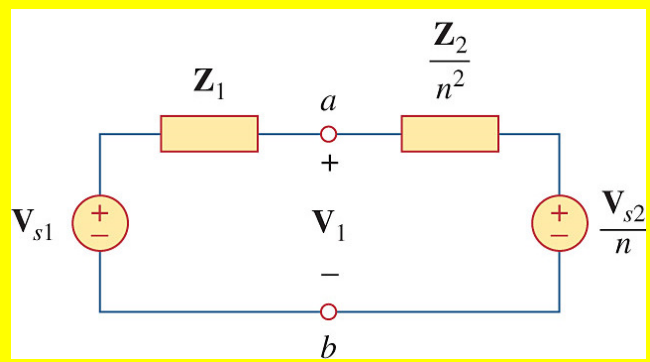
$$Z_{Th} = \frac{V_1}{I_1} = \frac{V_2}{n^2 I_2} = \boxed{\frac{Z_2}{n^2}}$$



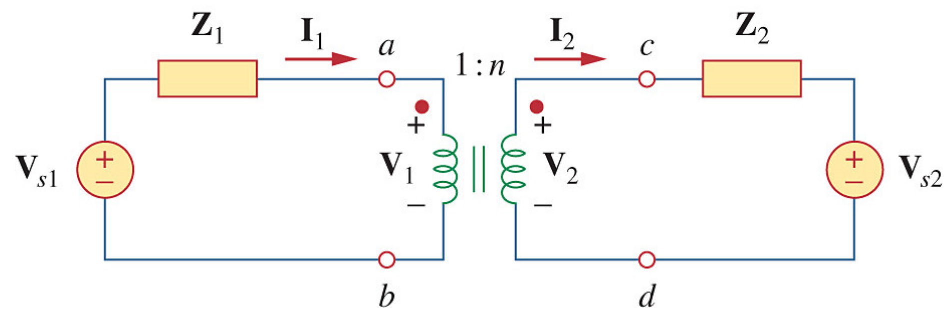
(b)

Equivalent Circuit

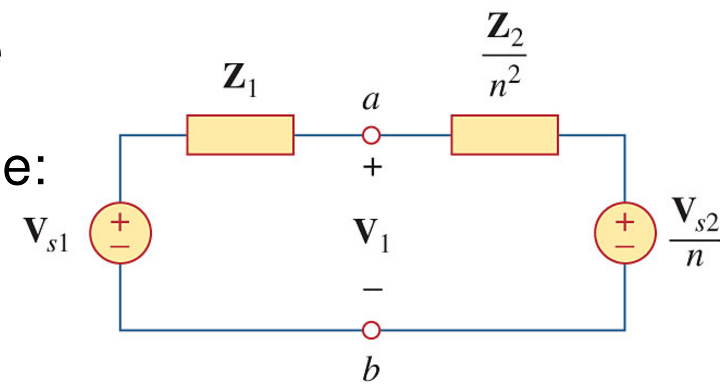
Reflecting the secondary to the primary



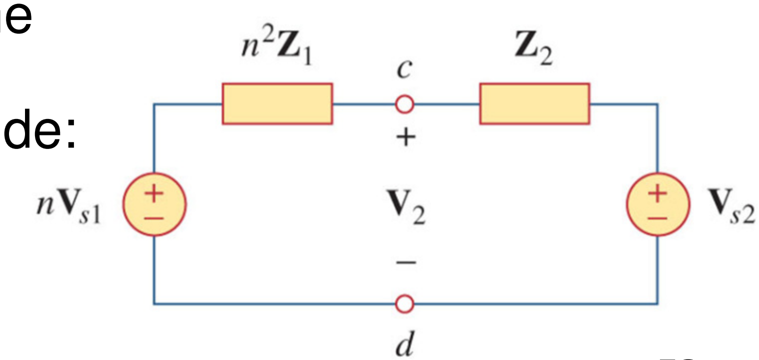
13.5 Ideal Transformers (13)



- The general rule for eliminating the transformer and reflecting the secondary circuit to the primary side:
  - Divide the secondary impedance by n^2
  - Divide the secondary voltage by n
  - Multiply the secondary current by n



