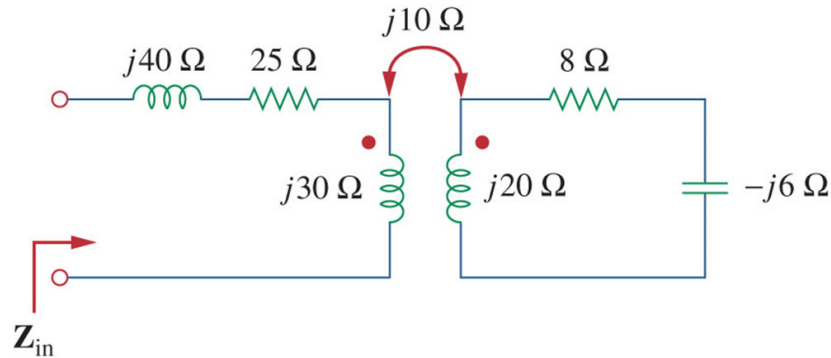


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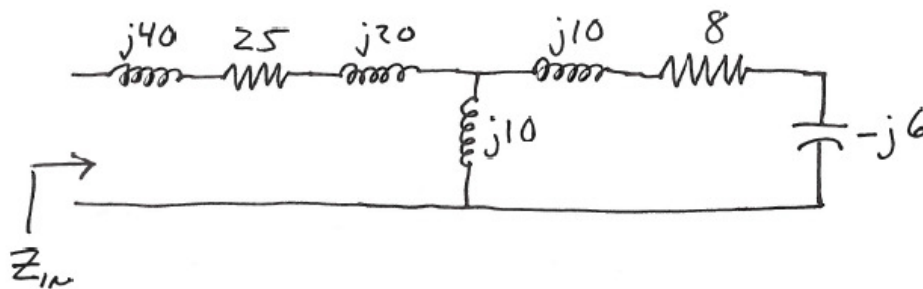
---

1. (Prob. 13.30 in text) For the circuit shown, find the input impedance by:
- Using the concept of reflected impedance  $Z_R$
  - Replacing the linear transformer by its T equivalent circuit



$$\begin{aligned}
 a) \quad Z_{in} &= Z_{primary} + Z_{reflected} \\
 &= (j40 + 25 + j30) + \frac{(\omega M)^2}{j\omega L_2 + 8 - j6} \\
 &= (25 + j70) + \frac{10^2}{j20 + 8 - j6} = \boxed{28.06 + j64.62 \, \Omega}
 \end{aligned}$$

$$\begin{aligned}
 b.) \quad T \text{ equivalent:} \quad L_a &= L_1 - M = 30 - 10 = 20 \\
 L_b &= L_2 - M = 20 - 10 = 10 \\
 L_c &= M = 10
 \end{aligned}$$

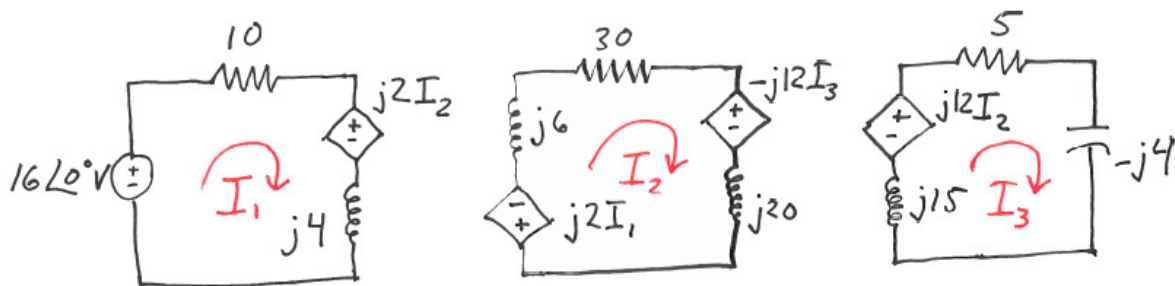
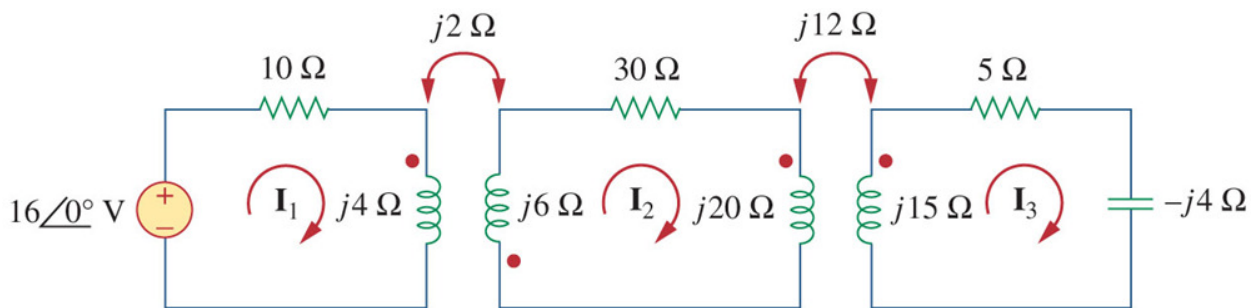


