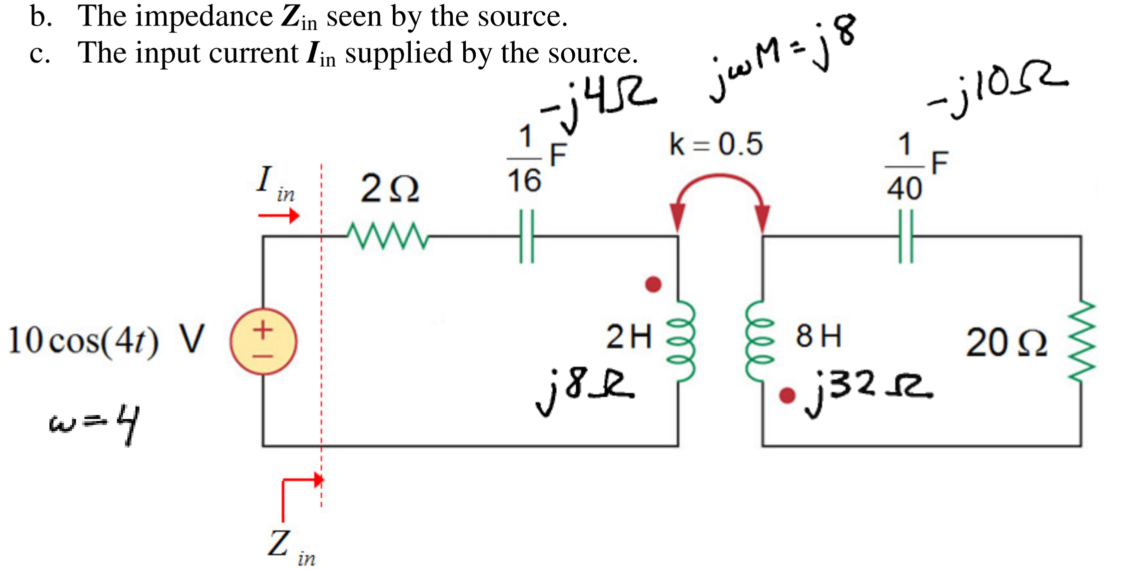
IUPUI ECE 202 Spring 2015:

Exam #1 (SOLUTION KEY) Name:

1. (20 points) For the circuit shown, determine:
   a. The mutual inductance $\boldsymbol{M}$.
   b. The impedance $\boldsymbol{Z}_{\text{in}}$ seen by the source.
   c. The input current $\boldsymbol{I}_{\text{in}}$ supplied by the source.



a) $M = k\sqrt{L_1 L_2} = 0.5\sqrt{2 \cdot 8} = \boxed{2}$

b) $Z_{IN} = Z_{primary} + \dfrac{(\omega M)^2}{Z_{SECONDARY}} = (2 - j4 + j8) + \dfrac{(2 \cdot 4)^2}{(20 + j32 - j10)}$

$= (2 + j4) + (1.448 - j1.5928) = \boxed{3.448 + j2.4072\ \Omega}$

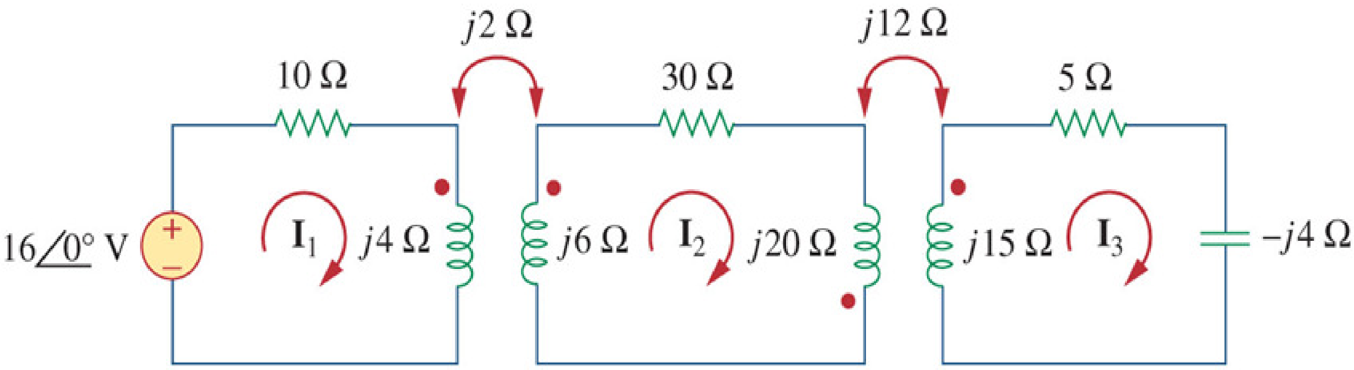
$= \boxed{4.205\angle 34.92^\circ\ \Omega}$

