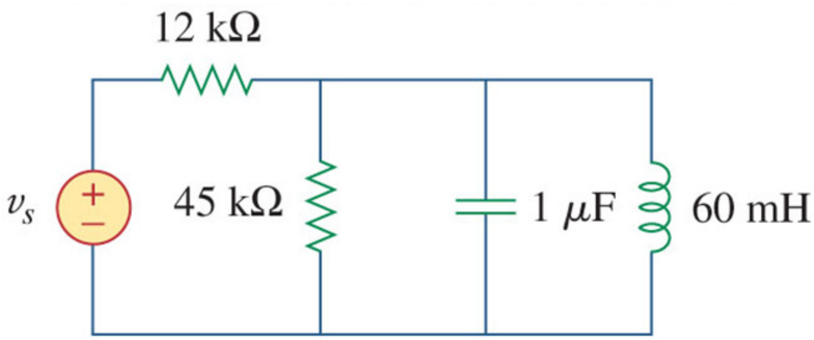
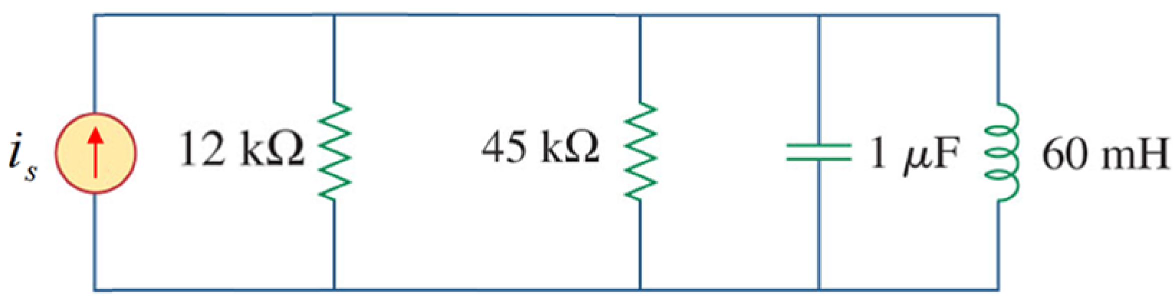
IUPUI ECE 202 Spring 2015:

Homework #5 (SOLUTIONS)   Name:

1. (Prob. 14.29 in text) Let $v_s = 20\cos(\omega t)$ V in the circuit below. Find resonant frequency $\boldsymbol{\omega_o}$, quality factor **Q**, and bandwidth **B**, as seen by the capacitor.



First, perform a source transform on the circuit and combine the parallel resistors:



$$i_s = \frac{20}{12}\cos\omega t = 1.66\cos\omega t$$

$$\mathbf{R} = 12\ \text{k}\Omega \,\|\, 45\ \text{k}\Omega\ \frac{(12)(45)}{(12+45)}\ \text{k}\Omega = 9.47\ \ \text{k}\Omega$$

(a) $$\omega_o = \frac{1}{\sqrt{\mathbf{LC}}} = \frac{1}{\sqrt{(60\times10^{-3})(1\times10^{-6})}} = \boxed{4.082\ \ \text{krad/s}}$$

(b) $$\mathbf{B} = \frac{1}{\mathbf{RC}} = \frac{1}{(9.47\times10^{3})(1\times10^{-6})} = \boxed{105.6\ \ \text{rad/s}}$$

(c) $$\mathbf{Q} = \frac{\omega_o}{\mathbf{B}} = \frac{4082}{105.6} = \boxed{38.7}$$

2. (Prob. 14.35 from Text) A parallel *RLC* circuit has R = 5 kΩ, L =8 mH, and C = 60 μF. Determine the following:

