ECE 202 Review (Ch 8 – 11)

IUPUI | SCHOOL OF ENGINEERING AND TECHNOLOGY
A PURDUE UNIVERSITY SCHOOL
Indianapolis

- Part I DC Circuits
  - Chapter 8: 2nd Order Circuits
- Part II AC Circuits
  - Chapter 9: Sinusoids and Phasors
  - Chapter 10: Sinusoidal Steady-State Analysis
  - Chapter 11: AC Power Analysis

8.1 Examples of 2nd order RCL circuit

8.2 Finding Initial and Final Values (X)

8.3 The source-free series RLC circuit

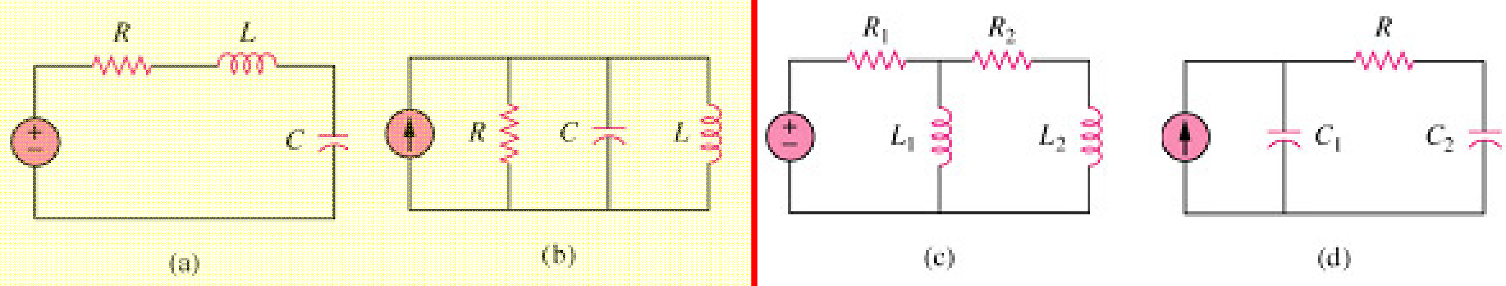
8.4 The source-free parallel RLC circuit

8.5 Step response of a series RLC circuit

8.6 Step response of a parallel RLC circuit

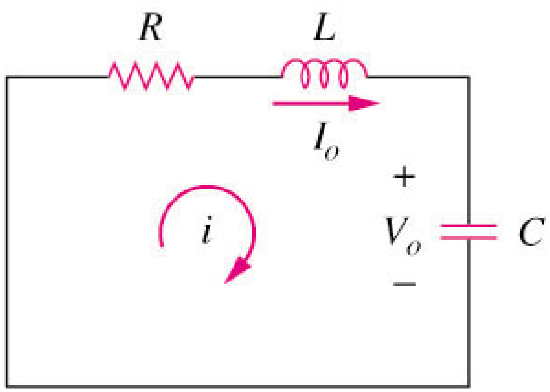
8.1 Examples of Second Order RLC circuits

A second-order circuit is characterized by a second-order differential equation. It consists of resistors and the equivalent of two energy storage elements.



RLC Series　RLC Parallel　RL T-config　RC Pi-config

8.3 Source-Free Series RLC Circuits (1)



- The solution of the source-free series RLC circuit is called as the natural response of the circuit.

Derived from KVL:

$$iR + L\frac{di}{dt} + \frac{1}{C}\int_{-\infty}^{t} i(\tau)d\tau = 0$$

$$R\frac{di}{dt} + L\frac{d^2i}{dt^2} + \frac{1}{C}i = 0$$

The 2nd order of expression

$$\frac{d^2i}{dt^2} + \frac{R}{L}\frac{di}{dt} + \frac{i}{LC} = 0$$

To find for voltage across capacitor

Substitute: $i = C\dfrac{dv}{dt}$

$$RC\frac{dv}{dt} + LC\frac{d^2v}{dt^2} + v = 0$$

Note: Same form if looking at voltage across capacitor!

$$\frac{d^2v}{dt^2} + \frac{R}{L}\frac{dv}{dt} + \frac{v}{LC} = 0$$

Where v is the voltage across the capacitor

8.3 Source-Free Series RLC Circuits (2)

Three possible solutions for the 2nd order differential equation:

$$\frac{d^2 i}{dt^2} + \frac{R}{L}\frac{di}{dt} + \frac{i}{LC} = 0$$

=> $$\frac{d^2 i}{dt^2} + 2\alpha\frac{di}{dt} + \omega_0^2 i = 0$$

where $\alpha = \frac{R}{2L}$ and $\omega_0 = \sqrt{\frac{1}{LC}}$

General 2nd order Form

The types of solutions for i(t) depend on the relative values of α and ω.

General Solution

$$i(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t}$$

$$s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2}$$

- If $\alpha > \omega_o$ ➔ over-damped
- If $\alpha = \omega_o$ ➔ critically-damped
- If $\alpha < \omega_o$ ➔ under-damped

8.3 Source-Free Series RLC Circuits (3)

There are three possible solutions for the following 2nd order differential equation:

$$\frac{d^2 i}{dt^2} + 2\alpha \frac{di}{dt} + \omega_0^2 i = 0 \qquad \alpha = \frac{R}{2L} \quad and \quad \omega_0 = \sqrt{\frac{1}{LC}}$$

1. If $\alpha > \omega_o$, over-damped case

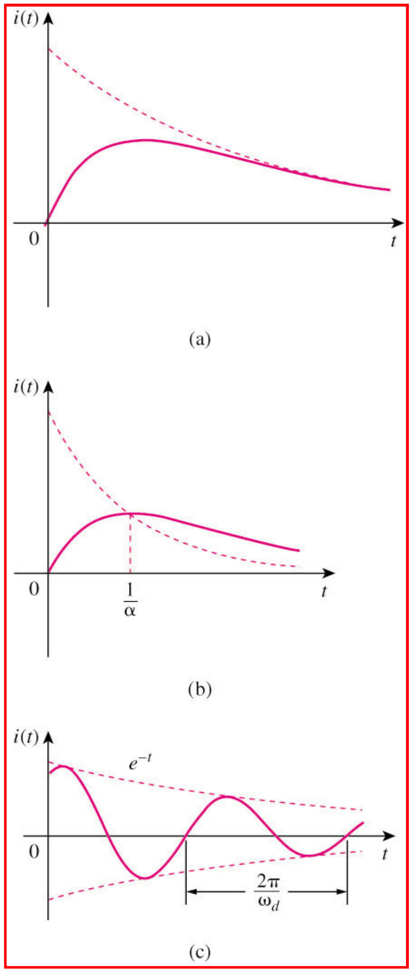
$$i(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t}$$ **where** $$s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2}$$

2. If $\alpha = \omega_o$, critical damped case

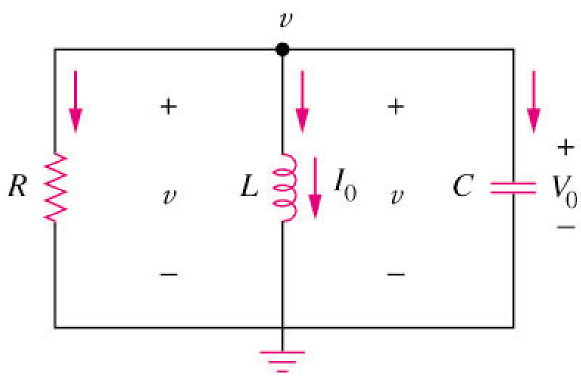
$$i(t) = (A_2 + A_1 t) e^{-\alpha t}$$ **where** $$s_{1,2} = -\alpha$$

3. If $\alpha < \omega_o$, under-damped case

$$i(t) = e^{-\alpha t}(B_1 \cos \omega_d t + B_2 \sin \omega_d t)$$ **where** $$\omega_d = \sqrt{\omega_0^2 - \alpha^2}$$



8.4 Source-Free Parallel RLC Circuits (1)



- Similar approach to series RLC analysis

Derived from KCL:

$$\frac{v}{R}+C\frac{dv}{dt}+\frac{1}{L}\int_{-\infty}^{t}v(\tau)d\tau=0$$

$$\frac{1}{R}\frac{dv}{dt}+C\frac{d^2v}{dt^2}+\frac{1}{L}v=0$$

The 2nd order of expression

$$\frac{d^2v}{dt^2}+\frac{1}{RC}\frac{dv}{dt}+\frac{1}{LC}v=0$$

To find for current across inductor

Substitute: $v=L\frac{di}{dt}$

$$\frac{L}{R}\frac{di}{dt}+LC\frac{d^2i}{dt^2}+i=0$$

Note: Same form if looking at current through inductor!

$$\frac{d^2i}{dt^2}+\frac{1}{RC}\frac{di}{dt}+\frac{1}{LC}i=0$$

Where i is the current through the inductor

8.4 Source-Free Parallel RLC Circuits (2)

There are three possible solutions for the following 2nd order differential equation:

$$\frac{d^2v}{dt^2} + 2\alpha\frac{dv}{dt} + \omega_0^2 v = 0 \quad \text{where} \quad \alpha = \frac{1}{2RC} \quad \text{and} \quad \omega_0 = \sqrt{\frac{1}{LC}}$$

1. If $\alpha > \omega_o$, over-damped case

$$v(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t}$$ **where** $$s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2}$$

2. If $\alpha = \omega_o$, critical damped case

$$v(t) = (A_2 + A_1 t)\, e^{-\alpha t}$$ **where** $$s_{1,2} = -\alpha$$

3. If $\alpha < \omega_o$, under-damped case

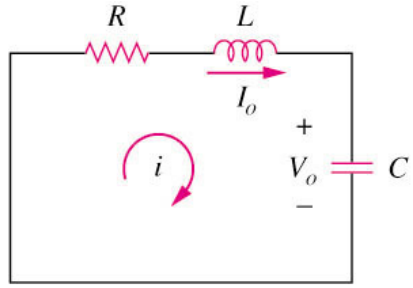
$$v(t) = e^{-\alpha t}(B_1 \cos \omega_d t + B_2 \sin \omega_d t)$$ **where** $$\omega_d = \sqrt{\omega_0^2 - \alpha^2}$$

Comparison Series / Parallel RLC Circuits

	Series RLC	Parallel RLC
Characteristic Equation	$\frac{d^2i}{dt^2} + 2\alpha\frac{di}{dt} + \omega_0^2 i = 0$	$\frac{d^2v}{dt^2} + 2\alpha\frac{dv}{dt} + \omega_0^2 v = 0$
Damping Factor	$\alpha = \frac{R}{2L}$	$\alpha = \frac{1}{2RC}$
Resonance Frequency	$\omega_0 = \sqrt{\frac{1}{LC}}$	$\omega_0 = \sqrt{\frac{1}{LC}}$
Over-damped $\alpha > \omega_o$	$i(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t}$ $s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2}$	$v(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t}$ $s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2}$
Critical damped $\alpha = \omega_o$,	$i(t) = (A_2 + A_1 t)\, e^{-\alpha t}$	$v(t) = (A_2 + A_1 t)\, e^{-\alpha t}$
Under-damped $\alpha < \omega_o$	$i(t) = e^{-\alpha t}(B_1 \cos\omega_d t + B_2 \sin\omega_d t)$ $\omega_d = \sqrt{\omega_0^2 - \alpha^2}$	$v(t) = e^{-\alpha t}(B_1 \cos\omega_d t + B_2 \sin\omega_d t)$ $\omega_d = \sqrt{\omega_0^2 - \alpha^2}$

Source-Free RLC Circuits

- Key to solving problems is to finding the initial conditions
  - Voltage doesn't change rapidly across a capacitor: $v(0^+) = v(0^-)$
  - Current doesn't change rapidly across an inductor: $i(0^+) = i(0^-)$
- Start by finding α and ω_o use these to find s_1 and s_2
  - Determine case (over / under / or critically damped)
- Apply initial conditions to the equations to find A_1 and A_1

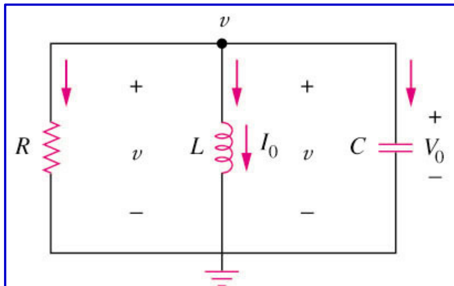


Series RLC

$$i(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t} \Rightarrow i(0^+) = A_1 + A_2$$

$$\frac{di(t)}{dt} = A_1 s_1 e^{s_1 t} + A_2 s_2 e^{s_2 t} \Rightarrow \frac{di(0^+)}{dt} = A_1 s_1 + A_2 s_2$$

$$\frac{di(0)}{dt} = -\frac{1}{L}(RI_0 + V_0)$$



Parallel RLC

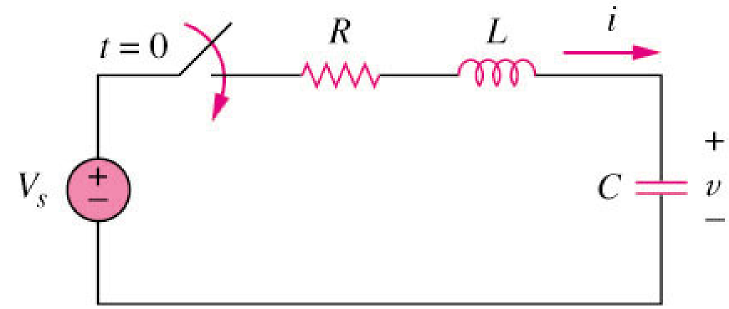
$$v(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t} \Rightarrow v(0^+) = A_1 + A_2$$

$$\frac{dv(t)}{dt} = A_1 s_1 e^{s_1 t} + A_2 s_2 e^{s_2 t} \Rightarrow \frac{dv(0^+)}{dt} = A_1 s_1 + A_2 s_2$$

$$\frac{dv(0)}{dt} = -\frac{1}{RC}(RI_0 + V_0)$$

8.5 Step-Response Series RLC Circuits (1)

- The step response is obtained by the sudden application of a dc source.



The 2nd order of expression

$$\underbrace{\frac{d^2v}{dt^2}+\frac{R}{L}\frac{dv}{dt}+\frac{v}{LC}}_{\textit{Natural Response}}=\frac{V_s}{LC}$$

The above equation has the same form as the equation for source-free series RLC circuit.

- The same coefficients (important in determining the frequency parameters).

8.5 Step-Response Series RLC Circuits (2)

The solution of the equation should have two components:
the transient response $v_t(t)$ & the steady-state response $v_{ss}(t)$:

$$v(t) = v_t(t) + v_{ss}(t)$$

This is the Voltage across the Capacitor !!

- The transient response v_t is the same as that for source-free case

$$v_t(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t}$$ (over-damped)

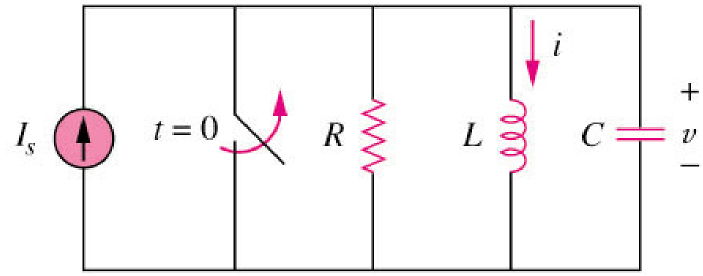
$$v_t(t) = (A_1 + A_2 t)e^{-\alpha t}$$ (critically damped)

$$v_t(t) = e^{-\alpha t}(A_1 \cos \omega_d t + A_2 \sin \omega_d t)$$ (under-damped)

- The steady-state response is the final value of v(t).
  - $v_{ss}(t) = v(\infty)$
- The values of A_1 and A_2 are obtained from the initial conditions:
  - $v(0)$ and $dv(0)/dt$.

8.6 Step-Response Parallel RLC Circuits (1)

- The step response is obtained by the sudden application of a dc source.



The 2nd order of expression

$$\frac{d^2 i}{dt^2} + \frac{1}{RC}\frac{di}{dt} + \frac{i}{LC} = \frac{I_s}{LC}$$

It has the same form as the equation for source-free parallel RLC circuit.

- The same coefficients (important in determining the frequency parameters).

8.6 Step-Response Parallel RLC Circuits (2)

The solution of the equation should have two components: the transient response $i_t(t)$ & the steady-state response $i_{ss}(t)$:

$$i(t) = i_t(t) + i_{ss}(t)$$

This is the Current through the Inductor !!

- The transient response i_t is the same as that for source-free case

$$i_t(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t} \quad \text{(over-damped)}$$

$$i_t(t) = (A_1 + A_2 t)e^{-\alpha t} \quad \text{(critical damped)}$$

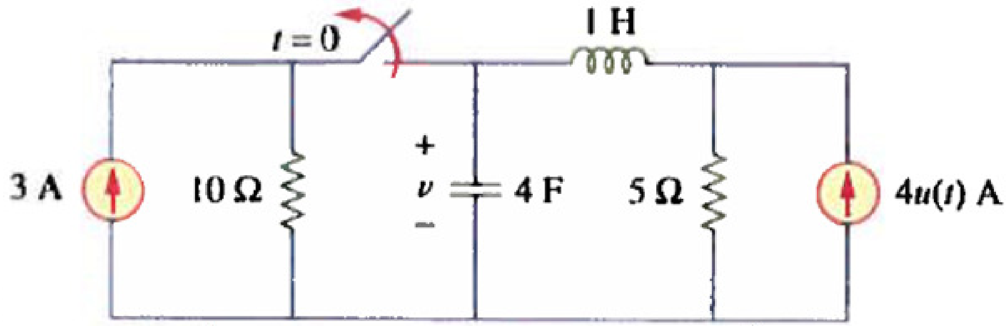
$$i_t(t) = e^{-\alpha t}(A_1 \cos \omega_d t + A_2 \sin \omega_d t) \quad \text{(under-damped)}$$

- The steady-state response is the final value of i(t).
  - $i_{ss}(t) = i(\infty) = I_s$
- The values of A_1 and A_2 are obtained from the initial conditions:
  - i(0) and di(0)/dt.

Chapter 8 Example (1)

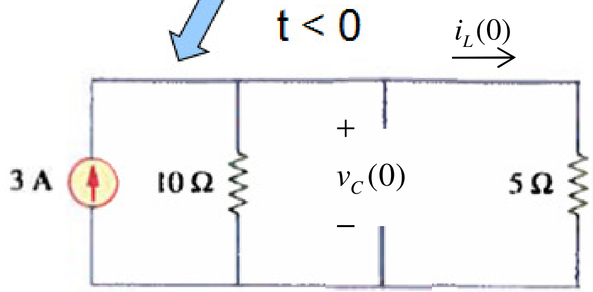
8.33 Find $v(t)$ for $t > 0$ in the circuit of Fig. 8.81.

Remember !!
Capacitor is "Open" to DC
Inductor is "Short" to DC



Before switch is opened

t < 0



Initial Voltage across Capacitor

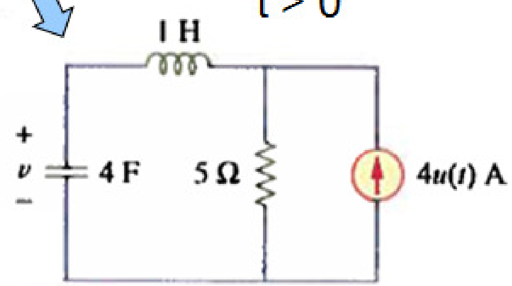
$$v_C(0) = \frac{5 \cdot 10}{5+10}(3) = 10$$

Initial Current through Inductor

$$i_L(0) = 2$$

After switch is opened

t > 0

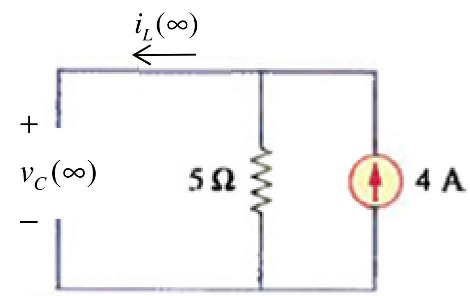


$$\alpha = \frac{R}{2L} = \frac{5}{(2)(1)} = 2.5$$

$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{(1)(4)}} = 0.5$$

$\alpha > \omega_0$ Over-damped Case

Steady State circuit ($t \rightarrow \infty$)



SS Voltage across Capacitor

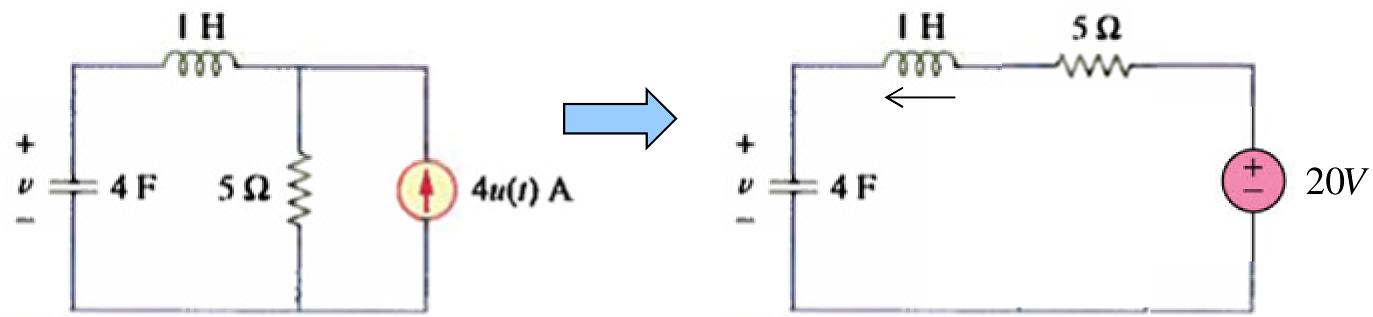
$$v_C(\infty) = (4)(5) = 20$$

SS Current through Inductor

$$i_L(\infty) = 0$$

Chapter 8 Example 8.33 (continued)

Circuit to be analyzed for response



Step Response of Series RLC Circuit

$$v(t) = v_t(t) + v_{ss}(t)$$

Voltage across Capacitor

$$v_t(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t}$$

(over-damped)

Known Parameters

$v_C(0) = 10$	$v_C(\infty) = 20$	$\alpha = 2.5$	$s_1 = 2.5 + \sqrt{(2.5)^2 - (0.5)^2} = 4.949$
$i_L(0) = 2$	$i_L(\infty) = 0$	$\omega_0 = 0.5$	$s_2 = 2.5 - \sqrt{(2.5)^2 - (0.5)^2} = 0.0505$

Voltage across Capacitor

$$v(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t} + 20$$

$$v(0) = A_1 + A_2 + 20 = 10$$

$$\boxed{A_1 + A_2 = -10}$$

Find Current through Inductor by taking derivative

$$i(0) = C\frac{dv(0)}{dt} \Rightarrow \frac{dv(0)}{dt} = \frac{i(0)}{C} = \frac{-2}{4}$$

Negative because in opposite direction

$$\frac{dv(0)}{dt} = \boxed{A_1 s_1 + A_2 s_2 = -0.5}$$

From here can solve for A1 and A2

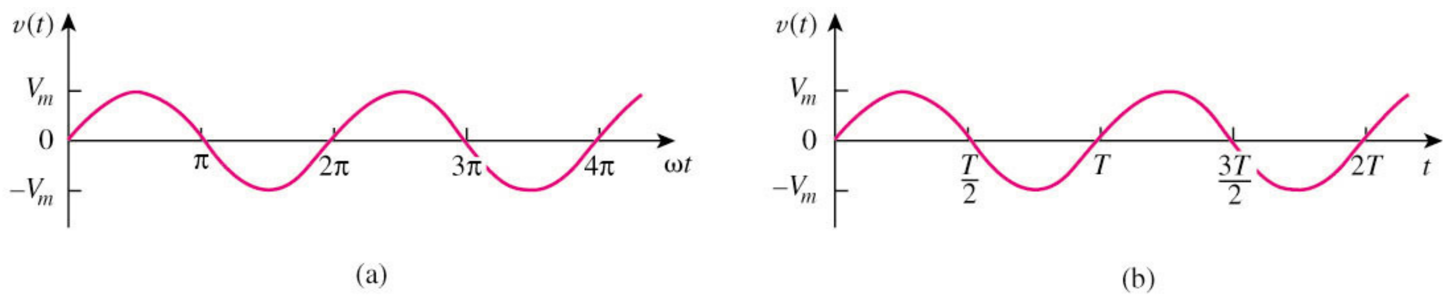
Sinusoids and Phasors Chapter 9 - Review

- 9.2 Sinusoids
- 9.3 Phasors
- 9.4 Phasor Relationships for Circuit Elements
- 9.5 Impedance and Admittance
- 9.6 Kirchhoff's Laws in the Frequency Domain
- 9.7 Impedance Combinations

9.2 Sinusoids (1)

- A sinusoid is a signal that has the form of the sine or cosine function.
- A general expression for the sinusoid,

$$v(t) = V_m \sin(\omega t + \phi)$$



where

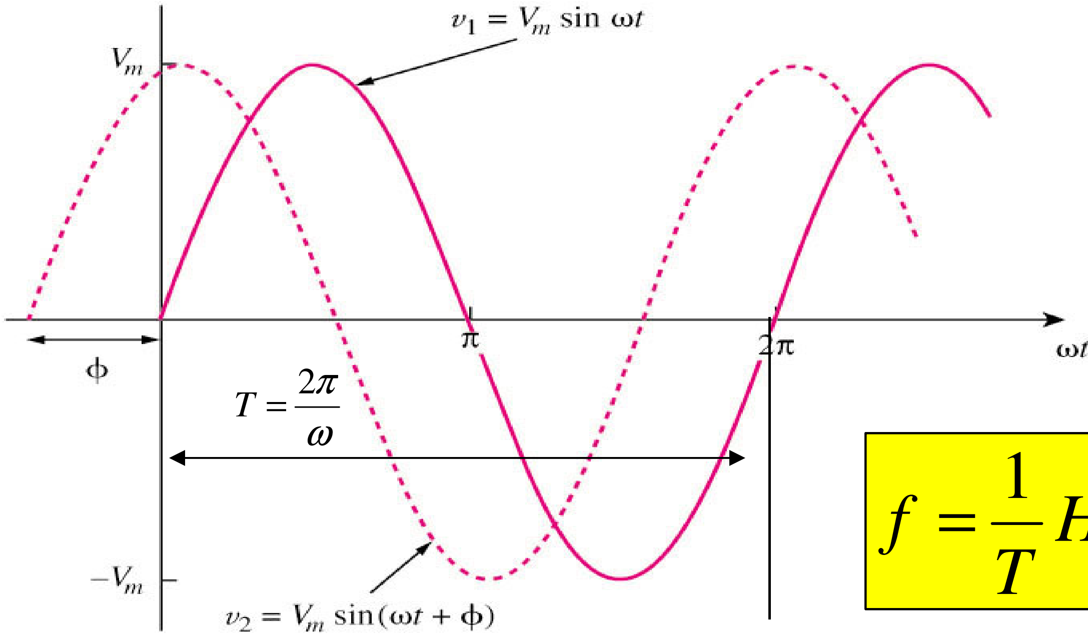
Vm = the **amplitude** of the sinusoid

ω = the angular frequency in radians/s

Φ = the phase

9.2 Sinusoids (2)

A periodic function is one that satisfies *v(t) = v(t + nT)*, for all t and for all integers n.



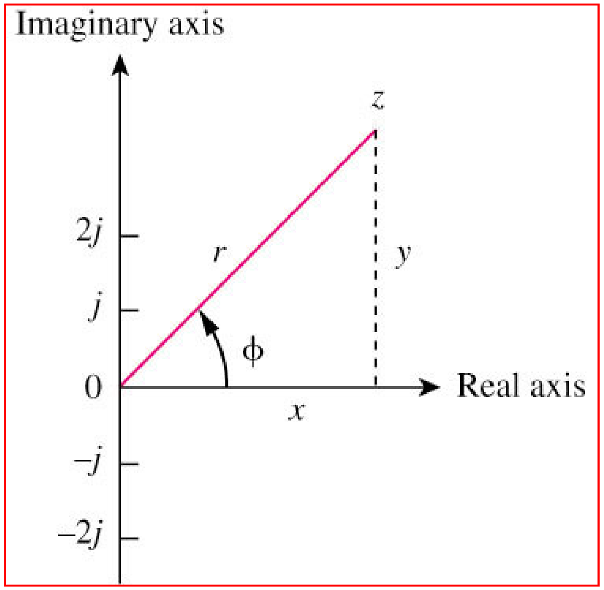
$$\boxed{f = \frac{1}{T} Hz} \quad \boxed{\omega = 2\pi f}$$

- Only two sinusoidal values with the same frequency can be compared by their amplitude and phase difference.
- If phase difference is zero, they are in phase; if phase difference is not zero, they are out of phase.

9.3 Phasors (1)

- A phasor is a complex number that represents the amplitude and phase of a sinusoid.

- It can be represented in one of the following three forms:



a. Rectangular $z = x + jy = r(\cos\phi + j\sin\phi)$

b. Polar $z = r\angle\phi$

c. Exponential $z = re^{j\phi}$

where $r = \sqrt{x^2 + y^2}$

$\phi = \tan^{-1}\dfrac{y}{x}$

Mathematic operation of complex number:

1. Addition $z_1 + z_2 = (x_1 + x_2) + j(y_1 + y_2)$
2. Subtraction $z_1 - z_2 = (x_1 - x_2) + j(y_1 - y_2)$
3. Multiplication $z_1 z_2 = r_1 r_2 \angle \phi_1 + \phi_2$
4. Division $\frac{z_1}{z_2} = \frac{r_1}{r_2} \angle \phi_1 - \phi_2$
5. Reciprocal $\frac{1}{z} = \frac{1}{r} \angle -\phi$
6. Square root $\sqrt{z} = \sqrt{r} \angle \phi/2$
7. Complex conjugate $z^* = x - jy = r \angle -\phi = re^{-j\phi}$
8. Euler's identity $e^{\pm j\phi} = \cos\phi \pm j\sin\phi$

9.3 Phasors (3)

- Transform a sinusoid to and from the time domain to the phasor domain:

$$v(t) = V_m \cos(\omega t + \phi) \longleftrightarrow V = V_m \angle \phi$$

(time domain) (phasor domain)

The differences between v(t) and V:

- v(t) is instantaneous or time-domain representation
  V is the frequency or phasor-domain representation.
- v(t) is time dependent, V is not.
- v(t) is always real with no complex term, V is generally complex.

Note: Phasor analysis applies only when frequency is constant; it is applied to two or more sinusoid signals only if they have the same frequency.

