Chapter 14, Frequency Response

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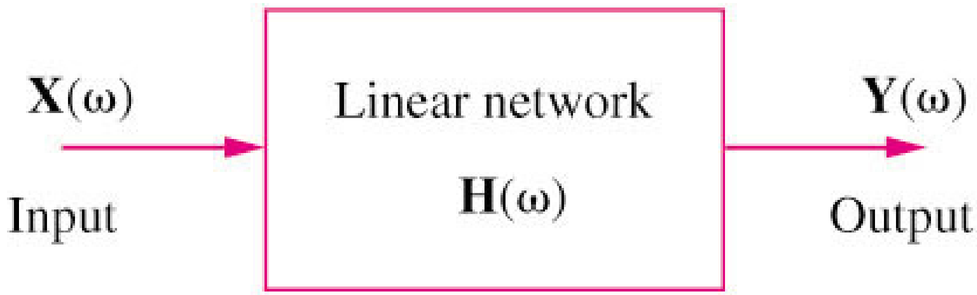
14.1 Introduction (1)

- Up to this point in our sinusoidal circuit analysis we have learned how to find voltages and currents in a circuit with a constant frequency source.
- If the amplitude of the sinusoidal source is held constant and the frequency is varied, we obtain the frequency response of the circuit.
- The frequency response of the circuit is the variation in it's behavior with change in signal frequency.
- A specific application of sinusoidal steady-state frequency response is in **electronic filters** that block out or eliminate signals with unwanted frequencies and pass signals of the desired frequencies.
- Filters are used in radio, TV and telephone systems to separate one broadcast frequency from another.

14.2 Transfer Function (1)

- The transfer function H(ω) of a circuit is the **frequency-dependent ratio** of a phasor output Y(ω) (an element voltage or current) to a phasor input X(ω) (source voltage or current).
- The frequency response of a circuit is the plot of the circuit's transfer function H(ω) versus ω, with ω varying from ω = 0 to ω = ∞



$$H(\omega) = \frac{Y(\omega)}{X(\omega)} = |H(\omega)|\angle\phi$$

14.2 Transfer Function (2)

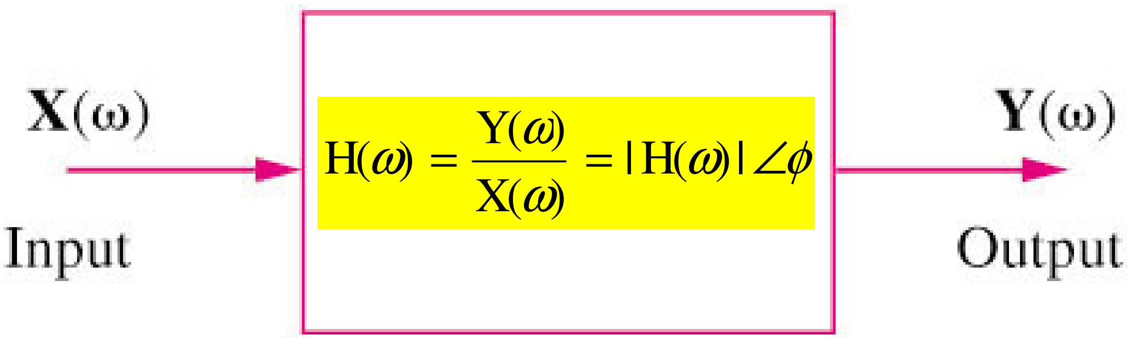
- There are four possible transfer functions:

$$\mathrm{H}(\omega) = \text{Voltage gain} = \frac{\mathrm{V_o}(\omega)}{\mathrm{V_i}(\omega)}$$

$$\mathrm{H}(\omega) = \text{Current gain} = \frac{\mathrm{I_o}(\omega)}{\mathrm{I_i}(\omega)}$$

$$\mathrm{H}(\omega) = \text{Transfer Impedance} = \frac{\mathrm{V_o}(\omega)}{\mathrm{I_i}(\omega)}$$

$$\mathrm{H}(\omega) = \text{Transfer Admittance} = \frac{\mathrm{I_o}(\omega)}{\mathrm{V_i}(\omega)}$$



14.2 Transfer Function (3)

- H(ω) is a complex quantity: $\mathrm{H}(\omega) = |\mathrm{H}(\omega)| \angle \phi$
  - |H(ω)| is the magnitude, <Φ is the phase angle.

$$H(\omega) = \frac{N(\omega)}{D(\omega)} \begin{array}{l} \longleftarrow \text{ Numerator polynomial} \\ \longleftarrow \text{ Denominator polynomial} \end{array}$$

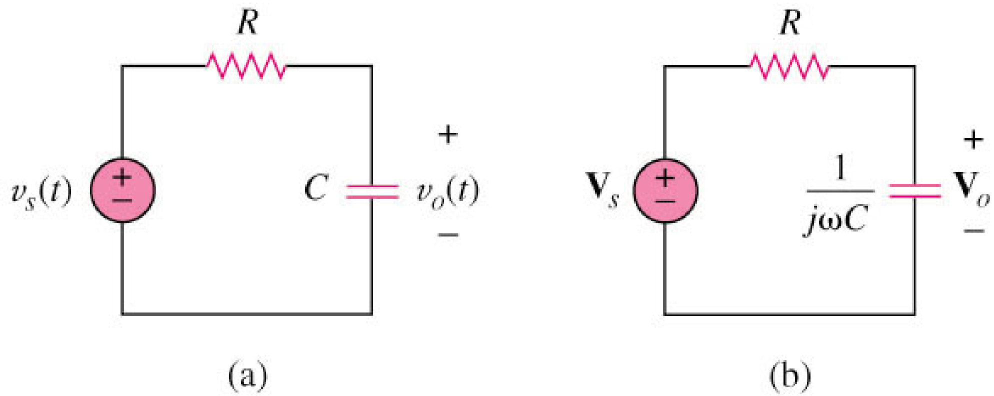
- Roots of *N(ω)=0* are called **zeros** of H(ω): $j\omega = z_1, z_2, \ldots$
- Roots *D(ω)=0* are called the **poles** of H(ω): $j\omega = p_1, p_2, \ldots$
- To avoid complex algebra, it is helpful to replace jω temporarily with s when working with the transfer function and replace s with jω a the end.

$$H(s) = \frac{N(s)}{D(s)} \begin{array}{l} \nearrow = 0 \quad \textit{When } N(s)=0 \textit{ at } \textbf{zeros} \ s = z_1, z_2, \ldots \\ \searrow = \infty \quad \textit{When } D(s)=0 \textit{ at } \textbf{poles} \ s = p_1, p_2, \ldots \end{array}$$

14.2 Transfer Function (4)

Example 14.1

For the RC circuit shown below, obtain the transfer function V_o/V_s and its frequency response.
Let $v_s = V_m \cos\omega t$.



14.2 Transfer Function (5)

Solution:

The transfer function is

NO ZEROS

$$H(s) = \frac{V_o}{V_s} = \frac{1/sC}{R + 1/sC} = \frac{1}{1 + sRC}$$

POLE "p_1" at s = - 1/RC

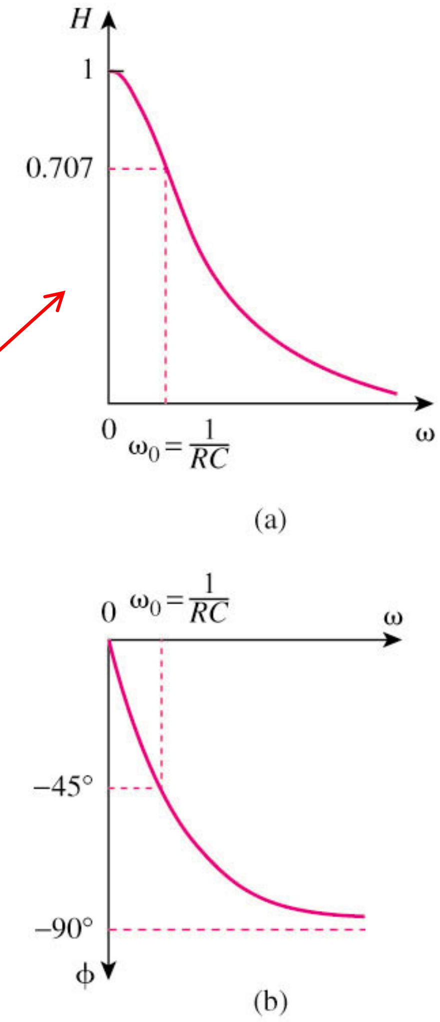
$$H(\omega) = \frac{1}{1 + j\omega RC}$$

The magnitude is $|H(\omega)| = \dfrac{1}{\sqrt{1 + (\omega/\omega_o)^2}}$

where $\omega_o = 1/RC$

The phase is $\phi = -\tan^{-1}\dfrac{\omega}{\omega_o}$

Lowpass Filter



14.2 Transfer Function (6)

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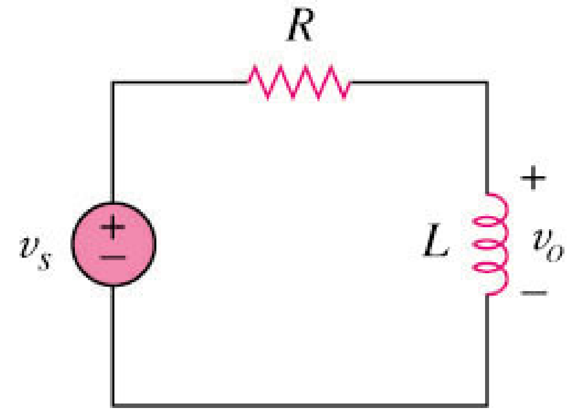
Practice Problem 14.1

Obtain the transfer function V_o/V_s of the RL circuit shown below, assuming $v_s = V_m\cos\omega t$. Sketch its frequency response.



14.2 Transfer Function (7)

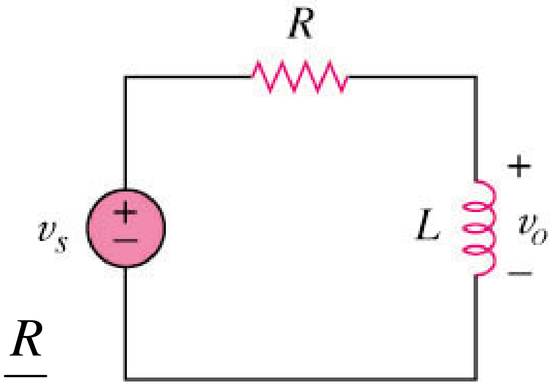
Solution:

$$H(s)=\frac{V_o}{V_s}=\frac{sL}{R+sL}=\frac{s\frac{L}{R}}{1+s\frac{L}{R}}$$

Zero z at s = 0

Pole p at s = $-\omega_0$

where $\omega_o=\frac{R}{L}$



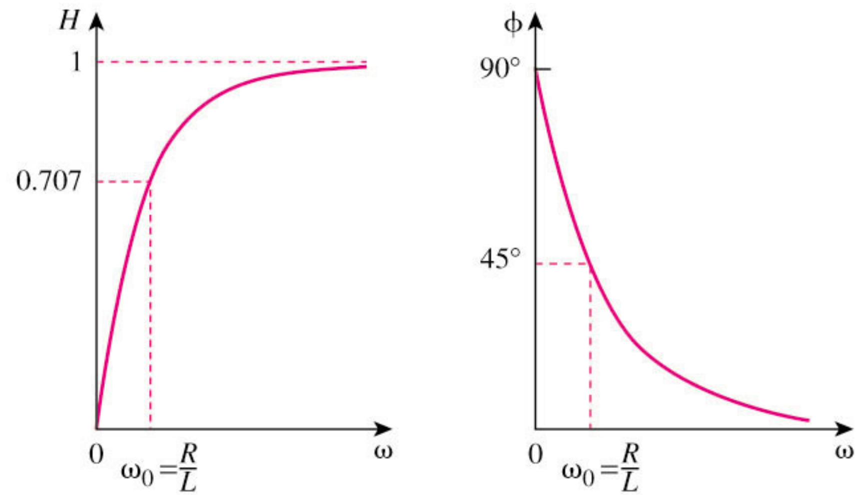
$$H(\omega)=\frac{j\omega/\omega_0}{1+j\omega/\omega_0}$$

Highpass Filter

The magnitude is

$$|\mathrm{H}(\omega)|=\frac{\omega/\omega_o}{\sqrt{1+(\omega/\omega_o)^2}}$$

The phase is $\phi=90°-\tan^{-1}(\frac{\omega}{\omega_o})$



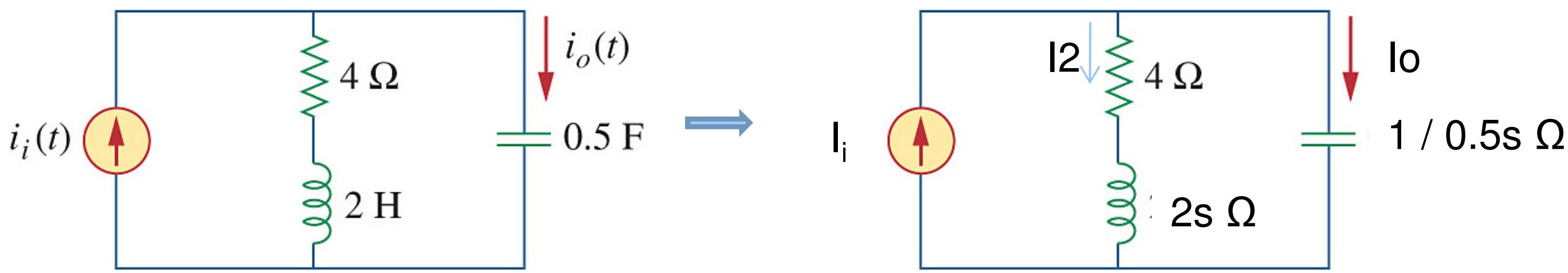
(a) (b)

At ω=0: H=0, Φ=90°
At ω=∞: H=1, Φ=0°
At $\omega=\omega_0$: H=1/√2, Φ=90°-45°=45°

14.2 Transfer Function (8)

Example 14.2

Find the current gain $I_o(\omega)/I_i(\omega)$ and its poles and zeros.



Substitute s for jω

(1) $I_i = I_2 + I_0$

(2) $$I_2(4+2s) = I_0\left(\frac{1}{0.5s}\right) \Rightarrow I_2 = I_0\left(\frac{\frac{1}{0.5s}}{4+2s}\right) = \frac{1}{0.5s(4+2s)} = \frac{1}{s(2+s)}$$

Substituting (2) in (1)

(3) $$I_i = I_2 + I_0 = I_0\left(1+\frac{1}{s(2+s)}\right) = I_0\left(\frac{s(2+s)+1}{s(2+s)}\right) = I_0\left(\frac{s^2+2s+1}{s(2+s)}\right)$$

$$\frac{I_0}{I_i} = \frac{s(2+s)}{s^2+2s+1}$$

zeros: $s(s+2) = 0 \rightarrow z_1 = 0,\ z_2 = -2$

poles: $s^2+2s+1 = (s+1)(s+1) = (s+1)^2 = 0$
$\rightarrow p = -1$ (double pole)

14.3 Decibel Scale (1)

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- It is common to use log and decibels to measure gain:
  - Eg: sound is measured (and experienced by humans) in a logarithmic fashion.

- Remember:
  - $\text{Log } P_1P_2 = \text{Log } P_1 + \text{Log } P_2$ *Multiplying → Adding*
  - $\text{Log } P_1 / P_2 = \text{Log } P_1 - \text{Log } P_2$ *Dividing → Subtracting*
  - $\text{Log } P^n = nLogP$ *Raise to N → Multiply by N*
  - $\text{Log } 1 = 0$

14.3 Decibel Scale (2)

- The 10 $\log_{10}$ is used for power while 20 $\log_{10}$ is used for voltage or current, because of the square relationship between them.
  - $G_{dB} = 10 \log_{10} P_2/P_1$
  - $G_{dB} = 20 \log_{10} V_2/V_1$; $G_{dB} = 20 \log_{10} I_2/I_1$
- The dB value is a logarithmic measurement of the ratio of one variable to another of the same type.
- Only voltage and current magnitudes are used in expressing G_{dB}. Negative signs and angels are handeled independently.

Power Gain: $G_{dB} = 10 \log_{10} P_2/P_1$

Voltage (or Current) Gain: $G_{dB} = 20 \log_{10} V_2/V_1$

$$P_{ave} = \frac{V_{rms}^2}{R} = I_{rms}^2 R$$

Squared relationship Causes "20" log

14.3 Decibel Scale (3)

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- Specific gain and their decibel values.

Power Gain

Half Power = -3 dB

2x Power = +3 dB

$10\log_{10}H$ (dB)	-30	-20	-10	-3	0	3	10	13	20	30
$\lvert H \rvert$	0.001	0.01	0.1	0.5	1	2	10	20	100	1000
$20\log_{10}H$ (dB)	-60	-40	-20	-6	0	6	20	26	40	60

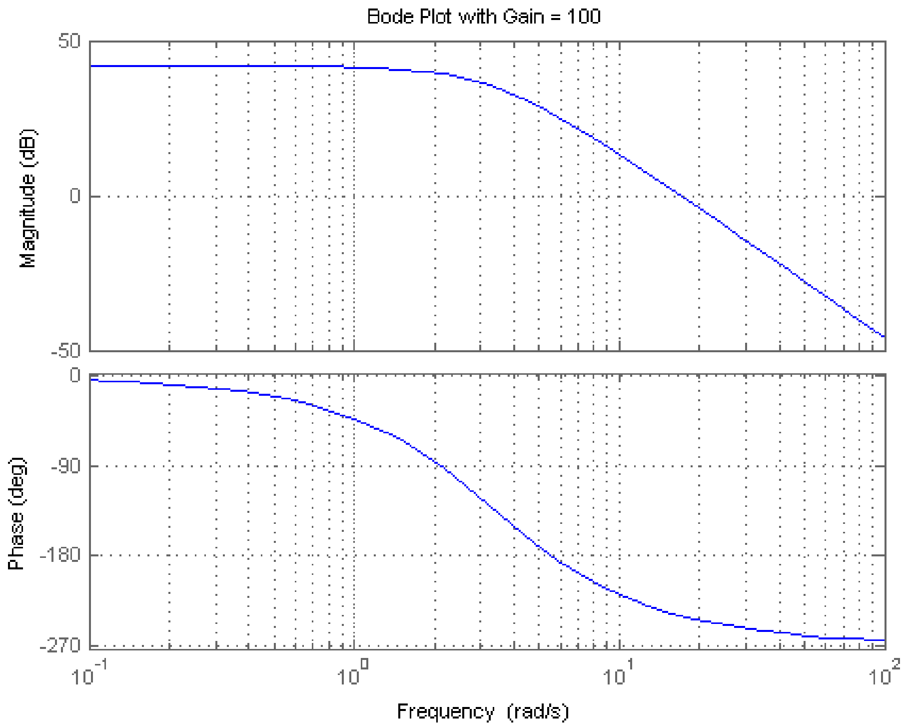
Voltage (or Current) Gain

Half Voltage = -6 dB

- The origin is where $\omega = 1$ or $\log\omega = 0$ and the gain is zero.

14.4 Bode Plots (1)

- The frequency range required in frequency response is often so wide that it is inconvenient to use a linear scale for the frequency axis.
- Bode plots are semilog plots of the magnitude (in decibels) and phase (in degrees) of a transfer function versus frequency (on log scale)



14.4 Bode Plots (2)

Transfer Function Representation

Zero/pole at origin
Gain
Simple zero
Quadratic zero

$$H(\omega) = \frac{K(j\omega)^{\pm 1}\left(1+\frac{j\omega}{z_1}\right)\left[1+j2\zeta_1\frac{\omega}{\omega_k}+\left(\frac{j\omega}{\omega_k}\right)^2\right]\cdots}{\left(1+\frac{j\omega}{p_1}\right)\left[1+j2\zeta_2\frac{\omega}{\omega_n}+\left(\frac{j\omega}{\omega_n}\right)^2\right]\cdots}$$

Simple pole
Quadratic pole

- *K is Gain at DC*
- *(jω) is zero while $(j\omega)^{-1}$ is pole or at the origin (DC)*
- *z_1 is simple zero (single)*
- *p_1 is simple pole (single)*
- *ω_K is a quadratic zero*
- *ω_n is a quadratic pole*
- The Bode plot starts with this form:
  - Remember: put the polynomials in the (1+ ….) form!