$$\begin{aligned}
 Z_{in} &= (j40 + 25 + j20) + (j10) \parallel (j10 + 8 - j6) \\
 &= (25 + j60) + \frac{(j10)(8 + j4)}{j10 + 8 + j4} = \boxed{28.06 + j64.62 \, \Omega}
 \end{aligned}$$

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2. (Prob. 13.35 from Text) Find currents  $I_1$ ,  $I_2$ , and  $I_3$  in the circuit below:



MESH #1:  $-16\angle 0^\circ + 10I_1 + j2I_2 + j4I_1 = 0 \Rightarrow (10+j4)I_1 + j2I_2 = 16\angle 0^\circ$

MESH #2:  $j2I_1 + j6I_2 + 30I_2 - j12I_3 + j20I_2 = 0 \Rightarrow j2I_1 + (30+j26)I_2 - j12I_3 = 0$

MESH #3:  $j15I_3 - j12I_2 + 5I_3 - j4I_3 = 0 \Rightarrow -j12I_2 + (5+j11)I_3 = 0$

$$\begin{bmatrix} (10+j4) & j2 & 0 \\ j2 & (30+j26) & -j12 \\ 0 & -j12 & (5+j11) \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 16\angle 0^\circ \\ 0 \\ 0 \end{bmatrix}$$

USE MATLAB (see OTHER PAGE)

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Problem 13.35 MATLAB output

```
>> A=[(10+4i) 2i 0; 2i (30+26i) -12i; 0 -12i (5+11i)]
```

A =

```
10.0000 + 4.0000i 0.0000 + 2.0000i 0.0000 + 0.0000i
0.0000 + 2.0000i 30.0000 + 26.0000i 0.0000 -12.0000i
0.0000 + 0.0000i 0.0000 -12.0000i 5.0000 + 11.0000i
```

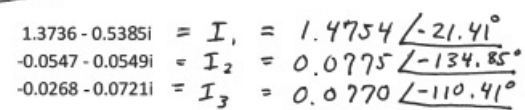
```
>> b=[16;0;0]
```

b =

```
16
0
0
```

```
>> x=A\b
```

x =



Handwritten solution for  $x$ :

$$\begin{aligned} 1.3736 - 0.5385i &= I_1 = 1.4754 \angle -21.41^\circ \\ -0.0547 - 0.0549i &= I_2 = 0.0775 \angle -134.85^\circ \\ -0.0268 - 0.0721i &= I_3 = 0.0770 \angle -110.41^\circ \end{aligned}$$

```
>> abs(x)
```

ans =

```
1.4754
0.0775
0.0770
```

```
>> angle(x)*180/pi()
```

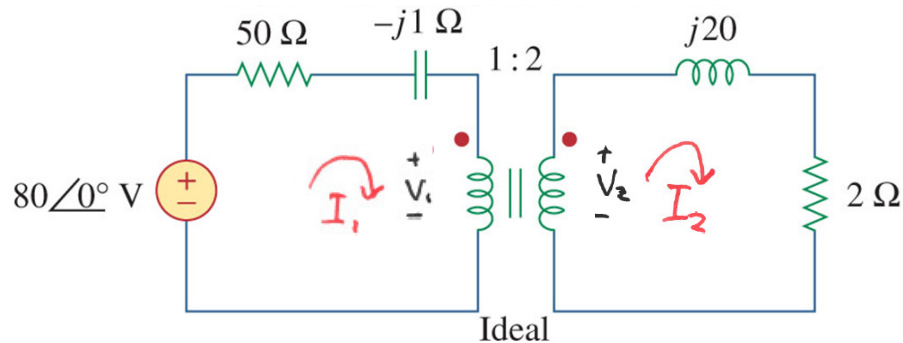
ans =

```
-21.4072
-134.8548
-110.4109
```

