IUPUI ECE 202 Spring 2015:

Exam #2 (SOLUTIONS) Name:

1. (15 points) For the following transfer function:
   a. (10 Points) Draw the magnitude Bode plot
   b. (3 Points) What type of filter is this (High Pass, Low Pass, Band Pass, or Band Stop) and what are the cutoff / corner frequencies (in rad/s)?
   c. (2 Points) Is this an Active or Passive filter?

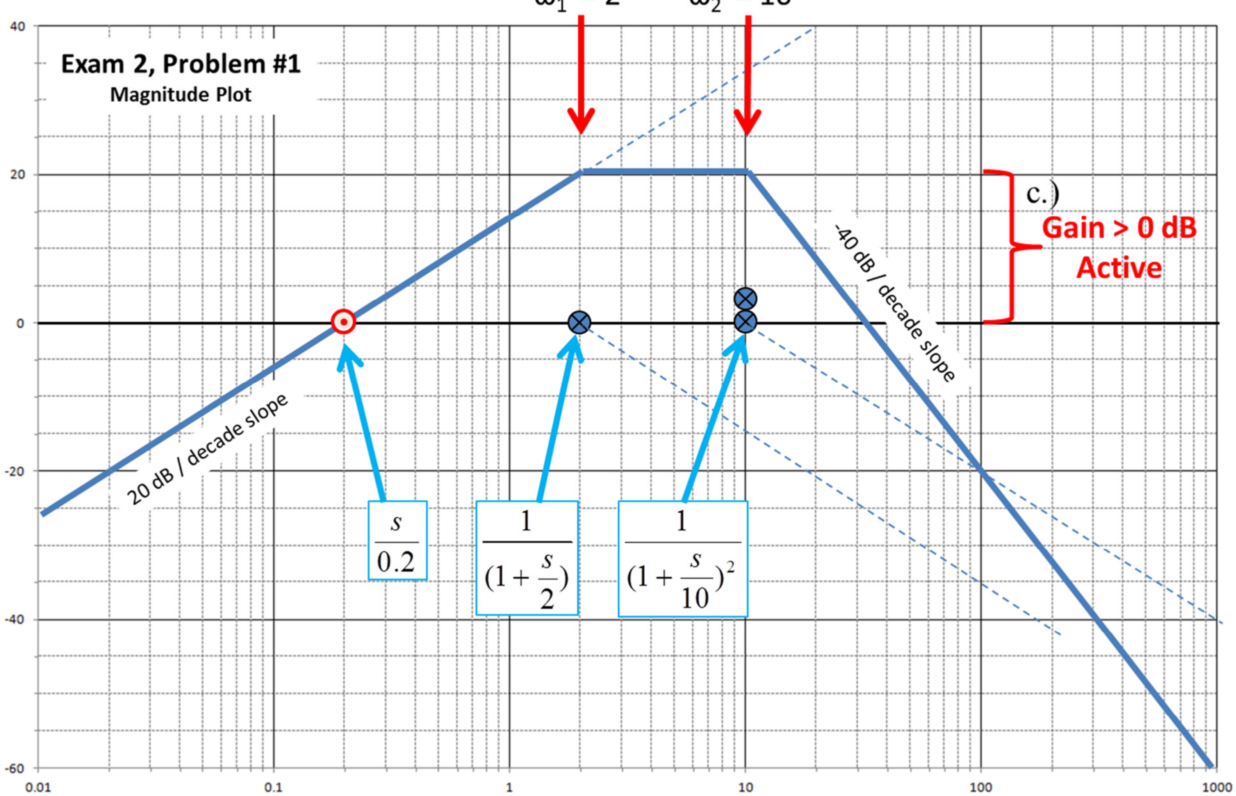
$$H(s) = \frac{1000s}{(s+2)(s+10)^2}$$

a.) $$H(s) = \frac{1000s}{(s+2)(10+s)^2} = \frac{1000s}{2(1+\frac{s}{2})10(1+\frac{s}{10})10(1+\frac{s}{10})}$$