14.4 Bode Plots (3)

- In constructing a Bode plot, plot each factor separately and then add them graphically.
  - Due to log function, multiplications in H(ω) become additions/subtractions in Bode plots, and therefore are easier to visualize.

$$H = \frac{K \cdot A \cdot B \cdot C}{D \cdot E}$$

Log(AB) = Log(A) + Log(B)

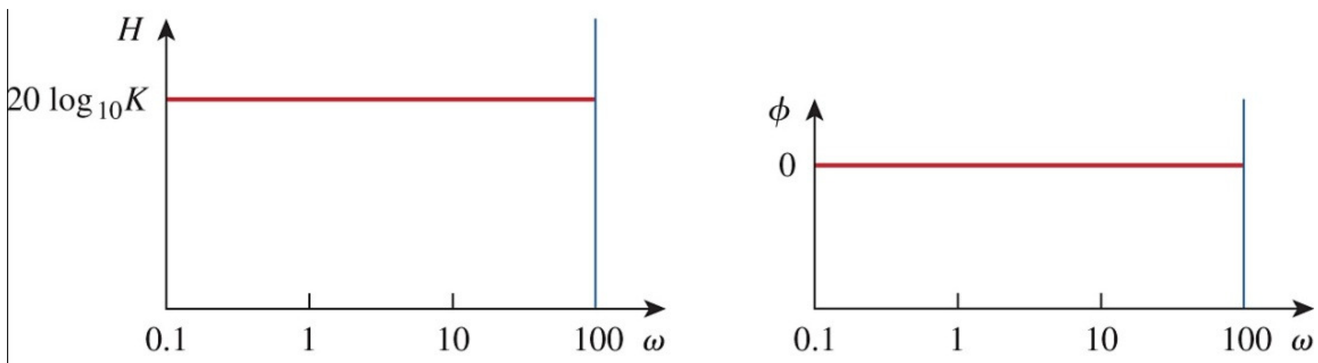
Log(A/B) = Log(A) - Log(B)

$$20\log_{10} H = 20\log_{10} K + 20\log_{10} A + 20\log_{10} B + 20\log_{10} C - 20\log_{10} D - 20\log_{10} E$$

  - Straight line Bode plots approximate the actual curve plots to a reasonable degree of accuracy.
  - Poles and Zeros of H(ω) are the "inflection" points of the Bode plot (slope of lines change).

14.4 Bode Plots (4)

- Constant term K is Gain at DC
  - Magnitude is 20 $\log_{10}$K and phase is 0°
  - Magnitude and phase are both constant with frequency
  - If K is negative, the magnitude remains 20 $\log_{10}|K|$, but the phase is -180°.



(a) (b)

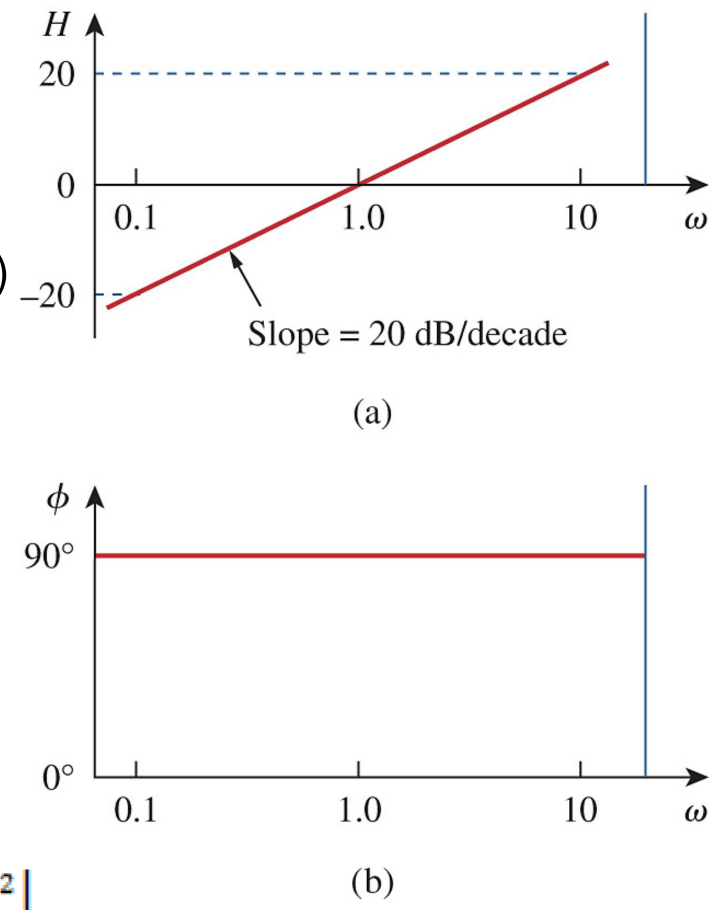
If K is negative

$\phi = -180^{\circ}$

$$H(\omega) = \frac{K(j\omega)^{\pm 1}\left(1+\frac{j\omega}{z_1}\right)\left[1+j2\zeta_1\frac{\omega}{\omega_k}+\left(\frac{j\omega}{\omega_k}\right)^2\right]\cdots}{\left(1+\frac{j\omega}{p_1}\right)\left[1+j2\zeta_2\frac{\omega}{\omega_n}+\left(\frac{j\omega}{\omega_n}\right)^2\right]\cdots}$$

14.4 Bode Plots (5)

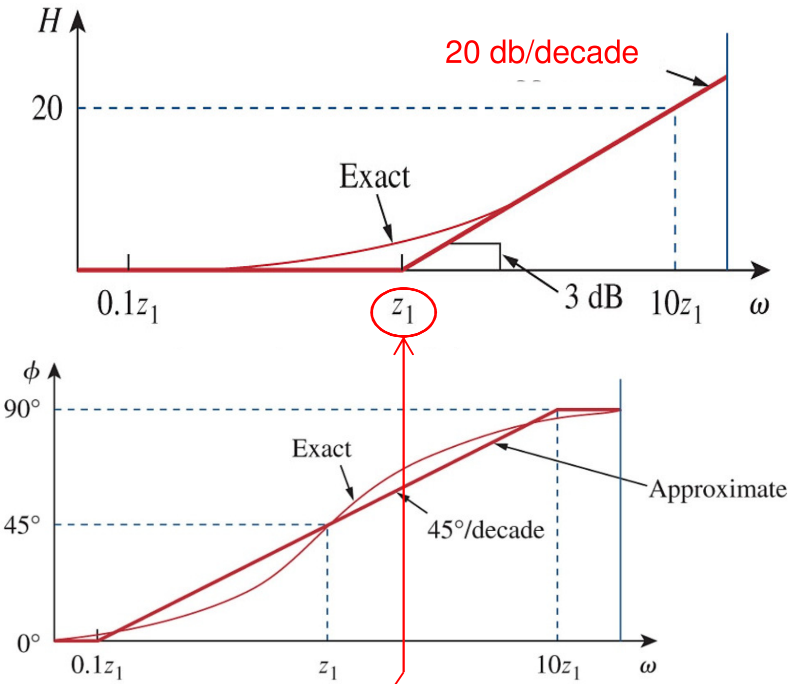
- Zero $j\omega$ at the origin (Positive Slope):
  - Magnitude is $20\log_{10}\omega$ and phase is 90°
  - The slope of the magnitude plot is 20 dB/decade
- Pole $(j\omega)^{-1}$ at the origin (Negative Slope)
  - The slope of the magnitude plot is -20 dB/decade
  - The phase is -90°
- In general for $(j\omega)^N$ where N is integer
  - The slope of the magnitude plot is 20N dB/decade
  - The phase is 90N degrees



$$H(\omega) = \frac{K(j\omega)^{\pm 1}\left(1+\frac{j\omega}{z_1}\right)\left[1+j2\zeta_1\frac{\omega}{\omega_k}+\left(\frac{j\omega}{\omega_k}\right)^2\right]\cdots}{\left(1+\frac{j\omega}{p_1}\right)\left[1+j2\zeta_2\frac{\omega}{\omega_n}+\left(\frac{j\omega}{\omega_n}\right)^2\right]\cdots}$$

14.4 Bode Plots (6)

- For simple zero $(1+ j\omega/z_1)$:
  - The magnitude is $20 \log_{10} |1 + j\omega/z_1|$
  - $\omega=z_1$ is corner or break frequency
  - $20 \log_{10} 1 = 0$ as $\omega \to 0$
  - $20 \log_{10} \omega/z_1 \to \infty$ as $\omega \to \infty$
  - The magnitude for $\omega > z_1$ has a slope of 20 dB/decade
  - The phase angel, Φ, is $\tan^{-1} \omega/z_1$
  - $\Phi = 45°$ for $\omega = z_1$
  - $\Phi \approx 0°$ for $\omega \le z_1/10$
  - $\Phi \approx 90°$ for $\omega \ge 10z_1$
  - The phase has a slope of 45°/decade

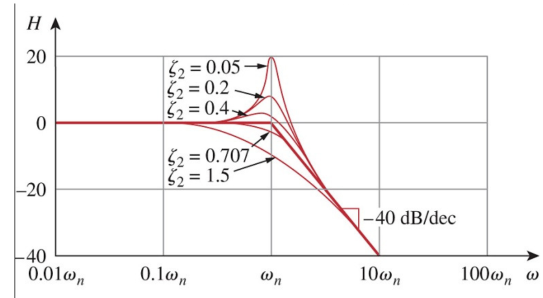


$$H(\omega) = \frac{K(j\omega)^{\pm 1}\left(1+\frac{j\omega}{z_1}\right)\left[1+j2\zeta_1\frac{\omega}{\omega_k}+\left(\frac{j\omega}{\omega_k}\right)^2\right]\cdots}{\left(1+\frac{j\omega}{p_1}\right)\left[1+j2\zeta_2\frac{\omega}{\omega_n}+\left(\frac{j\omega}{\omega_n}\right)^2\right]\cdots}$$

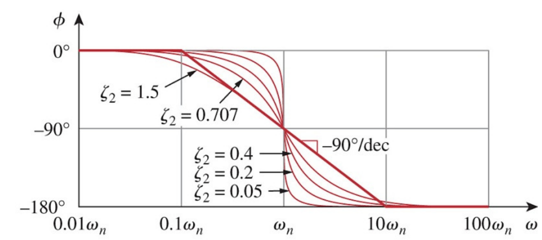
- For simple pole $1/(1+ j\omega/p_1)$:
  - The corner frequency is at $\omega=p_1$
  - The magnitude has a slope of -20 dB/decade
  - The phase has a slope of -45°/decade

14.4 Bode Plots (7)

- For a quadratic pole $1/[1+ j2\zeta_2\omega/\omega_n)+(j\omega/\omega_n)^2]$:
  - The magnitude $|H_{dB}|$ is $-20 \log_{10} |1 + j2\zeta_2\omega/\omega_n)+(j\omega/\omega_n)^2|$
  - The actual plot depends on the damping factor ζ_2
  - $|H_{dB}| = 0$ as $\omega \to 0$; $|H_{dB}| \to -40\log_{10}\omega/\omega_n \to \infty$ as $\omega \to \infty$
  - $\omega = \omega_n$ is the corner frequency
  - The magnitude for $\omega > \omega_n$ has a slope of -40 dB/decade
  - The phase angel, $\Phi = -\tan^{-1} [(2\zeta_2\omega/\omega_n)/(1- \omega^2/\omega_n^2)$
  - $\Phi = 0°$ for $\omega = 0$; $\Phi = -90°$ for $\omega = \omega_n$; $\Phi = -180°$ for $\omega \to \infty$
  - The phase plot has a slope of -90°/decade from $\omega_n/10$ to $10\omega_n$
  - The straight line approximation for both magnitude and phase plots for a quadratic pole are the same as those for a double pole



(a)



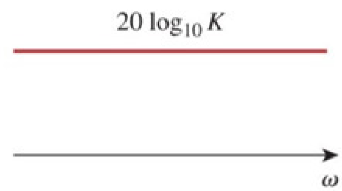
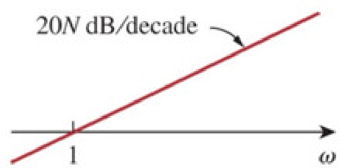
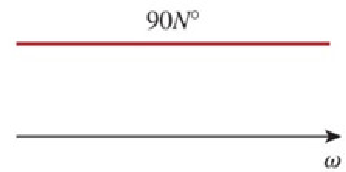
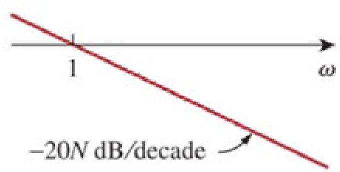
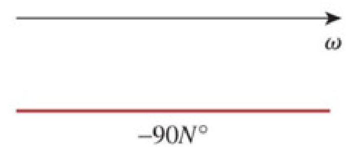
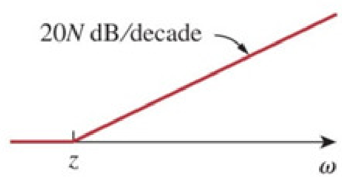
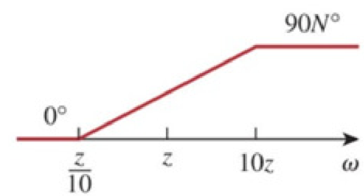
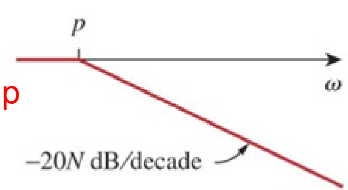
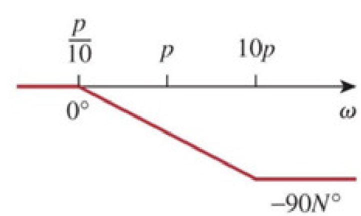
(b)

- For a quadratic zero $[1+ j2\zeta_1\omega/\omega_k)+(j\omega/\omega_k)^2]$:
  - The corner frequency is at $\omega=\omega_k$
  - The magnitude plot has a slope of 40 dB/decade for $\omega > \omega_k$
  - The phase plot has a slope of 90°/decade

$$H(\omega) = \frac{K(j\omega)^{\pm 1}\left(1+\frac{j\omega}{z_1}\right)\left[1+j2\zeta_1\frac{\omega}{\omega_k}+\left(\frac{j\omega}{\omega_k}\right)^2\right]\cdots}{\left(1+\frac{j\omega}{p_1}\right)\left[1+j2\zeta_2\frac{\omega}{\omega_n}+\left(\frac{j\omega}{\omega_n}\right)^2\right]\cdots}$$

Review Table 14.3 in the textbook:
Summary of Bode straight-line magnitude and phase plots.

Summary of Bode Plots
Straight Line Magnitude and Phase

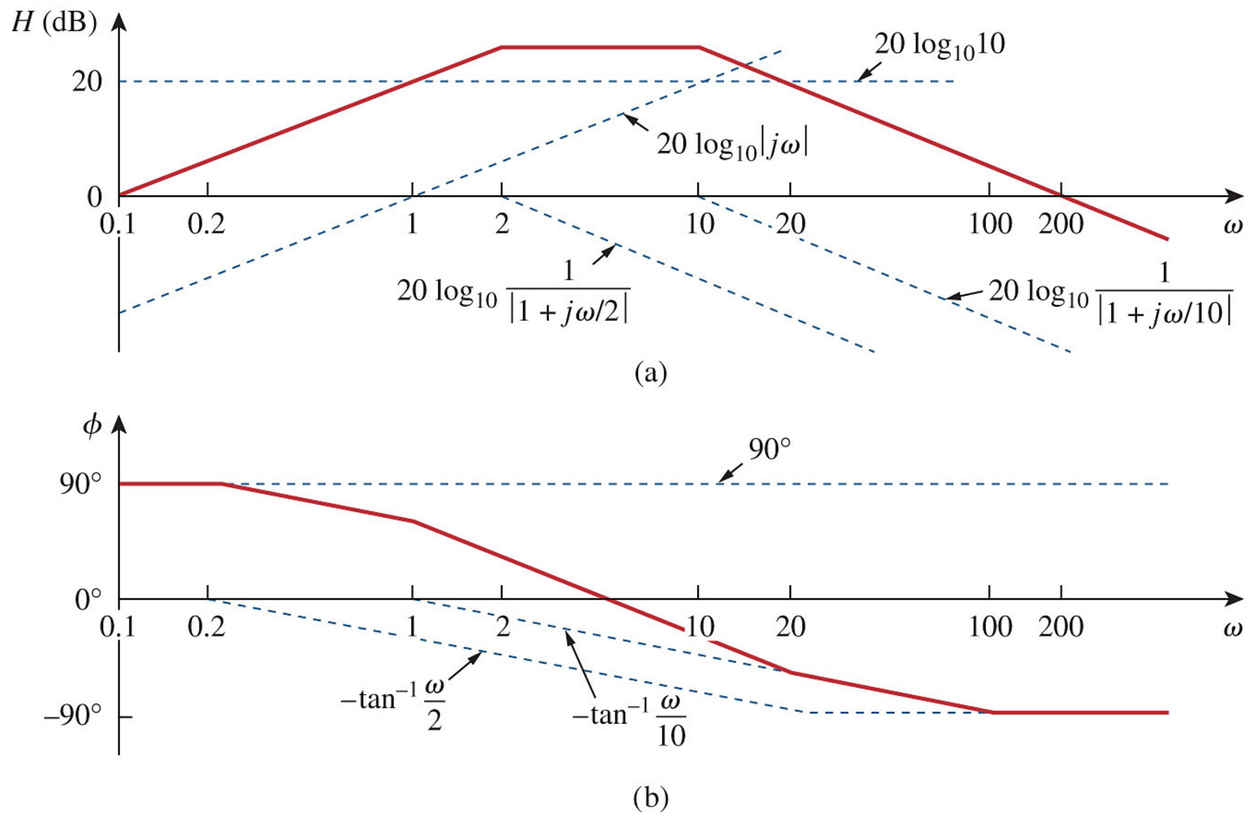
Factor		Magnitude	Phase
K	Gain = 20 Log_{10}K		
$(j\omega)^N$	Zero at jω=0 Slope 20*N/Decade Phase = 90*N		
$\dfrac{1}{(j\omega)^N}$	Pole at jω=0 Slope -20*N/Decade Phase = -90*N		
$\left(1 + \dfrac{j\omega}{z}\right)^N$	Zero at jω=p Slope 20*N/Decade after z Phase “Shifts” to 90*N		
$\dfrac{1}{(1 + j\omega/p)^N}$	Pole at jω=p Slope -20*N/Decade after p Phase “Shifts” to -90*N		

14.4 Bode Plots (8)

Example 14.3

Construct the Bode Plots for the transfer function H(ω):

$$H(\omega)=\frac{200j\omega}{(j\omega+2)(j\omega+10)}=\frac{200j\omega}{2(1+j\omega/2)10(1+j\omega/10)}=\frac{10j\omega}{(1+j\omega/2)(1+j\omega/10)}$$



14.4 Bode Plots (10)

Avoid this simple error

From this you can see a zero at 0 and poles at -2 and -10
However, the gain is NOT 200!

