Chapter 19: Two-Port Networks

19.1 Introduction (1)

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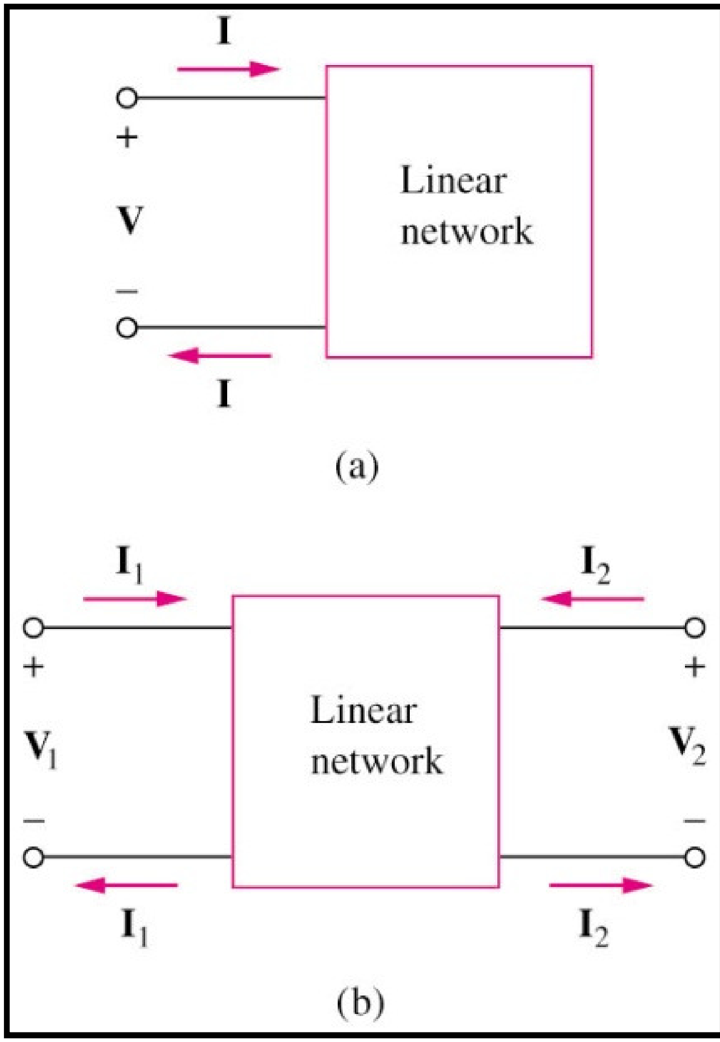
- A port is an access to the network and consists of a pair of terminals; the current entering one terminal leaves through the other terminal so that the net current entering the port equals zero.
- One port networks include two-terminal devices such as resistors, capacitors, and inductors.
- A two-port network has two separate ports for input and output.
- Two port networks include op amps, transistors and transformers.

19.1 Introduction (2)



One port or two terminal circuit

Two port or four terminal circuit

- **It is an electrical network with two separate ports for input and output.**
- **No independent sources.**

19.1 Introduction (3)

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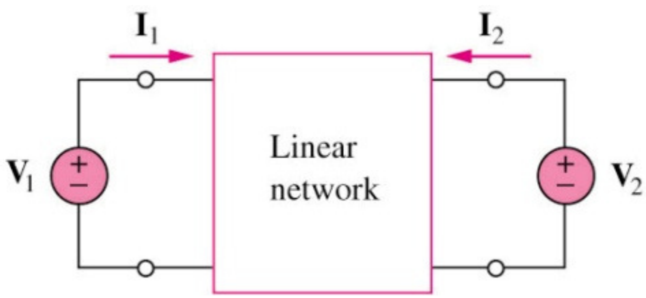
- Characterizing a two-port network requires that we relate the terminal quantities $\mathbf{V}_1$, $\mathbf{V}_2$, $\mathbf{I}_1$, $\mathbf{I}_2$ out of which two are independent. Six sets of voltage and current parameters will be derived.
- Two port networks are useful in communications, control systems, power systems, and electronics.
- They are used in electronics to model transistors and to facilitate cascaded design.
- Additionally, if we know the parameters of a two-port network it can be treated as a "black box" when embedded within a larger network.

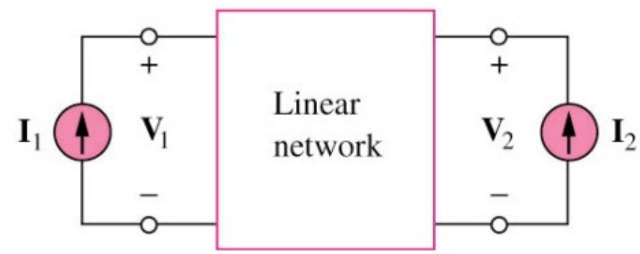
19.2 Impedance Parameters (1)

- Often called "**Z-parameters**" since their units are in **ohms** and they represent an impedance relationship between $\mathbf{V}_1$, $\mathbf{V}_2$, $\mathbf{I}_1$, $\mathbf{I}_2$ for the two port network shown below:



Voltage Driven Network

Assume no independent source in the network



Current Driven Network

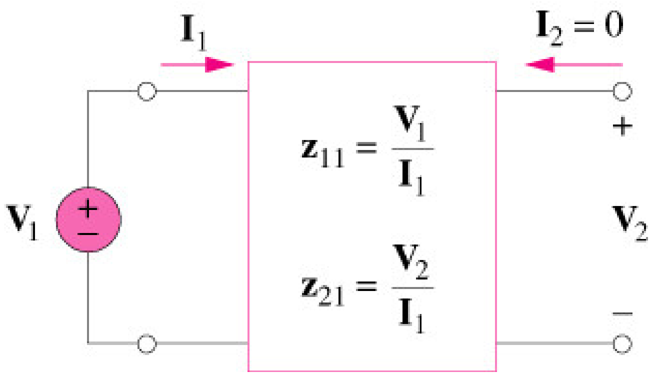
$$\mathrm{V}_1 = \mathrm{z}_{11}\mathrm{I}_1 + \mathrm{z}_{12}\mathrm{I}_2$$
$$\mathrm{V}_2 = \mathrm{z}_{21}\mathrm{I}_1 + \mathrm{z}_{22}\mathrm{I}_2$$

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

- Z-parameters are commonly used in filter synthesis, impedance matching networks design, and power distribution networks analysis.

19.2 Impedance Parameters (2)

The values of parameters can be evaluated by setting I_1=0 or I_2=0 (open circuit)

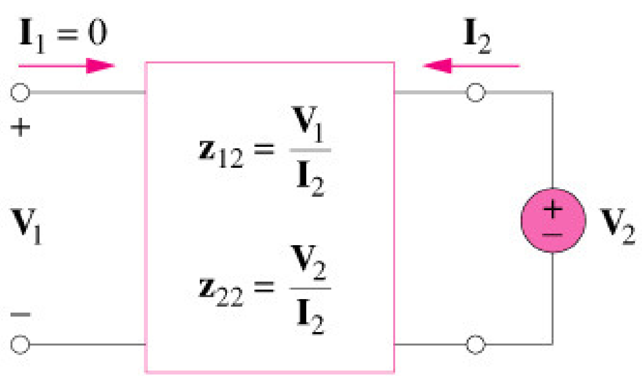


(a)

Setting $I_2 = 0$

$$z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0} \quad \text{and} \quad z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0}$$

z_{11} = Open-circuit input impedance
z_{21} = Open-circuit transfer impedance from port 2 to port 1



(b)

Setting $I_1 = 0$

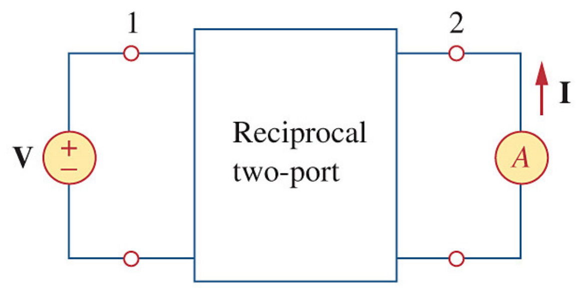
$$z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1=0} \quad \text{and} \quad z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0}$$

z12 = Open-circuit transfer impedance from port 1 to port 2
z22 = Open-circuit output impedance

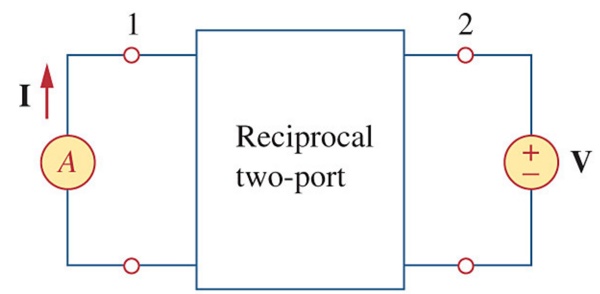
19.2 Impedance Parameters (3)

Properties of Z-parameters

- Symmetrical networks $z_{11} = z_{22}$
  - Implies a mirror like symmetry
- Reciprocal networks $z_{12} = z_{21}$
  - Any network made up entirely of resistors, capacitors, and inductors must be reciprocal.
  - Linear networks with no dependant sources are reciprocal.
  - Interchanging an ideal voltage source at one port with an ideal ammeter at the other port gives the same ammeter reading.

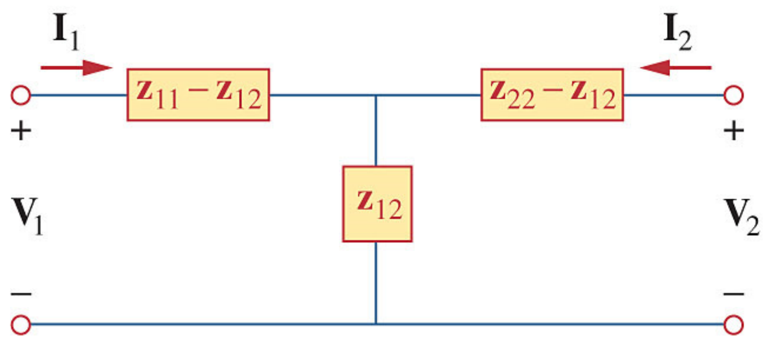


$$V = z_{12} I$$

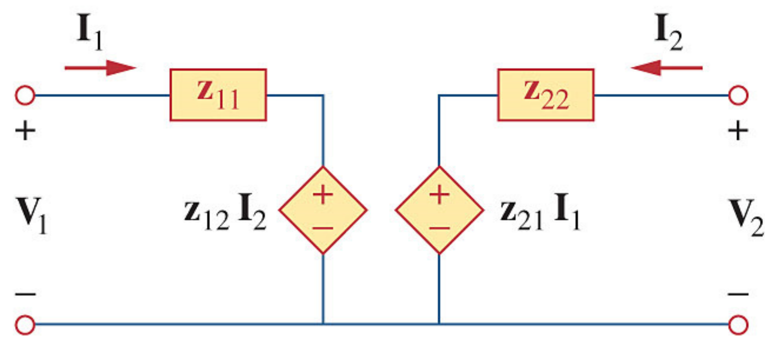


$$V = z_{21} I$$

19.2 Impedance Parameters (4)



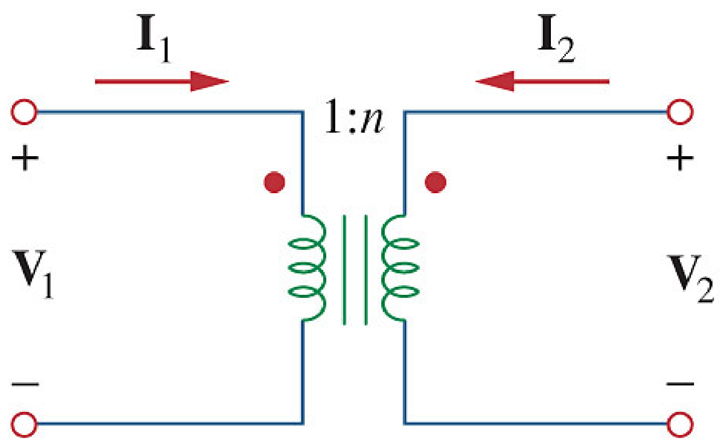
Reciprocal Network



General Network

- A reciprocal network can be replaced by the T-network shown above
- If not reciprocal, the General network is the T-equivalent.

19.2 Impedance Parameters (5)



- Note: some circuits do not have z-parameter equivalents. (they may have other 2-port equivalents, as we shall see)
- Consider an ideal transformer:

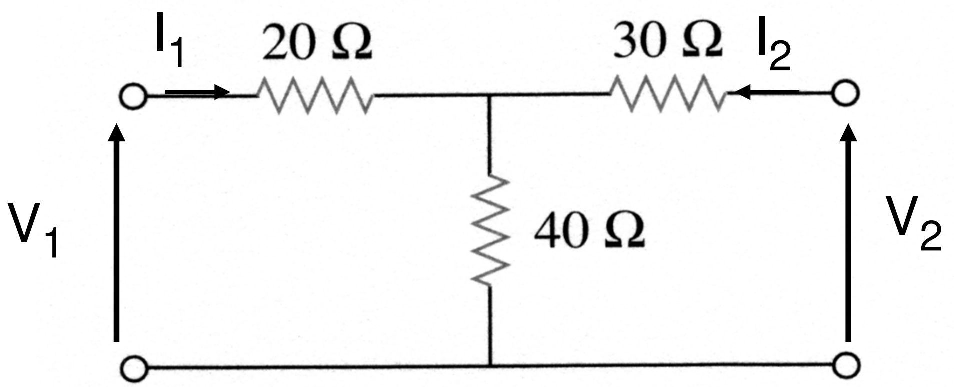
$$V_1 = V_2/n \text{ and } I_1 = -nI_2.$$

- This cannot be expressed by:

$$V_1 = z_{11}I_1 + z_{12}I_2$$
$$V_2 = z_{21}I_1 + z_{22}I_2$$

Example 19.1

Determine the z-parameters of the following circuit.



$$z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0} \quad \text{and} \quad z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0}$$

$$z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1=0} \quad \text{and} \quad z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0}$$

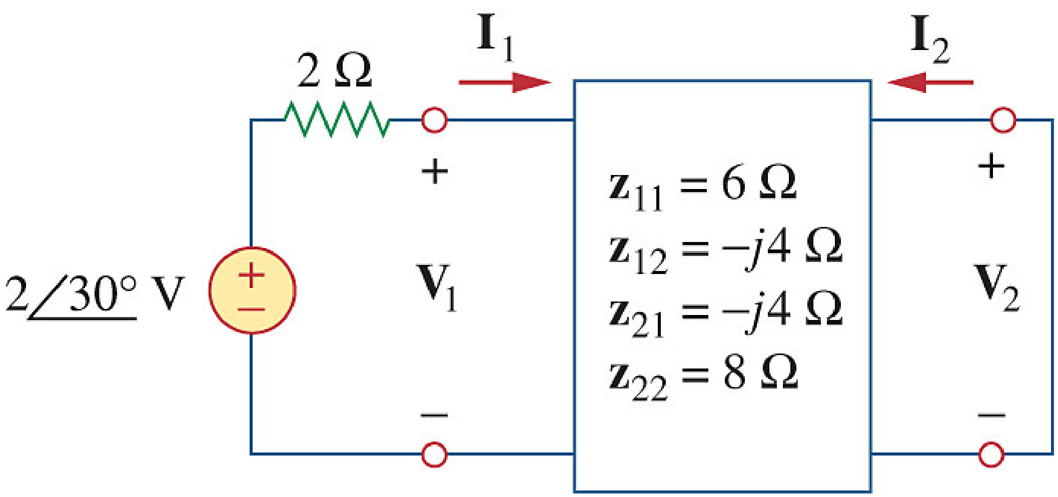
$$z = \begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix} \Omega$$

Answer: $$z = \begin{bmatrix} 60 & 40 \\ 40 & 70 \end{bmatrix} \Omega$$

19.2 Impedance Parameters (7)

Practice Problem 19.2

Determine I_1 and I_2 in the following circuit.

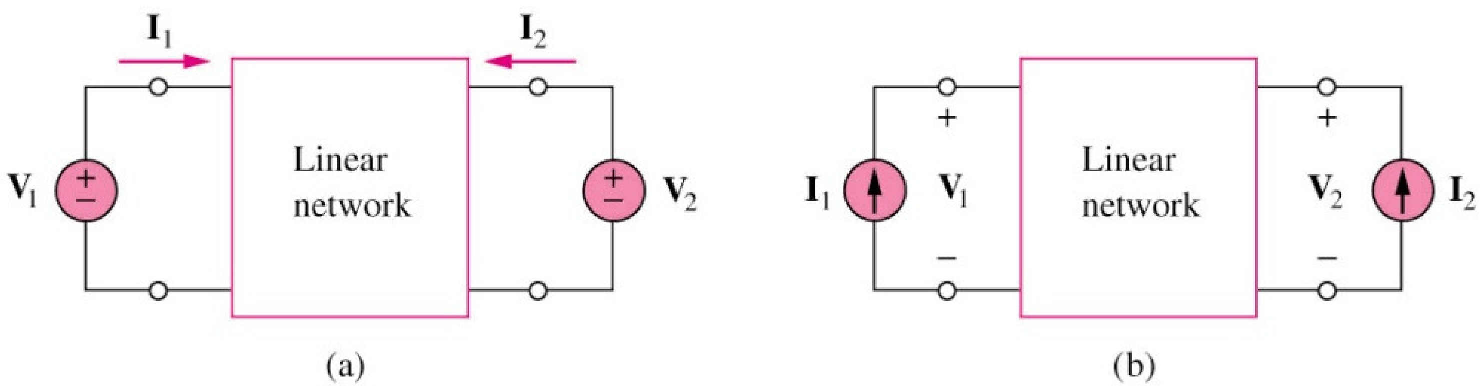


$$V_1 = z_{11}I_1 + z_{12}I_2$$
$$V_2 = z_{21}I_1 + z_{22}I_2$$

Answer:

$$I_1 = 200\angle 30^\circ \text{ mA}$$
$$I_2 = 100\angle 120^\circ \text{ mA}$$

19.3 Admittance Parameters (1)



Assume no independent source in the network

$$I_1 = y_{11}V_1 + y_{12}V_2$$
$$I_2 = y_{21}V_1 + y_{22}V_2$$

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = [y] \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

where the **y** terms are called the admittance parameters, or simply y parameters, and they have units of Siemens.

19.3 Admittance Parameters (2)

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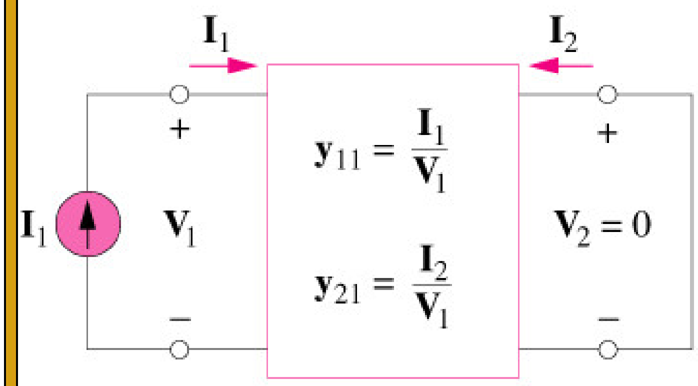
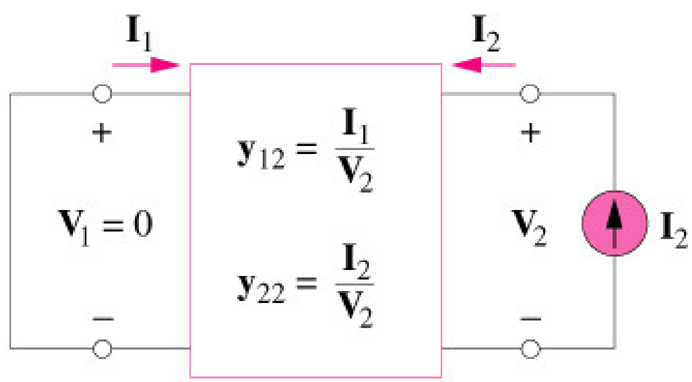


(a)



(b)

Setting $V_2 = 0$ (Shorting the output)

$$y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2=0} \quad \text{and} \quad y_{21} = \left.\frac{I_2}{V_1}\right|_{V_2=0}$$

y_{11} = Short-circuit input admittance
y_{21} = Short-circuit transfer admittance from port 1 to port 2

Setting $V_1 = 0$ (Shorting the input)

$$y_{12} = \left.\frac{I_1}{V_2}\right|_{V_1=0} \quad \text{and} \quad y_{22} = \left.\frac{I_2}{V_2}\right|_{V_1=0}$$

y_{12} = Short-circuit transfer admittance from port 2 to port 1
y_{22} = Short-circuit output admittance

19.3 Admittance Parameters (3)

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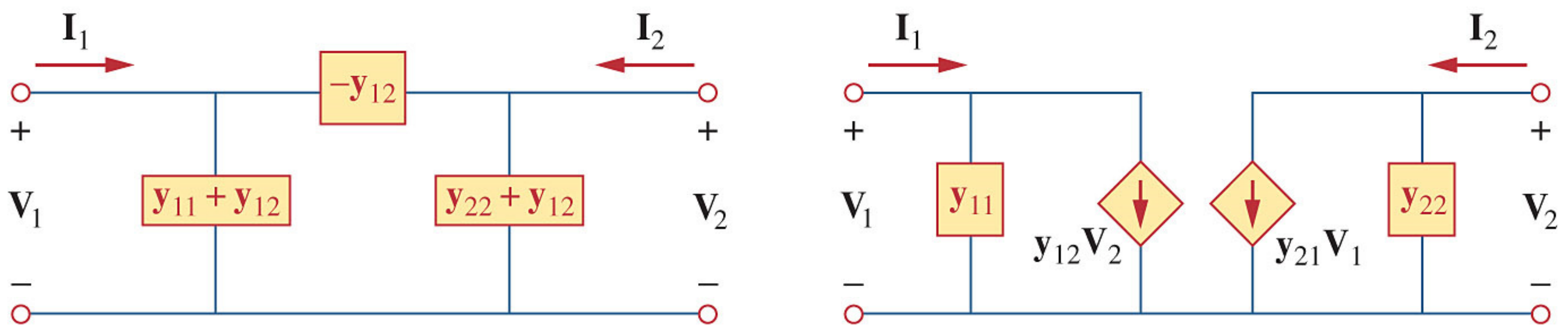


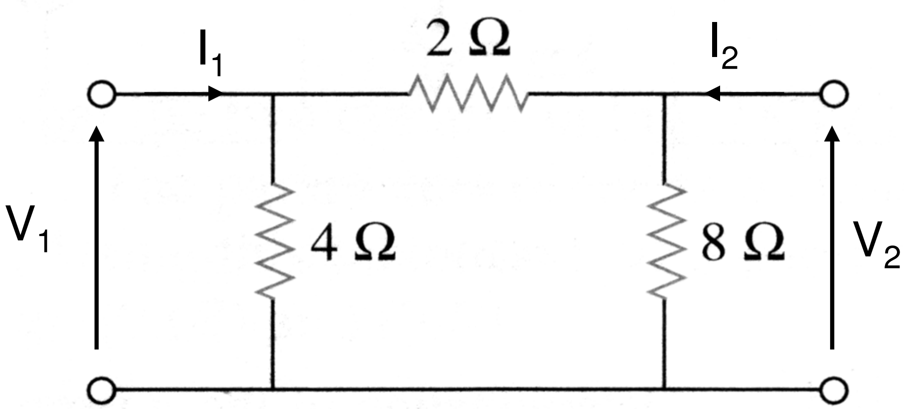
Reciprocal Network

General Network

•A reciprocal network (y_{12} = y_{21}) can be replaced by the Pi-network in figure (a).
•If not reciprocal, the network in figure (b) is the Pi-equivalent.

Example 19.3

Determine the y-parameters of the following circuit.



$$y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2=0} \quad \text{and} \quad y_{21} = \left.\frac{I_2}{V_1}\right|_{V_2=0}$$

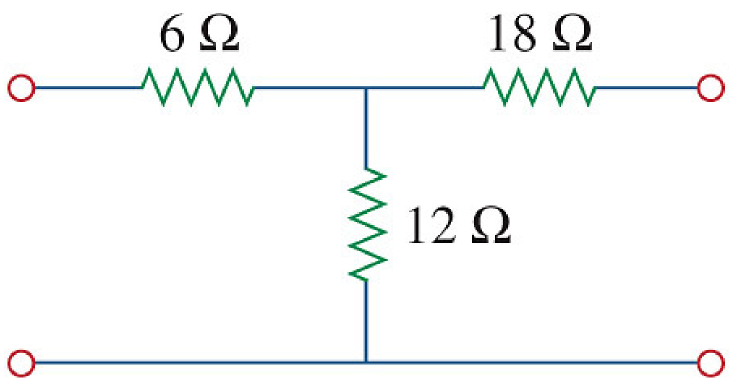
$$y_{12} = \left.\frac{I_1}{V_2}\right|_{V_1=0} \quad \text{and} \quad y_{22} = \left.\frac{I_2}{V_2}\right|_{V_1=0}$$

$$y = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} S$$

Answer: $y = \begin{bmatrix} 0.75 & -0.5 \\ -0.5 & 0.625 \end{bmatrix} S$

Practice Problem 19.3

Determine the y-parameters of the following circuit.



$$y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2=0} \quad \text{and} \quad y_{21} = \left.\frac{I_2}{V_1}\right|_{V_2=0}$$

$$y_{12} = \left.\frac{I_1}{V_2}\right|_{V_1=0} \quad \text{and} \quad y_{22} = \left.\frac{I_2}{V_2}\right|_{V_1=0}$$

$$y = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} S$$

Answer: $y = \begin{bmatrix} 75.77 & -30.3 \\ -30.3 & 45.47 \end{bmatrix} \text{mS}$

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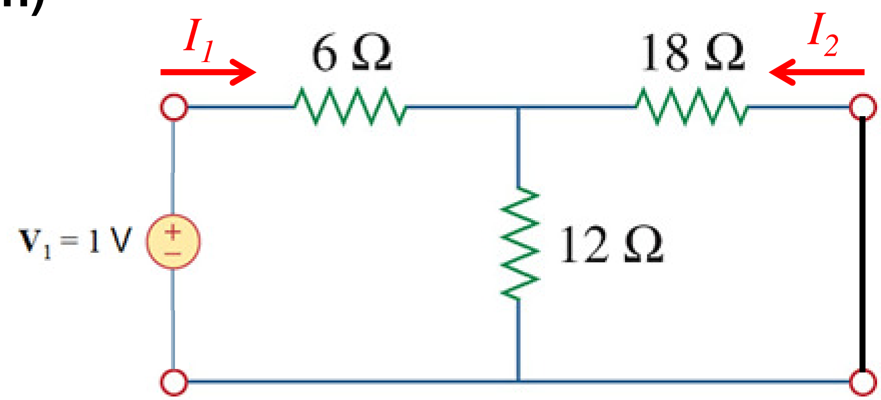
Practice Problem 19.3 (Solution)

- Short the output
- Put a 1 volt source at input
- Find I_1 and I_2



$$y_{11} = \left.\frac{I_1}{(1)}\right|_{V_2=0} \quad and \quad y_{21} = \left.\frac{I_2}{(1)}\right|_{V_2=0}$$

Find Input Impedance

$$Z_{in} = 6 + 12 \| 18 = 13.2$$

$$I_1 = \frac{V_1}{Z_{in}} = \frac{1}{13.2} = 0.07576$$

$$\boxed{y_{11} = 0.07576}$$

Similarly at Output

$$Z_{out} = 18 + 6 \| 12 = 22$$

$$I_2 = \frac{V_2}{Z_{in}} = \frac{1}{22} = 0.04545$$

$$\boxed{y_{22} = 0.04545}$$

Find I_2 from current divider equation

$$I_2 = \frac{-12}{12+18} I_1$$

$$I_2 = (-0.4)0.07576 = -0.0303$$

$$y_{21} = -0.0303$$

$$\boxed{y_{12} = y_{21} = -0.0303}$$

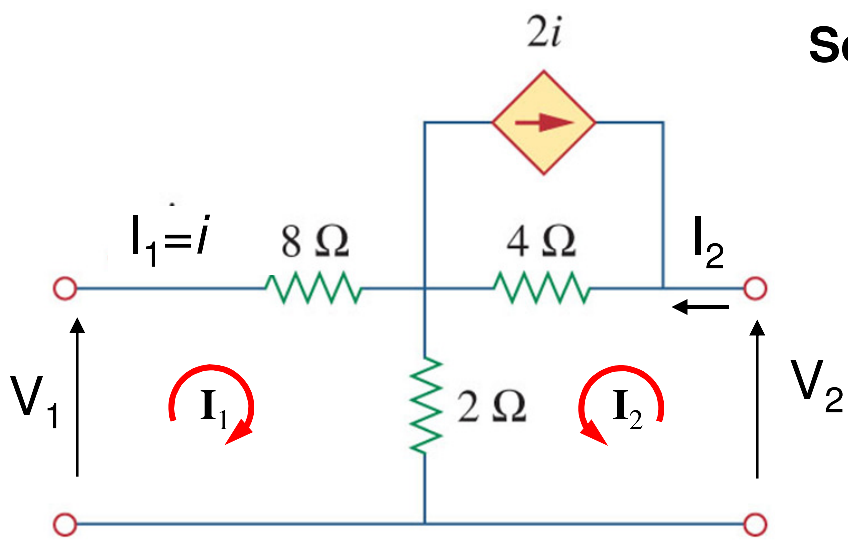
Reciprocal Network

19.3 Admittance Parameters (7)

Example 19.4

Determine the y-parameters of the following circuit.

$$I_1 = y_{11}V_1 + y_{12}V_2$$
$$I_2 = y_{21}V_1 + y_{22}V_2$$



Solution: Apply KVL

Mesh I_1: $V_1 = 8I_1 + 2(I_1 + I_2)$

$$V_1 = 10I_1 + 2I_2$$

Mesh I_2: $V_2 = 4(2i + I_2) + 2(I_1 + I_2)$

$$V_2 = 8I_1 + 4I_2 + 2I_1 + 2I_2$$
$$V_2 = 10I_1 + 6I_2$$

Subtract #1 from #2:

$$V_2 - V_1 = 0 + 4I_2$$
$$I_2 = -0.25V_1 + 0.25V_2$$

Substitute back into #1

$$V_1 = 10I_1 - 0.5V_1 + 0.5V_2$$
$$10I_1 = 1.5V_1 - 0.5V_2$$
$$I_1 = 0.15V_1 - 0.05V_2$$

Answer: $\mathrm{y} = \begin{bmatrix} 0.15 & -0.05 \\ -0.25 & 0.25 \end{bmatrix} \mathrm{S}$

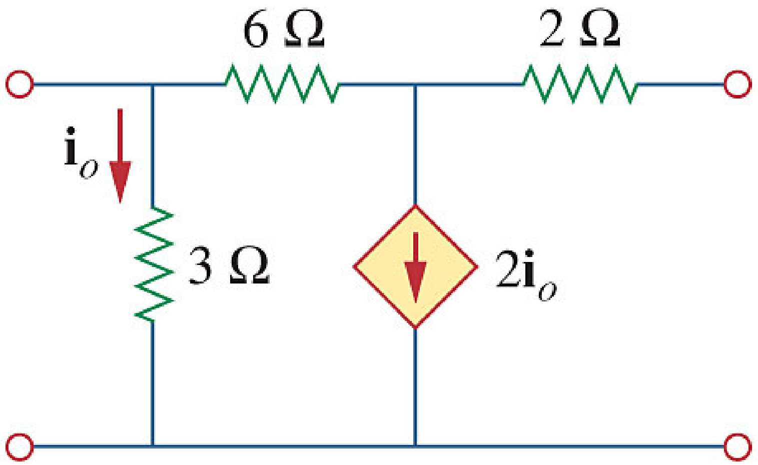
Note: Sometimes two port parameters will fall out directly from mesh equations.