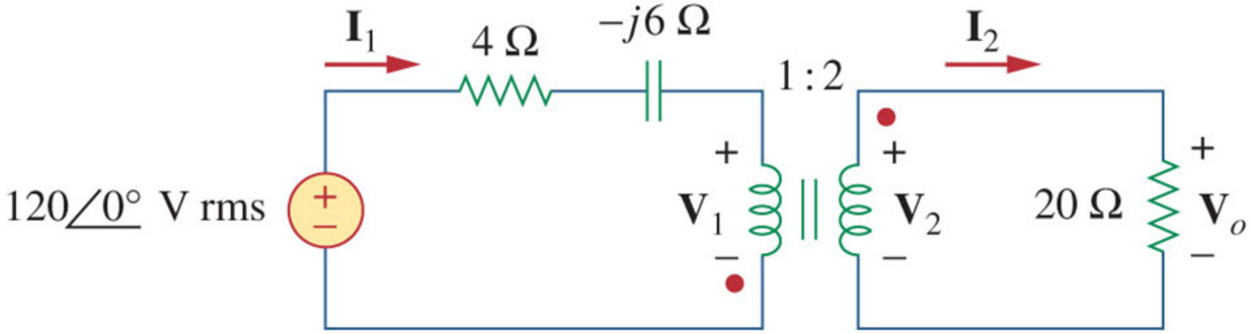
- The general rule for eliminating the transformer and reflecting the secondary circuit to the primary side:
  - Multiply the primary impedance by n^2
  - Multiply the primary voltage by n
  - Divide the primary current by n



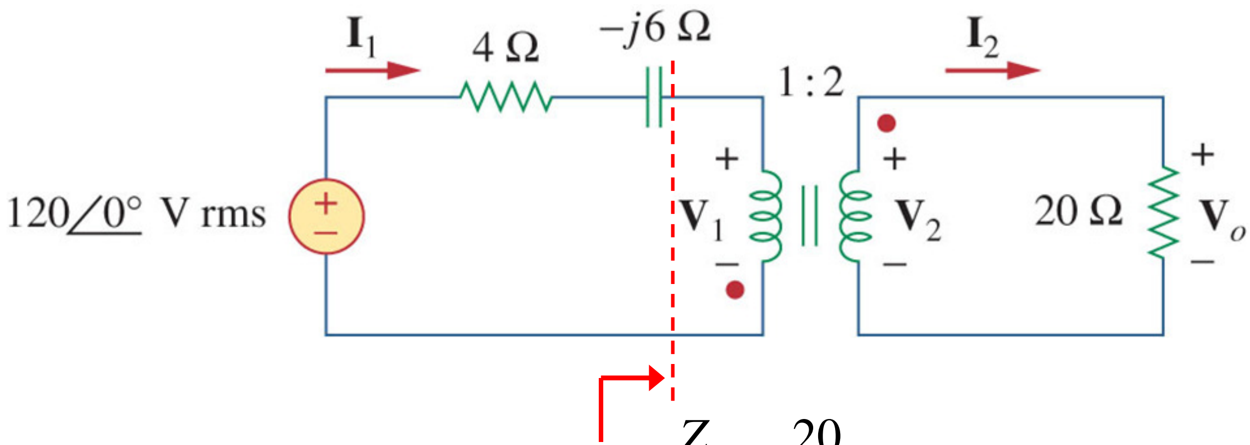
13.5 Ideal Transformer (14)

Example 13.8 (Textbook)

For the ideal transformer, find: (a) the source current I_1, (b) the output voltage V_0, and (c) the complex power supplied by the source



13.5 Ideal Transformer (14)



$$Z_R = \frac{Z_2}{n^2} = \frac{20}{2^2} = 5$$

Impedance seen by the Voltage source is:

$$Z_{in} = (4 - j6) + 5 = 9 - j6 = 10.82\angle -33.69^\circ \;\Omega$$

Input current I_1 is:

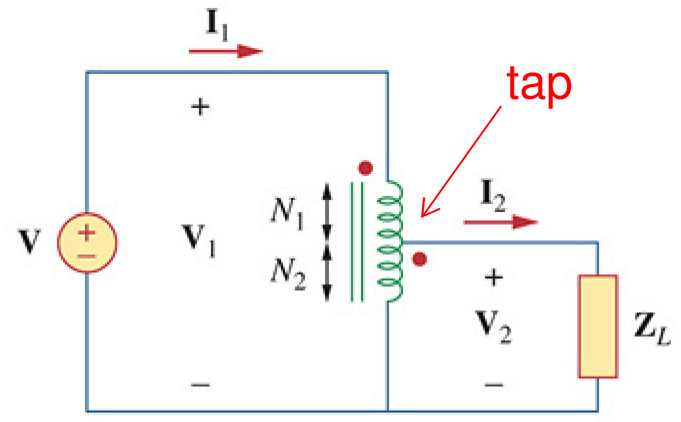
$$I_1 = \frac{V_s}{Z_{in}} = \frac{120\angle 0^\circ}{10.82\angle -33.69^\circ} = 11.09\angle 33.69^\circ \text{ A}$$

$$I_2 = -\frac{I_1}{n} = \frac{-11.09\angle 33.69^\circ}{2} = 5.55\angle -146.31^\circ \text{ A}$$