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3. (Prob. 13.42 from Text) For the circuit shown, determine the power absorbed by the  $2\ \Omega$  resistor. (Assume the  $80\text{ V}$  is a rms value).



$$V_2 = n V_1 = 2 V_1$$

$$I_2 = \frac{I_1}{n} = 0.5 I_1$$

$$\text{MESH \#1: } -80\angle 0^\circ + 50I_1 - j1I_1 + V_1 = 0 \Rightarrow (50-j)I_1 + V_1 = 80\angle 0^\circ$$

$$\text{MESH \#2: } -V_2 + j20I_2 + 2I_2 = 0 \Rightarrow (j20+2)\frac{I_1}{2} - 2V_1 = 0$$

$$(1+j10)I_1 - 2V_1 = 0$$

$$\begin{bmatrix} (50-j) & 1 \\ (1+j10) & -2 \end{bmatrix} \begin{bmatrix} I_1 \\ V_1 \end{bmatrix} = \begin{bmatrix} 80\angle 0^\circ \\ 0 \end{bmatrix}$$

USE MATLAB (SEE OTHER PAGE)

$$I_1 = 1.579 \angle -4.53^\circ \text{ A}$$

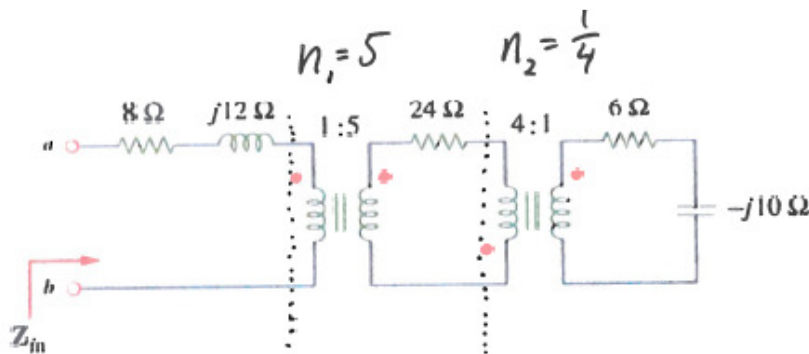
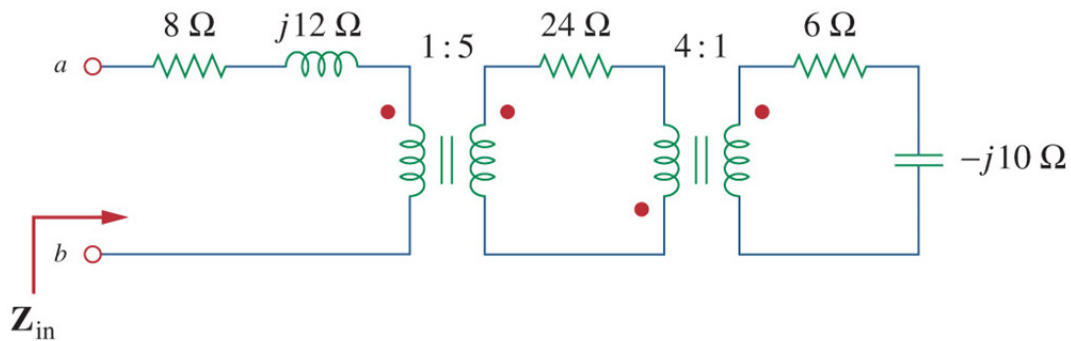
$$I_2 = 0.5 I_1 = 0.7896 \angle -4.53^\circ \text{ A}$$

POWER ABSORBED BY  $2\ \Omega$  RESISTOR IS

$$P = |I_2|^2 R = (0.7896)^2 (2) = \boxed{1.2469 \text{ W}}$$

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4. (Prob. 13.50 from Text) Calculate the input impedance for the network below:



**Figure 13.115**  
For Prob. 13.50.

$$Z_{r_2} = \frac{Z_{L_2}}{n_2^2} = \frac{6 - j10}{\left(\frac{1}{4}\right)^2} = 96 - j160$$

$$Z_{r_1} = \frac{Z_{L_1}}{n_1^2} = \frac{24 + (96 - j160)}{5^2}$$

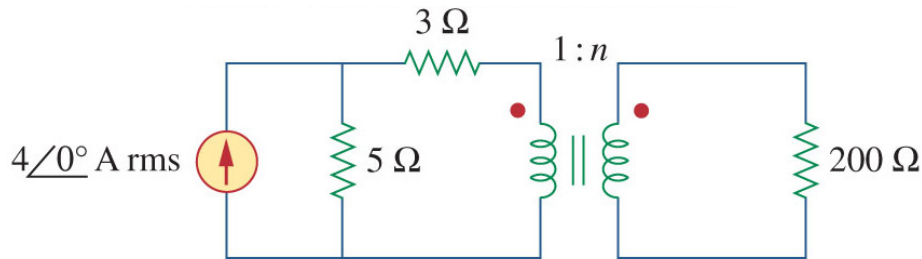
$$Z_{r_1} = 4.8 - j6.4$$

$$Z_{in} = (8 + j12) + (4.8 - j6.4)$$

$$\boxed{Z_{in} = 12.8 + j5.6 \, \Omega}$$

5. ("Based on" Prob. 13.53 from Text) Refer to the figure below for the following:

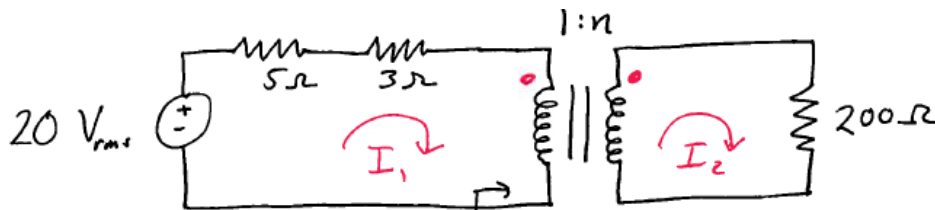
- Find the turns ratio  $n$  for maximum power supplied to the  $200\ \Omega$  load.
- Find the average power ( $P_{ave} = I_{rms}^2 R$ ) in the  $200\ \Omega$  load at this turns ratio.
- Find the average power in the  $200\ \Omega$  load if the turns ratio  $n = 6$ .



Dots "same"

$$V_2 = n V_1$$

$$I_2 = \frac{I_1}{n}$$



$$Z_R = \frac{Z_L}{n^2} = \frac{200}{n^2} = 8\ \Omega \text{ for max power transfer}$$

$$a.) n^2 = \frac{200}{8} = 25 \Rightarrow \boxed{n = 5}$$

$$b.) I_1 = \frac{20}{5+3+8} = 1.25; \quad I_2 = \frac{I_1}{n} = \frac{1.25}{5} = 0.25$$

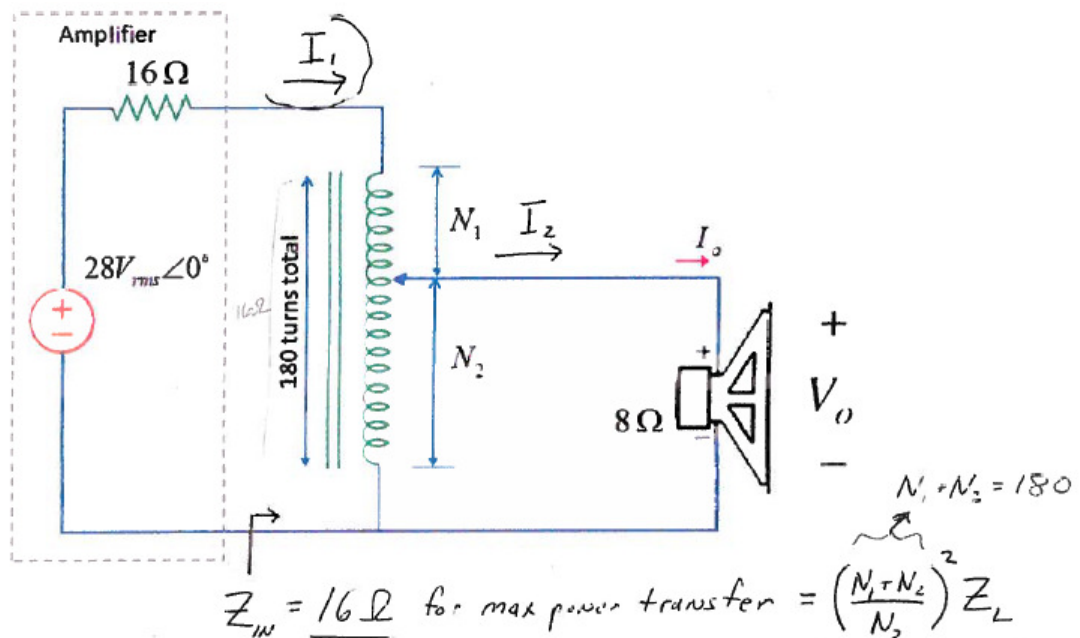
$$P_{ave} = I_2^2 R = (0.25)^2 200 = \boxed{12.5\text{ W}}$$