19.3 Admittance Parameters (8)

Practice problem 19.4

Practice Problem 19.4

Determine the y-parameters of the following circuit.



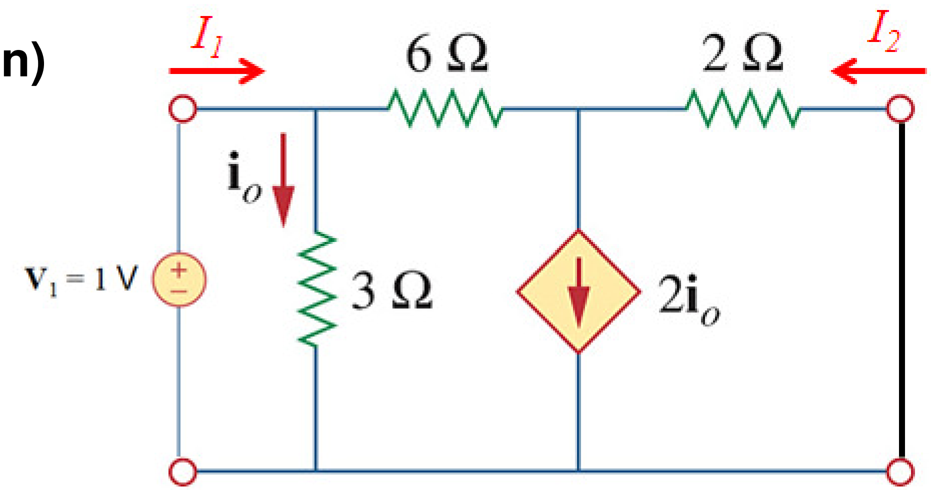
Answer: $y = \begin{bmatrix} 0.625 & -0.125 \\ 0.375 & 0.125 \end{bmatrix} S$

19.3 Admittance Parameters (9)

Practice problem 19.4

Practice Problem 19.4 (Solution)

- Short the output
- Put a 1 volt source at input
- Find I_1 and I_2



First find i_o:

$$i_0 = \frac{1}{3}$$

Dependent current source is then 2/3, find I_1 by repetitive source transformations of the dependant current source

$$I_1 = 0.625 \quad \Rightarrow \boxed{y_{11} = 0.625}$$

Next find current across 6 Ω resistor $I_{6\Omega}$:

$$I_{6\Omega} = 0.625 - \frac{1}{3}$$

$$I_2 + I_{6\Omega} = 2i_0$$

$$I_2 = 2i_0 - I_{6\Omega} = \frac{2}{3} - \left(0.625 - \frac{1}{3}\right) = 0.375 \quad \Rightarrow \boxed{y_{12} = 0.375}$$

Z and Y Parameters
Comparison

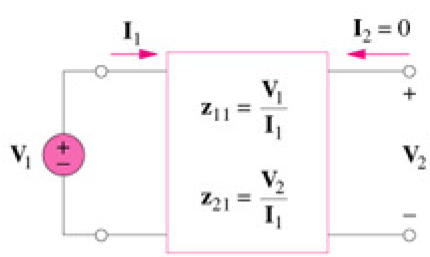
Z-Parameters

$$V_1 = z_{11}I_1 + z_{12}I_2$$
$$V_2 = z_{21}I_1 + z_{22}I_2$$

- **Open** one port (I_1=0 or I_2=0)
- Connect a source to the other port
- Solve to find z-parameters

$$z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0} \quad \text{and} \quad z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0}$$

$$z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1=0} \quad \text{and} \quad z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0}$$



I = 0
Open Circuit

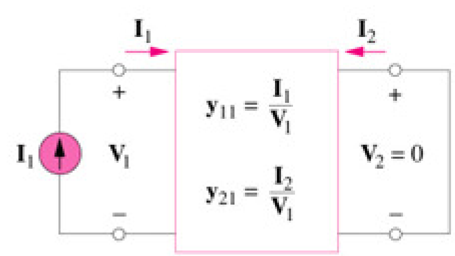
Y-Parameters

$$I_1 = y_{11}V_1 + y_{12}V_2$$
$$I_2 = y_{21}V_1 + y_{22}V_2$$

- **Short** one port (V_1=0 or V_2=0)
- Connect a source to the other port
- Solve to find y-parameters

$$y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2=0} \quad \text{and} \quad y_{21} = \left.\frac{I_2}{V_1}\right|_{V_2=0}$$

$$y_{12} = \left.\frac{I_1}{V_2}\right|_{V_1=0} \quad \text{and} \quad y_{22} = \left.\frac{I_2}{V_2}\right|_{V_1=0}$$



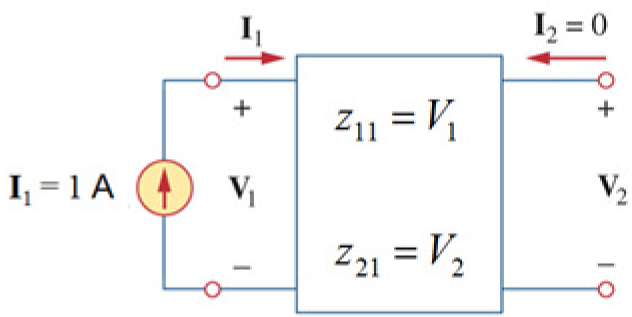
V = 0
Short Circuit

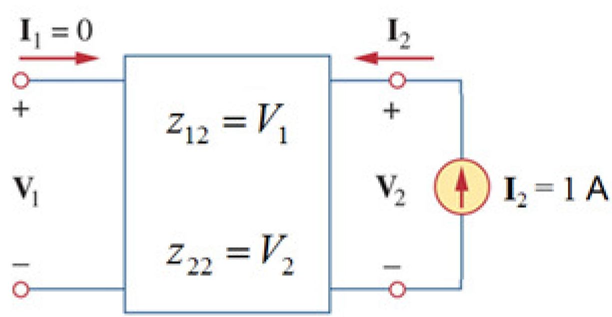
Z and Y parameters

Alternative method (1 Amp / 1 Volt sources)

Z-Parameters

- Open circuit one port
- Put a 1 Amp current source at other port
- Resulting voltages are the z-parameters

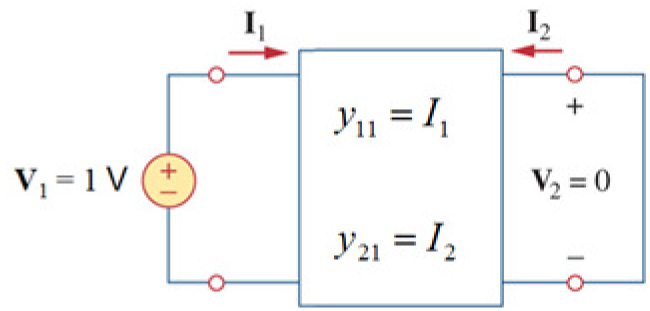


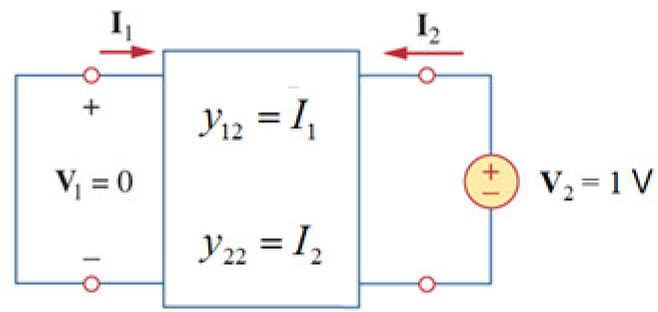


$$\begin{aligned} V_1 &= z_{11} I_1 + z_{12} I_2 \\ V_2 &= z_{21} I_1 + z_{22} I_2 \end{aligned}$$

Y-Parameters

- Short circuit one port
- Put a 1 Volt voltage source at other port
- Resulting current are the y-parameters

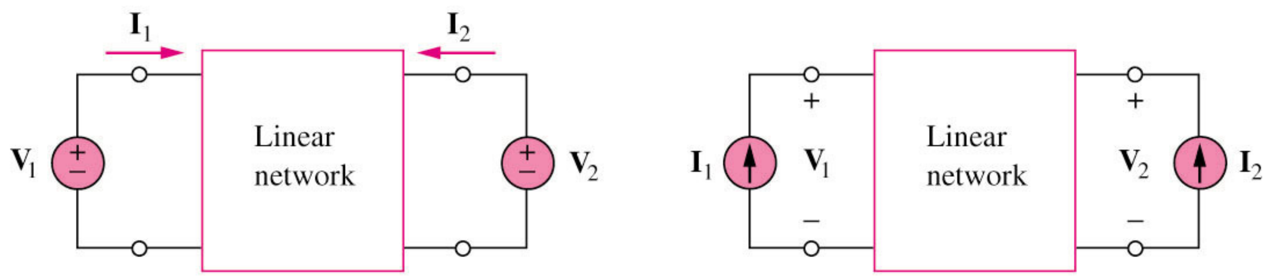




$$\begin{aligned} I_1 &= y_{11} V_1 + y_{12} V_2 \\ I_2 &= y_{21} V_1 + y_{22} V_2 \end{aligned}$$

19.4 Hybrid Parameters (1)

•The z and y parameters of a two-port network do not always exist. Therefore, there is a need to develop another set of parameters based on making V_1 and I_2 the dependent variables.



Assume no independent source in the network

$$V_1 = h_{11}I_1 + h_{12}V_2$$
$$I_2 = h_{21}I_1 + h_{22}V_2$$

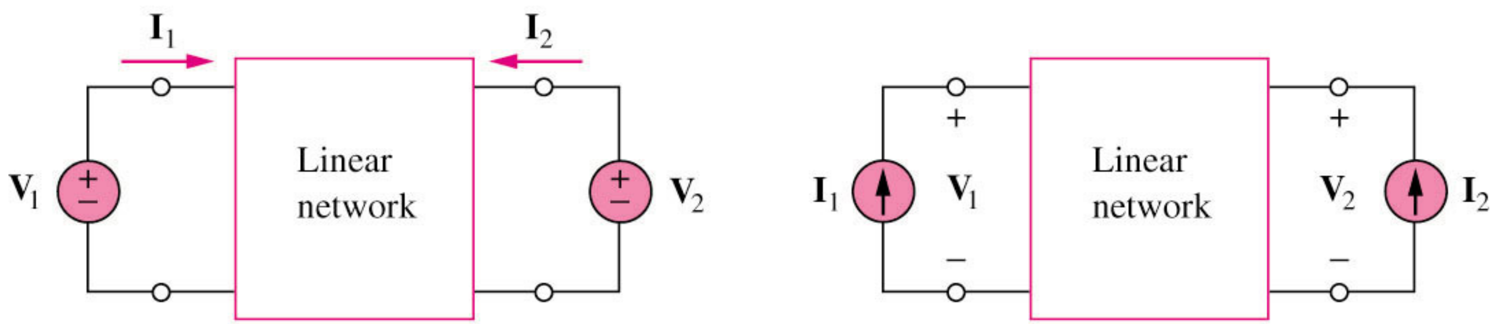
$$\rightarrow \begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ V_2 \end{bmatrix} = [h] \begin{bmatrix} I_1 \\ V_2 \end{bmatrix}$$

where the **h** terms are called the hybrid parameters, or simply h parameters.

•Hybrid parameters are very useful for describing electronic devices such as transistors because it is much easier to measure the h parameters of these devices than to measure their z or y parameters.

•The ideal transformer can also be described by h parameters.

19.4 Hybrid Parameters (2)



Assume no independent source in the network

$$h_{11} = \left.\frac{V_1}{I_1}\right|_{V_2=0}$$

h_{11}= short-circuit input impedance (Ω)

$$h_{21} = \left.\frac{I_2}{I_1}\right|_{V_2=0}$$

h_{21} = short-circuit forward current gain

$$h_{12} = \left.\frac{V_1}{V_2}\right|_{I_1=0}$$

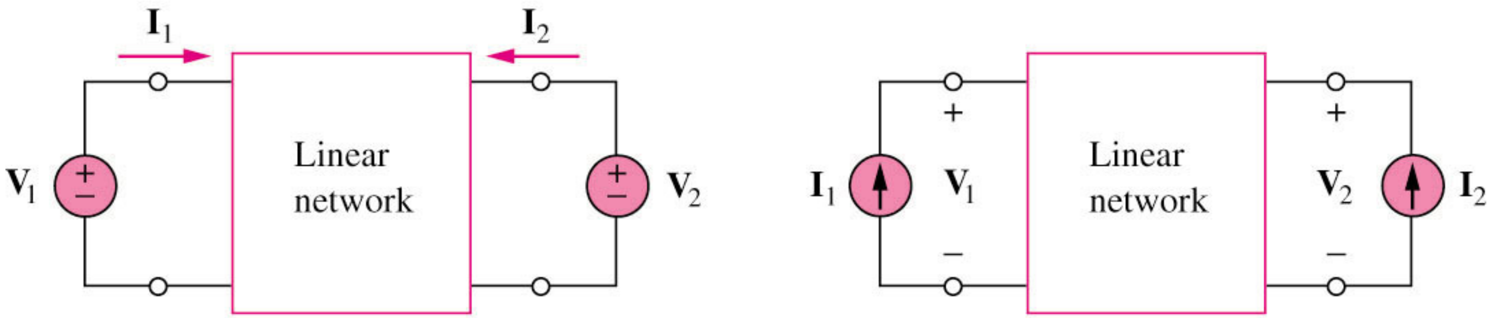
h_{12} = open-circuit reverse voltage-gain

$$h_{22} = \left.\frac{I_2}{V_2}\right|_{I_1=0}$$

h_{22} = open-circuit output admittance (S)

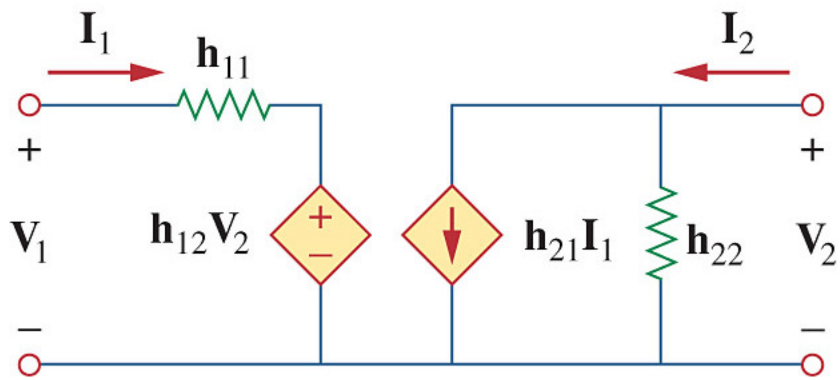
- Note that the h parameters represent an impedance, voltage gain, current gain, and admittance, thereby the term hybrid parameters.
- For reciprocal network, $h_{12} = -h_{21}$

19.4 Hybrid Parameters (3)



Assume no independent source in the network

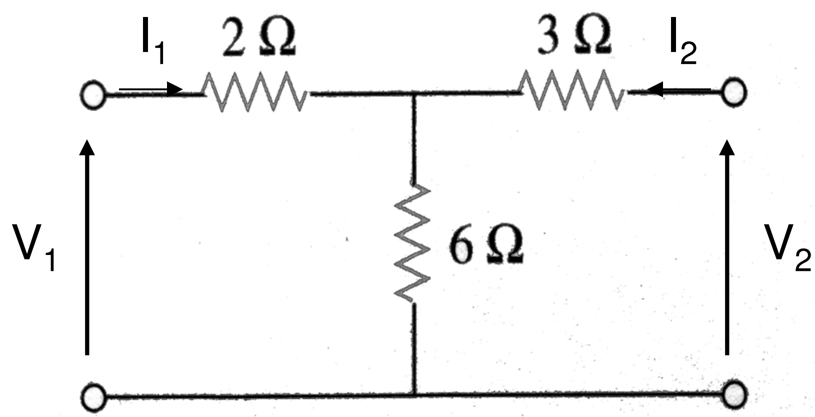
Hybrid model of a two-port network:



19.4 Hybrid Parameters (4)

Example 19.5:

Determine the h-parameters of the following circuit.



$$V_1 = h_{11} I_1 + h_{12} V_2$$
$$I_2 = h_{21} I_1 + h_{22} V_2$$

$$h_{11} = \left.\frac{V_1}{I_1}\right|_{V_2=0} \quad \text{and} \quad h_{21} = \left.\frac{I_2}{I_1}\right|_{V_2=0}$$

$$h_{12} = \left.\frac{V_1}{V_2}\right|_{I_1=0} \quad \text{and} \quad h_{22} = \left.\frac{I_2}{V_2}\right|_{I_1=0}$$

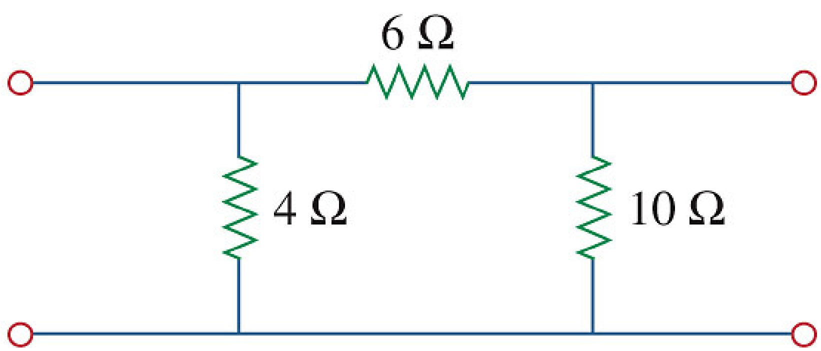
$$h = \begin{bmatrix} h_{11}\Omega & h_{12} \\ h_{21} & h_{22}S \end{bmatrix}$$

Answer: $h = \begin{bmatrix} 4\Omega & \frac{2}{3} \\ -\frac{2}{3} & \frac{1}{9}S \end{bmatrix}$

19.4 Hybrid Parameters (5)

Practice Problem 19.5:

Determine the h-parameters of the following circuit.



$$h_{11} = \left.\frac{V_1}{I_1}\right|_{V_2=0} \quad \text{and} \quad h_{21} = \left.\frac{I_2}{I_1}\right|_{V_2=0}$$

$$h_{12} = \left.\frac{V_1}{V_2}\right|_{I_1=0} \quad \text{and} \quad h_{22} = \left.\frac{I_2}{V_2}\right|_{I_1=0}$$

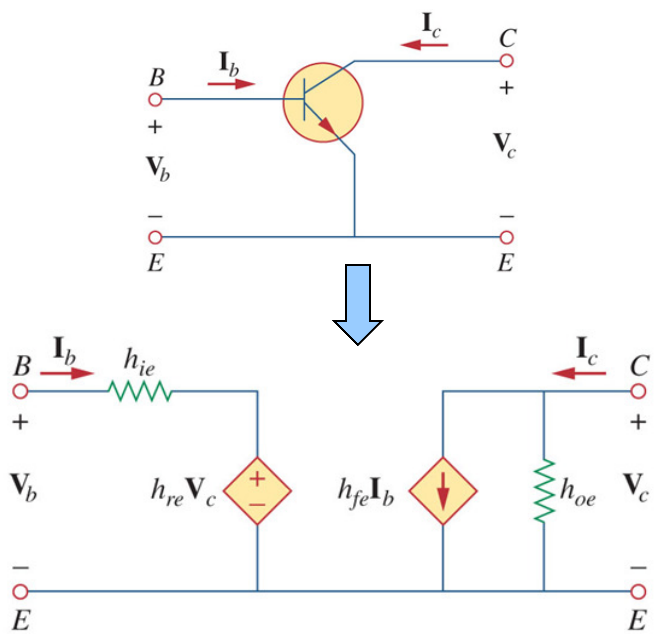
$$h = \begin{bmatrix} h_{11}\Omega & h_{12} \\ h_{21} & h_{22}S \end{bmatrix}$$

Answer: $h = \begin{bmatrix} 2.4\Omega & 0.4 \\ -0.4 & 0.2S \end{bmatrix}$

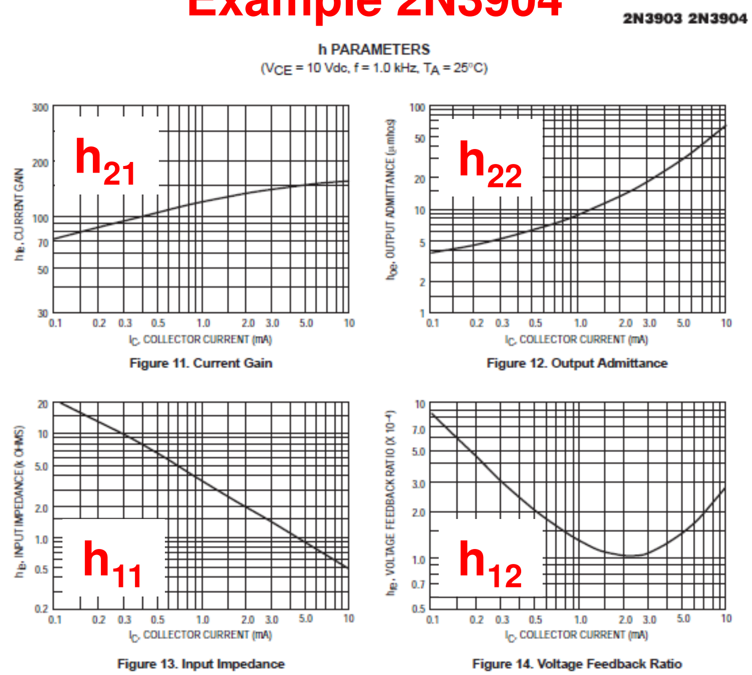
19.9.1 Transistor Circuits (1)

Hybrid Parameters

- H-parameters are often used to model transistor circuits
- The h-parameters vary depending on biasing conditions
- Parameters are given different subscripts:
  - h_{11} → h_{ie} = Base input impedance
  - h_{12} → h_{re} = Reverse voltage feedback ration
  - h_{21} → h_{fe} = Base-collector current gain
  - h_{22} → h_{oe} = Output admittance



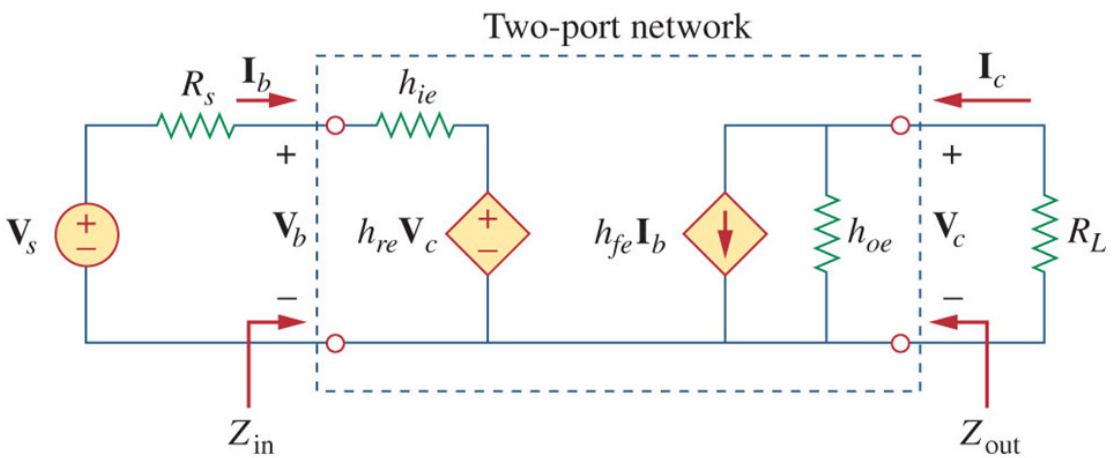
Example 2N3904



19.9.1 Transistor Circuits (2)

Hybrid Parameters

- H parameters are often found in manufacturers spec sheets
- Provide ability to calculate the exact voltage gain, input impedance, and output impedance of the transistor.



Input Impedance

$$\boxed{Z_{in} = \frac{V_b}{I_b} = h_{ie} - \frac{h_{re}h_{fe}R_L}{1 + h_{oe}R_L}}$$

Output Impedance

$$\boxed{Z_{out} = \left.\frac{V_c}{I_c}\right|_{V_s=0} = \frac{R_s + h_{ie}}{(R_s + h_{ie})h_{oe} - h_{re}h_{fe}}}$$

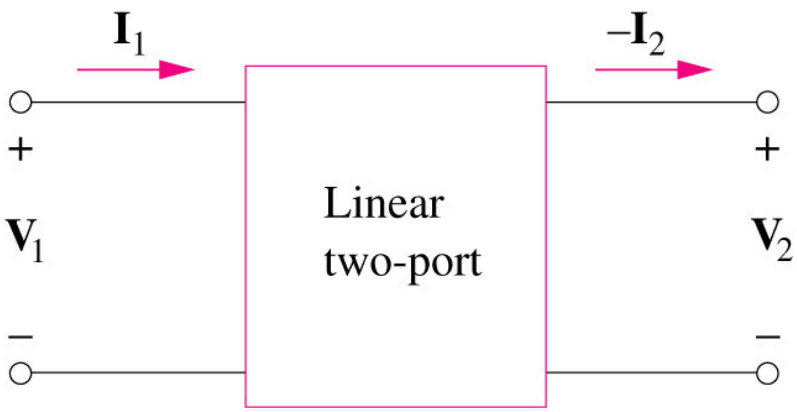
Current Gain

$$\boxed{A_i = \frac{I_c}{I_b} = \frac{h_{fe}}{1 + h_{oe}R_L}}$$

Voltage Gain

$$\boxed{A_v = \frac{V_c}{V_b} = \frac{-h_{fe}R_L}{h_{ie} + (h_{ie}h_{oe} - h_{re}h_{fe})R_L}}$$

19.5 Transmission Parameters (1)



Assume no independent source in the network

$$V_1 = AV_2 - BI_2$$
$$I_1 = CV_2 - DI_2$$

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix} = [T] \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$

where the **T** terms are called the transmission parameters, or simply T or ABCD parameters.

•Note that $-I_2$ is used since the current is considered to be leaving the network. It is logical to think of I_2 as leaving the two-port; this is customary convention in the power industry.