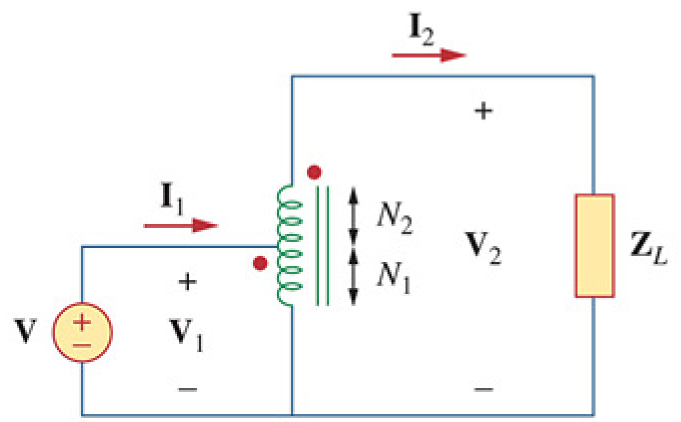
$$V_o = 20 I_2 = 20(5.55\angle -146.31^\circ) = 110.9\angle -146.31^\circ \text{ V}$$

13.6 Ideal Auto-Transformers (1)

- An **autotransformer** is a transformer in which both the primary and the secondary are in a single winding
- A connection point called a *tap* is separates the primary and secondary.
- The tap is often adjustable to provide a desired turns ratio.
- An adjustable tap provides a variable voltage to the load
- A disadvantage of the autotransformer is it provides *no electrical isolation*



Step Down Auto-Transformer



Step Up Auto-Transformer

13.6 Ideal Auto-Transformers (2)

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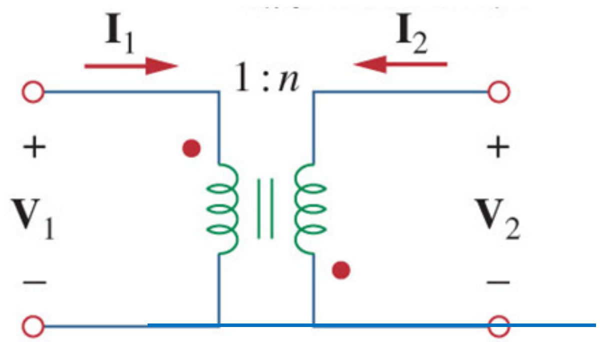
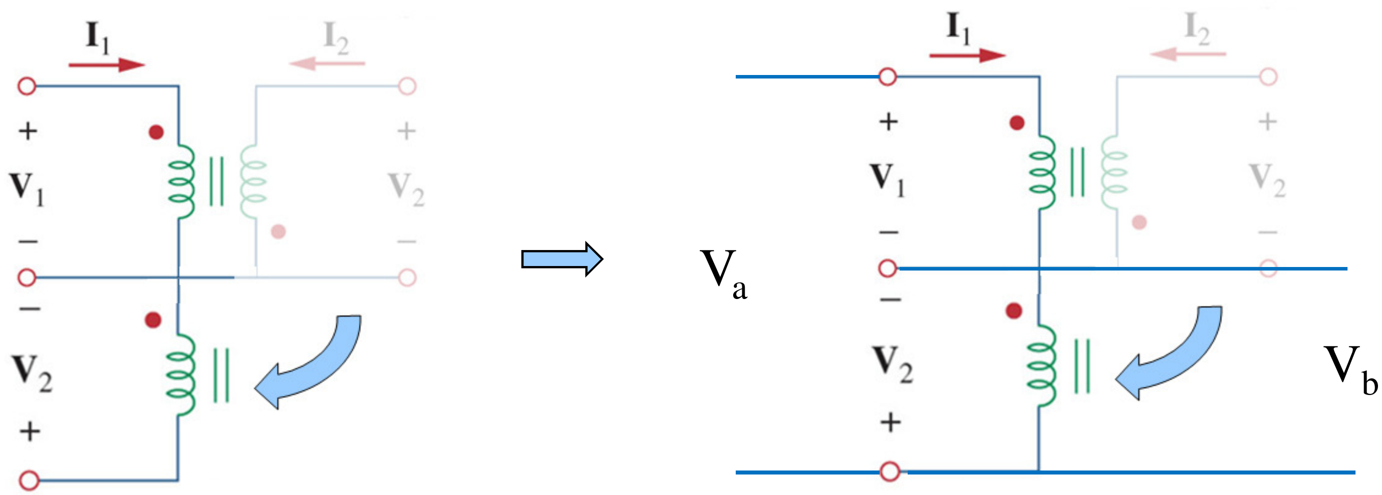
- In analyzing the autotransformer, consider the following circuit:



From earlier we know the following relationship

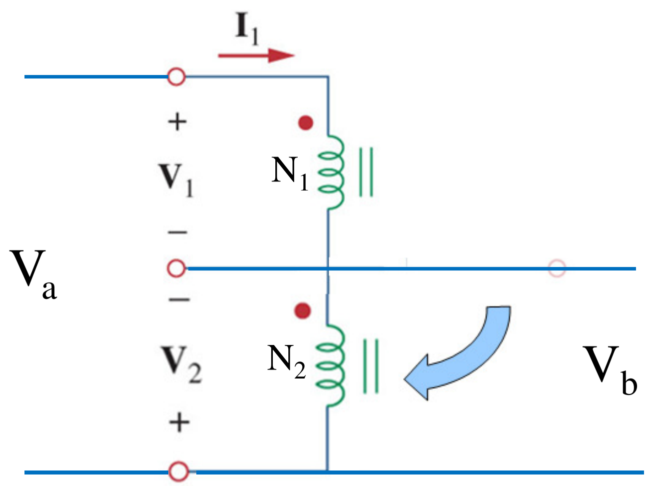
$$V_2 = -nV_1$$

- If we flip the secondary side underneath the primary, we can create an autotransformer as shown



13.6 Ideal Auto-Transformers (3)

- Analysis of this circuit provides the following results:



Primary Side

$$V_a = V_1 - V_2$$

$$V_a = V_1 + nV_1$$

$$V_a = (1+n)V_1$$

Secondary Side

$$V_b = -V_2 = nV_1$$

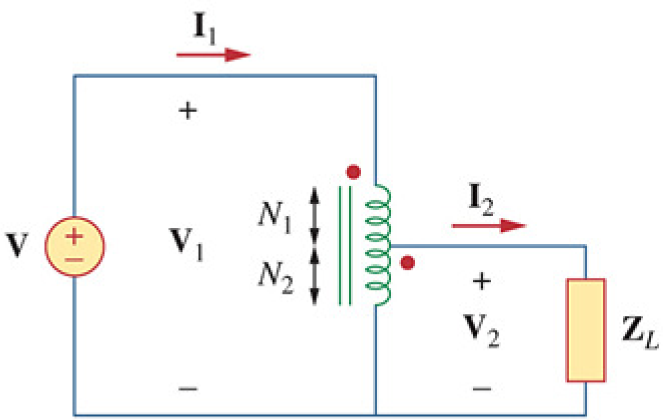
Ratio Primary/Secondary

$$\frac{V_a}{V_b} = \frac{1+n}{n} = \frac{N_1 + N_2}{N_2} \Rightarrow \boxed{V_b = \left(\frac{N_2}{N_1 + N_2}\right) V_a}$$

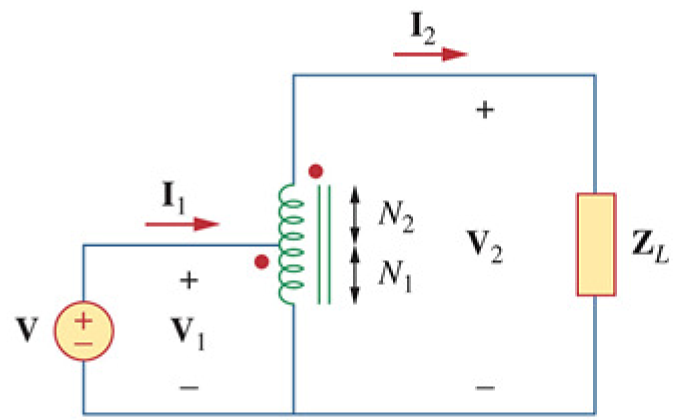
Notice, this looks like a Voltage Divider !