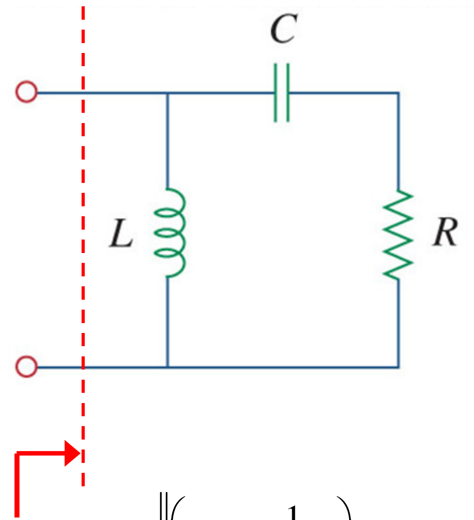
   a. The resonant frequency $\boldsymbol{\omega}_o$
   b. The bandwidth **B**
   c. The quality factor **Q**

$$\text{(a)} \quad \omega_o = \frac{1}{\sqrt{\mathbf{LC}}} = \frac{1}{\sqrt{(8\times10^{-3})(60\times10^{-6})}} = \boxed{1.443 \quad \text{krad/s}}$$

$$\text{(b)} \quad \mathbf{B} = \frac{1}{\mathbf{RC}} = \frac{1}{(5\times10^{3})(60\times10^{-6})} = \boxed{3.33 \quad \text{rad/s}}$$

$$\text{(c)} \quad \mathbf{Q} = \frac{\omega_o}{\mathbf{B}} = \frac{1443}{3.33} = \boxed{433}$$

3. (Prob. 14.38 from Text) Find the resonant frequency of the circuit in the figure below.



$$Z_{in} = j\omega L \,\Big\|\, \left(R + \frac{1}{j\omega C} \right)$$

Find the complex impedance and set imaginary part = 0

$$Z_{in} = j\omega L \,\Big\|\, \left(R - j\frac{1}{\omega C} \right) = \frac{(j\omega L)\left(R - j\frac{1}{\omega C} \right)}{j\omega L + R - j\frac{1}{\omega C}}$$

$$Z_{in} = \frac{\left(j\omega LR + \frac{L}{C} \right)}{R + j\left(\omega L - \frac{1}{\omega C} \right)} \bullet \frac{\left(R - j\left(\omega L - \frac{1}{\omega C} \right) \right)}{R - j\left(\omega L - \frac{1}{\omega C} \right)}$$

$$Z_{in} = \frac{\left(j\omega LR^2 + \frac{LR}{C} \right) - j\left(j\omega LR + \frac{L}{C} \right)\left(\omega L - \frac{1}{\omega C} \right)}{R^2 + \left(\omega L - \frac{1}{\omega C} \right)^2}$$

$$\operatorname{Im}[Z_{in}] = \frac{\boxed{\omega LR^2 - \left(\frac{L}{C} \right)\left(\omega L - \frac{1}{\omega C} \right)}}{R^2 + \left(\omega L - \frac{1}{\omega C} \right)^2} = 0$$

$$\omega LR^2 - \left(\frac{L}{C} \right)\left(\omega L - \frac{1}{\omega C} \right) = 0$$

$$\omega R^2 - \left(\frac{1}{C} \right)\left(\omega L - \frac{1}{\omega C} \right) = 0$$

$$\omega R^2 C - \left(\omega L - \frac{1}{\omega C} \right) = 0$$

$$\omega^2 R^2 C^2 - \left(\omega^2 LC - 1 \right) = 0$$

$$\omega^2 R^2 C^2 - \omega^2 LC + 1 = 0$$

$$\omega^2 \left(LC - R^2 C^2 \right) = 1$$

$$\boxed{\omega = \frac{1}{\sqrt{LC - R^2 C^2}}}$$

4. (Prob. 14.53 from Text) Design a series *RLC* type **bandpass** filter with cutoff frequencies of 10 kHz and 11 kHz. Assuming ***C* = 80 pF** (80 x 10^{-12}), find ***R***, ***L***, and ***Q***. **Draw** the circuit

Convert frequency into angular frequency:

$$f_1 = 10 \times 10^3 \text{ Hz} \quad \Rightarrow \quad \omega_1 = 2\pi \cdot f_1 = 62{,}832$$

$$f_2 = 11 \times 10^3 \text{ Hz} \quad \Rightarrow \quad \omega_2 = 2\pi \cdot f_2 = 69{,}115$$