19.5 Transmission Parameters (2)

- These two-port transmission parameters provide a measure of how a circuit transmits voltage and current form a source to a load.
- They are useful in the analysis of transmission lines and are therefore called transmission parameters.
- They are also known as ABCD parameters and are used in the design of telephone systems, microwave networks, and radars.

$$A = \left.\frac{V_1}{V_2}\right|_{I_2=0}$$

A=open-circuit voltage ratio

$$C = \left.\frac{I_1}{V_2}\right|_{I_2=0}$$

C= open-circuit transfer admittance (S)

$$B = -\left.\frac{V_1}{I_2}\right|_{V_2=0}$$

B= negative short-circuit transfer impedance (Ω)

$$D = -\left.\frac{I_1}{I_2}\right|_{V_2=0}$$

D=negative short-circuit current ratio

19.5 Transmission Parameters (3)

Solving for Transmission Parameters

- To find the transmission parameters, analyze the circuit as follows:
- Perform the analysis with the output Open Circuited (I_2=0)

$$\begin{aligned} V_1 &= AV_2 - \cancel{BI_2}^{\,0} \\ I_1 &= CV_2 - \cancel{DI_2}^{\,0} \end{aligned} \Rightarrow \begin{aligned} V_1 &= AV_2 \\ I_1 &= CV_2 \end{aligned} \Rightarrow \begin{aligned} A &= \frac{V_1}{V_2} \\ C &= \frac{I_1}{V_2} \end{aligned}$$

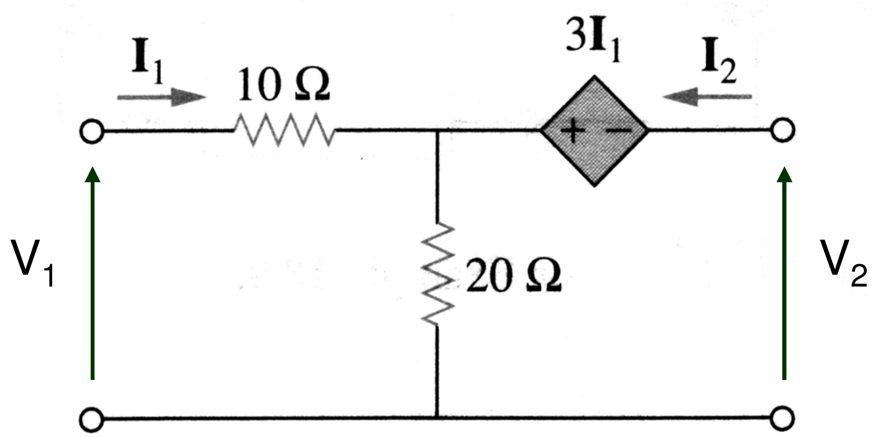
- Perform the analysis with the output Short Circuited(V_2=0)

$$\begin{aligned} V_1 &= \cancel{AV_2}^{\,0} - BI_2 \\ I_1 &= \cancel{CV_2}^{\,0} - DI_2 \end{aligned} \Rightarrow \begin{aligned} V_1 &= -BI_2 \\ I_1 &= -DI_2 \end{aligned} \Rightarrow \begin{aligned} B &= -\frac{V_1}{I_2} \\ D &= -\frac{I_1}{I_2} \end{aligned}$$

19.5 Transmission Parameters (4)

Example 19.8

Determine the T-parameters of the following circuit.



$$V_1 = AV_2 - BI_2$$
$$I_1 = CV_2 - DI_2$$

Apply KVL

$$V_1 = 10I_1 + 20(I_1 + I_2)$$
$$V_2 = -3I_1 + 20(I_1 + I_2)$$

$$V_1 = \frac{30}{17}V_2 - \frac{260}{17}I_2$$
$$I_1 = \frac{1}{17}V_2 - \frac{20}{17}I_2$$

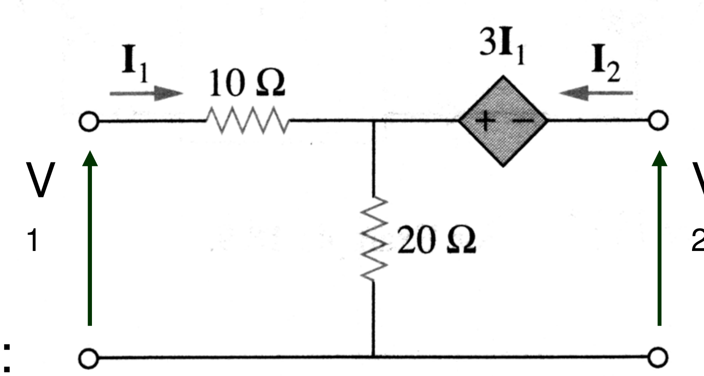
Answer: $$T = \begin{bmatrix} 1.765 & 15.294\Omega \\ 0.059S & 1.176 \end{bmatrix}$$

19.5 Transmission Parameters (5)

Example 19.8

From KVL:

$$V_1 = 10I_1 + 20(I_1 + I_2) = 30I_1 + 20I_2$$
$$V_2 = -3I_1 + 20(I_1 + I_2) = 17I_1 + 20I_2$$



If we “open circuit” the output we get:

$$V_1 = 30I_1 + 20\cancelto{0}{I_2} \qquad V_1 = 30I_1$$
$$V_2 = 17I_1 + 20\cancelto{0}{I_2} \qquad V_2 = 17I_1$$

$$A = \frac{V_1}{V_2} = \frac{30I_1}{17I_1} = \frac{30}{17} = 1.765$$
$$C = \tfrac{1}{17} = 0.0588$$

If we “short circuit” the output we get:

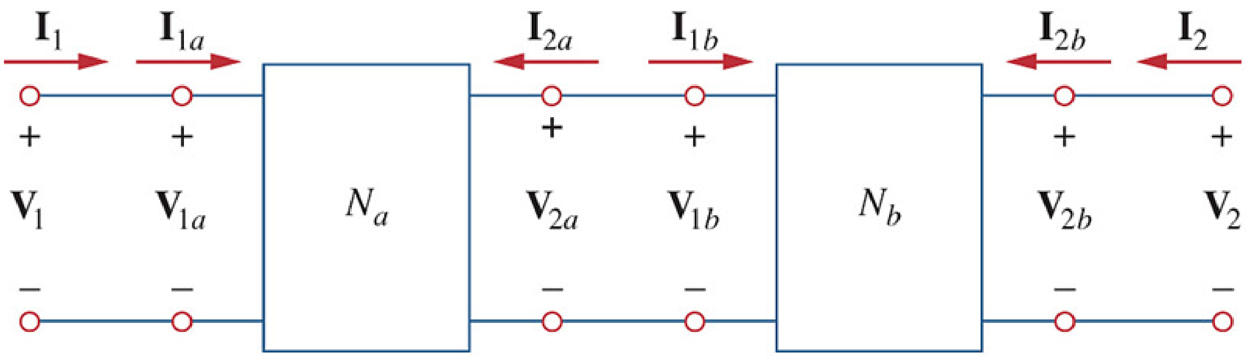
$$V_1 = 30I_1 + 20I_2 \qquad V_1 = 30I_1 + 20I_2$$
$$\cancelto{0}{V_2} = 17I_1 + 20I_2 \qquad 0 = 17I_1 + 20I_2$$

$$V_1 = 30\left(\tfrac{-20}{17}\right)I_2 + 20I_2$$
$$I_1 = \tfrac{-20}{17}I_2$$

$$B = -\frac{V_1}{I_2} = -\frac{\left(30\left(\tfrac{-20}{17}\right) + 20\right)I_2}{I_2} = 15.29$$
$$D = -\frac{I_1}{I_2} = \frac{20}{17} = 1.176$$

19.5 Transmission Parameters (6)

- Transmission Parameters can be cascaded with the result found through simple matrix multiplication



$$\begin{bmatrix} V_{1a} \\ I_{1a} \end{bmatrix} = \begin{bmatrix} A_a & B_a \\ C_a & D_a \end{bmatrix} \begin{bmatrix} V_{2a} \\ -I_{2a} \end{bmatrix} \qquad \begin{bmatrix} V_{1b} \\ I_{1b} \end{bmatrix} = \begin{bmatrix} A_b & B_b \\ C_b & D_b \end{bmatrix} \begin{bmatrix} V_{2b} \\ -I_{2b} \end{bmatrix}$$

$$-I_{2a} = I_{1b}$$

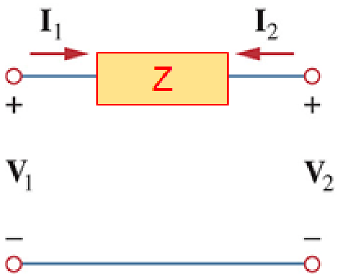
$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A_a & B_a \\ C_a & D_a \end{bmatrix} \begin{bmatrix} A_b & B_b \\ C_b & D_b \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$

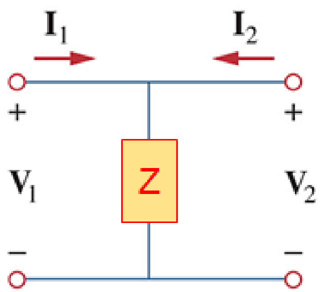
$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A' & B' \\ C' & D' \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$

19.5 Transmission Parameters (7)

Properties: Building Block Circuits

Consider the following simple circuits



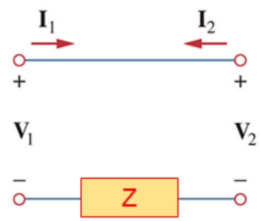


We can find their T Parameters to be:

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} 1 & Z \\ 0 & 1 \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ \frac{1}{Z} & 1 \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$

Note, following is equivalent

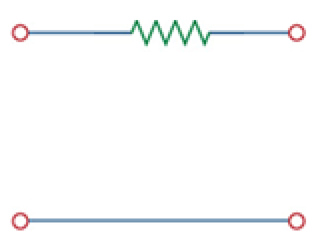


19.5 Transmission Parameters (8)

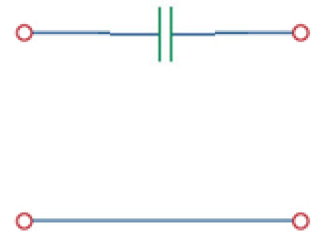
Properties: Building Block Circuits

- We can use this to construct the following “building block T parameters” to find the T parameters for any ladder type circuit.

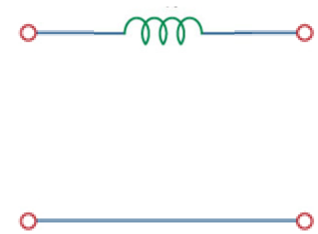
$$\begin{bmatrix} 1 & R \\ 0 & 1 \end{bmatrix}$$



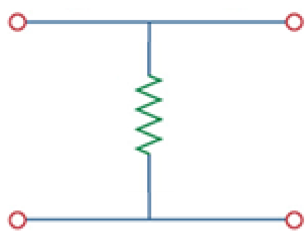
$$\begin{bmatrix} 1 & \frac{1}{sC} \\ 0 & 1 \end{bmatrix}$$



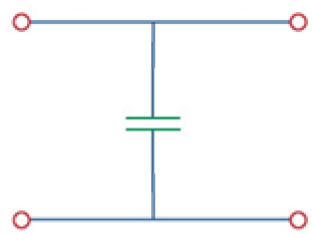
$$\begin{bmatrix} 1 & sL \\ 0 & 1 \end{bmatrix}$$



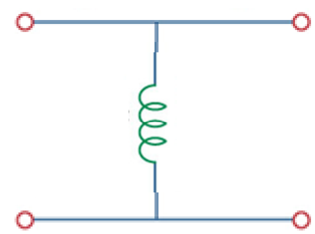
$$\begin{bmatrix} 1 & 0 \\ \frac{1}{R} & 1 \end{bmatrix}$$



$$\begin{bmatrix} 1 & 0 \\ sC & 1 \end{bmatrix}$$



$$\begin{bmatrix} 1 & 0 \\ \frac{1}{sL} & 1 \end{bmatrix}$$



19.5 Transmission Parameters (9)

Properties: Transfer function / Thevenin Equivalent

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- The "A" parameter can be used to provide the inverse of the voltage Transfer Function H(s).

$$\boxed{A = \left.\frac{V_1}{V_2}\right|_{I_2=0} = \frac{1}{H(s)}}$$

- Parameters "A" and "B" can be used to find a relationship between the Open Circuit Voltage (V_2) and the Short Circuit Current ($-I_2$).
- We can us this to find the parameters for the Thevenin Equivalent Circuit.

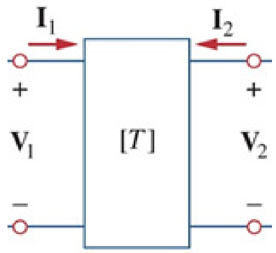
$$\boxed{A = \left.\frac{V_1}{V_2}\right|_{I_2=0} = \frac{V_1}{V_{oc}}}$$

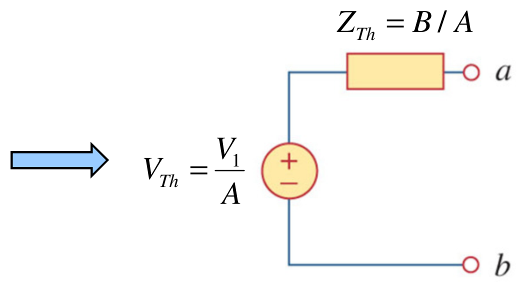
$$\boxed{V_{Th} = V_{oc} = \frac{V_1}{A}}$$

$$\boxed{Z_{Th} = \frac{V_{oc}}{I_{sc}} = \frac{B}{A}}$$

$$\boxed{B = -\left.\frac{V_1}{I_2}\right|_{V_2=0} = \frac{V_1}{I_{sc}}}$$

$$\boxed{I_N = I_{sc} = \frac{V_1}{B}}$$



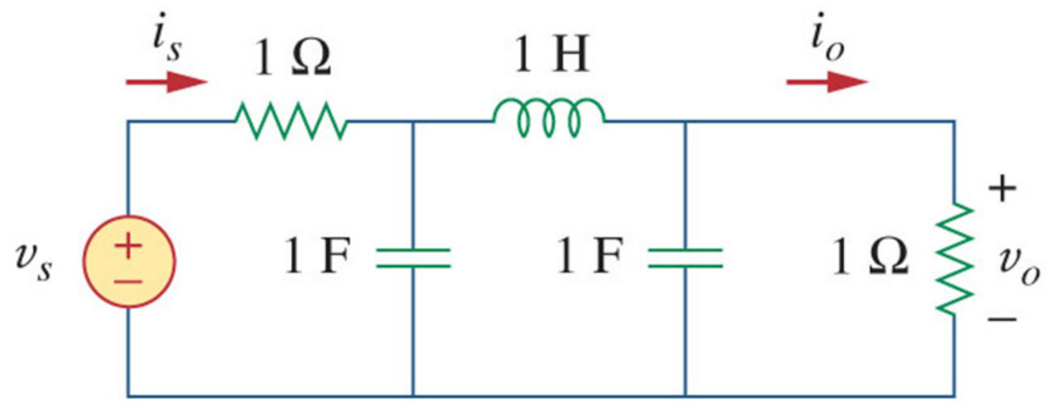


19.5 Transmission Parameters (10)

Transfer Function - Example

Problem 16.80(a)

Find the transfer function $V_o(s)/V_s(s)$ for the following circuit



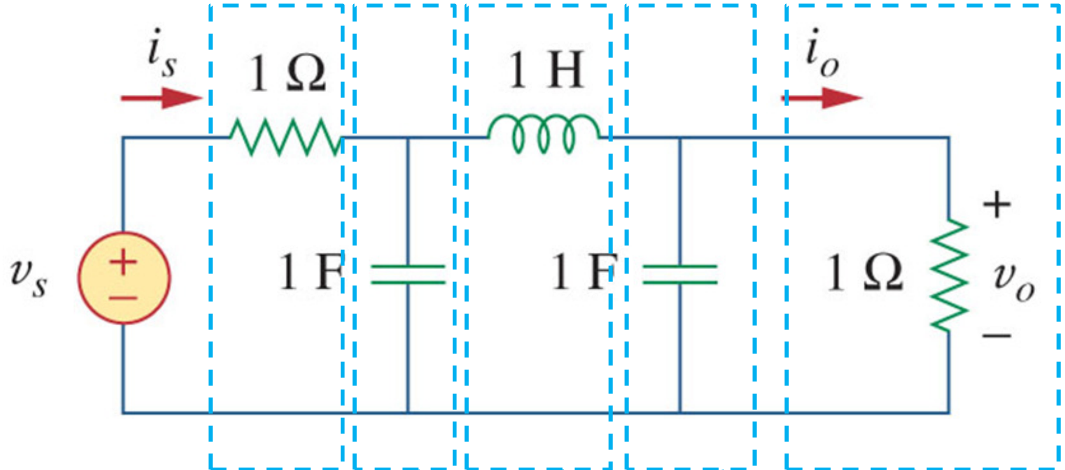
Answer: $$H(s) = \frac{1}{s^3 + 2s^2 + 3s + 2}$$

19.5 Transmission Parameters (11)

Transfer Function - Example

Problem 16.80(a) Solution:

a) Break up the circuit into a series of cascaded series and shunt components
b) Find the composite “T” parameters for the circuit
c) Use the relationship between the parameter “A” and the Transfer function



$$\begin{bmatrix} V_s \\ I_s \end{bmatrix} = \underbrace{\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ s & 1 \end{bmatrix}\begin{bmatrix} 1 & s \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ s & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}}_{\begin{bmatrix} A & B \\ C & D \end{bmatrix}}\begin{bmatrix} V_o \\ I_o \end{bmatrix}$$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} \rightarrow \boxed{A = \left.\frac{V_1}{V_2}\right|_{I_2=0} = \frac{1}{H(s)}}$$

19.5 Transmission Parameters (12)

Transfer Function - Example

Finding the combined T-matrix

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ s & 1 \end{bmatrix}\begin{bmatrix} 1 & s \\ 0 & 1 \end{bmatrix}\underbrace{\begin{bmatrix} 1 & 0 \\ s & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}}$$

$$\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ s & 1 \end{bmatrix}\underbrace{\begin{bmatrix} 1 & s \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ (s+1) & 1 \end{bmatrix}}$$

$$\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}\underbrace{\begin{bmatrix} 1 & 0 \\ s & 1 \end{bmatrix}\begin{bmatrix} 1+s(s+1) & s \\ (s+1) & 1 \end{bmatrix}}$$

$$\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1+s(s+1) & s \\ s+s^2(s+1)+(s+1) & s^2 \end{bmatrix}$$

$$\underbrace{\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} s^2+s+1 & s \\ s^3+s^2+2s+1 & s^2 \end{bmatrix}}$$

$$\begin{bmatrix} s^3+2s^2+3s+2 & s+s^2 \\ s^3+s^2+2s+1 & s^2 \end{bmatrix}$$

The transfer function can be found directly from the Transmission Parameter “A” !

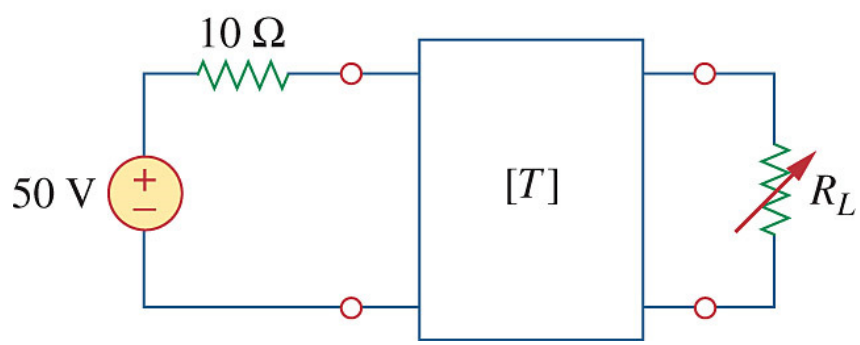
$$\boxed{A = \left.\frac{V_1}{V_2}\right|_{I_2=0} = \frac{1}{H(s)}}$$

$$H(s) = \frac{1}{s^3+2s^2+3s+2}$$

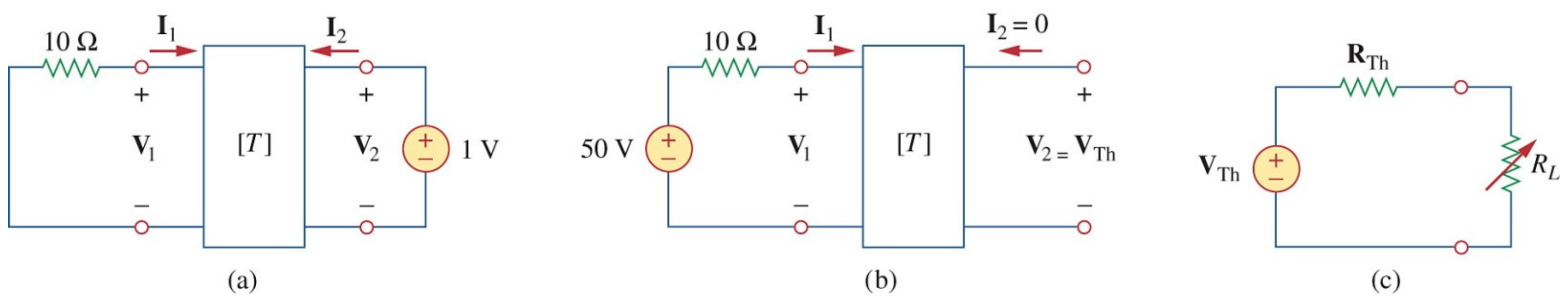
Example 19.9

The ABCD parameters of the two-port network at right are

$$T = \begin{bmatrix} 4 & 20\ \Omega \\ 0.1S & 2 \end{bmatrix}$$



The output port is connected to a variable load for maximum power transfer. Find R_L and the maximum power transferred.

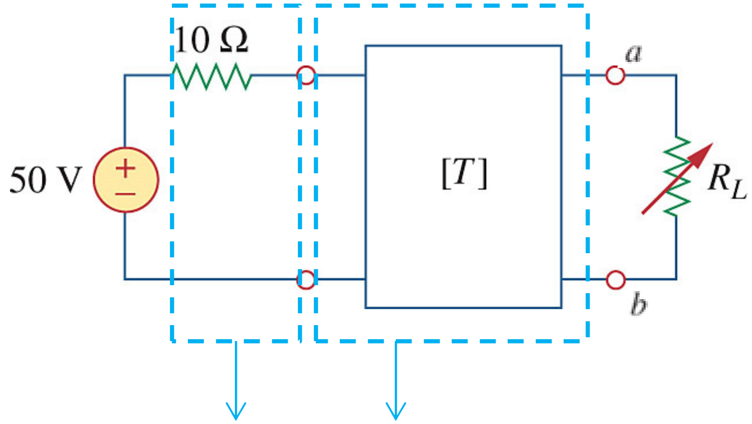


Answer: V_{TH} = 10V V; R_L = 8Ω; Pm = 3.125W.

19.5 Transmission Parameters (14)

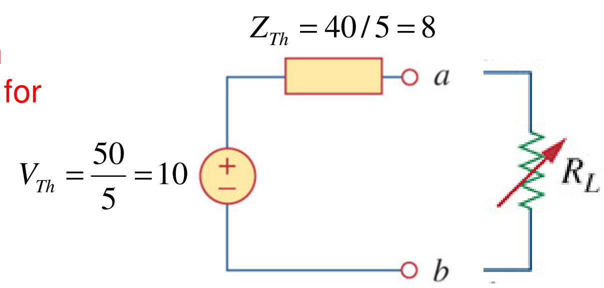
Solution: Example 19.9

a) Cascade the Series Resistor with the network
b) Find the composite "T" parameters for the circuit
c) Use the relationships to find V_{Th} and Z_{Th}



$$[T'] = \begin{bmatrix} 1 & 10 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 4 & 20 \\ 0.1 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 40 \\ 0.1 & 2 \end{bmatrix}$$

Find the Thevenin Equivalent Circuit for the source



For Max Power Transfer

$$\boxed{R_L = Z_{Th} = 8\ \Omega}$$

$$P_{\max} = I^2 R_L$$

$$P_{\max} = \left(\frac{V_{Th}}{R_L + Z_{Th}} \right)^2 R_L$$

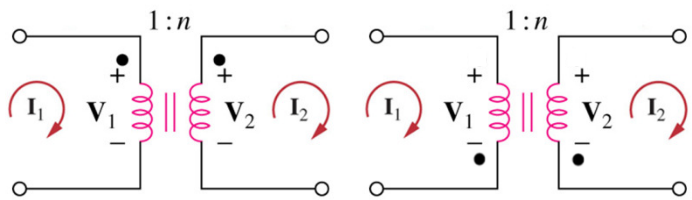
$$\boxed{P_{\max} = \left(\frac{10}{16} \right)^2 8 = 3.125\,\mathrm{W}}$$

19.5 Transmission Parameters (15)

Properties: Building Block Circuits – Ideal Transformer

- We can also use these “building blocks” to model ideal transformers. Remember from Chapter 13

Dots “Same” → “+”

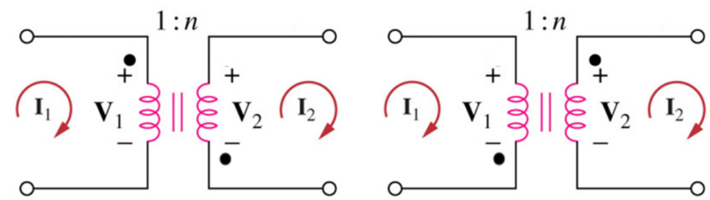


$$V_2 = nV_1$$

$$I_2 = \frac{I_1}{n}$$

$$\begin{bmatrix} \frac{1}{n} & 0 \\ 0 & n \end{bmatrix}$$

Dots “Different” → “-”



$$V_2 = -nV_1$$

$$I_2 = -\frac{I_1}{n}$$

$$\begin{bmatrix} -\frac{1}{n} & 0 \\ 0 & -n \end{bmatrix}$$

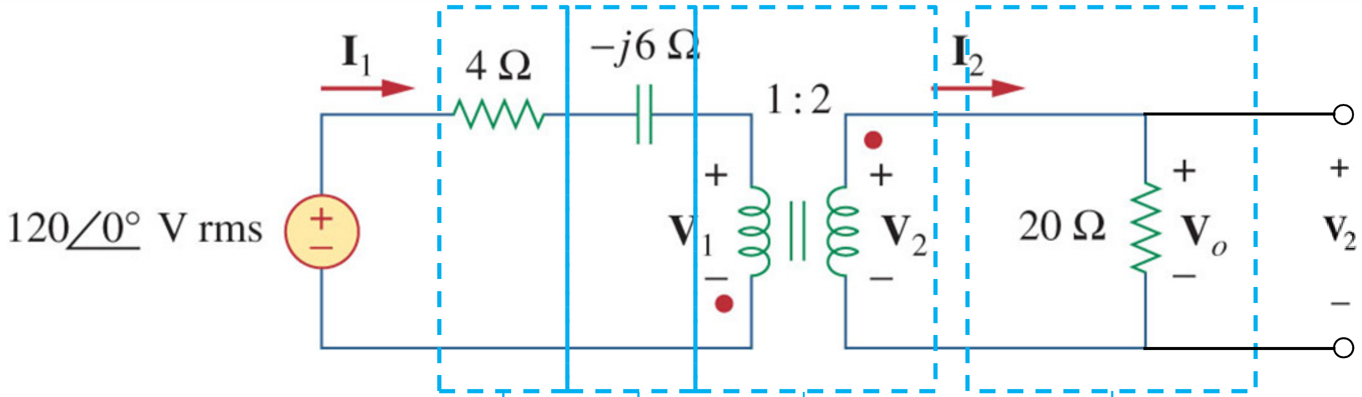
T - parameters

$$V_1 = AV_2 - BI_2$$

$$I_1 = CV_2 - DI_2$$

19.5 Transmission Parameters (16)

Example 13.8 Revisited



Cascaded T parameters →

$$\underset{a}{\begin{bmatrix} 1 & 4 \\ 0 & 1 \end{bmatrix}}\underset{b}{\begin{bmatrix} 1 & -j6 \\ 0 & 1 \end{bmatrix}}\underset{c}{\begin{bmatrix} -\frac{1}{2} & 0 \\ 0 & -2 \end{bmatrix}}\quad\underset{d}{\begin{bmatrix} 1 & 0 \\ 0.05 & 1 \end{bmatrix}}$$

Using MATLAB: →

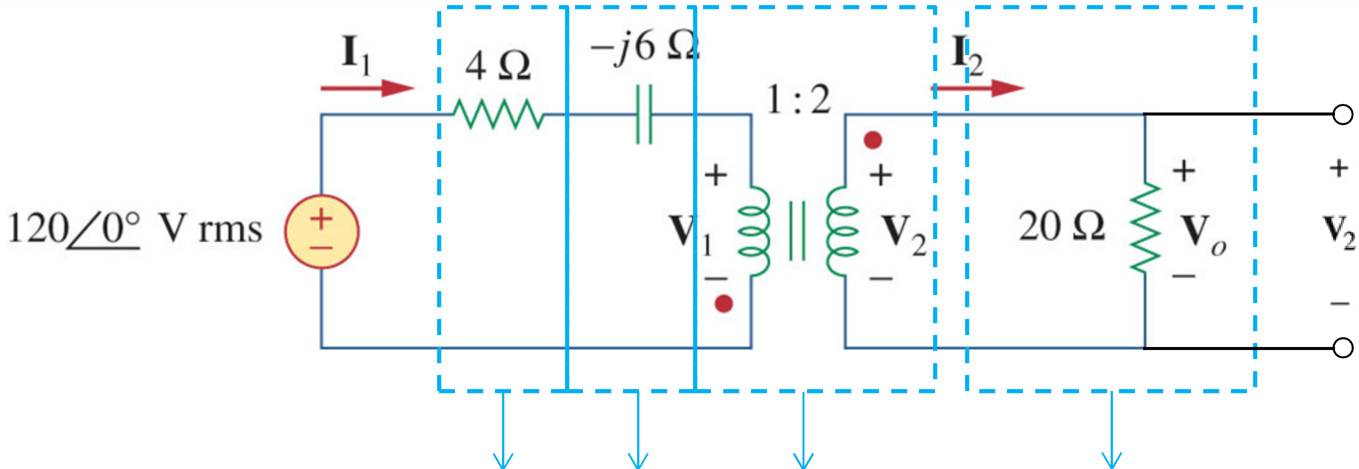
```
>> T=a*b*c*d

T =

  -0.9000 + 0.6000i  -8.0000 +12.0000i
  -0.1000 + 0.0000i  -2.0000 + 0.0000i
```

$$V_0 = V_2 = \frac{1}{A}V_1 = \left(\frac{1}{\boxed{-0.9+0.6j}}\right)(120\angle 0°) = \boxed{110.94\angle -146.31°\text{V}}$$