$$H(s) = \frac{5s}{(1+\frac{s}{2})(1+\frac{s}{10})^2} = \boxed{\frac{\frac{s}{0.2}}{(1+\frac{s}{2})(1+\frac{s}{10})^2}}$$

b.) **Band Pass**

$\omega_1 = 2$ $\omega_2 = 10$

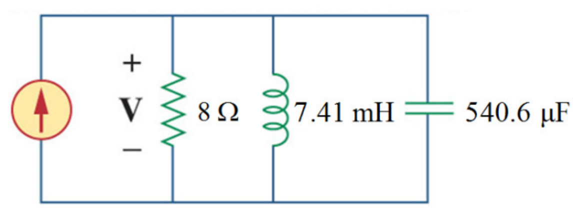


2. (20 points) Design a ***parallel RLC*** circuit to act as an audio band pass filter for the frequency range of **63.2 to 100 Hz** (*Band 4 of a 16 band graphic equalizer*). The filter should pass signals contained within this band and have **8 Ohm** impedance at resonance.
   a) (4 Points) Determine the ***center frequency*** and the quality factor "**Q**"
   b) (8 Points) Determine the component values for **R, L**, and **C** and **Draw the Circuit**
   c) (6 Points) Scale this filter up to *Band 7* so its center frequency is now at **317 Hz** and find the new values **R', L'**, and **C'**.
   d) (2) Points) For this new circuit find the new bandwidth **B'**

a.)
$$\mathrm{B}_{\mathrm{Hz}} = 100 - 63.2 = 36.8 \ \text{Hz}$$
$$f_o = \sqrt{f_1 f_2} = \sqrt{(63.2)(100)} = \boxed{79.5 \ \text{Hz}}$$
$$Q = \frac{\omega_0}{\mathrm{B}} = \frac{f}{\mathrm{B}_{\mathrm{Hz}}} = \frac{79.5}{36.8} = \boxed{2.16}$$

Alternate Solution:
$$\omega_1 = 2\pi \times f_1 = 2\pi \times 63.2 = 397.1 \ \text{rad/s}$$
$$\omega_2 = 2\pi \times f_2 = 2\pi \times 100 = 628.3 \ \text{rad/s}$$
$$\omega_o = \sqrt{\omega_1 \omega_2} = \sqrt{(397.1)(628.3)} = \boxed{499.5 \ \text{rad/s}}$$
$$\mathrm{B} = \omega_2 - \omega_1 = 628.3 - 397.1 = 231.2 \ \text{rad/s}$$
$$Q = \frac{\omega_0}{\mathrm{B}} = \frac{499.5}{231.2} = \boxed{2.16}$$

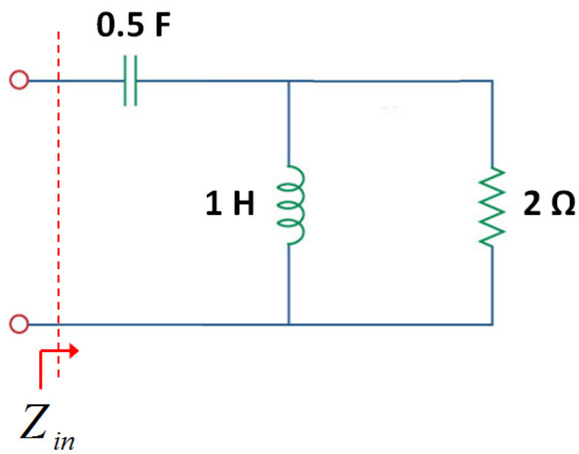
b.)
$$\boxed{R = 8 \ \Omega}$$
$$Q = \frac{R}{\omega_0 L} = \omega_0 RC$$
$$L = \frac{R}{\omega_0 Q} = \frac{R}{(2\pi f_0)Q} = \frac{8}{(2\pi \times 79.5)(2.16)} = \boxed{7.41 \times 10^{-3} \ \text{H or } 7.41 \ \text{mH}}$$
$$C = \frac{Q}{\omega_0 R} = \frac{2.16}{(2\pi \times 79.5)(8)} = \boxed{540.6 \times 10^{-6} \ \text{F or } 540.6 \ \mu\text{F}}$$