13.6 Ideal Auto-Transformers (4)

- The voltage / current relationships for the lossless ideal autotransformer are as follows:



Step Down Auto-Transformer



Step Up Auto-Transformer

Similar to Voltage Divider equation

$$V_2 = \frac{N_2}{N_1 + N_2} V_1 \qquad V_2 = \frac{N_1 + N_2}{N_1} V_1$$

Inverse Relation

$$I_2 = \frac{N_1 + N_2}{N_2} I_1 \qquad I_2 = \frac{N_1}{N_1 + N_2} I_1$$

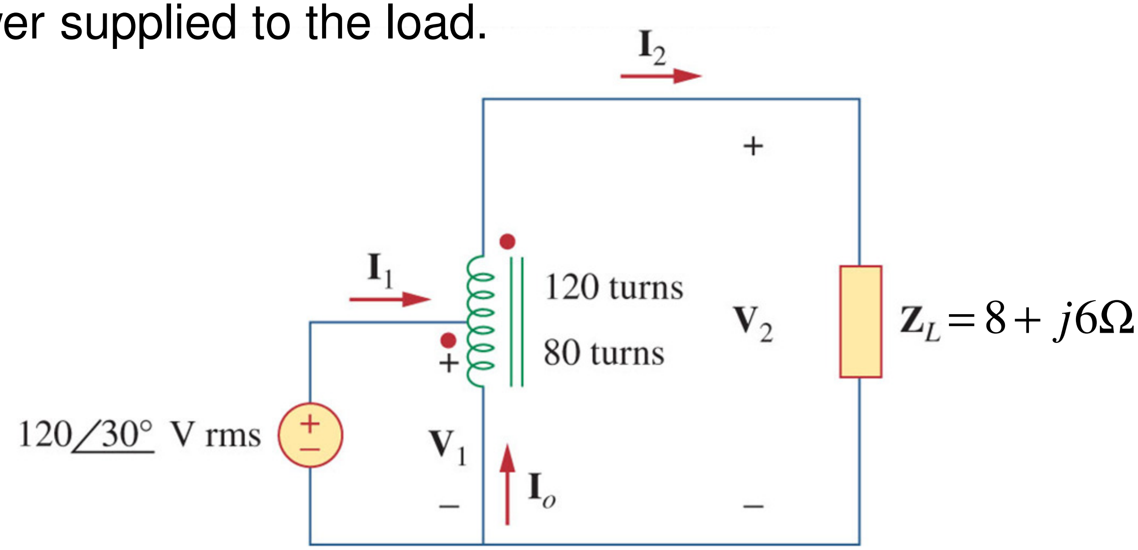
Derive from V/I

$$Z_{in} = \left(\frac{N_1 + N_2}{N_2}\right)^2 Z_L \qquad Z_{in} = \left(\frac{N_1}{N_1 + N_2}\right)^2 Z_L$$

13.6 Ideal Auto-Transformers (5)

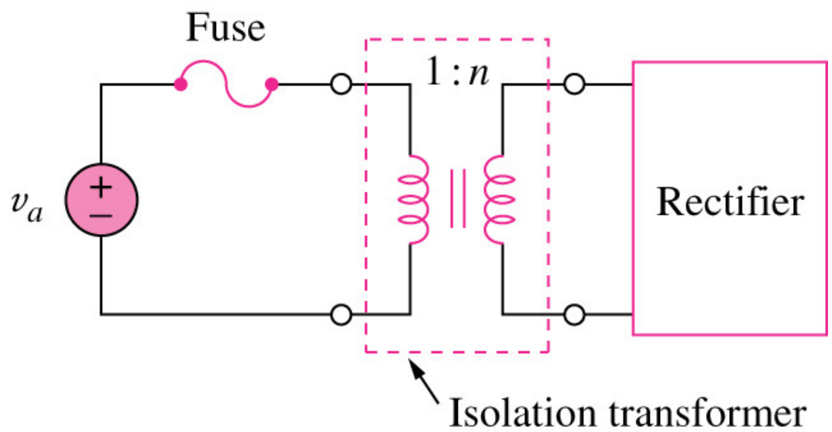
Example 13.11 (Textbook)