Find the Bandwidth, center frequency and quality factor "Q":

$$B = \omega_2 - \omega_1 = 69{,}115 - 62{,}832 = 6{,}283$$

$$\omega_o = \sqrt{\omega_1 \omega_2} = \sqrt{62{,}832 \times 69{,}115} = 65{,}899$$

$$Q = \frac{\omega_o}{B} = \frac{65{,}899}{6{,}283} = \boxed{10.488}$$

Use equation for resonant frequency and known value of C to find L:

$$\omega_o = \frac{1}{\sqrt{LC}} \quad \Rightarrow \quad \omega_o^2 = \frac{1}{LC} \quad \Rightarrow \quad L = \frac{1}{C\omega_o^2}$$

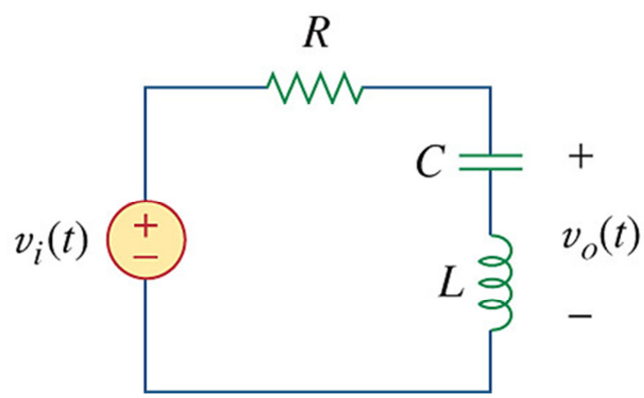
$$L = \frac{1}{80 \times 10^{-12} \times (65{,}899)^2} = \boxed{2.878 \text{ H}}$$

Find R from bandwidth equation:

$$B = \frac{R}{L} \quad \Rightarrow \quad R = BL = 6{,}283 \times 2.878 = \boxed{18{,}086\ \Omega}$$

5. (Prob. 14.54 from Text) Design passive **bandstop** filter with ω_o = 10 rad/s and **Q** = 20. **Draw** the circuit.

Series RLC Bandstop (notch) filter (look at output voltage across L and C)



$$\mathbf{B} = \frac{\omega_o}{\mathbf{Q}} = \frac{10 \quad \text{rad/s}}{20} = 0.5 \quad \text{rad/s}$$

Let $\boxed{R = 1\ \Omega}$

$$\mathbf{B} = \frac{R}{L} \quad \Rightarrow \quad L = \frac{R}{\mathbf{B}} = \frac{1}{0.5} = \boxed{2 \quad \text{H}}$$

$$\omega_o = \frac{1}{\sqrt{\mathbf{LC}}} \quad \Rightarrow \quad C = \frac{1}{L\,\omega_o^{\,2}} = \frac{1}{2(10)^2} = \boxed{0.005 \quad \text{F}}$$

Choosing different values of R gives the following results:

R	L=(2)R	C = 1/(L x ω_0^2)
1 Ω	2 H	5 mF
5 Ω	10 H	1 mF
8 Ω	16 H	625 μF

6. (Prob. 14.67 from Text) Design an **active lowpass** filter with dc gain of 0.25 and corner frequency of 500 Hz. (Remember $\boldsymbol{\omega} = 2\pi \boldsymbol{f}$)

Start with the equations for Gain and cutoff frequency and "choose" a value for the feedback resistor R_f (there can be more than one answer to this design problem:

$$Gain = \frac{R_f}{R_i} = 0.25 \qquad \omega_c = \frac{1}{R_f C_f} = 2\pi \cdot 500$$