✓

19.5 Transmission Parameters (17)

Example 13.8 Revisited

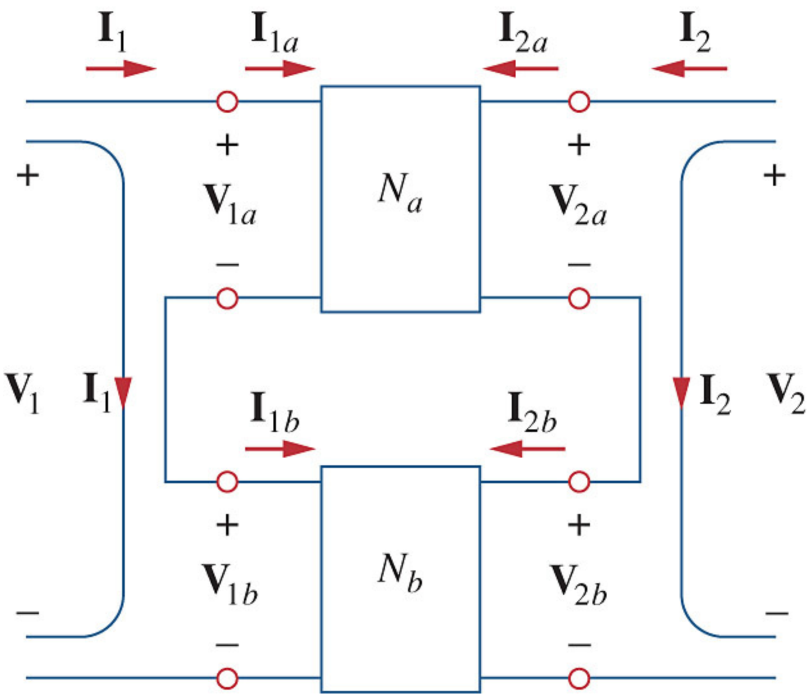



```
>> T=a*b*c*d

T =

  -0.9000 + 0.6000i  -8.0000 +12.0000i
  -0.1000 + 0.0000i  -2.0000 + 0.0000i
```

$$I_1 = CV_2 = \boxed{(-0.1)}(110.94\angle -146.31°) = \boxed{11.09\angle 33.69° \text{ A}}$$

$$I_2 = \frac{V_2}{20} = \frac{(110.94\angle -146.31°)}{20} = \boxed{5.55\angle -146.31° \text{ A}}$$

19.7 Interconnection of Networks (1)

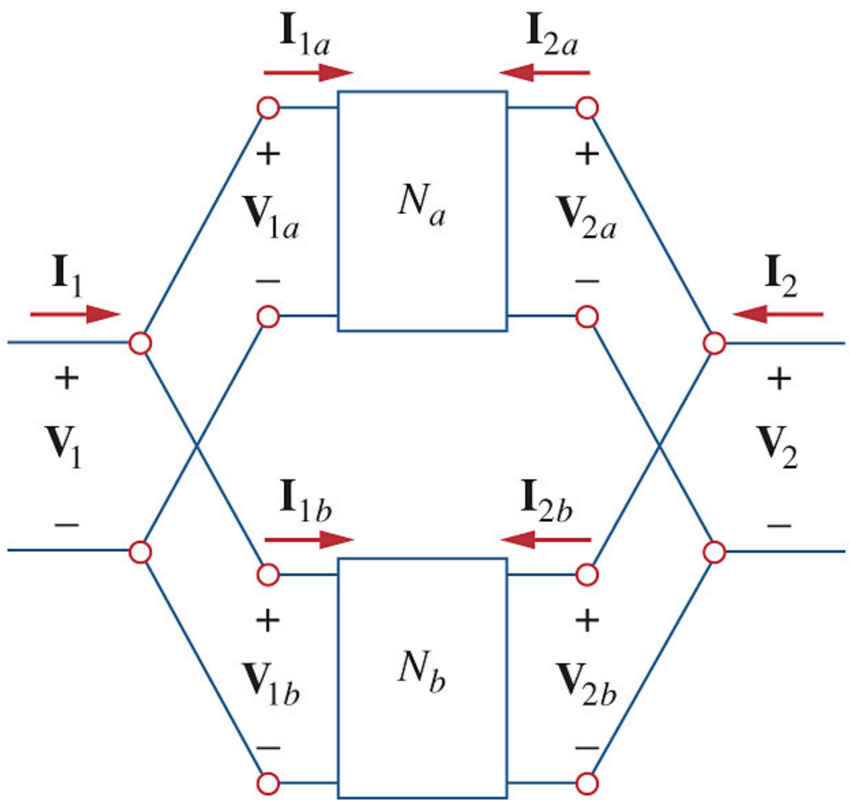


Series Connection of two-port networks:

For Impedances; ADD matrices.

$Z = Z_a + Z_b$

19.7 Interconnection of Networks (2)



Parallel Connection of two-port networks:

For Admittances; ADD matrices.

$$Y = Y_a + Y_b$$

19.6 Relationships Between Networks

- Use this table to convert between two port parameters

	z		y		h		T	
z	z_{11}	z_{12}	$\frac{y_{22}}{\Delta_y}$	$-\frac{y_{12}}{\Delta_y}$	$\frac{\Delta_h}{h_{22}}$	$\frac{h_{12}}{h_{22}}$	$\frac{A}{C}$	$\frac{\Delta_T}{C}$
	z_{21}	z_{22}	$-\frac{y_{21}}{\Delta_y}$	$\frac{y_{11}}{\Delta_y}$	$-\frac{h_{21}}{h_{22}}$	$\frac{1}{h_{22}}$	$\frac{1}{C}$	$\frac{D}{C}$
y	$\frac{z_{22}}{\Delta_z}$	$-\frac{z_{12}}{\Delta_z}$	y_{11}	y_{12}	$\frac{1}{h_{11}}$	$-\frac{h_{12}}{h_{11}}$	$\frac{D}{B}$	$-\frac{\Delta_T}{B}$
	$-\frac{z_{21}}{\Delta_z}$	$\frac{z_{11}}{\Delta_z}$	y_{21}	y_{22}	$\frac{h_{21}}{h_{11}}$	$\frac{\Delta_h}{h_{11}}$	$-\frac{1}{B}$	$\frac{A}{B}$
h	$\frac{\Delta_z}{z_{22}}$	$\frac{z_{12}}{z_{22}}$	$\frac{1}{y_{11}}$	$-\frac{y_{12}}{y_{11}}$	h_{11}	h_{12}	$\frac{B}{D}$	$\frac{\Delta_T}{D}$
	$-\frac{z_{21}}{z_{22}}$	$\frac{1}{z_{22}}$	$\frac{y_{21}}{y_{11}}$	$\frac{\Delta_y}{y_{11}}$	h_{21}	h_{22}	$-\frac{1}{D}$	$\frac{C}{D}$
T	$\frac{z_{11}}{z_{21}}$	$\frac{\Delta_z}{z_{21}}$	$-\frac{y_{22}}{y_{21}}$	$-\frac{1}{y_{21}}$	$-\frac{\Delta_h}{h_{21}}$	$-\frac{h_{11}}{h_{21}}$	A	B
	$\frac{1}{z_{21}}$	$\frac{z_{22}}{z_{21}}$	$-\frac{\Delta_y}{y_{21}}$	$-\frac{y_{11}}{y_{21}}$	$-\frac{h_{22}}{h_{21}}$	$-\frac{1}{h_{21}}$	C	D

$\Delta_y = y_{11}y_{22} - y_{12}y_{21}$ $\quad$ $\Delta_z = z_{11}z_{22} - z_{12}z_{21}$ $\quad$ $\Delta_h = h_{11}h_{22} - h_{12}h_{21}$ $\quad$ $\Delta_T = AD - BC$

Chapter 19 Review

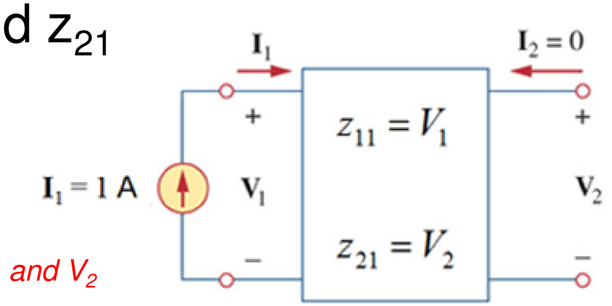
Z-Parameters

- Parameters: $$\boxed{\begin{aligned} V_1 &= z_{11}I_1 + z_{12}I_2 \\ V_2 &= z_{21}I_1 + z_{22}I_2 \end{aligned}}$$

- Open circuit the **output** to find z_{11} and z_{21}

$$\begin{aligned} V_1 &= z_{11}I_1 + \cancelto{0}{z_{12}I_2} \\ V_2 &= z_{21}I_1 + \cancelto{0}{z_{22}I_2} \end{aligned} \Rightarrow \begin{aligned} V_1 &= z_{11}I_1 \\ V_2 &= z_{21}I_1 \end{aligned}$$

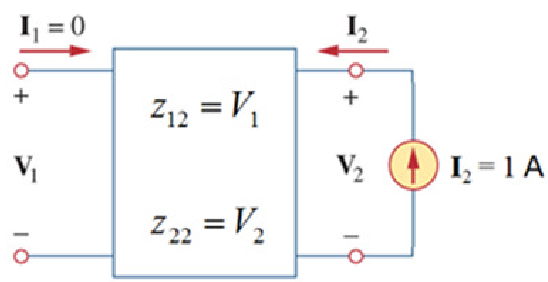
Set I_1 = 1 then solve for V_1 and V_2



- Open circuit the **input** to find z_{21} and z_{22}

$$\begin{aligned} V_1 &= \cancelto{0}{z_{11}I_1} + z_{12}I_2 \\ V_2 &= \cancelto{0}{z_{21}I_1} + z_{22}I_2 \end{aligned} \Rightarrow \begin{aligned} V_1 &= z_{12}I_2 \\ V_2 &= z_{22}I_2 \end{aligned}$$

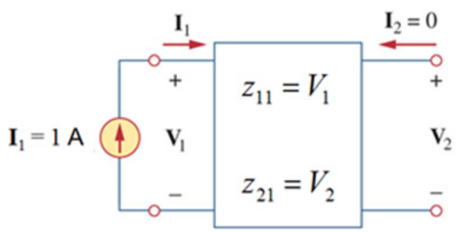
Set I_2 = 1 then solve for V_1 and V_2

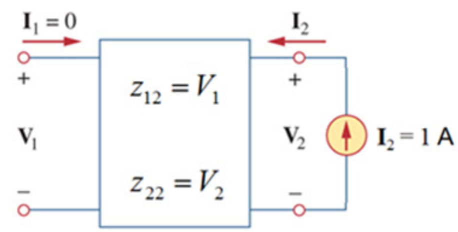


Chapter 19 Review

Z-Parameters (Given a circuit, find Z-parameters)

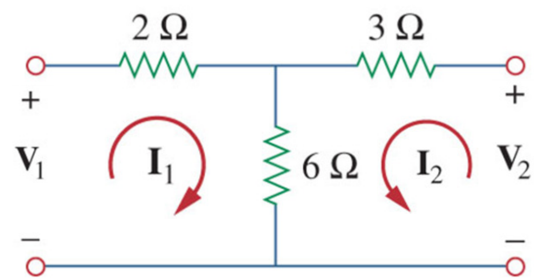
- Solving problems to find z-parameters:
  1. Refer to definition, apply 1 amp source at input and output with opposite port left open (see previous slide)





  2. Sometimes, KVL (mesh current equations) will cause z-parameters to fall right out! :

$$V_1 = 2I_1 + 6(I_1 + I_2) = 8I_1 + 6I_2$$
$$V_2 = 6(I_1 + I_2) + 3I_2 = 6I_1 + 9I_2$$
$$\Downarrow$$
$$z = \begin{bmatrix} 8 & 6 \\ 6 & 9 \end{bmatrix} \Omega$$

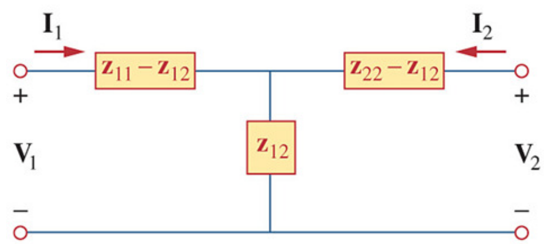


This mesh defined in counter clockwise direction for convenience

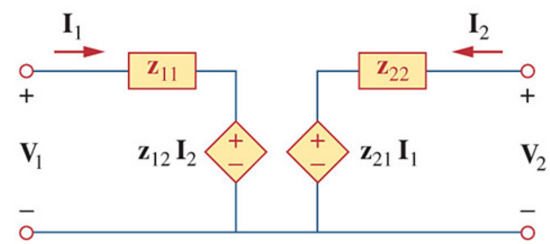
Chapter 19 Review

Z-Parameters (Given Z parameters, find circuit parameters)

- If given, z-parameters can use following techniques to find other circuit parameters (V_1, V_2, I_1, I_2, etc.):
  1. Apply the model and solve the circuit:



Reciprocal Network



General Network

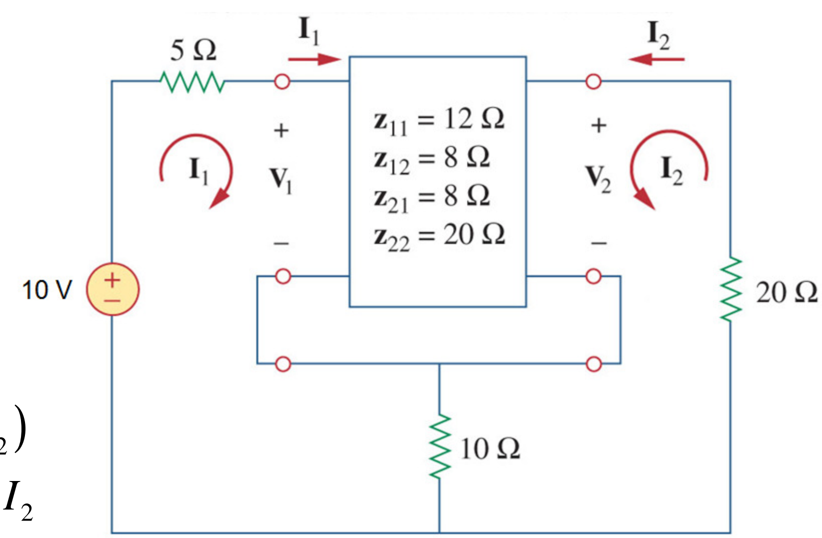
  2. Substitute the defining equations into your analysis:

Mesh Analysis

$$10 = 5I_1 + V_1 + 10(I_1 + I_2)$$
$$0 = V_2 + 10(I_1 + I_2) + 20I_2$$

Substitute for V_1 and V_2

$$10 = 5I_1 + (12I_1 + 8I_2) + 10(I_1 + I_2)$$
$$0 = (8I_1 + 20I_2) + 10(I_1 + I_2) + 20I_2$$

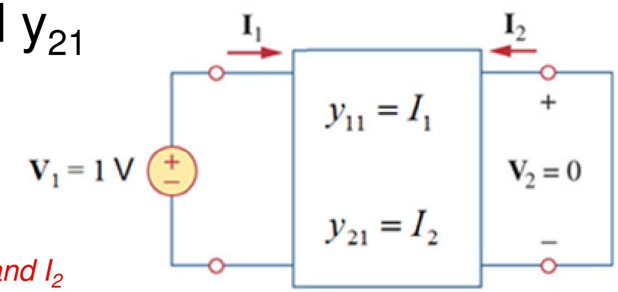


Chapter 19 Review
Y-Parameters

- Parameters: $$\boxed{\begin{aligned} I_1 &= y_{11}V_1 + y_{12}V_2 \\ I_2 &= y_{21}V_1 + y_{22}V_2 \end{aligned}}$$

- Short circuit the **output** to find y_{11} and y_{21}

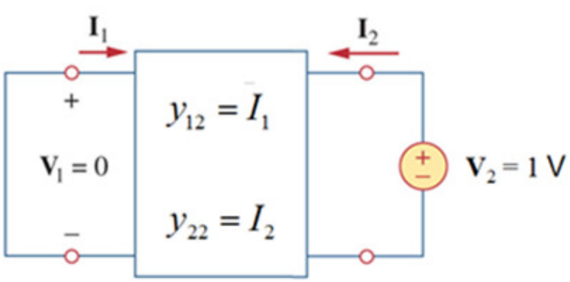
$$I_1 = y_{11}V_1 + \cancelto{0}{y_{12}V_2} \qquad \Rightarrow \qquad I_1 = y_{11}V_1$$
$$I_2 = y_{21}V_1 + \cancelto{0}{y_{22}V_2} \qquad \Rightarrow \qquad I_2 = y_{21}V_1$$



Set $V_1 = 1$ then solve for I_1 and I_2

- Short circuit the **input** to find y_{21} and y_{22}

$$I_1 = \cancelto{0}{y_{11}V_1} + y_{12}V_2 \qquad \Rightarrow \qquad I_1 = y_{12}V_2$$
$$I_2 = \cancelto{0}{y_{21}V_1} + y_{22}V_2 \qquad \Rightarrow \qquad I_2 = y_{22}V_2$$

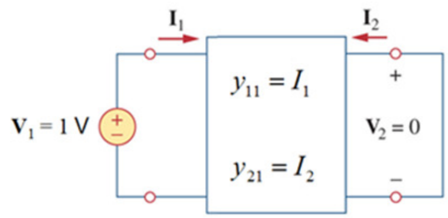


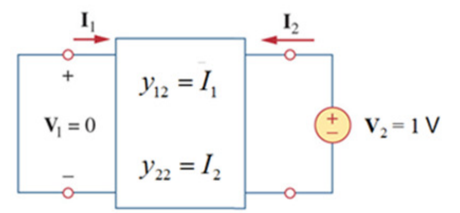
Set $V_2 = 1$ then solve for I_1 and I_2

Chapter 19 Review

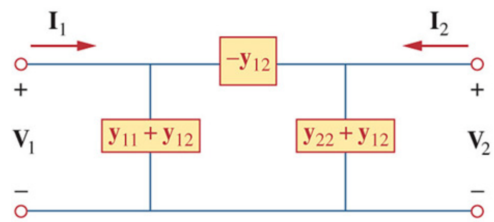
Y-Parameters (Solving Problems)

- To solve Y-parameter problems, can use these techniques
  1. Apply method from previous slide. Apply 1 Volt source at input and output while shorting opposite port

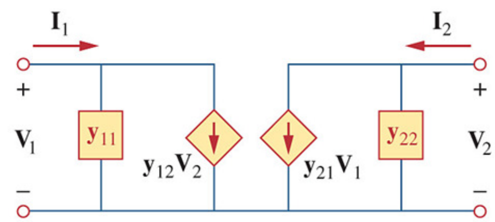




  2. If given Y parameters can apply the model and solve the circuit:



Reciprocal Network



General Network

  3. Make it easy on yourself! Use conversions from Z → Y or Y → Z

$$\begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix} = \left(\frac{1}{\Delta_y}\right)\begin{bmatrix} y_{22} & -y_{12} \\ -y_{21} & y_{11} \end{bmatrix}$$

$$\Delta_y = y_{11}y_{22} - y_{12}y_{21}$$

$$\begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} = \left(\frac{1}{\Delta_z}\right)\begin{bmatrix} z_{22} & -z_{12} \\ -z_{21} & z_{11} \end{bmatrix}$$

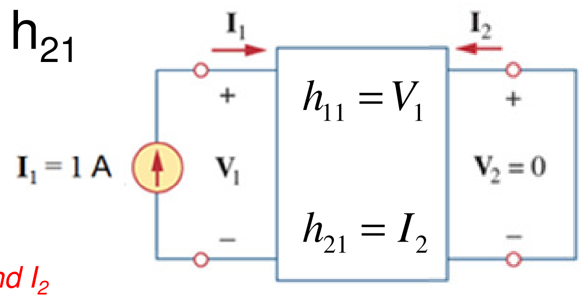
$$\Delta_z = z_{11}z_{22} - z_{12}z_{21}$$

Chapter 19 Review
H-Parameters

- Parameters (hybrid of z and y): $\boxed{\begin{aligned} V_1 &= h_{11}I_1 + h_{12}V_2 \\ I_2 &= h_{21}I_1 + h_{22}V_2 \end{aligned}}$

- Short circuit the **output** to find h_{11} and h_{21}

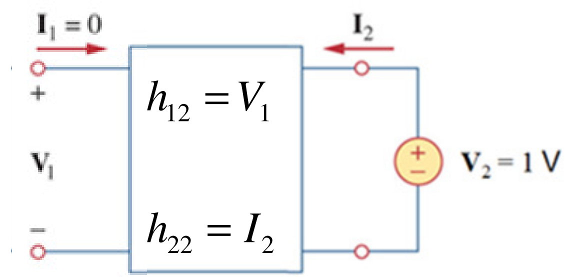
$$\begin{aligned} V_1 &= h_{11}I_1 + \cancelto{0}{h_{12}V_2} \\ I_2 &= h_{21}I_1 + \cancelto{0}{h_{22}V_2} \end{aligned} \quad \Rightarrow \quad \begin{aligned} V_1 &= h_{11}I_1 \\ I_2 &= h_{21}I_1 \end{aligned}$$



Set I_1 = 1 then solve for V_1 and I_2

- Open circuit the **input** to find h_{21} and h_{22}

$$\begin{aligned} V_1 &= \cancelto{0}{h_{11}I_1} + h_{12}V_2 \\ I_2 &= \cancelto{0}{h_{21}I_1} + h_{22}V_2 \end{aligned} \quad \Rightarrow \quad \begin{aligned} V_1 &= h_{12}V_2 \\ I_2 &= h_{22}V_2 \end{aligned}$$

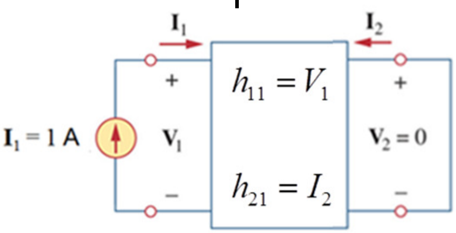


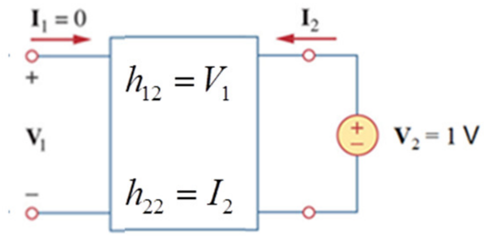
Set V_2 = 1 then solve for V_1 and I_2

Chapter 19 Review

H-Parameters (Solving Problems)

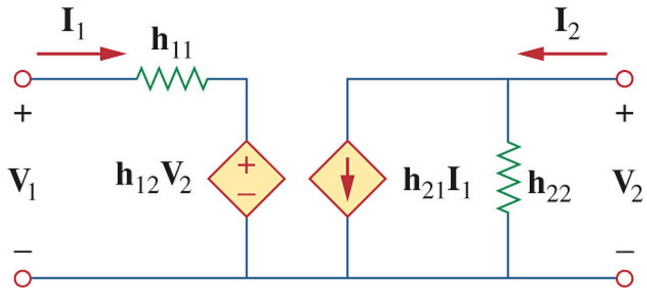
- To solve H-parameter problems, can use these techniques
  1. Apply methods from previous slide.





  2. H parameters can be found by performing a set of tests on the device
     a) Shorting the output and applying a current
     b) Leaving the input open and applying a voltage across the output

  3. If given H parameters can apply the model and solve the circuit:

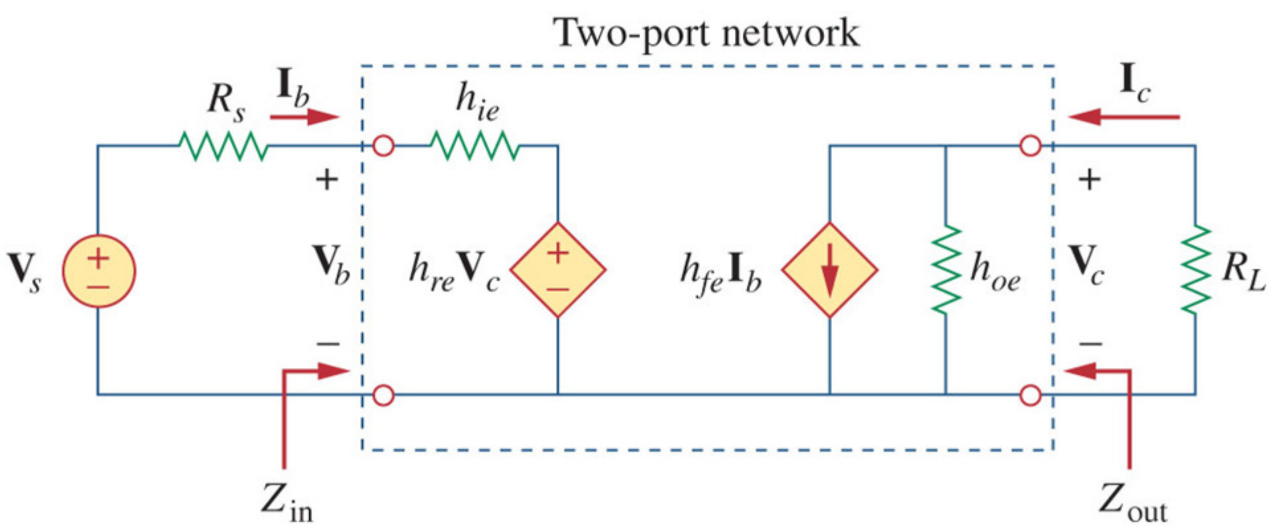


  4. If helpful, use conversion tables

Chapter 19 Review

H-Parameters (Transistor Model)

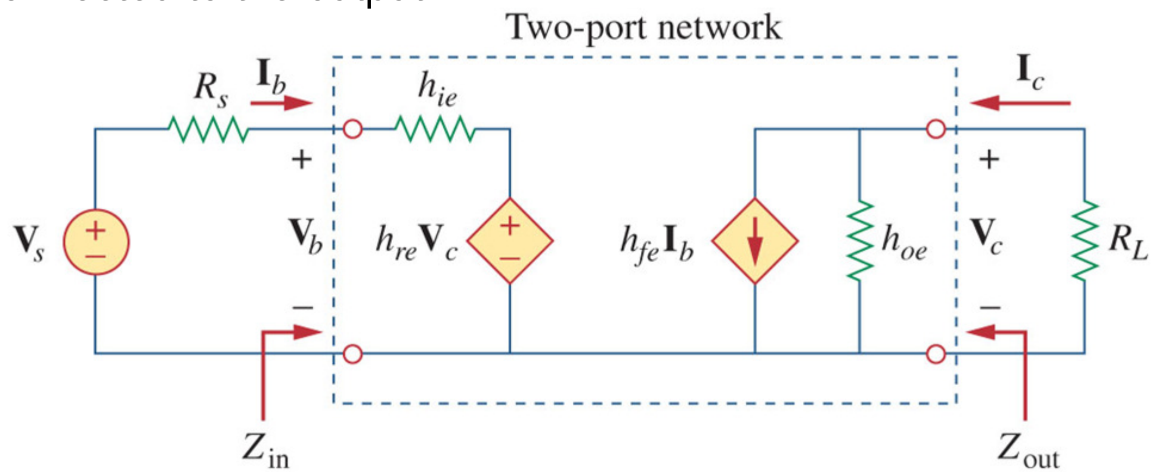
- H parameters are often used in modeling transistors
- Parameters vary depending on biasing conditions
- Spec sheets often use different subscripts:
  - h_{11} → h_{ie} = Base input impedance
  - h_{12} → h_{re} = Reverse voltage feedback ration
  - h_{21} → h_{fe} = Base-collector current gain
  - h_{22} → h_{oe} = Output admittance



Chapter 19 Review

H-Parameters (Transistor Model)

- Equations for calculating input impedance, output impedance, voltage gain, and current gain for simple transistor circuit:
  - V_s and R_s can be the Thevenin equivalent source driving the input.
  - R_L can be the input impedance looking into the load of the circuit connected to the output