9.3 Phasors (5)

Relationship between differential, integral operation in phasor listed as follow:

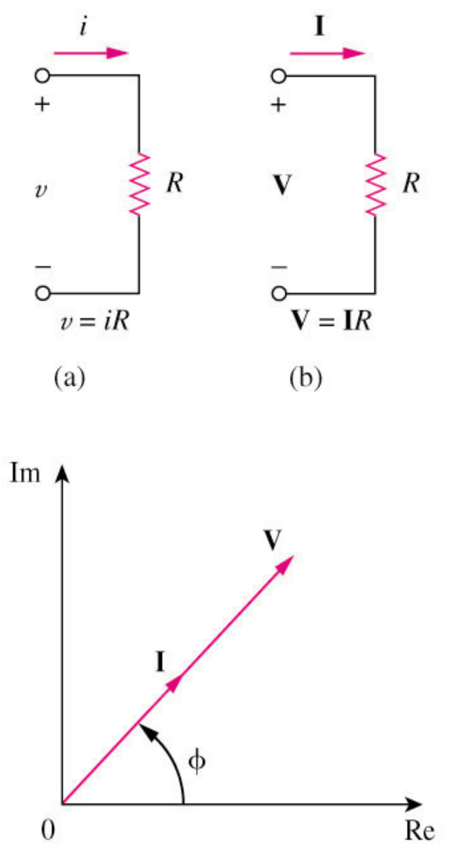
$$v(t) \longleftrightarrow \mathbf{V} = V\angle\phi$$

$$\frac{dv}{dt} \longleftrightarrow j\omega \mathbf{V}$$

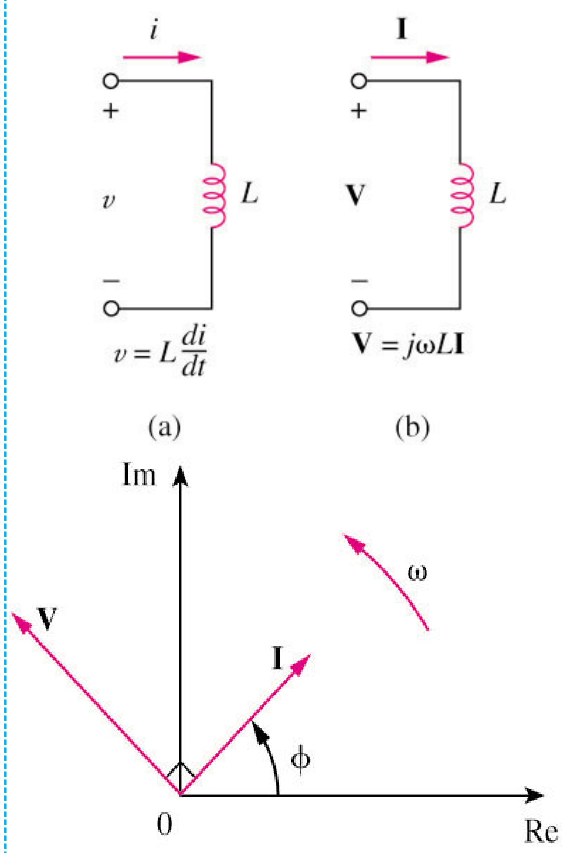
$$\int v\,dt \longleftrightarrow \frac{\mathbf{V}}{j\omega}$$

9.4 Phasor Relationships for Circuit Elements (1)

Resistors



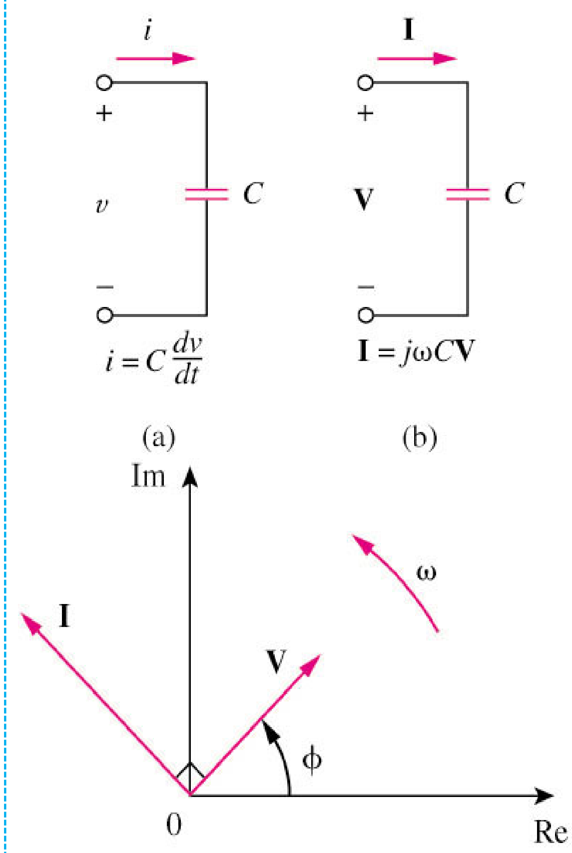
Inductors



"ELI" the

Voltage Lead Current
For Inductors "L" (ELI)

Capacitors



"ICE" man

Current Lead Voltage
For Capacitors "C" (ICE)

Aside: ELI the ICE man & the derivative

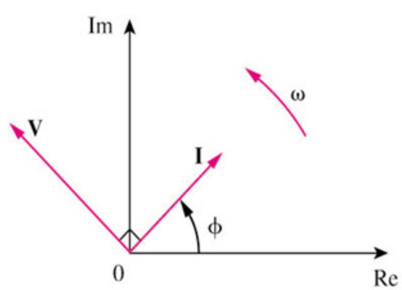
- The phrase “ELI the ICE man” can be used to remember the relationships of voltage & current for inductors & capacitors

E L I ↓ ↓ ↓

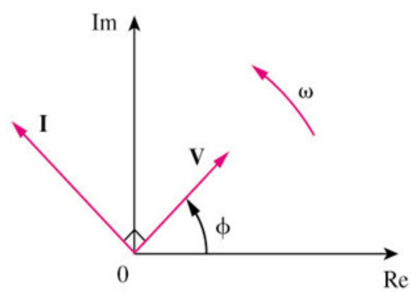
$$v = L\frac{di}{dt}$$

I C E ↓ ↓ ↓

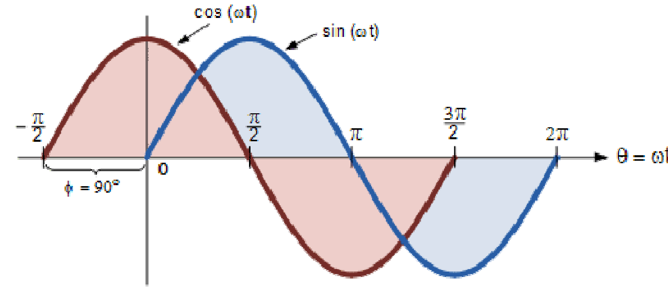
$$i = C\frac{dv}{dt}$$



Voltage leads current



Current leads voltage



Note:

1. Derivative of sin(t) is cos(t).
2. Observe the instantaneous “slope” of sin(t). It’s cos(t)!
3. Derivative introduces a 90° shift
4. Now look at equations for L and C and this explains the 90° shift
5. Also, should be easier now to remember:

$$\frac{d}{dt}\left[\sin(\omega t)\right] = \omega\cos(\omega t)$$

$$\frac{d}{dt}\left[\cos(\omega t)\right] = -\omega\sin(\omega t)$$

9.4 Phasor Relationships for Circuit Elements (2)

Summary of voltage-current relationship		
Element	Time domain	Frequency domain
R	$v = Ri$	$V = RI$
L	$v = L\frac{di}{dt}$	$V = j\omega LI$
C	$i = C\frac{dv}{dt}$	$V = \frac{I}{j\omega C}$

9.5 Impedance and Admittance (1)

- The impedance Z of a circuit is the ratio of the phasor voltage V to the phasor current I, measured in ohms Ω.

$$Z = \frac{V}{I} = R + jX$$

Positive X is for L and negative X is for C.

- The admittance Y is the reciprocal of impedance, measured in siemens (S).

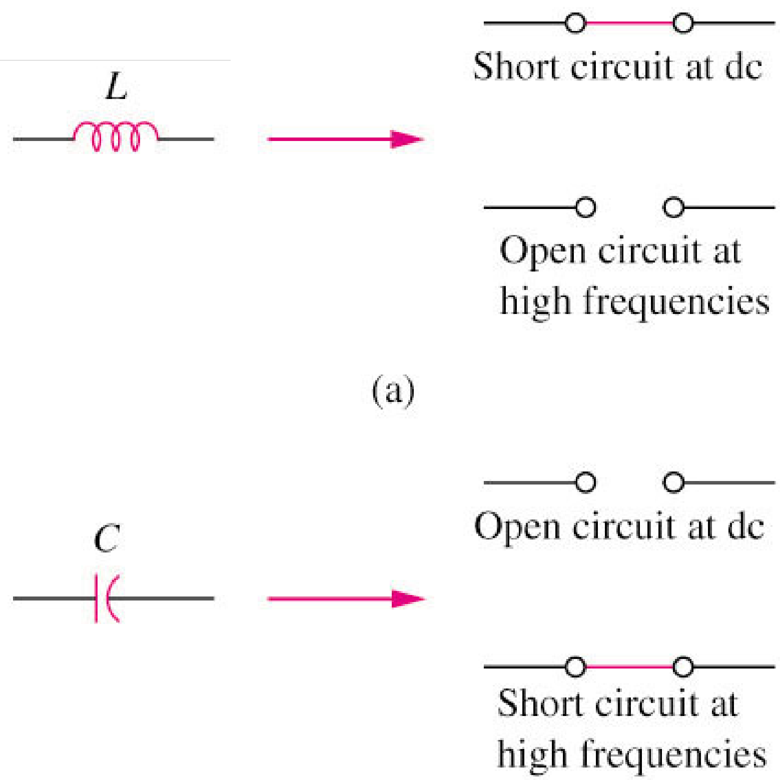
$$Y = \frac{1}{Z} = \frac{I}{V}$$

9.5 Impedance and Admittance (2)

Impedances and admittances of passive elements		
Element	**Impedance**	**Admittance**
R	$Z = R$	$Y = \dfrac{1}{R}$
L	$Z = j\omega L$	$Y = \dfrac{1}{j\omega L}$
C	$Z = \dfrac{1}{j\omega C}$	$Y = j\omega C$

9.5 Impedance and Admittance (3)

$$Z = j\omega L$$



$$\omega = 0; Z = 0$$

$$\omega \to \infty; Z \to \infty$$

(a)

$$Z = \frac{1}{j\omega C}$$

$$\omega = 0; Z \to \infty$$

$$\omega \to \infty; Z = 0$$

(b)

9.6 Kirchhoff's Laws in the Frequency Domain (1)

After we know how to convert RLC components from time to phasor domain, we can transform a time domain circuit into a phasor/frequency domain circuit.

Hence, we can apply the KCL laws and other theorems to directly set up phasor equations involving our target variable(s) for solving.

9.6 Kirchhoff's Laws in the Frequency Domain (2)

- Both KVL and KCL are hold in the phasor domain or more commonly called frequency domain.
- Moreover, the variables to be handled are phasors, which are complex numbers.
- All the mathematical operations involved are now in complex domain.

9.7 Impedance Combinations (1)

- The following principles used for DC circuit analysis all apply to AC circuit.
- For example:
  a. voltage division
  b. current division
  c. circuit reduction
  d. impedance equivalence
  e. Y-Δ transformation

10.1 Basic Approach

10.2 Nodal Analysis

10.3 Mesh Analysis

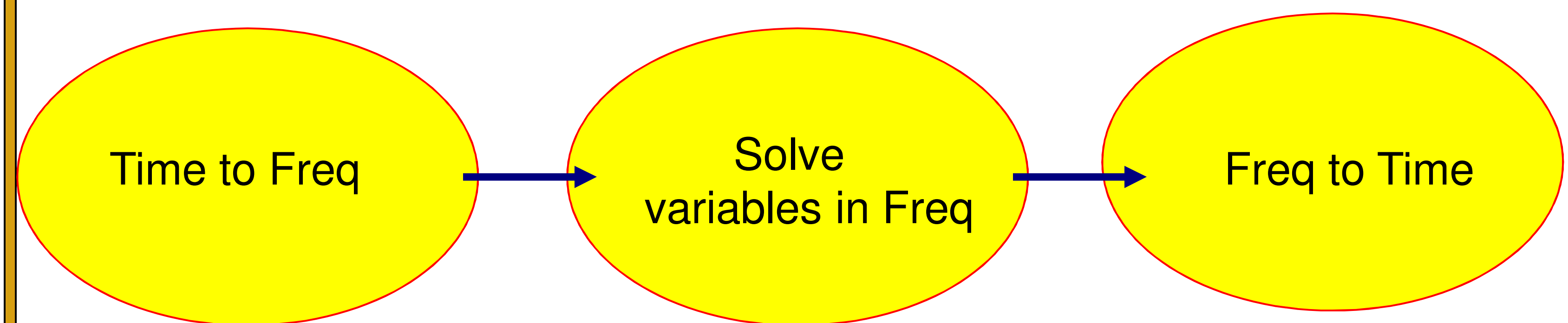
10.4 Superposition Theorem

10.5 Source Transformation

10.6 Thevenin and Norton Equivalent Circuits

Steps to Analyze AC Circuits:

1. Transform the circuit to the phasor or frequency domain.
2. Solve the problem using circuit techniques (nodal analysis, mesh analysis, superposition, etc.).
3. Transform the resulting phasor to the time domain.

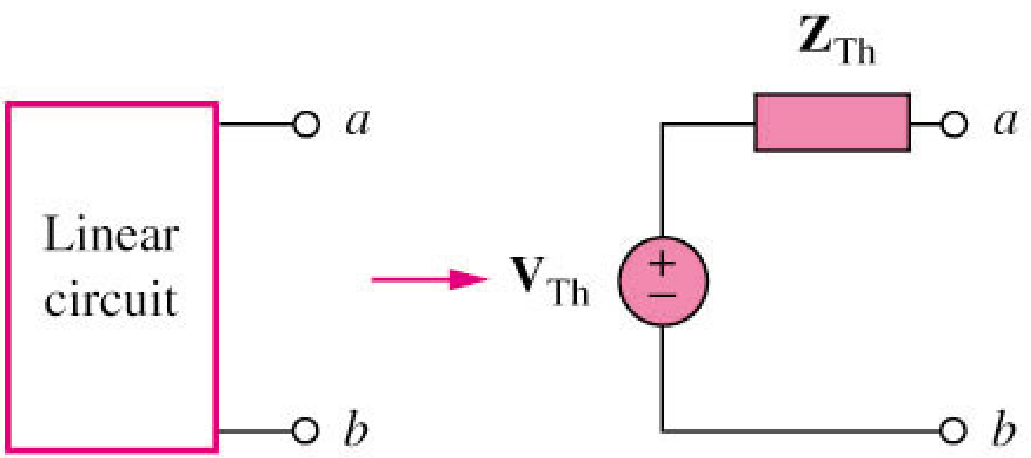


10.4 Superposition Theorem (1)

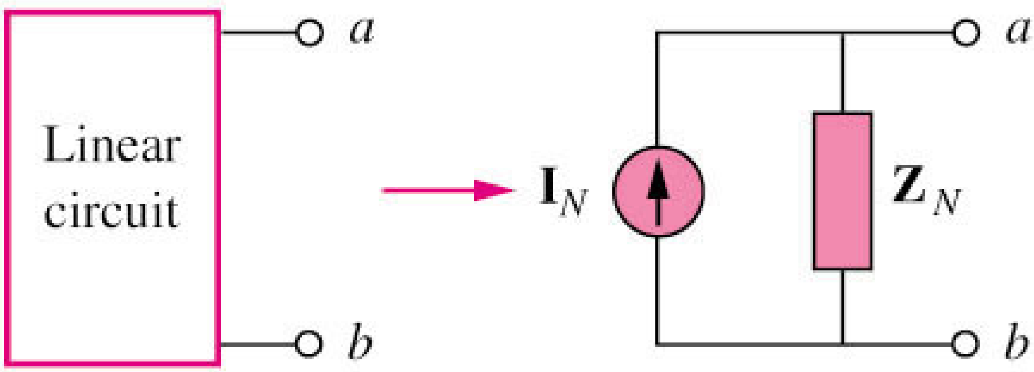
When a circuit has sources operating at different frequencies,

- The separate phasor circuit for each frequency must be solved independently, and
- The total response is the sum of time-domain responses of all the individual phasor circuits.

10.6 Thevenin and Norton Equivalent Circuits (1)



Thevenin transform



Norton transform

Chapter 10 Example (1)

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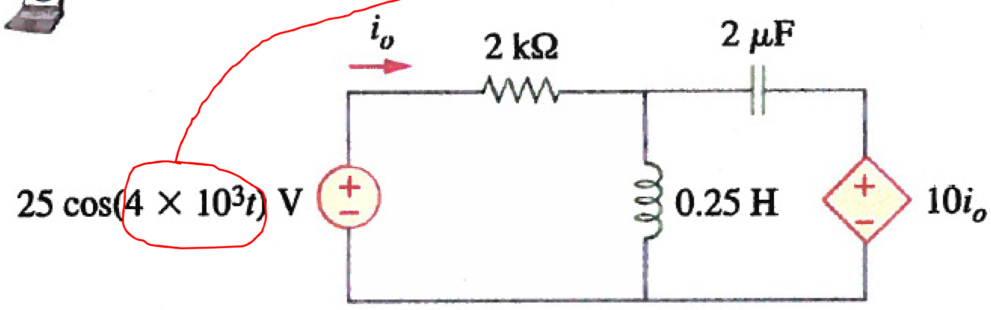
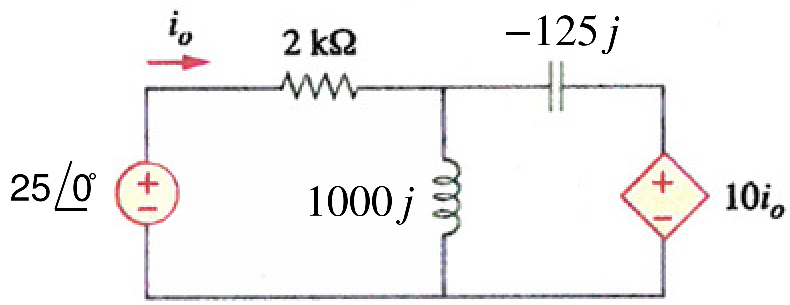
10.5 Find i_o in the circuit of Fig. 10.54.



$$\omega = 4000$$

$$Z_L = j\omega L = j4000(0.25) = 1000j$$

$$Z_C = \frac{1}{j\omega C} = \frac{1}{j4000(2\mu)} = -125j$$



Mesh 1

$$-25 + 2000i_1 + 1000j(i_1 - i_2) = 0$$

$$(2000 + 1000j)i_1 - (1000j)i_2 = 25$$

Mesh 2

$$1000j(i_2 - i_1) + (-125j)i_2 + 10i_1 = 0$$

$$(10 - 1000j)i_1 + (875j)i_2 = 0$$

$$\begin{bmatrix} 2000+1000j & -1000j \\ 10-1000j & 875j \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} 25 \\ 0 \end{bmatrix}$$

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11.2 Instantaneous and Average Power
11.3 Maximum Average Power Transfer
11.4 Effective or RMS Value
11.5 Apparent Power and Power Factor
11.6 Complex Power

11.2 Instantaneous and Average Power (1)

- The instantaneously power, p(t), absorbed by an element is the product of the instantaneous voltage v(t) across the element and the instantaneous current i(t) through it.

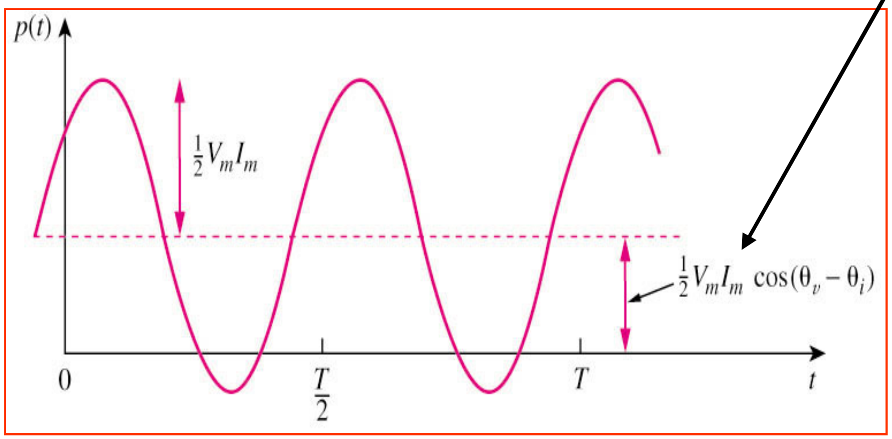
$$p(t) = v(t)\,i(t) = V_m I_m \cos(\omega t + \theta_v)\cos(\omega t + \theta_i)$$

$$= \underbrace{\frac{1}{2} V_m I_m \cos(\theta_v - \theta_i)}_{\text{Constant power}} + \underbrace{\frac{1}{2} V_m I_m \cos(2\omega t + \theta_v + \theta_i)}_{\text{Sinusoidal power at } 2\omega t}$$

11.2 Instantaneous and Average Power (2)

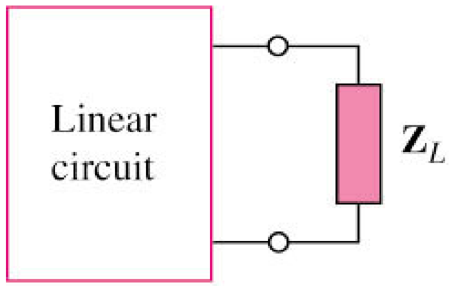
- The average power, P, *is the average of the instantaneous power over one period.*

$$P = \frac{1}{T}\int_0^T p(t)\,dt = \frac{1}{2}V_m\, I_m \cos(\theta_v - \theta_i)$$

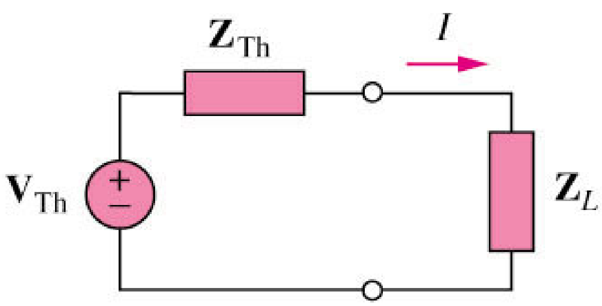


1. **P is not time dependent.**
2. **When $\theta_v = \theta_i$, it is a purely resistive load case.**
3. **When $\theta_v - \theta_i = \pm 90o$, it is a purely reactive load case.**
4. **P = 0 means that the circuit absorbs no average power.**

11.3 Maximum Average Power Transfer (1)



(a)



(b)

Maximum power is transferred to the load if the load impedance is the complex conjugate of the Thevenin impedance.

$$Z_{TH} = R_{TH} + jX_{TH}$$

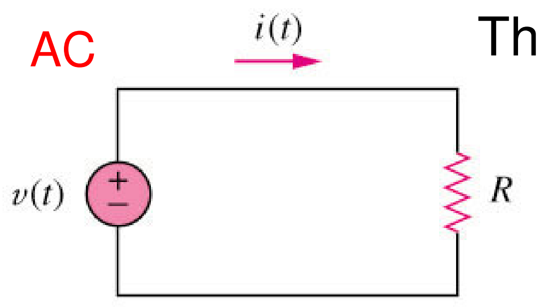
$$\boxed{Z_L = Z_{Th}^* = R_{TH} - jX_{TH}}$$

$$P_{max} = \frac{|V_{TH}|^2}{8R_{TH}}$$

When $Z_L = Z_{Th}^*$ we say the load is “matched” to the source.

11.4 Effective or RMS Value (1)

AC



(a)

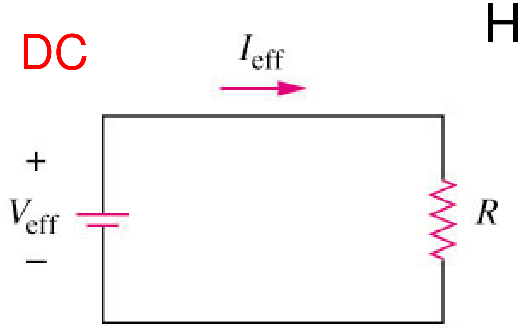
The total power dissipated by R is given by:

$$P = \frac{1}{T}\int_0^T i^2 R dt = \frac{R}{T}\int_0^T i^2 dt = I_{rms}^2 R$$

Hence, I_{eff} is equal to:

$$I_{eff} = \sqrt{\frac{1}{T}\int_0^T i^2 \, dt} = I_{rms}$$

DC



(b)

The rms value is a constant itself which depending on the shape of the function i(t).

The effective value of a periodic current is the dc current that delivers the same average power to a resistor as the periodic current.

11.5 Apparent Power and Power Factor (1)

- Apparent Power, S, is the product of the r.m.s. values of voltage and current.
- It is measured in volt-amperes or VA to distinguish it from the average or real power which is measured in watts.

$$P = \underbrace{V_{rms}\, I_{rms}}_{\text{Apparent Power, S}} \cos(\theta_v - \theta_i) = S \underbrace{\cos(\theta_v - \theta_i)}_{\text{Power Factor, pf}}$$

- Power factor is the cosine of the phase difference between the voltage and current. It is also the cosine of the angle of the load impedance.

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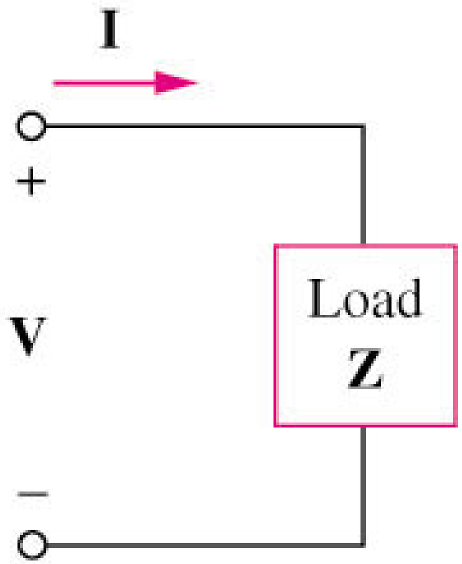
11.5 Apparent Power and Power Factor (2)

Load Type	Power Factor Angle	Power Factor
Purely resistive load (R)	$\theta_v - \theta_i = 0$, $Pf = 1$	P/S = 1, all power are consumed
Purely reactive load (L or C)	$\theta_v - \theta_i = \pm 90^o$, $pf = 0$	P = 0, no real power consumption
Resistive and reactive load (R and L/C)	$\theta_v - \theta_i > 0$ $\theta_v - \theta_i < 0$	• Lagging - inductive load • Leading - capacitive load

11.6 Complex Power (1)

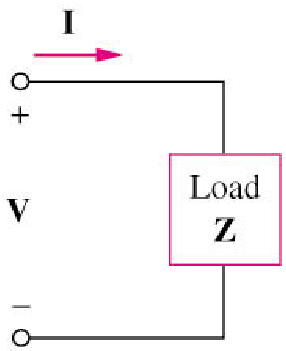
Complex power **S** is the product of the voltage and the complex conjugate of the current:



$$\mathbf{V} = V_m \angle \theta_v \qquad \mathbf{I} = I_m \angle \theta_i$$

$$\frac{1}{2} \mathbf{V}\, \mathbf{I}^* = V_{rms}\, I_{rms} \angle \theta_v - \theta_i$$

11.6 Complex Power (2)



$$S = \frac{1}{2} V I^* = V_{rms}\, I_{rms} \angle \theta_v - \theta_i$$

$$\Rightarrow S = \underbrace{V_{rms}\, I_{rms} \cos(\theta_v - \theta_i)}_{P} + j\, \underbrace{V_{rms}\, I_{rms} \sin(\theta_v - \theta_i)}_{Q}$$

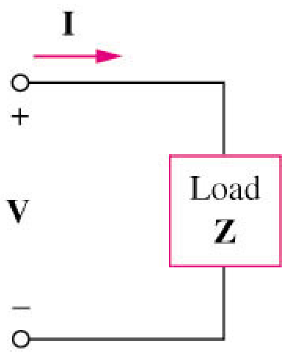
S **=** **P** **+ j** **Q**

P: is the average power in watts delivered to a load and it is the only useful power.

Q: is the reactive power exchange between the source and the reactive part of the load. It is measured in VAR.

- $Q = 0$ for *resistive loads* (unity pf).
- $Q < 0$ for *capacitive loads* (leading pf).
- $Q > 0$ for *inductive loads* (lagging pf).

11.6 Complex Power (3)



$$\Rightarrow S = \underbrace{V_{rms}\, I_{rms} \cos(\theta_v - \theta_i)}_{P} + j\, \underbrace{V_{rms}\, I_{rms} \sin(\theta_v - \theta_i)}_{Q}$$

$$\mathbf{S} = P + j\,Q$$

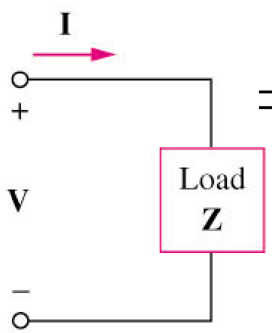
Apparent Power, $S = |\mathbf{S}| = V_{rms} * I_{rms} = \sqrt{P^2 + Q^2}$

Real power, $P = \mathrm{Re}(\mathbf{S}) = S\cos(\theta_v - \theta_i)$

Reactive Power, $Q = \mathrm{Im}(\mathbf{S}) = S\sin(\theta_v - \theta_i)$

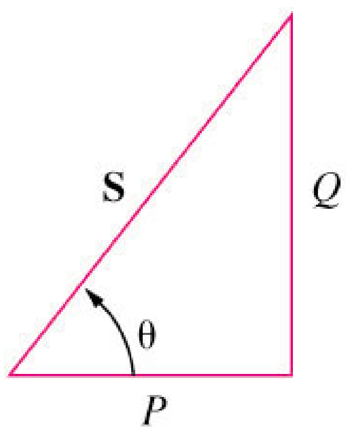
Power factor, $pf = P/S = \cos(\theta_v - \theta_i)$

11.6 Complex Power (4)

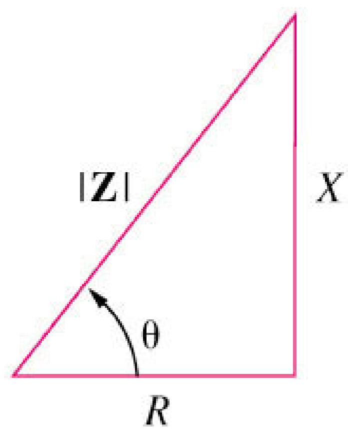


$$\Rightarrow S = \underbrace{V_{rms}\, I_{rms} \cos(\theta_v - \theta_i)}_{P} + j\, \underbrace{V_{rms}\, I_{rms} \sin(\theta_v - \theta_i)}_{Q}$$

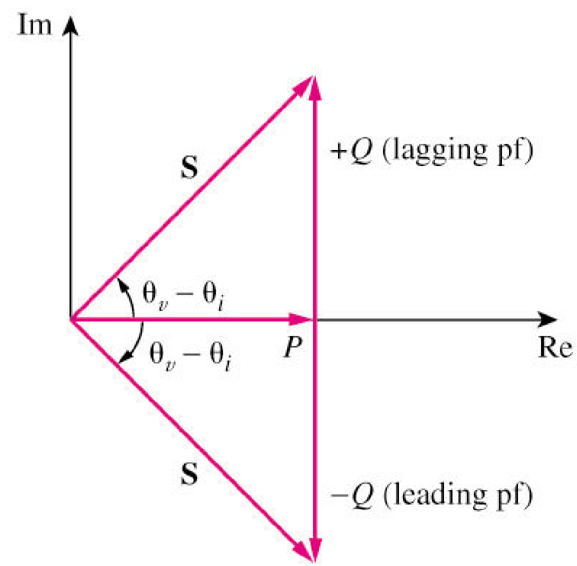
$$\mathbf{S} = \mathbf{P} + \mathbf{j}\ \mathbf{Q}$$



Power Triangle



Impedance Triangle



Power Factor

Homework #1
Due in class Wednesday, Jan 21

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- Problems:
  - 3.23 (can use Matlab or any other matrix solver to solve the set of linear equations)
  - 3.51
  - 4.41
  - 5.27
  - 7.39a
  - 7.53a
  - 8.49
  - 10.1

No Class on Monday the 19th !

