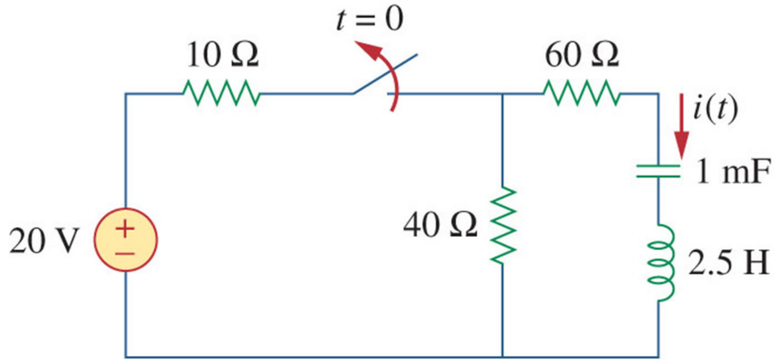
IUPUI ECE 202 Spring 2015:

Homework #9 (SOLUTION KEY)   Name:

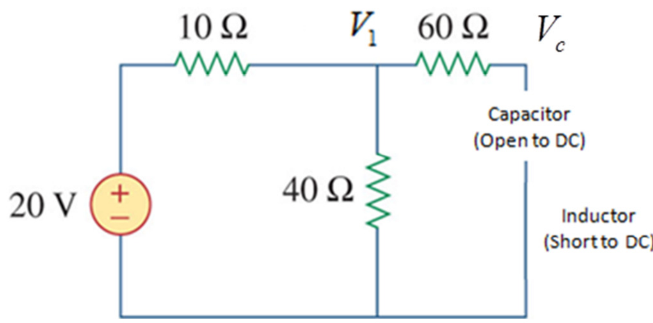
1. (Prob. 16.20 in text) Find $i(t)$ for $t > 0$ in the circuit below:
   Hint: Should be able to factor the quadratic into a double pole.



First find initial conditions (circuit for t < 0)

$$V_1 = \frac{40}{40+10}(20) = 16 \text{ V}$$

$$V_c = V_1 = 16 \text{ V}$$



Initial capacitor voltage is 16 V. Now use Laplace Model of Capacitor with initial conditions. Model circuit for t > 0

Loop equation:

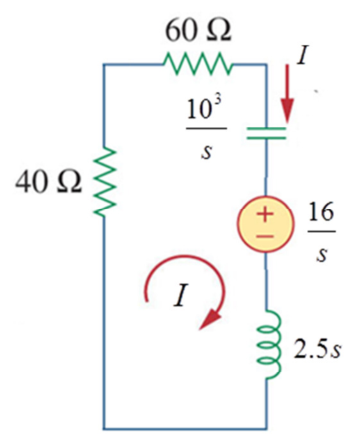
$$I(40) + I(60) + I\left(\frac{10^3}{s}\right) + \frac{16}{s} + I(2.5s) = 0$$

$$I\left(100 + \frac{10^3}{s} + 2.5s\right) = -\frac{16}{s}$$

$$I\left(2.5s^2 + 100s + 1000\right) = -16$$

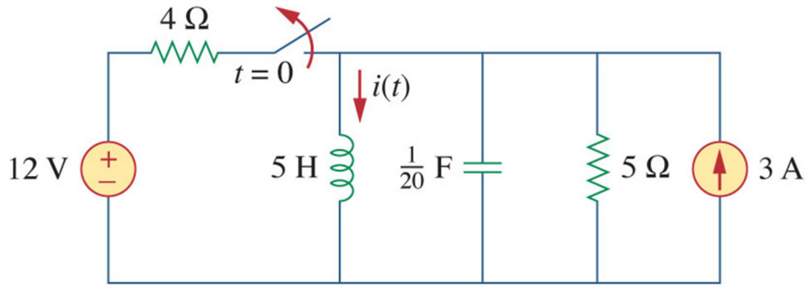
$$I\left(s^2 + 40s + 400\right) = \frac{-16}{2.5} = -6.4$$

$$I = \frac{-6.4}{\left(s^2 + 40s + 400\right)} = \frac{-6.4}{(s+20)^2}$$



$$\boxed{i(t) = \mathcal{L}^{-1}\left[\frac{-6.4}{(s+20)^2}\right] = -6.4te^{-20t} \text{ A} \quad \text{for } t > 0}$$

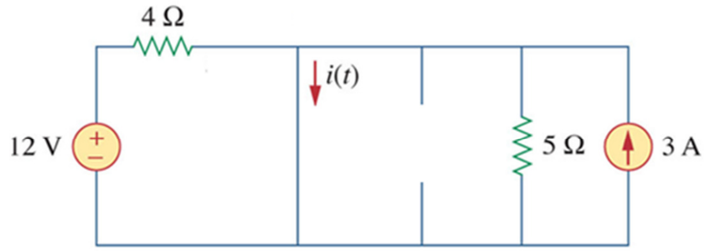
2. (Prob. 16.43 from Text) Find $i(t)$ for $t > 0$ in the circuit below:



First find initial conditions (circuit for t < 0). Capacitor is like an open, inductor like a short. Initial voltage across capacitor = 0. Initial current through inductor is 3 + 12/4 = 6 Amps

$$i_l(0^-) = \frac{12}{4} + 3 = 6$$

$$v_c(0^-) = 0$$



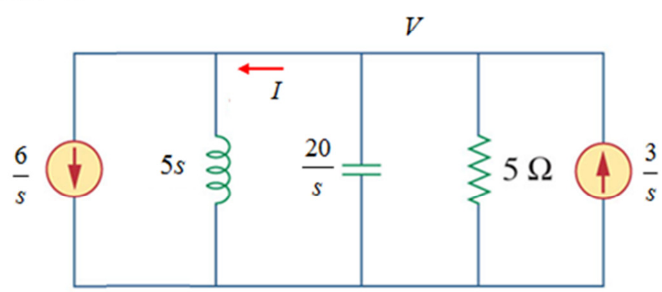
Next use the initial conditions to construct the t > 0 model of the circuit. Sum the currents into the single node to find V:

$$V\left(\frac{1}{5s} + \frac{s}{20} + \frac{1}{5}\right) + \frac{6}{s} = \frac{3}{s}$$

$$V\left(\frac{1}{5} + \frac{s^2}{20} + \frac{s}{5}\right) = -3$$

$$V\left(4 + s^2 + 4s\right) = -60$$

$$V = \frac{-60}{s^2 + 4s + 4} = \frac{-60}{(s+2)^2}$$



Current I is the sum of the current through the inductor and initial condition current source

$$I = \frac{-60}{(s+2)^2} \cdot \frac{1}{5s} + \frac{6}{s} = \frac{6}{s} - \frac{12}{s(s+2)^2} = \frac{6}{s} + \frac{k_0}{s} + \frac{k_1}{(s+2)^2} + \frac{k_2}{(s+2)}$$

Substitute 1 in for s to find the last k_2

Residue method

$$6 - \frac{12}{(0+2)^2} = 6 + k_0 \Rightarrow \boxed{k_0 = -3}$$

Residue method

$$-\frac{12}{-2} = \boxed{k_1 = 6}$$

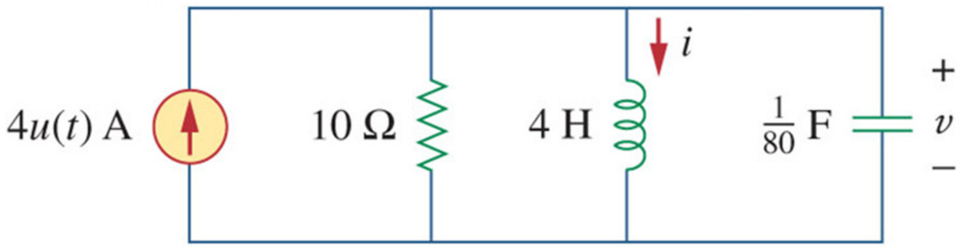
$$\frac{6}{1} - \frac{12}{1(1+2)^2} = \frac{6}{1} - \frac{3}{1} + \frac{6}{(1+2)^2} + \frac{k_2}{(1+2)}$$

$$-4 = -9 + 2 + k_2 \Rightarrow \boxed{k_2 = 3}$$

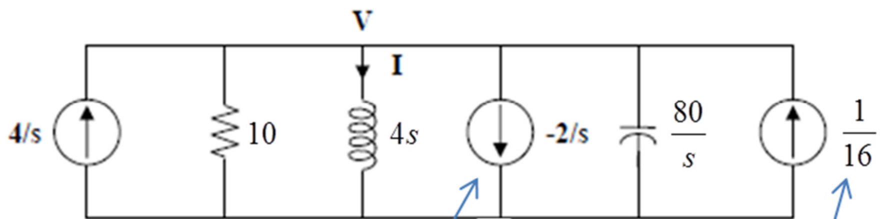
$$I = \frac{6}{s} - \frac{3}{s} + \frac{6}{(s+2)^2} + \frac{3}{(s+2)} \quad \Rightarrow \quad \boxed{i(t) = 3u(t) + 6te^{-2t} + 3e^{-2t} \text{ A for } t > 0}$$

3. (Prob. 16.63 from Text) Consider the parallel RLC circuit shown below. Find $v(t)$ for $t > 0$ given the following initial conditions: $v(0) = 5$ V and $i(0) = -2$ A:

Hint: May need to use the "Complete the square" method



First construct the s-domain form of the circuit with the initial conditions:



Inductor Initial Condition

$$\frac{i_l(0^-)}{s} = -\frac{2}{s}$$

Capacitor Initial Condition

$$v_c(0^-)C = 5\left(\frac{1}{80}\right) = \frac{1}{16}$$

Summing current into the node

$$\frac{4}{s}+\frac{2}{s}+\frac{1}{16} = V\left(\frac{1}{10}+\frac{1}{4s}+\frac{s}{80}\right)$$

$$6+\frac{s}{16} = V\left(\frac{s}{10}+\frac{1}{4}+\frac{s^2}{80}\right)$$

$$480+5s = V\left(8s+20+s^2\right)$$

$$V = \frac{5s+480}{\left(s^2+8s+20\right)}$$

Completing the square

$$s^2+8s+20 = s^2+2as+\left(a^2+\omega^2\right)$$

$$a = 4$$

$$20 = 4^2+\omega^2 \quad \Rightarrow \quad \omega = 2$$

$$V = \frac{5s+480}{(s+4)^2+2} = \frac{5(s+4)+460}{(s+4)^2+2} = \frac{5(s+4)}{(s+4)^2+2}+\frac{(230)(2)}{(s+4)^2+2}$$

$$v(t) = 5\mathcal{L}^{-1}\left[\frac{(s+4)}{(s+4)^2+2}\right]+230\mathcal{L}^{-1}\left[\frac{(2)}{(s+4)^2+2}\right] = \boxed{5e^{-4t}\cos(2t)+230e^{-4t}\sin(2t)\text{ V for } t>0}$$