c.)
$$K_m = 1 \quad \text{no change in magnitude}$$
$$K_f = \frac{f'}{f} = \frac{317}{79.5} = 3.99 \quad \text{or} \quad \frac{\omega'}{\omega} = \frac{2\pi \times 317}{499.5} = 3.99$$
$$R' = \boxed{8 \ \Omega}$$
$$C' = \frac{1}{K_f} C = \frac{540.6 \ \mu\text{F}}{3.99} = \boxed{135.6 \ \mu\text{F}}$$
$$L' = \frac{1}{K_f} L = \frac{7.41 \ \text{mH}}{3.99} = \boxed{1.86 \ \text{mH}}$$

d.)
$$\mathrm{B}' = K_f \mathrm{B} = 3.99 \times (36.8 \ \text{Hz}) = \boxed{146.8 \ \text{Hz or } 922.6 \ \text{rad/s}}$$

3. (15 points) Find the resonant frequency for the circuit shown below:



$$Z_{in} = \frac{1}{j\omega C} + j\omega L \| R = \frac{1}{j\omega(0.5)} + \frac{(j\omega)(2)}{2 + j\omega}$$

$$Z_{in} = -j\frac{2}{\omega} + \frac{2j\omega}{2 + j\omega} \cdot \frac{(2 - j\omega)}{(2 - j\omega)} = -j\frac{2}{\omega} + \frac{2\omega^2 + 4j\omega}{2^2 + \omega^2}$$

$$Z_{in} = \frac{2\omega^2}{4 + \omega^2} + j\underbrace{\left(\frac{4\omega}{4 + \omega^2} - \frac{2}{\omega}\right)}$$

$$\boxed{\mathrm{Im}_{Z_{in}} = \frac{4\omega}{4 + \omega^2} - \frac{2}{\omega} = 0 \quad \text{At resonance}}$$

$$\frac{4\omega}{4 + \omega^2} = \frac{2}{\omega}$$

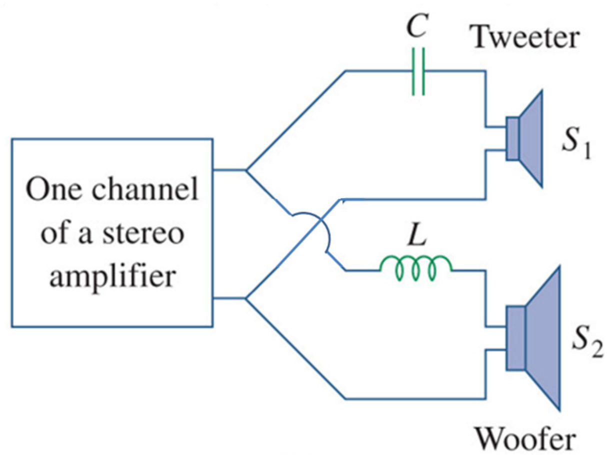
$$4\omega^2 = 2(4 + \omega^2)$$

$$2\omega^2 = 4 + \omega^2$$

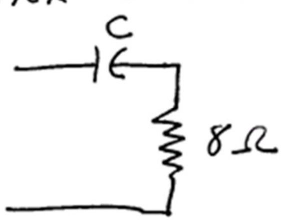
$$\omega^2 = 4$$

$$\boxed{\omega = 2}$$

4. (15 points) Design a crossover circuit for a speaker with a Woofer & Tweeter shown below. Assume the impedance of each speaker is **8 Ω** and let the crossover frequency be **2.5 KHz**. Determine the values for **C** and **L**. (assume no interaction between the two branches).