Gain "K" is 10

$$H(\omega) = \frac{200\, j\omega}{(j\omega + 2)(j\omega + 10)} = \frac{10\, j\omega}{(1 + j\omega/2)(1 + j\omega/10)}$$

Must put in correct (1+…) form to get correct gain

14.4 Bode Plots (9)
Alternate Method

Steps:

1. $H(\omega)$ polynomials must be in defined form (1+ ….)
2. Find value at very small ω(DC gain)
3. Find zeros and poles
4. Plot inflection points for gain
5. Determine inflection points for phase
6. Piece-wise plot in sections between inflection points

** Our text teaches superposition over the whole range of omega.

14.4 Bode Plots (10) - Example for Gain

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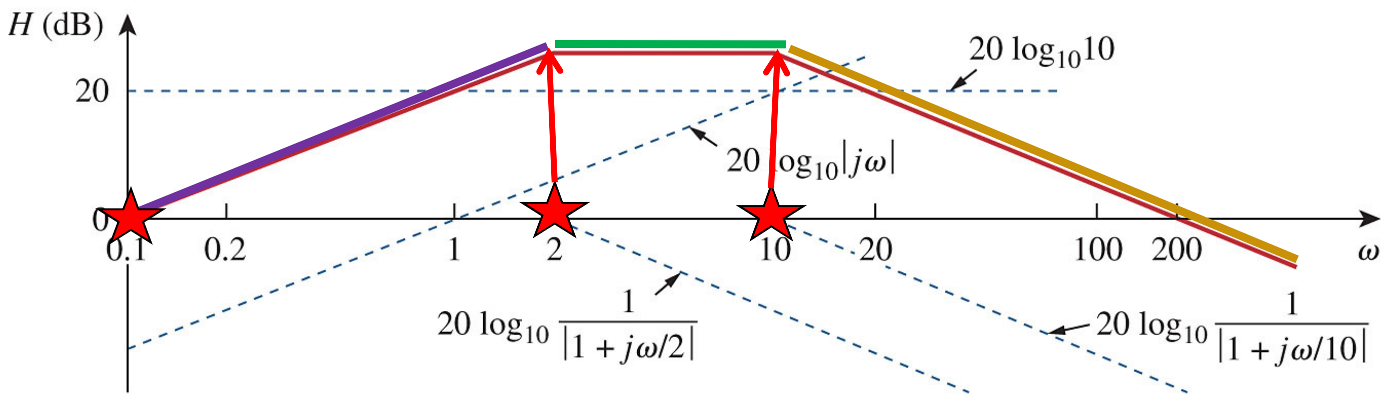
Steps:

$$H(\omega) = \frac{200 j\omega}{(j\omega+2)(j\omega+10)} = \frac{10 j\omega}{(1+j\omega/2)(1+j\omega/10)}$$

1. Put equation in the right form:
2. Find gain at very small ω (DC gain) $20\log_{10}10 + 20\log_{10}|j\omega| \sim 0$
   a) Here it is also useful to find at ω = 1

   $20\log_{10}10 + 20\log_{10}|j| - 20\log_{10}|1+j/2| - 20\log_{10}|1+j/10| \sim 20$
3. Find zeros and poles z=0, p1=2, p2=10
4. Find inflection points for gain (red stars and arrows)
5. Plot, piece-wise in sections between inflection points, summing influence at each inflection piont:
   a) Between 0 and 2, the slope is positive responding to the first zero at the origin
   b) Between 2 and 10 the first pole cancels first zero and makes the magnitude flat
   c) Between 10 and infinity, the 2nd pole dominates, the slope becomes negative



(a)

14.4 Bode Plots (11) - Phase

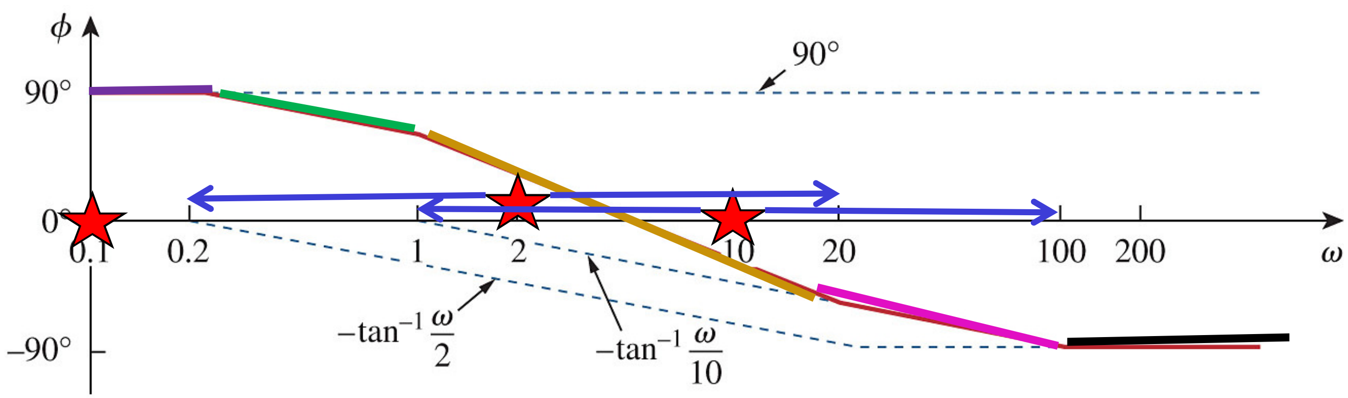
- The main difference in phase plotting is that the inflection points are one decade before and after the pole or zero frequency
  - For example: a zero at 10 will provide inflection points at 1 and 100!

14.4 Bode Plots (12) - Example for Phase

Steps:

$$H(\omega) = \frac{200 j\omega}{(j\omega+2)(j\omega+10)} = \frac{10 j\omega}{(1+ j\omega/2)(1+ j\omega/10)}$$

1. Put equation in the right form:
2. Find value at very small ω (DC phase) Phase of (jω) = 90 deg.
3. Find zeros and poles z=0, p1=2, p2=10
4. Find inflection points phase ±1 decade (red stars and blue arrows)
5. Plot, piece-wise in sections between inflection points, summing influence at each inflection piont:
   a) Between 0 and 0.2, the phase is 90 responding to the first zero at the origin
   b) Between 0.2 and 1 the first pole cancels first zero reducing the phase slope
   c) Between 1 and 20, the both poles are active, the slope becomes more negative
   d) Between 20 and 100, only the last pole is dominant and the slope becomes less negative
   e) After 100, the asymptotic phase is -90 degrees (one zero, two poles)



(b)

14.4 Bode Plots (13)

Practice Problem 14.3

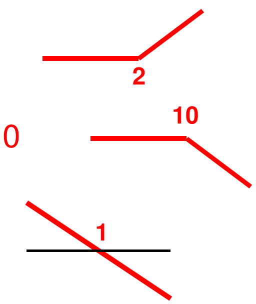
Construct the Bode Plots for the transfer function H(s):

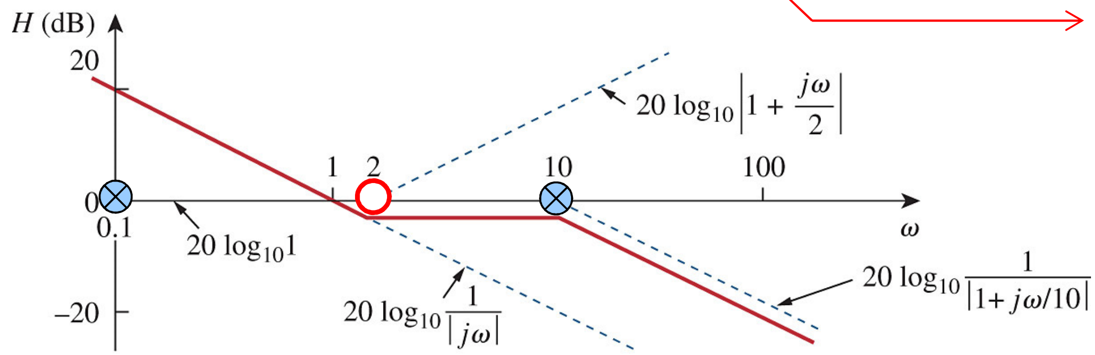
$$\mathrm{H}(s)=\frac{5(s+2)}{s(s+10)}=\frac{5*2(1+s/2)}{10\,s(1+s/10)}=\frac{\boxed{(1+s/2)}}{\boxed{s}\boxed{(1+s/10)}}$$

Simple zero at 2

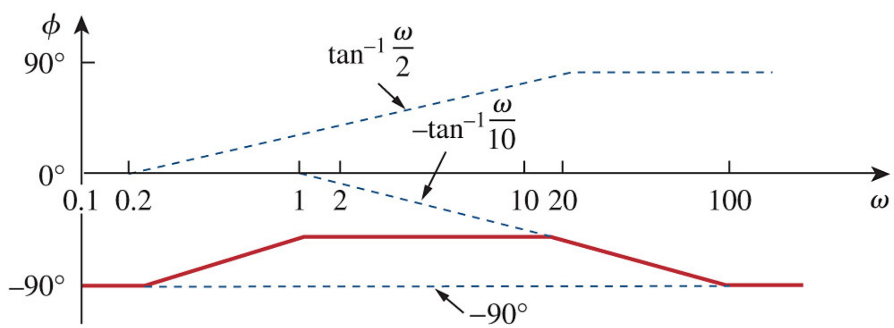
Simple pole at 10

Pole at origin





(a)



(b)

14.4 Bode Plots

Another Example Problem

Problem 14.4 Sketch the Bode plots, both magnitude and phase, given the following transfer function in the s-domain.

$$H(s) = \frac{(90)(s+1)(s+10)}{s(s+3)(s+30)}$$

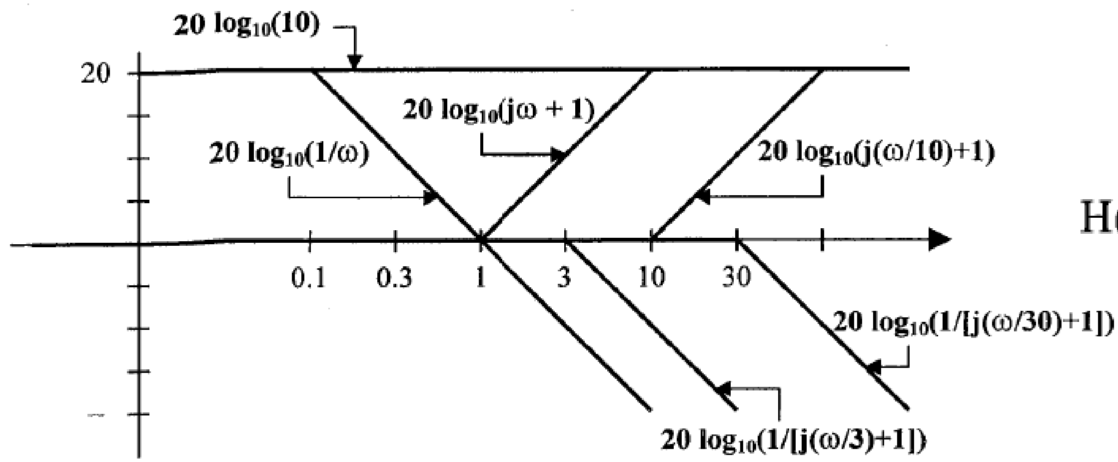
First, we need to modify the transfer function so that it is in a form that is easy to plot.

$$H(s) = \frac{(90)(s+1)(s+10)}{s(s+3)(s+30)} = \frac{(90)(10)\left(\frac{s}{1}+1\right)\left(\frac{s}{10}+1\right)}{(3)(30)(s)\left(\frac{s}{3}+1\right)\left(\frac{s}{30}+1\right)} = \frac{(10)\left(\frac{s}{1}+1\right)\left(\frac{s}{10}+1\right)}{(s)\left(\frac{s}{3}+1\right)\left(\frac{s}{30}+1\right)}$$

- Constant term K = 10: $20\text{Log}_{10}(10) = 20$
- 2 Simple zero's at 1 and 10
- Pole at the origin
- 2 Simple pole's at 3 and 30

14.4 Bode Plots

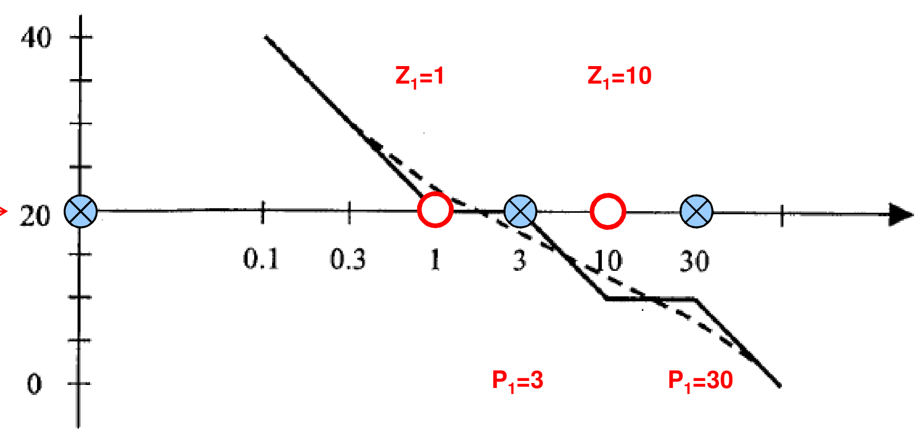
Another Example Problem



$$H(s) = \frac{(10)\left(\frac{s}{1}+1\right)\left(\frac{s}{10}+1\right)}{(s)\left(\frac{s}{3}+1\right)\left(\frac{s}{30}+1\right)}$$

Now, combine, or add, the curves to acquire the composite **magnitude (dB) plot** of the transfer function. Note that the dashed curve shows the approximation to the actual curve.

Gain of 10:
$20 \log_{10} 10 =$ **20**



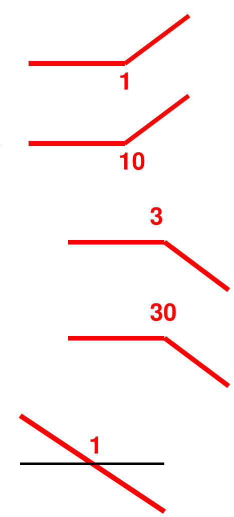
Simple zero at 1

Simple zero at 10

Simple pole at 3

Simple pole at 30

Pole at origin



14.4 Bode Plots (14)

Reverse case, get Transfer Function from Bode Plot

Practice Problem 14.6

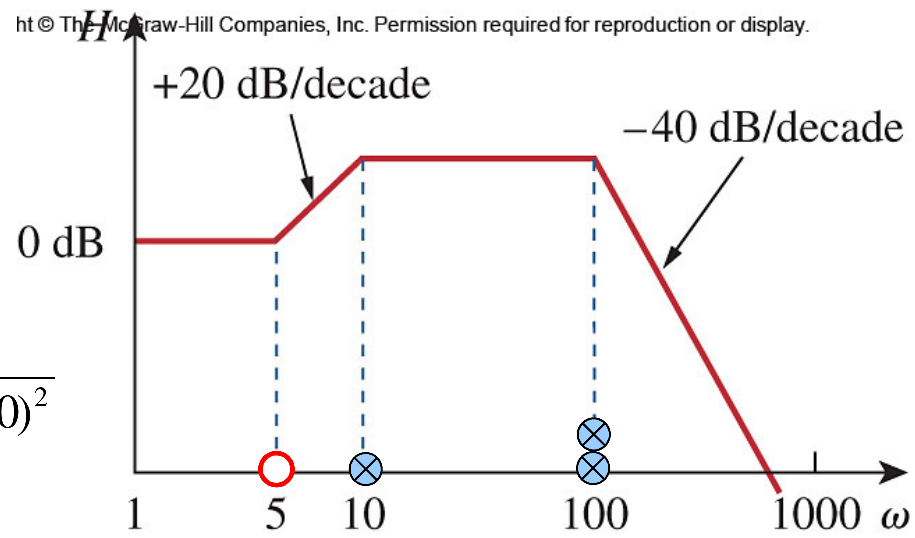
Obtain the transfer function H(ω) corresponding to the Bode plot shown below:

A zero at ω=5: 1+j ω/5
(due to upward turn)

A pole at ω=10: $\frac{1}{1+j\omega/10}$
(due to downward turn)

Two poles at ω=100: $\frac{1}{(1+j\omega/100)^2}$

(due to double downward turn)



Therefore,

$$H(\omega) = \frac{(1+j\omega/5)}{(1+j\omega/10)(1+j\omega/100)^2} = \frac{(1/5)(5+j\omega)}{(1/100000)(10+j\omega)(100+j\omega)^2} = \frac{20000(s+5)}{(s+10)(s+100)^2}$$

14.4 Bode Plots (15)

- Check for small and large values of ω to confirm asymptotic behavior of gain and phase.
- adding zeros will increase slope
  - Double zeros, slope increase twice
- Adding poles will decrease slope
  - Double poles, slope decreases twice
- Check out this website: http://www.facstaff.bucknell.edu/mastascu/eControlHTML/Freq/Freq5.html

Using MATLAB to find Bode Plots

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- Basic Summary:
  - Enter numerator and denominator in as polynomial arrays:

From previous exercise

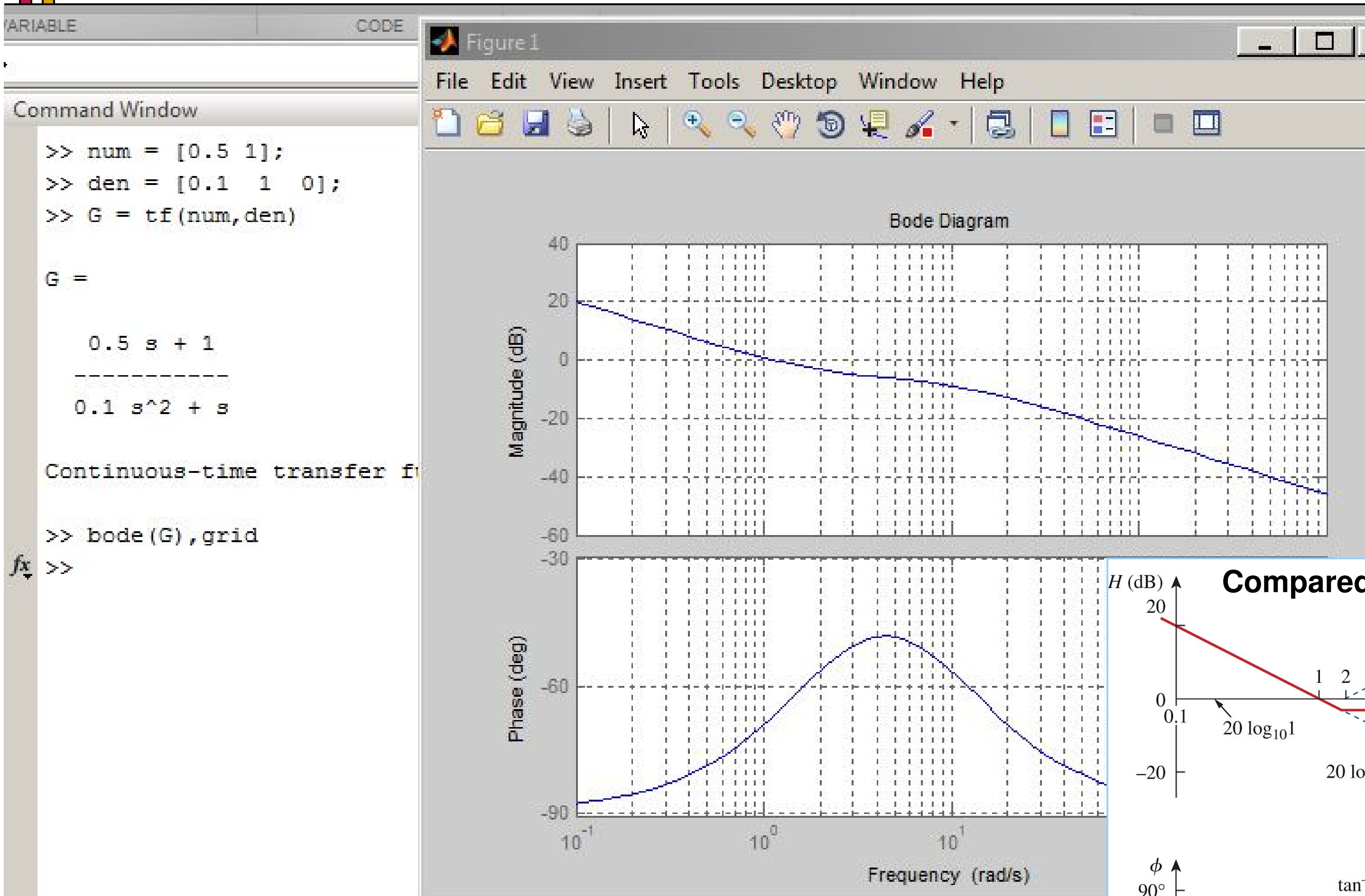
$$H(s) = \frac{(1+s/2)}{s(1+s/10)} = \frac{0.5s+1}{0.1s^2+s+1}$$

```
num = [ 0.5  1 ];
den = [ 0.1  1  1 ];
```

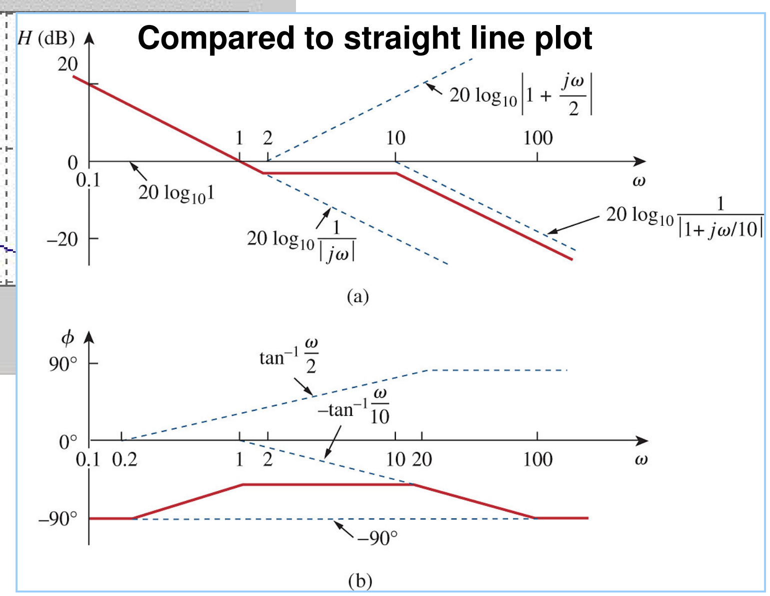
  - Store as a transfer function: `G = tf(num,den);`
  - Plot the bode diagram: `bode(G), grid`

- Website for video on using MATLAB to find Bode Plots:
  - http://www.youtube.com/watch?v=-HMhKVZ0EtQ

MATLAB Example:



MATLAB SCREENSHOT



Homework #4

Due in Class Wednesday Feb 18, 2015

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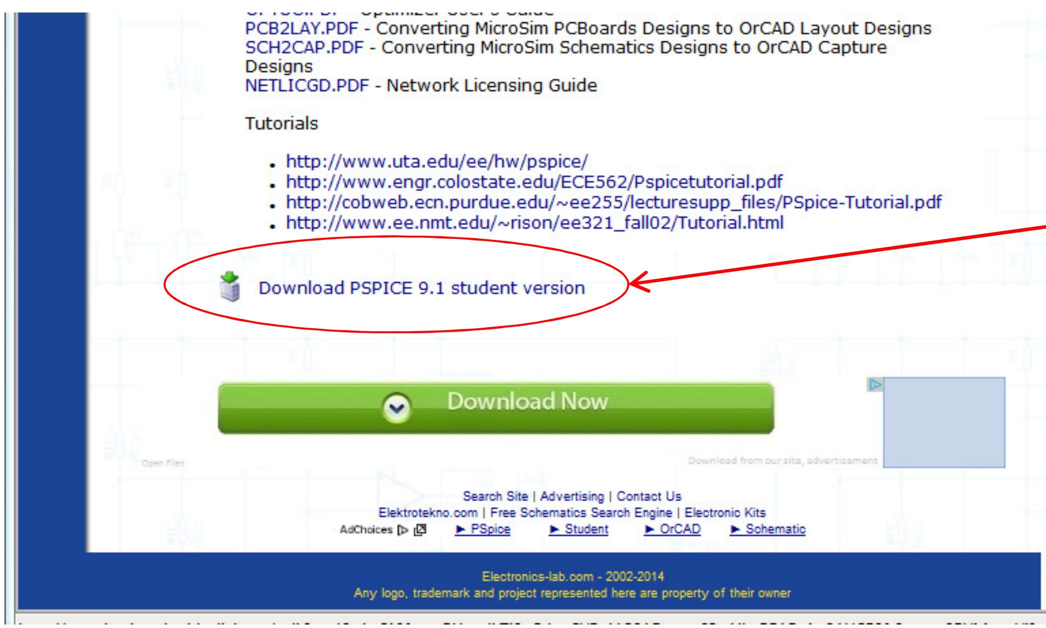
- 14.5
- 14.6
- 14.17
- 14.19
- 14.22 (Also use MATLAB to generate the Bode plot from your derived transfer function)

!! Due at beginning of class Wednesday Feb 18th !!