ECE 202 Review (Ch 8 – 11)

- Part I DC Circuits
  - Chapter 8: 2nd Order Circuits
- Part II AC Circuits
  - Chapter 9: Sinusoids and Phasors
  - Chapter 10: Sinusoidal Steady-State Analysis
  - Chapter 11: AC Power Analysis

Second-Order Circuits

Chapter 8 - Review

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8.1 Examples of 2nd order RCL circuit

8.2 Finding Initial and Final Values (X)

8.3 The source-free series RLC circuit

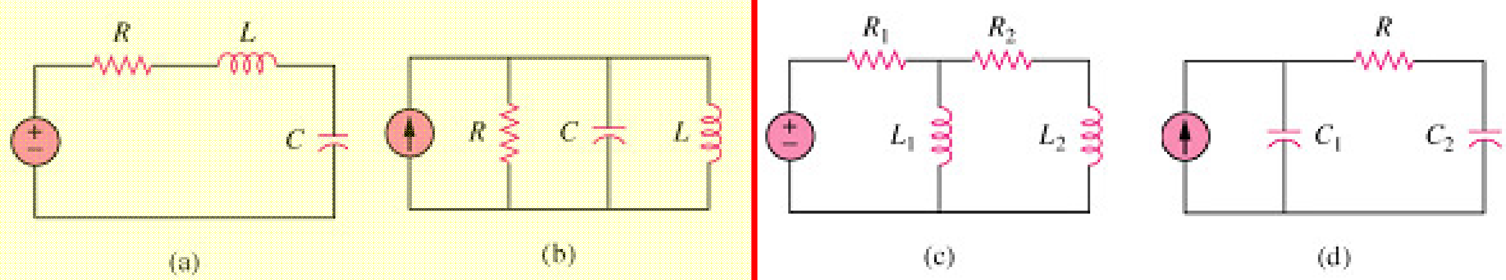
8.4 The source-free parallel RLC circuit

8.5 Step response of a series RLC circuit

8.6 Step response of a parallel RLC circuit

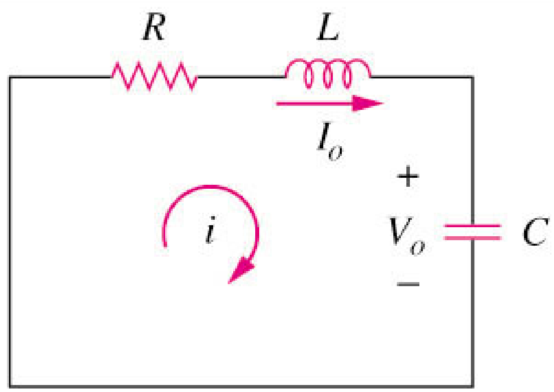
8.1 Examples of Second Order RLC circuits

A second-order circuit is characterized by a <u>second-order differential equation</u>. It consists of <u>resistors</u> and the equivalent of <u>two energy storage elements</u>.



RLC Series | RLC Parallel | RL T-config | RC Pi-config

8.3 Source-Free Series RLC Circuits (1)



- The solution of the source-free series RLC circuit is called as the natural response of the circuit.

The 2nd order of expression

Derived from KVL:

$$iR + L\frac{di}{dt} + \frac{1}{C}\int_{-\infty}^{t} i(\tau)d\tau = 0$$

$$R\frac{di}{dt} + L\frac{d^2 i}{dt^2} + \frac{1}{C}i = 0$$

$$\frac{d^2 i}{dt^2} + \frac{R}{L}\frac{di}{dt} + \frac{i}{LC} = 0$$

Note: Same form if looking at voltage across capacitor!

To find for voltage across capacitor

Substitute: $i = C\frac{dv}{dt}$

$$RC\frac{dv}{dt} + LC\frac{d^2 v}{dt^2} + v = 0$$

$$\frac{d^2 v}{dt^2} + \frac{R}{L}\frac{dv}{dt} + \frac{v}{LC} = 0$$

Where v is the voltage across the capacitor

8.3 Source-Free Series RLC Circuits (2)

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Three possible solutions for the 2nd order differential equation:

$$\frac{d^2 i}{dt^2}+\frac{R}{L}\frac{di}{dt}+\frac{i}{LC}=0$$

=> $$\frac{d^2 i}{dt^2}+2\alpha\frac{di}{dt}+\omega_0^2 i=0$$

where $\alpha=\frac{R}{2L}$ *and* $\omega_0=\sqrt{\frac{1}{LC}}$

General 2nd order Form

The types of solutions for i(t) depend on the relative values of α and ω.

General Solution

$$i(t)=A_1e^{s_1 t}+A_2e^{s_2 t}$$

$$s_{1,2}=-\alpha\pm\sqrt{\alpha^2-\omega_0^2}$$

- If $\alpha > \omega_o$ → over-damped
- If $\alpha = \omega_o$ → critically-damped
- If $\alpha < \omega_o$ → under-damped

8.3 Source-Free Series RLC Circuits (3)

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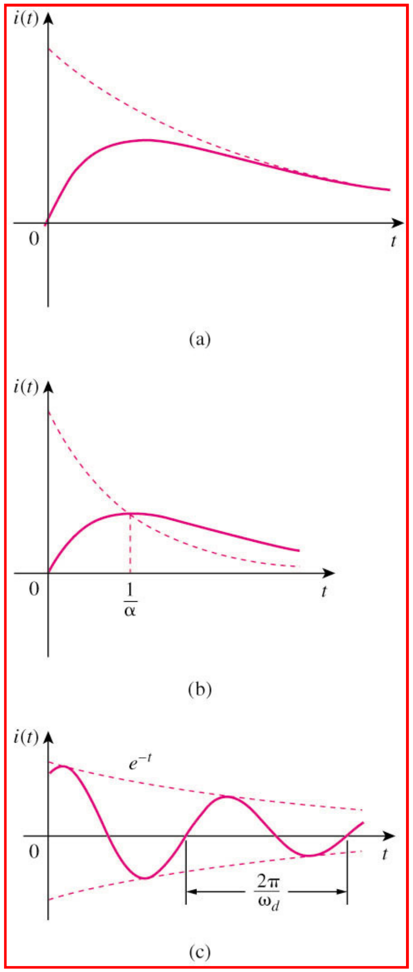
There are three possible solutions for the following 2nd order differential equation:

$$\frac{d^2 i}{dt^2} + 2\alpha \frac{di}{dt} + \omega_0^2 i = 0 \qquad \alpha = \frac{R}{2L} \quad and \quad \omega_0 = \sqrt{\frac{1}{LC}}$$



1. If $\alpha > \omega_o$, over-damped case

$$i(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t} \quad \textbf{where} \quad s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2}$$

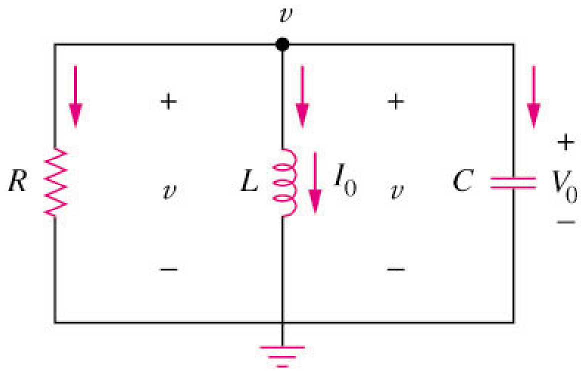
2. If $\alpha = \omega_o$, critical damped case

$$i(t) = (A_2 + A_1 t)e^{-\alpha t} \quad \textbf{where} \quad s_{1,2} = -\alpha$$

3. If $\alpha < \omega_o$, under-damped case

$$i(t) = e^{-\alpha t}(B_1 \cos \omega_d t + B_2 \sin \omega_d t) \quad \textbf{where} \quad \omega_d = \sqrt{\omega_0^2 - \alpha^2}$$

8.4 Source-Free Parallel RLC Circuits (1)



- Similar approach to series RLC analysis

Derived from KCL:

$$\frac{v}{R}+C\frac{dv}{dt}+\frac{1}{L}\int_{-\infty}^{t}v(\tau)d\tau=0$$

$$\frac{1}{R}\frac{dv}{dt}+C\frac{d^2v}{dt^2}+\frac{1}{L}v=0$$

The 2nd order of expression

$$\frac{d^2v}{dt^2}+\frac{1}{RC}\frac{dv}{dt}+\frac{1}{LC}v=0$$

To find for current across inductor

Substitute: $v=L\frac{di}{dt}$

$$\frac{L}{R}\frac{di}{dt}+LC\frac{d^2i}{dt^2}+i=0$$

Note: Same form if looking at current through inductor!

$$\frac{d^2i}{dt^2}+\frac{1}{RC}\frac{di}{dt}+\frac{1}{LC}i=0$$

Where i is the current through the inductor

8.4 Source-Free Parallel RLC Circuits (2)

There are three possible solutions for the following 2nd order differential equation:

$$\frac{d^2v}{dt^2} + 2\alpha\frac{dv}{dt} + \omega_0^2 v = 0 \quad \text{where} \quad \alpha = \frac{1}{2RC} \quad \text{and} \quad \omega_0 = \sqrt{\frac{1}{LC}}$$

1. If $\alpha > \omega_o$, over-damped case

$$v(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t} \quad \textbf{where} \quad s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2}$$

2. If $\alpha = \omega_o$, critical damped case

$$v(t) = (A_2 + A_1 t)\, e^{-\alpha t} \quad \textbf{where} \quad s_{1,2} = -\alpha$$

3. If $\alpha < \omega_o$, under-damped case

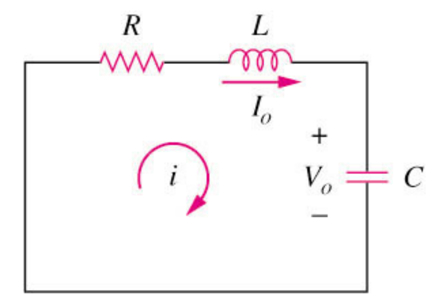
$$v(t) = e^{-\alpha t}(B_1 \cos \omega_d t + B_2 \sin \omega_d t) \quad \textbf{where} \quad \omega_d = \sqrt{\omega_0^2 - \alpha^2}$$

Comparison Series / Parallel RLC Circuits

	Series RLC	Parallel RLC
Characteristic Equation	$\frac{d^2i}{dt^2} + 2\alpha\frac{di}{dt} + \omega_0^2 i = 0$	$\frac{d^2v}{dt^2} + 2\alpha\frac{dv}{dt} + \omega_0^2 v = 0$
Damping Factor	$\alpha = \frac{R}{2L}$	$\alpha = \frac{1}{2RC}$
Resonance Frequency	$\omega_0 = \sqrt{\frac{1}{LC}}$	$\omega_0 = \sqrt{\frac{1}{LC}}$
Over-damped $\alpha > \omega_o$	$i(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t}$ $s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_0^{\ 2}}$	$v(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t}$ $s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_0^{\ 2}}$
Critical damped $\alpha = \omega_o$,	$i(t) = (A_2 + A_1 t)\, e^{-\alpha t}$	$v(t) = (A_2 + A_1 t)\, e^{-\alpha t}$
Under-damped $\alpha < \omega_o$	$i(t) = e^{-\alpha t}(B_1 \cos \omega_d t + B_2 \sin \omega_d t)$ $\omega_d = \sqrt{\omega_0^2 - \alpha^2}$	$v(t) = e^{-\alpha t}(B_1 \cos \omega_d t + B_2 \sin \omega_d t)$ $\omega_d = \sqrt{\omega_0^2 - \alpha^2}$

Source-Free RLC Circuits

- Key to solving problems is to finding the initial conditions
  - Voltage doesn't change rapidly across a capacitor: $v(0^+) = v(0^-)$
  - Current doesn't change rapidly across an inductor: $i(0^+) = i(0^-)$
- Start by finding α and ω_o use these to find s_1 and s_2
  - Determine case (over / under / or critically damped)
- Apply initial conditions to the equations to find A_1 and A_1

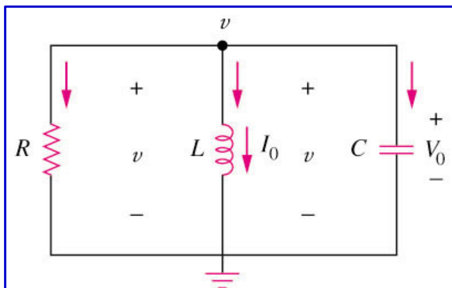


Series RLC

$$i(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t} \Rightarrow i(0^+) = A_1 + A_2$$

$$\frac{di(t)}{dt} = A_1 s_1 e^{s_1 t} + A_2 s_2 e^{s_2 t} \Rightarrow \frac{di(0^+)}{dt} = A_1 s_1 + A_2 s_2$$

$$\frac{di(0)}{dt} = -\frac{1}{L}(RI_0 + V_0)$$



Parallel RLC

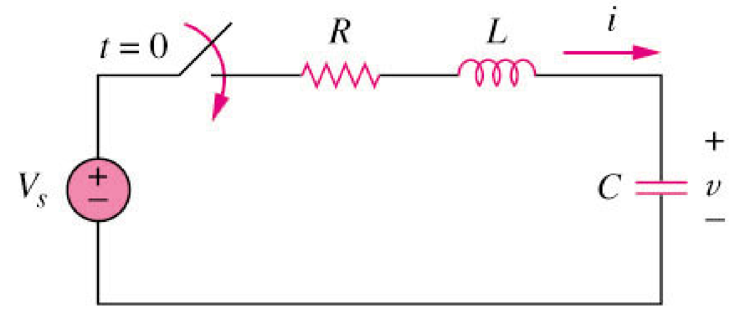
$$v(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t} \Rightarrow v(0^+) = A_1 + A_2$$

$$\frac{dv(t)}{dt} = A_1 s_1 e^{s_1 t} + A_2 s_2 e^{s_2 t} \Rightarrow \frac{dv(0^+)}{dt} = A_1 s_1 + A_2 s_2$$

$$\frac{dv(0)}{dt} = -\frac{1}{RC}(RI_0 + V_0)$$

8.5 Step-Response Series RLC Circuits (1)

- The step response is obtained by the sudden application of a dc source.



The 2nd order of expression

$$\underbrace{\frac{d^2v}{dt^2}+\frac{R}{L}\frac{dv}{dt}+\frac{v}{LC}}_{\textit{Natural Response}}=\frac{V_s}{LC}$$

The above equation has the same form as the equation for source-free series RLC circuit.

- The same coefficients (important in determining the frequency parameters).

8.5 Step-Response Series RLC Circuits (2)

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The solution of the equation should have two components:
the transient response $v_t(t)$ & the steady-state response $v_{ss}(t)$:

$$v(t) = v_t(t) + v_{ss}(t)$$

This is the Voltage across the Capacitor !!

- The transient response v_t is the same as that for source-free case

$$v_t(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t} \quad \text{(over-damped)}$$

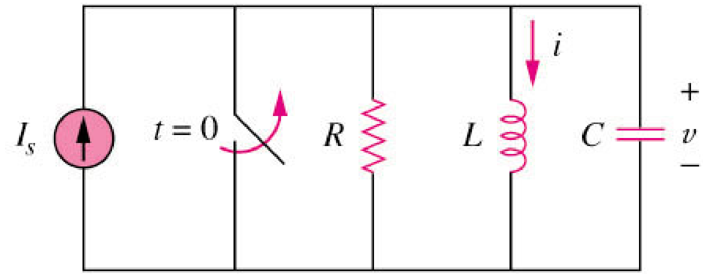
$$v_t(t) = (A_1 + A_2 t)e^{-\alpha t} \quad \text{(critically damped)}$$

$$v_t(t) = e^{-\alpha t}(A_1 \cos \omega_d t + A_2 \sin \omega_d t) \quad \text{(under-damped)}$$

- The steady-state response is the final value of v(t).
  - $v_{ss}(t) = v(\infty)$
- The values of A_1 and A_2 are obtained from the initial conditions:
  - v(0) and dv(0)/dt.

8.6 Step-Response Parallel RLC Circuits (1)

- The step response is obtained by the sudden application of a dc source.



The 2nd order of expression

$$\frac{d^2 i}{dt^2} + \frac{1}{RC}\frac{di}{dt} + \frac{i}{LC} = \frac{I_s}{LC}$$

It has the same form as the equation for source-free parallel RLC circuit.

- The same coefficients (important in determining the frequency parameters).

8.6 Step-Response Parallel RLC Circuits (2)

The solution of the equation should have two components: the transient response $i_t(t)$ & the steady-state response $i_{ss}(t)$:

$$i(t) = i_t(t) + i_{ss}(t)$$

This is the Current through the Inductor !!

- The transient response i_t is the same as that for source-free case

$$i_t(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t}$$ (over-damped)

$$i_t(t) = (A_1 + A_2 t)e^{-\alpha t}$$ (critical damped)

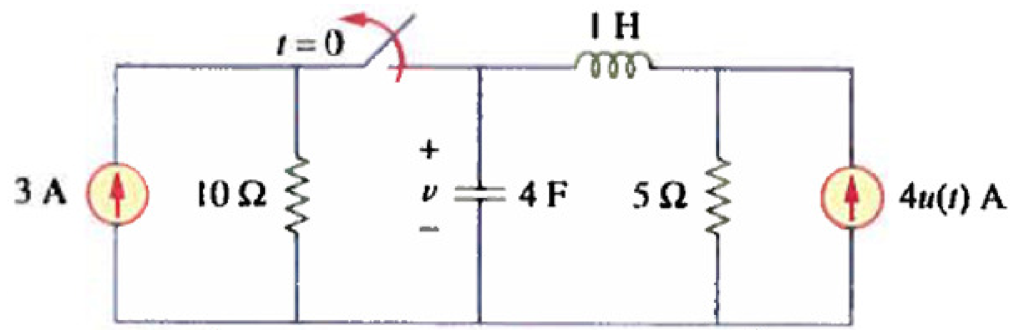
$$i_t(t) = e^{-\alpha t}(A_1 \cos \omega_d t + A_2 \sin \omega_d t)$$ (under-damped)

- The steady-state response is the final value of i(t).
  - $i_{ss}(t) = i(\infty) = I_s$
- The values of A_1 and A_2 are obtained from the initial conditions:
  - i(0) and di(0)/dt.

Chapter 8 Example (1)

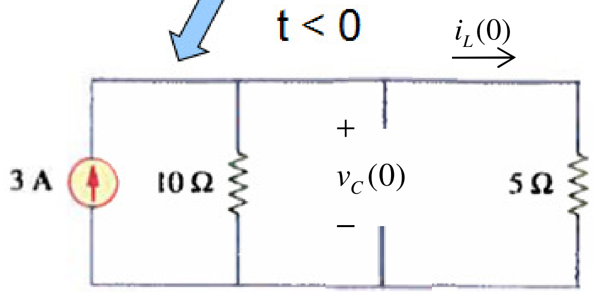
8.33 Find $v(t)$ for $t > 0$ in the circuit of Fig. 8.81.

Remember !!
Capacitor is "Open" to DC
Inductor is "Short" to DC



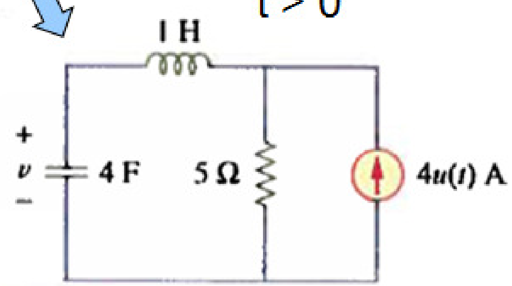
Before switch is opened

t < 0

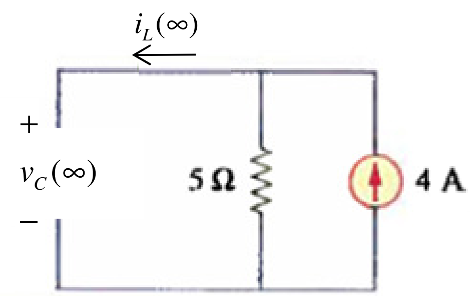


After switch is opened

t > 0



Steady State circuit (t → ∞)



Initial Voltage across Capacitor

$$v_C(0) = \frac{5 \cdot 10}{5+10}(3) = 10$$

Initial Current through Inductor

$$i_L(0) = 2$$

$$\alpha = \frac{R}{2L} = \frac{5}{(2)(1)} = 2.5$$

$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{(1)(4)}} = 0.5$$

$\alpha > \omega_0$ Over-damped Case

SS Voltage across Capacitor

$$v_C(\infty) = (4)(5) = 20$$

SS Current through Inductor

$$i_L(\infty) = 0$$

Chapter 8 Example 8.33 (continued)

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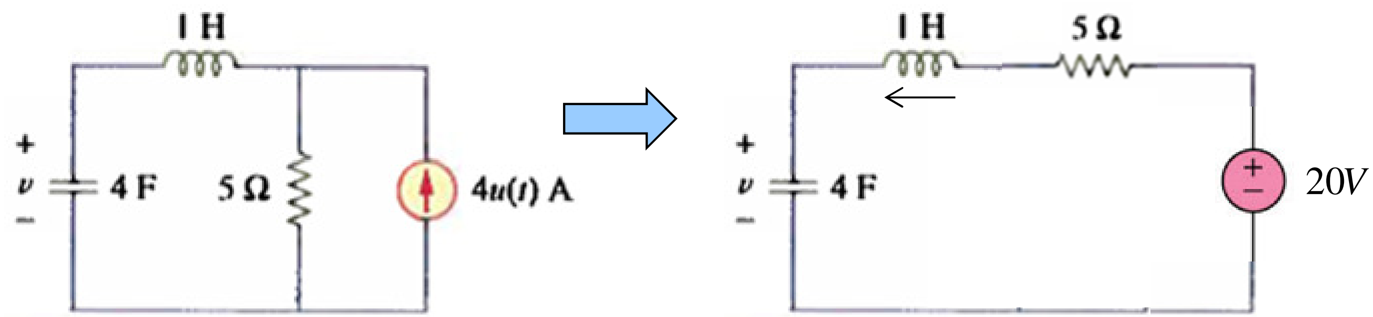
Circuit to be analyzed for response



Step Response of Series RLC Circuit

$$v(t) = v_t(t) + v_{ss}(t)$$

Voltage across Capacitor

$$v_t(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t}$$

(over-damped)

Known Parameters

$v_C(0) = 10$ $\quad v_C(\infty) = 20$ $\quad \alpha = 2.5$

$i_L(0) = 2$ $\quad i_L(\infty) = 0$ $\quad \omega_0 = 0.5$

$\Rightarrow$

$s_1 = 2.5 + \sqrt{(2.5)^2 - (0.5)^2} = 4.949$

$s_2 = 2.5 - \sqrt{(2.5)^2 - (0.5)^2} = 0.0505$

Voltage across Capacitor

$$v(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t} + 20$$

$$v(0) = A_1 + A_2 + 20 = 10$$

$$\boxed{A_1 + A_2 = -10}$$

Find Current through Inductor by taking derivative

$$i(0) = C\frac{dv(0)}{dt} \Rightarrow \frac{dv(0)}{dt} = \frac{i(0)}{C} = \frac{-2}{4}$$

Negative because in opposite direction

$$\frac{dv(0)}{dt} = \boxed{A_1 s_1 + A_2 s_2 = -0.5}$$

From here can solve for A1 and A2

Sinusoids and Phasors Chapter 9 - Review

9.2 Sinusoids

9.3 Phasors

9.4 Phasor Relationships for Circuit Elements

9.5 Impedance and Admittance

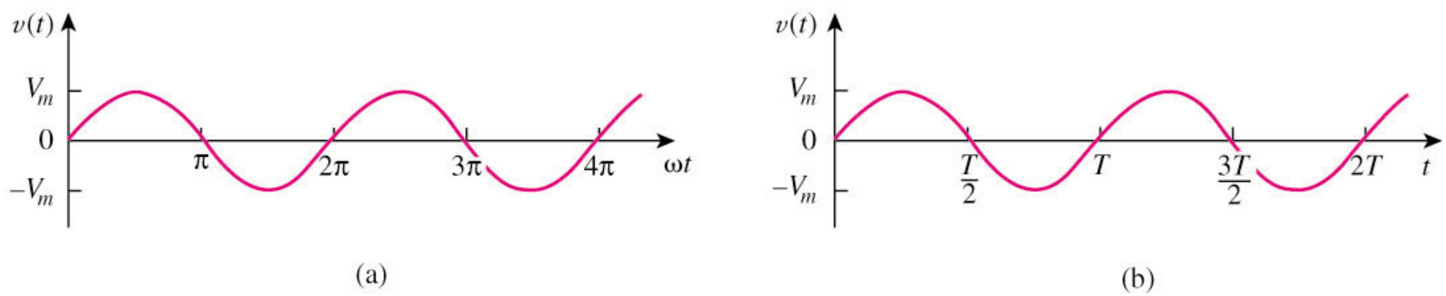
9.6 Kirchhoff's Laws in the Frequency Domain

9.7 Impedance Combinations

9.2 Sinusoids (1)

- A sinusoid is a signal that has the form of the sine or cosine function.
- A general expression for the sinusoid,

$$v(t) = V_m \sin(\omega t + \phi)$$



where

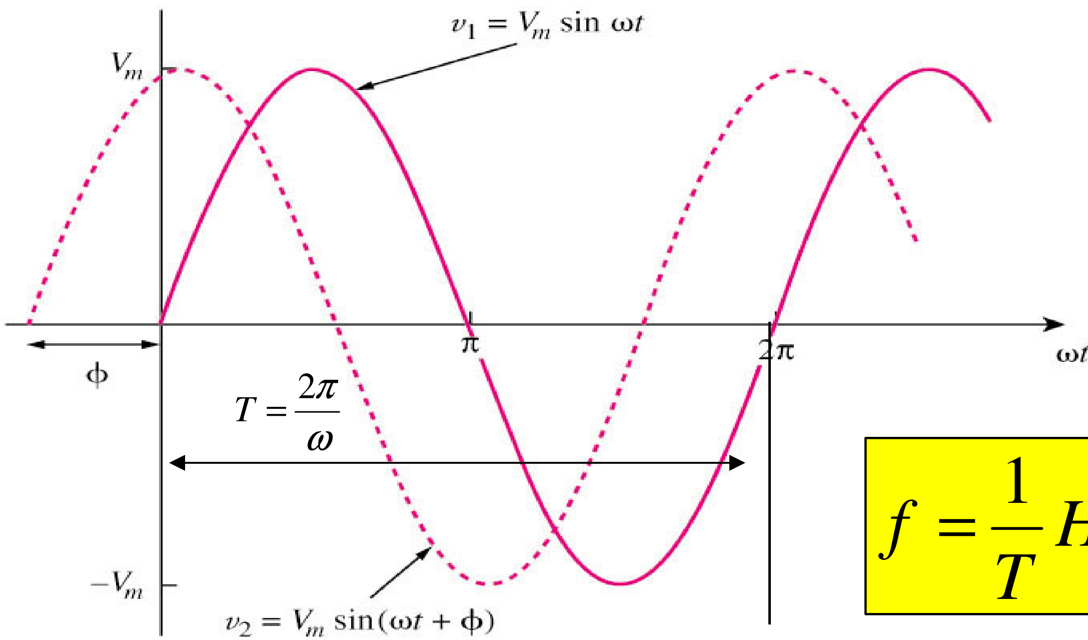
Vm = the **amplitude** of the sinusoid

ω = the angular frequency in radians/s

Φ = the phase

9.2 Sinusoids (2)

A periodic function is one that satisfies *v(t) = v(t + nT)*, for all t and for all integers n.



$$f = \frac{1}{T} Hz$$

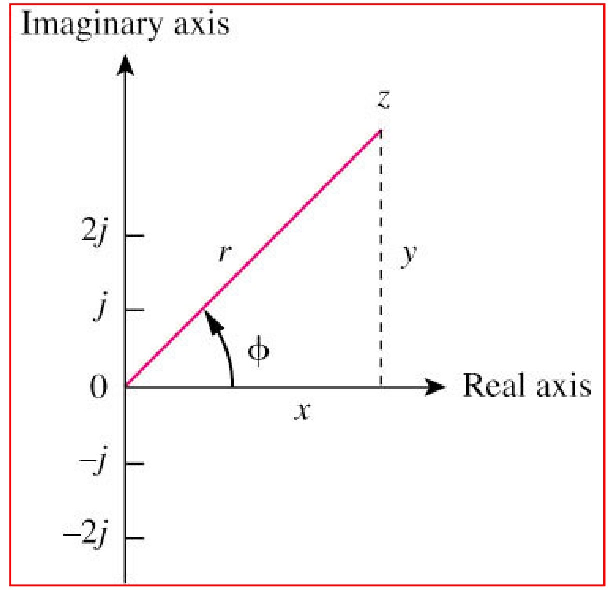
$$\omega = 2\pi f$$

- Only two sinusoidal values with the same frequency can be compared by their amplitude and phase difference.
- If phase difference is zero, they are in phase; if phase difference is not zero, they are out of phase.

9.3 Phasors (1)

- A phasor is a complex number that represents the amplitude and phase of a sinusoid.

- It can be represented in one of the following three forms:



a. Rectangular $z = x + jy = r(\cos\phi + j\sin\phi)$

b. Polar $z = r\angle\phi$

c. Exponential $z = re^{j\phi}$

where
$$r = \sqrt{x^2 + y^2}$$
$$\phi = \tan^{-1}\frac{y}{x}$$

Mathematic operation of complex number:

1. Addition $z_1 + z_2 = (x_1 + x_2) + j(y_1 + y_2)$
2. Subtraction $z_1 - z_2 = (x_1 - x_2) + j(y_1 - y_2)$
3. Multiplication $z_1 z_2 = r_1 r_2 \angle \phi_1 + \phi_2$
4. Division $\frac{z_1}{z_2} = \frac{r_1}{r_2} \angle \phi_1 - \phi_2$
5. Reciprocal $\frac{1}{z} = \frac{1}{r} \angle -\phi$
6. Square root $\sqrt{z} = \sqrt{r} \angle \phi/2$
7. Complex conjugate $z^* = x - jy = r \angle -\phi = re^{-j\phi}$
8. Euler's identity $e^{\pm j\phi} = \cos\phi \pm j\sin\phi$

9.3 Phasors (3)

- Transform a sinusoid to and from the time domain to the phasor domain:

$$v(t) = V_m \cos(\omega t + \phi) \longleftrightarrow V = V_m \angle\phi$$

(time domain) (phasor domain)

The differences between v(t) and V:

- v(t) is instantaneous or time-domain representation
  V is the frequency or phasor-domain representation.
- v(t) is time dependent, V is not.
- v(t) is always real with no complex term, V is generally complex.

Note: Phasor analysis applies only when frequency is constant; it is applied to two or more sinusoid signals only if they have the same frequency.