TWEETER CIRCUIT:



HIGH PASS FILTER

$$\omega_c = \frac{1}{RC}$$

$$f_c = 2.5\times10^3 \text{ Hz}$$

$$C = \frac{1}{\omega_c R}$$

$$\omega_c = 2\pi f_c = 15.7\times10^3 \tfrac{rad}{s}$$

$$C = \frac{1}{(15.7\times10^3)(8)} = \boxed{\begin{aligned} &7.96\times10^{-6} F \\ =\ &7.96 \mu F \end{aligned}}$$

WOOFER CIRCUIT

LOW PASS FILTER
$f_c = 2.5$ kHz

$$\omega_c = 2\pi f_c = \frac{R}{L}$$

$$L = \frac{R}{2\pi f_c}$$

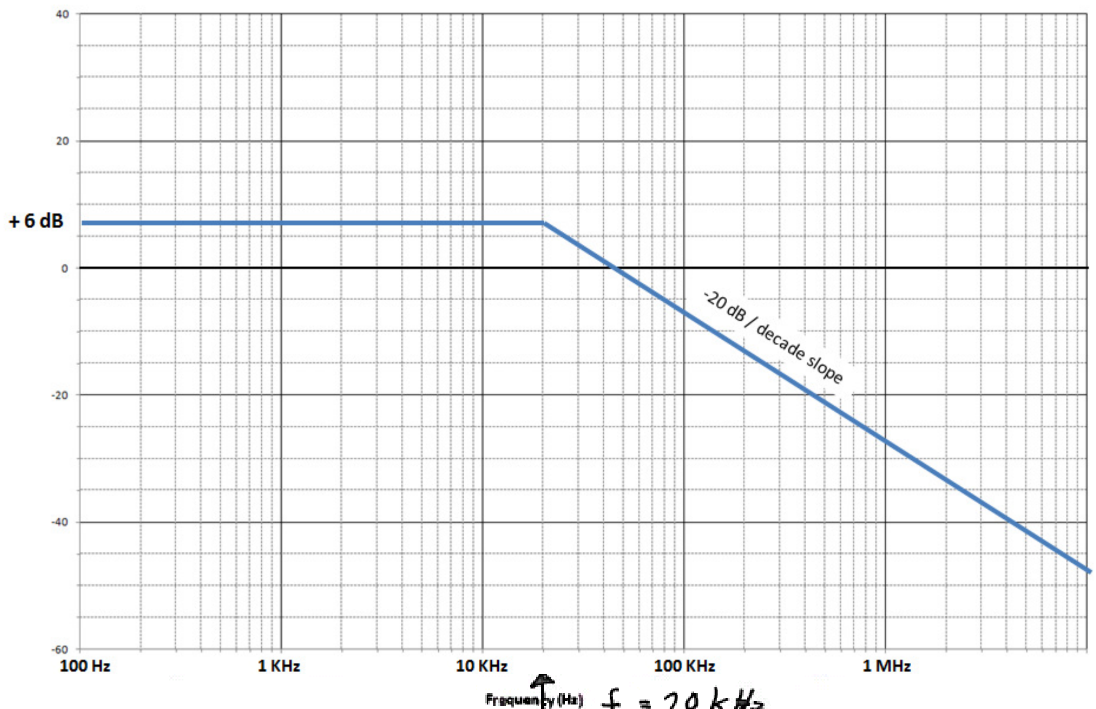
$$L = \frac{8}{2\pi\cdot(2.5\times10^3)} = \boxed{\begin{aligned} &509.3\times10^{-6} H \\ =\ &509.3 \mu H \end{aligned}}$$

5. (20 points) Design and **active** filter to give the frequency response shown below. Use only one OPAMP in the circuit. Determine the **component values** and **draw the circuit**.
(Note the response is shown in Hz !)

GAIN = +6dB

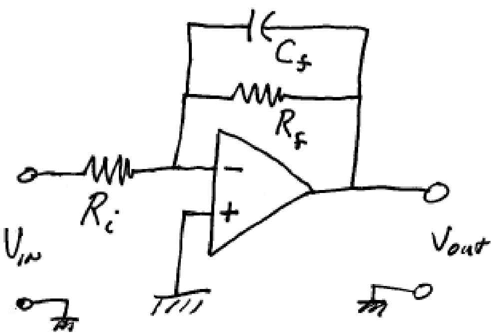
$6dB = 20\log_{10} K$

$K = 10^{\frac{6}{20}} = 2$



$f_c = 20\,kHz$

LOW PASS FILTER.