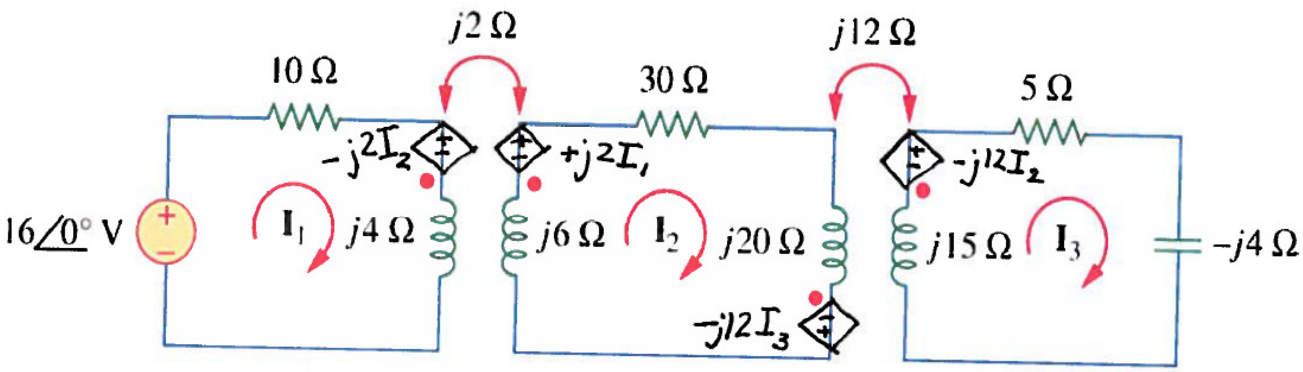
c) $I_{IN} = \dfrac{10\angle 0^\circ}{4.205\angle 34.92^\circ} = \boxed{2.378\angle -34.92^\circ\ A}$

or

$\boxed{1.9498 - j1.3613\ A}$

2. (15 points) Write a set of equations necessary to calculate $\mathbf{I}_1$, $\mathbf{I}_2$, and $\mathbf{I}_3$. DO NOT SOLVE THE SET OF EQUATIONS.