Late Homework will not be graded

Resources

- MATLAB: Available on IUanyWare site:
  - http://iuanyware.iu.edu/
- PSPICE 9.1 Student version can be downloaded from the following web site:
  - http://www.electronics-lab.com/downloads/schematic/013



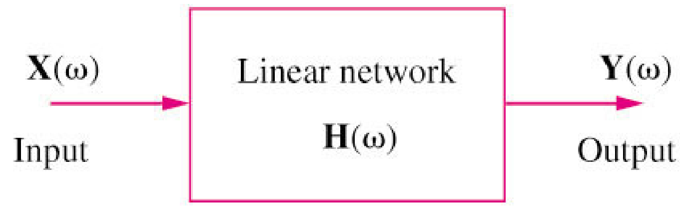
Scroll to bottom of the page and click on the link “Download PSICE 9.1 student version”

Chapter 14.2 Review

Transfer Function

- Basic idea is it's the ratio of an output to an input:
  - V_{out} / V_{in} ; I_{out} / I_{in} ; etc.

$$H(\omega) = \frac{Y(\omega)}{X(\omega)}$$



- Use circuit analysis techniques we've already covered:
  - Try Source Transformation, Parallel/Series Impedance, and Voltage Divider equations first.
  - If necessary apply Mesh / Node Voltage and substitution
- Put into a "nice" form (1+s/a) or $a_0+a_1s+a_2s^2$…:

$$H(s) = \frac{N(s)}{D(s)} = \underbrace{\frac{s(z_0+s)(z_1+s)(\ldots)}{(p_0+s)(p_1+s)(\ldots)}}_{\begin{array}{c}\text{Looks nice}\\ \text{(Can spot zeros / poles easily)}\\ \text{(Do not use for GAIN!)}\end{array}} \text{ or } \underbrace{\frac{s(1+\frac{S}{z1})(1+\frac{S}{z2})(\ldots)}{(1+\frac{S}{p1})(1+\frac{S}{p2})(\ldots)}}_{\begin{array}{c}\text{Bode Plot Form}\\ \text{(gives correct gain)}\end{array}} \text{ or } \underbrace{\frac{a_0+a_1s+a_2s^2+\ldots}{b_0+b_1s+b_2s^2+\ldots}}_{\begin{array}{c}\text{MATLAB}\\ \text{Likes this form}\\ \text{Good for spotting limits}\end{array}}$$

14.4 Bode Plot

Additional Tips (Zero at Origin)

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- Consider the following Transfer Function:

$$H(s) = \frac{200s}{(s+2)(s+10)} = \frac{200s}{2(1+s/2)10(1+s/10)} = \frac{10s}{(1+s/2)(1+s/10)}$$

  - We could find Gain K = 10 and plot as $20\text{Log}_{10}(10)$ = 20 dB
  - Add to a zero at origin which has a 20 dB/Decade slope at ω=1

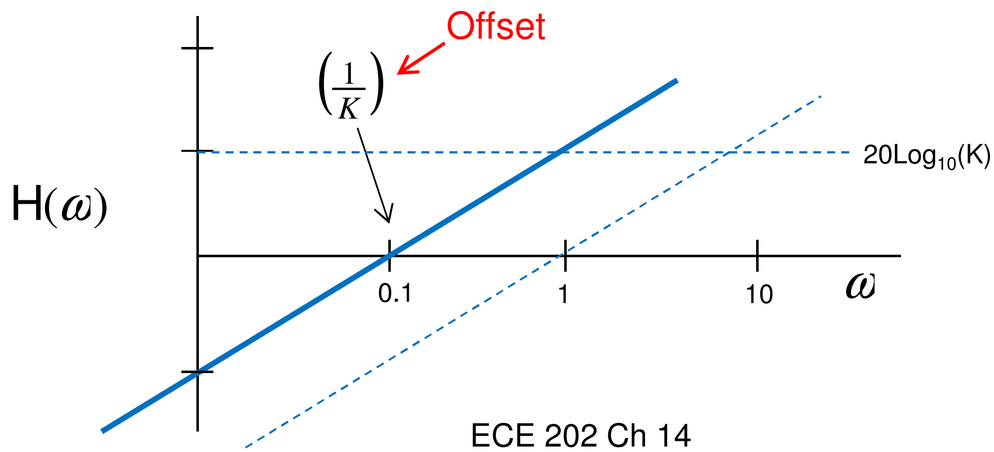
- Alternatively:
  - Combine the gain with the zero as follows:

$$H(s) = \frac{10s}{(1+s/2)(1+s/10)} = \frac{s/(\frac{1}{10})}{(1+s/2)(1+s/10)}$$

$$\boxed{Ks \rightarrow \frac{s}{\left(\frac{1}{K}\right)}}$$

Offset

  - Can now draw zero at origin offset by (1 / the Gain “K”)



Note: for double zero at origin Offset by sqrt(1/k) with slope +40 db/Decade

$$Ks^2 \rightarrow \left(\frac{s}{\left(\sqrt{\frac{1}{K}}\right)}\right)^2$$

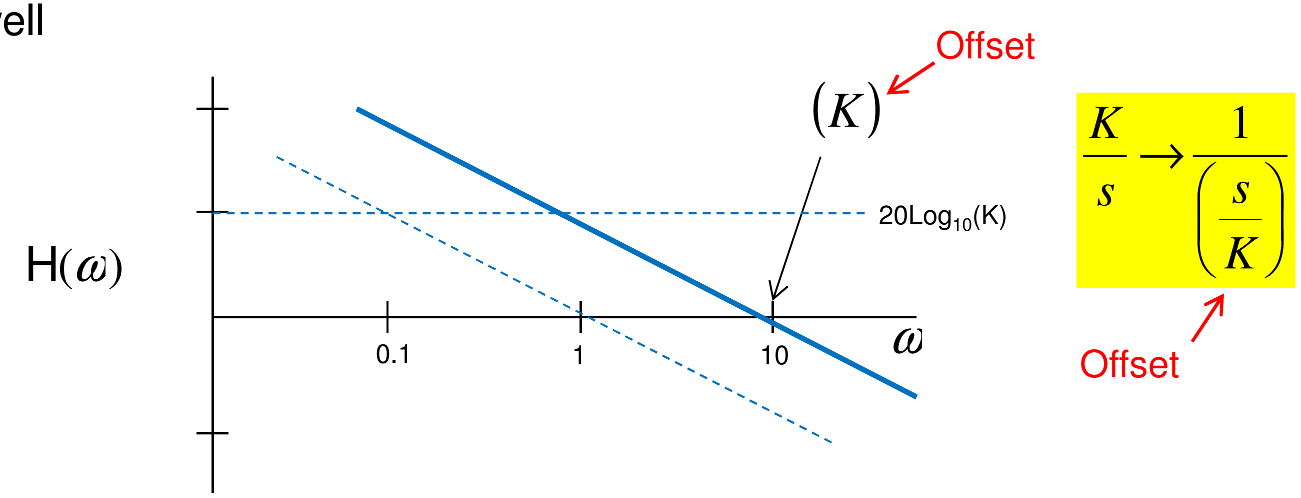
14.4 Bode Plot

Additional Tips (Pole at Origin)

- Similarly, consider a pole at the origin:

$$H(s) = \frac{10}{s(1+s/2)} = \frac{10}{s(1+s/2)} \times \frac{\frac{1}{10}}{\frac{1}{10}} = \frac{1}{\left(\frac{s}{10}\right)(1+s/2)}$$

  - Can now draw a pole at origin offset by the “pole at origin” location as well

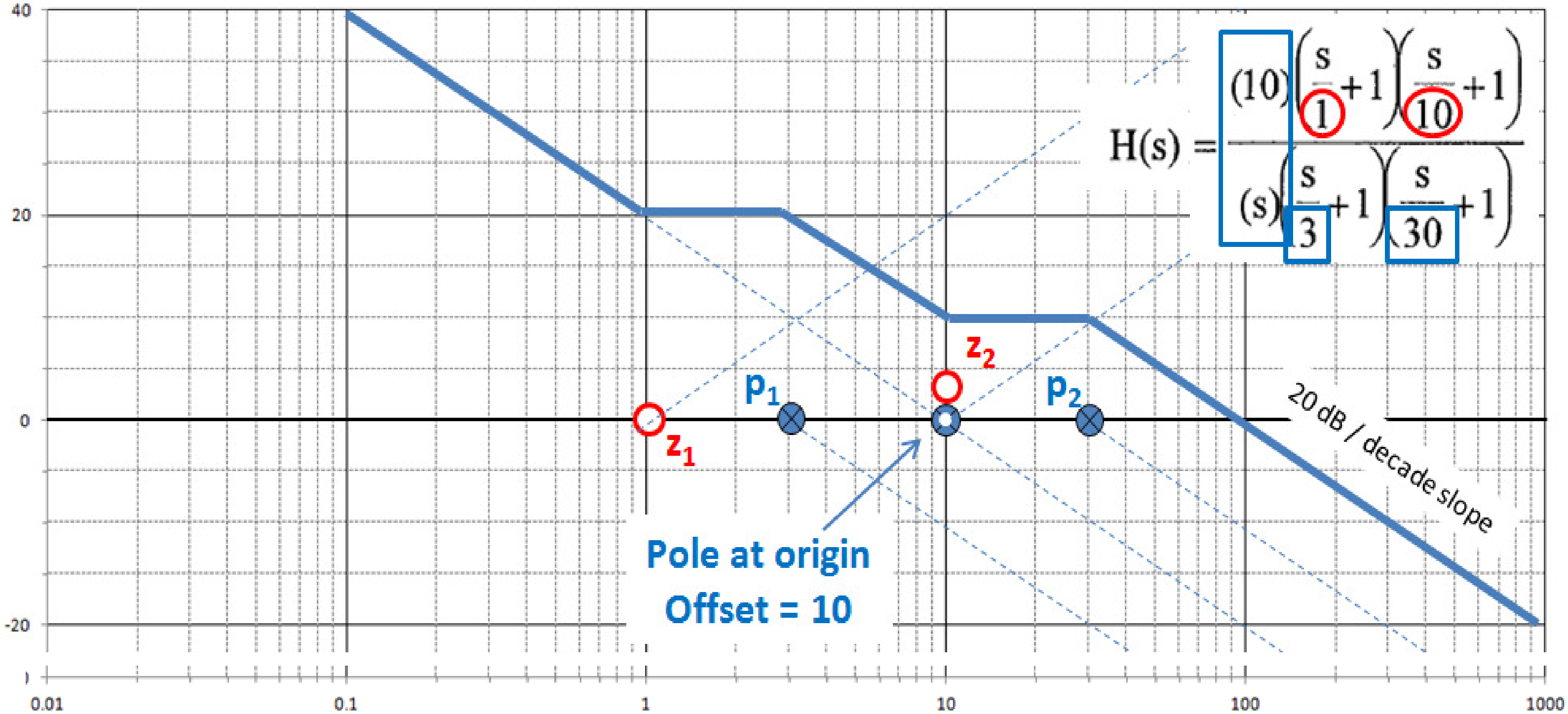


Chapter 14.4 Review

More on Bode Plots

- Make sure in proper form!
  - Proper form required for proper gain
  - Use locations of zeros & poles as "Inflection Points"
    - Zeros contribute to +20 dB / decade slope
    - Poles contribute to -20 db / decade slope
- Example:

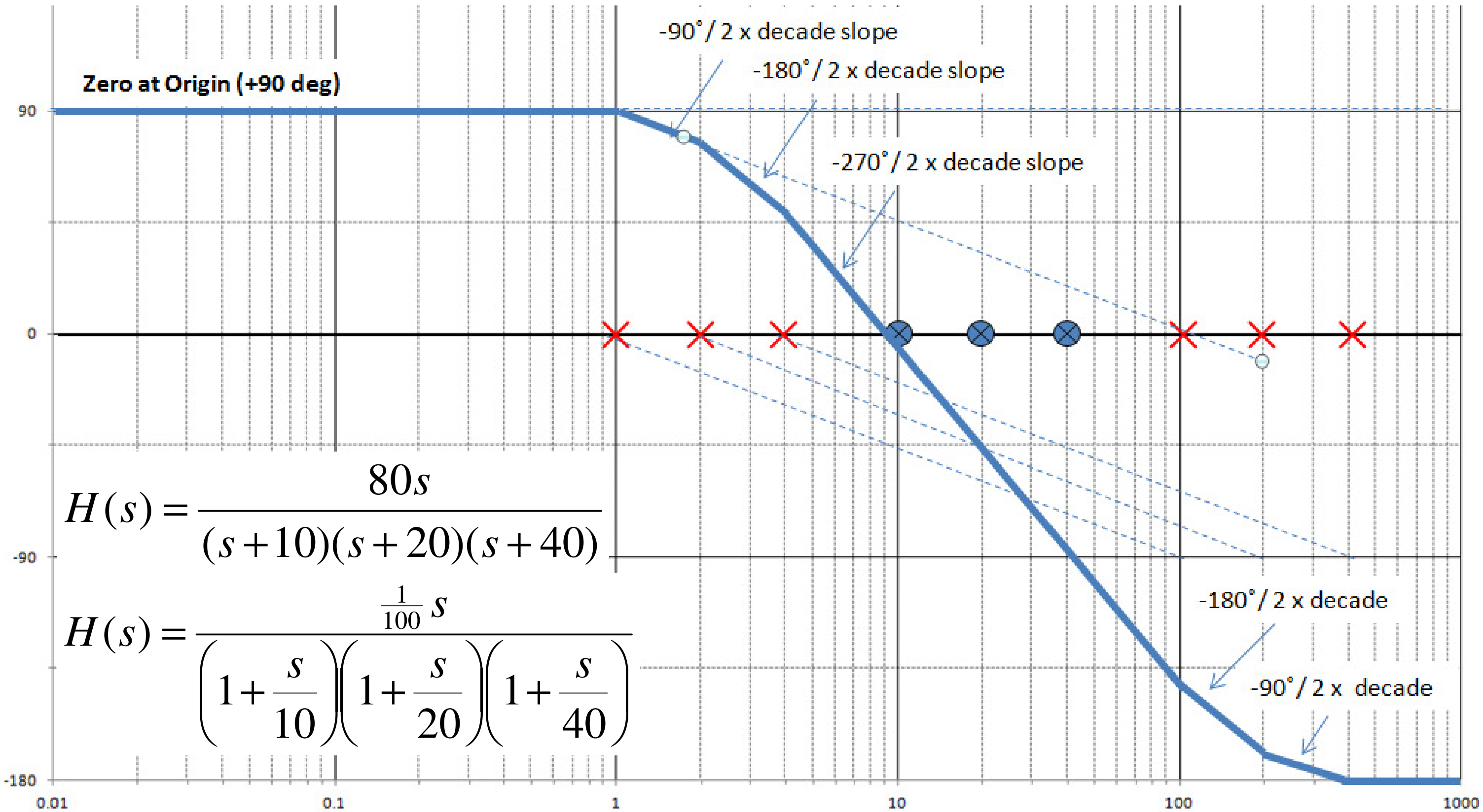
$$H(s)=\frac{s(1+\frac{S}{z1})(1+\frac{S}{z2})(\ldots)}{(1+\frac{S}{p1})(1+\frac{S}{p2})(\ldots)}$$



Chapter 14 Review

Bode Plot – Phase Diagram

- Inflection points occur 1 decade below and above pole/zero
- Review Homework Solutions



$$H(s) = \frac{80s}{(s+10)(s+20)(s+40)}$$

$$H(s) = \frac{\frac{1}{100}s}{\left(1+\frac{s}{10}\right)\left(1+\frac{s}{20}\right)\left(1+\frac{s}{40}\right)}$$

Chapter 14 Review

Bode Plot – Phase Diagram Hint

Sum up the phases for each pole/zero to check final answer

Zeros contribute +90° ; Poles contribute -90°

$$H(s)=\frac{\frac{1}{100}s}{\left(1+\frac{s}{10}\right)\left(1+\frac{s}{20}\right)\left(1+\frac{s}{40}\right)}$$

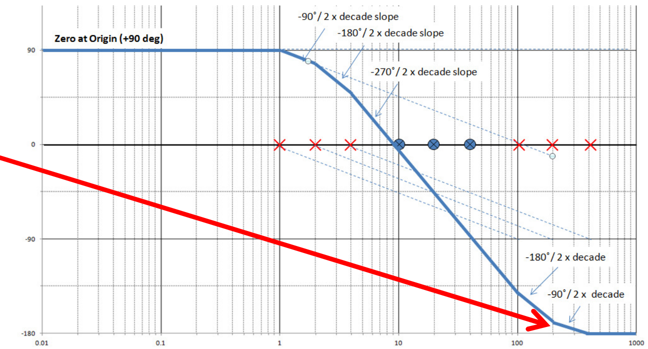
$\frac{1}{100}s \rightarrow +90$

$\left(1+\frac{s}{10}\right) \rightarrow -90$

$\left(1+\frac{s}{20}\right) \rightarrow -90$

$\left(1+\frac{s}{40}\right) \rightarrow -90$

- 180°



Factor		Magnitude	Phase
K	Gain = 20 Log_{10}K	$20 \log_{10} K$	$0°$
$(j\omega)^N$	Zero at jω=0 Slope 20*N/Decade Phase = 90*N	$20N$ dB/decade	$90N°$
$\frac{1}{(j\omega)^N}$	Pole at jω=0 Slope -20*N/Decade Phase = -90*N	$-20N$ dB/decade	$-90N°$
$\left(1+\frac{j\omega}{z}\right)^N$	Zero at jω=p Slope 20*N/Decade after z Phase "Shifts" to 90*N	$20N$ dB/decade	$0°$ to $90N°$ ($\frac{z}{10}$, z, $10z$)
$\frac{1}{(1+j\omega/p)^N}$	Pole at jω=p Slope -20*N/Decade after p Phase "Shifts" to -90*N	$-20N$ dB/decade	$0°$ to $-90N°$ ($\frac{p}{10}$, p, $10p$)

Phase as s → ∞	
Zero at origin	→ +90
Pole at origin	→ -90
Zero	→ +90
Pole	→ -90

14.5 Series Resonance (1)

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- Resonance occurs in any system that has a complex conjugate pairs of poles and is the cause of oscillations of stored energy from one from to the other.
- Circuit must have at least one inductor and one capacitor.
- Resonance is a condition in an RLC circuit in which the capacitive and inductive reactances are equal in magnitude, thereby resulting in a purely resistive impedance.
- The imaginary part of the impedance is = 0 (the L and C cancel each other).
- Resonance circuits are useful in many applications including signal filtering and frequency selection in radio and TV receivers.

14.5 Series Resonance (2)

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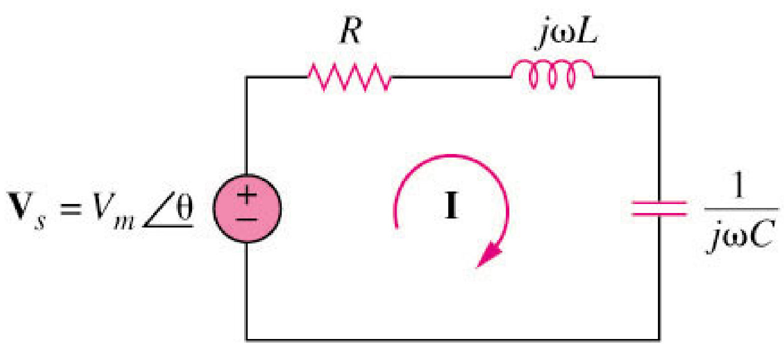
Resonance is a condition in an RLC circuit in which the capacitive and inductive reactances are equal in magnitude, thereby resulting in a purely resistive impedance.