Relationship between differential, integral operation in phasor listed as follow:

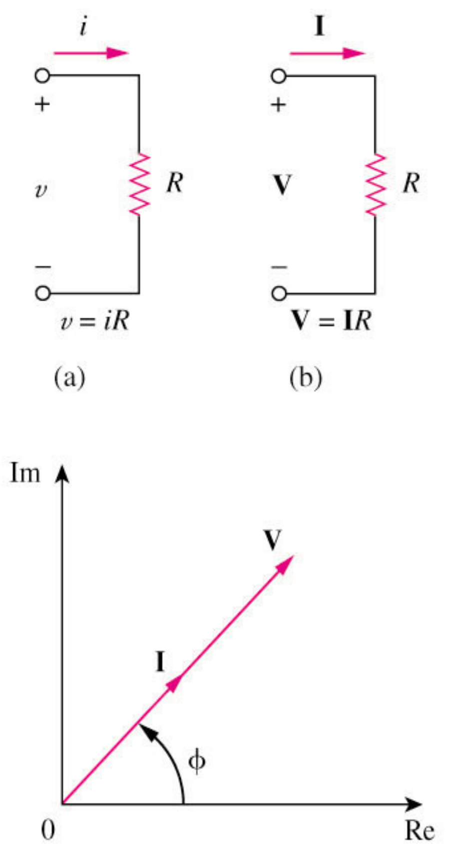
$$v(t) \longleftrightarrow \mathbf{V} = V\angle\phi$$

$$\frac{dv}{dt} \longleftrightarrow j\omega\mathbf{V}$$

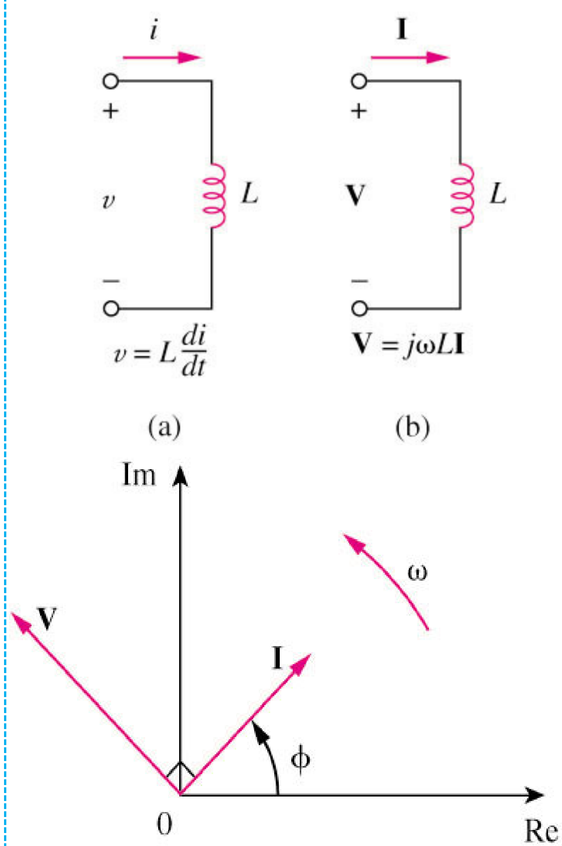
$$\int v\,dt \longleftrightarrow \frac{\mathbf{V}}{j\omega}$$

9.4 Phasor Relationships for Circuit Elements (1)

Resistors



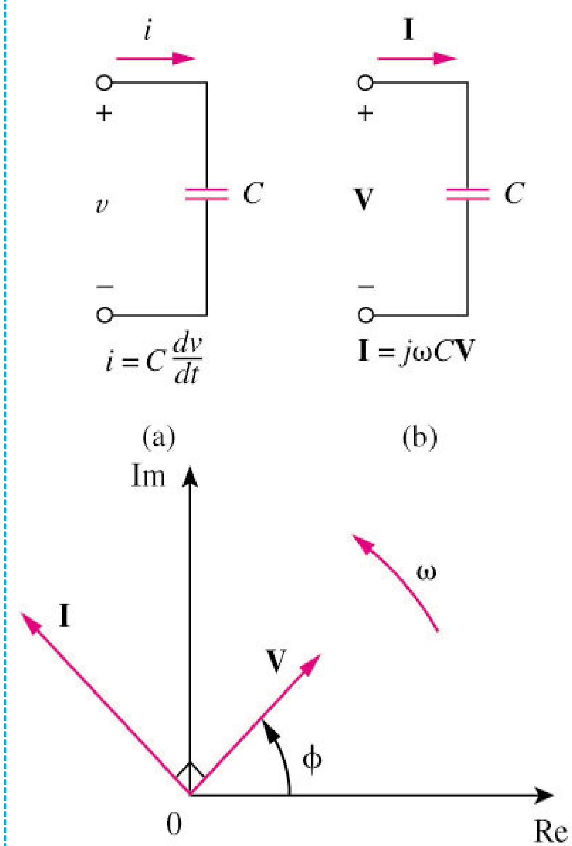
Inductors



"ELI" *the*

Voltage Lead Current
For Inductors "L" (ELI)

Capacitors



"ICE" *man*

Current Lead Voltage
For Capacitors "C" (ICE)

Aside: ELI the ICE man & the derivative

- The phrase "ELI the ICE man" can be used to remember the relationships of voltage & current for inductors & capacitors

E L I

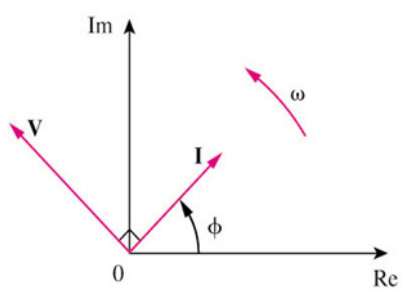
↓ ↓ ↓

$$v = L\frac{di}{dt}$$

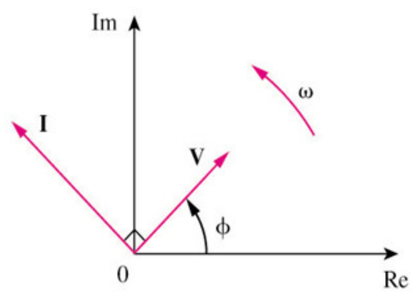
I C E

↓ ↓ ↓

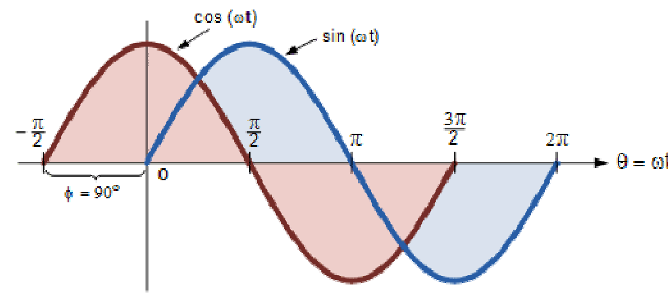
$$i = C\frac{dv}{dt}$$



Voltage leads current



Current leads voltage



Note:

1. Derivative of sin(t) is cos(t).
2. Observe the instantaneous "slope" of sin(t). It's cos(t)!
3. Derivative introduces a 90° shift
4. Now look at equations for L and C and this explains the 90° shift
5. Also, should be easier now to remember:

$$\frac{d}{dt}[\sin(\omega t)] = \omega\cos(\omega t)$$

$$\frac{d}{dt}[\cos(\omega t)] = -\omega\sin(\omega t)$$

9.4 Phasor Relationships for Circuit Elements (2)

Summary of voltage-current relationship		
Element	Time domain	Frequency domain
R	$v = Ri$	$V = RI$
L	$v = L\frac{di}{dt}$	$V = j\omega LI$
C	$i = C\frac{dv}{dt}$	$V = \frac{I}{j\omega C}$

9.5 Impedance and Admittance (1)

- The impedance Z of a circuit is the ratio of the phasor voltage V to the phasor current I, measured in ohms Ω.

$$Z = \frac{V}{I} = R + jX$$

Positive X is for L and negative X is for C.

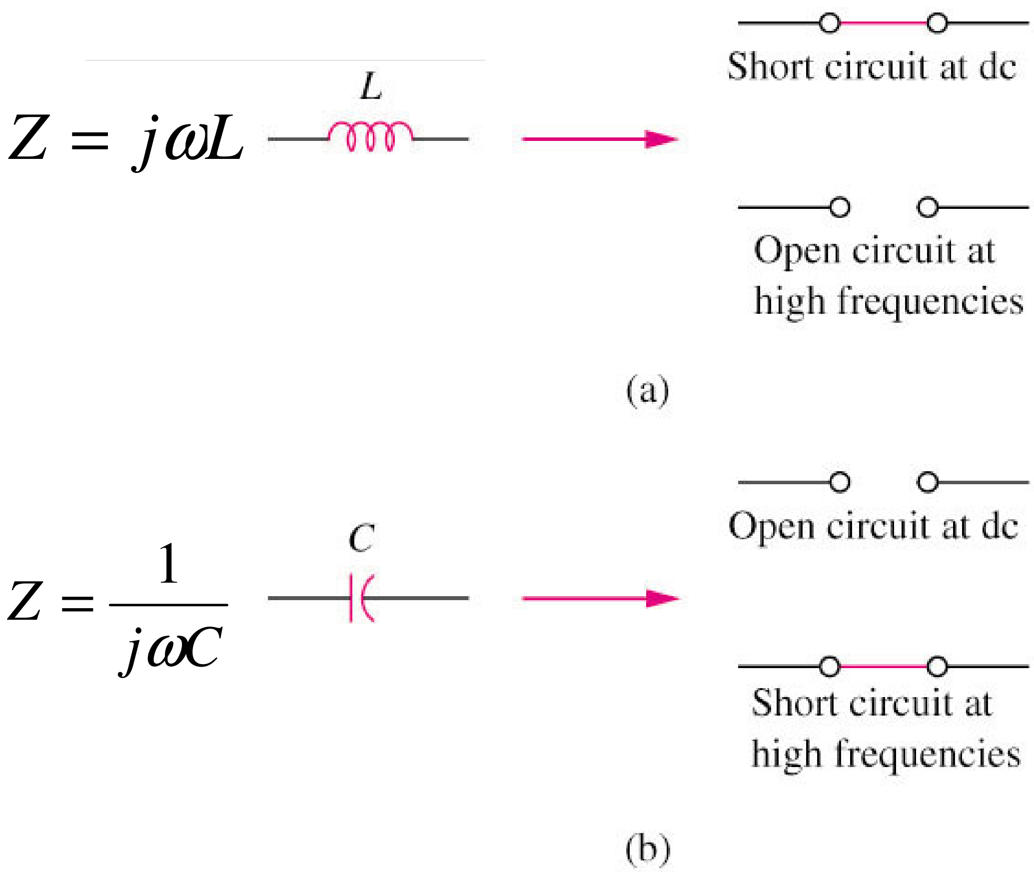
- The admittance Y is the reciprocal of impedance, measured in siemens (S).

$$Y = \frac{1}{Z} = \frac{I}{V}$$

9.5 Impedance and Admittance (2)

Impedances and admittances of passive elements		
Element	**Impedance**	**Admittance**
R	$Z = R$	$Y = \dfrac{1}{R}$
L	$Z = j\omega L$	$Y = \dfrac{1}{j\omega L}$
C	$Z = \dfrac{1}{j\omega C}$	$Y = j\omega C$

9.5 Impedance and Admittance (3)



$$\omega = 0; Z = 0$$

$$\omega \to \infty; Z \to \infty$$

$$\omega = 0; Z \to \infty$$

$$\omega \to \infty; Z = 0$$

9.6 Kirchhoff's Laws in the Frequency Domain (1)

After we know how to convert RLC components from time to phasor domain, we can transform a time domain circuit into a phasor/frequency domain circuit.

Hence, we can apply the KCL laws and other theorems to directly set up phasor equations involving our target variable(s) for solving.

9.6 Kirchhoff's Laws in the Frequency Domain (2)

- Both KVL and KCL are hold in the phasor domain or more commonly called frequency domain.
- Moreover, the variables to be handled are phasors, which are complex numbers.
- All the mathematical operations involved are now in complex domain.

9.7 Impedance Combinations (1)

- The following principles used for DC circuit analysis all apply to AC circuit.

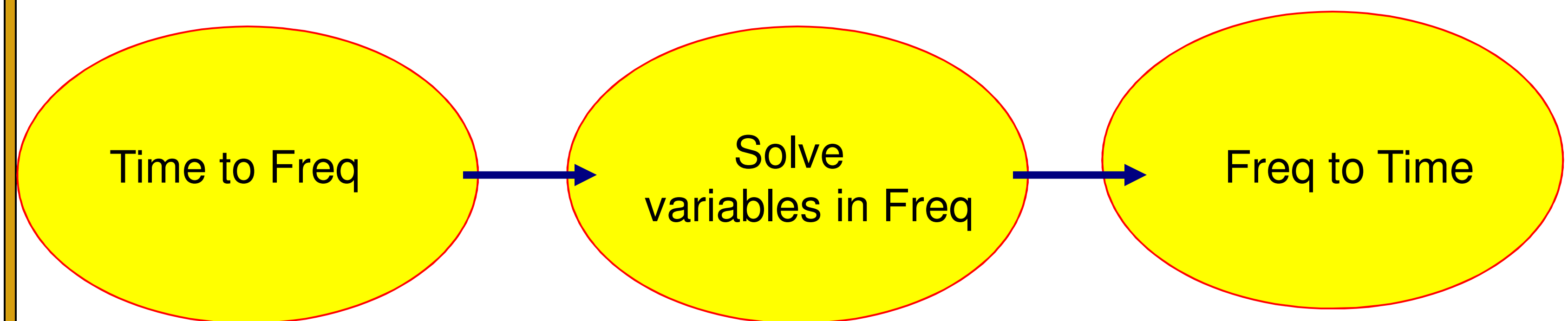
- For example:
  a. voltage division
  b. current division
  c. circuit reduction
  d. impedance equivalence
  e. Y-Δ transformation

Sinusoidal Steady-State Analysis
Chapter 10 Review

10.1 Basic Approach
10.2 Nodal Analysis
10.3 Mesh Analysis
10.4 Superposition Theorem
10.5 Source Transformation
10.6 Thevenin and Norton Equivalent Circuits

Steps to Analyze AC Circuits:

1. Transform the circuit to the phasor or frequency domain.
2. Solve the problem using circuit techniques (nodal analysis, mesh analysis, superposition, etc.).
3. Transform the resulting phasor to the time domain.



10.4 Superposition Theorem (1)

When a circuit has sources operating at different frequencies,

- The separate phasor circuit for each frequency must be solved independently, and
- The total response is the sum of time-domain responses of all the individual phasor circuits.

10.6 Thevenin and Norton Equivalent Circuits (1)

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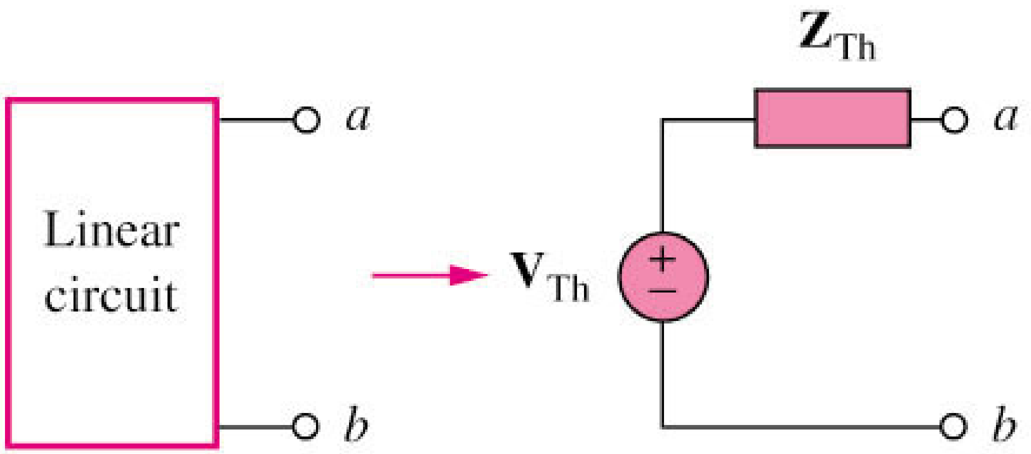
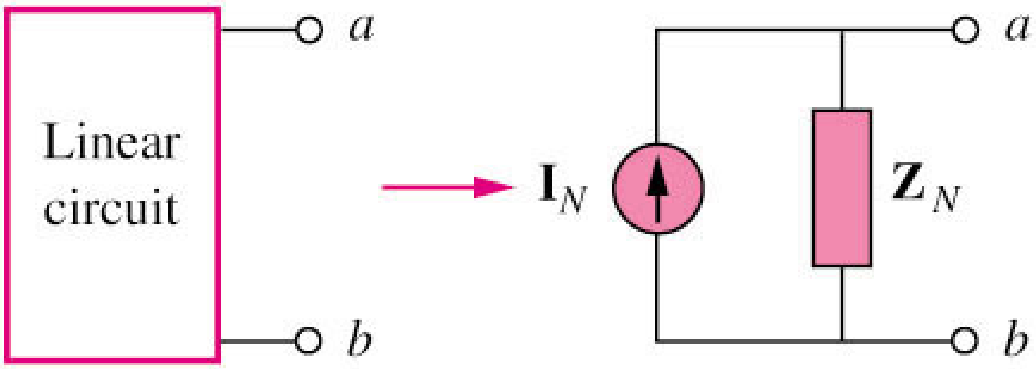


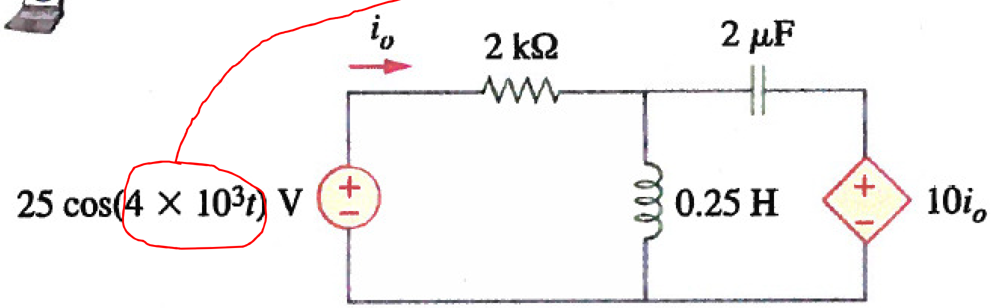
Thevenin transform



Norton transform

Chapter 10 Example (1)

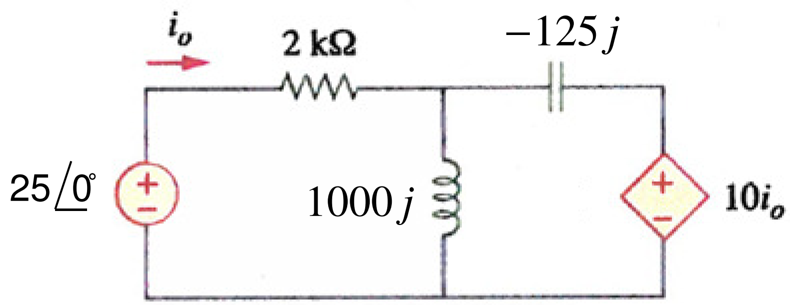
10.5 Find i_o in the circuit of Fig. 10.54.



$$\omega = 4000$$

$$Z_L = j\omega L = j4000(0.25) = 1000j$$

$$Z_C = \frac{1}{j\omega C} = \frac{1}{j4000(2\mu)} = -125j$$



Mesh 1

$$-25 + 2000i_1 + 1000j(i_1 - i_2) = 0$$

$$(2000 + 1000j)i_1 - (1000j)i_2 = 25$$

Mesh 2

$$1000j(i_2 - i_1) + (-125j)i_2 + 10i_1 = 0$$

$$(10 - 1000j)i_1 + (875j)i_2 = 0$$

$$\begin{bmatrix} 2000+1000j & -1000j \\ 10-1000j & 875j \end{bmatrix}\begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} 25 \\ 0 \end{bmatrix}$$

AC Power Analysis
Chapter 11 Review

11.2 Instantaneous and Average Power
11.3 Maximum Average Power Transfer
11.4 Effective or RMS Value
11.5 Apparent Power and Power Factor
11.6 Complex Power

11.2 Instantaneous and Average Power (1)

- The instantaneously power, p(t), absorbed by an element is the product of the instantaneous voltage v(t) across the element and the instantaneous current i(t) through it.

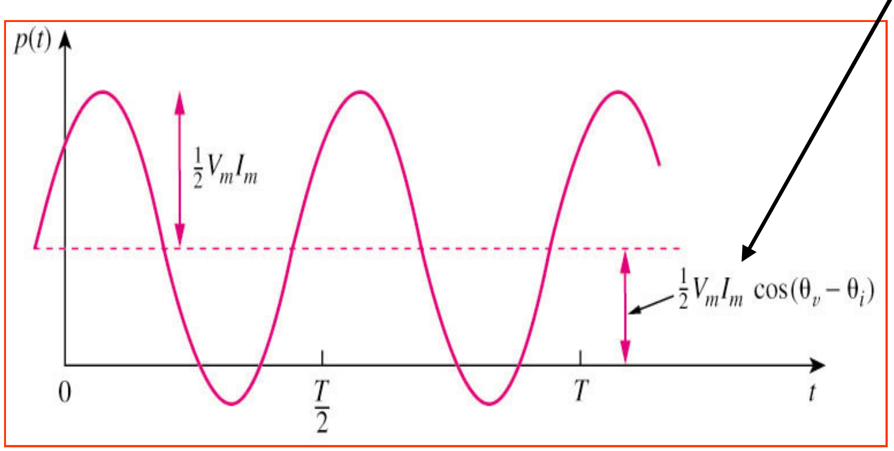
$$p(t) = v(t)\, i(t) = V_m I_m \cos(\omega t + \theta_v) \cos(\omega t + \theta_i)$$

$$= \underbrace{\frac{1}{2} V_m I_m \cos(\theta_v - \theta_i)}_{\text{Constant power}} + \underbrace{\frac{1}{2} V_m I_m \cos(2\omega t + \theta_v + \theta_i)}_{\text{Sinusoidal power at } 2\omega t}$$

11.2 Instantaneous and Average Power (2)

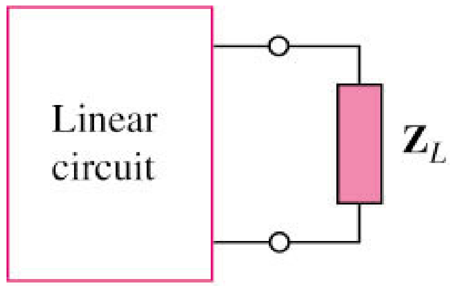
- The average power, P, *is the average of the instantaneous power over one period.*

$$\boxed{P = \frac{1}{T}\int_0^T p(t)\,dt = \frac{1}{2}V_m\, I_m \cos(\theta_v - \theta_i)}$$

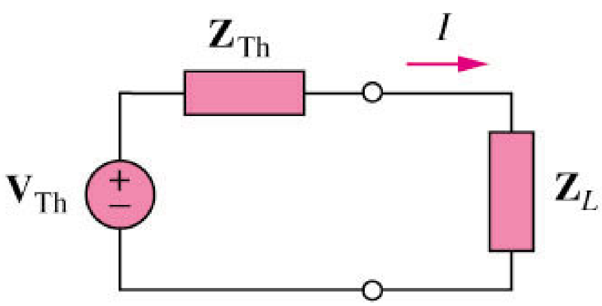


1. **P is not time dependent.**
2. **When $\theta_v = \theta_i$, it is a purely resistive load case.**
3. **When $\theta_v - \theta_i = \pm 90o$, it is a purely reactive load case.**
4. **P = 0 means that the circuit absorbs no average power.**

11.3 Maximum Average Power Transfer (1)



(a)



(b)

Maximum power is transferred to the load if the load impedance is the complex conjugate of the Thevenin impedance.

$$Z_{TH} = R_{TH} + jX_{TH}$$

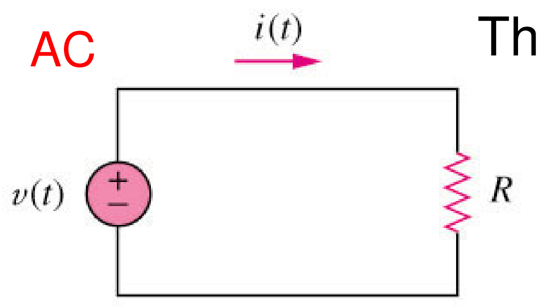
$$\boxed{Z_L = Z_{Th}^* = R_{TH} - jX_{TH}}$$

$$P_{max} = \frac{|V_{TH}|^2}{8R_{TH}}$$

When $Z_L = Z_{Th}^*$ we say the load is “matched” to the source.

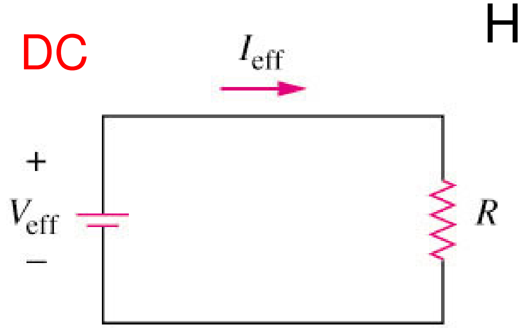
11.4 Effective or RMS Value (1)

AC



(a)

DC



(b)

The total power dissipated by R is given by:

$$P = \frac{1}{T}\int_0^T i^2 R dt = \frac{R}{T}\int_0^T i^2 dt = I_{rms}^2 R$$

Hence, I_{eff} is equal to: $$\boxed{I_{eff} = \sqrt{\frac{1}{T}\int_0^T i^2\, dt} = I_{rms}}$$

The rms value is a constant itself which depending on the shape of the function i(t).

The effective value of a periodic current is the dc current that delivers the same average power to a resistor as the periodic current.

11.5 Apparent Power and Power Factor (1)

- Apparent Power, S, is the product of the r.m.s. values of voltage and current.
- It is measured in volt-amperes or VA to distinguish it from the average or real power which is measured in watts.

$$P = V_{rms} I_{rms} \cos(\theta_v - \theta_i) = \underbrace{S}_{\text{Apparent Power, S}} \underbrace{\cos(\theta_v - \theta_i)}_{\text{Power Factor, pf}}$$

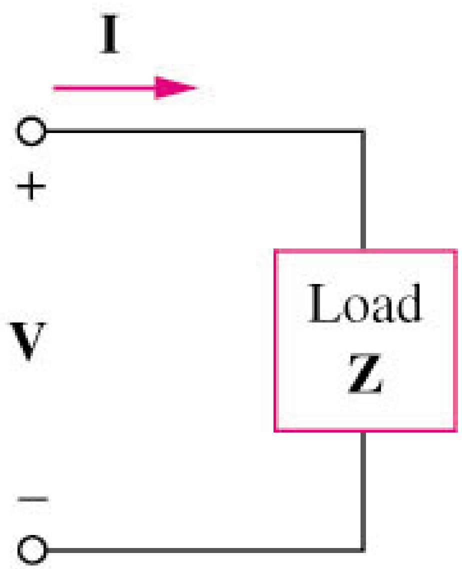
- Power factor is the cosine of the phase difference between the voltage and current. It is also the cosine of the angle of the load impedance.

11.5 Apparent Power and Power Factor (2)

Load Type	Power Factor Angle	Power Factor
Purely resistive load (R)	$\theta_v - \theta_i = 0$, $Pf = 1$	P/S = 1, all power are consumed
Purely reactive load (L or C)	$\theta_v - \theta_i = \pm 90^o$, $pf = 0$	P = 0, no real power consumption
Resistive and reactive load (R and L/C)	$\theta_v - \theta_i > 0$ $\theta_v - \theta_i < 0$	• Lagging - inductive load • Leading - capacitive load

11.6 Complex Power (1)

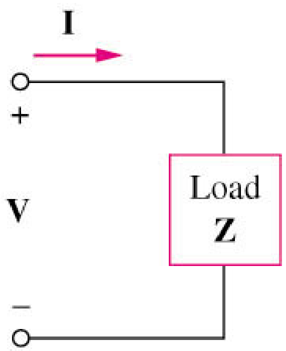
Complex power **S** is the product of the voltage and the complex conjugate of the current:



$$\mathbf{V} = V_m \angle \theta_v \qquad \mathbf{I} = I_m \angle \theta_i$$

$$\frac{1}{2} \mathrm{V}\, \mathrm{I}^* = \mathrm{V}_{\mathrm{rms}}\ \mathrm{I}_{\mathrm{rms}}\ \angle\, \theta_v - \theta_i$$

11.6 Complex Power (2)



$$S = \frac{1}{2} V I^* = V_{rms} I_{rms} \angle \theta_v - \theta_i$$

$$\Rightarrow S = \underbrace{V_{rms} I_{rms} \cos(\theta_v - \theta_i)}_{P} + j \underbrace{V_{rms} I_{rms} \sin(\theta_v - \theta_i)}_{Q}$$

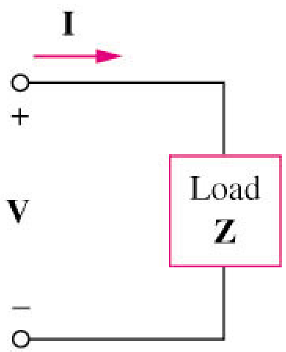
***S* = P + j Q**

P: is the <u>average power in watts</u> delivered to a load and it is the only useful power.

Q: is the <u>reactive power exchange</u> between the source and the reactive part of the load. It is measured in VAR.

- Q = 0 for *resistive loads* (unity pf).
- Q < 0 for *capacitive loads* (leading pf).
- Q > 0 for *inductive loads* (lagging pf).