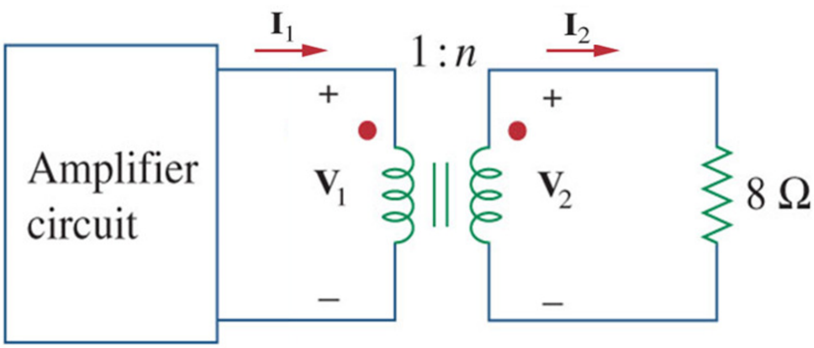


MESH #1: $(10+j4)I_1 - j2I_2 = 16\angle 0°$

MESH #2: $-j2I_1 + (30+j26)I_2 + j12I_3 = 0$

MESH #4: $+j12I_2 + (5+j11)I_3 = 0$

3. (25 points) A transformer is used to match an amplifier with an 8 Ω load. The Thevenin equivalent of the amplifier is: $\mathbf{V}_{th}$ = 10 V_{rms}, and $\mathbf{Z}_{th}$ = 128 Ω.
   a.) Find the required turns ratio n for maximum power supplied to the 8 Ω load.
   b.) Determine the currents $\mathbf{I}_1$ and $\mathbf{I}_2$.
   c.) Determine the voltages $\mathbf{V}_1$ and $\mathbf{V}_2$.
   d.) Find the average power ($P_{ave} = I_{rms}^2 R$) in the 8 Ω load at this turns ratio.
   e.) If the amplifier signal was biased at 5 V DC (that is 5 V DC is added to the signal) how would this affect the average power absorbed by the 8 Ω load?



a) For maximum power transfer, set the input impedance $Z_{in} = Z_{th}$

$$Z_{in} = \frac{Z_L}{n^2} = \frac{8}{n^2} = 128 \quad \Rightarrow \quad n^2 = \frac{8}{128} = \frac{1}{16} \quad \Rightarrow \quad \boxed{n = \frac{1}{4} = 0.25}$$

b) $$I_1 = \frac{V_{th}}{(Z_{th} + Z_{in})} = \frac{10}{(128 + 128)} = \boxed{0.0391 \text{ A} \quad \text{or} \quad 39.1 \text{ mA}}$$

$$I_2 = \frac{I_1}{n} = \frac{0.0391}{0.25} = \boxed{0.156 \text{ A} \quad \text{or} \quad 156 \text{ mA}}$$

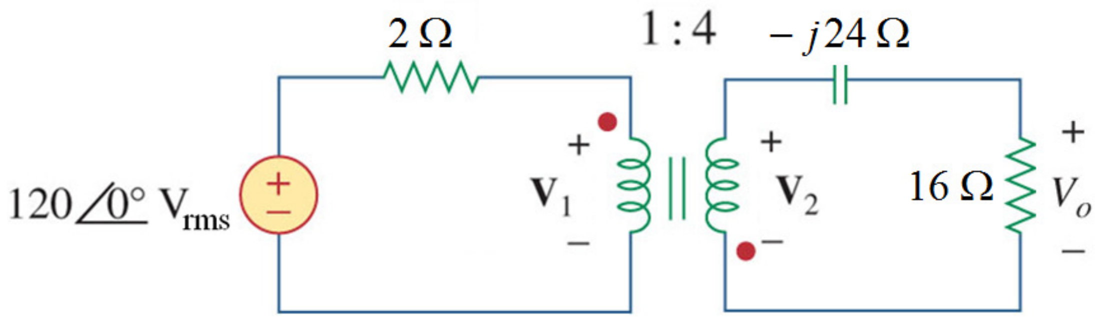
c) $$V_1 = I_1 Z_{in} = (0.0391)(128) = \boxed{5.0 \text{ V}} \quad (\text{Note: at max power} = \tfrac{1}{2} V_1)$$

$$V_2 = nV_1 = (0.25)(5) = \boxed{1.25 \text{ V}}$$

d) $$P_{ave} = I_1^2 Z_L = (0.156)^2 (8) = 0.195 \text{ W} \quad \text{or} \quad 195 \text{ mW}$$

e) No affect, transformers isolate DC current. Only AC current is coupled into the secondary.

4. (20 points) For the ideal transformer circuit shown,
   a. Find the voltage $\boldsymbol{V}_o$ across the 16 Ω resistor.
   b. Find the complex power $\boldsymbol{S} = \boldsymbol{V}_{rms}\,\boldsymbol{I}^*_{rms}$ supplied by the source.