Input Impedance

$$\boxed{Z_{in} = \frac{V_b}{I_b} = h_{ie} - \frac{h_{re}h_{fe}R_L}{1 + h_{oe}R_L}}$$

Output Impedance

$$\boxed{Z_{out} = \left.\frac{V_c}{I_c}\right|_{V_s=0} = \frac{R_s + h_{ie}}{(R_s + h_{ie})h_{oe} - h_{re}h_{fe}}}$$

Current Gain

$$\boxed{A_i = \frac{I_c}{I_b} = \frac{h_{fe}}{1 + h_{oe}R_L}}$$

Voltage Gain

$$\boxed{A_v = \frac{V_c}{V_b} = \frac{-h_{fe}R_L}{h_{ie} + (h_{ie}h_{oe} - h_{re}h_{fe})R_L}}$$

Chapter 19 Review

Transmission ("T") Parameters

- Parameters: $\boxed{\begin{aligned} V_1 &= AV_2 - BI_2 \\ I_1 &= CV_2 - DI_2 \end{aligned}}$

- Perform the analysis with the **output** Open Circuited (I_2=0)

$$\begin{aligned} V_1 &= AV_2 - \cancel{BI_2}^{\,0} \\ I_1 &= CV_2 - \cancel{DI_2}^{\,0} \end{aligned} \quad \Rightarrow \quad \begin{aligned} V_1 &= AV_2 \\ I_1 &= CV_2 \end{aligned} \quad \Rightarrow \quad \begin{aligned} A &= \frac{V_1}{V_2} \\ C &= \frac{I_1}{V_2} \end{aligned}$$

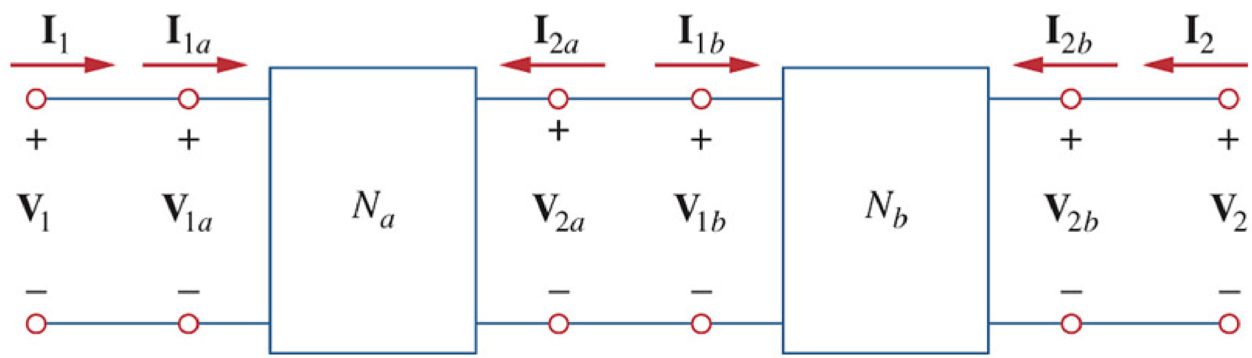
- Perform the analysis with the **output** Short Circuited(V_2=0)

$$\begin{aligned} V_1 &= \cancel{AV_2}^{\,0} - BI_2 \\ I_1 &= \cancel{CV_2}^{\,0} - DI_2 \end{aligned} \quad \Rightarrow \quad \begin{aligned} V_1 &= -BI_2 \\ I_1 &= -DI_2 \end{aligned} \quad \Rightarrow \quad \begin{aligned} B &= -\frac{V_1}{I_2} \\ D &= -\frac{I_1}{I_2} \end{aligned}$$

Chapter 19 Review

Transmission (“T”) Parameters (Cascading)

- Primary benefit of “T”-Parameters is their ability to be cascaded.



$$\begin{bmatrix} V_{1a} \\ I_{1a} \end{bmatrix} = \begin{bmatrix} A_a & B_a \\ C_a & D_a \end{bmatrix} \begin{bmatrix} V_{2a} \\ -I_{2a} \end{bmatrix} \qquad \begin{bmatrix} V_{1b} \\ I_{1b} \end{bmatrix} = \begin{bmatrix} A_b & B_b \\ C_b & D_b \end{bmatrix} \begin{bmatrix} V_{2b} \\ -I_{2b} \end{bmatrix}$$

$$-I_{2a} = I_{1b}$$

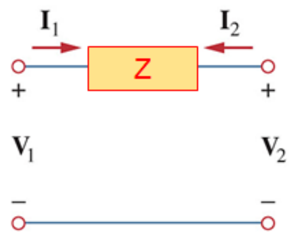
$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A_a & B_a \\ C_a & D_a \end{bmatrix} \begin{bmatrix} A_b & B_b \\ C_b & D_b \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$

$$\boxed{\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A' & B' \\ C' & D' \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}}$$

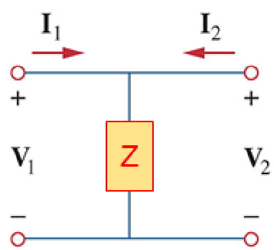
Chapter 19 Review

T - Parameters (Building Block models)

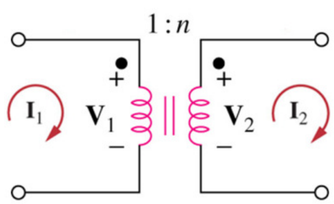
- We can create “building block” models of components by finding their T-parameters and use the cascading property to find the T-parameters for the complete circuit/system.



$$\Rightarrow \begin{bmatrix} 1 & Z \\ 0 & 1 \end{bmatrix}$$



$$\Rightarrow \begin{bmatrix} 1 & 0 \\ \frac{1}{Z} & 1 \end{bmatrix}$$



Dots “same”

$$\Rightarrow \begin{bmatrix} \frac{1}{n} & 0 \\ 0 & n \end{bmatrix}$$

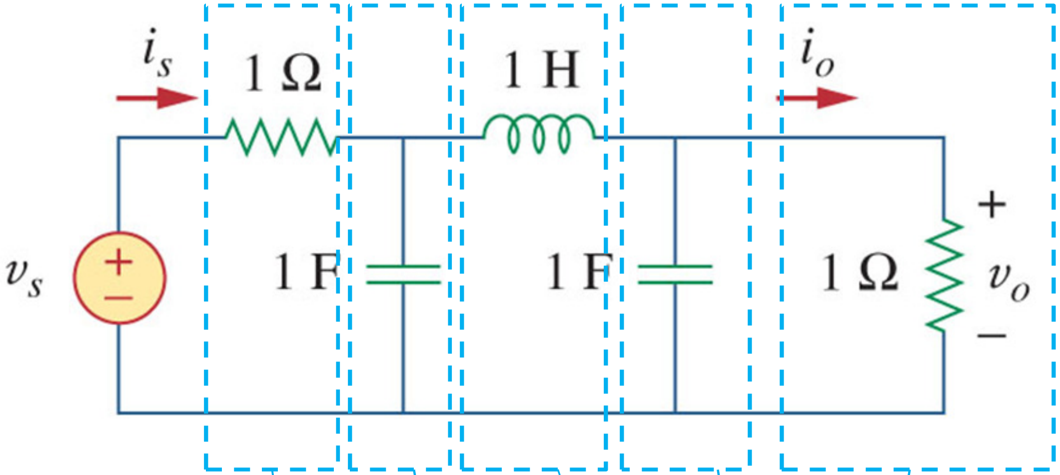
Dots “opposite”

$$\begin{bmatrix} -\frac{1}{n} & 0 \\ 0 & -n \end{bmatrix}$$

Chapter 19 Review

T - Parameters (Building Block models)

- With "Building Block" approach, circuits can be broke up into discrete components and analyzed using T-parameters



$$\begin{bmatrix} V_s \\ I_s \end{bmatrix} = \underbrace{\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ s & 1 \end{bmatrix}\begin{bmatrix} 1 & s \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ s & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}}_{\begin{bmatrix} A & B \\ C & D \end{bmatrix}}\begin{bmatrix} V_o \\ I_o \end{bmatrix}$$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} \longrightarrow \boxed{A = \left.\frac{V_1}{V_2}\right|_{I_2=0} = \frac{1}{H(s)}}$$

Chapter 19 Review

T - Parameters (Useful Properties)

- The T parameters give us useful properties in the analysis of circuits:
  - Open Circuit Voltage Transfer Function:

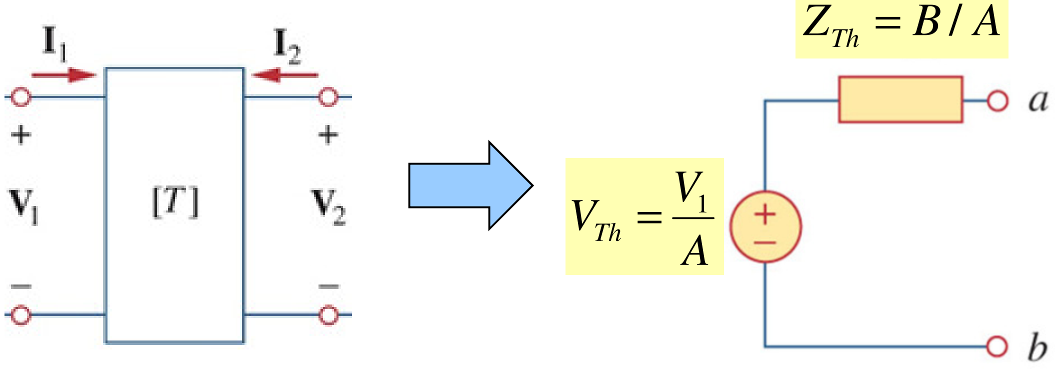
$$A = \left.\frac{V_1}{V_2}\right|_{I_2=0} = \frac{1}{H(s)}$$

$$H(s) = \frac{1}{A}$$

  - Thevenin Equivalent Circuit (Replace circuit as a source)

$$Z_{Th} = B / A$$

$$V_{Th} = \frac{V_1}{A}$$



Chapter 19 Review

Conversion between Parameters

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Indianapolis

- Conversion tables exists to convert between parameters

	z		y		h		T	
z	z_{11}	z_{12}	$\frac{y_{22}}{\Delta_y}$	$-\frac{y_{12}}{\Delta_y}$	$\frac{\Delta_h}{h_{22}}$	$\frac{h_{12}}{h_{22}}$	$\frac{A}{C}$	$\frac{\Delta_T}{C}$
	z_{21}	z_{22}	$-\frac{y_{21}}{\Delta_y}$	$\frac{y_{11}}{\Delta_y}$	$-\frac{h_{21}}{h_{22}}$	$\frac{1}{h_{22}}$	$\frac{1}{C}$	$\frac{D}{C}$
y	$\frac{z_{22}}{\Delta_z}$	$-\frac{z_{12}}{\Delta_z}$	y_{11}	y_{12}	$\frac{1}{h_{11}}$	$-\frac{h_{12}}{h_{11}}$	$\frac{D}{B}$	$-\frac{\Delta_T}{B}$
	$-\frac{z_{21}}{\Delta_z}$	$\frac{z_{11}}{\Delta_z}$	y_{21}	y_{22}	$\frac{h_{21}}{h_{11}}$	$\frac{\Delta_h}{h_{11}}$	$-\frac{1}{B}$	$\frac{A}{B}$
h	$\frac{\Delta_z}{z_{22}}$	$\frac{z_{12}}{z_{22}}$	$\frac{1}{y_{11}}$	$-\frac{y_{12}}{y_{11}}$	h_{11}	h_{12}	$\frac{B}{D}$	$\frac{\Delta_T}{D}$
	$-\frac{z_{21}}{z_{22}}$	$\frac{1}{z_{22}}$	$\frac{y_{21}}{y_{11}}$	$\frac{\Delta_y}{y_{11}}$	h_{21}	h_{22}	$-\frac{1}{D}$	$\frac{C}{D}$
T	$\frac{z_{11}}{z_{21}}$	$\frac{\Delta_z}{z_{21}}$	$-\frac{y_{22}}{y_{21}}$	$-\frac{1}{y_{21}}$	$-\frac{\Delta_h}{h_{21}}$	$-\frac{h_{11}}{h_{21}}$	A	B
	$\frac{1}{z_{21}}$	$\frac{z_{22}}{z_{21}}$	$-\frac{\Delta_y}{y_{21}}$	$-\frac{y_{11}}{y_{21}}$	$-\frac{h_{22}}{h_{21}}$	$-\frac{1}{h_{21}}$	C	D

$\Delta_y = y_{11}y_{22} - y_{12}y_{21}$ $\Delta_z = z_{11}z_{22} - z_{12}z_{21}$ $\Delta_h = h_{11}h_{22} - h_{12}h_{21}$ $\Delta_T = AD - BC$

Chapter 19: Two-Port Networks

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19.1 Introduction (1)

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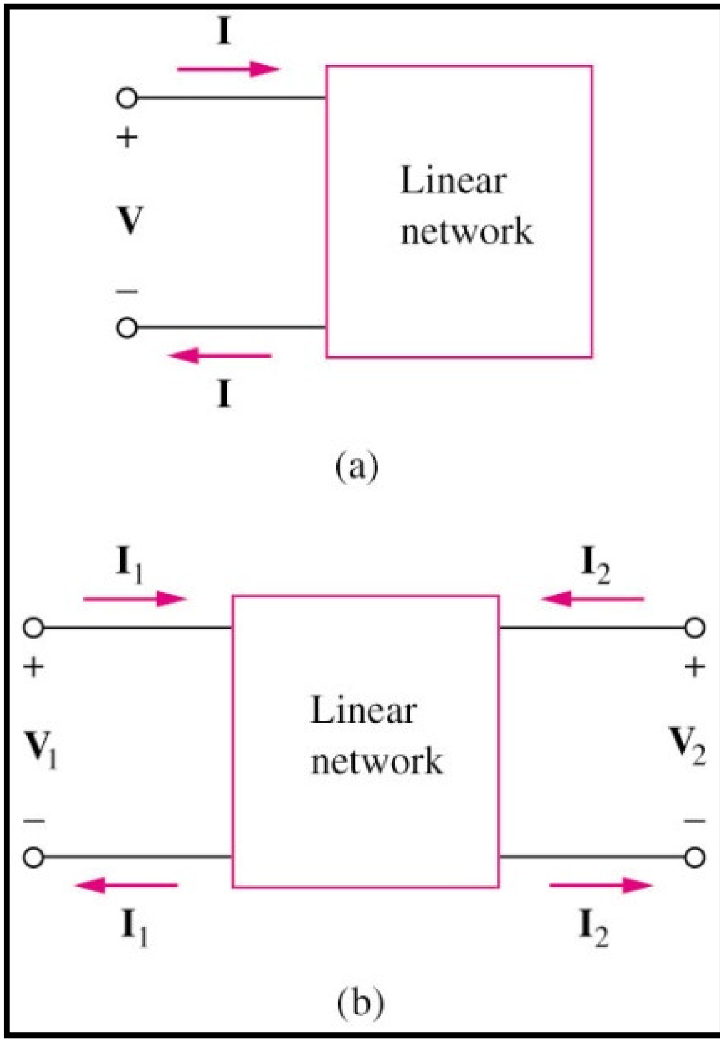
- A port is an access to the network and consists of a pair of terminals; the current entering one terminal leaves through the other terminal so that the net current entering the port equals zero.
- One port networks include two-terminal devices such as resistors, capacitors, and inductors.
- A two-port network has two separate ports for input and output.
- Two port networks include op amps, transistors and transformers.

19.1 Introduction (2)



One port or two terminal circuit

Two port or four terminal circuit

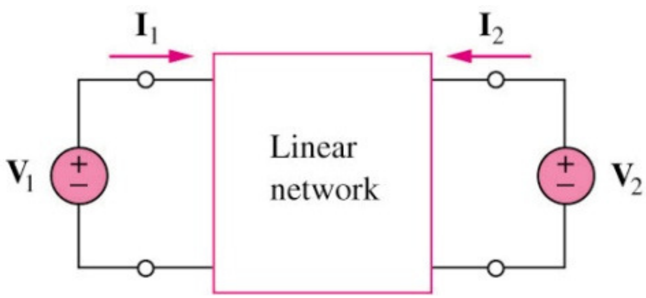
- **It is an electrical network with two separate ports for input and output.**
- **No independent sources.**

19.1 Introduction (3)

- Characterizing a two-port network requires that we relate the terminal quantities $\mathbf{V}_1$, $\mathbf{V}_2$, $\mathbf{I}_1$, $\mathbf{I}_2$ out of which two are independent. Six sets of voltage and current parameters will be derived.
- Two port networks are useful in communications, control systems, power systems, and electronics.
- They are used in electronics to model transistors and to facilitate cascaded design.
- Additionally, if we know the parameters of a two-port network it can be treated as a "black box" when embedded within a larger network.

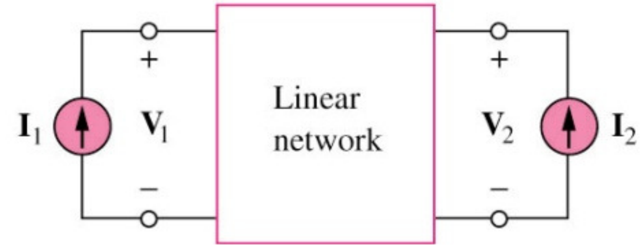
19.2 Impedance Parameters (1)

- Often called “**Z-parameters**” since their units are in **ohms** and they represent an impedance relationship between $\mathbf{V}_1$, $\mathbf{V}_2$, $\mathbf{I}_1$, $\mathbf{I}_2$ for the two port network shown below:



Voltage Driven Network

Assume no independent source in the network



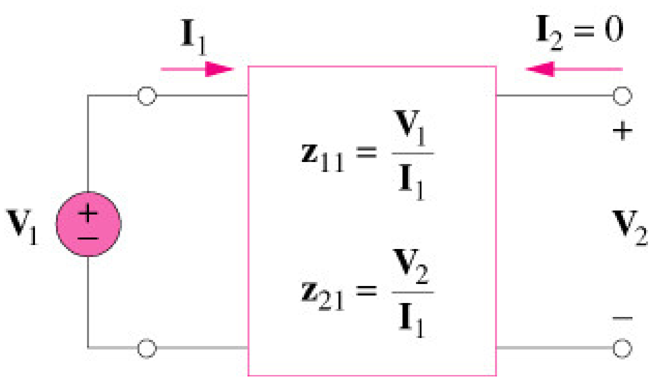
Current Driven Network

$$\begin{aligned} V_1 &= z_{11} I_1 + z_{12} I_2 \\ V_2 &= z_{21} I_1 + z_{22} I_2 \end{aligned} \Rightarrow \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

- Z-parameters are commonly used in filter synthesis, impedance matching networks design, and power distribution networks analysis.

19.2 Impedance Parameters (2)

The values of parameters can be evaluated by setting I_1=0 or I_2=0 (open circuit)

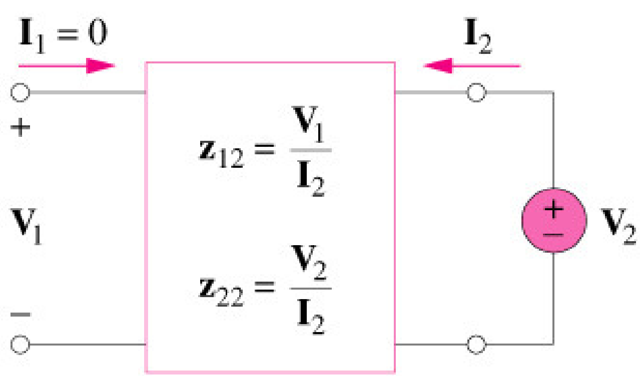


(a)

Setting I_2 = 0

$$z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0} \quad \text{and} \quad z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0}$$

z_{11} = Open-circuit input impedance
z_{21} = Open-circuit transfer impedance from port 2 to port 1



(b)

Setting I_1 = 0

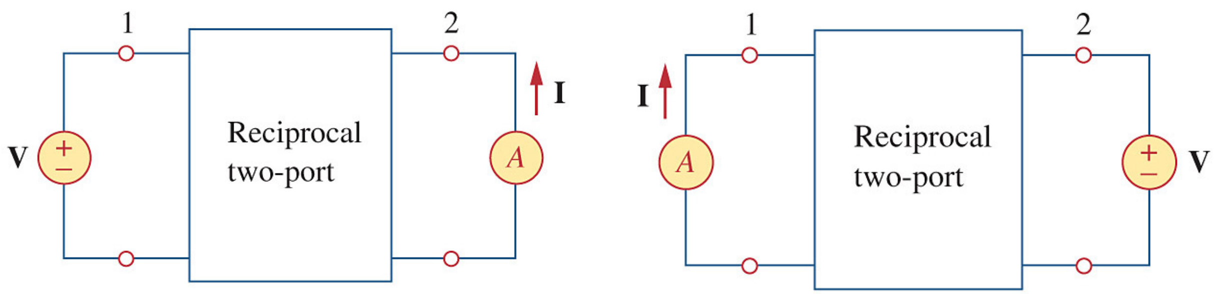
$$z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1=0} \quad \text{and} \quad z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0}$$

z12 = Open-circuit transfer impedance from port 1 to port 2
z22 = Open-circuit output impedance

19.2 Impedance Parameters (3)

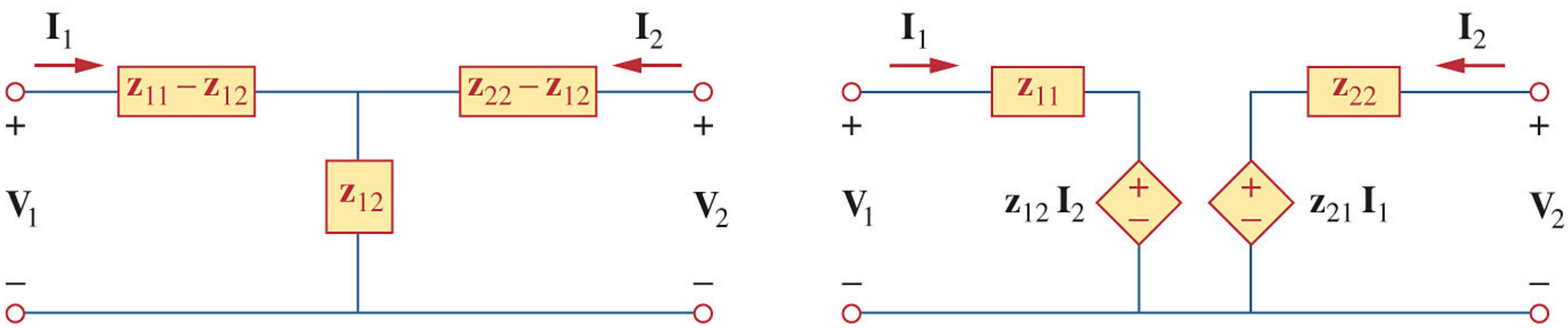
Properties of Z-parameters

- Symmetrical networks $z_{11} = z_{22}$
  - Implies a mirror like symmetry
- Reciprocal networks $z_{12} = z_{21}$
  - Any network made up entirely of resistors, capacitors, and inductors must be reciprocal.
  - Linear networks with no dependant sources are reciprocal.
  - Interchanging an ideal voltage source at one port with an ideal ammeter at the other port gives the same ammeter reading.



$$V = z_{12} I \qquad\qquad V = z_{21} I$$

19.2 Impedance Parameters (4)

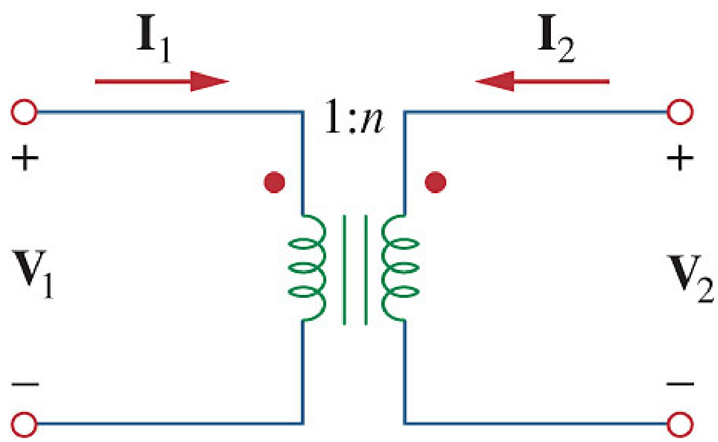


Reciprocal Network

General Network

- A reciprocal network can be replaced by the T-network shown above
- If not reciprocal, the General network is the T-equivalent.

19.2 Impedance Parameters (5)



- Note: some circuits do not have z-parameter equivalents. (they may have other 2-port equivalents, as we shall see)
- Consider an ideal transformer:

$$V_1 = V_2/n \quad \text{and} \quad I_1 = -nI_2.$$

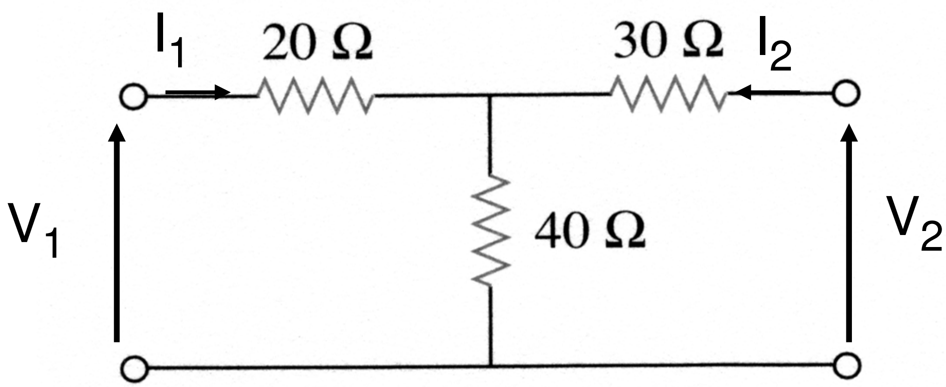
- This cannot be expressed by:

$$V_1 = z_{11}I_1 + z_{12}I_2$$
$$V_2 = z_{21}I_1 + z_{22}I_2$$

19.2 Impedance Parameters (6)

Example 19.1

Determine the z-parameters of the following circuit.



$$z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0} \quad \text{and} \quad z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0}$$

$$z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1=0} \quad \text{and} \quad z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0}$$

$$z = \begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix} \Omega$$

Answer: $$z = \begin{bmatrix} 60 & 40 \\ 40 & 70 \end{bmatrix} \Omega$$

19.2 Impedance Parameters (7)

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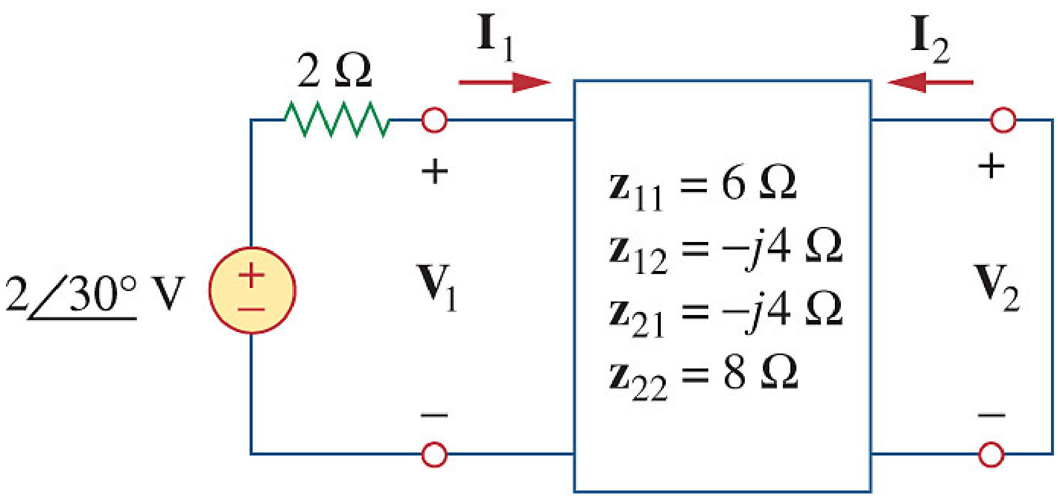
Practice Problem 19.2

Determine I_1 and I_2 in the following circuit.