11.6 Complex Power (3)



$$\Rightarrow S = \underbrace{V_{rms}\, I_{rms} \cos(\theta_v - \theta_i)}_{P} + j\, \underbrace{V_{rms}\, I_{rms} \sin(\theta_v - \theta_i)}_{Q}$$

S = P + j Q

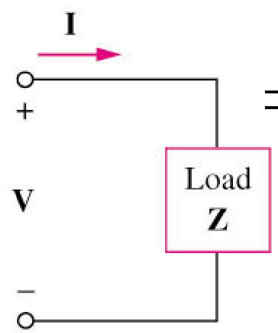
Apparent Power, S = |**S**| = Vrms*Irms $= \sqrt{P^2 + Q^2}$

Real power, P = Re(**S**) = S $\cos(\theta_v - \theta_i)$

Reactive Power, Q = Im(**S**) = S $\sin(\theta_v - \theta_i)$

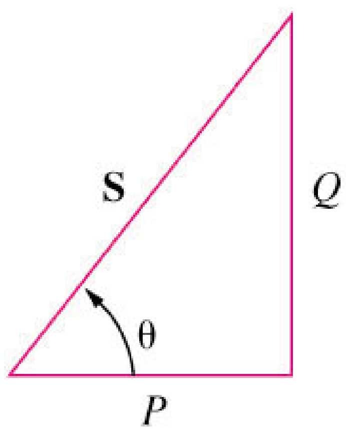
Power factor, pf = P/S = $\cos(\theta_v - \theta_i)$

11.6 Complex Power (4)

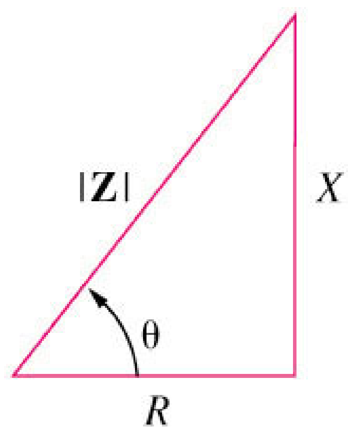


$$\Rightarrow S = \underbrace{V_{rms}\, I_{rms}\, \cos(\theta_v - \theta_i)}_{P} + j\, \underbrace{V_{rms}\, I_{rms}\, \sin(\theta_v - \theta_i)}_{Q}$$

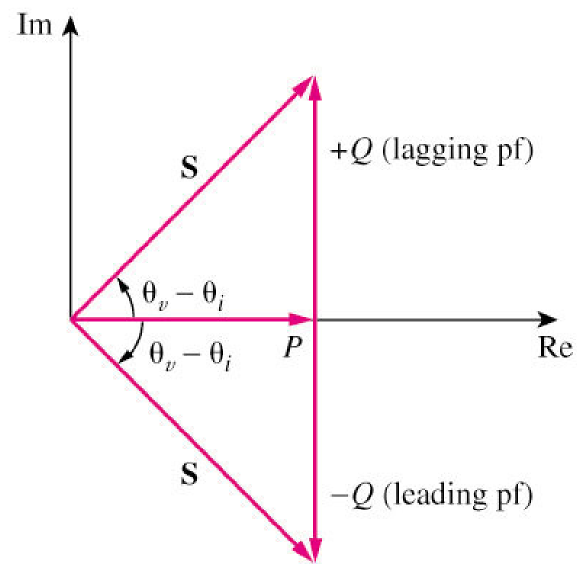
$$S = P + j\, Q$$



Power Triangle



Impedance Triangle



Power Factor

Homework #1
Due in class Wednesday, Jan 21

- Problems:
  - 3.23 (can use Matlab or any other matrix solver to solve the set of linear equations)
  - 3.51
  - 4.41
  - 5.27
  - 7.39a
  - 7.53a
  - 8.49
  - 10.1

No Class on Monday the 19th !

ECE 202 Review (Ch 8 – 11)

- Part I DC Circuits
  - Chapter 8: 2nd Order Circuits
- Part II AC Circuits
  - Chapter 9: Sinusoids and Phasors
  - Chapter 10: Sinusoidal Steady-State Analysis
  - Chapter 11: AC Power Analysis

Second-Order Circuits

Chapter 8 - Review

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8.1 Examples of 2nd order RCL circuit

8.2 Finding Initial and Final Values (X)

8.3 The source-free series RLC circuit

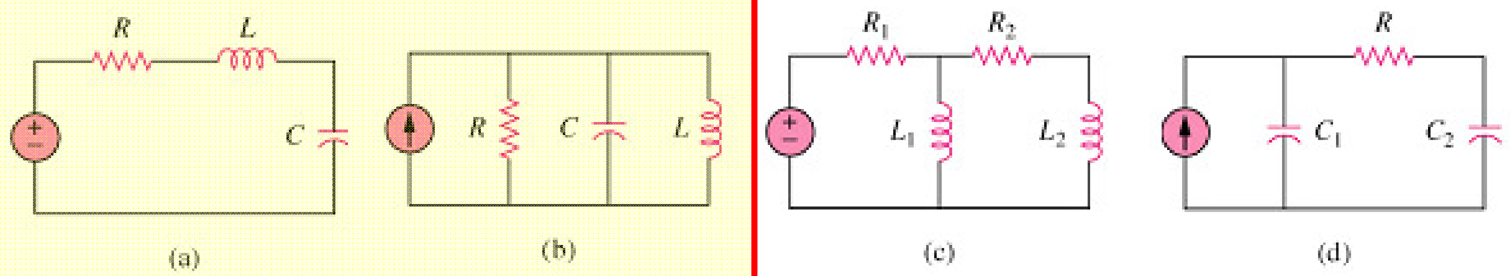
8.4 The source-free parallel RLC circuit

8.5 Step response of a series RLC circuit

8.6 Step response of a parallel RLC circuit

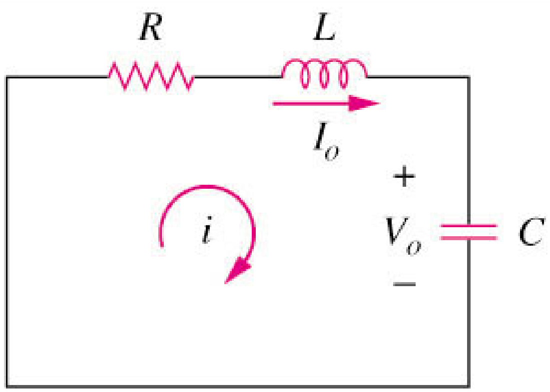
8.1 Examples of Second Order RLC circuits

A second-order circuit is characterized by a second-order differential equation. It consists of resistors and the equivalent of two energy storage elements.



RLC Series | RLC Parallel | RL T-config | RC Pi-config

8.3 Source-Free Series RLC Circuits (1)



- The solution of the source-free series RLC circuit is called as the natural response of the circuit.

The 2nd order of expression

Derived from KVL:

$$iR + L\frac{di}{dt} + \frac{1}{C}\int_{-\infty}^{t} i(\tau)d\tau = 0$$

$$R\frac{di}{dt} + L\frac{d^2i}{dt^2} + \frac{1}{C}i = 0$$

$$\frac{d^2i}{dt^2} + \frac{R}{L}\frac{di}{dt} + \frac{i}{LC} = 0$$

Note: Same form if looking at voltage across capacitor!

To find for voltage across capacitor

Substitute: $i = C\frac{dv}{dt}$

$$RC\frac{dv}{dt} + LC\frac{d^2v}{dt^2} + v = 0$$

$$\frac{d^2v}{dt^2} + \frac{R}{L}\frac{dv}{dt} + \frac{v}{LC} = 0$$

Where v is the voltage across the capacitor

8.3 Source-Free Series RLC Circuits (2)

Three possible solutions for the 2nd order differential equation:

$$\frac{d^2 i}{dt^2} + \frac{R}{L}\frac{di}{dt} + \frac{i}{LC} = 0$$

=> $$\frac{d^2 i}{dt^2} + 2\alpha \frac{di}{dt} + \omega_0^2 i = 0$$

General 2nd order Form

where $\alpha = \frac{R}{2L}$ *and* $\omega_0 = \sqrt{\frac{1}{LC}}$

The types of solutions for i(t) depend on the relative values of α and ω.

General Solution

$$i(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t}$$

$$s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2}$$

- If $\alpha > \omega_o$ ➔ over-damped
- If $\alpha = \omega_o$ ➔ critically-damped
- If $\alpha < \omega_o$ ➔ under-damped

8.3 Source-Free Series RLC Circuits (3)

There are three possible solutions for the following 2nd order differential equation:

$$\frac{d^2i}{dt^2}+2\alpha\frac{di}{dt}+\omega_0^2 i=0 \qquad \alpha=\frac{R}{2L} \quad and \quad \omega_0=\sqrt{\frac{1}{LC}}$$

1. If $\alpha > \omega_o$, over-damped case

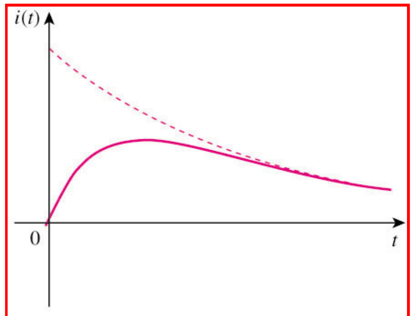
$$i(t)=A_1e^{s_1t}+A_2e^{s_2t}$$ **where** $$s_{1,2}=-\alpha\pm\sqrt{\alpha^2-\omega_0{}^2}$$

2. If $\alpha = \omega_o$, critical damped case

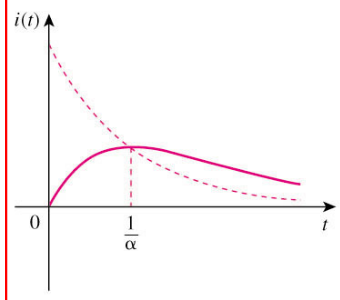
$$i(t)=(A_2+A_1t)e^{-\alpha t}$$ **where** $$s_{1,2}=-\alpha$$

3. If $\alpha < \omega_o$, under-damped case

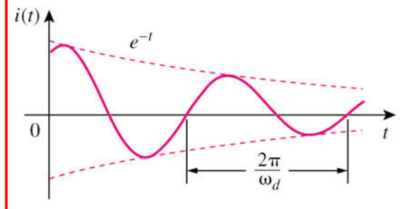
$$i(t)=e^{-\alpha t}(B_1\cos\omega_d t+B_2\sin\omega_d t)$$ **where** $$\omega_d=\sqrt{\omega_0^2-\alpha^2}$$



(a)

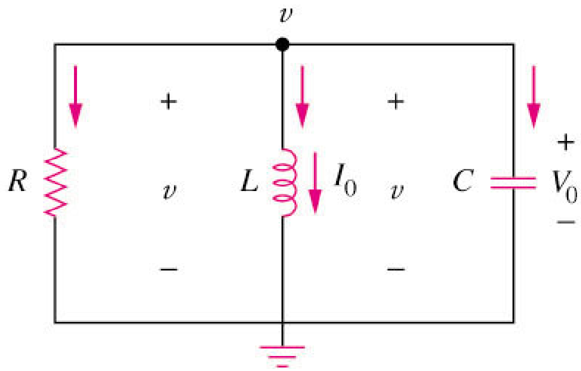


(b)



(c)

8.4 Source-Free Parallel RLC Circuits (1)



- Similar approach to series RLC analysis

Derived from KCL:

$$\frac{v}{R}+C\frac{dv}{dt}+\frac{1}{L}\int_{-\infty}^{t} v(\tau)d\tau=0$$

$$\frac{1}{R}\frac{dv}{dt}+C\frac{d^2v}{dt^2}+\frac{1}{L}v=0$$

The 2nd order of expression

$$\frac{d^2v}{dt^2}+\frac{1}{RC}\frac{dv}{dt}+\frac{1}{LC}v=0$$

To find for current across inductor

Substitute: $v = L\frac{di}{dt}$

$$\frac{L}{R}\frac{di}{dt}+LC\frac{d^2i}{dt^2}+i=0$$

Note: Same form if looking at current through inductor!

$$\frac{d^2i}{dt^2}+\frac{1}{RC}\frac{di}{dt}+\frac{1}{LC}i=0$$

Where i is the current through the inductor

8.4 Source-Free Parallel RLC Circuits (2)

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There are three possible solutions for the following 2nd order differential equation:

$$\frac{d^2v}{dt^2} + 2\alpha\frac{dv}{dt} + \omega_0^2 v = 0 \quad \text{where} \quad \alpha = \frac{1}{2RC} \quad \text{and} \quad \omega_0 = \sqrt{\frac{1}{LC}}$$

1. If $\alpha > \omega_o$, over-damped case

$$v(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t} \quad \textbf{where} \quad s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2}$$

2. If $\alpha = \omega_o$, critical damped case

$$v(t) = (A_2 + A_1 t)\, e^{-\alpha t} \quad \textbf{where} \quad s_{1,2} = -\alpha$$

3. If $\alpha < \omega_o$, under-damped case

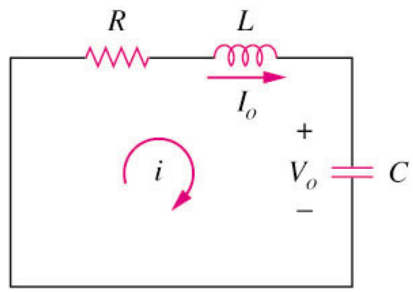
$$v(t) = e^{-\alpha t}(B_1 \cos \omega_d t + B_2 \sin \omega_d t) \quad \textbf{where} \quad \omega_d = \sqrt{\omega_0^2 - \alpha^2}$$

Comparison Series / Parallel RLC Circuits

	Series RLC	Parallel RLC
Characteristic Equation	$\frac{d^2 i}{dt^2} + 2\alpha \frac{di}{dt} + \omega_0^2 i = 0$	$\frac{d^2 v}{dt^2} + 2\alpha \frac{dv}{dt} + \omega_0^2 v = 0$
Damping Factor	$\alpha = \frac{R}{2L}$	$\alpha = \frac{1}{2RC}$
Resonance Frequency	$\omega_0 = \sqrt{\frac{1}{LC}}$	$\omega_0 = \sqrt{\frac{1}{LC}}$
Over-damped $\alpha > \omega_o$	$i(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t}$ $s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2}$	$v(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t}$ $s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2}$
Critical damped $\alpha = \omega_o$,	$i(t) = (A_2 + A_1 t)\, e^{-\alpha t}$	$v(t) = (A_2 + A_1 t)\, e^{-\alpha t}$
Under-damped $\alpha < \omega_o$	$i(t) = e^{-\alpha t}(B_1 \cos \omega_d t + B_2 \sin \omega_d t)$ $\omega_d = \sqrt{\omega_0^2 - \alpha^2}$	$v(t) = e^{-\alpha t}(B_1 \cos \omega_d t + B_2 \sin \omega_d t)$ $\omega_d = \sqrt{\omega_0^2 - \alpha^2}$

Source-Free RLC Circuits

- Key to solving problems is to finding the initial conditions
  - Voltage doesn't change rapidly across a capacitor: $v(0^+) = v(0^-)$
  - Current doesn't change rapidly across an inductor: $i(0^+) = i(0^-)$
- Start by finding α and ω_o use these to find s_1 and s_2
  - Determine case (over / under / or critically damped)
- Apply initial conditions to the equations to find A_1 and A_1

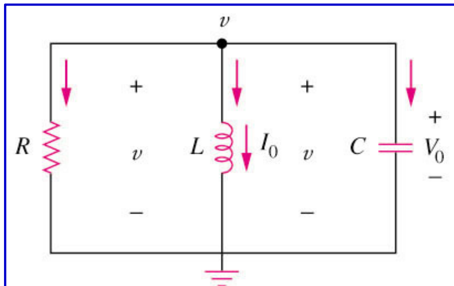


Series RLC

$$i(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t} \Rightarrow i(0^+) = A_1 + A_2$$

$$\frac{di(t)}{dt} = A_1 s_1 e^{s_1 t} + A_2 s_2 e^{s_2 t} \Rightarrow \frac{di(0^+)}{dt} = A_1 s_1 + A_2 s_2$$

$$\frac{di(0)}{dt} = -\frac{1}{L}(RI_0 + V_0)$$



Parallel RLC

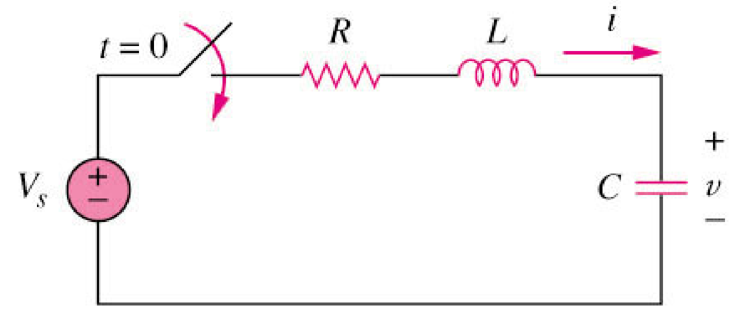
$$v(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t} \Rightarrow v(0^+) = A_1 + A_2$$

$$\frac{dv(t)}{dt} = A_1 s_1 e^{s_1 t} + A_2 s_2 e^{s_2 t} \Rightarrow \frac{dv(0^+)}{dt} = A_1 s_1 + A_2 s_2$$

$$\frac{dv(0)}{dt} = -\frac{1}{RC}(RI_0 + V_0)$$

8.5 Step-Response Series RLC Circuits (1)

- The step response is obtained by the sudden application of a dc source.



The 2nd order of expression

$$\overbrace{\frac{d^2v}{dt^2}+\frac{R}{L}\frac{dv}{dt}+\frac{v}{LC}}^{\textit{Natural Response}}=\frac{V_s}{LC}$$

The above equation has the same form as the equation for source-free series RLC circuit.

- The same coefficients (important in determining the frequency parameters).

8.5 Step-Response Series RLC Circuits (2)

The solution of the equation should have two components: the transient response $v_t(t)$ & the steady-state response $v_{ss}(t)$:

$$v(t) = v_t(t) + v_{ss}(t)$$

This is the Voltage across the Capacitor !!

- The transient response v_t is the same as that for source-free case

$$v_t(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t}$$ (over-damped)

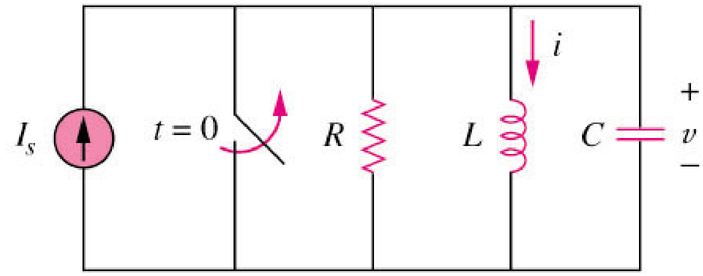
$$v_t(t) = (A_1 + A_2 t) e^{-\alpha t}$$ (critically damped)

$$v_t(t) = e^{-\alpha t}(A_1 \cos \omega_d t + A_2 \sin \omega_d t)$$ (under-damped)

- The steady-state response is the final value of v(t).
  - $v_{ss}(t) = v(\infty)$
- The values of A_1 and A_2 are obtained from the initial conditions:
  - v(0) and dv(0)/dt.

8.6 Step-Response Parallel RLC Circuits (1)

- The step response is obtained by the sudden application of a dc source.



The 2nd order of expression

$$\frac{d^2 i}{dt^2} + \frac{1}{RC}\frac{di}{dt} + \frac{i}{LC} = \frac{I_s}{LC}$$

It has the same form as the equation for source-free parallel RLC circuit.

- The same coefficients (important in determining the frequency parameters).

8.6 Step-Response Parallel RLC Circuits (2)

The solution of the equation should have two components: the transient response $i_t(t)$ & the steady-state response $i_{ss}(t)$:

$$i(t) = i_t(t) + i_{ss}(t)$$

This is the Current through the Inductor !!

- The transient response i_t is the same as that for source-free case

$$i_t(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t}$$ (over-damped)

$$i_t(t) = (A_1 + A_2 t)e^{-\alpha t}$$ (critical damped)

$$i_t(t) = e^{-\alpha t}(A_1 \cos \omega_d t + A_2 \sin \omega_d t)$$ (under-damped)

- The steady-state response is the final value of i(t).
  - $i_{ss}(t) = i(\infty) = I_s$
- The values of A_1 and A_2 are obtained from the initial conditions:
  - i(0) and di(0)/dt.

Chapter 8 Example (1)

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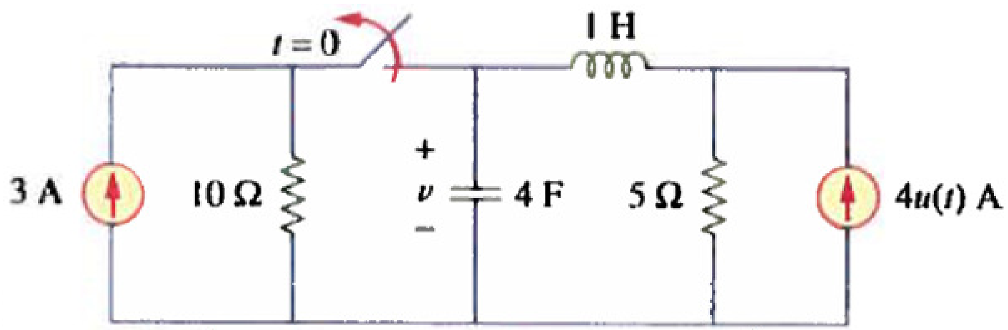
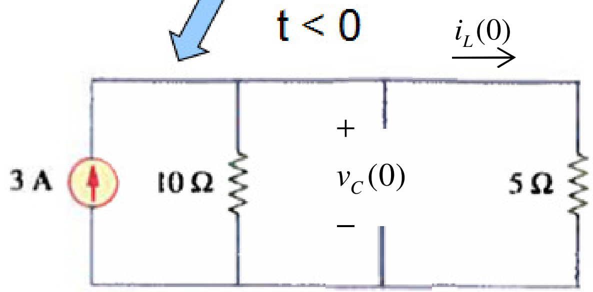
8.33 Find $v(t)$ for $t > 0$ in the circuit of Fig. 8.81.



Remember !!
Capacitor is "Open" to DC
Inductor is "Short" to DC

Before switch is opened

t < 0



Initial Voltage across Capacitor

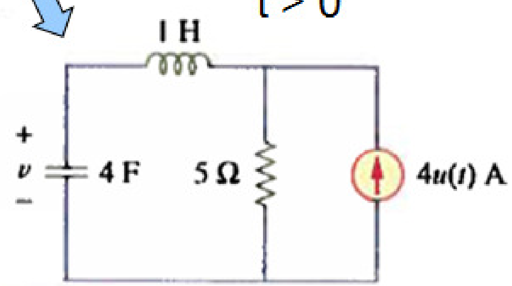
$$v_C(0) = \frac{5\cdot 10}{5+10}(3) = 10$$

Initial Current through Inductor

$$i_L(0) = 2$$

After switch is opened

t > 0

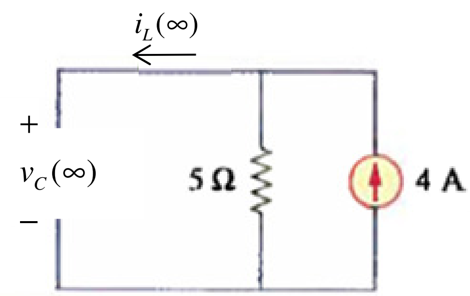


$$\alpha = \frac{R}{2L} = \frac{5}{(2)(1)} = 2.5$$

$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{(1)(4)}} = 0.5$$

$\alpha > \omega_0$ Over-damped Case

Steady State circuit ($t \to \infty$)



SS Voltage across Capacitor

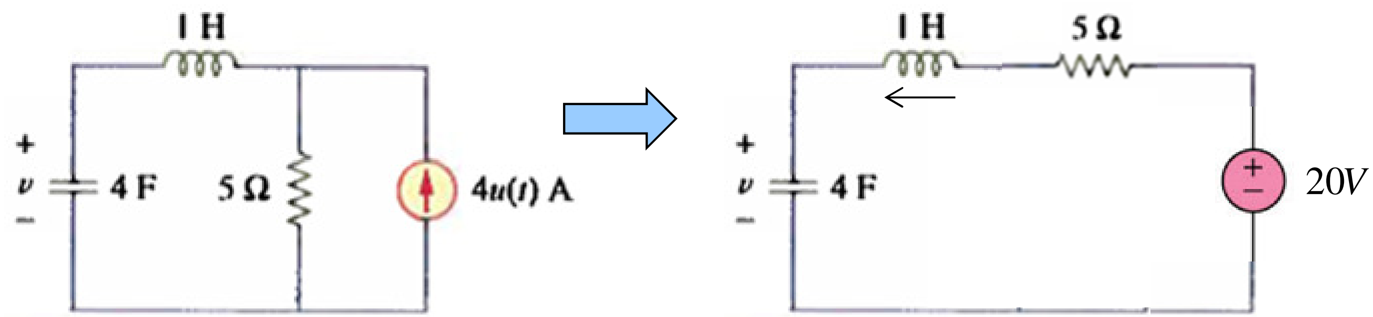
$$v_C(\infty) = (4)(5) = 20$$

SS Current through Inductor

$$i_L(\infty) = 0$$

Chapter 8 Example 8.33 (continued)

Circuit to be analyzed for response



Step Response of Series RLC Circuit

$$v(t) = v_t(t) + v_{ss}(t)$$

Voltage across Capacitor

$$v_t(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t}$$ (over-damped)

Known Parameters

$v_C(0) = 10$	$v_C(\infty) = 20$	$\alpha = 2.5$	$s_1 = 2.5 + \sqrt{(2.5)^2 - (0.5)^2} = 4.949$
$i_L(0) = 2$	$i_L(\infty) = 0$	$\omega_0 = 0.5$	$s_2 = 2.5 - \sqrt{(2.5)^2 - (0.5)^2} = 0.0505$

Voltage across Capacitor

$$v(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t} + 20$$

$$v(0) = A_1 + A_2 + 20 = 10$$

$$\boxed{A_1 + A_2 = -10}$$

Find Current through Inductor by taking derivative

$$i(0) = C\frac{dv(0)}{dt} \Rightarrow \frac{dv(0)}{dt} = \frac{i(0)}{C} = \frac{-2}{4}$$

Negative because in opposite direction

$$\frac{dv(0)}{dt} = \boxed{A_1 s_1 + A_2 s_2 = -0.5}$$

From here can solve for A1 and A2

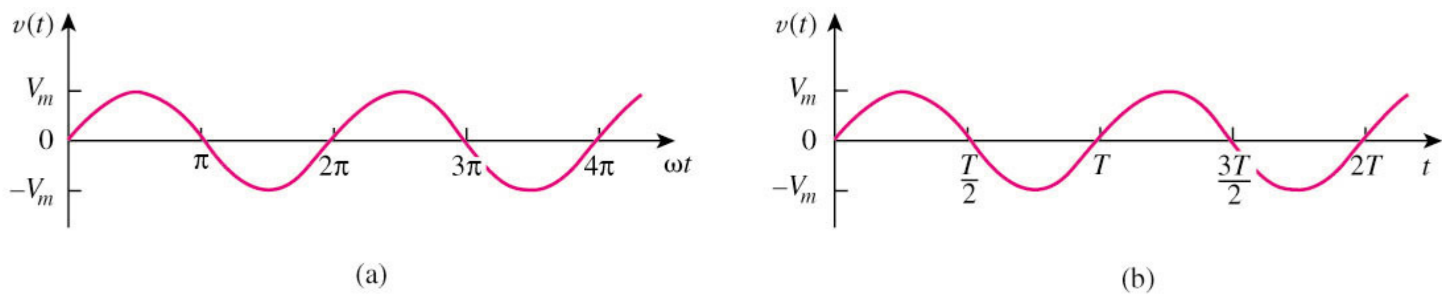
Sinusoids and Phasors Chapter 9 - Review

9.2 Sinusoids
9.3 Phasors
9.4 Phasor Relationships for Circuit Elements
9.5 Impedance and Admittance
9.6 Kirchhoff's Laws in the Frequency Domain
9.7 Impedance Combinations

9.2 Sinusoids (1)

- A sinusoid is a signal that has the form of the sine or cosine function.
- A general expression for the sinusoid,

$$v(t) = V_m \sin(\omega t + \phi)$$



where

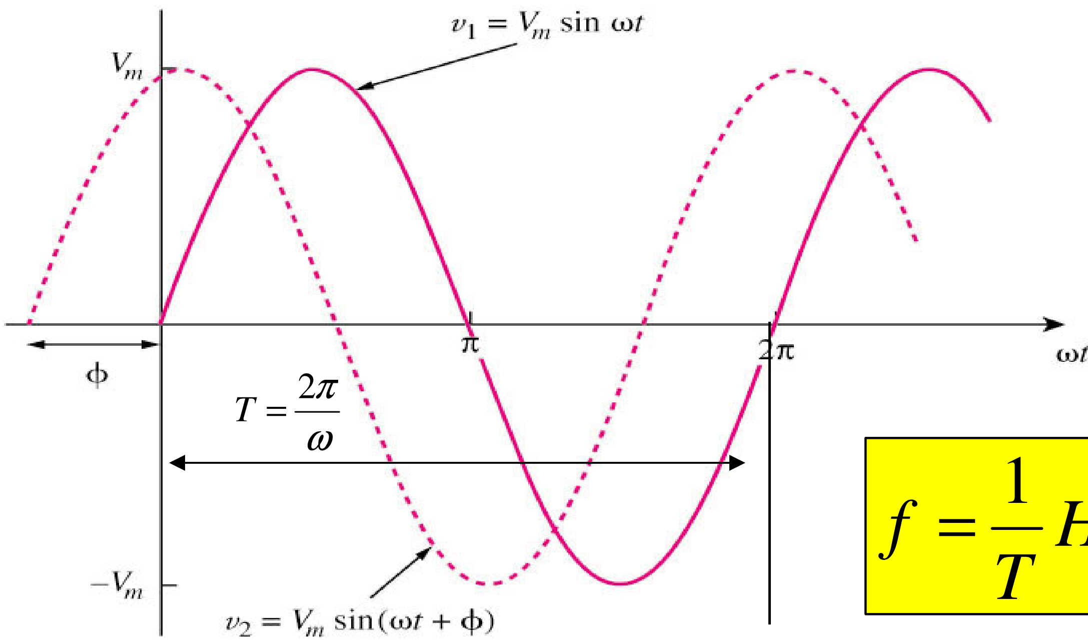
Vm = the **amplitude** of the sinusoid

ω = the angular frequency in radians/s

Φ = the phase

9.2 Sinusoids (2)

A periodic function is one that satisfies *v(t) = v(t + nT)*, for all t and for all integers n.



$$f = \frac{1}{T} Hz$$

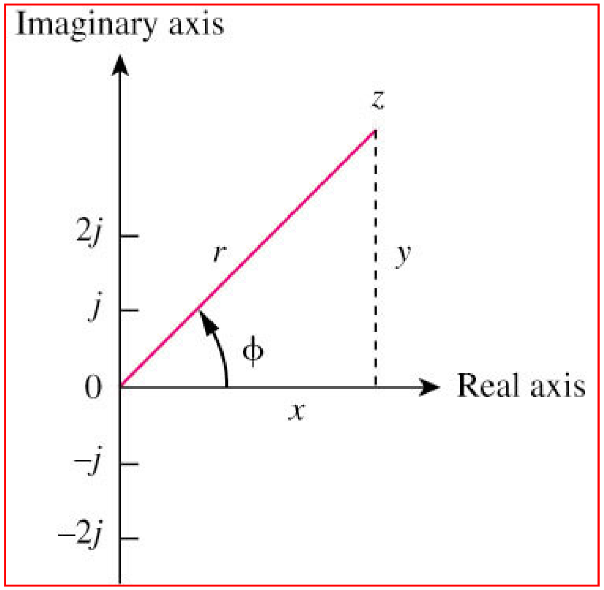
$$\omega = 2\pi f$$

- Only two sinusoidal values with the same frequency can be compared by their amplitude and phase difference.
- If phase difference is zero, they are in phase; if phase difference is not zero, they are out of phase.

9.3 Phasors (1)

- A phasor is a complex number that represents the amplitude and phase of a sinusoid.