Resonance frequency:

$$\omega_o = \frac{1}{\sqrt{LC}} \text{rad/s}$$

$$f_o = \frac{1}{2\pi\sqrt{LC}} \text{Hz}$$

$$Z = R + j\underbrace{(\omega L - \frac{1}{\omega C})}$$

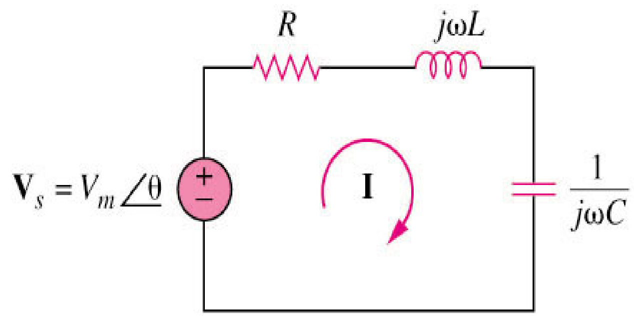
Resonance occurs when the imaginary term goes to zero! →

$$(\omega L - \frac{1}{\omega C}) = 0 \Rightarrow \omega L = \frac{1}{\omega C} \Rightarrow \omega^2 = \frac{1}{LC}$$

14.5 Series Resonance (3)

At resonance:

1. The impedance is purely resistive, Z = R; The LC series combination acts like a short circuit and the entire voltage is across R.
2. The supply voltage V_s and the current I are in in phase, so the power factor is unity: cos θ = 1.
3. The magnitude of the transfer function H(ω) = Z(ω) is minimum.
4. The inductor voltage and capacitor voltage can be much more than the source voltage.



$$Z = R + j\underbrace{\left(\omega L - \frac{1}{\omega C}\right)}_{\text{Goes to Zero At Resonance}}$$

$$|V_L| = \frac{V_m}{R}\omega_0 L = QV_m$$

$$|V_C| = \frac{V_m}{R}\frac{1}{\omega_0 C} = QV_m$$

Q = quality factor

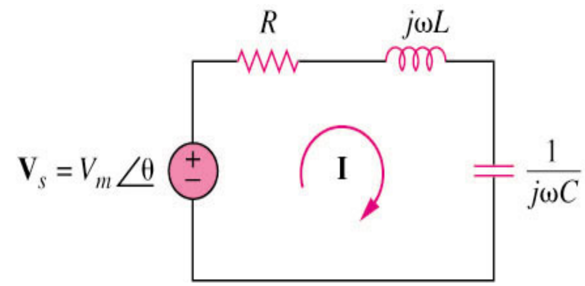
14.5 Series Resonance (4)

Frequency Response / Max Power

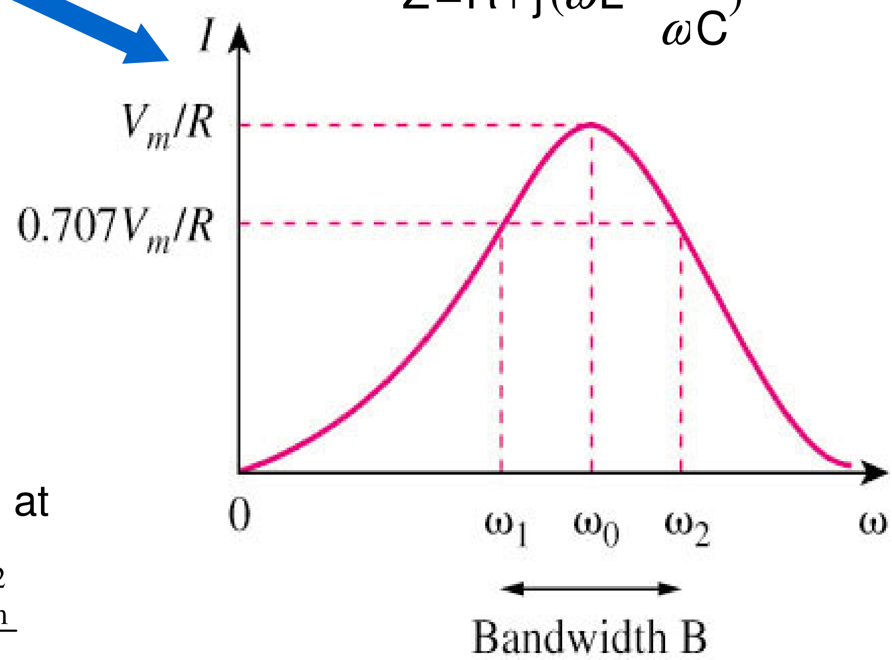
The frequency response of the resonance circuit current magnitude is

$$|I_m| = \frac{V_m}{\sqrt{R^2 + \underbrace{(\omega L - 1/\omega C)^2}_{}}}$$

Maximum current occurs at resonance



$$Z = R + j(\omega L - \frac{1}{\omega C})$$



The average power absorbed by the RLC circuit is

$$P(\omega) = \frac{1}{2} I_m{}^2 R$$

The highest power dissipated occurs at resonance, $I = V_m/R$:

$$P(\omega_o) = \frac{1}{2}\frac{V_m^2}{R}$$

14.5 Series Resonance (5)

Half-Power Bandwidth

Half-power frequencies ω_1 and ω_2 are frequencies at which the dissipated power is half the maximum value:

$$P(\omega_1) = P(\omega_2) = \frac{1}{2}\left(\frac{V_m}{|Z|}\right)^2 R = \frac{1}{2} P(\omega_0) = \frac{1}{2}\left(\frac{V_m}{\sqrt{2}R}\right)^2 R$$

Note: Current magnitude |I| is $V_m / \sqrt{2}$ R at half power frequencies. Therefore, the half-power frequencies can be obtained by setting |Z| equal to $\sqrt{2}$ R:

$$\sqrt{R^2 + (\omega L - \frac{1}{\omega C})^2} = \sqrt{2}R$$

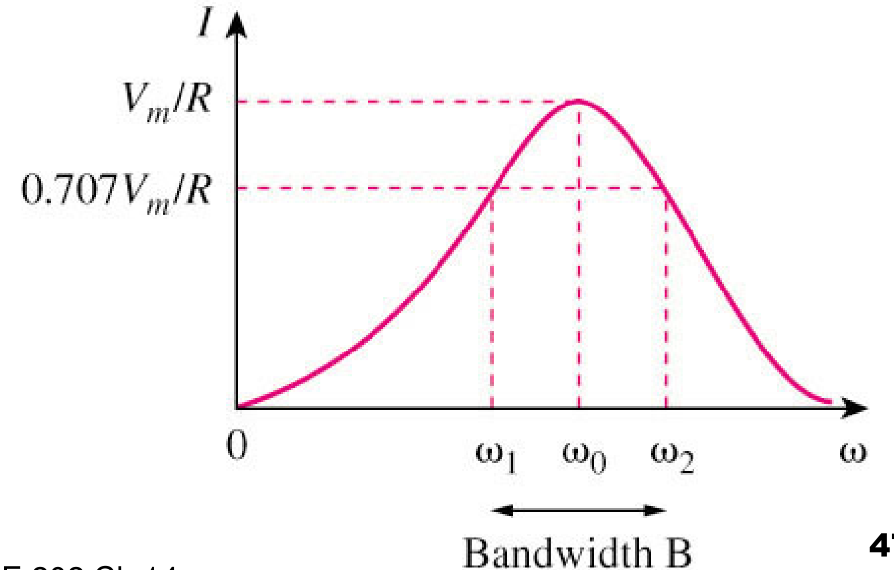
Bandwidth $B = \omega_2 - \omega_1$

Solving for ω we obtain:

$$\omega_1 = -\frac{R}{2L} + \sqrt{(\frac{R}{2L})^2 + \frac{1}{LC}}$$

$$\omega_2 = \frac{R}{2L} + \sqrt{(\frac{R}{2L})^2 + \frac{1}{LC}}$$

$$\omega_o = \sqrt{\omega_1 \omega_2}$$



14.5 Series Resonance (6)

Quality Factor "Q"

- The quality factor "Q" describes the sharpness of the resonance in a resonant circuit and is defined by the following:

$$Q = 2\pi \frac{\text{Peak energy stored in the circuit}}{\text{Energy dissipated by the circuit in one period at resonance}}$$

- At resonance, the reactive energy oscillates between the inductor and the capacitor
- In a series RLC circuit, the peak energy stored is: $\frac{1}{2}LI^2$ while the energy dissipated in one period is $\frac{1}{2}I^2R(1/f_0)$. Therefore,

$$Q = 2\pi \frac{\frac{1}{2}LI^2}{\frac{1}{2}I^2R(1/f_0)} = \frac{2\pi f_0 L}{R} = \frac{\omega_0 L}{R} = \frac{1}{\omega_0 CR}$$

$$Q = \frac{\omega_0 L}{R} = \frac{1}{\omega_0 CR}$$

$$Q = \frac{\omega_o}{B}$$

The Quality Factor is dimensionless

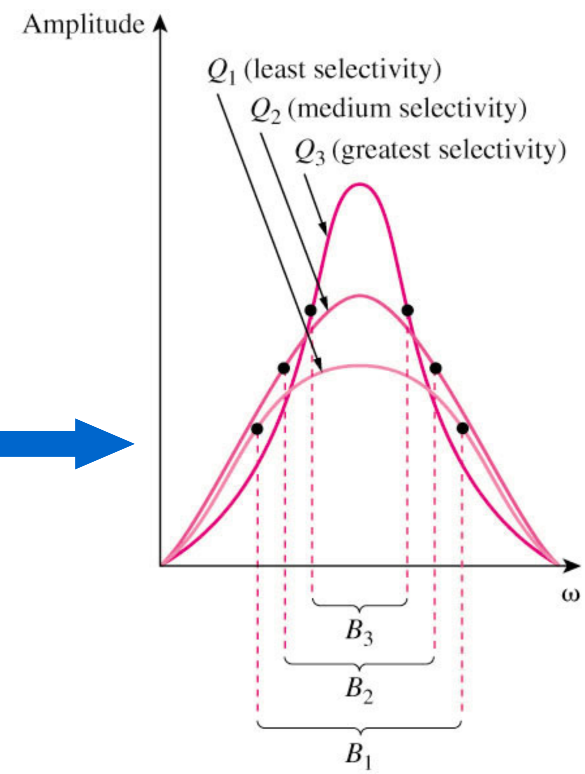
14.5 Series Resonance (7)

Relationships B, Q, ω_o

B, Q and ω_o have the following relationships:

$$B = \frac{R}{L} = \frac{\omega_o}{Q} = \omega_o^2 CR$$

- The quality factor is the ratio of its resonant frequency to its bandwidth.
- The selectivity of an RLC circuit is the ability of the circuit to respond to a certain frequency and / or discriminate against all other frequencies.
- If the bandwidth is narrow, the quality factor of the resonant circuit must be high.
- If the bandwidth is wide, the quality factor must be low.
- A resonant circuit is designed to operate at or near its resonant frequency.



$$Q = \frac{\omega_o}{B}$$

14.5 Series Resonance (8)

High Q Approximations

- For a high-Q circuit: $Q > 10$
- In high-Q circuits the half power frequencies are, for all practical purposes, symmetrical around the resonant frequency.
- The half power frequencies can be approximated as:

$$\omega_1 = \omega_o - \frac{B}{2}; \quad \omega_2 = \omega_o + \frac{B}{2}$$

- Resonant circuit is characterized by five related parameters:
  - 1,2) Two half-power frequencies, ω1 and ω2
  - 3) Resonant frequency ω0
  - 4) Bandwidth B
  - 5) Quality factor Q

14.5 Series Resonance (9)

Practice Problem 14.7

A series-connected circuit has R = 4 Ω and L = 25 mH.

a. Calculate the value of C that will produce a quality factor of 50.

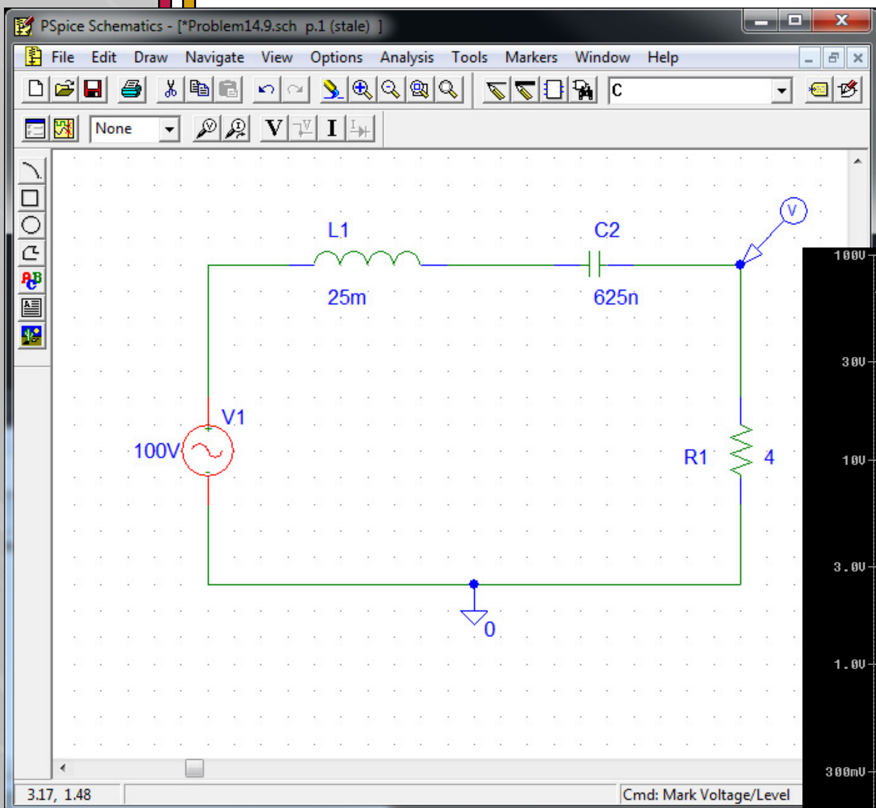
b. Find ω_1 and ω_2, and B.

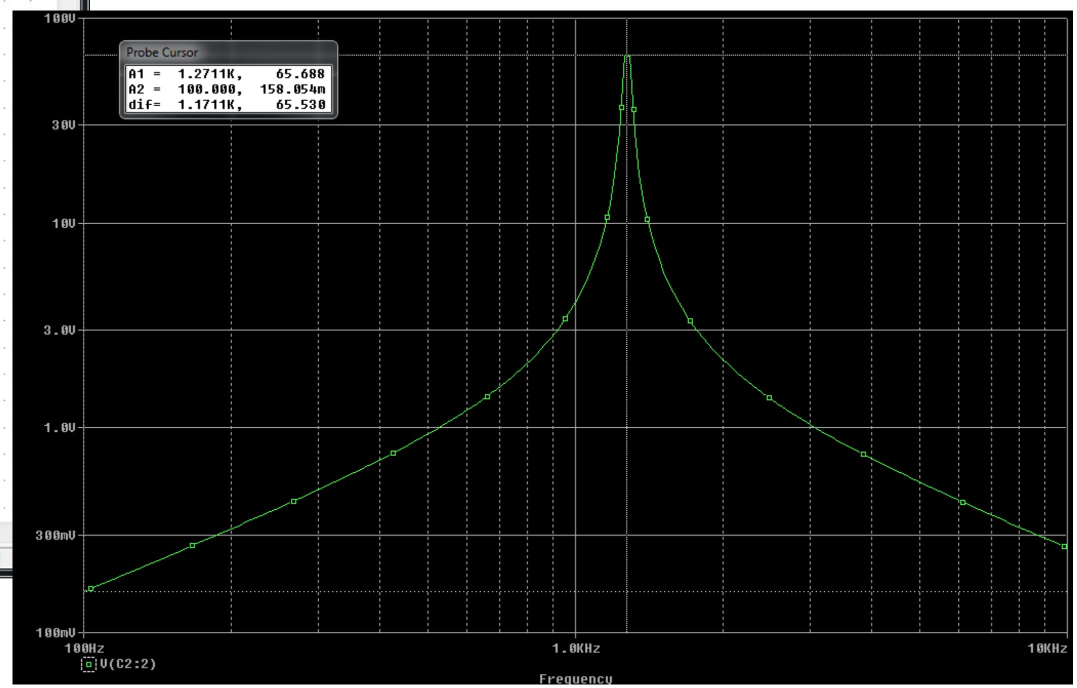
c. Determine the average power dissipated at $\omega = \omega_o, \omega_1, \omega_2$. Take $V_m = 100V$.

14.5 Series Resonance (9)

PSpice output for Practice Problem 14.7

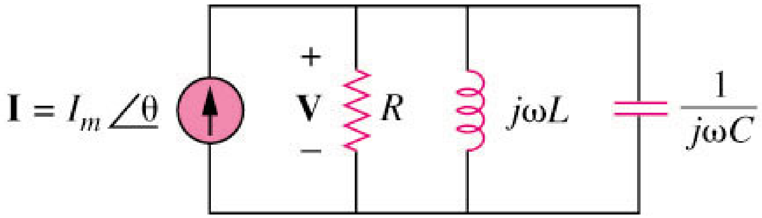
Resonant Frequency $\omega_o = 8000.$ $f_0 = 1273$ Hz





14.6 Parallel Resonance (1)

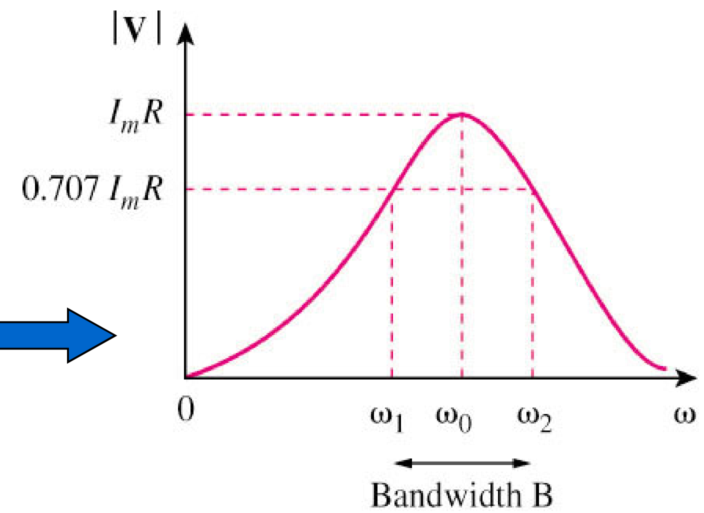
Parallel RLC Circuit



The parallel RLC circuit is the dual of the series RLC circuit. The admittance is:

$$Y = H(\omega) = \frac{I}{V} = \frac{1}{R} + j\omega C + \frac{1}{j\omega L}$$

$$Y = \frac{1}{R} + j(\omega C - \frac{1}{\omega L})$$

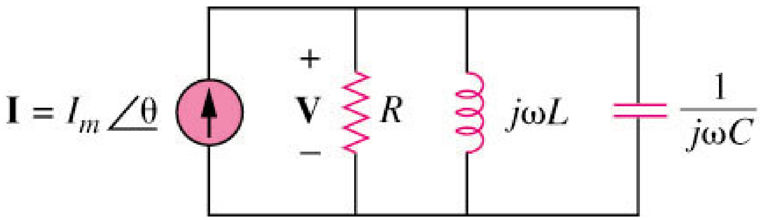


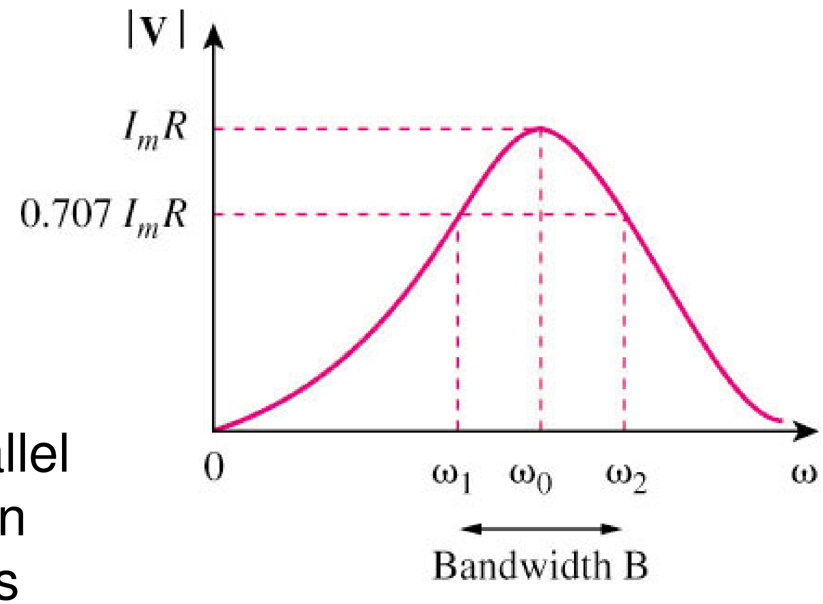
Resonance occurs when the imaginary part of Y is zero:

$$\omega C - \frac{1}{\omega L} = 0 \qquad \omega_o = \frac{1}{\sqrt{LC}} \text{rad/s or } f_o = \frac{1}{2\pi\sqrt{LC}} Hz$$

14.6 Parallel Resonance (2)

Parallel RLC Circuit





Note that at resonance, the parallel LC combination acts like an open circuit, so the entire current flows through R.