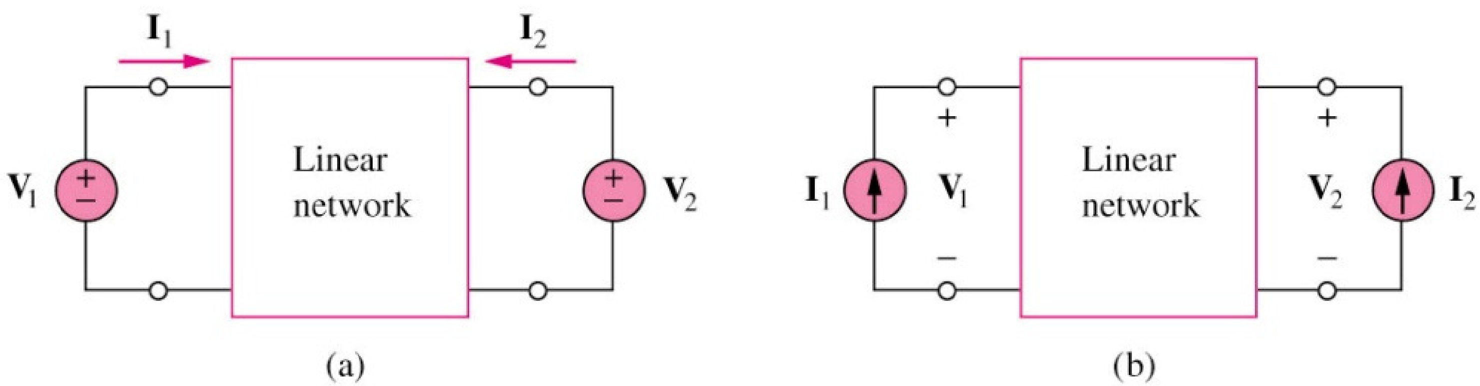


$$V_1 = z_{11}I_1 + z_{12}I_2$$
$$V_2 = z_{21}I_1 + z_{22}I_2$$

Answer:

$$I_1 = 200\angle 30°\ \text{mA}$$
$$I_2 = 100\angle 120°\ \text{mA}$$

19.3 Admittance Parameters (1)



Assume no independent source in the network

$$I_1 = y_{11}V_1 + y_{12}V_2$$
$$I_2 = y_{21}V_1 + y_{22}V_2$$

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = [y] \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

where the **y** terms are called the admittance parameters, or simply y parameters, and they have units of Siemens.

19.3 Admittance Parameters (2)

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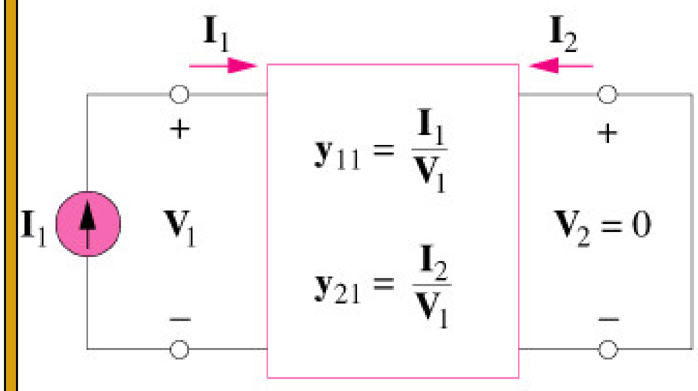
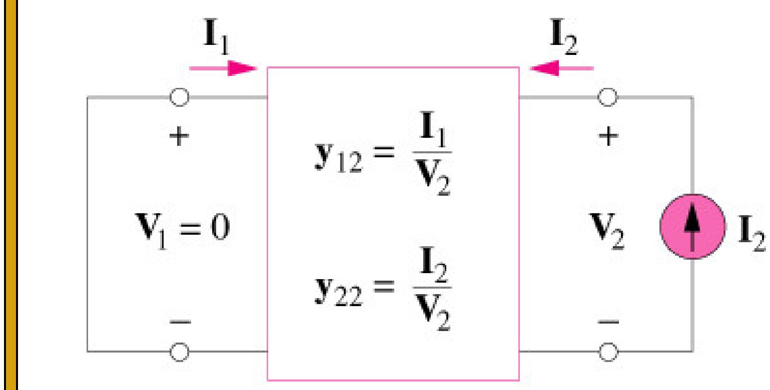


(a)



(b)

Setting $V_2 = 0$ (Shorting the output)

$$y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2=0} \quad \text{and} \quad y_{21} = \left.\frac{I_2}{V_1}\right|_{V_2=0}$$

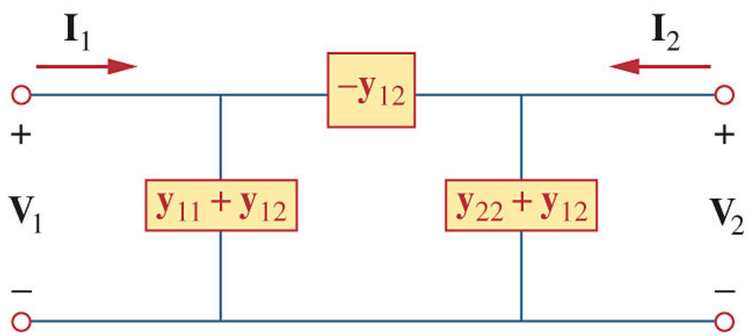
y_{11} = Short-circuit input admittance
y_{21} = Short-circuit transfer admittance from port 1 to port 2

Setting $V_1 = 0$ (Shorting the input)

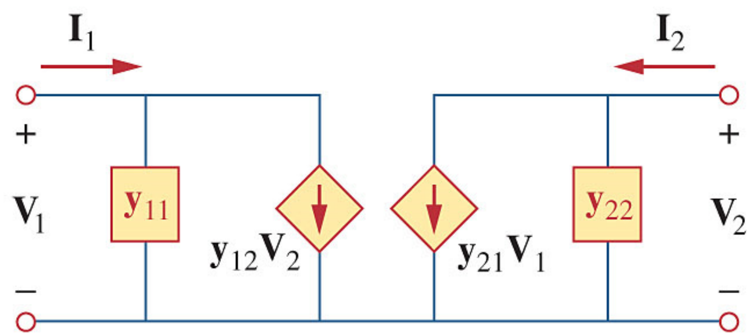
$$y_{12} = \left.\frac{I_1}{V_2}\right|_{V_1=0} \quad \text{and} \quad y_{22} = \left.\frac{I_2}{V_2}\right|_{V_1=0}$$

y_{12} = Short-circuit transfer admittance from port 2 to port 1
y_{22} = Short-circuit output admittance

19.3 Admittance Parameters (3)



Reciprocal Network



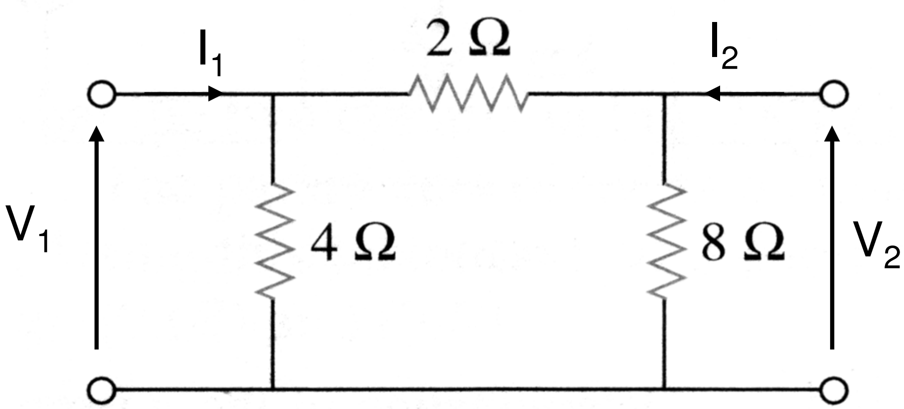
General Network

- A reciprocal network ($y_{12} = y_{21}$) can be replaced by the Pi-network in figure (a).
- If not reciprocal, the network in figure (b) is the Pi-equivalent.

19.3 Admittance Parameters (4)

Example 19.3

Determine the y-parameters of the following circuit.



$$y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2=0} \quad \text{and} \quad y_{21} = \left.\frac{I_2}{V_1}\right|_{V_2=0}$$

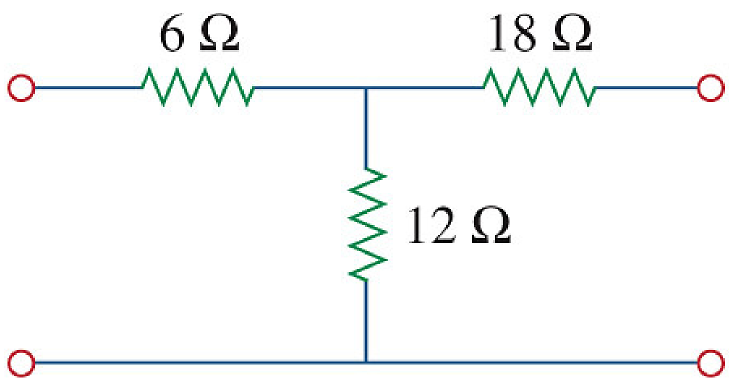
$$y_{12} = \left.\frac{I_1}{V_2}\right|_{V_1=0} \quad \text{and} \quad y_{22} = \left.\frac{I_2}{V_2}\right|_{V_1=0}$$

$$y = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} S$$

Answer: $y = \begin{bmatrix} 0.75 & -0.5 \\ -0.5 & 0.625 \end{bmatrix} S$

Practice Problem 19.3

Determine the y-parameters of the following circuit.



$$y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2=0} \quad \text{and} \quad y_{21} = \left.\frac{I_2}{V_1}\right|_{V_2=0}$$

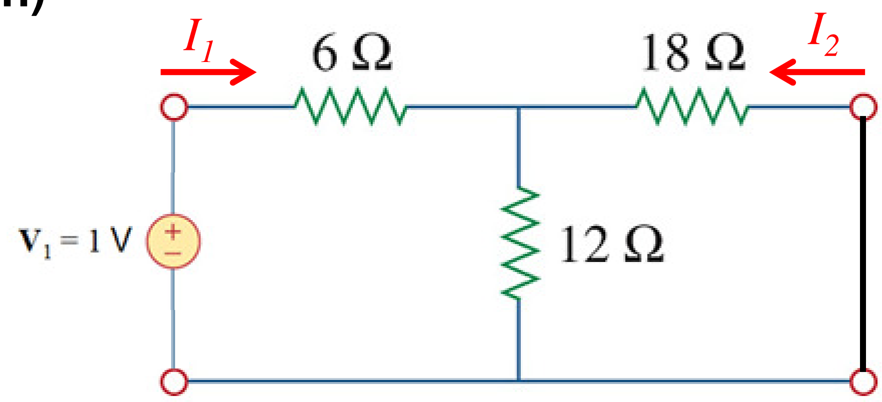
$$y_{12} = \left.\frac{I_1}{V_2}\right|_{V_1=0} \quad \text{and} \quad y_{22} = \left.\frac{I_2}{V_2}\right|_{V_1=0}$$

$$y = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} S$$

Answer: $y = \begin{bmatrix} 75.77 & -30.3 \\ -30.3 & 45.47 \end{bmatrix} \text{mS}$

Practice Problem 19.3 (Solution)

- Short the output
- Put a 1 volt source at input
- Find I_1 and I_2



$$y_{11} = \frac{I_1}{(1)}\bigg|_{V_2=0} \quad and \quad y_{21} = \frac{I_2}{(1)}\bigg|_{V_2=0}$$

Find Input Impedance

$$Z_{in} = 6 + 12 \| 18 = 13.2$$

$$I_1 = \frac{V_1}{Z_{in}} = \frac{1}{13.2} = 0.07576$$

$$\boxed{y_{11} = 0.07576}$$

Similarly at Output

$$Z_{out} = 18 + 6 \| 12 = 22$$

$$I_2 = \frac{V_2}{Z_{in}} = \frac{1}{22} = 0.04545$$

$$\boxed{y_{22} = 0.04545}$$

Find I_2 from current divider equation

$$I_2 = \frac{-12}{12+18} I_1$$

$$I_2 = (-0.4)0.07576 = -0.0303$$

$$y_{21} = -0.0303$$

$$\boxed{y_{12} = y_{21} = -0.0303}$$

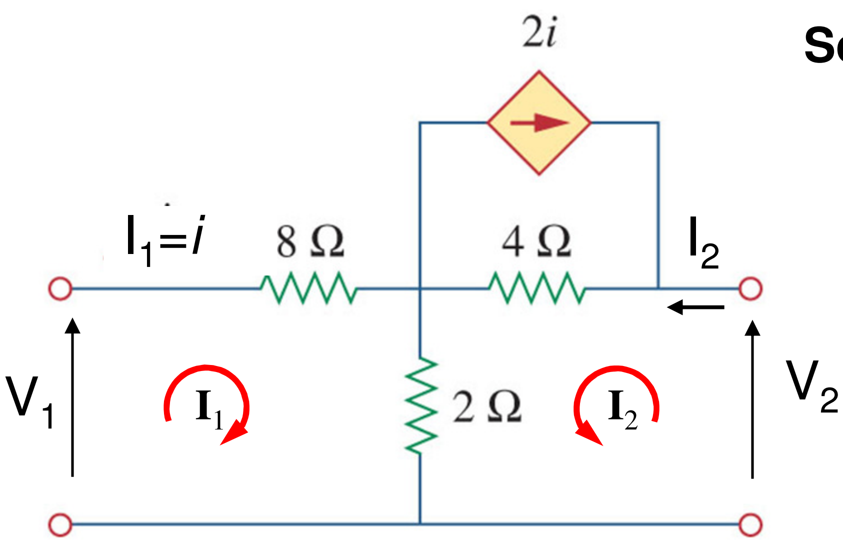
Reciprocal Network

19.3 Admittance Parameters (7)

Example 19.4

Determine the y-parameters of the following circuit.

$$\boxed{\begin{aligned} I_1 &= y_{11}V_1 + y_{12}V_2 \\ I_2 &= y_{21}V_1 + y_{22}V_2 \end{aligned}}$$



Solution: Apply KVL

Mesh I_1: $V_1 = 8I_1 + 2(I_1 + I_2)$

$$V_1 = 10I_1 + 2I_2$$

Mesh I_2: $V_2 = 4(2i + I_2) + 2(I_1 + I_2)$

$$V_2 = 8I_1 + 4I_2 + 2I_1 + 2I_2$$

$$V_2 = 10I_1 + 6I_2$$

Subtract #1 from #2:

$$V_2 - V_1 = 0 + 4I_2$$

$$I_2 = -0.25V_1 + 0.25V_2$$

Substitute back into #1

$$V_1 = 10I_1 - 0.5V_1 + 0.5V_2$$

$$10I_1 = 1.5V_1 - 0.5V_2$$

$$I_1 = 0.15V_1 - 0.05V_2$$

Answer: $\boxed{\mathbf{y} = \begin{bmatrix} 0.15 & -0.05 \\ -0.25 & 0.25 \end{bmatrix} \text{S}}$

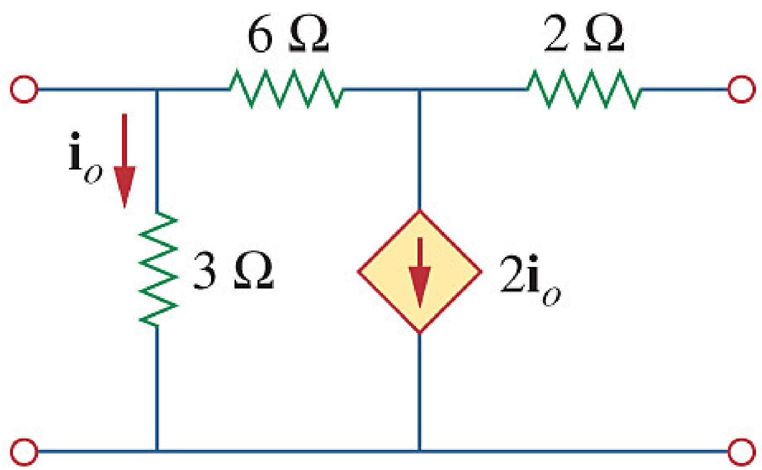
Note: Sometimes two port parameters will fall out directly from mesh equations.

19.3 Admittance Parameters (8)

Practice problem 19.4

Practice Problem 19.4

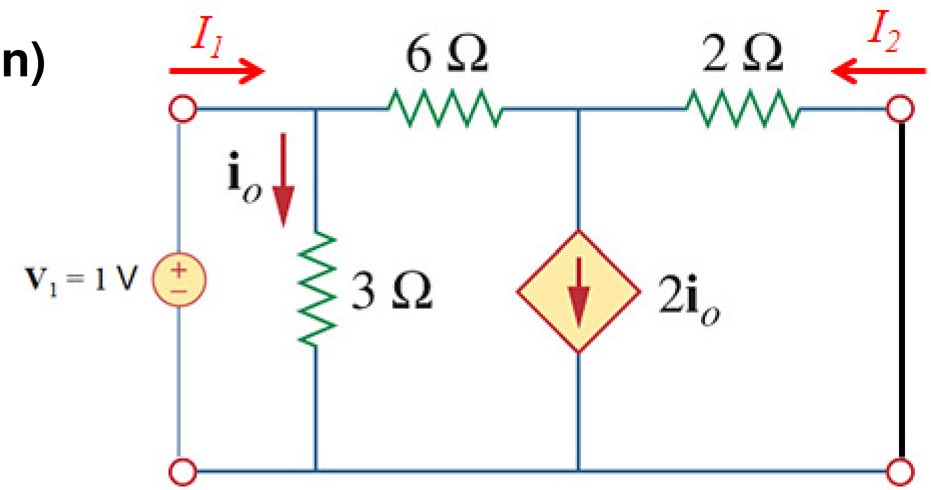
Determine the y-parameters of the following circuit.



Answer: $y = \begin{bmatrix} 0.625 & -0.125 \\ 0.375 & 0.125 \end{bmatrix} S$

Practice Problem 19.4 (Solution)

- Short the output
- Put a 1 volt source at input
- Find I_1 and I_2



First find i_o:

$$i_0 = \frac{1}{3}$$

Dependent current source is then 2/3, find I_1 by repetitive source transformations of the dependant current source

$$I_1 = 0.625 \quad \Rightarrow \boxed{y_{11} = 0.625}$$

Next find current across 6 Ω resistor $I_{6\Omega}$:

$$I_{6\Omega} = 0.625 - \frac{1}{3}$$

$$I_2 + I_{6\Omega} = 2i_0$$

$$I_2 = 2i_0 - I_{6\Omega} = \frac{2}{3} - \left(0.625 - \frac{1}{3}\right) = 0.375 \quad \Rightarrow \boxed{y_{12} = 0.375}$$

Z and Y Parameters
Comparison

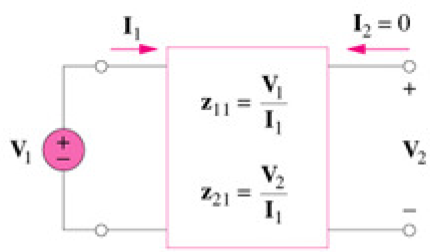
Z-Parameters

$$V_1 = z_{11} I_1 + z_{12} I_2$$
$$V_2 = z_{21} I_1 + z_{22} I_2$$

- **Open** one port (I_1=0 or I_2=0)
- Connect a source to the other port
- Solve to find z-parameters

$$z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0} \quad \text{and} \quad z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0}$$

$$z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1=0} \quad \text{and} \quad z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0}$$



I = 0
Open Circuit

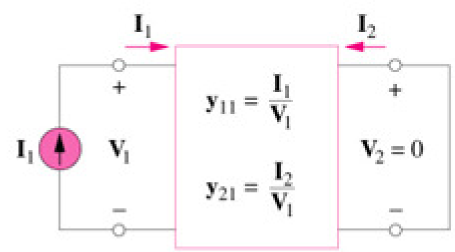
Y-Parameters

$$I_1 = y_{11} V_1 + y_{12} V_2$$
$$I_2 = y_{21} V_1 + y_{22} V_2$$

- **Short** one port (V_1=0 or V_2=0)
- Connect a source to the other port
- Solve to find y-parameters

$$y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2=0} \quad \text{and} \quad y_{21} = \left.\frac{I_2}{V_1}\right|_{V_2=0}$$

$$y_{12} = \left.\frac{I_1}{V_2}\right|_{V_1=0} \quad \text{and} \quad y_{22} = \left.\frac{I_2}{V_2}\right|_{V_1=0}$$



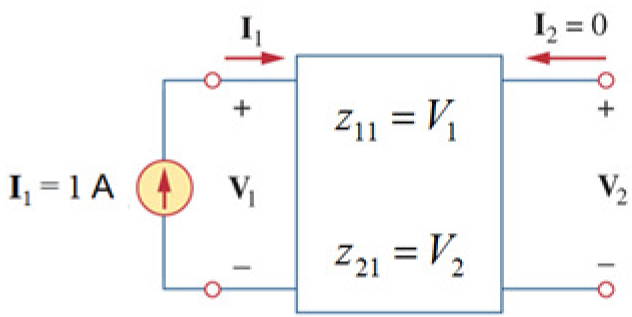
V = 0
Short Circuit

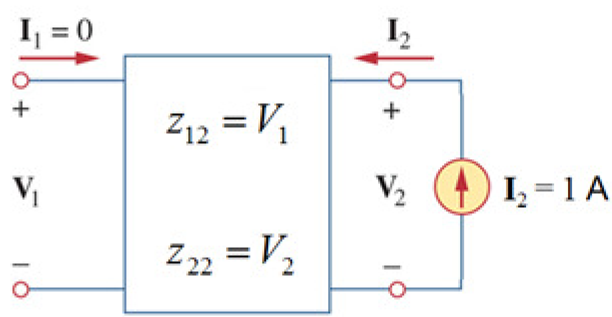
Z and Y parameters

Alternative method (1 Amp / 1 Volt sources)

Z-Parameters

- Open circuit one port
- Put a 1 Amp current source at other port
- Resulting voltages are the z-parameters

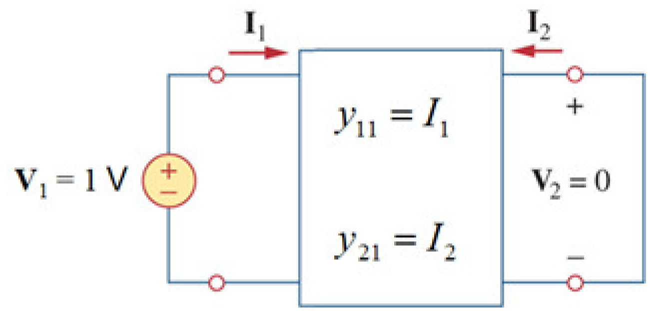


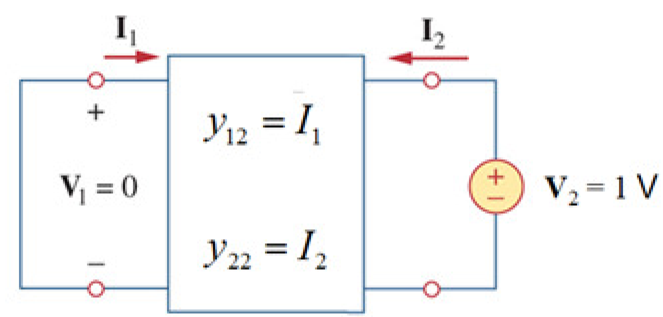


$$V_1 = z_{11}I_1 + z_{12}I_2$$
$$V_2 = z_{21}I_1 + z_{22}I_2$$

Y-Parameters

- Short circuit one port
- Put a 1 Volt voltage source at other port
- Resulting current are the y-parameters

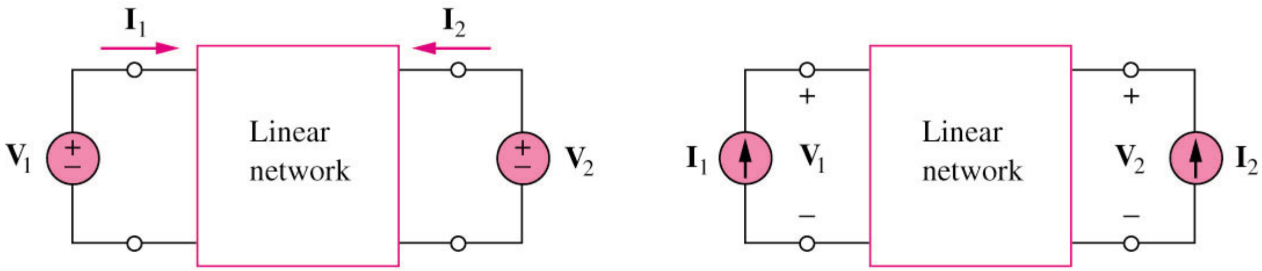




$$I_1 = y_{11}V_1 + y_{12}V_2$$
$$I_2 = y_{21}V_1 + y_{22}V_2$$

19.4 Hybrid Parameters (1)

•The z and y parameters of a two-port network do not always exist. Therefore, there is a need to develop another set of parameters based on making V_1 and I_2 the dependent variables.



Assume no independent source in the network

$$V_1 = h_{11}I_1 + h_{12}V_2$$
$$I_2 = h_{21}I_1 + h_{22}V_2$$

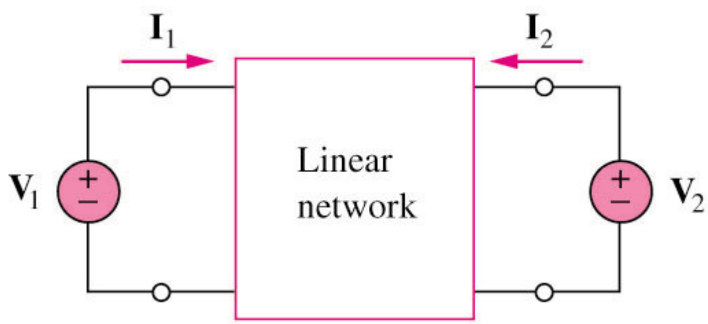
$$\begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ V_2 \end{bmatrix} = [h] \begin{bmatrix} I_1 \\ V_2 \end{bmatrix}$$

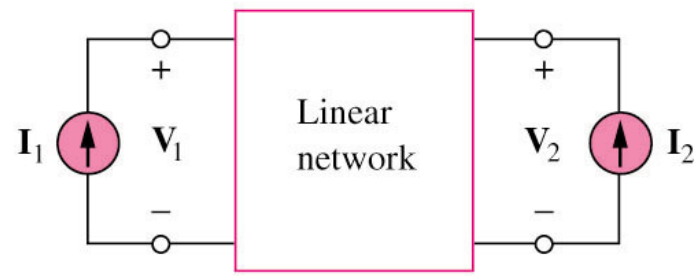
where the **h** terms are called the hybrid parameters, or simply h parameters.

•Hybrid parameters are very useful for describing electronic devices such as transistors because it is much easier to measure the h parameters of these devices than to measure their z or y parameters.

•The ideal transformer can also be described by h parameters.