$\omega_c = 2\pi f_c = 2\pi \cdot 20\,kHz$

$\omega_c = 125.7 \times 10^3\ rad/sec$

VOLTAGE GAIN $= \frac{R_f}{R_i} = 2$

CHOOSE ONE RESISTOR, CALCULATE THE OTHER.

i.e. $\boxed{R_i = 10k\Omega} \Rightarrow \boxed{R_f = 20k\Omega}$

CAPACITANCE FOUND BY:

$$\omega_c = \frac{1}{R_f C_f} \Rightarrow C_f = \frac{1}{\omega_c R_f} = \frac{1}{(125.7\times10^3)(20\times10^3)} = \boxed{397.9\times10^{-12}F} = \boxed{397.9pF}$$

∴ MAY HAVE DIFFERENT VALUES DEPENDING ON CHOICE OR R_i R_f.

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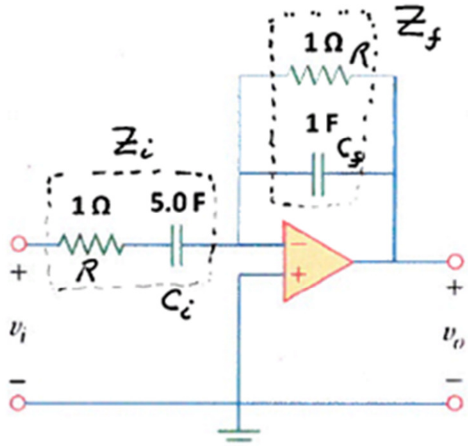
6. (15 Points) For the circuit shown below:
   a) (8 Points) Find the transfer function **H(s)**.
   b) (4 Points) Put **H(s)** in proper Bode form $(1+\frac{s}{p_1})$ … and find the cutoff frequency(s).
   c) (3 Points) What type of filter is this (High Pass, Low Pass, Band Pass, or Band Stop)?



a.) $H(s) = -\frac{Z_f}{Z_i} = -Z_f\left(\frac{1}{Z_i}\right)$

$Z_f = R \,||\, \frac{1}{sC_f}$

$Z_f = \frac{R(\frac{1}{sC_f})}{R+\frac{1}{sC_f}} = \frac{R}{sRC_f+1}$

$Z_i = R + \frac{1}{sC_i}$

$H(s) = -\left(\frac{R}{sRC_f+1}\right)\cdot\frac{1}{R+\frac{1}{sC_i}} = \frac{-sRC_i}{(sRC_f+1)(sRC_i+1)}$

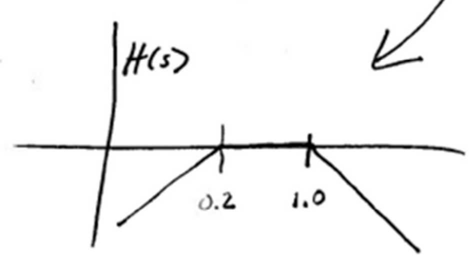
$= \frac{-5s}{(s\cdot 1\cdot 1+1)(s\cdot 1\cdot 5+1)} = \boxed{\frac{-5s}{(s+1)(5s+1)}}$

zero offset at 0.2

b.) $H(s) = \frac{-5s}{(1+s)(1+\frac{s}{1/5})} = \frac{-\frac{s}{1/5}}{(1+s)(1+\frac{s}{0.2})} = \boxed{\frac{-\frac{s}{0.2}}{(1+s)(1+\frac{s}{0.2})}}$

$p=1$ $\quad p=0.2$

$\boxed{\omega_1 = 0.2\ ;\ \omega_2 = 1.0}$



c.) BAND PASS