- It can be represented in one of the following three forms:



a. Rectangular $z = x + jy = r(\cos\phi + j\sin\phi)$

b. Polar $z = r\angle\phi$

c. Exponential $z = re^{j\phi}$

where $r = \sqrt{x^2 + y^2}$

$\phi = \tan^{-1}\frac{y}{x}$

Mathematic operation of complex number:

1. Addition $z_1 + z_2 = (x_1 + x_2) + j(y_1 + y_2)$
2. Subtraction $z_1 - z_2 = (x_1 - x_2) + j(y_1 - y_2)$
3. Multiplication $z_1 z_2 = r_1 r_2 \angle \phi_1 + \phi_2$
4. Division $\dfrac{z_1}{z_2} = \dfrac{r_1}{r_2} \angle \phi_1 - \phi_2$
5. Reciprocal $\dfrac{1}{z} = \dfrac{1}{r} \angle -\phi$
6. Square root $\sqrt{z} = \sqrt{r} \angle \phi/2$
7. Complex conjugate $z^* = x - jy = r \angle -\phi = re^{-j\phi}$
8. Euler's identity $e^{\pm j\phi} = \cos\phi \pm j\sin\phi$

9.3 Phasors (3)

- Transform a sinusoid to and from the time domain to the phasor domain:

$$v(t) = V_m \cos(\omega t + \phi) \longleftrightarrow \mathbf{V} = V_m \angle\phi$$

(time domain) (phasor domain)

The differences between v(t) and V:

- v(t) is instantaneous or time-domain representation
  V is the frequency or phasor-domain representation.
- v(t) is time dependent, V is not.
- v(t) is always real with no complex term, V is generally complex.

Note: Phasor analysis applies only when frequency is constant; it is applied to two or more sinusoid signals only if they have the same frequency.

9.3 Phasors (5)

Relationship between differential, integral operation in phasor listed as follow:

$$v(t) \longleftrightarrow \mathbf{V} = V\angle\phi$$

$$\frac{dv}{dt} \longleftrightarrow j\omega\mathbf{V}$$

$$\int v\,dt \longleftrightarrow \frac{\mathbf{V}}{j\omega}$$

9.4 Phasor Relationships for Circuit Elements (1)

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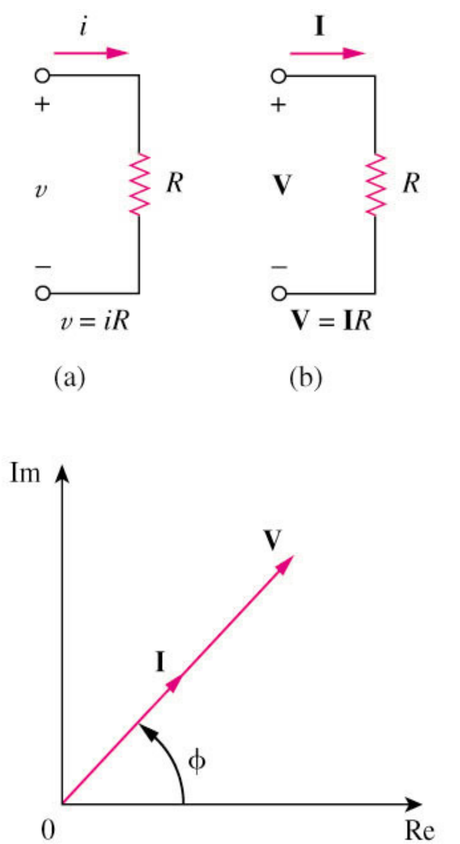
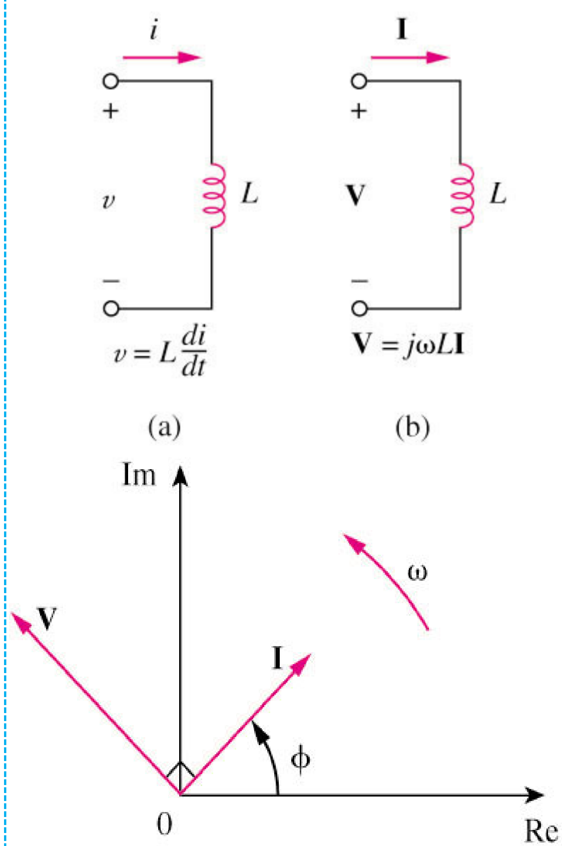
Resistors



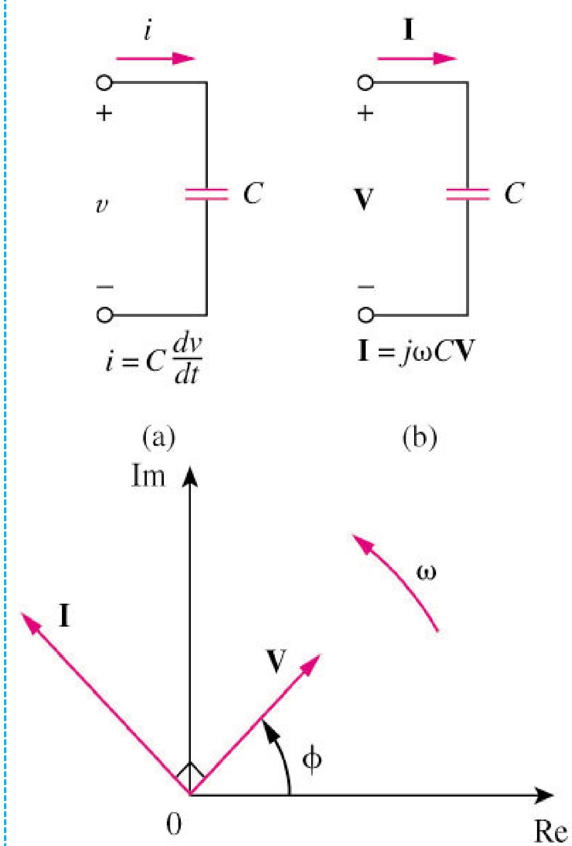
Inductors



"ELI" the

Voltage Lead Current
For Inductors "L" (ELI)

Capacitors



"ICE" man

Current Lead Voltage
For Capacitors "C" (ICE)

Aside: ELI the ICE man & the derivative

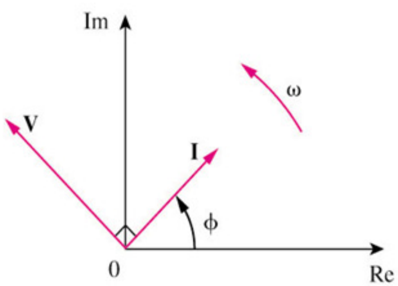
- The phrase "ELI the ICE man" can be used to remember the relationships of voltage & current for inductors & capacitors

E L I

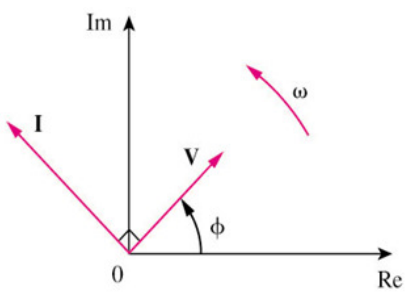
$$v = L\frac{di}{dt}$$

I C E

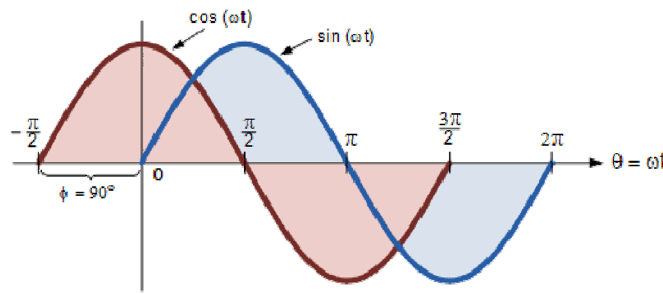
$$i = C\frac{dv}{dt}$$



Voltage leads current



Current leads voltage



Note:

1. Derivative of sin(t) is cos(t).
2. Observe the instantaneous "slope" of sin(t). It's cos(t)!
3. Derivative introduces a 90° shift
4. Now look at equations for L and C and this explains the 90° shift
5. Also, should be easier now to remember:

$$\frac{d}{dt}\left[\sin(\omega t)\right] = \omega\cos(\omega t)$$

$$\frac{d}{dt}\left[\cos(\omega t)\right] = -\omega\sin(\omega t)$$

9.4 Phasor Relationships for Circuit Elements (2)

Summary of voltage-current relationship		
Element	Time domain	Frequency domain
R	$v = Ri$	$V = RI$
L	$v = L\frac{di}{dt}$	$V = j\omega LI$
C	$i = C\frac{dv}{dt}$	$V = \frac{I}{j\omega C}$

9.5 Impedance and Admittance (1)

- The impedance Z of a circuit is the ratio of the phasor voltage V to the phasor current I, measured in ohms Ω.

$$Z = \frac{V}{I} = R + jX$$

Positive X is for L and negative X is for C.

- The admittance Y is the reciprocal of impedance, measured in siemens (S).

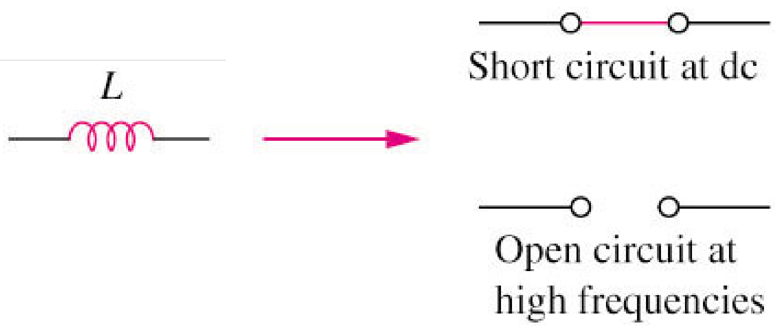
$$Y = \frac{1}{Z} = \frac{I}{V}$$

9.5 Impedance and Admittance (2)

Impedances and admittances of passive elements		
Element	**Impedance**	**Admittance**
R	$Z = R$	$Y = \dfrac{1}{R}$
L	$Z = j\omega L$	$Y = \dfrac{1}{j\omega L}$
C	$Z = \dfrac{1}{j\omega C}$	$Y = j\omega C$

9.5 Impedance and Admittance (3)

$$Z = j\omega L$$

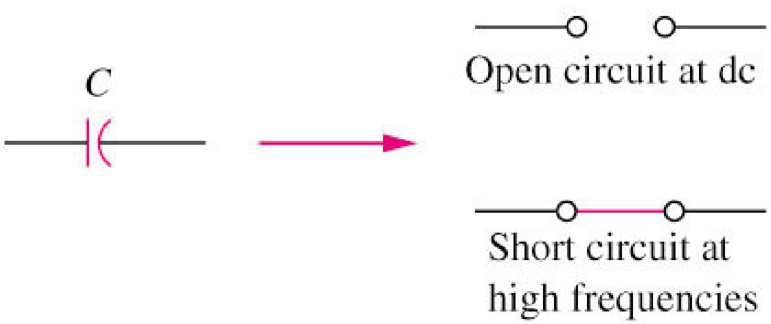


$$\omega = 0; Z = 0$$

$$\omega \to \infty; Z \to \infty$$

(a)

$$Z = \frac{1}{j\omega C}$$



$$\omega = 0; Z \to \infty$$

$$\omega \to \infty; Z = 0$$

(b)

9.6 Kirchhoff's Laws in the Frequency Domain (1)

After we know how to convert RLC components from time to phasor domain, we can transform a time domain circuit into a phasor/frequency domain circuit.

Hence, we can apply the KCL laws and other theorems to directly set up phasor equations involving our target variable(s) for solving.

9.6 Kirchhoff's Laws in the Frequency Domain (2)

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- Both KVL and KCL are hold in the phasor domain or more commonly called frequency domain.
- Moreover, the variables to be handled are phasors, which are complex numbers.
- All the mathematical operations involved are now in complex domain.

9.7 Impedance Combinations (1)

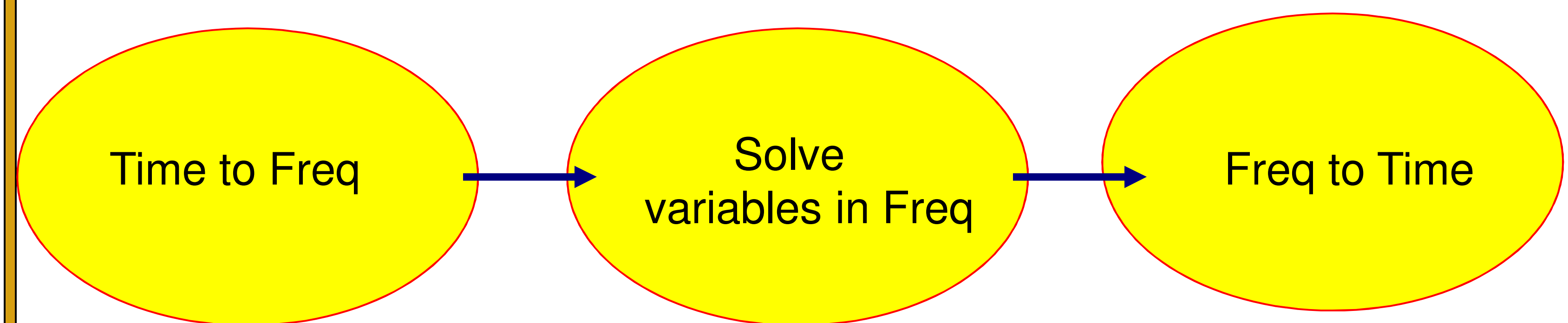
- The following principles used for DC circuit analysis all apply to AC circuit.
- For example:
  a. voltage division
  b. current division
  c. circuit reduction
  d. impedance equivalence
  e. Y-Δ transformation

Sinusoidal Steady-State Analysis
Chapter 10 Review

- 10.1 Basic Approach
- 10.2 Nodal Analysis
- 10.3 Mesh Analysis
- 10.4 Superposition Theorem
- 10.5 Source Transformation
- 10.6 Thevenin and Norton Equivalent Circuits

Steps to Analyze AC Circuits:

1. Transform the circuit to the phasor or frequency domain.
2. Solve the problem using circuit techniques (nodal analysis, mesh analysis, superposition, etc.).
3. Transform the resulting phasor to the time domain.



10.4 Superposition Theorem (1)

When a circuit has sources operating at different frequencies,

- The separate phasor circuit for each frequency must be solved independently, and
- The total response is the sum of time-domain responses of all the individual phasor circuits.

10.6 Thevenin and Norton Equivalent Circuits (1)

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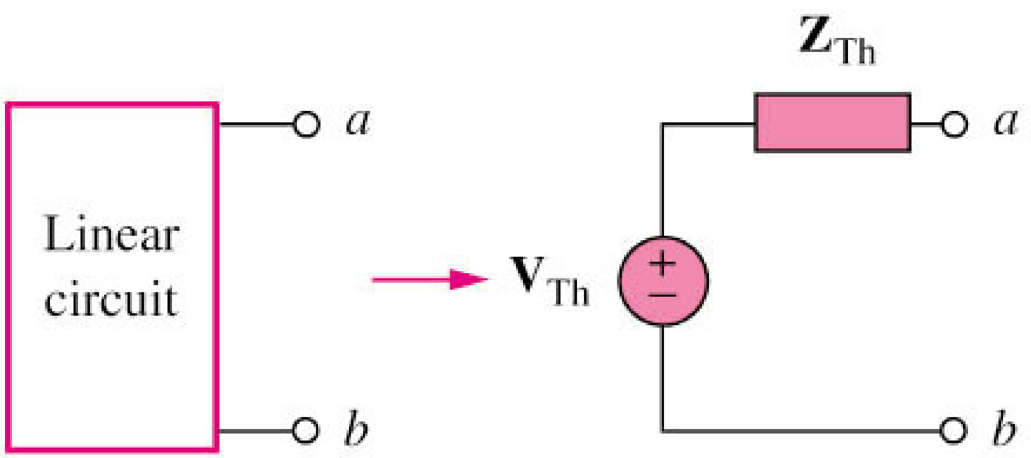
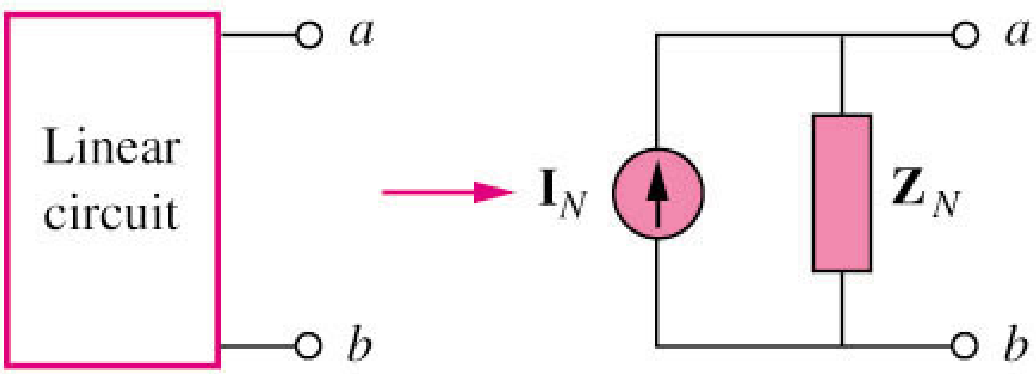


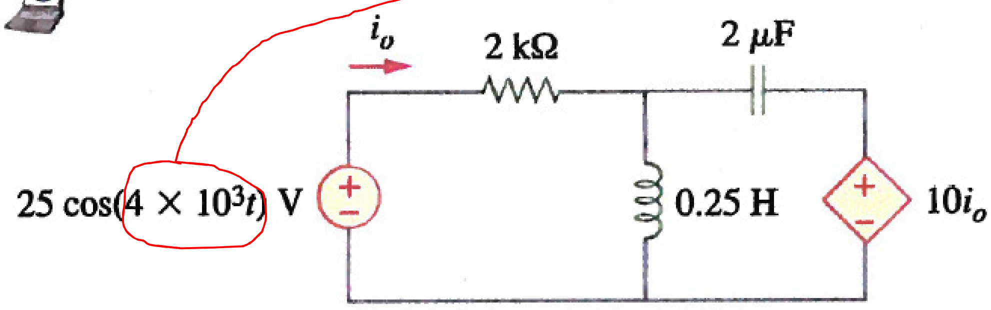
Thevenin transform



Norton transform

Chapter 10 Example (1)

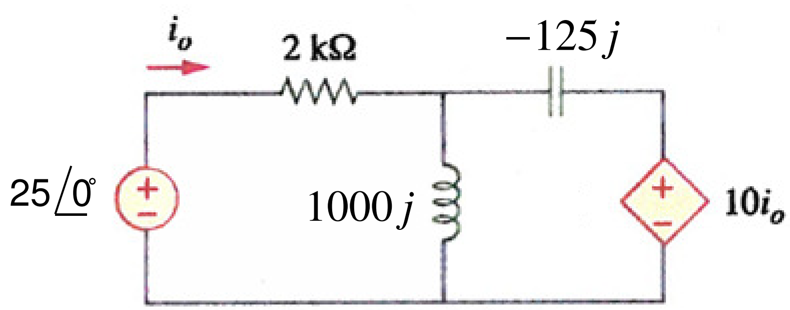
10.5 Find i_o in the circuit of Fig. 10.54.



$$\omega = 4000$$

$$Z_L = j\omega L = j4000(0.25) = 1000j$$

$$Z_C = \frac{1}{j\omega C} = \frac{1}{j4000(2\mu)} = -125j$$



Mesh 1

$$-25 + 2000i_1 + 1000j(i_1 - i_2) = 0$$

$$(2000 + 1000j)i_1 - (1000j)i_2 = 25$$

Mesh 2

$$1000j(i_2 - i_1) + (-125j)i_2 + 10i_1 = 0$$

$$(10 - 1000j)i_1 + (875j)i_2 = 0$$

$$\begin{bmatrix} 2000+1000j & -1000j \\ 10-1000j & 875j \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} 25 \\ 0 \end{bmatrix}$$

AC Power Analysis
Chapter 11 Review

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11.2 Instantaneous and Average Power
11.3 Maximum Average Power Transfer
11.4 Effective or RMS Value
11.5 Apparent Power and Power Factor
11.6 Complex Power

11.2 Instantaneous and Average Power (1)

- The instantaneously power, p(t), absorbed by an element is the product of the instantaneous voltage v(t) across the element and the instantaneous current i(t) through it.

$$p(t) = v(t)\,i(t) = V_m I_m \cos(\omega t + \theta_v)\cos(\omega t + \theta_i)$$

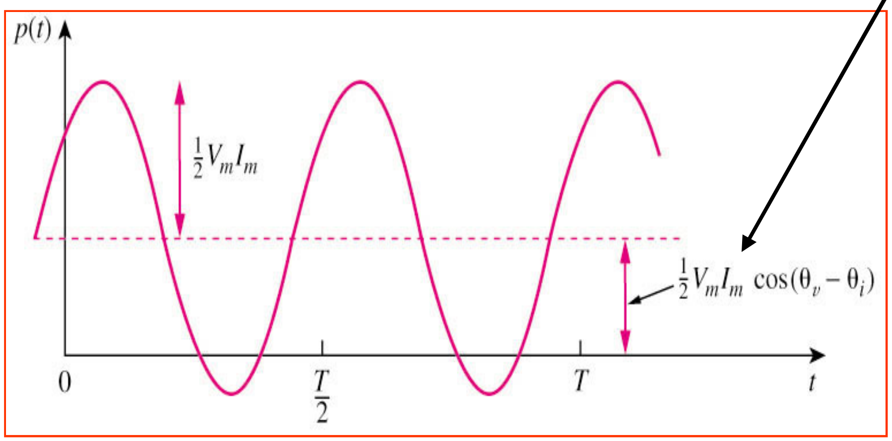
$$= \underbrace{\frac{1}{2}V_m I_m \cos(\theta_v - \theta_i)}_{\text{Constant power}} + \underbrace{\frac{1}{2}V_m I_m \cos(2\omega t + \theta_v + \theta_i)}_{\text{Sinusoidal power at } 2\omega t}$$

Constant power **Sinusoidal power at 2ωt**

11.2 Instantaneous and Average Power (2)

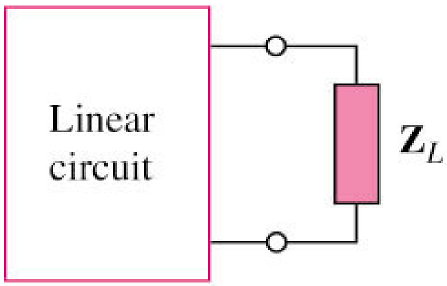
- The average power, P, *is the average of the instantaneous power over one period.*

$$\boxed{P = \frac{1}{T}\int_0^T p(t)\,dt = \frac{1}{2} V_m\, I_m \cos\left(\theta_v - \theta_i\right)}$$

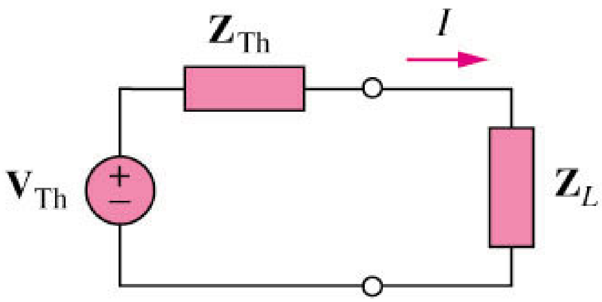


1. **P is not time dependent.**
2. **When $\theta_v = \theta_i$, it is a purely resistive load case.**
3. **When $\theta_v - \theta_i = \pm 90o$, it is a purely reactive load case.**
4. **P = 0 means that the circuit absorbs no average power.**

11.3 Maximum Average Power Transfer (1)



(a)



(b)

Maximum power is transferred to the load if the load impedance is the complex conjugate of the Thevenin impedance.

$$Z_{TH} = R_{TH} + jX_{TH}$$

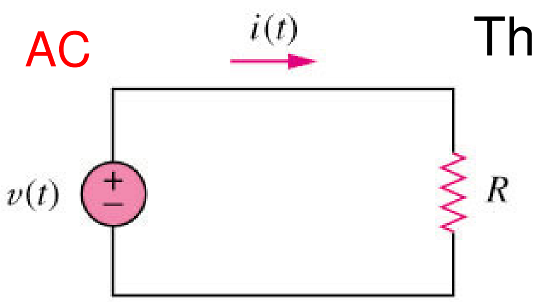
$$\boxed{Z_L = Z_{Th}^* = R_{TH} - jX_{TH}}$$

$$P_{max} = \frac{|V_{TH}|^2}{8R_{TH}}$$

When $Z_L = Z_{Th}^*$ we say the load is “matched” to the source.

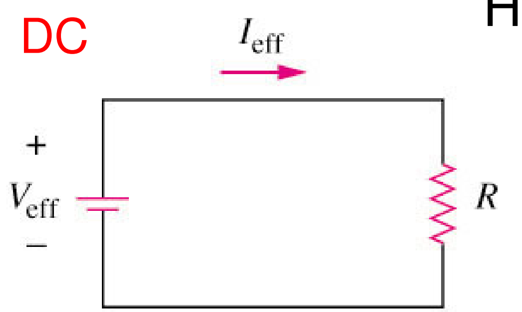
11.4 Effective or RMS Value (1)

AC



(a)

DC



(b)

The total power dissipated by R is given by:

$$P = \frac{1}{T}\int_0^T i^2 R dt = \frac{R}{T}\int_0^T i^2 dt = I_{rms}^2 R$$

Hence, I_{eff} is equal to:

$$I_{eff} = \sqrt{\frac{1}{T}\int_0^T i^2\, dt} = I_{rms}$$

The rms value is a constant itself which depending on the shape of the function i(t).

The effective value of a periodic current is the dc current that delivers the same average power to a resistor as the periodic current.

11.5 Apparent Power and Power Factor (1)

- Apparent Power, S, is the product of the r.m.s. values of voltage and current.
- It is measured in volt-amperes or VA to distinguish it from the average or real power which is measured in watts.

$$P = V_{rms}\, I_{rms} \cos(\theta_v - \theta_i) = \underbrace{S}_{\text{Apparent Power, S}}\, \underbrace{\cos(\theta_v - \theta_i)}_{\text{Power Factor, pf}}$$

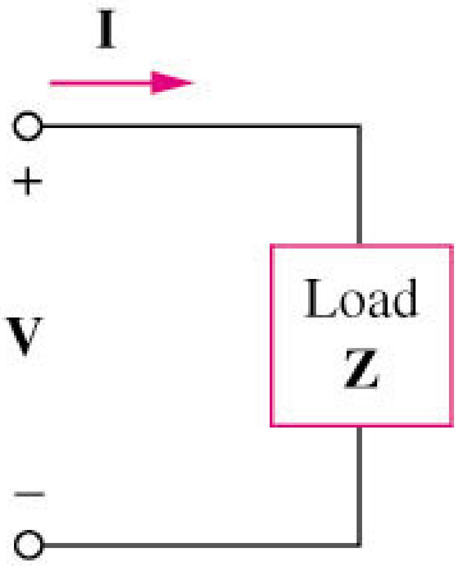
- Power factor is the cosine of the phase difference between the voltage and current. It is also the cosine of the angle of the load impedance.

11.5 Apparent Power and Power Factor (2)

Load Type	Power Factor Angle	Power Factor
Purely resistive load (R)	$\theta_v - \theta_i = 0$, $Pf = 1$	P/S = 1, all power are consumed
Purely reactive load (L or C)	$\theta_v - \theta_i = \pm 90^o$, $pf = 0$	P = 0, no real power consumption
Resistive and reactive load (R and L/C)	$\theta_v - \theta_i > 0$ $\theta_v - \theta_i < 0$	• Lagging - inductive load • Leading - capacitive load

11.6 Complex Power (1)

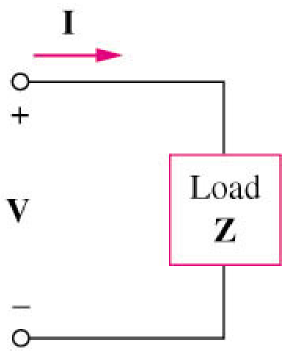
Complex power **S** is the product of the voltage and the complex conjugate of the current:



$$\mathbf{V} = V_m \angle \theta_v \qquad \mathbf{I} = I_m \angle \theta_i$$

$$\frac{1}{2}\mathbf{V}\,\mathbf{I}^* = V_{rms}\, I_{rms}\, \angle\, \theta_v - \theta_i$$

11.6 Complex Power (2)



$$S = \frac{1}{2} V I^* = V_{rms} I_{rms} \angle \theta_v - \theta_i$$

$$\Rightarrow S = \underbrace{V_{rms} I_{rms} \cos(\theta_v - \theta_i)}_{P} + j \underbrace{V_{rms} I_{rms} \sin(\theta_v - \theta_i)}_{Q}$$

S **=** **P** **+ j** **Q**

P: is the average power in watts delivered to a load and it is the only useful power.

Q: is the reactive power exchange between the source and the reactive part of the load. It is measured in VAR.

- $Q = 0$ for *resistive loads* (unity pf).
- $Q < 0$ for *capacitive loads* (leading pf).
- $Q > 0$ for *inductive loads* (lagging pf).