19.4 Hybrid Parameters (2)





Assume no independent source in the network

$$h_{11} = \left.\frac{V_1}{I_1}\right|_{V_2=0}$$

h_{11}= short-circuit input impedance (Ω)

$$h_{21} = \left.\frac{I_2}{I_1}\right|_{V_2=0}$$

h_{21} = short-circuit forward current gain

$$h_{12} = \left.\frac{V_1}{V_2}\right|_{I_1=0}$$

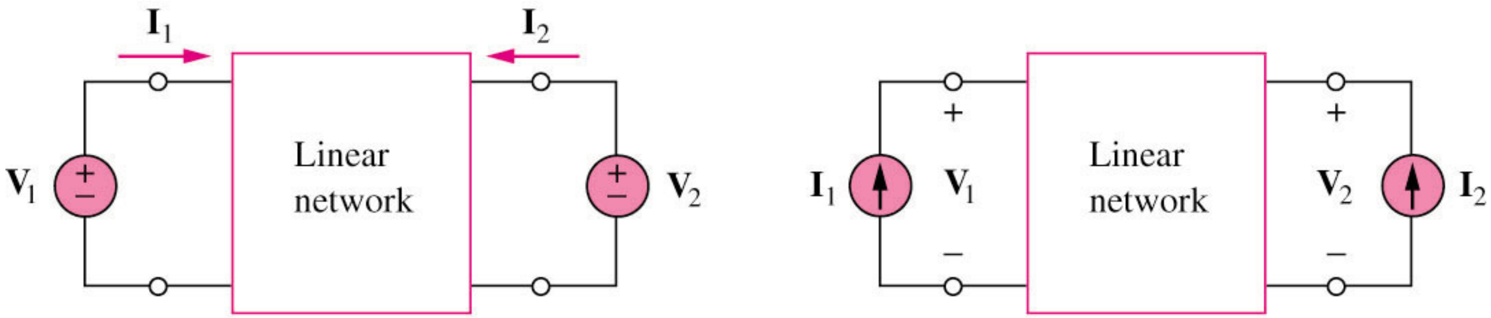
h_{12} = open-circuit reverse voltage-gain

$$h_{22} = \left.\frac{I_2}{V_2}\right|_{I_1=0}$$

h_{22} = open-circuit output admittance (S)

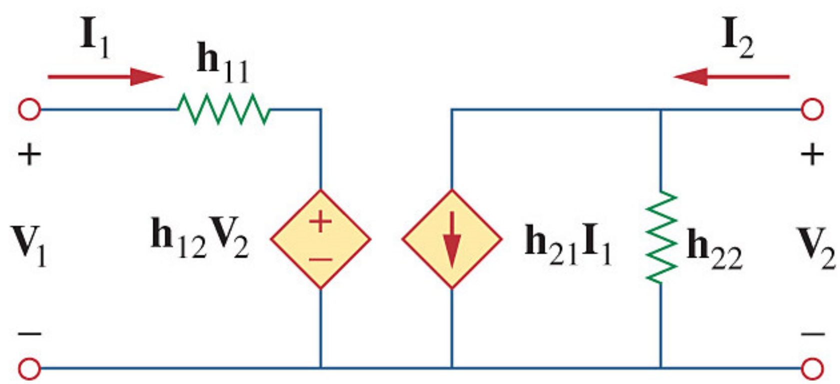
- Note that the h parameters represent an impedance, voltage gain, current gain, and admittance, thereby the term hybrid parameters.
- For reciprocal network, $h_{12} = -h_{21}$

19.4 Hybrid Parameters (3)



Assume no independent source in the network

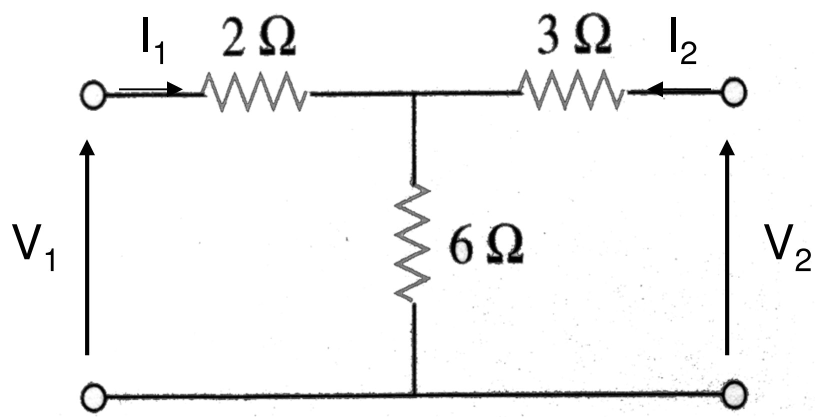
Hybrid model of a two-port network:



19.4 Hybrid Parameters (4)

Example 19.5:

Determine the h-parameters of the following circuit.



$$V_1 = h_{11}I_1 + h_{12}V_2$$
$$I_2 = h_{21}I_1 + h_{22}V_2$$

$$h_{11} = \left.\frac{V_1}{I_1}\right|_{V_2=0} \quad \text{and} \quad h_{21} = \left.\frac{I_2}{I_1}\right|_{V_2=0}$$

$$h_{12} = \left.\frac{V_1}{V_2}\right|_{I_1=0} \quad \text{and} \quad h_{22} = \left.\frac{I_2}{V_2}\right|_{I_1=0}$$

$$\Downarrow$$

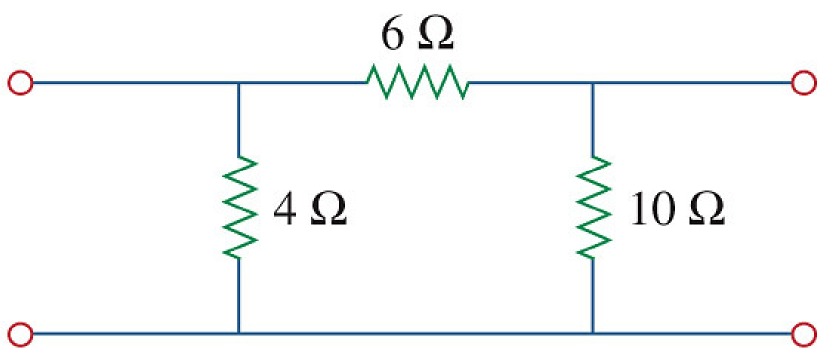
$$h = \begin{bmatrix} h_{11}\Omega & h_{12} \\ h_{21} & h_{22}S \end{bmatrix}$$

Answer: $h = \begin{bmatrix} 4\Omega & \frac{2}{3} \\ -\frac{2}{3} & \frac{1}{9}S \end{bmatrix}$

19.4 Hybrid Parameters (5)

Practice Problem 19.5:

Determine the h-parameters of the following circuit.



$$h_{11} = \left.\frac{V_1}{I_1}\right|_{V_2=0} \quad \text{and} \quad h_{21} = \left.\frac{I_2}{I_1}\right|_{V_2=0}$$

$$h_{12} = \left.\frac{V_1}{V_2}\right|_{I_1=0} \quad \text{and} \quad h_{22} = \left.\frac{I_2}{V_2}\right|_{I_1=0}$$

$$h = \begin{bmatrix} h_{11}\Omega & h_{12} \\ h_{21} & h_{22}S \end{bmatrix}$$

Answer: $h = \begin{bmatrix} 2.4\Omega & 0.4 \\ -0.4 & 0.2S \end{bmatrix}$

19.9.1 Transistor Circuits (1)

Hybrid Parameters

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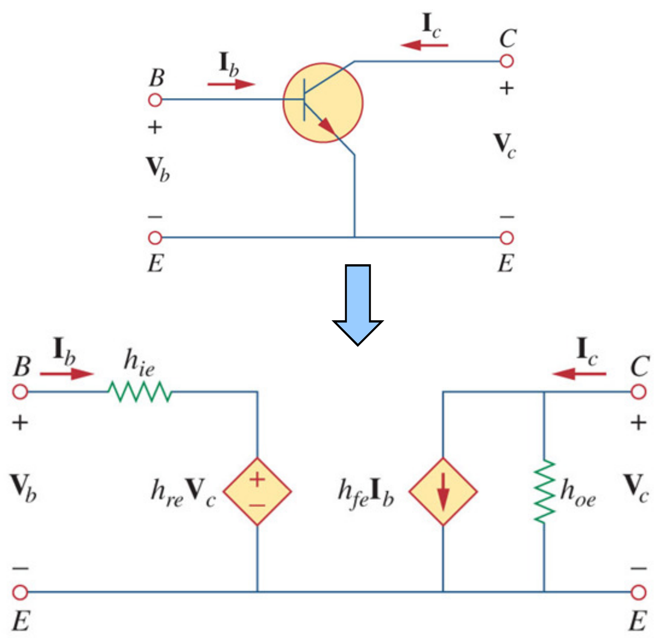
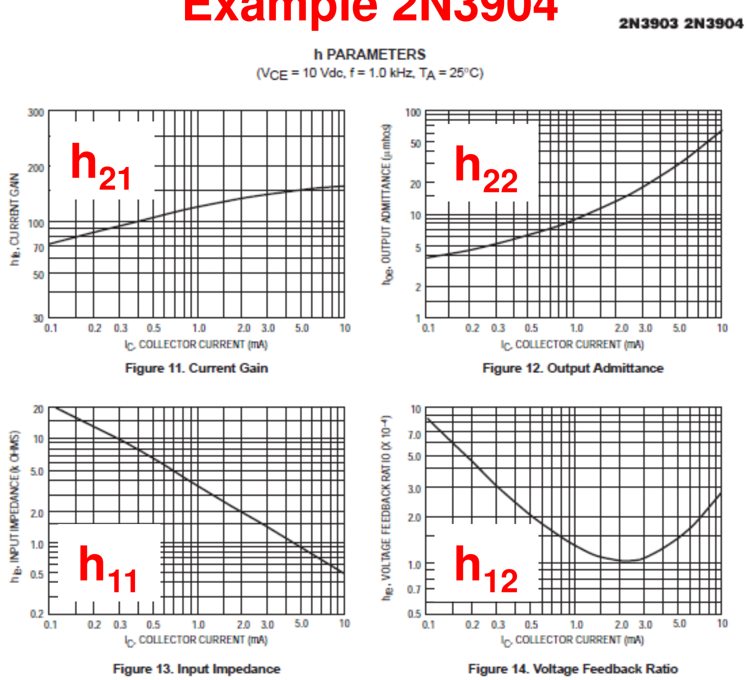
- H-parameters are often used to model transistor circuits
- The h-parameters vary depending on biasing conditions
- Parameters are given different subscripts:
  - h_{11} → h_{ie} = Base input impedance
  - h_{12} → h_{re} = Reverse voltage feedback ration
  - h_{21} → h_{fe} = Base-collector current gain
  - h_{22} → h_{oe} = Output admittance



Example 2N3904



19.9.1 Transistor Circuits (2)

Hybrid Parameters

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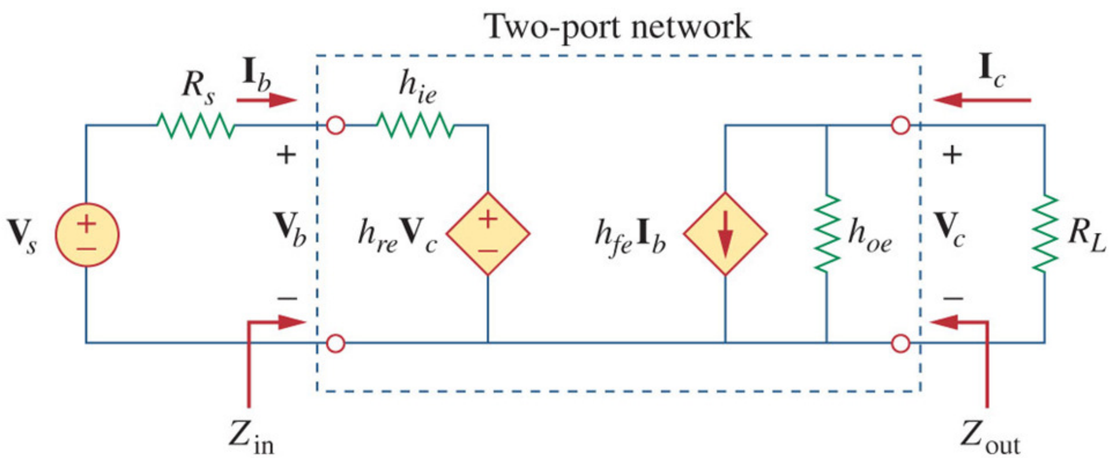
- H parameters are often found in manufacturers spec sheets
- Provide ability to calculate the exact voltage gain, input impedance, and output impedance of the transistor.



Input Impedance

$$Z_{in} = \frac{V_b}{I_b} = h_{ie} - \frac{h_{re}h_{fe}R_L}{1 + h_{oe}R_L}$$

Output Impedance

$$Z_{out} = \left.\frac{V_c}{I_c}\right|_{V_s=0} = \frac{R_s + h_{ie}}{(R_s + h_{ie})h_{oe} - h_{re}h_{fe}}$$

Current Gain

$$A_i = \frac{I_c}{I_b} = \frac{h_{fe}}{1 + h_{oe}R_L}$$

Voltage Gain

$$A_v = \frac{V_c}{V_b} = \frac{-h_{fe}R_L}{h_{ie} + (h_{ie}h_{oe} - h_{re}h_{fe})R_L}$$

19.5 Transmission Parameters (1)

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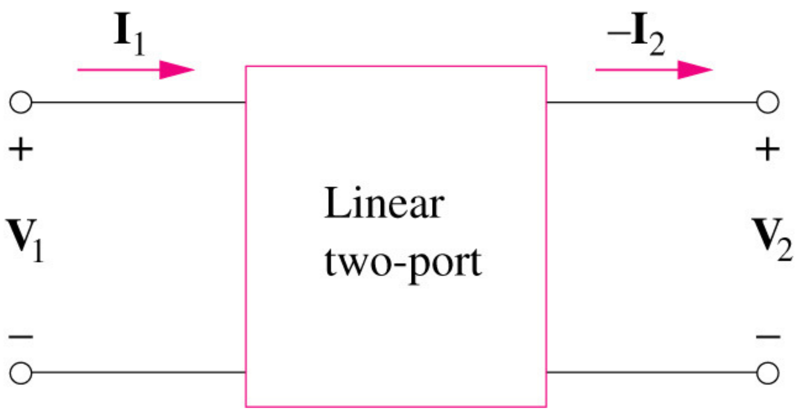

Assume no independent source in the network

$$V_1 = AV_2 - BI_2$$
$$I_1 = CV_2 - DI_2$$

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix} = [T] \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$

where the **T** terms are called the transmission parameters, or simply T or ABCD parameters.

•Note that $-I_2$ is used since the current is considered to be leaving the network. It is logical to think of I_2 as leaving the two-port; this is customary convention in the power industry.

19.5 Transmission Parameters (2)

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- These two-port transmission parameters provide a measure of how a circuit transmits voltage and current form a source to a load.
- They are useful in the analysis of transmission lines and are therefore called transmission parameters.
- They are also known as ABCD parameters and are used in the design of telephone systems, microwave networks, and radars.

$$A = \left.\frac{V_1}{V_2}\right|_{I_2=0}$$

A=open-circuit voltage ratio

$$C = \left.\frac{I_1}{V_2}\right|_{I_2=0}$$

C= open-circuit transfer admittance (S)

$$B = -\left.\frac{V_1}{I_2}\right|_{V_2=0}$$

B= negative short-circuit transfer impedance (Ω)

$$D = -\left.\frac{I_1}{I_2}\right|_{V_2=0}$$

D=negative short-circuit current ratio

19.5 Transmission Parameters (3)

Solving for Transmission Parameters

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- To find the transmission parameters, analyze the circuit as follows:
- Perform the analysis with the output Open Circuited (I_2=0)

$$\begin{aligned} V_1 &= AV_2 - \cancel{BI_2}^{\,0} \\ I_1 &= CV_2 - \cancel{DI_2}^{\,0} \end{aligned} \Rightarrow \begin{aligned} V_1 &= AV_2 \\ I_1 &= CV_2 \end{aligned} \Rightarrow \begin{aligned} A &= \frac{V_1}{V_2} \\ C &= \frac{I_1}{V_2} \end{aligned}$$

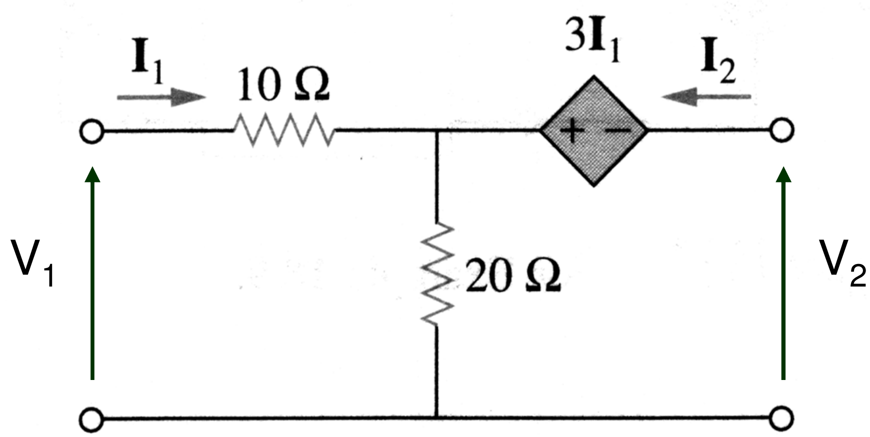
- Perform the analysis with the output Short Circuited(V_2=0)

$$\begin{aligned} V_1 &= \cancel{AV_2}^{\,0} - BI_2 \\ I_1 &= \cancel{CV_2}^{\,0} - DI_2 \end{aligned} \Rightarrow \begin{aligned} V_1 &= -BI_2 \\ I_1 &= -DI_2 \end{aligned} \Rightarrow \begin{aligned} B &= -\frac{V_1}{I_2} \\ D &= -\frac{I_1}{I_2} \end{aligned}$$

19.5 Transmission Parameters (4)

Example 19.8

Determine the T-parameters of the following circuit.



$$V_1 = AV_2 - BI_2$$
$$I_1 = CV_2 - DI_2$$

Apply KVL

$$V_1 = 10I_1 + 20(I_1 + I_2)$$
$$V_2 = -3I_1 + 20(I_1 + I_2)$$

$$V_1 = \frac{30}{17}V_2 - \frac{260}{17}I_2$$
$$I_1 = \frac{1}{17}V_2 - \frac{20}{17}I_2$$

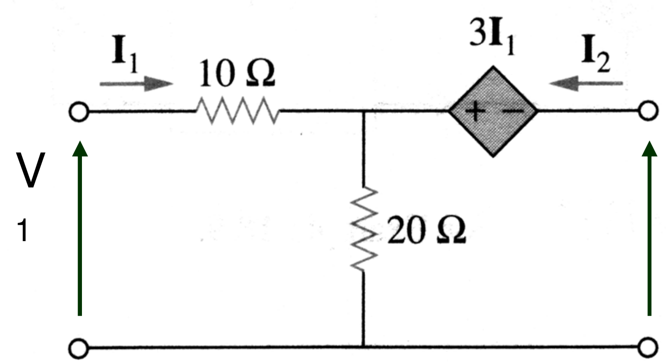
Answer: $$T = \begin{bmatrix} 1.765 & 15.294\Omega \\ 0.059S & 1.176 \end{bmatrix}$$

19.5 Transmission Parameters (5)

Example 19.8

From KVL:

$$V_1 = 10I_1 + 20(I_1 + I_2) = 30I_1 + 20I_2$$
$$V_2 = -3I_1 + 20(I_1 + I_2) = 17I_1 + 20I_2$$



If we "open circuit" the output we get:

$$V_1 = 30I_1 + 20\cancel{I_2}^{\,0} \qquad V_1 = 30I_1$$
$$V_2 = 17I_1 + 20\cancel{I_2}^{\,0} \qquad V_2 = 17I_1$$

$$A = \frac{V_1}{V_2} = \frac{30I_1}{17I_1} = \frac{30}{17} = 1.765$$
$$C = \tfrac{1}{17} = 0.0588$$

If we "short circuit" the output we get:

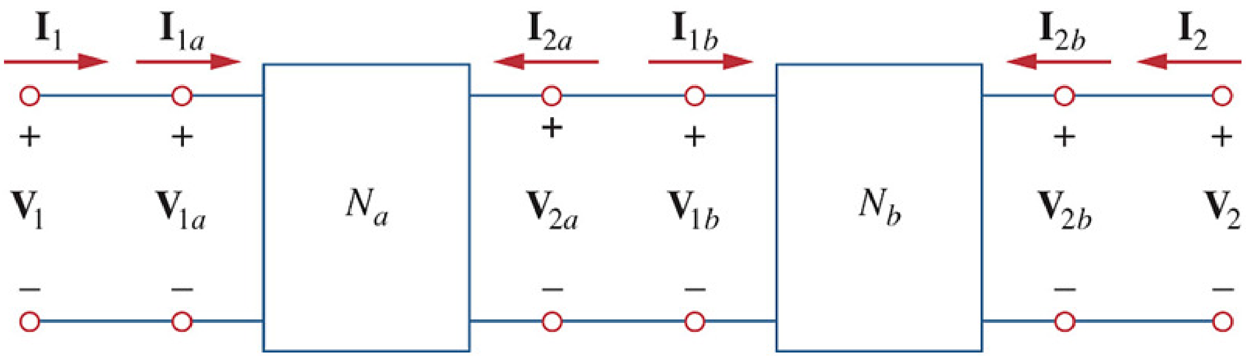
$$V_1 = 30I_1 + 20I_2 \qquad V_1 = 30I_1 + 20I_2$$
$$\cancel{V_2}^{\,0} = 17I_1 + 20I_2 \qquad 0 = 17I_1 + 20I_2$$

$$V_1 = 30\left(\tfrac{-20}{17}\right)I_2 + 20I_2$$
$$I_1 = \tfrac{-20}{17}I_2$$

$$B = -\frac{V_1}{I_2} = -\frac{\left(30\left(\tfrac{-20}{17}\right) + 20\right)I_2}{I_2} = 15.29$$
$$D = -\frac{I_1}{I_2} = \frac{20}{17} = 1.176$$

19.5 Transmission Parameters (6)

- Transmission Parameters can be cascaded with the result found through simple matrix multiplication



$$\begin{bmatrix} V_{1a} \\ I_{1a} \end{bmatrix} = \begin{bmatrix} A_a & B_a \\ C_a & D_a \end{bmatrix} \begin{bmatrix} V_{2a} \\ -I_{2a} \end{bmatrix} \qquad \begin{bmatrix} V_{1b} \\ I_{1b} \end{bmatrix} = \begin{bmatrix} A_b & B_b \\ C_b & D_b \end{bmatrix} \begin{bmatrix} V_{2b} \\ -I_{2b} \end{bmatrix}$$

$$-I_{2a} = I_{1b}$$

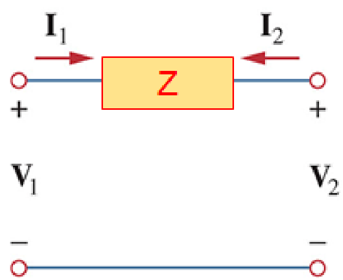
$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A_a & B_a \\ C_a & D_a \end{bmatrix} \begin{bmatrix} A_b & B_b \\ C_b & D_b \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$

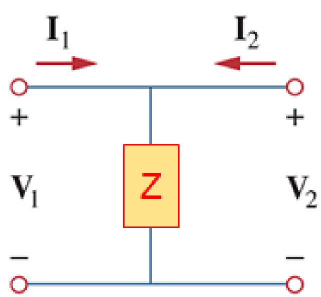
$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A' & B' \\ C' & D' \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$

19.5 Transmission Parameters (7)

Properties: Building Block Circuits

Consider the following simple circuits



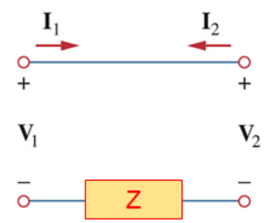


We can find their T Parameters to be:

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} 1 & Z \\ 0 & 1 \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ \frac{1}{Z} & 1 \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$

Note, following is equivalent

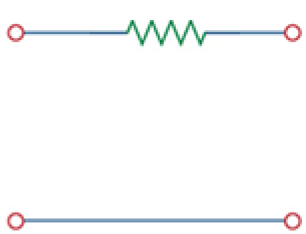


19.5 Transmission Parameters (8)

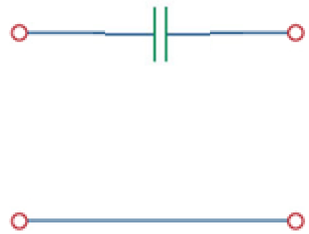
Properties: Building Block Circuits

- We can use this to construct the following "building block T parameters" to find the T parameters for any ladder type circuit.

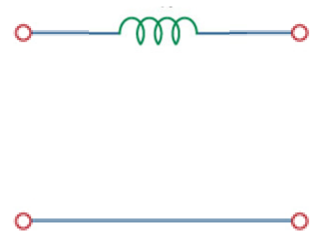
$$\begin{bmatrix} 1 & R \\ 0 & 1 \end{bmatrix}$$



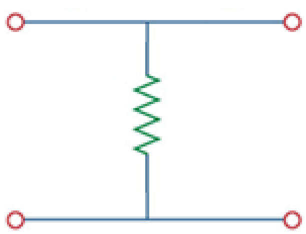
$$\begin{bmatrix} 1 & \frac{1}{sC} \\ 0 & 1 \end{bmatrix}$$



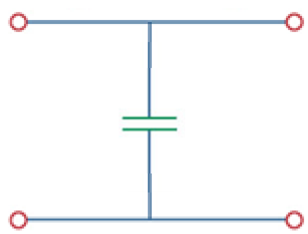
$$\begin{bmatrix} 1 & sL \\ 0 & 1 \end{bmatrix}$$



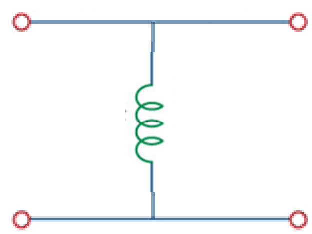
$$\begin{bmatrix} 1 & 0 \\ \frac{1}{R} & 1 \end{bmatrix}$$



$$\begin{bmatrix} 1 & 0 \\ sC & 1 \end{bmatrix}$$



$$\begin{bmatrix} 1 & 0 \\ \frac{1}{sL} & 1 \end{bmatrix}$$



19.5 Transmission Parameters (9)

Properties: Transfer function / Thevenin Equivalent

- The "A" parameter can be used to provide the inverse of the voltage Transfer Function H(s).

$$\boxed{A = \left.\frac{V_1}{V_2}\right|_{I_2=0} = \frac{1}{H(s)}}$$

- Parameters "A" and "B" can be used to find a relationship between the Open Circuit Voltage (V_2) and the Short Circuit Current ($-I_2$).
- We can us this to find the parameters for the Thevenin Equivalent Circuit.

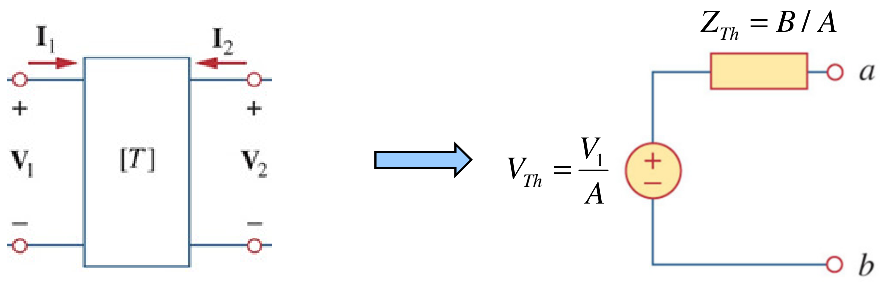
$$\boxed{A = \left.\frac{V_1}{V_2}\right|_{I_2=0} = \frac{V_1}{V_{oc}}}$$

$$\boxed{V_{Th} = V_{oc} = \frac{V_1}{A}}$$

$$\boxed{Z_{Th} = \frac{V_{oc}}{I_{sc}} = \frac{B}{A}}$$

$$\boxed{B = -\left.\frac{V_1}{I_2}\right|_{V_2=0} = \frac{V_1}{I_{sc}}}$$

$$\boxed{I_N = I_{sc} = \frac{V_1}{B}}$$

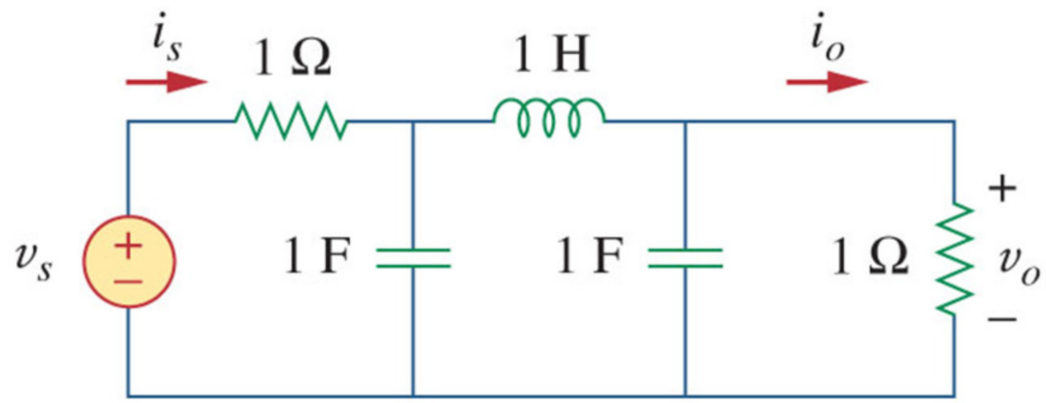


19.5 Transmission Parameters (10)

Transfer Function - Example

Problem 16.80(a)

Find the transfer function $V_o(s)/V_s(s)$ for the following circuit



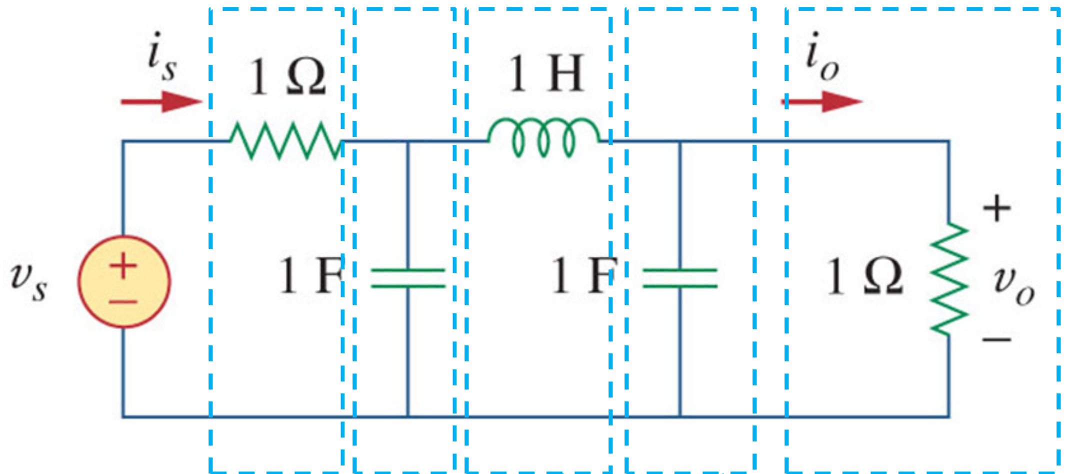
Answer: $$H(s) = \frac{1}{s^3 + 2s^2 + 3s + 2}$$

19.5 Transmission Parameters (11)

Transfer Function - Example

Problem 16.80(a) Solution:

a) Break up the circuit into a series of cascaded series and shunt components
b) Find the composite “T” parameters for the circuit
c) Use the relationship between the parameter “A” and the Transfer function



$$\begin{bmatrix} V_s \\ I_s \end{bmatrix} = \underbrace{\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ s & 1 \end{bmatrix}\begin{bmatrix} 1 & s \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ s & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}}_{\begin{bmatrix} A & B \\ C & D \end{bmatrix}}\begin{bmatrix} V_o \\ I_o \end{bmatrix}$$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} \longrightarrow \boxed{A = \left.\frac{V_1}{V_2}\right|_{I_2=0} = \frac{1}{H(s)}}$$

19.5 Transmission Parameters (12)

Transfer Function - Example

Finding the combined T-matrix

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ s & 1 \end{bmatrix}\begin{bmatrix} 1 & s \\ 0 & 1 \end{bmatrix}\underbrace{\begin{bmatrix} 1 & 0 \\ s & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}}$$

$$\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ s & 1 \end{bmatrix}\underbrace{\begin{bmatrix} 1 & s \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ (s+1) & 1 \end{bmatrix}}$$

$$\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}\underbrace{\begin{bmatrix} 1 & 0 \\ s & 1 \end{bmatrix}\begin{bmatrix} 1+s(s+1) & s \\ (s+1) & 1 \end{bmatrix}}$$

$$\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1+s(s+1) & s \\ s+s^2(s+1)+(s+1) & s^2 \end{bmatrix}$$

$$\underbrace{\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} s^2+s+1 & s \\ s^3+s^2+2s+1 & s^2 \end{bmatrix}}$$

$$\begin{bmatrix} s^3+2s^2+3s+2 & s+s^2 \\ s^3+s^2+2s+1 & s^2 \end{bmatrix}$$

The transfer function can be found directly from the Transmission Parameter "A" !

$$\boxed{A = \left.\frac{V_1}{V_2}\right|_{I_2=0} = \frac{1}{H(s)}}$$

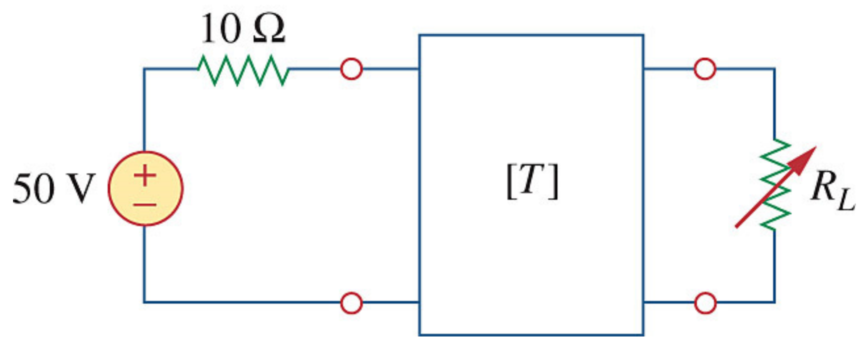
$$H(s) = \frac{1}{s^3+2s^2+3s+2}$$

19.5 Transmission Parameters (13)

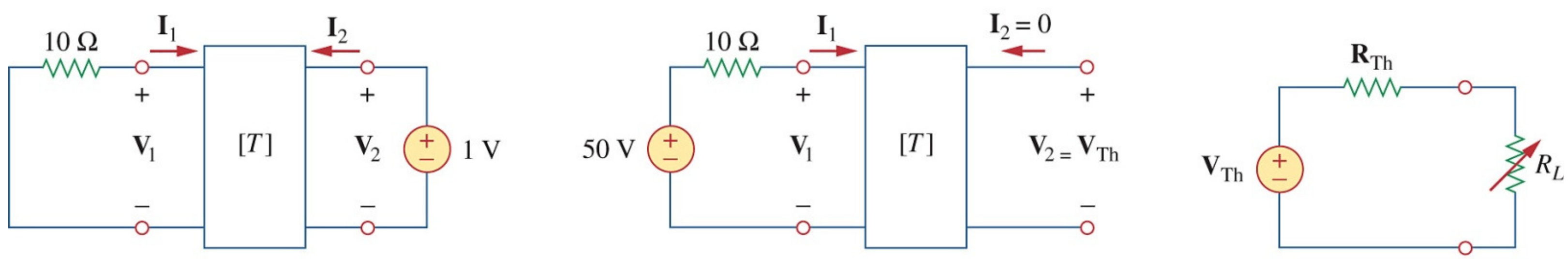
Example 19.9

The ABCD parameters of the two-port network at right are

$$T = \begin{bmatrix} 4 & 20\ \Omega \\ 0.1S & 2 \end{bmatrix}$$



The output port is connected to a variable load for maximum power transfer. Find R_L and the maximum power transferred.