$$c.) Z_R = \frac{200}{(6)^2} = 5.555\ \Omega \Rightarrow I_1 = \frac{20}{(5+3)+(5.555)} = 1.475$$

$$I_2 = \frac{1.475}{6} = 0.2459 \Rightarrow P_{ave} = (0.2459)^2 (200) = \boxed{12.09\text{ W}}$$

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6. An audio amplifier with an internal Thevenin impedance of  $16\ \Omega$  uses a source matching autotransformer shown below to match an  $8\ \Omega$  speaker for maximum power transfer.
- Find the location of the tap  $N_2$  for maximum power transfer to the speaker.
  - Find the Voltage  $V_o$  and Current  $I_o$  delivered to the speaker.
  - Find the average power  $P_{ave}$  delivered to the speaker.
  - Find the location of the tap  $N_2$  if we replaced speaker with a  $4\ \Omega$  speaker.
  - Find the average power  $P_{ave}$  delivered to the  $4\ \Omega$  speaker.



$$a.) \quad 16 = \left( \frac{180}{N_2} \right)^2 8 \Rightarrow \frac{16}{8} = \left( \frac{180}{N_2} \right)^2 \Rightarrow \sqrt{2} = \frac{180}{N_2}$$

$$N_2 = \frac{180}{\sqrt{2}} = \boxed{127.3 \text{ turns}}$$

$$b.) \quad I_1 = \frac{28}{16 + 16} = 0.875 \text{ A} \Rightarrow I_2 = \left( \frac{N_1 + N_2}{N_2} \right) I_1 = \frac{180}{\left( \frac{180}{\sqrt{2}} \right)} 0.875$$

$\uparrow$   $Z_{in}$  for max power is  $16\ \Omega$

$$\boxed{I_2 = 1.24 \text{ A} = I_o}$$

$$c.) \quad P_{ave} = I_o V_o = (1.24)(9.9) = \boxed{12.25 \text{ W}}$$

$$\boxed{V_o = I_o R = (1.24)(8) = 9.9 \text{ V}}$$

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$$d) \quad 16 = \left(\frac{180}{N_2}\right)^2 4 \Rightarrow 4 = \left(\frac{180}{N_2}\right)^2 \Rightarrow 2 = \frac{180}{N_2} \Rightarrow \boxed{N_2 = 90 \text{ turns}}$$

$$e) \quad I_1 = \frac{28}{16+16} = 0.875 \text{ A} \Rightarrow I_2 = \left(\frac{N_1+N_2}{N_2}\right) I_1 = \left(\frac{180}{90}\right)(0.875)$$

$$I_2 = \boxed{1.75 \text{ A} = I_o}$$

$$V_o = I_o R = (1.75)(4) = \boxed{7.0 \text{ V}}$$

$$P_{ave} = I_o V_o = (1.75)(7.0) = \boxed{12.25 \text{ W}}$$

Note: should be same as part c  
because we have set  $N_2$  for  
max power transfer.

Lossless transformer

$$P_{ave} = I_1^2 Z_{in} = (0.875)^2 (16)$$

$$\boxed{P_{ave} = 12.25 \text{ W also!}}$$