11.6 Complex Power (3)

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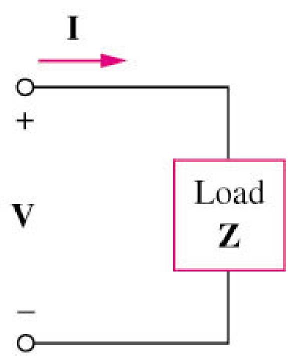


$$\Rightarrow \mathbf{S} = \underbrace{V_{rms}\, I_{rms} \cos(\theta_v - \theta_i)}_{P} + j\, \underbrace{V_{rms}\, I_{rms} \sin(\theta_v - \theta_i)}_{Q}$$

$$\mathbf{S} = P + j\,Q$$

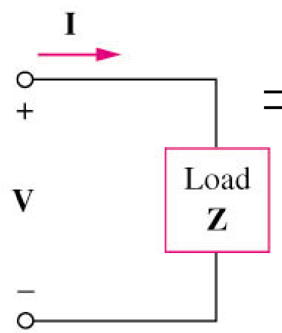
Apparent Power, $S = |\mathbf{S}| = \text{Vrms*Irms} = \sqrt{P^2 + Q^2}$

Real power, $P = \text{Re}(\mathbf{S}) = S\cos(\theta_v - \theta_i)$

Reactive Power, $Q = \text{Im}(\mathbf{S}) = S\sin(\theta_v - \theta_i)$

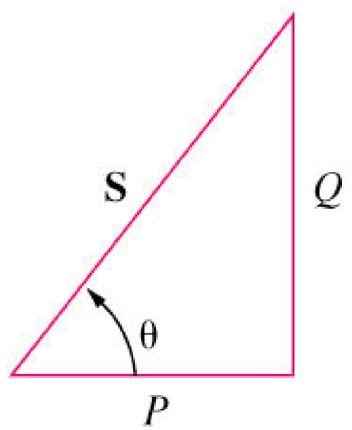
Power factor, $pf = P/S = \cos(\theta_v - \theta_i)$

11.6 Complex Power (4)

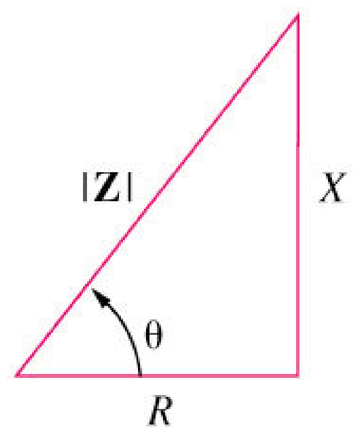


$$\Rightarrow S = \underbrace{V_{rms}\, I_{rms} \cos(\theta_v - \theta_i)}_{P} + j\, \underbrace{V_{rms}\, I_{rms} \sin(\theta_v - \theta_i)}_{Q}$$

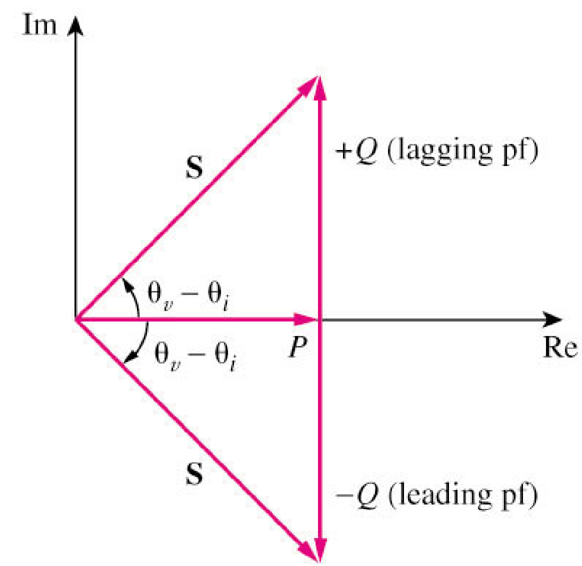
S = P + j Q



Power Triangle



Impedance Triangle



Power Factor

Homework #1
Due in class Wednesday, Jan 21

- Problems:
  - 3.23 (can use Matlab or any other matrix solver to solve the set of linear equations)
  - 3.51
  - 4.41
  - 5.27
  - 7.39a
  - 7.53a
  - 8.49
  - 10.1

No Class on Monday the 19th !

ECE 202 Review (Ch 8 – 11)

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- Part I DC Circuits
  - Chapter 8: 2nd Order Circuits
- Part II AC Circuits
  - Chapter 9: Sinusoids and Phasors
  - Chapter 10: Sinusoidal Steady-State Analysis
  - Chapter 11: AC Power Analysis

Second-Order Circuits

Chapter 8 - Review

8.1 Examples of 2nd order RCL circuit

8.2 Finding Initial and Final Values (X)

8.3 The source-free series RLC circuit

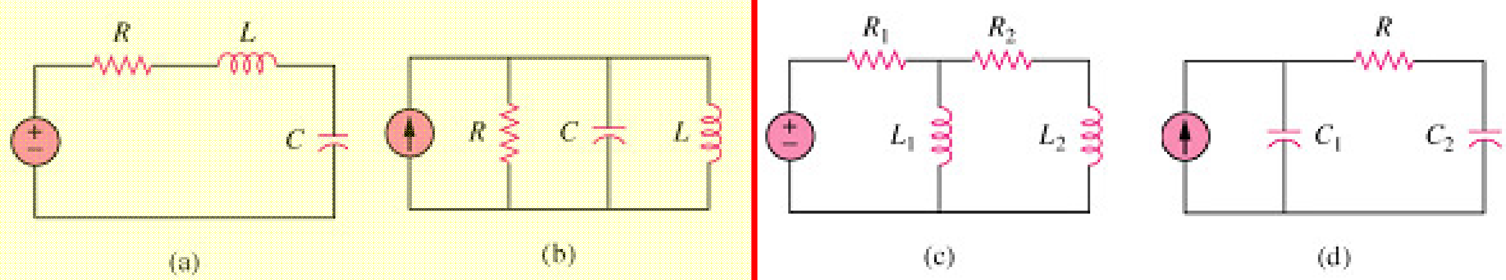
8.4 The source-free parallel RLC circuit

8.5 Step response of a series RLC circuit

8.6 Step response of a parallel RLC circuit

8.1 Examples of Second Order RLC circuits

A second-order circuit is characterized by a second-order differential equation. It consists of resistors and the equivalent of two energy storage elements.



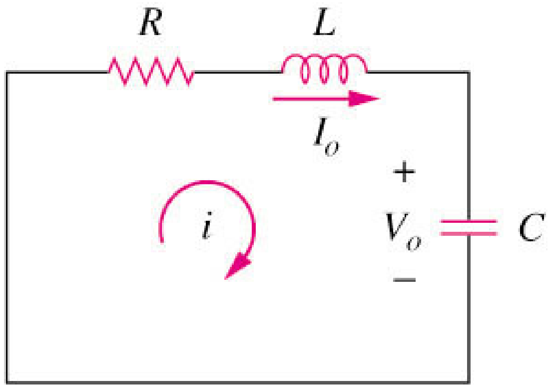
RLC Series

RLC Parallel

RL T-config

RC Pi-config

8.3 Source-Free Series RLC Circuits (1)



- The solution of the source-free series RLC circuit is called as the natural response of the circuit.

The 2nd order of expression

Derived from KVL:

$$iR + L\frac{di}{dt} + \frac{1}{C}\int_{-\infty}^{t} i(\tau)d\tau = 0$$

$$R\frac{di}{dt} + L\frac{d^2i}{dt^2} + \frac{1}{C}i = 0$$

$$\frac{d^2i}{dt^2} + \frac{R}{L}\frac{di}{dt} + \frac{i}{LC} = 0$$

Note: Same form if looking at voltage across capacitor!

To find for voltage across capacitor

Substitute: $i = C\frac{dv}{dt}$

$$RC\frac{dv}{dt} + LC\frac{d^2v}{dt^2} + v = 0$$

$$\frac{d^2v}{dt^2} + \frac{R}{L}\frac{dv}{dt} + \frac{v}{LC} = 0$$

Where v is the voltage across the capacitor

8.3 Source-Free Series RLC Circuits (2)

Three possible solutions for the 2nd order differential equation:

$$\frac{d^2 i}{dt^2} + \frac{R}{L}\frac{di}{dt} + \frac{i}{LC} = 0$$

=> $$\frac{d^2 i}{dt^2} + 2\alpha \frac{di}{dt} + \omega_0^2 i = 0$$

General 2nd order Form

where $\alpha = \frac{R}{2L}$ *and* $\omega_0 = \sqrt{\frac{1}{LC}}$

The types of solutions for i(t) depend on the relative values of α and ω.

General Solution

$$i(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t}$$

$$s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2}$$

- If $\alpha > \omega_o$ ➔ over-damped
- If $\alpha = \omega_o$ ➔ critically-damped
- If $\alpha < \omega_o$ ➔ under-damped

8.3 Source-Free Series RLC Circuits (3)

There are three possible solutions for the following 2nd order differential equation:

$$\frac{d^2i}{dt^2}+2\alpha\frac{di}{dt}+\omega_0^2 i=0 \qquad \alpha=\frac{R}{2L} \quad and \quad \omega_0=\sqrt{\frac{1}{LC}}$$

1. If $\alpha > \omega_o$, over-damped case

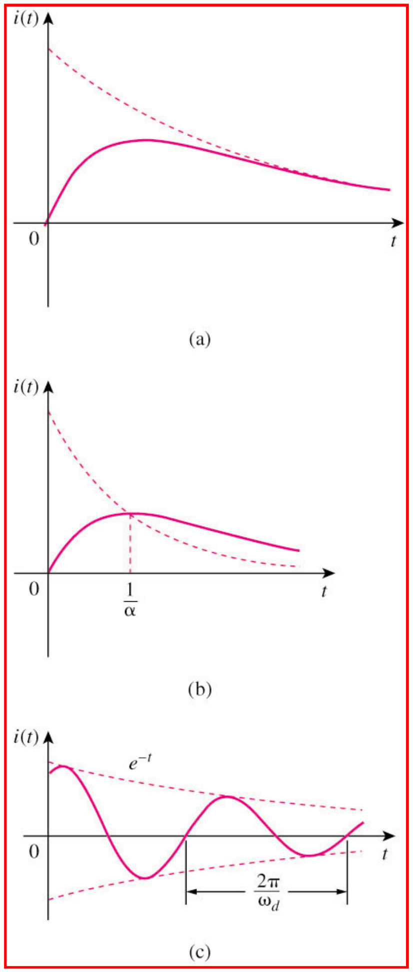
$$i(t)=A_1e^{s_1t}+A_2e^{s_2t}$$ **where** $$s_{1,2}=-\alpha\pm\sqrt{\alpha^2-\omega_0^2}$$

2. If $\alpha = \omega_o$, critical damped case

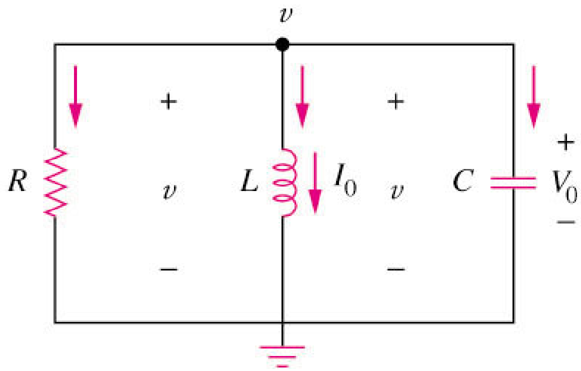
$$i(t)=(A_2+A_1t)e^{-\alpha t}$$ **where** $$s_{1,2}=-\alpha$$

3. If $\alpha < \omega_o$, under-damped case

$$i(t)=e^{-\alpha t}(B_1\cos\omega_d t+B_2\sin\omega_d t)$$ **where** $$\omega_d=\sqrt{\omega_0^2-\alpha^2}$$



8.4 Source-Free Parallel RLC Circuits (1)



- Similar approach to series RLC analysis

Derived from KCL:

$$\frac{v}{R}+C\frac{dv}{dt}+\frac{1}{L}\int_{-\infty}^{t} v(\tau)d\tau=0$$

$$\frac{1}{R}\frac{dv}{dt}+C\frac{d^2v}{dt^2}+\frac{1}{L}v=0$$

The 2nd order of expression

$$\frac{d^2v}{dt^2}+\frac{1}{RC}\frac{dv}{dt}+\frac{1}{LC}v=0$$

To find for current across inductor

Substitute: $v = L\frac{di}{dt}$

$$\frac{L}{R}\frac{di}{dt}+LC\frac{d^2i}{dt^2}+i=0$$

Note: Same form if looking at current through inductor!

$$\frac{d^2i}{dt^2}+\frac{1}{RC}\frac{di}{dt}+\frac{1}{LC}i=0$$

Where i is the current through the inductor

8.4 Source-Free Parallel RLC Circuits (2)

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There are three possible solutions for the following 2nd order differential equation:

$$\frac{d^2v}{dt^2} + 2\alpha\frac{dv}{dt} + \omega_0^2 v = 0 \quad \text{where} \quad \alpha = \frac{1}{2RC} \quad \text{and} \quad \omega_0 = \sqrt{\frac{1}{LC}}$$

1. If $\alpha > \omega_o$, over-damped case

$$v(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t}$$ **where** $$s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2}$$

2. If $\alpha = \omega_o$, critical damped case

$$v(t) = (A_2 + A_1 t)\, e^{-\alpha t}$$ **where** $$s_{1,2} = -\alpha$$

3. If $\alpha < \omega_o$, under-damped case

$$v(t) = e^{-\alpha t}(B_1 \cos \omega_d t + B_2 \sin \omega_d t)$$ **where** $$\omega_d = \sqrt{\omega_0^2 - \alpha^2}$$

Comparison Series / Parallel RLC Circuits

	Series RLC	Parallel RLC
Characteristic Equation	$\frac{d^2 i}{dt^2} + 2\alpha \frac{di}{dt} + \omega_0^2 i = 0$	$\frac{d^2 v}{dt^2} + 2\alpha \frac{dv}{dt} + \omega_0^2 v = 0$
Damping Factor	$\alpha = \frac{R}{2L}$	$\alpha = \frac{1}{2RC}$
Resonance Frequency	$\omega_0 = \sqrt{\frac{1}{LC}}$	$\omega_0 = \sqrt{\frac{1}{LC}}$
Over-damped $\alpha > \omega_o$	$i(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t}$ $s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2}$	$v(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t}$ $s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2}$
Critical damped $\alpha = \omega_o$,	$i(t) = (A_2 + A_1 t)\, e^{-\alpha t}$	$v(t) = (A_2 + A_1 t)\, e^{-\alpha t}$
Under-damped $\alpha < \omega_o$	$i(t) = e^{-\alpha t}(B_1 \cos \omega_d t + B_2 \sin \omega_d t)$ $\omega_d = \sqrt{\omega_0^2 - \alpha^2}$	$v(t) = e^{-\alpha t}(B_1 \cos \omega_d t + B_2 \sin \omega_d t)$ $\omega_d = \sqrt{\omega_0^2 - \alpha^2}$

Source-Free RLC Circuits

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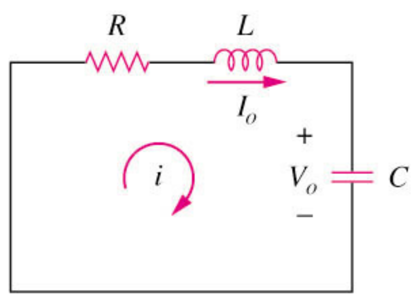
- Key to solving problems is to finding the initial conditions
  - Voltage doesn't change rapidly across a capacitor: $v(0^+) = v(0^-)$
  - Current doesn't change rapidly across an inductor: $i(0^+) = i(0^-)$
- Start by finding α and ω_o use these to find s_1 and s_2
  - Determine case (over / under / or critically damped)
- Apply initial conditions to the equations to find A_1 and A_1

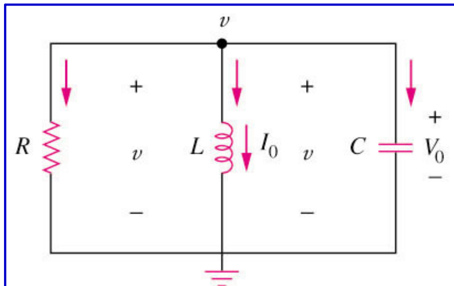


Series RLC

$$i(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t} \Rightarrow i(0^+) = A_1 + A_2$$

$$\frac{di(t)}{dt} = A_1 s_1 e^{s_1 t} + A_2 s_2 e^{s_2 t} \Rightarrow \frac{di(0^+)}{dt} = A_1 s_1 + A_2 s_2$$

$$\frac{di(0)}{dt} = -\frac{1}{L}(RI_0 + V_0)$$



Parallel RLC

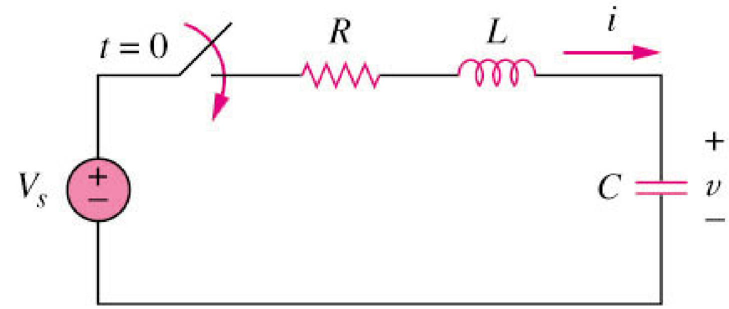
$$v(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t} \Rightarrow v(0^+) = A_1 + A_2$$

$$\frac{dv(t)}{dt} = A_1 s_1 e^{s_1 t} + A_2 s_2 e^{s_2 t} \Rightarrow \frac{dv(0^+)}{dt} = A_1 s_1 + A_2 s_2$$

$$\frac{dv(0)}{dt} = -\frac{1}{RC}(RI_0 + V_0)$$

8.5 Step-Response Series RLC Circuits (1)

- The step response is obtained by the sudden application of a dc source.



The 2nd order of expression

$$\underbrace{\frac{d^2v}{dt^2}+\frac{R}{L}\frac{dv}{dt}+\frac{v}{LC}}_{\textit{Natural Response}}=\frac{V_s}{LC}$$

The above equation has the same form as the equation for source-free series RLC circuit.

- The same coefficients (important in determining the frequency parameters).

8.5 Step-Response Series RLC Circuits (2)

The solution of the equation should have two components: the transient response $v_t(t)$ & the steady-state response $v_{ss}(t)$:

$$v(t) = v_t(t) + v_{ss}(t)$$

This is the Voltage across the Capacitor !!

- The transient response v_t is the same as that for source-free case

$$v_t(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t} \quad \text{(over-damped)}$$

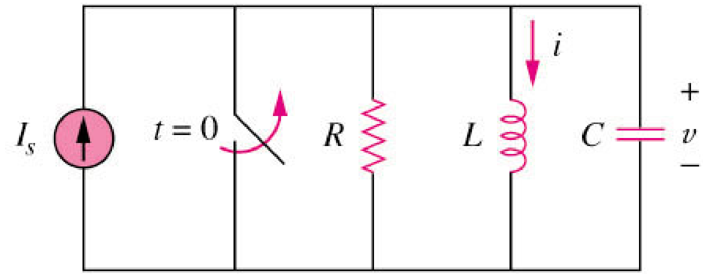
$$v_t(t) = (A_1 + A_2 t) e^{-\alpha t} \quad \text{(critically damped)}$$

$$v_t(t) = e^{-\alpha t}(A_1 \cos \omega_d t + A_2 \sin \omega_d t) \quad \text{(under-damped)}$$

- The steady-state response is the final value of v(t).
  - $v_{ss}(t) = v(\infty)$
- The values of A_1 and A_2 are obtained from the initial conditions:
  - v(0) and dv(0)/dt.

8.6 Step-Response Parallel RLC Circuits (1)

- The step response is obtained by the sudden application of a dc source.



The 2nd order of expression

$$\frac{d^2 i}{dt^2} + \frac{1}{RC}\frac{di}{dt} + \frac{i}{LC} = \frac{I_s}{LC}$$

It has the same form as the equation for source-free parallel RLC circuit.

- The same coefficients (important in determining the frequency parameters).

8.6 Step-Response Parallel RLC Circuits (2)

The solution of the equation should have two components: the transient response $i_t(t)$ & the steady-state response $i_{ss}(t)$:

$$i(t) = i_t(t) + i_{ss}(t)$$

This is the Current through the Inductor !!

- The transient response i_t is the same as that for source-free case

$$i_t(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t} \qquad \text{(over-damped)}$$

$$i_t(t) = (A_1 + A_2 t) e^{-\alpha t} \qquad \text{(critical damped)}$$

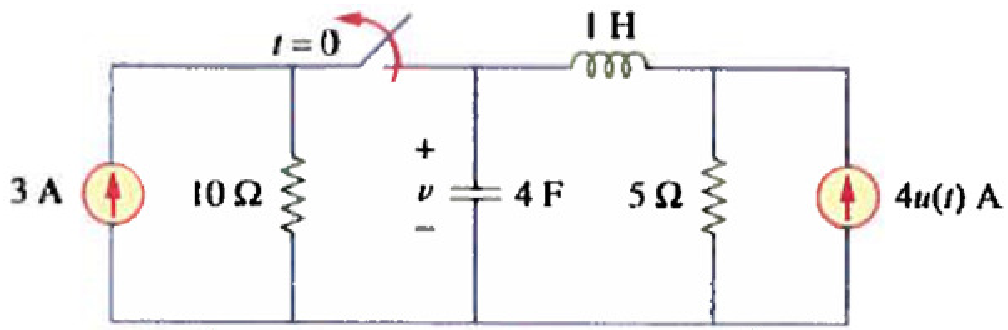
$$i_t(t) = e^{-\alpha t}(A_1 \cos \omega_d t + A_2 \sin \omega_d t) \qquad \text{(under-damped)}$$

- The steady-state response is the final value of i(t).
  - $i_{ss}(t) = i(\infty) = I_s$
- The values of A_1 and A_2 are obtained from the initial conditions:
  - i(0) and di(0)/dt.

Chapter 8 Example (1)

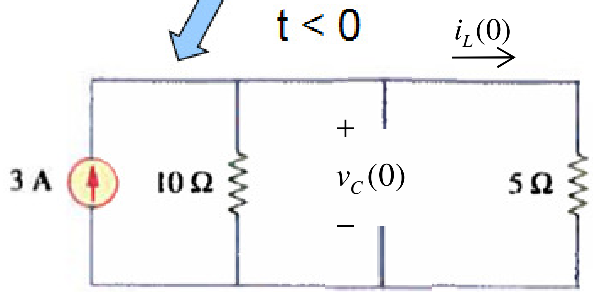
8.33 Find $v(t)$ for $t > 0$ in the circuit of Fig. 8.81.

Remember !!
Capacitor is "Open" to DC
Inductor is "Short" to DC



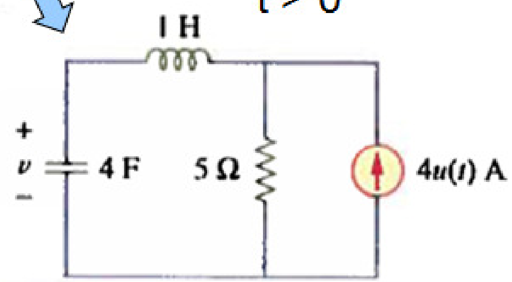
Before switch is opened

t < 0

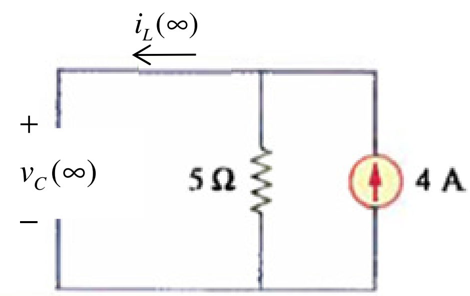


After switch is opened

t > 0



Steady State circuit (t → ∞)



Initial Voltage across Capacitor

$$v_C(0) = \frac{5 \cdot 10}{5+10}(3) = 10$$

Initial Current through Inductor

$$i_L(0) = 2$$

$$\alpha = \frac{R}{2L} = \frac{5}{(2)(1)} = 2.5$$

$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{(1)(4)}} = 0.5$$

$\alpha > \omega_0$ Over-damped Case

SS Voltage across Capacitor

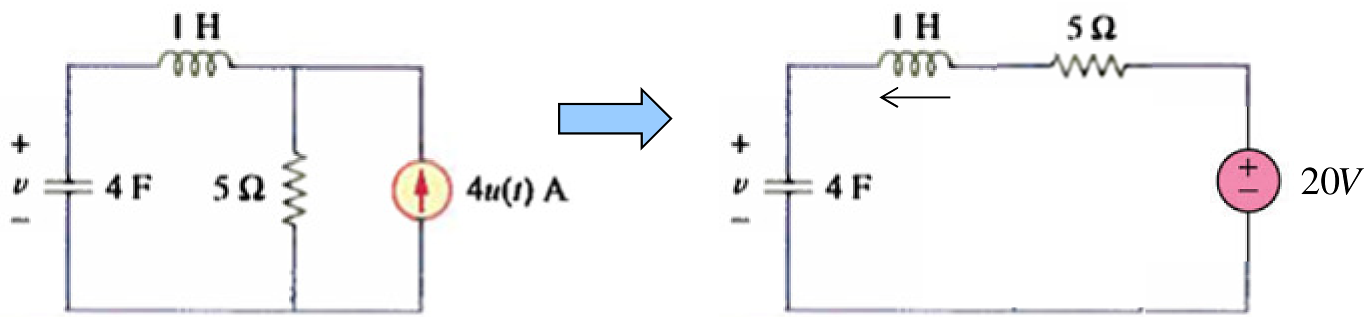
$$v_C(\infty) = (4)(5) = 20$$

SS Current through Inductor

$$i_L(\infty) = 0$$

Chapter 8 Example 8.33 (continued)

Circuit to be analyzed for response



Step Response of Series RLC Circuit

$$v(t) = v_t(t) + v_{ss}(t)$$

Voltage across Capacitor

$$v_t(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t}$$

(over-damped)

Known Parameters

$v_C(0) = 10$	$v_C(\infty) = 20$	$\alpha = 2.5$	$s_1 = 2.5 + \sqrt{(2.5)^2 - (0.5)^2} = 4.949$
$i_L(0) = 2$	$i_L(\infty) = 0$	$\omega_0 = 0.5$	$s_2 = 2.5 - \sqrt{(2.5)^2 - (0.5)^2} = 0.0505$

Voltage across Capacitor

$$v(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t} + 20$$

$$v(0) = A_1 + A_2 + 20 = 10$$

$$\boxed{A_1 + A_2 = -10}$$

Find Current through Inductor by taking derivative

$$i(0) = C\frac{dv(0)}{dt} \Rightarrow \frac{dv(0)}{dt} = \frac{i(0)}{C} = \frac{-2}{4}$$

Negative because in opposite direction

$$\frac{dv(0)}{dt} = \boxed{A_1 s_1 + A_2 s_2 = -0.5}$$

From here can solve for A1 and A2

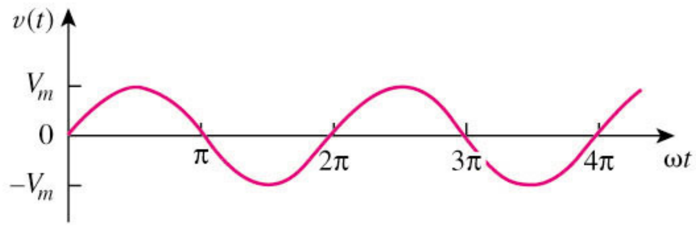
Sinusoids and Phasors Chapter 9 - Review

9.2 Sinusoids
9.3 Phasors
9.4 Phasor Relationships for Circuit Elements
9.5 Impedance and Admittance
9.6 Kirchhoff's Laws in the Frequency Domain
9.7 Impedance Combinations

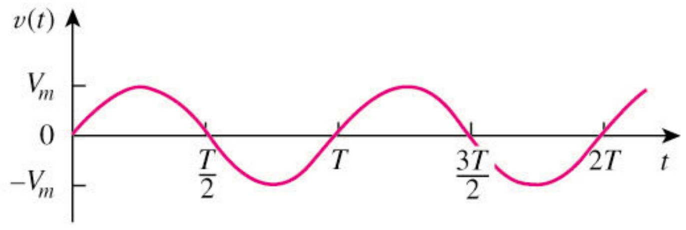
9.2 Sinusoids (1)

- A sinusoid is a signal that has the form of the sine or cosine function.
- A general expression for the sinusoid,

$$v(t) = V_m \sin(\omega t + \phi)$$



(a)



(b)

where

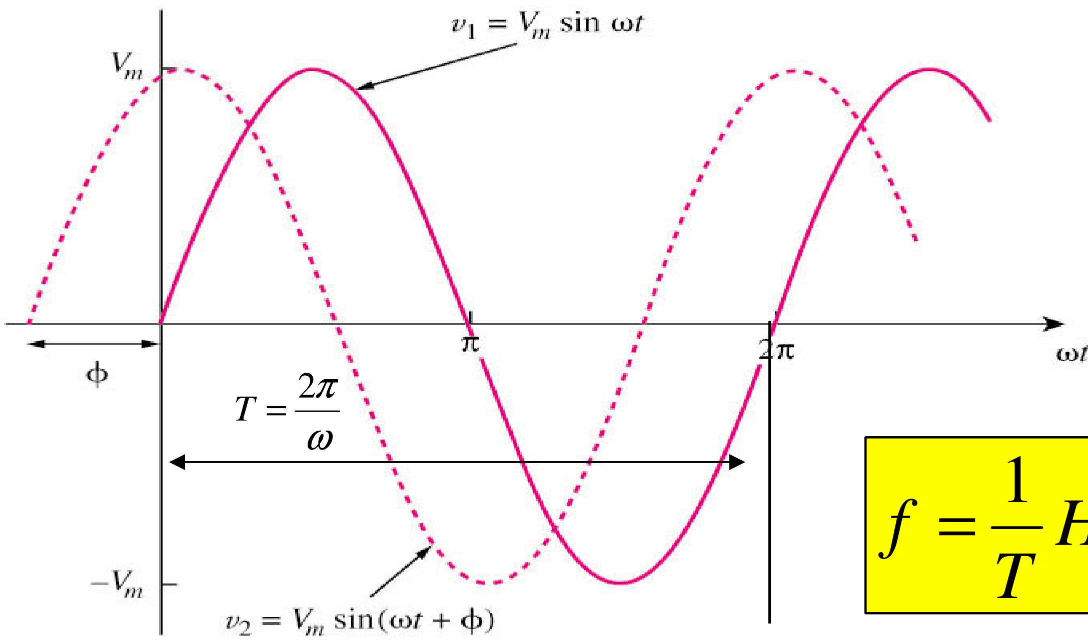
Vm = the **amplitude** of the sinusoid

ω = the angular frequency in radians/s

Φ = the phase

9.2 Sinusoids (2)

A <u>periodic function</u> is one that satisfies *v(t) = v(t + nT)*, for all t and for all integers n.



$$f = \frac{1}{T} Hz$$

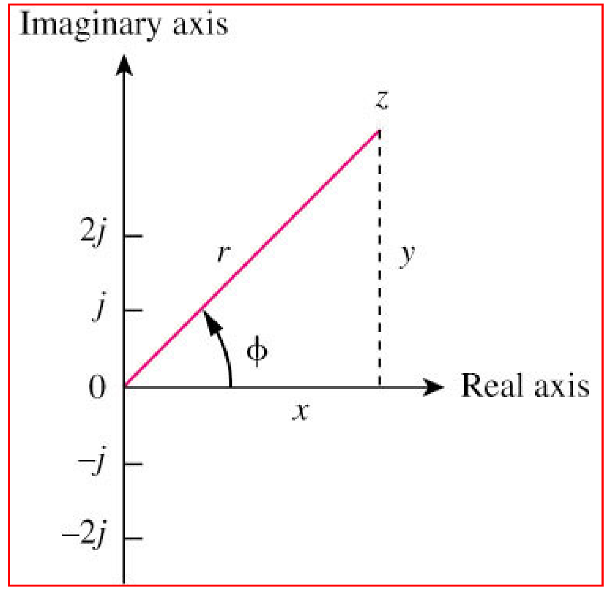
$$\omega = 2\pi f$$

- Only two sinusoidal values with the <u>same frequency</u> can be compared by their amplitude and phase difference.
- If phase difference is zero, they are in phase; if phase difference is not zero, they are out of phase.

9.3 Phasors (1)

- A phasor is a complex number that represents the amplitude and phase of a sinusoid.