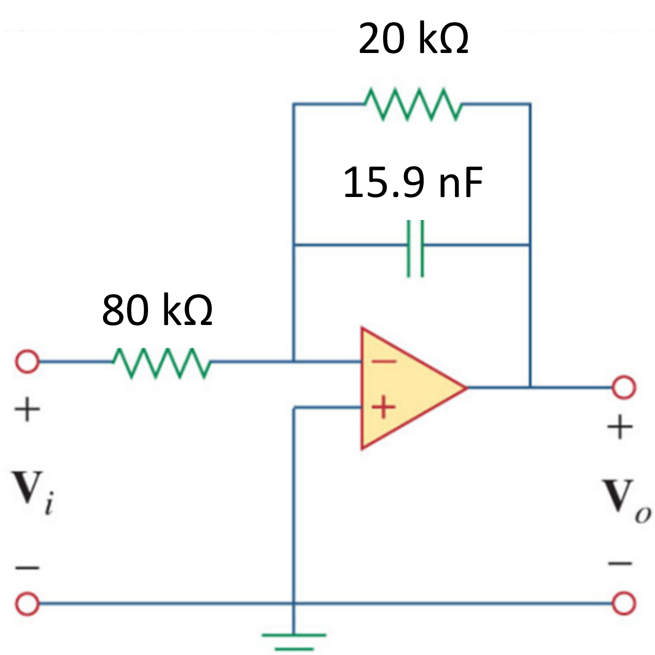
Choose $\boxed{R_f = 20 \text{ k}\Omega}$

