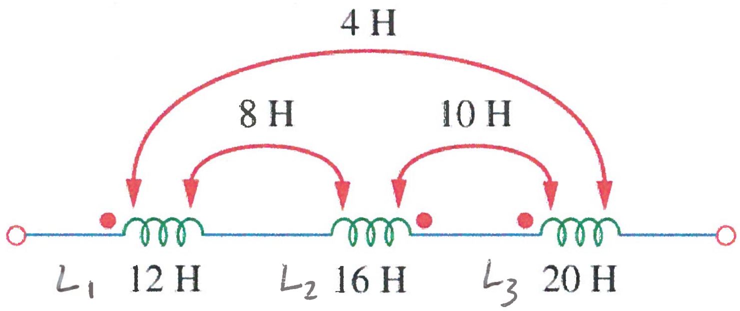
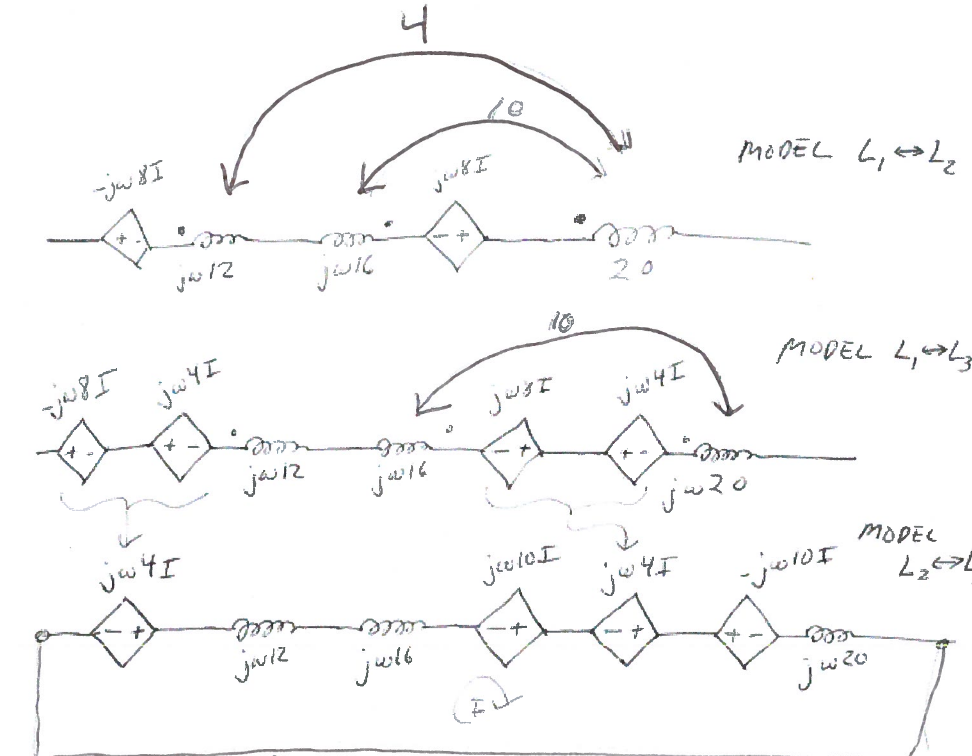
# Problem 13.1

For the three coupled coils, calculate the total inductance:




SUM VOLTAGE DROPS

$$-j\omega 4I + j\omega 12I + j\omega 16I - j\omega 10I - j\omega 4I - j\omega 10I + j\omega 20I = 0$$

$$-j\omega I \underbrace{(-4 + 12 + 16 - 10 - 4 - 10 + 20)} = -j\omega I L_{eq} = 0$$