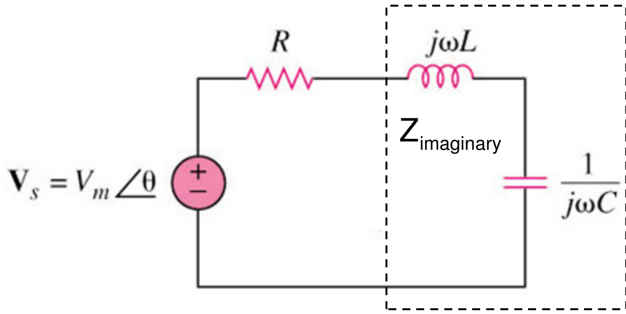
Also, the inductor and capacitor current can be much more than the source current.

14.5 & 14.6 Resonance

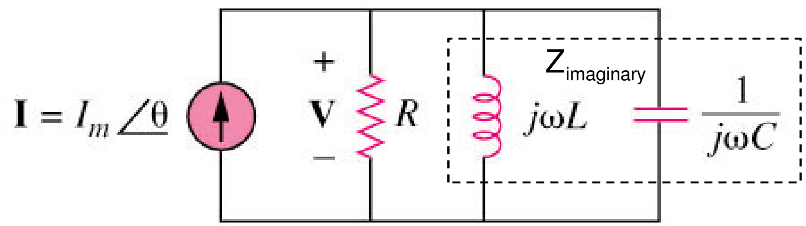
Series and Parallel

At Resonance, impedance is purely real !!

Series Resonant Circuit



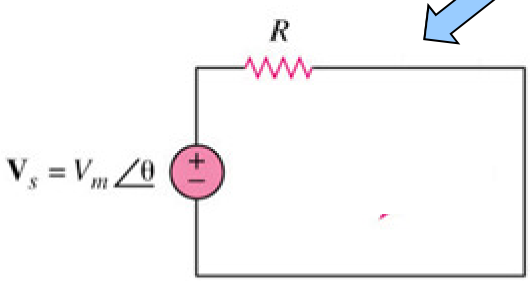
Parallel Resonant Circuit

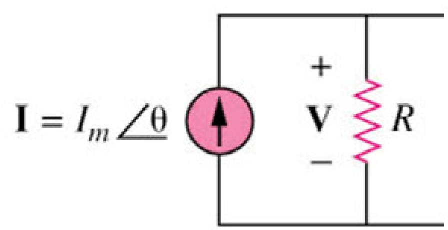


$$Z_{\text{imaginary}} = j\left(\omega L - \frac{1}{\omega C}\right)$$

$$Z_{\text{imaginary}} = j\omega L \,||\, \frac{1}{j\omega C} = \frac{j\omega L \times \frac{1}{j\omega C}}{j\omega L + \frac{1}{j\omega C}} = \frac{\frac{L}{C}}{j\left(\omega L - \frac{1}{\omega C}\right)}$$

$$\boxed{\omega_o = \frac{1}{\sqrt{LC}} \text{ rad/s}}$$





14.5 & 14.6 Resonance

Quality Factor "Q"

- The quality factor "Q" describes the sharpness of the resonance.
- "Q" is the ratio of center frequency to the bandwidth.
  - Higher the "Q", narrower the bandwidth
  - Lower the "Q", wider the bandwidth

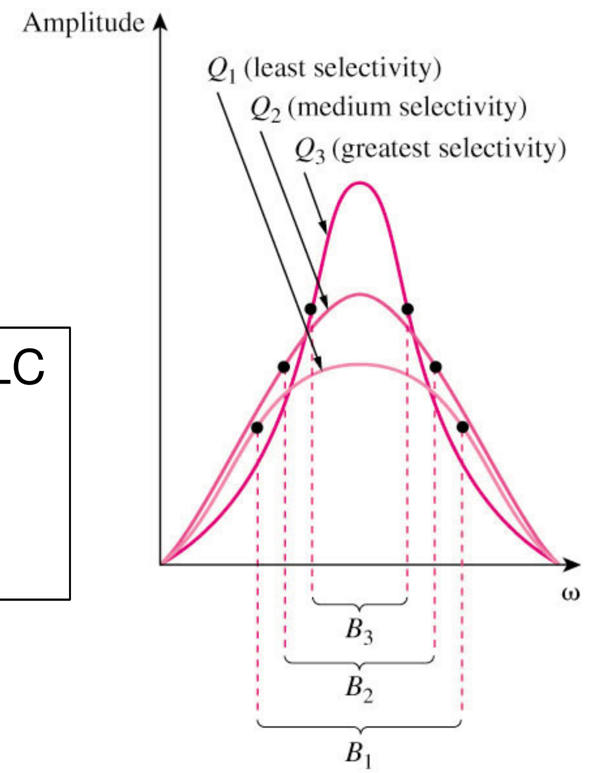
$$Q = \frac{\omega_o}{B}$$

"Q" for Series RLC

$$\frac{\omega_o L}{R} \quad or \quad \frac{1}{\omega_o RC}$$

"Q" for Parallel RLC

$$\frac{R}{\omega_o L} \quad or \quad \omega_o RC$$



The Quality Factor is dimensionless

14.5 & 14.6 Resonance

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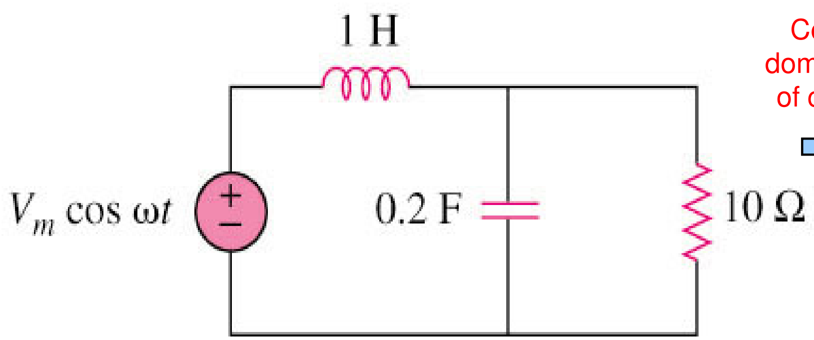
Summary of series and parallel resonance circuits

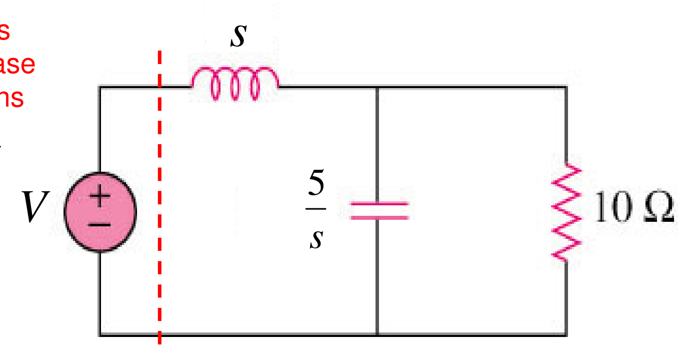
characteristic	*Series circuit*	*Parallel circuit*	
ω_o	$\frac{1}{\sqrt{LC}}$ or $\sqrt{\omega_1\omega_2}$	$\frac{1}{\sqrt{LC}}$ or $\sqrt{\omega_1\omega_2}$	same
Q	$\frac{\omega_o L}{R}$ or $\frac{1}{\omega_o RC}$	$\frac{R}{\omega_o L}$ or $\omega_o RC$	inverse
B	$\frac{\omega_o}{Q}$	$\frac{\omega_o}{Q}$	same
B	$\frac{R}{L}$ or $\omega_o^2 RC$	$\frac{1}{RC}$ or $\omega_o^2 \frac{L}{R}$	
$\omega_1,\ \omega_2$	$\omega_o\sqrt{1+(\frac{1}{2Q})^2} \pm \frac{\omega_o}{2Q}$	$\omega_o\sqrt{1+(\frac{1}{2Q})^2} \pm \frac{\omega_o}{2Q}$	same
if $Q \geq 10,\ \omega_1,\ \omega_2$	$\omega_1 = \omega_o - \frac{B}{2}$; $\omega_2 = \omega_o + \frac{B}{2}$	$\omega_1 = \omega_o - \frac{B}{2}$; $\omega_2 = \omega_o + \frac{B}{2}$	same

Similar to Practice Problem 14.9

Calculate the resonant frequency of the circuit in the figure shown below.



Convert to s domain for ease of calculations



Lets look at the input impedance

$$Z_{in} = s + \frac{5}{s} \bigg\| 10$$

$$Z_{in} = s + \frac{\left(\frac{5}{s}\right)10}{\frac{5}{s}+10} = s + \frac{50}{5+10s} = s + \frac{10}{1+2s}$$

Lets convert back to jω so we can find ω for resonance

$$Z_{in} = j\omega + \frac{10}{1+2j\omega}$$

$$Z_{in} = j\omega + \frac{10}{1+2j\omega} \bullet \frac{(1-2j\omega)}{(1-2j\omega)}$$

Multiply by conjugate to get j out of denominator

$$Z_{in} = j\omega + \frac{10}{(1+4\omega^2)}(1-2j\omega)$$

Separate into Real & imaginary

$$Z_{in} = \underbrace{\frac{10}{(1+4\omega^2)}}_{\text{Real Part}} + \underbrace{j\omega\left(1-\frac{20}{(1+4\omega^2)}\right)}_{\text{Imaginary Part}}$$

Imaginary = 0 at Resonance !

Therefore: $$1-\frac{20}{(1+4\omega^2)} = 0$$

Solving for ω $$1+4\omega^2 = 20$$

$$4\omega^2 = 19$$

$$\omega = \sqrt{\frac{19}{4}} = \frac{\sqrt{19}}{2} = 2.179$$

14.7 Passive Filters (1)

- A filter is a circuit designed to pass signals with desired frequencies and reject (attenuate) others.
- Filters are used in radio and TV receivers to allow selection of one desired signal out of a multitude of broadcast signals.
- A filter is a passive filter if it consists only of passive elements R, L, and C components.
- Active filters which will be covered later, consists of active elements such as transistors and op-amps in addition to the passive elements.

14.7 Passive Filters (2)

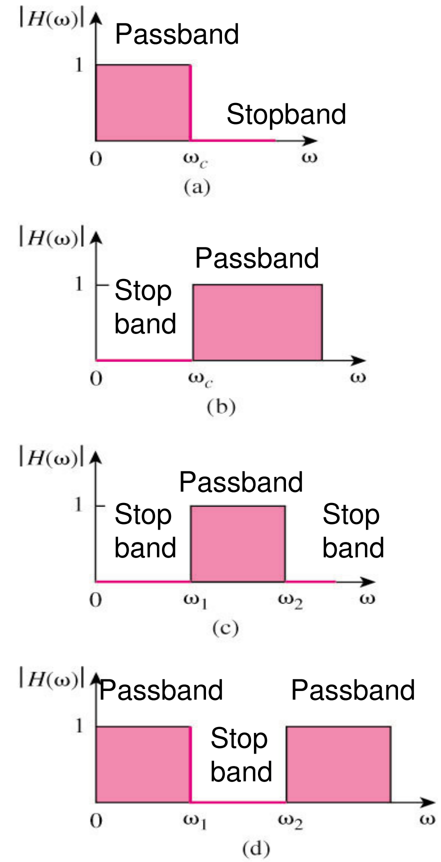
- There are four types of filters:
  1. Low pass filter passes low frequencies and stops high frequencies
  2. High pass filter passes high frequencies and stops low frequencies
  3. Bandpass filter passes frequencies within a frequency band and blocks or attenuates frequencies outside the band.
  4. Bandstop filter passes frequencies outside a frequency band and blocks or attenuates frequencies within the band.

1. Lowpass

2. Highpass

3. Bandpass

4. Bandstop



14.7 Passive Filters (3)

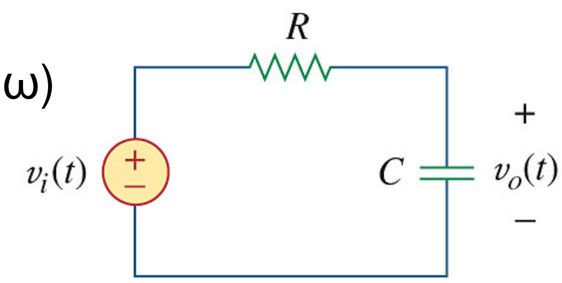
Summary of characteristics of ideal filters:

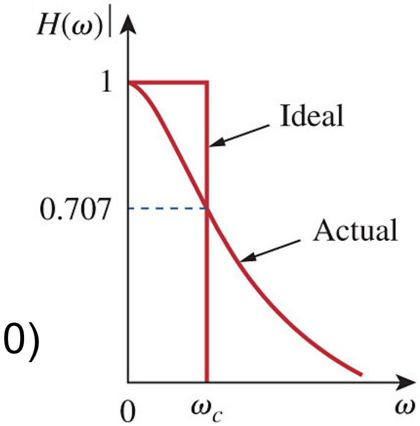
Type of Filter	H(0)	H(∞)	$H(\omega_c)$ or $H(\omega_0)$
Lowpass	1	0	$1/\sqrt{2}$
Highpass	0	1	$1/\sqrt{2}$
Bandpass	0	0	1
Bandstop	1	1	0

Lowpass Filter:

Lowpass Filter

The transfer function H(ω) is:





$$H(\omega)=\frac{V_o}{V_i}=\frac{\frac{1}{j\omega C}}{R+1/j\omega C}=\frac{1}{1+j\omega RC}$$

(Note: H(0) = 1; H(∞) = 0)

The half-power frequency known as the cutoff frequency ω_c is obtained by setting the magnitude of H(ω) equal to 1/ √2 of its maximum value.

$$H(\omega_c)=\frac{1}{\sqrt{1+\omega_C{}^2R^2C^2}}=\frac{1}{\sqrt{2}} \Rightarrow \omega_c=\frac{1}{RC}$$

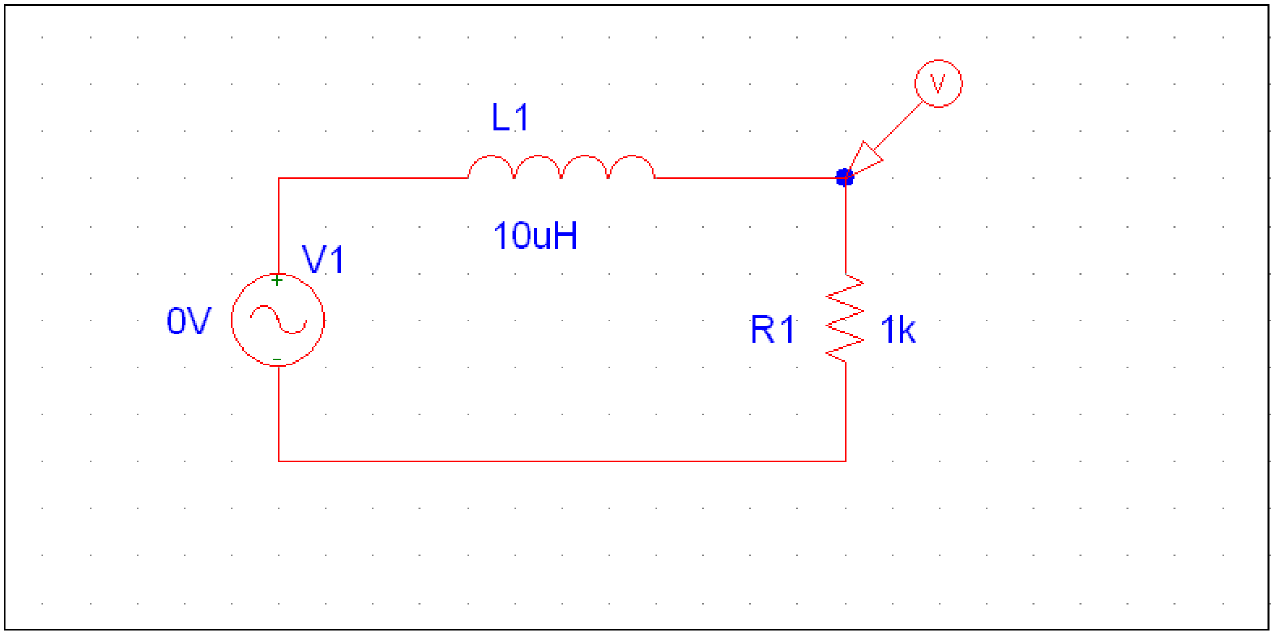
The cutoff frequency (or rolloff frequency) is the frequency at which the transfer function H drops in magnitude to 70.71% of its maximum value. It is also regarded as the frequency at which the power dissipated in the circuit is half its maximum value.

14.7 Passive Filters (5)

Lowpass Filter

A lowpass filter can also be formed when the output V_0 of an RL circuit is taken off the resistor:

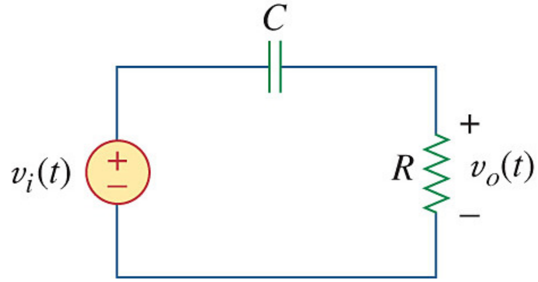
Note: $H(0) = 1$, $H(\infty) = 0$



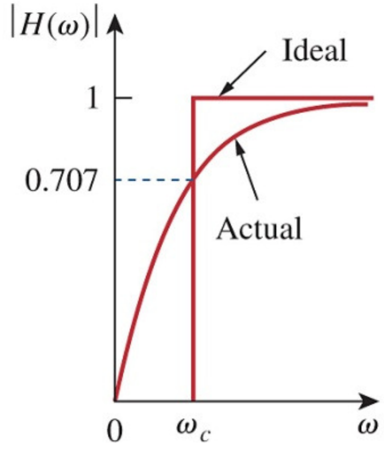
14.7 Passive Filters (6)
Highpass Filter

Highpass Filter:

The transfer function is: $H(\omega)=\dfrac{V_o}{V_s}=\dfrac{R}{R+\dfrac{1}{j\omega C}}=\dfrac{j\omega RC}{1+j\omega RC}$



Highpass Filter



Note: H(0) =0, H(∞)=1

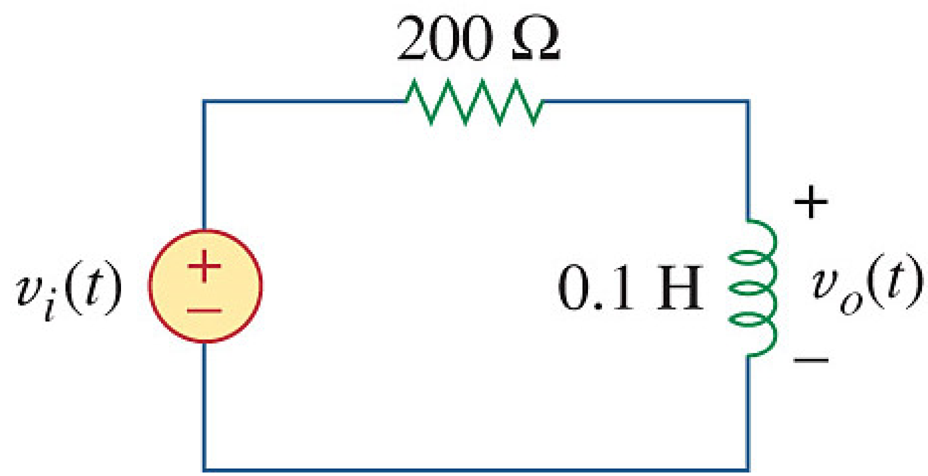
The cutoff frequency (ω_C) is defined by setting the magnitude of H equal to 1/√2, or 0.707 from peak.