- It can be represented in one of the following three forms:



a. Rectangular $z = x + jy = r(\cos\phi + j\sin\phi)$

b. Polar $z = r\angle\phi$

c. Exponential $z = re^{j\phi}$

where
$$r = \sqrt{x^2 + y^2}$$
$$\phi = \tan^{-1}\frac{y}{x}$$

Mathematic operation of complex number:

1. Addition $z_1 + z_2 = (x_1 + x_2) + j(y_1 + y_2)$
2. Subtraction $z_1 - z_2 = (x_1 - x_2) + j(y_1 - y_2)$
3. Multiplication $z_1 z_2 = r_1 r_2 \angle \phi_1 + \phi_2$
4. Division $\dfrac{z_1}{z_2} = \dfrac{r_1}{r_2} \angle \phi_1 - \phi_2$
5. Reciprocal $\dfrac{1}{z} = \dfrac{1}{r} \angle -\phi$
6. Square root $\sqrt{z} = \sqrt{r} \angle \phi/2$
7. Complex conjugate $z^* = x - jy = r \angle -\phi = re^{-j\phi}$
8. Euler's identity $e^{\pm j\phi} = \cos\phi \pm j\sin\phi$

9.3 Phasors (3)

- Transform a sinusoid to and from the time domain to the phasor domain:

$$v(t) = V_m \cos(\omega t + \phi) \longleftrightarrow V = V_m \angle \phi$$

(time domain) (phasor domain)

The differences between v(t) and V:

- v(t) is instantaneous or time-domain representation
  V is the frequency or phasor-domain representation.
- v(t) is time dependent, V is not.
- v(t) is always real with no complex term, V is generally complex.

Note: Phasor analysis applies only when frequency is constant; it is applied to two or more sinusoid signals only if they have the same frequency.

9.3 Phasors (5)

Relationship between differential, integral operation in phasor listed as follow:

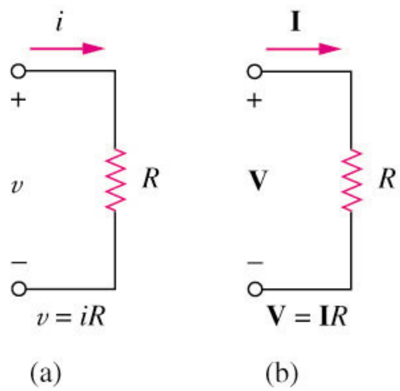
$$v(t) \longleftrightarrow \mathbf{V} = V\angle\phi$$

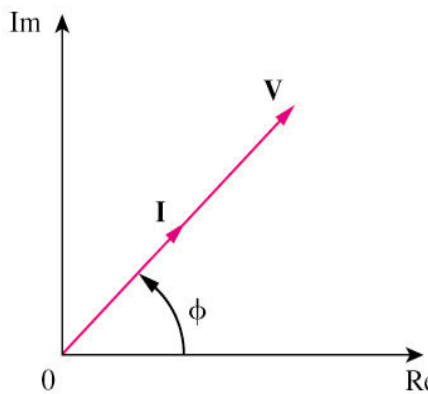
$$\frac{dv}{dt} \longleftrightarrow j\omega\mathbf{V}$$

$$\int v\,dt \longleftrightarrow \frac{\mathbf{V}}{j\omega}$$

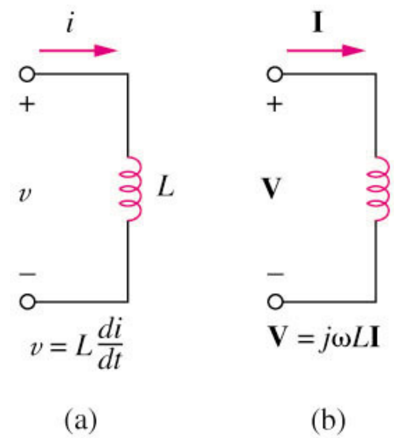
9.4 Phasor Relationships for Circuit Elements (1)

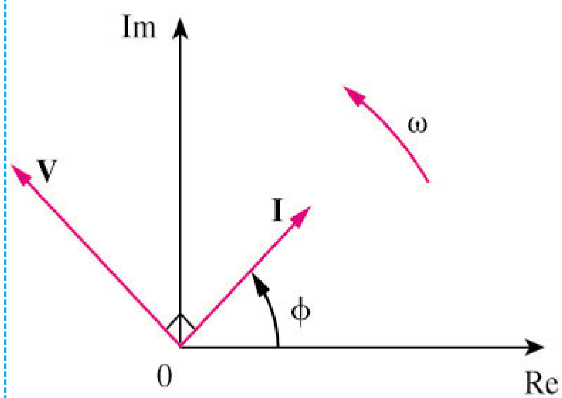
Resistors





Inductors

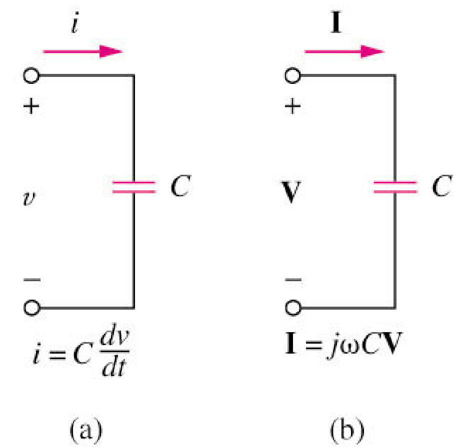


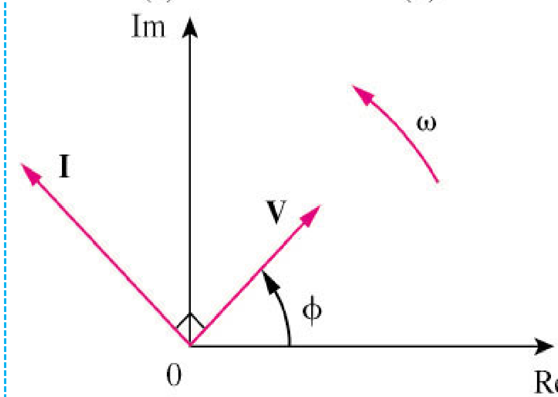


"ELI" *the*

Voltage Lead Current
For Inductors "L" (ELI)

Capacitors





"ICE" *man*

Current Lead Voltage
For Capacitors "C" (ICE)

Aside: ELI the ICE man & the derivative

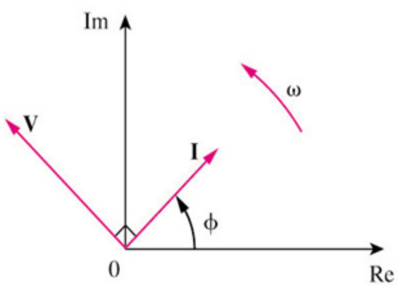
- The phrase "ELI the ICE man" can be used to remember the relationships of voltage & current for inductors & capacitors

E L I

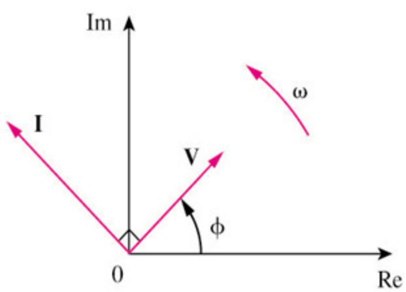
$$v = L\frac{di}{dt}$$

I C E

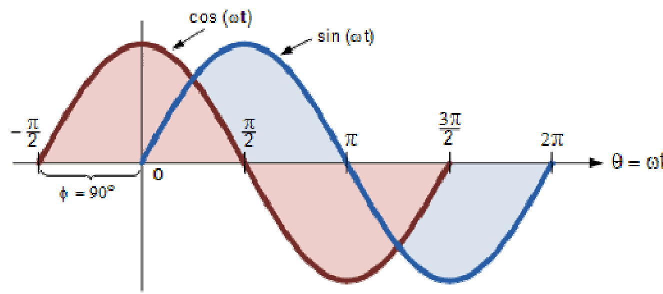
$$i = C\frac{dv}{dt}$$



Voltage leads current



Current leads voltage



Note:
1. Derivative of sin(t) is cos(t).
2. Observe the instantaneous "slope" of sin(t). It's cos(t)!
3. Derivative introduces a 90° shift
4. Now look at equations for L and C and this explains the 90° shift
5. Also, should be easier now to remember:

$$\frac{d}{dt}\left[\sin(\omega t)\right] = \omega\cos(\omega t)$$

$$\frac{d}{dt}\left[\cos(\omega t)\right] = -\omega\sin(\omega t)$$

9.4 Phasor Relationships for Circuit Elements (2)

Summary of voltage-current relationship		
Element	Time domain	Frequency domain
R	$v = Ri$	$V = RI$
L	$v = L\dfrac{di}{dt}$	$V = j\omega LI$
C	$i = C\dfrac{dv}{dt}$	$V = \dfrac{I}{j\omega C}$

9.5 Impedance and Admittance (1)

- The impedance Z of a circuit is the ratio of the phasor voltage V to the phasor current I, measured in ohms Ω.

$$Z = \frac{V}{I} = R + jX$$

Positive X is for L and negative X is for C.

- The admittance Y is the reciprocal of impedance, measured in siemens (S).

$$Y = \frac{1}{Z} = \frac{I}{V}$$

9.5 Impedance and Admittance (2)

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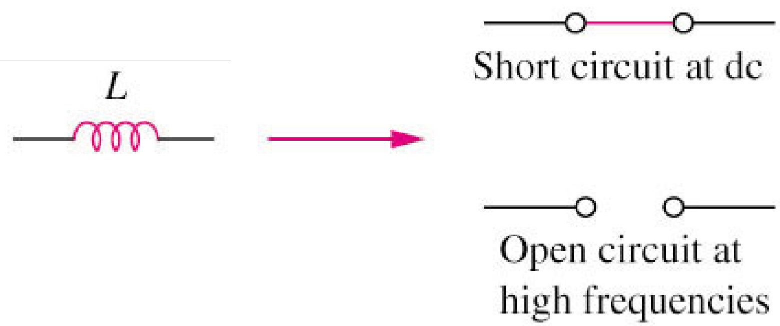
Impedances and admittances of passive elements		
Element	**Impedance**	**Admittance**
R	$Z = R$	$Y = \frac{1}{R}$
L	$Z = j\omega L$	$Y = \frac{1}{j\omega L}$
C	$Z = \frac{1}{j\omega C}$	$Y = j\omega C$

9.5 Impedance and Admittance (3)

$$Z = j\omega L$$

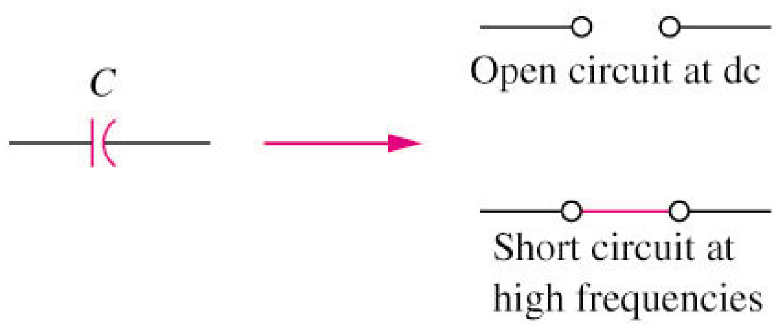


$$\omega = 0; Z = 0$$

$$\omega \to \infty; Z \to \infty$$

(a)

$$Z = \frac{1}{j\omega C}$$



$$\omega = 0; Z \to \infty$$

$$\omega \to \infty; Z = 0$$

(b)

9.6 Kirchhoff's Laws in the Frequency Domain (1)

After we know how to convert RLC components from time to phasor domain, we can transform a time domain circuit into a phasor/frequency domain circuit.

Hence, we can apply the KCL laws and other theorems to directly set up phasor equations involving our target variable(s) for solving.

9.6 Kirchhoff's Laws in the Frequency Domain (2)

- Both KVL and KCL are hold in the phasor domain or more commonly called frequency domain.
- Moreover, the variables to be handled are phasors, which are complex numbers.
- All the mathematical operations involved are now in complex domain.

9.7 Impedance Combinations (1)

- The following principles used for DC circuit analysis all apply to AC circuit.
- For example:
  a. voltage division
  b. current division
  c. circuit reduction
  d. impedance equivalence
  e. Y-Δ transformation

Sinusoidal Steady-State Analysis Chapter 10 Review

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10.1 Basic Approach
10.2 Nodal Analysis
10.3 Mesh Analysis
10.4 Superposition Theorem
10.5 Source Transformation
10.6 Thevenin and Norton Equivalent Circuits

10.1 Basic Approach

Steps to Analyze AC Circuits:

1. Transform the circuit to the phasor or frequency domain.
2. Solve the problem using circuit techniques (nodal analysis, mesh analysis, superposition, etc.).
3. Transform the resulting phasor to the time domain.

Time to Freq → Solve variables in Freq → Freq to Time

10.4 Superposition Theorem (1)

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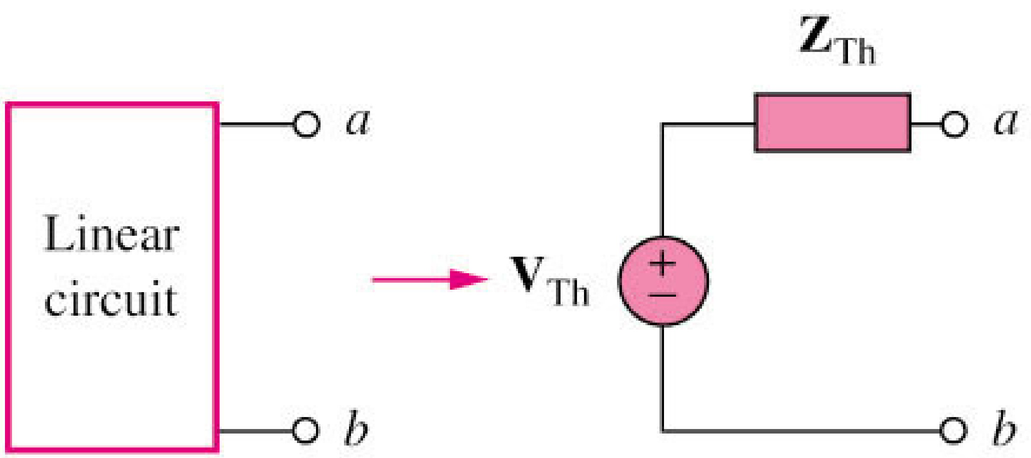
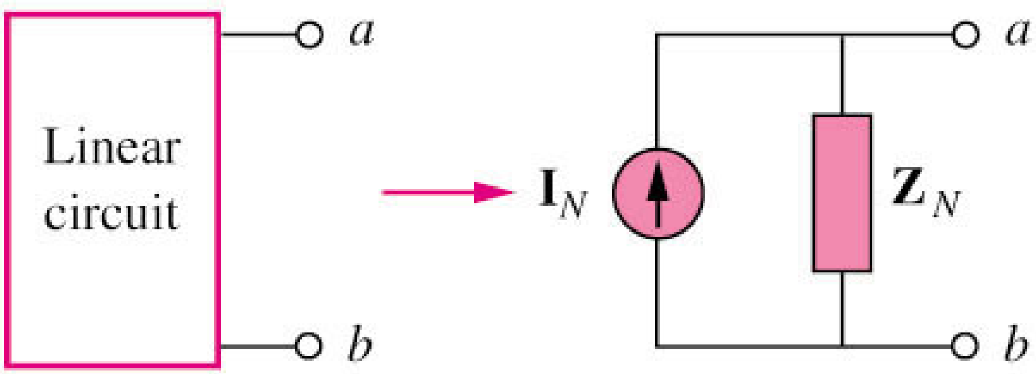
When a circuit has sources operating at different frequencies,

- The separate phasor circuit for each frequency must be solved independently, and
- The total response is the sum of time-domain responses of all the individual phasor circuits.

10.6 Thevenin and Norton Equivalent Circuits (1)



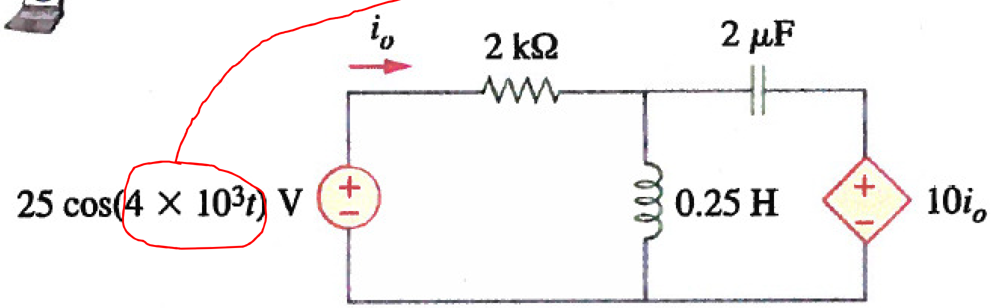
Thevenin transform



Norton transform

Chapter 10 Example (1)

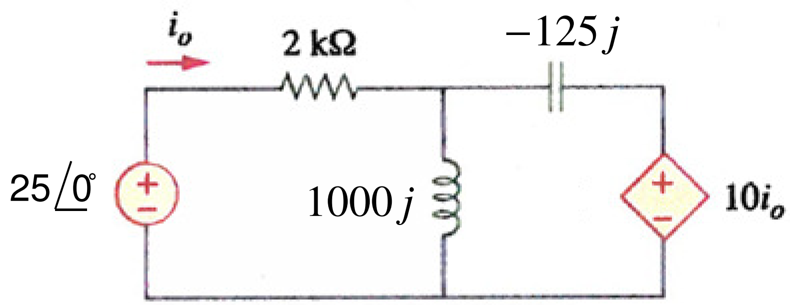
10.5 Find i_o in the circuit of Fig. 10.54.



$$\omega = 4000$$

$$Z_L = j\omega L = j4000(0.25) = 1000j$$

$$Z_C = \frac{1}{j\omega C} = \frac{1}{j4000(2\mu)} = -125j$$



Mesh 1

$$-25 + 2000i_1 + 1000j(i_1 - i_2) = 0$$

$$(2000 + 1000j)i_1 - (1000j)i_2 = 25$$

Mesh 2

$$1000j(i_2 - i_1) + (-125j)i_2 + 10i_1 = 0$$

$$(10 - 1000j)i_1 + (875j)i_2 = 0$$

$$\begin{bmatrix} 2000+1000j & -1000j \\ 10-1000j & 875j \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} 25 \\ 0 \end{bmatrix}$$

11.2 Instantaneous and Average Power (1)

- The instantaneously power, p(t), absorbed by an element is the product of the instantaneous voltage v(t) across the element and the instantaneous current i(t) through it.

$$p(t) = v(t)\,i(t) = V_m I_m \cos(\omega t + \theta_v)\cos(\omega t + \theta_i)$$

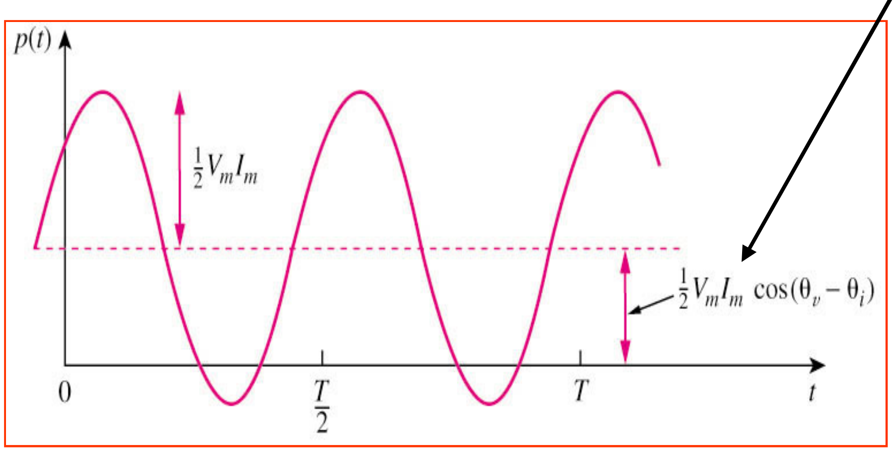
$$= \frac{1}{2} V_m I_m \cos(\theta_v - \theta_i) + \frac{1}{2} V_m I_m \cos(2\omega t + \theta_v + \theta_i)$$

Constant power **Sinusoidal power at 2ωt**

11.2 Instantaneous and Average Power (2)

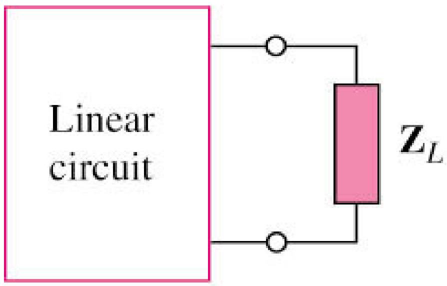
- The average power, P, *is the average of the instantaneous power over one period.*

$$\boxed{P = \frac{1}{T}\int_0^T p(t)\,dt = \frac{1}{2} V_m\, I_m \cos(\theta_v - \theta_i)}$$

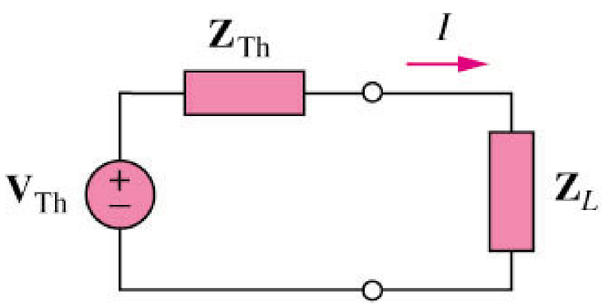


1. **P is not time dependent.**
2. **When $\theta_v = \theta_i$, it is a purely resistive load case.**
3. **When $\theta_v - \theta_i = \pm 90o$, it is a purely reactive load case.**
4. **P = 0 means that the circuit absorbs no average power.**

11.3 Maximum Average Power Transfer (1)



(a)



(b)

Maximum power is transferred to the load if the load impedance is the complex conjugate of the Thevenin impedance.

$$Z_{TH} = R_{TH} + jX_{TH}$$

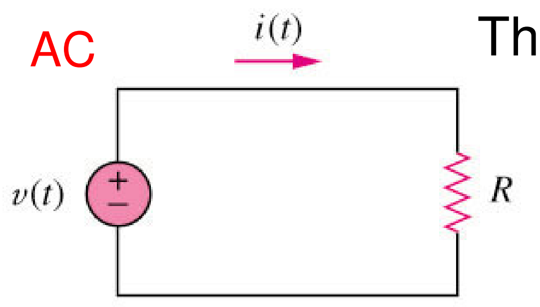
$$\boxed{Z_L = Z_{Th}^* = R_{TH} - jX_{TH}}$$

$$P_{max} = \frac{|V_{TH}|^2}{8R_{TH}}$$

When $Z_L = Z_{Th}^*$ we say the load is "matched" to the source.

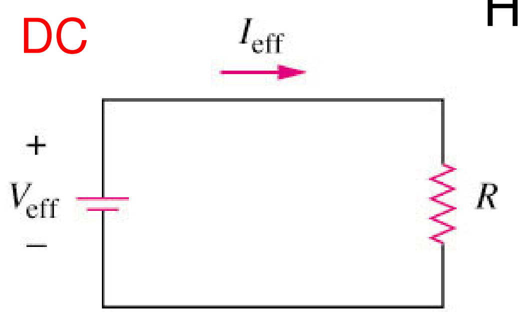
11.4 Effective or RMS Value (1)

AC



(a)

DC



(b)

The total power dissipated by R is given by:

$$P = \frac{1}{T}\int_0^T i^2 R dt = \frac{R}{T}\int_0^T i^2 dt = I_{rms}^2 R$$

Hence, I_{eff} is equal to:

$$\boxed{I_{eff} = \sqrt{\frac{1}{T}\int_0^T i^2\, dt} = I_{rms}}$$

The rms value is a constant itself which depending on the shape of the function i(t).

The effective value of a periodic current is the dc current that delivers the same average power to a resistor as the periodic current.

11.5 Apparent Power and Power Factor (1)

- Apparent Power, S, is the product of the r.m.s. values of voltage and current.
- It is measured in volt-amperes or VA to distinguish it from the average or real power which is measured in watts.

$$P = \underbrace{V_{rms}\, I_{rms}}_{\text{Apparent Power, S}} \cos(\theta_v - \theta_i) = S \underbrace{\cos(\theta_v - \theta_i)}_{\text{Power Factor, pf}}$$

- Power factor is the cosine of the phase difference between the voltage and current. It is also the cosine of the angle of the load impedance.

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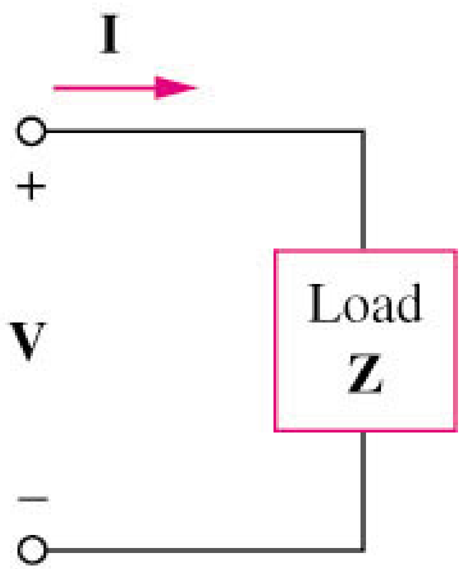
11.5 Apparent Power and Power Factor (2)

Load Type	Power Factor Angle	Power Factor
Purely resistive load (R)	$\theta_v - \theta_i = 0$, $Pf = 1$	P/S = 1, all power are consumed
Purely reactive load (L or C)	$\theta_v - \theta_i = \pm 90^o$, $pf = 0$	P = 0, no real power consumption
Resistive and reactive load (R and L/C)	$\theta_v - \theta_i > 0$ $\theta_v - \theta_i < 0$	• Lagging - inductive load • Leading - capacitive load

11.6 Complex Power (1)

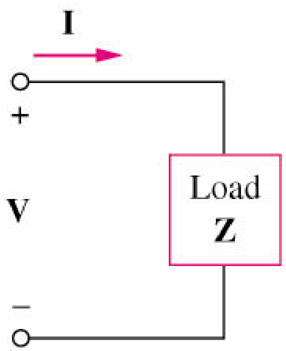
Complex power **S** is the product of the voltage and the complex conjugate of the current:



$$\mathbf{V} = V_m \angle \theta_v \qquad \mathbf{I} = I_m \angle \theta_i$$

$$\frac{1}{2} \mathrm{V}\, \mathrm{I}^* = \mathrm{V}_{rms}\, \mathrm{I}_{rms} \angle \theta_v - \theta_i$$

11.6 Complex Power (2)



$$S = \frac{1}{2} V\, I^* = V_{rms}\, I_{rms} \angle \theta_v - \theta_i$$

$$\Rightarrow S = \underbrace{V_{rms}\, I_{rms} \cos(\theta_v - \theta_i)}_{P} + j\, \underbrace{V_{rms}\, I_{rms} \sin(\theta_v - \theta_i)}_{Q}$$

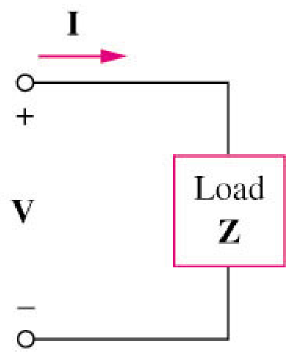
***S* = P + j Q**

P: is the <u>average power in watts</u> delivered to a load and it is the only useful power.

Q: is the <u>reactive power exchange</u> between the source and the reactive part of the load. It is measured in VAR.

- Q = 0 for *resistive loads* (unity pf).
- Q < 0 for *capacitive loads* (leading pf).
- Q > 0 for *inductive loads* (lagging pf).

11.6 Complex Power (3)



$$\Rightarrow S = \underbrace{V_{rms}\, I_{rms} \cos(\theta_v - \theta_i)}_{P} + j\, \underbrace{V_{rms}\, I_{rms} \sin(\theta_v - \theta_i)}_{Q}$$

$$\mathbf{S} = P + jQ$$

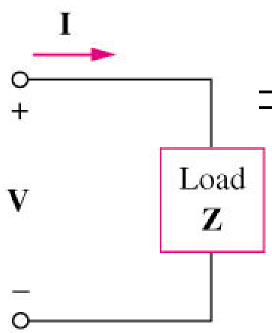
Apparent Power, $S = |\mathbf{S}| = V_{rms} * I_{rms} = \sqrt{P^2 + Q^2}$

Real power, $P = \mathrm{Re}(\mathbf{S}) = S \cos(\theta_v - \theta_i)$

Reactive Power, $Q = \mathrm{Im}(\mathbf{S}) = S \sin(\theta_v - \theta_i)$

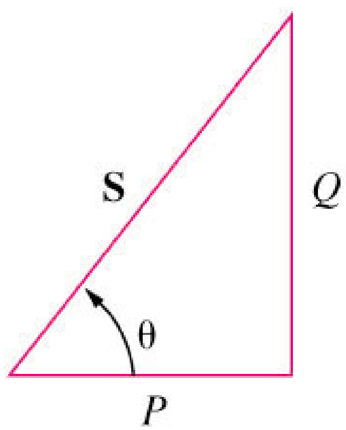
Power factor, $pf = P/S = \cos(\theta_v - \theta_i)$

11.6 Complex Power (4)

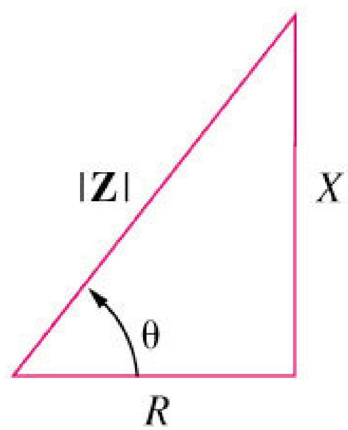


$$\Rightarrow S = \underbrace{V_{rms}\, I_{rms} \cos(\theta_v - \theta_i)}_{P} + j\, \underbrace{V_{rms}\, I_{rms} \sin(\theta_v - \theta_i)}_{Q}$$

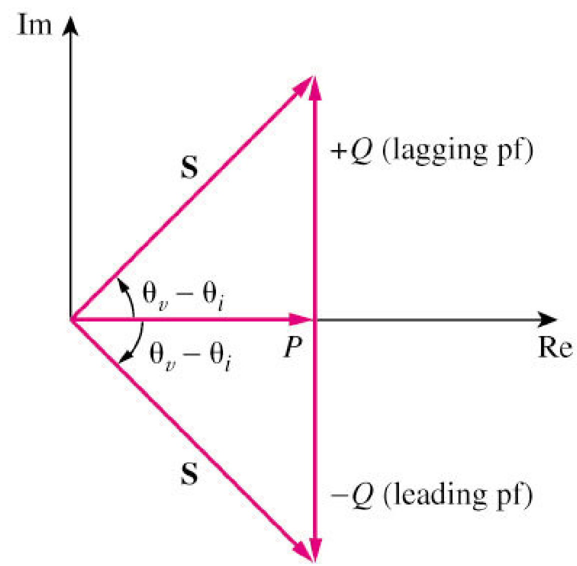
$$\mathbf{S} = P + j\,Q$$



Power Triangle



Impedance Triangle



Power Factor

Homework #1
Due in class Wednesday, Jan 21

- Problems:
  - 3.23 (can use Matlab or any other matrix solver to solve the set of linear equations)
  - 3.51
  - 4.41
  - 5.27
  - 7.39a
  - 7.53a
  - 8.49
  - 10.1

No Class on Monday the 19th !