FIND EQUIVALENT INDUCTANCE

$$\boxed{L_{eq} = 20}$$

DOT'S SAME

DOT'S OPPOSIT

ALT SOLUTION

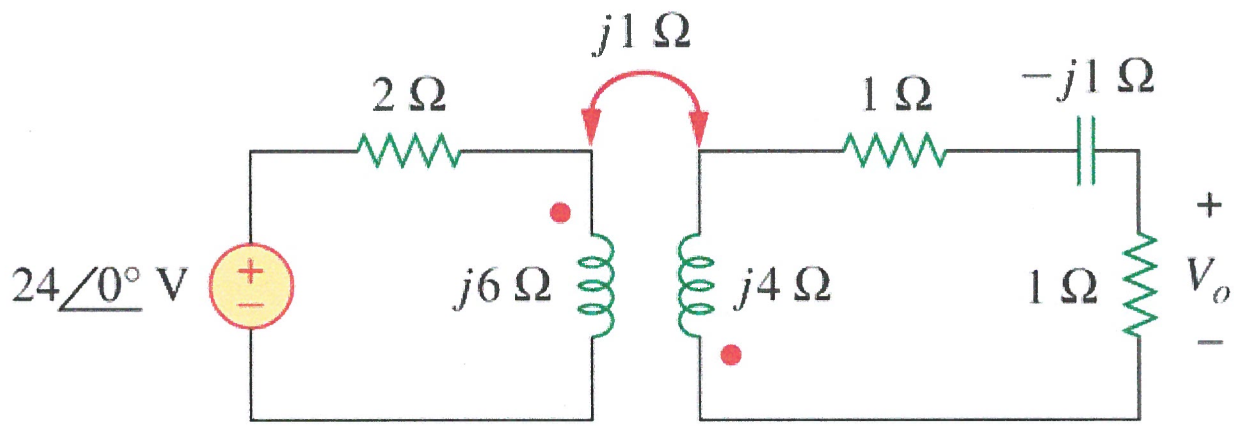
$$L_{eq} = L_1 + L_2 + L_3 - 2M_{12} + 2M_{13} - 2M_{23}$$

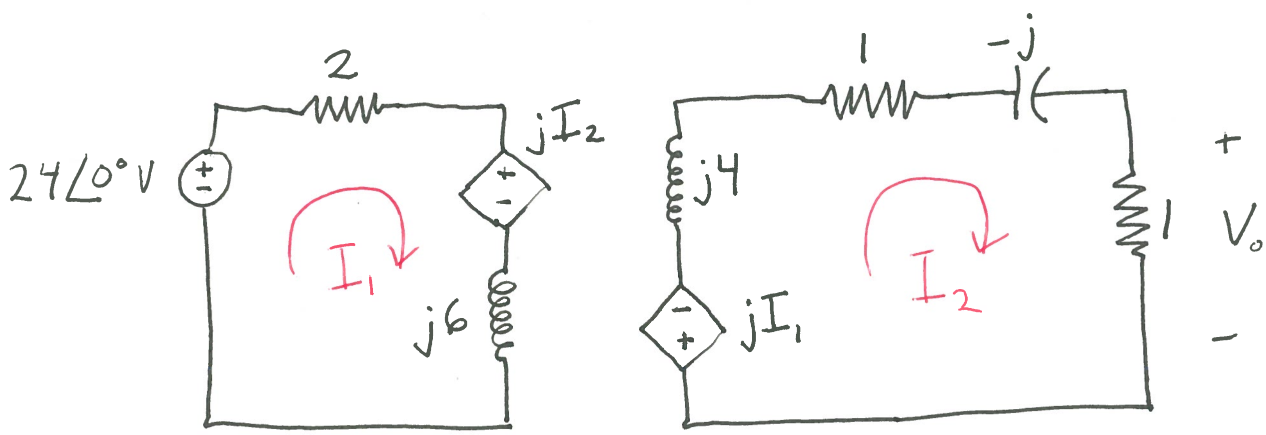
$$= 12 + 16 + 20 - 2(8) + 2(4) - 2(10)$$

$$\boxed{L_{eq} = 20}$$

## Problem 13.7

For the circuit shown below find $V_o$




Loop 1: $-24 + 2I_1 + jI_2 + j6I_1 = 0 \Rightarrow (2 + j6)I_1 + jI_2 = 24$

Loop 2: $jI_1 + j4I_2 + I_2 - jI_2 + I_2 = 0 \Rightarrow jI_1 + (2 + j3)I_2 = 0$

$$I_1 = (-3 + j2)I_2$$

SUBSTITUTING BACK INTO LOOP 1 EQ$^{N}$

$$(2 + j6)(-3 + j2)I_2 + jI_2 = 24$$

$$(-6 - j18 + j4 - 12)I_2 + jI_2 = 24$$

$$(-18 - j13)I_2 = 24$$

$$I_2 = \frac{-24}{(18 + j13)} = -0.876 + j0.633 = 1.081\angle 144.16^\circ\ A$$

$$\boxed{V_o = 1 \times I_2 = 1.081\angle 144.16^\circ\ V}$$

## Problem 13.7: (MATLAB output)

$$\underbrace{\begin{bmatrix} (2+j6) & j \\ j & (2+j3) \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} I_1 \\ I_2 \end{bmatrix}}_{x} = \underbrace{\begin{bmatrix} 24 \\ 0 \end{bmatrix}}_{b}$$