Therefore the cutoff frequency: $\omega_c=\dfrac{1}{RC}$

14.7 Passive Filters (7)
Highpass Filter

The highpass can also be formed with RL circuit when the output voltage is taken off the inductor:

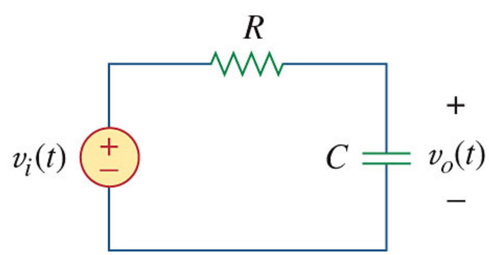
Note: H(0) =0, H(∞)=1

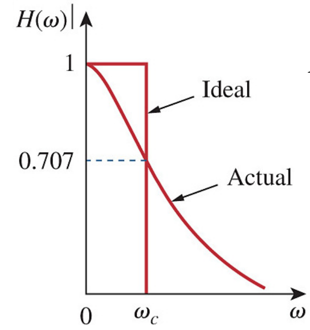


14.7 Passive Filters

High Pass / Low Pass Summary

Simple RC Low pass Filter



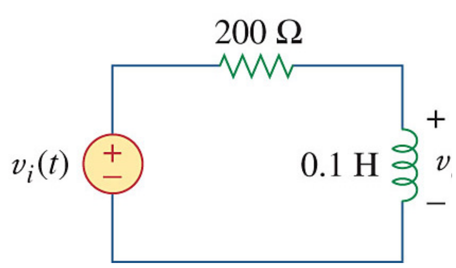


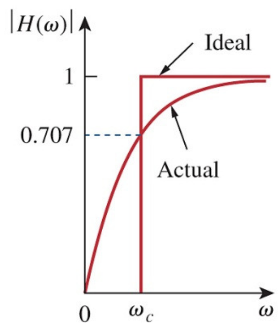
$$H(s) = \frac{V_o}{V_i} = \frac{\frac{1}{sC}}{R + 1/sC} = \frac{1}{1 + sRC} = \frac{1}{1 + \frac{s}{\frac{1}{RC}}}$$

$$\omega_c = \frac{1}{RC}$$

Can make into High Pass by looking across resistor

Simple RL High pass Filter





$$H(s) = \frac{V_o}{V_i} = \frac{sL}{R + sL} = \frac{s}{\frac{R}{L} + s} = \frac{\frac{s}{\left(\frac{R}{L}\right)}}{1 + \frac{s}{\left(\frac{R}{L}\right)}}$$

$$\omega_c = \frac{R}{L}$$

Can make into Low Pass by looking across resistor

Bandpass Filter:

$$\mathbf{H}(\omega) = \frac{V_o}{V_i} = \frac{R}{R + j(\omega L - 1/\omega C)}$$

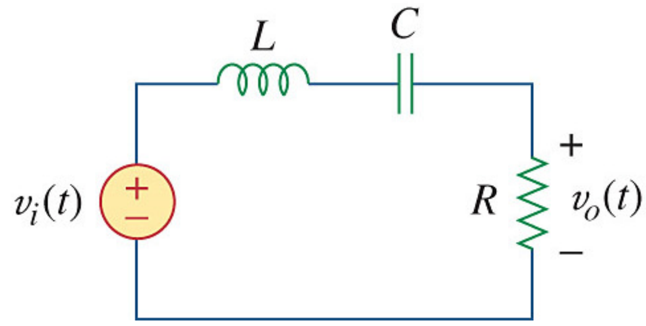
$$\omega_1 = -\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}$$

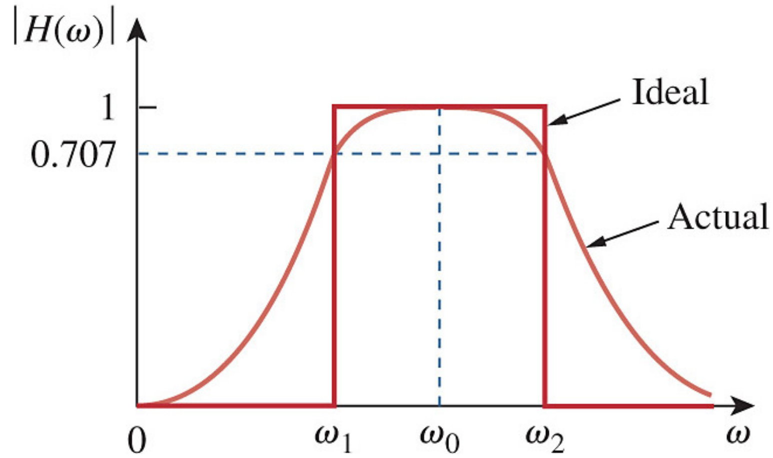
$$\omega_2 = \frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}$$

$$Q = \frac{\omega_o L}{R} = \frac{1}{\omega_o CR}$$

$$\omega_o = \frac{1}{\sqrt{LC}}$$

$$B = \frac{R}{L} = \frac{\omega_o}{Q} = \omega_o^2 CR$$





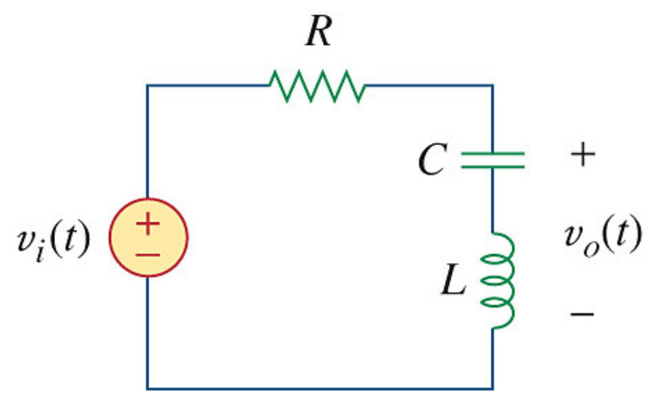
Note: $H(0) = 0$, $H(\omega_0) = 1$, $H(\infty) = 0$

A bandpass filter can also be formed by cascading a lowpass filter (where $\omega_2 = \omega_c$) with the highpass filter (where $\omega_1 = \omega_c$).

14.7 Passive Filters (9)

Bandstop Filter

Bandstop Filter:
(Also known as the bandreject or notch filter)

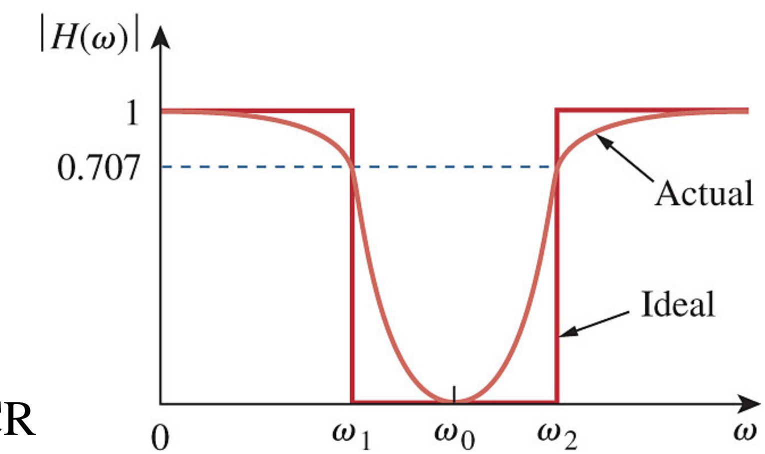


$$\mathbf{H}(\omega) = \frac{V_o}{V_i} = \frac{j(\omega L - 1/\omega C)}{R + j(\omega L - 1/\omega C)}$$

$$\omega_1 = -\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}$$

$$\omega_2 = \frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}} \qquad Q = \frac{\omega_o L}{R} = \frac{1}{\omega_o CR}$$

$$\omega_o = \frac{1}{\sqrt{LC}} \qquad B = \frac{R}{L} = \frac{\omega_o}{Q} = \omega_o^2 CR$$



Note: $H(0)=1$; $H(\omega_0)=0$; $H(\infty)=1$

14.7 Passive Filters (10)
Bandstop Filter

- For bandstop filters:
  - The center frequency $\omega_{0,}$ is called the frequency of rejection
  - And the bandwidth B is known as bandwidth of rejection

14.7 Passive Filters (11)

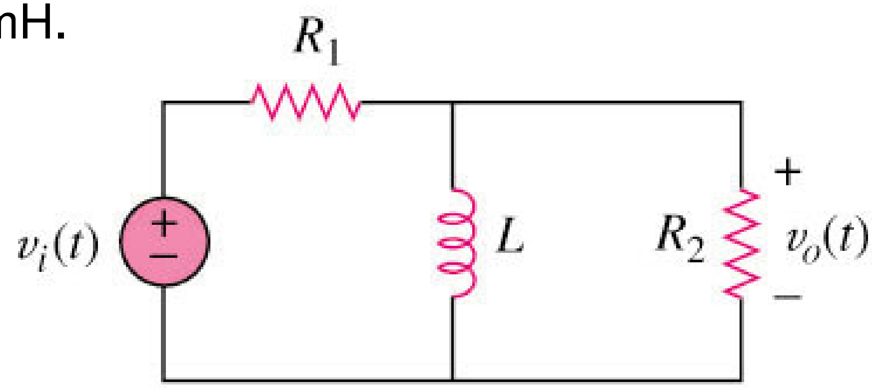
In concluding this section, we should note that:

1. Maximum gain of a passive filter is unity. To generate a gain grater than unity, an active filter should be used.
2. There are other ways to get the passive filters.
3. Many other filter types have sharper responses as well as complex frequency responses.

14.7 Passive Filters (12)

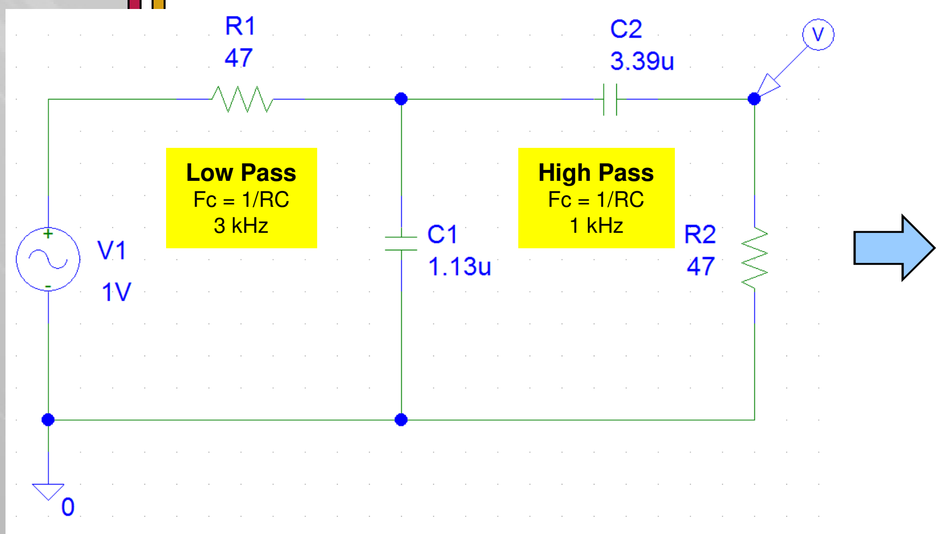
Practice Problem 14.10

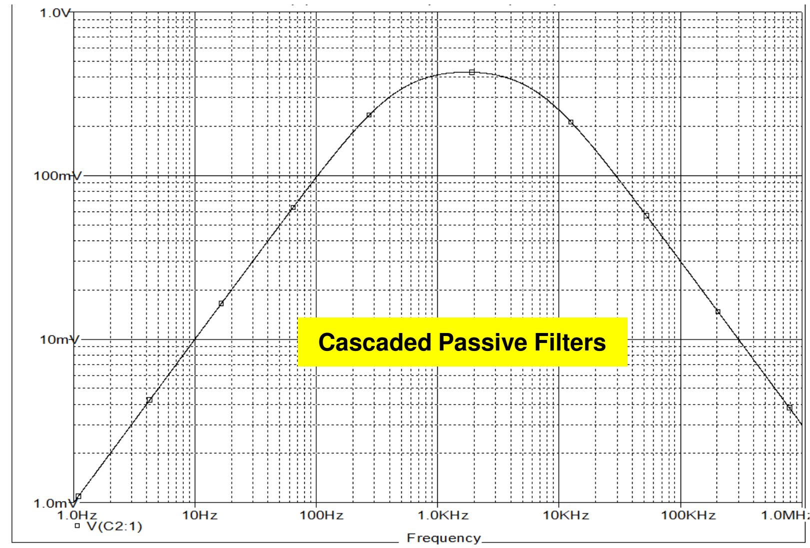
For the circuit below, obtain the transfer function $V_o(\omega)/V_i(\omega)$. Identify the type of filter the circuit represents and determine the corner frequency. Take R_1 = 100 = R_2, and L = 2 mH.



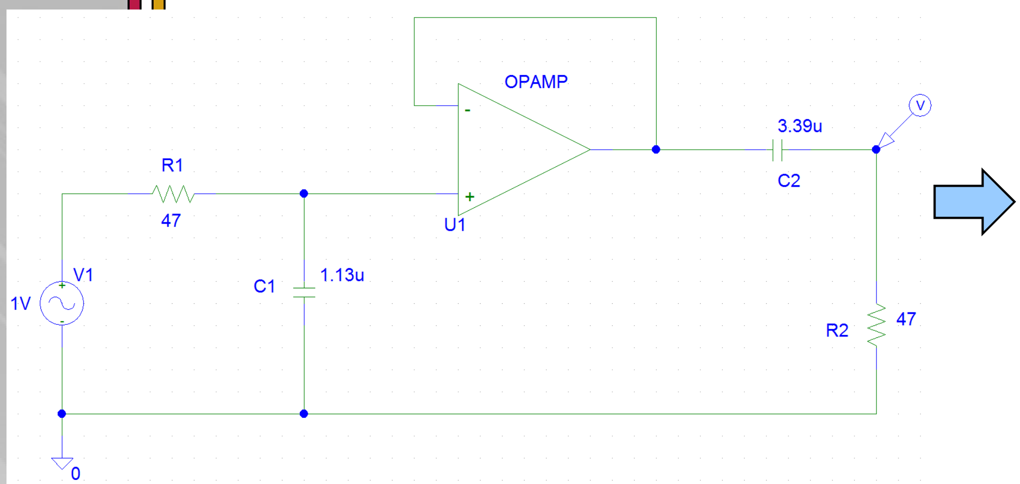
14.7 Passive Filters

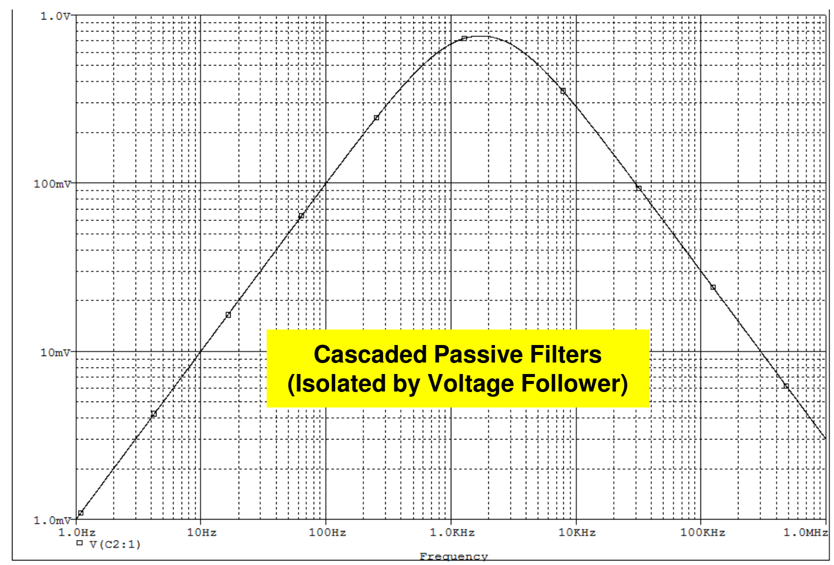
Cascading Filters & loading (aside)





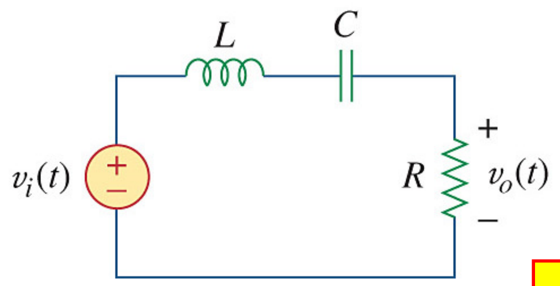
Voltage Follower isolates two filters

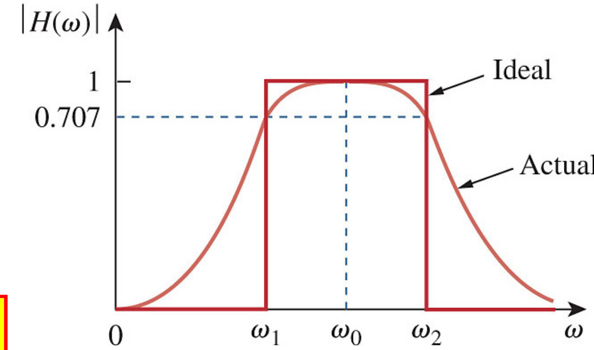




These are simply Resonant Circuits!

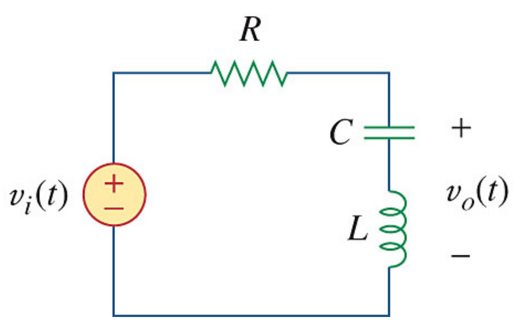
Band Pass Filter

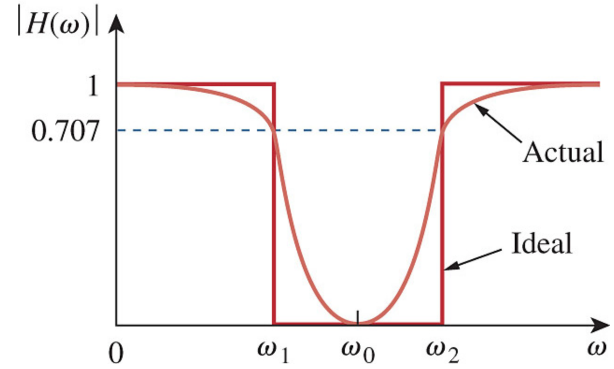




$$\omega_o = \frac{1}{\sqrt{LC}}\text{ rad/s}$$

Band Stop Filter





Ask yourself:
- How would you change the center frequency?
- How would you change the bandwidth?

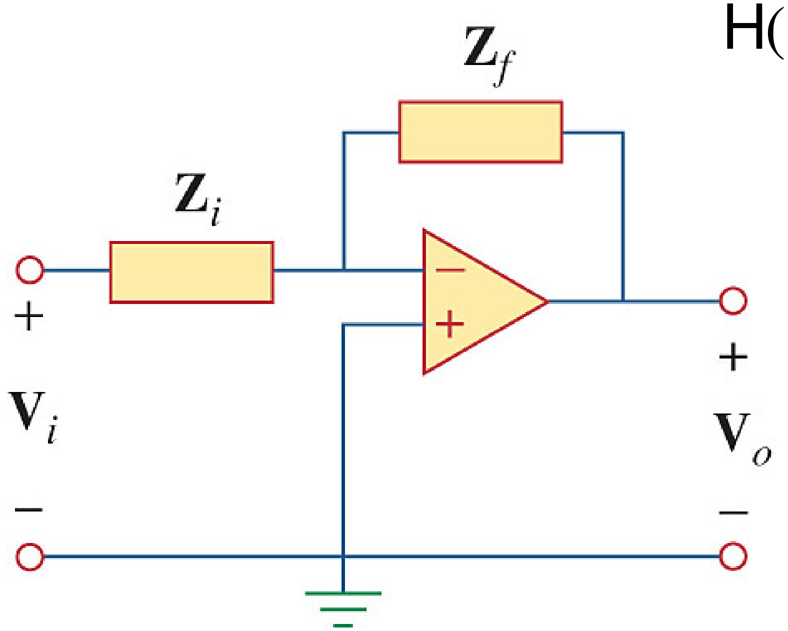
14.8 Active Filters (1)

- Active filters have transistors, op-amps, resistors and capacitors
- Advantages:
  - Gain can be greater than unity
  - Eliminates inductors (which can be large and bulky), and are more difficult to make in Integrated Circuits (ICs)
  - Low frequency range (such as audio) is difficult to achieve without active filters
  - Op-amps can act as buffers to isolate one filter stage in a cascade from the next
- Disadvantages:
  - Maximum limit about 100kHz
  - High gain can introduce instability (oscillations)

14.8 Active Filters (2)
General Form

- Either Z_f or Z_i must be reactive (contain capacitor)
- Transfer function is always

$$H(\omega) = \frac{V_o}{V_i} = -\frac{Z_f}{Z_i}$$

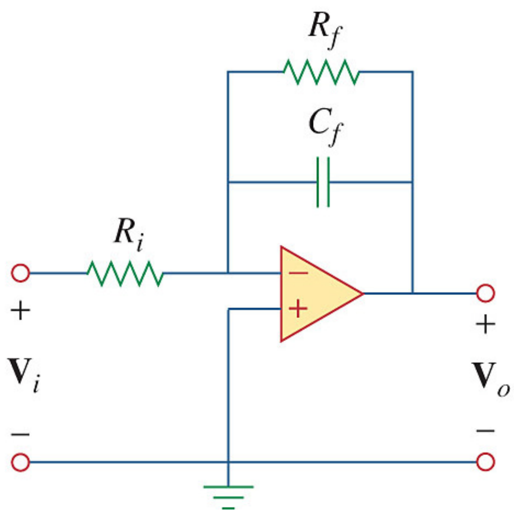


14.8 Active Filters (3)

Lowpass Filter

First order Lowpass Filter

$$Z_f = R_f \| \frac{1}{sC_f} = \frac{\frac{R_f}{sC_f}}{R_f + 1/sC_f} = \frac{R_f}{(1 + sC_f R_f)}$$



The transfer function is

$$H(s) = -\frac{Z_f}{Z_i} = -\frac{R_f}{R_i} \frac{1}{(1 + s R_f C_f)}$$

Similar to equation for passive filter version except there is a low frequency gain of $-R_f / R_i$

The corner frequency is: $\omega_c = \dfrac{1}{R_f C_f}$

Corner frequency does not depend on R_i so several inputs with different R_i could be summed without changing the corner frequency.