DOTS DIFFERENT

$V_2 = -nV_1 = -4V_1$

$I_2 = -\frac{I_1}{n} = -\frac{I_1}{4}$

a.) MESH #1: $2I_1 + V_1 = 120$

MESH #2: $(16-j24)I_2 = V_2$

SUBS FOR $V_2 I_2$: $(16-24)\left(\frac{-I_1}{4}\right) = -4V_1$

$(4-j6)I_1 = 4V_1 \Rightarrow \left(1+j\frac{3}{2}\right)I_1 = V_1$

SUBS INTO #1: $2I_1 + \left(1-j\frac{3}{2}\right)I_1 = 120$

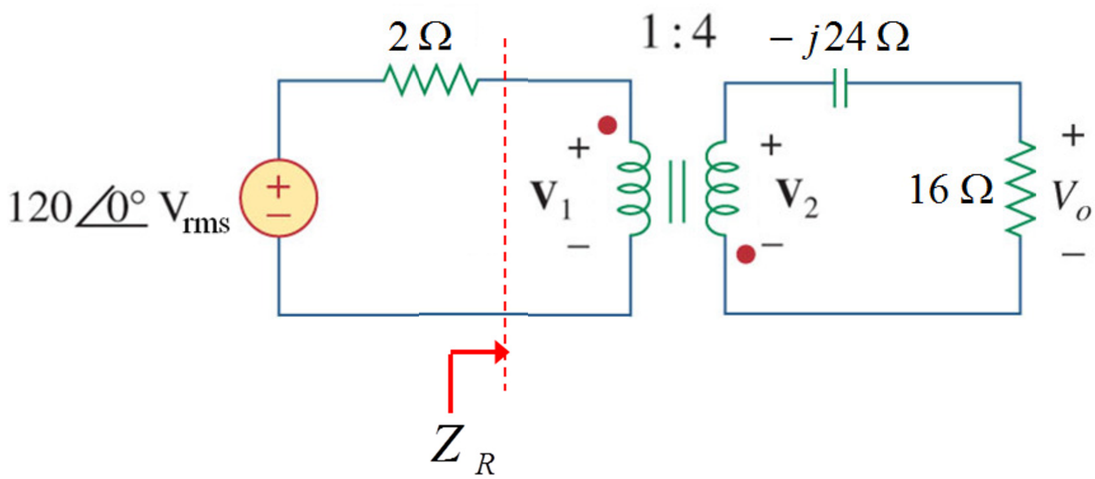
$I_1 = \frac{120}{\left(3-j\frac{3}{2}\right)} = 32+j16\ A$

$I_2 = -\frac{I_1}{4} = -\frac{(32+j16)}{4} = -8-j4 = 8.944\angle -153.4°$

$V_o = I_2 R = I_2(16) = \boxed{\begin{aligned}&-128-j64\ V_{rms}\\ =\ &143.11\angle -153.4°\ V_{rms}\end{aligned}}$

b) $S = VI^*_{rms} = (120\angle 0°)I_1^* = (120\angle 0°)(32-j16) = (120\angle 0°)(35.77\angle -26.57)$

$\boxed{\begin{aligned}S &= 4293.2\angle -26.59°\ W\\ &\text{or}\\ S &= 3840 - j1920\ W\end{aligned}}$