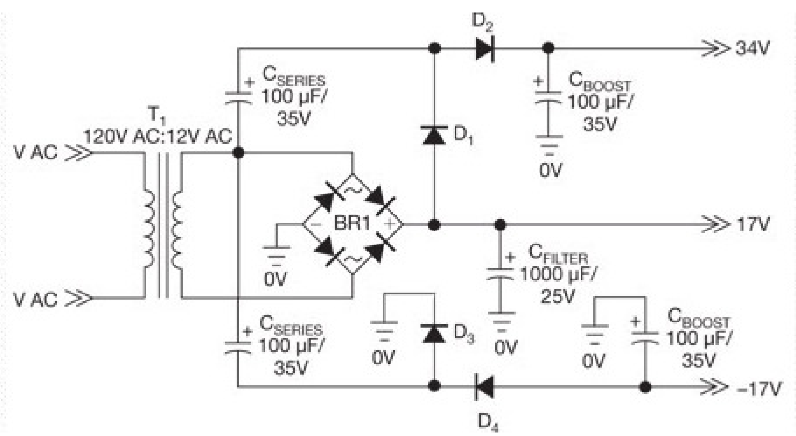
For the autotransformer below, find: (a) the currents I_1 , I_2 , I_0, (b) the complex power supplied to the load.

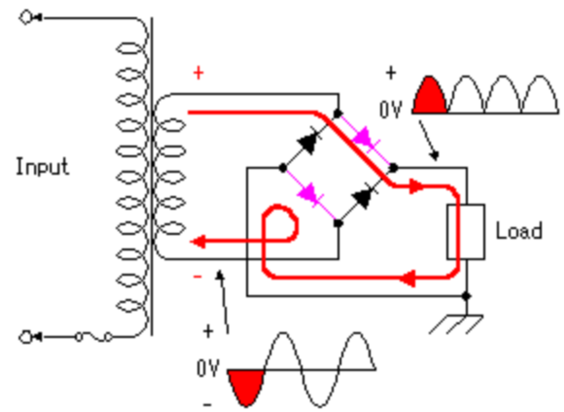


13.9 Applications (1)

- Transformer as an Isolation Device to isolate ac supply from a rectifier

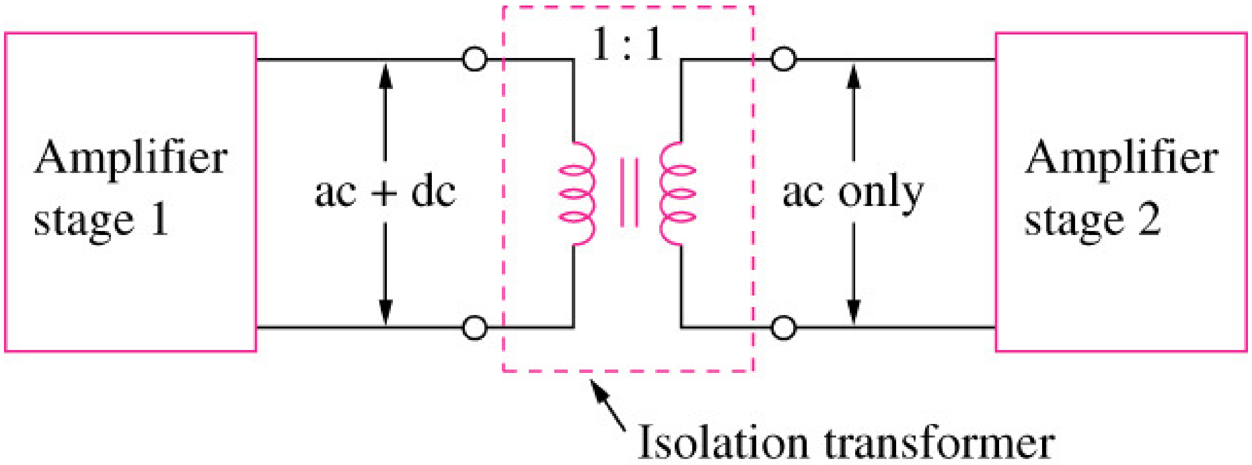






13.9 Applications (2)

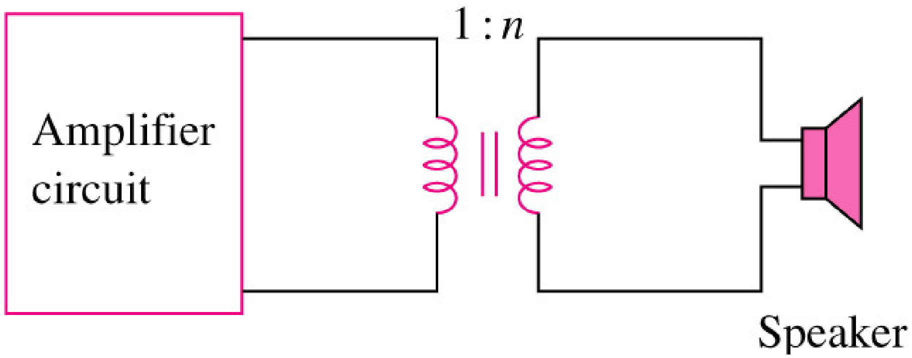
- Transformer as an Isolation Device to isolate dc between two amplifier stages.