14.8 Active Filters (4)
Highpass Filter

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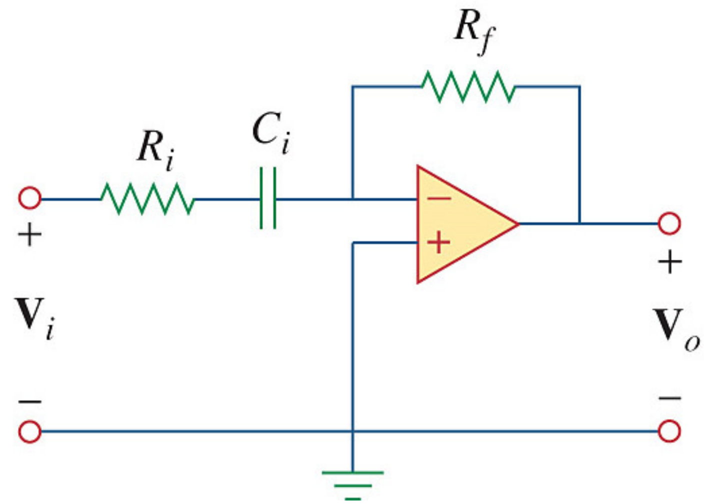
First order Highpass Filter



$$Z_i = \mathrm{R_i} + 1/\,s\mathrm{C_i}$$

The transfer function is

$$\mathrm{H}(s) = -\frac{\mathrm{Z_f}}{\mathrm{Z_i}} = -\frac{s\,\mathrm{R_f C_i}}{\left(1+s\,\mathrm{R_i C_i}\right)}$$

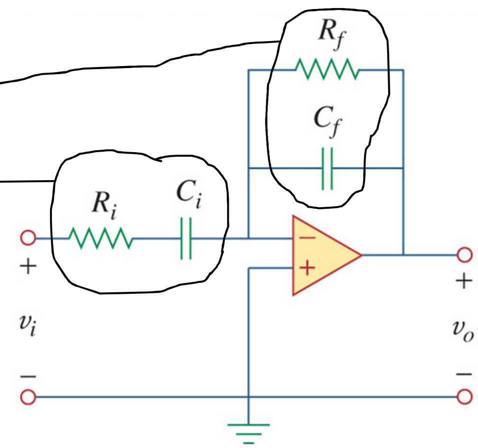
Similar to equation for passive filter version except there is a high frequency gain of $-\mathrm{R_f} / \mathrm{R_i}$

The corner frequency is: $\omega_c = \dfrac{1}{\mathrm{R_i C_i}}$

14.8 Active Filters (3)

Band Pass Filter Example:

- Analysis of the following Circuit



$$H(s) = \frac{V_o}{V_i} = -\frac{Z_f}{Z_i}$$

$$Z_f = R_f \parallel \left(\frac{1}{sC_f}\right) = \frac{R_f}{sR_fC_f + 1} \qquad Z_i = R_i + \left(\frac{1}{sC_i}\right)$$

$$H(s) = \frac{-R_f}{sR_fC_f + 1} \times \frac{1}{R_i + \left(\frac{1}{sC_i}\right)} = \frac{-R_f}{sR_fC_f + 1} \times \frac{sC_i}{sR_iC_i + 1} = \frac{-sR_fC_i}{(sR_fC_f + 1)(sR_iC_i + 1)}$$

Put into the following Form:

$$H(s) = \frac{-\frac{s}{\omega_{fi}}}{(1 + \frac{s}{\omega_f})(1 + \frac{s}{\omega_i})}$$

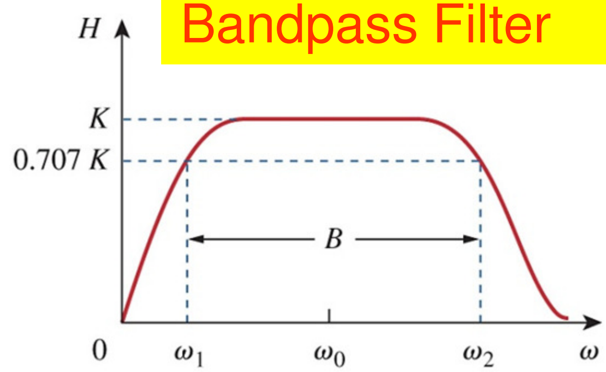
$\omega_{fi} = \frac{1}{R_f C_i}$ Zero at Origin offset by: ω_{fi}

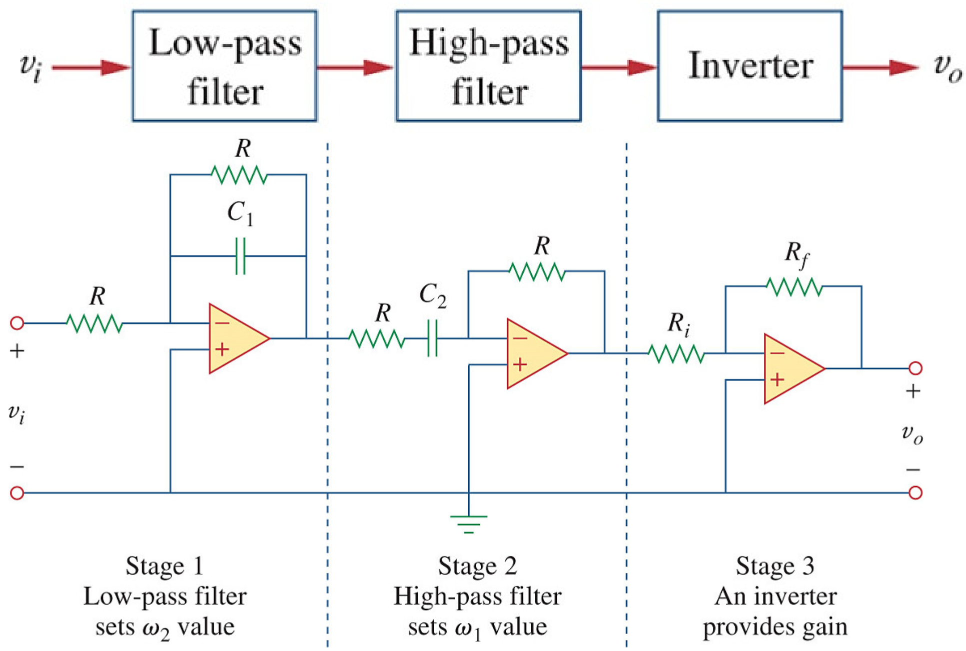
$\omega_i = \frac{1}{R_i C_i}$ Pole at: ω_i

$\omega_f = \frac{1}{R_f C_f}$ Pole at: ω_f

14.8 Active Filters (5)
Bandpass Construction

Bandpass Filter





14.8 Active Filters (6)
Bandpass Transfer Function

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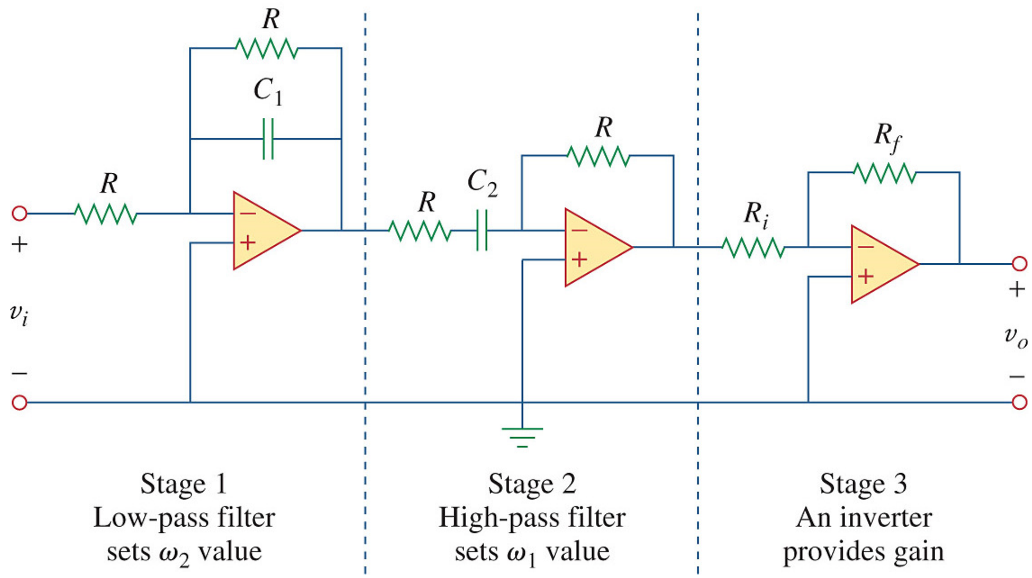


The transfer function is

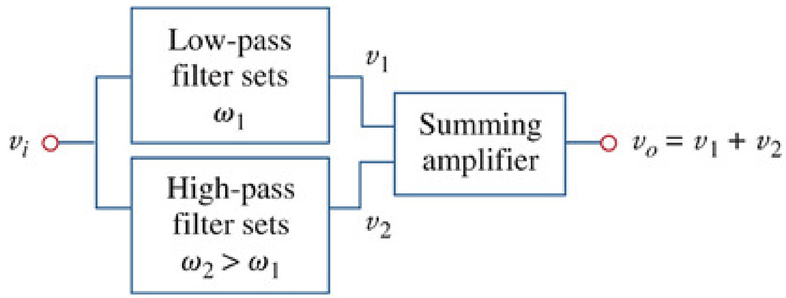
$$H(\omega) = \frac{V_o}{V_i} = \left(-\frac{1}{1+j\omega RC_1}\right)\left(-\frac{j\omega RC_2}{1+j\omega RC_2}\right)\left(-\frac{R_f}{R_i}\right) = \left(-\frac{R_f}{R_i}\right)\frac{j\omega RC_2}{(1+j\omega RC_1)(1+j\omega RC_2)}$$

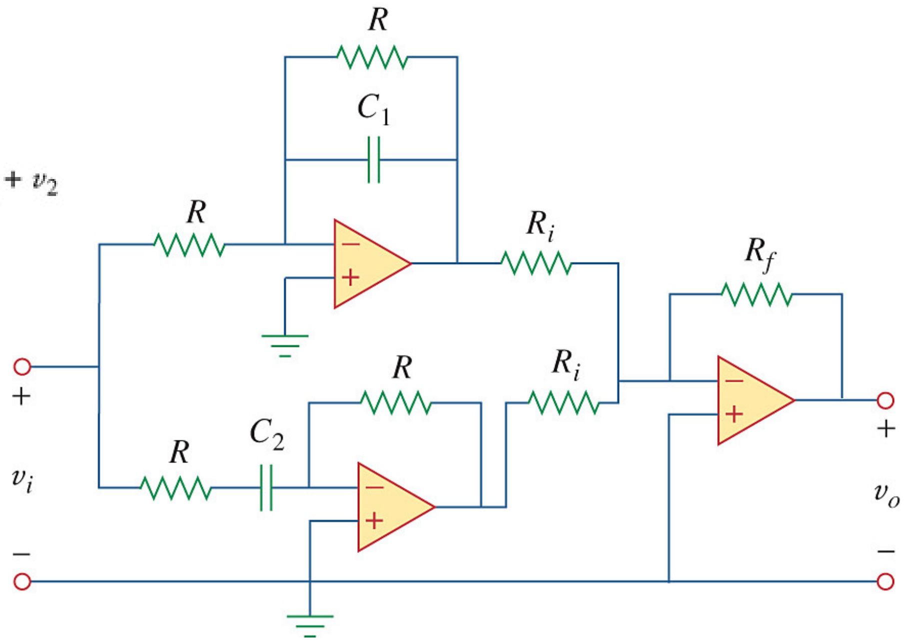
Lowpass section sets upper corner frequency: $\omega_2 = \dfrac{1}{RC_1}$

Highpass section sets lower corner frequency: $\omega_1 = \dfrac{1}{RC_2}$

14.8 Active Filters (8)

Bandreject Transfer Function





The transfer function is

$$H(\omega) = \frac{V_o}{V_i} = \left(-\frac{1}{1+j\omega RC_1} - \frac{j\omega RC_2}{1+j\omega RC_2}\right)\left(-\frac{R_f}{R_i}\right) = \left(\frac{R_f}{R_i}\right)\left(\frac{1}{1+j\omega RC_1} + \frac{j\omega RC_2}{1+j\omega RC_2}\right)$$

The corner frequencies are: $\omega_2 = \dfrac{1}{RC_1}$ and $\omega_1 = \dfrac{1}{RC_2}$

Homework #5

Due in Class Wednesday Feb 25, 2015

- 14.29
- 14.35
- 14.38
- 14.53
- 14.54
- 14.67
- 14.68 (Voltage Gain +6dB, corner frequency = 3000 Hz)

!! UPDATE !!

!! Due at beginning of class Wednesday Feb 25th !!

Late Homework will not be graded

Homework #6

Due in Class Monday March 3, 2015

- 14.82
- 14.83
- Design Problem (See Handout)

!! Due at beginning of class Monday March 3rd !!

Late Homework will not be graded

14.9 Scaling (1)

- Sometimes it is convenient to work with circuit element values of 1 ohm, 1 Henry, and 1 Farad and transform the values to realistic values by scaling.
- Magnitude scaling
  - The process of increasing all impedances by a factor, frequency response remaining unchanged.
- Frequency scaling
  - The process of shifting the frequency response of a network up or down the frequency axis while leaving the impedance the same.

14.9 Scaling (2)
Magnitude Scaling

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$Z'_R = K_m Z_R = K_m R$

$Z'_L = K_m Z_L = j\omega K_m L$

$Z'_C = K_m Z_C = K_m / j\omega C = 1 / (j\omega C / K_m)$

Consider: $\omega_o = \dfrac{1}{\sqrt{LC}}$

After Magnitude scaling by K_m:

$$\omega'_o = \frac{1}{\sqrt{L'C'}} = \frac{1}{\sqrt{K_m L(C/K_m)}} = \frac{1}{\sqrt{LC}}$$

Similarly, the quality factor, bandwidth and transfer function are not affected by magnitude scaling.

14.9 Scaling (3)
Frequency Scaling

- In frequency scaling, the frequency is multiplied by a factor K_f while keeping the impedance the same

$Z'_R = Z_R = R$ (no changes to resistors)

$Z_L = j(\omega K_f)L'$; therefore $L' = L/K_f$

$Z_C = 1/[\,j(\omega K_f)C']$; therefore $C' = C/K_f$

Consider: $\omega_o = \dfrac{1}{\sqrt{LC}}$

After Frequency scaling by K_f:

$$\omega'_o = \frac{1}{\sqrt{L'C'}} = \frac{1}{\sqrt{(L/K_f)(C/K_f)}} = \frac{K_f}{\sqrt{LC}} = K_f\,\omega_o$$

14.9 Scaling (4)

Frequency Scaling Summary

Magnitude Scaling

$$K_m = \frac{R'}{R}$$

Frequency Scaling

$$K_f = \frac{\omega'_o}{\omega_o} = \frac{f'_o}{f_o} = \frac{B'}{B}$$

Component / Parameter values

$$R' = K_m R \qquad \omega'_o = K_f \omega_o$$

$$L' = \frac{K_m}{K_f} L \qquad B' = K_f B$$

$$C' = \frac{1}{K_m K_f} C \qquad Q' = Q$$

Chapter 14

End of Chapter Review

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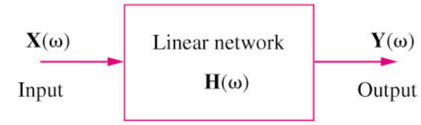
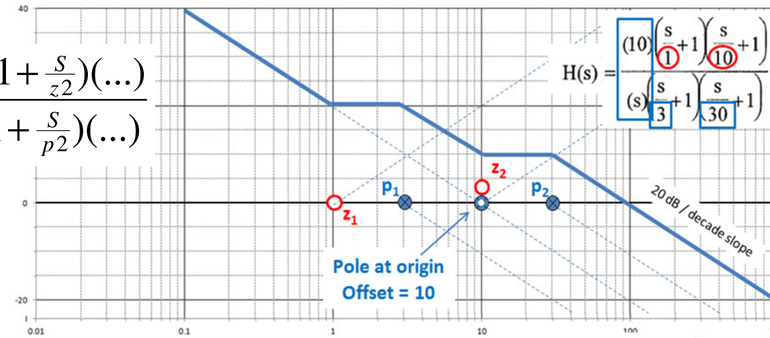
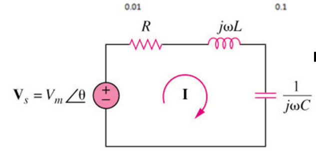
- Transfer Function
- Bode Plot
  - Magnitude
  - Phase
- Resonant Circuits
  - Series / Parallel
  - "Q" & Bandwidth
- Passive Filters
  - Low Pass / High Pass
  - Band Pass / Band Stop
- Active Filters
- Scaling

$$H(\omega)=\frac{Y(\omega)}{X(\omega)}$$

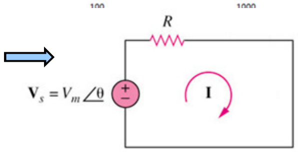


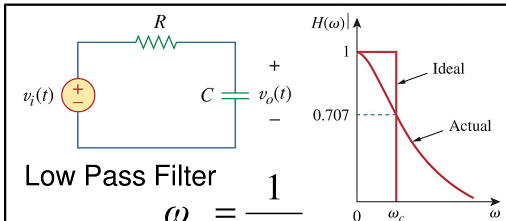
$$H(s)=\frac{s(1+\frac{S}{z1})(1+\frac{S}{z2})(\ldots)}{(1+\frac{S}{p1})(1+\frac{S}{p2})(\ldots)}$$





$$\omega_o=\frac{1}{\sqrt{LC}}\ \text{rad/s}$$

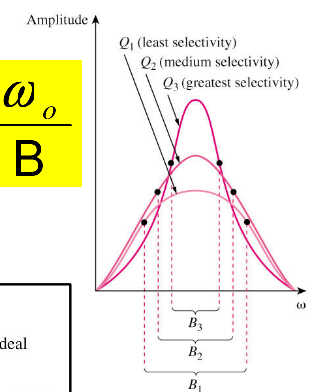




Low Pass Filter

$$\omega_c=\frac{1}{RC}$$

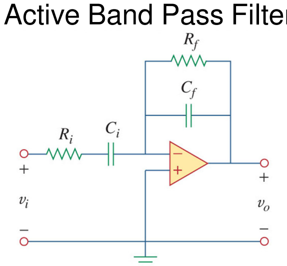
$$Q=\frac{\omega_o}{B}$$



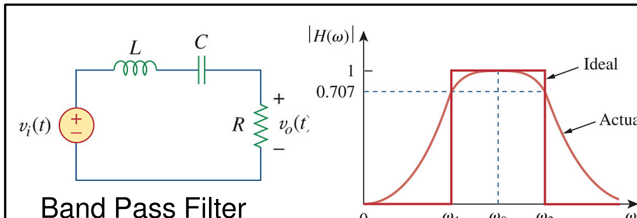
Component / Parameter values

$R'=K_mR$	$\omega'_o=K_f\omega_o$
$L'=\frac{K_m}{K_f}L$	$B'=K_fB$
$C'=\frac{1}{K_mK_f}C$	$Q'=Q$

Active Band Pass Filter



$$H(s)=\frac{V_o}{V_i}=-\frac{Z_f}{Z_i}$$



Band Pass Filter

$$\omega_o=\frac{1}{\sqrt{LC}}$$