Alternate Solution using reflected impedance Z_R

$$a) \qquad Z_R = \frac{Z_L}{n^2} = \frac{16 - j24}{4^2} = 1 - j1.5$$

Find the current $\mathbf{I}_1$ on the primary side

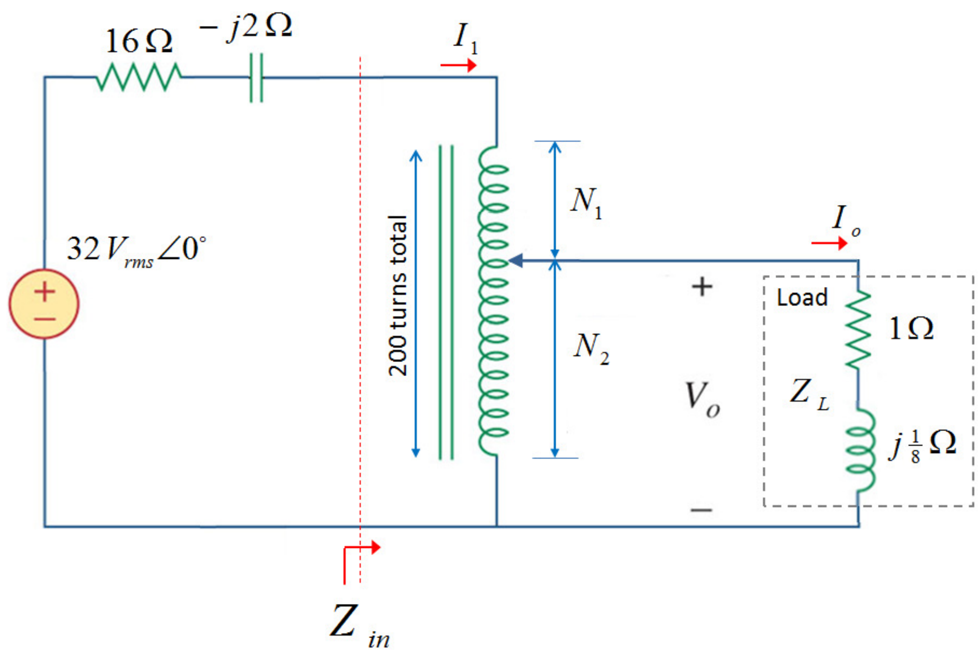
$$I_1 = \frac{120}{2 + (1 - j1.5)} = 32 + j16 \ \text{A}$$

$$I_2 = -\frac{I_1}{n} = -\frac{32 + j16}{4} = -8 - j4 \ \text{A}$$

$$V_o = I_2(16) = (-8 - j4)(16) = \boxed{-128 - j64 \ \text{V} \quad \text{or} \quad 143.11\angle -153.43° \ \text{V}}$$

$b)$ Find complex power same as before

5. (20 points) In the figure below the tap is initially to $N_2 = 50$.
   a. Find the impedance $\boldsymbol{Z}_{\text{in}}$ seen looking into the primary side.
   b. Find the current $\boldsymbol{I}_o$ delivered to the load.
   c. Find the complex power $\boldsymbol{S}$ delivered to the load.
   d. Find the complex power $\boldsymbol{S}$ delivered to the load if $N_2 = 100$.



a) $$Z_{in} = \left(\frac{N_1 + N_2}{N_2}\right)^2 Z_L = \left(\frac{200}{50}\right)^2 \left(1 + \tfrac{1}{8} j\right) = \boxed{(16 + j2)\ \Omega}$$

b) $$I_1 = \frac{32}{(16 - j2 + Z_{in})} = \frac{32}{(16 - j2 + 16 + j2)} = \frac{32}{(16 + 16)} = 1.0\text{ A}$$

$$I_O = \left(\frac{N_1 + N_2}{N_2}\right) I_1 = \frac{200}{50}(1.0) = \boxed{4.0\text{ A}}$$

c) $$S = V_O I_O^* = |I_O|^2 Z_L = (4)^2 \left(1 + j\tfrac{1}{8}\right)$$

$$S = \boxed{16 + j2\ \text{W} \quad \text{or} \quad 16.125\angle 7.125^\circ\ \text{W}}$$

$$d)\quad Z_{in} = \left(\frac{N_1 + N_2}{N_2}\right)^2 Z_L = \left(\frac{200}{100}\right)^2 \left(1 + \tfrac{1}{8} j\right) = \left(4 + j0.5\right)\ \Omega$$

$$I_1 = \frac{32}{\left(16 - j2 + Z_{in}\right)} = \frac{32}{\left(16 - j2 + 4 + j0.5\right)} = \frac{32}{\left(20 - j1.5\right)}$$

$$I_1 = 1.59 + j0.119\ \text{A} \quad \text{or} \quad 1.596\angle 4.29^\circ\ \text{A}$$

$$I_O = \left(\frac{N_1 + N_2}{N_2}\right) I_1 = \frac{200}{100}\left(1.59 + j0.119\right)$$

$$I_O = 3.182 + \text{j}0.239\ \text{A} \quad \text{or} \quad 3.191\angle 4.289^\circ$$

$$S = V_O I_O^{*} = \left|I_O\right|^2 Z_L = \left(3.191\right)^2 \left(1 + j\tfrac{1}{8}\right)$$

$$S = \boxed{10.18 + j1.27\ \text{W} \quad \text{or} \quad 10.26\angle 7.125^\circ\ \ \text{W}}$$