- Biasing is the application of a DC voltage to a transistor amplifier to produce a desired mode of operation.
- Each amplifier stage can be biased separately to operate in a particular mode.

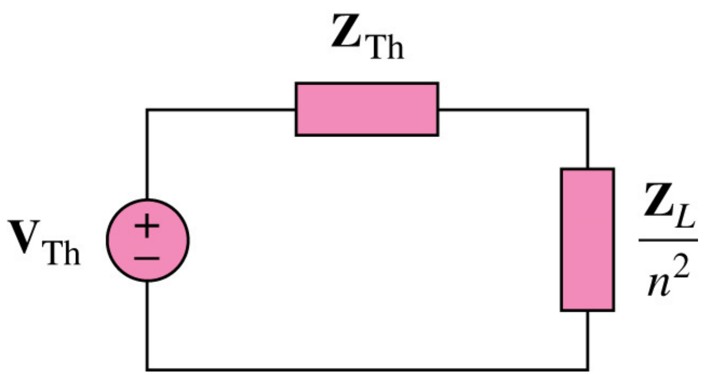
13.9 Applications (3)

- Transformer as a Matching Device



Using an ideal transformer to match the speaker to the amplifier

Equivalent circuit



13.9 Applications (4)

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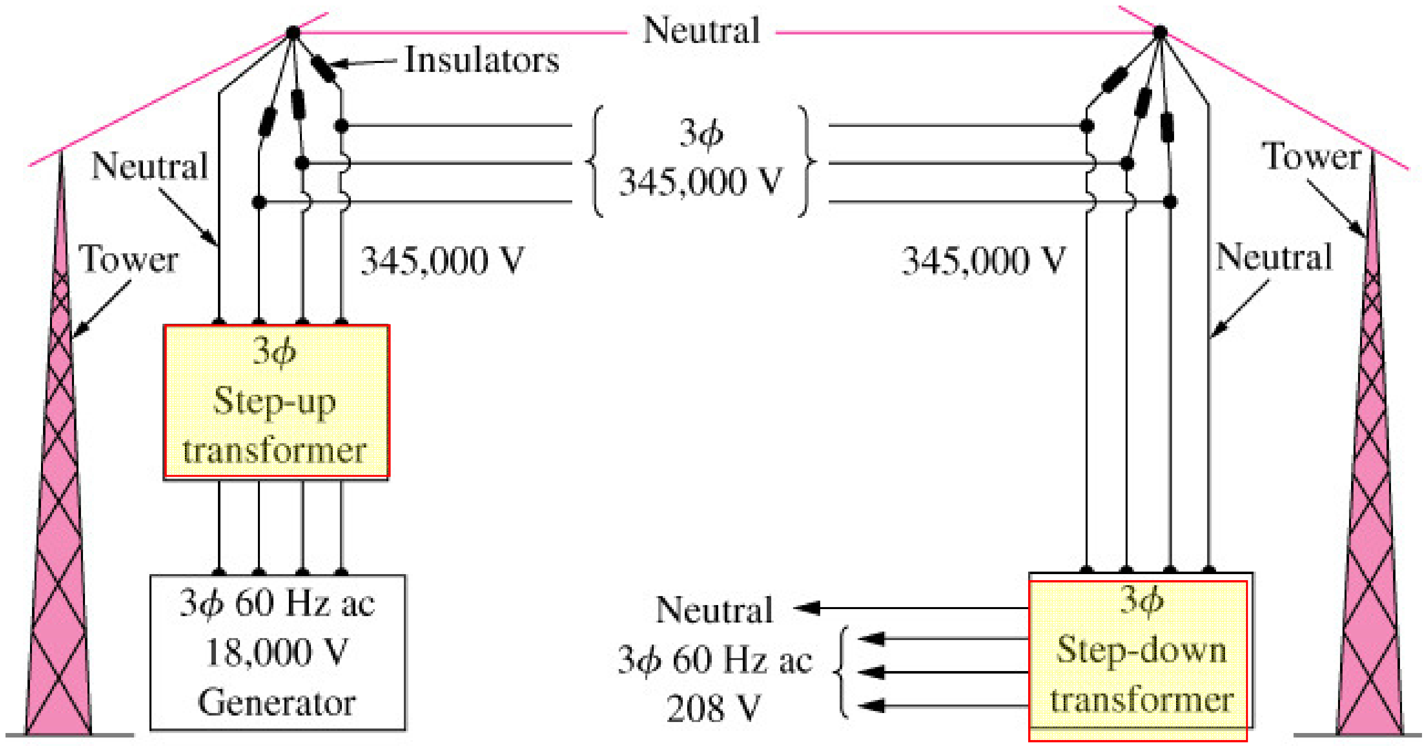
Practice Problem 13.16 (Textbook)