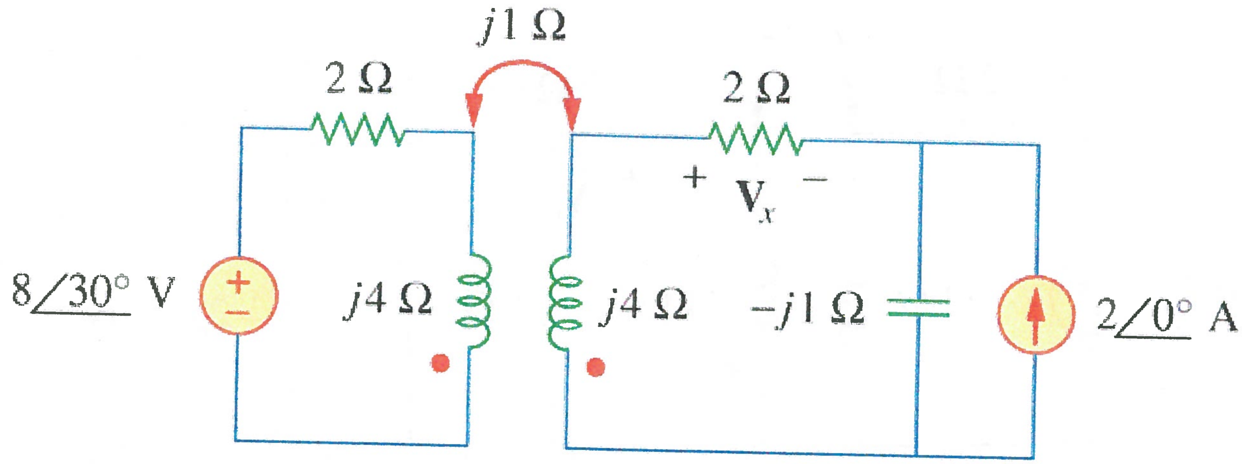
```
>> A=[(2+6i) i ; i (2+3i)]

A =

   2.0000 + 6.0000i   0.0000 + 1.0000i
   0.0000 + 1.0000i   2.0000 + 3.0000i

>> b=[24;0]

b =

    24
     0

>> x=A\b

>> abs(x)

ans =

    3.8973
    1.0809

>> angle(x)*180/pi()

ans =

  -69.5277
  144.1623
```

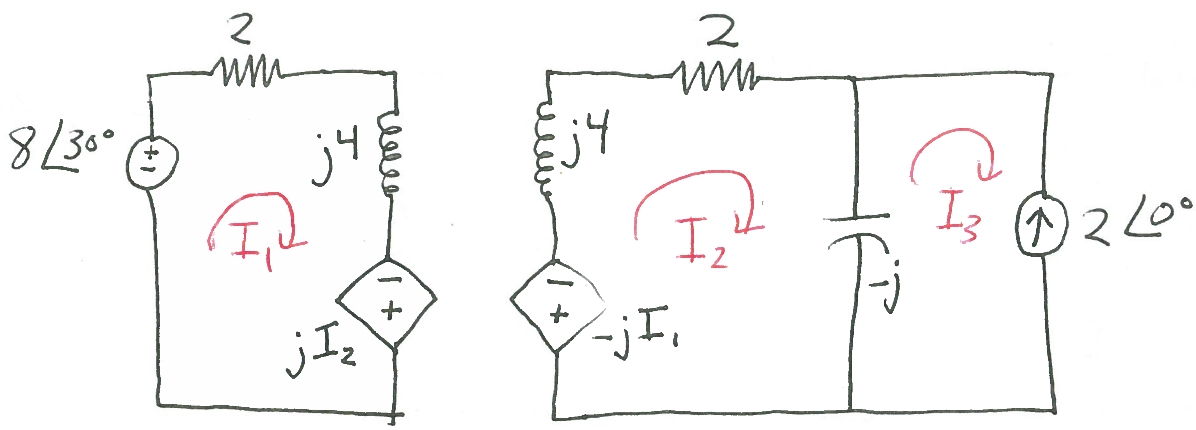
$$I_2 = 1.0809 \angle 144.1623^\circ \text{ A}$$

$$\boxed{V_o = (1) I_2 = 1.0809 \angle 144.1623 \text{ V}}$$

# Problem 13.9

Find $V_x$ for the circuit below:




$I_3 = -2\angle 0°$

Loop 1: $-8\angle 30° + 2I_1 + j4I_1 - jI_2 = 0 \Rightarrow (2+j4)I_1 - jI_2 = 8\angle 30°$

Loop 2: $-jI_1 + j4I_2 + 2I_2 - j(I_2 - \overbrace{(-2\angle 0°)}^{I_3}) = 0$

$-jI_1 + j3I_2 + 2I_2 - j2 = 0 \Rightarrow -jI_1 + (2+j3)I_2 = j2$

Substituting into #1 $\qquad I_1 = (3-j2)I_2 - 2$

$$(2+j4)[(3-j2)I_2 - 2] - jI_2 = 8\angle 30°$$

$$(6-j4)I_2 - 4 + (8+j12)I_2 - j8 - jI_2 = 8\angle 30°$$

$$(14+j7)I_2 = 8\angle 30° + (4+j8) \Rightarrow I_2 = \frac{8\angle 30° + (4+j8)}{(14+j7)} = 1.037\angle 21.12°$$

$$\boxed{V_x = 2I_2 = 2.074\angle 21.12°\ V}$$

## Problem 13.9: (MATLAB Output)

```
>> A=[(2+4i) -i ; -i (2+3i)]

A =

   2.0000 + 4.0000i   0.0000 - 1.0000i
   0.0000 - 1.0000i   2.0000 + 3.0000i

>> b=[8*exp(30*pi()*i/180); 2i]

b =

   6.9282 + 4.0000i
   0.0000 + 2.0000i

>> x=A\b

x =

   1.6489 - 0.8142i
   0.9673 + 0.3735i

>> V=2*x

V =

   3.2979 - 1.6284i
   1.9347 + 0.7470i

>> abs(V)

ans =

    3.6780
    2.0738

>> angle(V)*180/pi()

ans =

  -26.2792
   21.1113
```

$1.6489 - 0.8142i = I_1$

$0.9673 + 0.3735i = I_2$

$1.9347 + 0.7470i = 2 \cdot I_2 = V_x$

$$\boxed{V_x = 2.0738 \angle 21.1113^\circ \text{ V}}$$

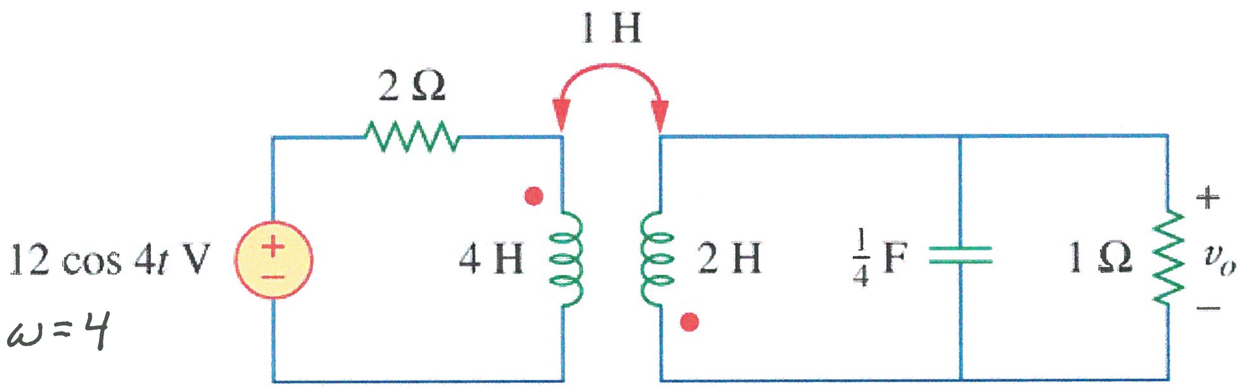
## Problem 13.24

In the circuit below:

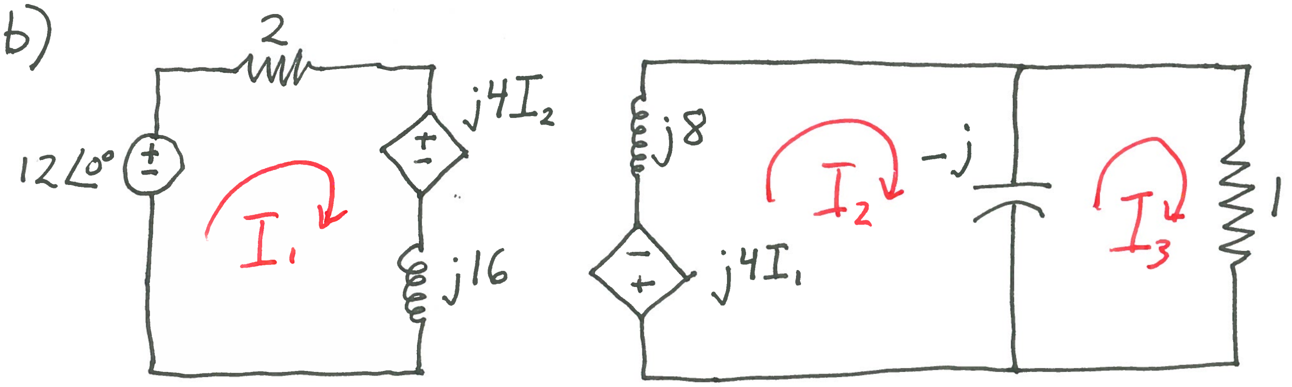
(a) Find the coupling coefficient
(b) Calculate $v_o$
(c) Determine the energy stored in the coupled inductors at t = 2 sec.

a) $k = M/\sqrt{L_1 L_2} = \dfrac{1}{\sqrt{(4)(2)}}$

$$\boxed{k = 0.3535}$$




b)




MESH #1: $-12 + 2I_1 + j4I_2 + j16I_1 \Rightarrow (2+j16)I_1 + j4I_2 = 12$

MESH #2: $j4I_1 + j7I_2 + jI_3 = 0$

MESH #3: $jI_2 + (1-j)I_3 = 0$

USING MATLAB:

$$\begin{bmatrix} (2+j16) & j4 & 0 \\ j4 & j7 & j \\ 0 & j & (1-j) \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 12 \\ 0 \\ 0 \end{bmatrix}$$

## Problem 13.24 (MATLAB Output)

```
>> A=[(2+16i) 4i 0; 4i 7i i; 0 i (1-i)]

A =

   2.0000 +16.0000i   0.0000 + 4.0000i   0.0000 + 0.0000i
   0.0000 + 4.0000i   0.0000 + 7.0000i   0.0000 + 1.0000i
   0.0000 + 0.0000i   0.0000 + 1.0000i   1.0000 - 1.0000i

>> b=[12;0;0]

b =

    12
     0
     0

>> x=A\b

x =

   0.1304 - 0.8447i
  -0.0991 + 0.4439i
   0.1724 + 0.2715i

>> abs(x)

ans =

    0.8547
    0.4548
    0.3216

>> angle(x)*180/pi()

ans =

  -81.2264
  102.5877
   57.5877
```

TO FIND $V_o$

$$V_o = i_3 (1\Omega)$$

$0.1304 - 0.8447i = I_1 = 0.8547\angle -81.2264$

$-0.0991 + 0.4439i = I_2 = 0.4548\angle 102.5877$

$0.1724 + 0.2715i = I_3 = 0.3216\angle 57.5877$

$0.8547 = |i_1|$

$0.4548 = |i_2|$

$0.3216 = |i_3|$

$-81.2264 = \angle$ of $i_1$

$102.5877 = \angle$ of $i_2$

$57.5877 = \angle$ of $i_3$

$$i_1 = 0.8547\cos(4t - 81.2264°)$$

$$i_2 = 0.4548\cos(4t + 102.5877°)$$

$$i_3 = 0.3216\cos(4t + 57.5877°)$$

$$\boxed{V_o = 0.3216\cos(4t + 57.5877°)\ V}$$

c.) Energy at $t = 2$ sec

$$4t = 4(2) = 8 \text{ rad} = 98.37^\circ$$

$$i_1 = 0.8547 \cos(98.37^\circ - 81.23^\circ) = 0.8167$$

$$i_2 = 0.4548 \cos(98.37^\circ + 102.5877^\circ) = -0.4247$$

CURRENTS <u>BOTH</u> <u>ENTER</u> DOTTED TERMINALS

$$\therefore \quad w = \frac{1}{2}L_1 i_1^2 + \frac{1}{2}L_2 i_2^2 + M i_1 i_2$$

$$= \frac{1}{2}(4)(0.8167)^2 + \frac{1}{2}(2)(-0.4247)^2 + (1)(0.8167)(-0.4247)$$

$$= 1.334 + 0.1804 - 0.3469$$

$$\boxed{w = 1.168 \text{ J}} \quad @ \ \underline{t = 2 \text{ sec}}$$