$$R_i = \frac{R_f}{0.25} = \frac{20{,}000}{0.25} = \boxed{80 \text{ k}\Omega}$$

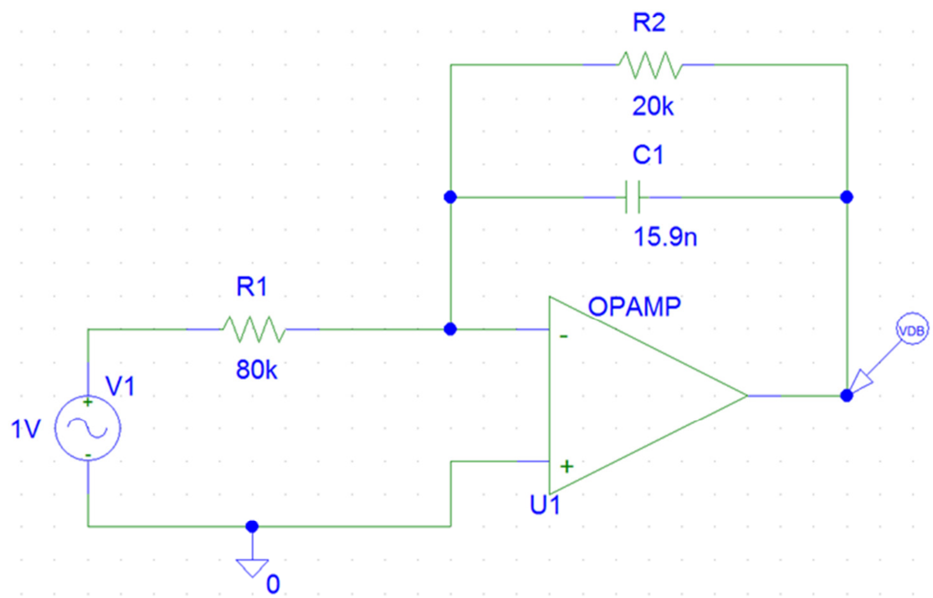
$$C_f = \frac{1}{\omega_c R_f}$$

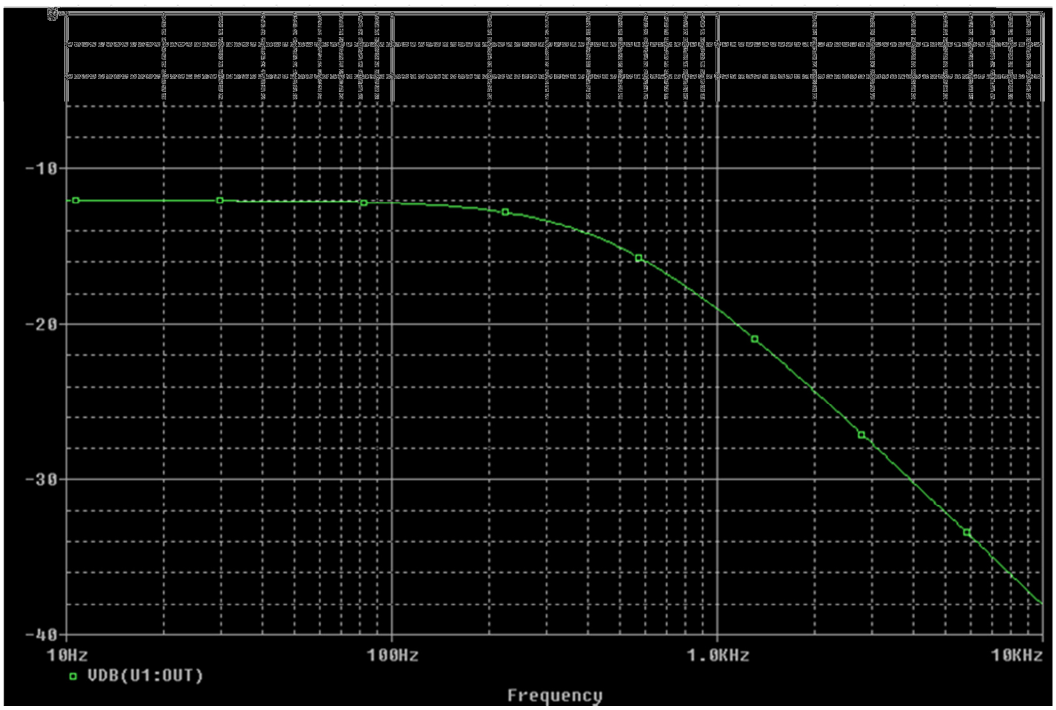
$$C_f = \frac{1}{2\pi \cdot 500 \times 20{,}000}$$

$$\boxed{C_f = 15.9 \times 10^{-9} \text{ F}}$$



Problem 14.67 - Pspice Simulation





7. ("Based on" Prob. 14.68 from Text) Design an **active highpass** filter with dc voltage gain of +6 dB and corner frequency of 3000 Hz. (Remember $\boldsymbol{\omega} = 2\pi \boldsymbol{f}$)

A voltage gain of +6 dB is equivalent to a 2x voltage gain ($V_{out} = 2\ V_{in}$)

$$20 \log_{10} \mathrm{H} = +6 \quad \Rightarrow \quad \mathrm{H} = 10^{6/20} \approx 2$$

The magnitude of the voltage gain as $\omega \rightarrow \infty$ is

$$|H(\infty)| = \frac{R_f}{R_i} = 2 \quad \Rightarrow \quad R_f = (2)R_i$$

Note: We can ignore the 180 degree phase shift caused by going into the inverting terminal if all we care about is the amplitude.

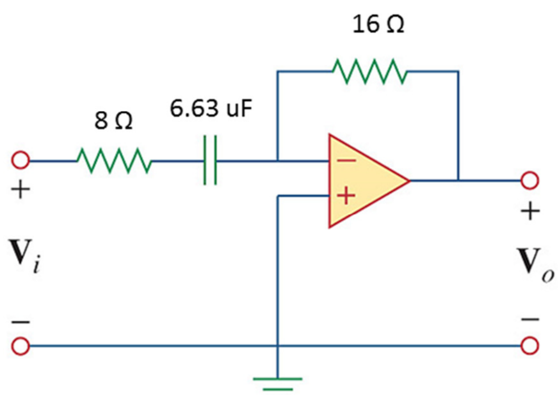
Choose a value for the input impedance R_i

$$R_i = 8\ \Omega$$
$$R_i = (2)R_i = 16\ \Omega$$

The corner frequency now can be used to find the capacitance value:

$$\omega_c = 2\pi \cdot 3000 = 18850 \quad \text{rad/s}$$

$$\omega_c = \frac{1}{R_i C_i} \quad \Rightarrow \quad C_i = \frac{1}{R_i \omega_c} = \frac{1}{(8)(18850)} = 6.63 \times 10^{-6}\ \mathrm{F}$$



Choosing different values of R_i gives the following results:

Ri	C=1/(Riω$_c$)	Rf
8 Ω	6.63 uF	16 Ω
16 Ω	3.32 uF	32Ω
150 Ω	354 nF	300Ω

Problem 14.68 - Pspice Simulation