Calculate the turns ratio of an ideal transformer required to match a 400Ω load to a source with internal impedance of 2.5kΩ. Find the load voltage when the source voltage is 30V.

Ans: $n = 0.4$; $V_L = 6V$

13.9 Applications (5)

- A typical power distribution system



Homework #3

Due beginning of class Wednesday Feb 4, 2015

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- 13.30
- 13.35
- 13.42
- 13.50
- 13.53 (modified)
- Autotransformer (See handout)

Exam over Chapter 13 on Monday Feb 9

Chapter 13
Equation / Analysis Summary

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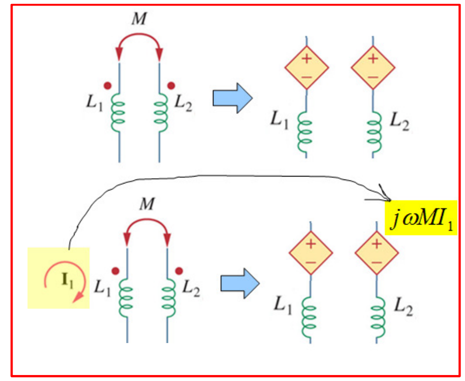
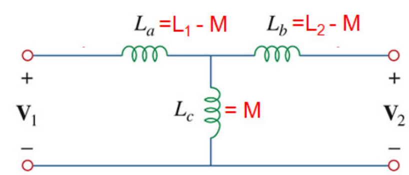
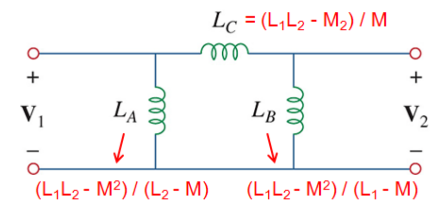
- Series Aiding $L = L_1 + L_2 + 2M$ / Opposing $L = L_1 + L_2 - 2M$
- Dot Convention Model



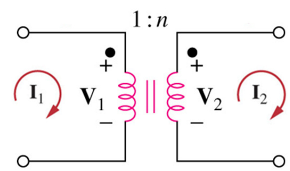
- Coupling coefficient "k" $M = k\sqrt{L_1 L_2}$
- Linear Transformer
  - Input Impedance: $Z_{in} = Z_{primary} + \dfrac{\omega^2 M^2}{Z_{Secondary}}$
  - Reflected Impedance: $Z_{reflected} = \dfrac{\omega^2 M^2}{Z_{Secondary}}$
  - Equivalent T or π Circuits:





- Ideal Transformer
  - K = 1, L_1, L_2, → ∞
  - Lossless ($S_1 = S_2$)
  - Voltage / Current Relationship:
    - Dots "same" = +n, Dots "diff" = -n
  - Complex Power: $S_1 = V_1 I_1^* = V_2 I_2^* = S_2$

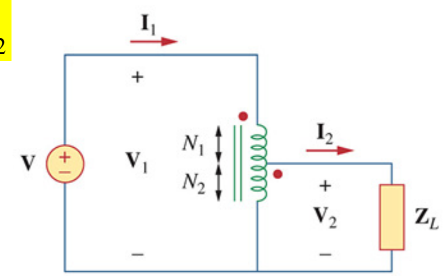
Turns Ratio $n = \dfrac{N_2}{N_1}$



$V_2 = nV_1$ $\quad I_2 = \dfrac{I_1}{n}$ $\quad Z_{in} = \dfrac{Z_L}{n^2}$

$P_{ave} = |I_2|^2 R_L$

- Autotransformer
  - Adjustable "tap"
  - No electrical Isolation
  - Voltage Divider like relationship



$V_2 = \dfrac{N_2}{N_1 + N_2} V_1$

$I_2 = \dfrac{N_1 + N_2}{N_2} I_1$