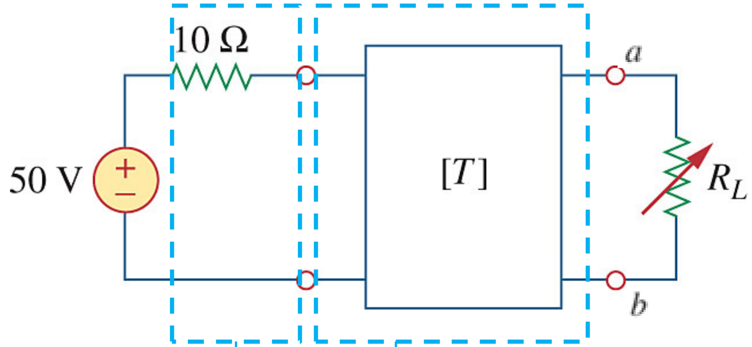
(a) (b) (c)

Answer: V_{TH} = 10V V; R_L = 8Ω; Pm = 3.125W.

19.5 Transmission Parameters (14)

Solution: Example 19.9

a) Cascade the Series Resistor with the network
b) Find the composite “T” parameters for the circuit
c) Use the relationships to find V_{Th} and Z_{Th}

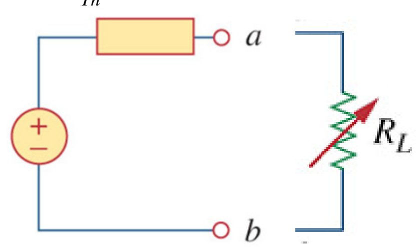


$$[T'] = \begin{bmatrix} 1 & 10 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 4 & 20 \\ 0.1 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 40 \\ 0.1 & 2 \end{bmatrix}$$

Find the Thevenin Equivalent Circuit for the source

$Z_{Th} = 40/5 = 8$

$V_{Th} = \frac{50}{5} = 10$



For Max Power Transfer

$$\boxed{R_L = Z_{Th} = 8\ \Omega}$$

$$P_{max} = I^2 R_L$$

$$P_{max} = \left(\frac{V_{Th}}{R_L + Z_{Th}}\right)^2 R_L$$

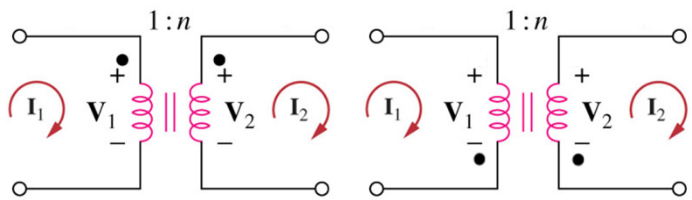
$$\boxed{P_{max} = \left(\frac{10}{16}\right)^2 8 = 3.125\,\text{W}}$$

19.5 Transmission Parameters (15)

Properties: Building Block Circuits – Ideal Transformer

- We can also use these “building blocks” to model ideal transformers. Remember from Chapter 13

Dots “Same” → “+”

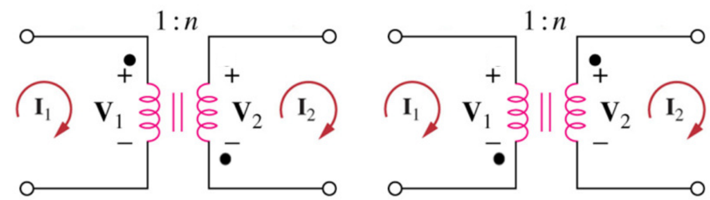


$$V_2 = nV_1$$
$$I_2 = \frac{I_1}{n}$$

⇓

$$\begin{bmatrix} \frac{1}{n} & 0 \\ 0 & n \end{bmatrix}$$

Dots “Different” → “-”



$$V_2 = -nV_1$$
$$I_2 = -\frac{I_1}{n}$$

⇓

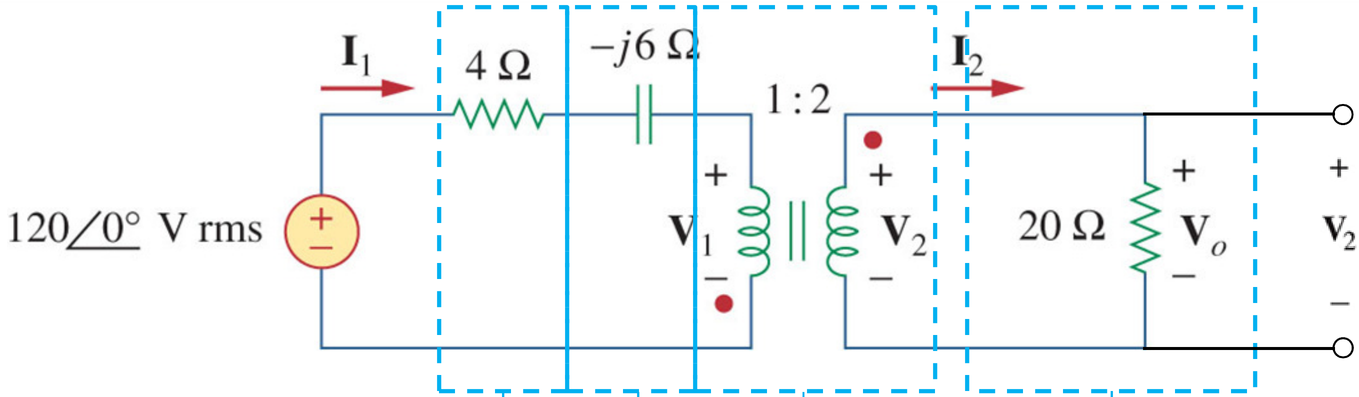
$$\begin{bmatrix} -\frac{1}{n} & 0 \\ 0 & -n \end{bmatrix}$$

T - parameters

$$V_1 = AV_2 - BI_2$$
$$I_1 = CV_2 - DI_2$$

19.5 Transmission Parameters (16)

Example 13.8 Revisited



Cascaded T parameters →

$$\begin{bmatrix}1 & 4\\ 0 & 1\end{bmatrix}\begin{bmatrix}1 & -j6\\ 0 & 1\end{bmatrix}\begin{bmatrix}-\frac{1}{2} & 0\\ 0 & -2\end{bmatrix}\begin{bmatrix}1 & 0\\ 0.05 & 1\end{bmatrix}$$

a b c d

Using MATLAB: →

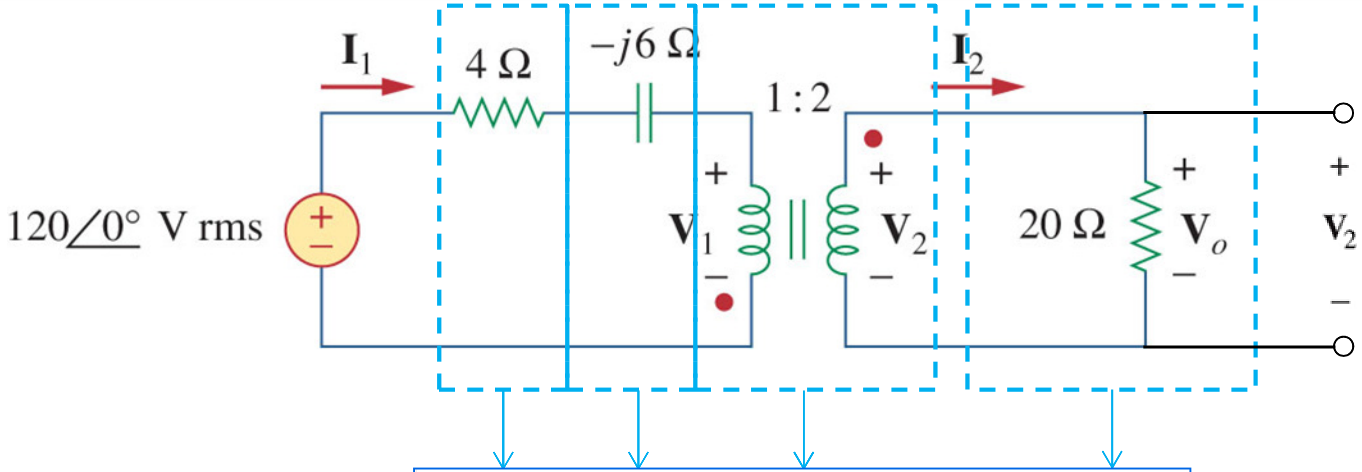
```
>> T=a*b*c*d

T =

  -0.9000 + 0.6000i  -8.0000 +12.0000i
  -0.1000 + 0.0000i  -2.0000 + 0.0000i
```

$$V_0 = V_2 = \frac{1}{A}V_1 = \left(\frac{1}{\boxed{-0.9+0.6j}}\right)(120\angle 0°) = \boxed{110.94\angle -146.31° \text{V}}$$

✓

19.5 Transmission Parameters (17)

Example 13.8 Revisited

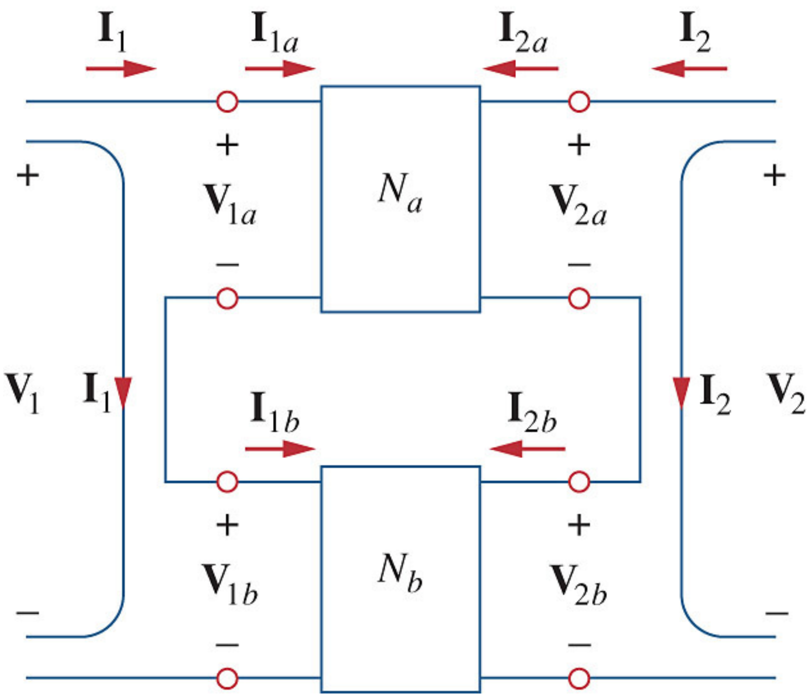



```
>> T=a*b*c*d

T =

  -0.9000 + 0.6000i  -8.0000 +12.0000i
  -0.1000 + 0.0000i  -2.0000 + 0.0000i
```

$$I_1 = CV_2 = \boxed{(-0.1)}(110.94\angle -146.31°) = \boxed{11.09\angle 33.69°\ \text{A}}$$

$$I_2 = \frac{V_2}{20} = \frac{(110.94\angle -146.31°)}{20} = \boxed{5.55\angle -146.31°\ \text{A}}$$

19.7 Interconnection of Networks (1)

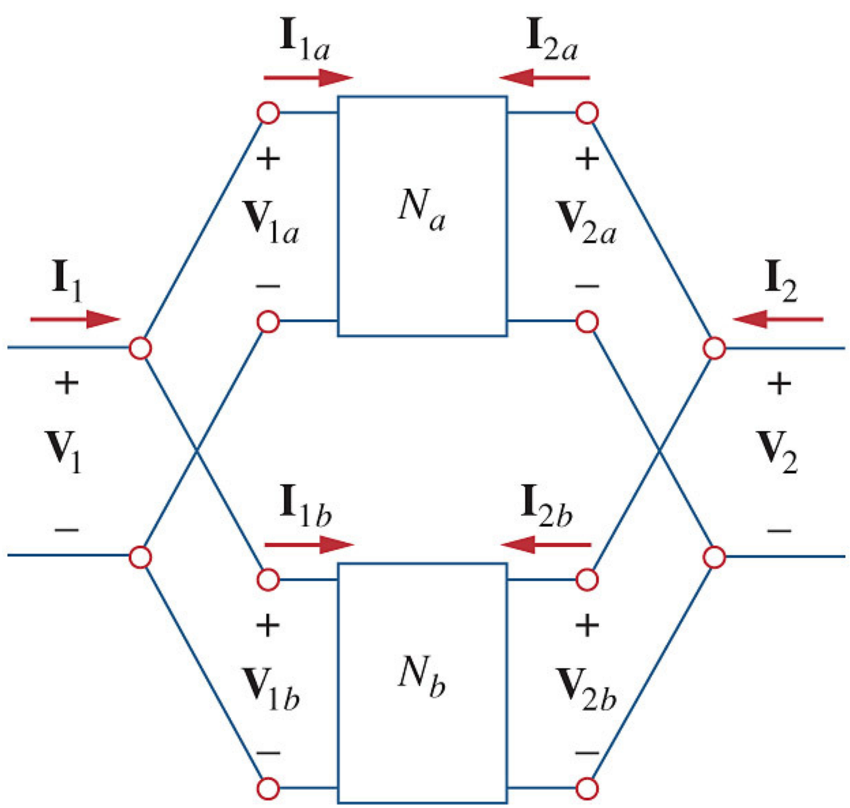


Series Connection of two-port networks:

For Impedances; ADD matrices.

$Z = Z_a + Z_b$

19.7 Interconnection of Networks (2)



Parallel Connection of two-port networks:

For Admittances; ADD matrices.

$Y = Y_a + Y_b$

19.6 Relationships Between Networks

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- Use this table to convert between two port parameters

	z		y		h		T	
z	z_{11}	z_{12}	$\frac{y_{22}}{\Delta_y}$	$-\frac{y_{12}}{\Delta_y}$	$\frac{\Delta_h}{h_{22}}$	$\frac{h_{12}}{h_{22}}$	$\frac{A}{C}$	$\frac{\Delta_T}{C}$
	z_{21}	z_{22}	$-\frac{y_{21}}{\Delta_y}$	$\frac{y_{11}}{\Delta_y}$	$-\frac{h_{21}}{h_{22}}$	$\frac{1}{h_{22}}$	$\frac{1}{C}$	$\frac{D}{C}$
y	$\frac{z_{22}}{\Delta_z}$	$-\frac{z_{12}}{\Delta_z}$	y_{11}	y_{12}	$\frac{1}{h_{11}}$	$-\frac{h_{12}}{h_{11}}$	$\frac{D}{B}$	$-\frac{\Delta_T}{B}$
	$-\frac{z_{21}}{\Delta_z}$	$\frac{z_{11}}{\Delta_z}$	y_{21}	y_{22}	$\frac{h_{21}}{h_{11}}$	$\frac{\Delta_h}{h_{11}}$	$-\frac{1}{B}$	$\frac{A}{B}$
h	$\frac{\Delta_z}{z_{22}}$	$\frac{z_{12}}{z_{22}}$	$\frac{1}{y_{11}}$	$-\frac{y_{12}}{y_{11}}$	h_{11}	h_{12}	$\frac{B}{D}$	$\frac{\Delta_T}{D}$
	$-\frac{z_{21}}{z_{22}}$	$\frac{1}{z_{22}}$	$\frac{y_{21}}{y_{11}}$	$\frac{\Delta_y}{y_{11}}$	h_{21}	h_{22}	$-\frac{1}{D}$	$\frac{C}{D}$
T	$\frac{z_{11}}{z_{21}}$	$\frac{\Delta_z}{z_{21}}$	$-\frac{y_{22}}{y_{21}}$	$-\frac{1}{y_{21}}$	$-\frac{\Delta_h}{h_{21}}$	$-\frac{h_{11}}{h_{21}}$	A	B
	$\frac{1}{z_{21}}$	$\frac{z_{22}}{z_{21}}$	$-\frac{\Delta_y}{y_{21}}$	$-\frac{y_{11}}{y_{21}}$	$-\frac{h_{22}}{h_{21}}$	$-\frac{1}{h_{21}}$	C	D

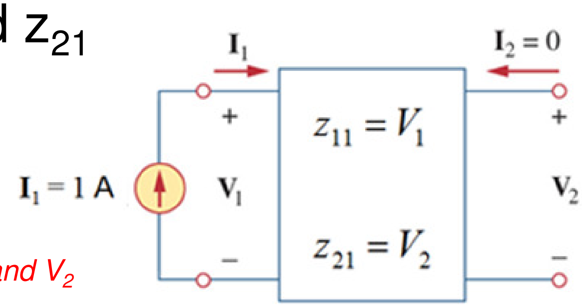
$\Delta_y = y_{11}y_{22} - y_{12}y_{21}$ $\quad \Delta_z = z_{11}z_{22} - z_{12}z_{21}$ $\quad \Delta_h = h_{11}h_{22} - h_{12}h_{21}$ $\quad \Delta_T = AD - BC$

- Parameters: $\boxed{\begin{aligned} V_1 &= z_{11}I_1 + z_{12}I_2 \\ V_2 &= z_{21}I_1 + z_{22}I_2 \end{aligned}}$

- Open circuit the **output** to find z_{11} and z_{21}

$$V_1 = z_{11}I_1 + \cancel{z_{12}I_2}^{0} \qquad\Rightarrow\qquad V_1 = z_{11}I_1$$
$$V_2 = z_{21}I_1 + \cancel{z_{22}I_2}^{0} \qquad\Rightarrow\qquad V_2 = z_{21}I_1$$

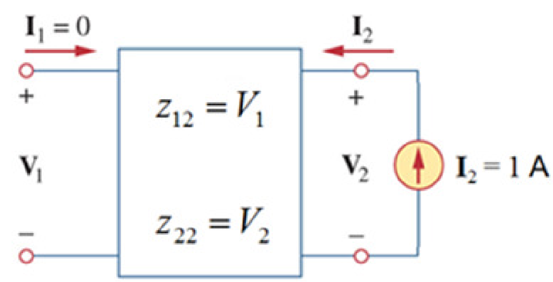
Set I_1 = 1 then solve for V_1 and V_2



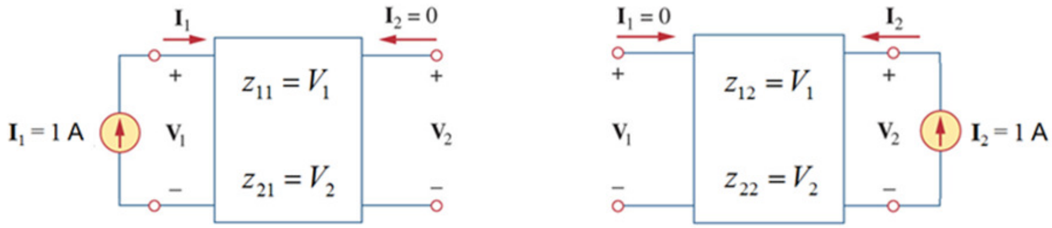
- Open circuit the **input** to find z_{21} and z_{22}

$$V_1 = \cancel{z_{11}I_1}^{0} + z_{12}I_2 \qquad\Rightarrow\qquad V_1 = z_{12}I_2$$
$$V_2 = \cancel{z_{21}I_1}^{0} + z_{22}I_2 \qquad\Rightarrow\qquad V_2 = z_{22}I_2$$

Set I_2 = 1 then solve for V_1 and V_2

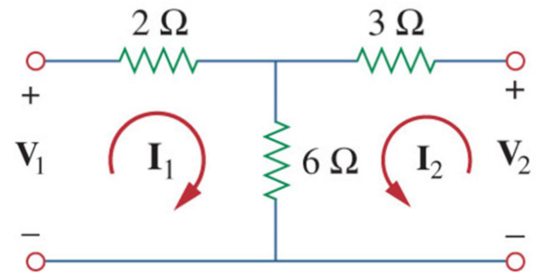


- Solving problems to find z-parameters:
  1. Refer to definition, apply 1 amp source at input and output with opposite port left open (see previous slide)



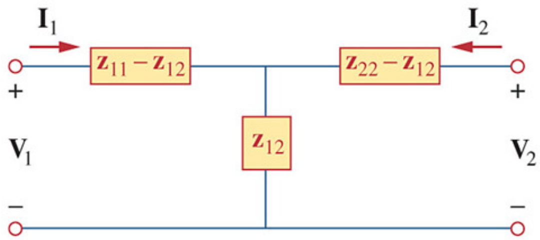
  2. Sometimes, KVL (mesh current equations) will cause z-parameters to fall right out! :

$$V_1 = 2I_1 + 6(I_1 + I_2) = 8I_1 + 6I_2$$
$$V_2 = 6(I_1 + I_2) + 3I_2 = 6I_1 + 9I_2$$
$$\Downarrow$$
$$z = \begin{bmatrix} 8 & 6 \\ 6 & 9 \end{bmatrix} \Omega$$

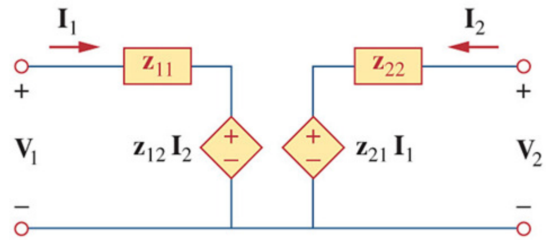


This mesh defined in counter clockwise direction for convenience

- If given, z-parameters can use following techniques to find other circuit parameters (V_1, V_2, I_1, I_2, etc.):
  1. Apply the model and solve the circuit:



Reciprocal Network



General Network

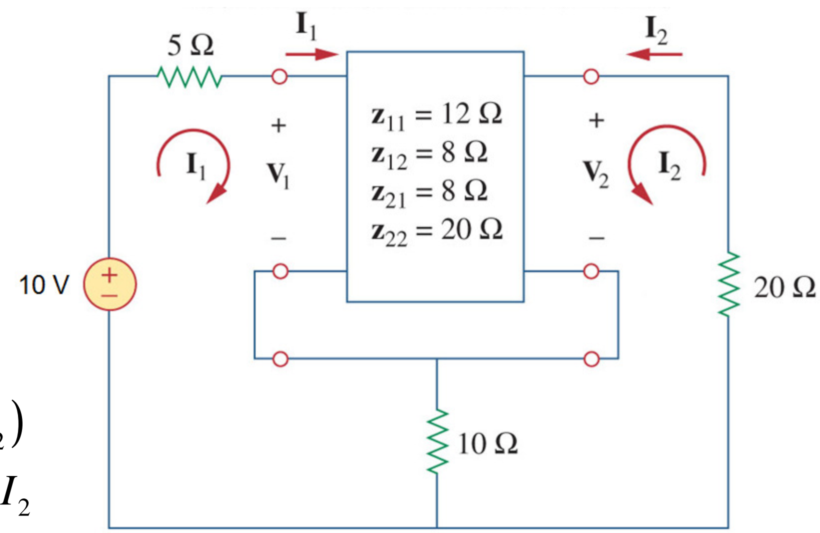
  2. Substitute the defining equations into your analysis:

Mesh Analysis

$$10 = 5I_1 + V_1 + 10(I_1 + I_2)$$
$$0 = V_2 + 10(I_1 + I_2) + 20I_2$$

Substitute for V_1 and V_2

$$10 = 5I_1 + (12I_1 + 8I_2) + 10(I_1 + I_2)$$
$$0 = (8I_1 + 20I_2) + 10(I_1 + I_2) + 20I_2$$



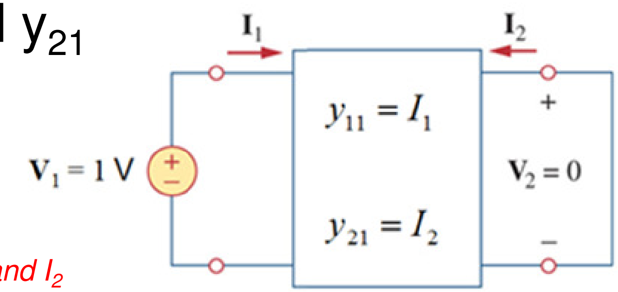
Chapter 19 Review

Y-Parameters

- Parameters: $$\boxed{\begin{aligned} I_1 &= y_{11}V_1 + y_{12}V_2 \\ I_2 &= y_{21}V_1 + y_{22}V_2 \end{aligned}}$$

- Short circuit the **output** to find y_{11} and y_{21}

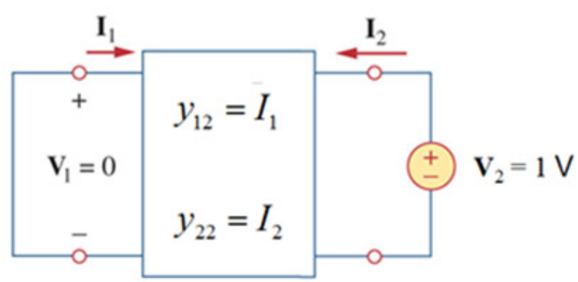
$$I_1 = y_{11}V_1 + \cancelto{0}{y_{12}V_2} \qquad I_2 = y_{21}V_1 + \cancelto{0}{y_{22}V_2} \quad \Rightarrow \quad I_1 = y_{11}V_1 \qquad I_2 = y_{21}V_1$$



Set V_1 = 1 then solve for I_1 and I_2

- Short circuit the **input** to find y_{21} and y_{22}

$$I_1 = \cancelto{0}{y_{11}V_1} + y_{12}V_2 \qquad I_2 = \cancelto{0}{y_{21}V_1} + y_{22}V_2 \quad \Rightarrow \quad I_1 = y_{12}V_2 \qquad I_2 = y_{22}V_2$$

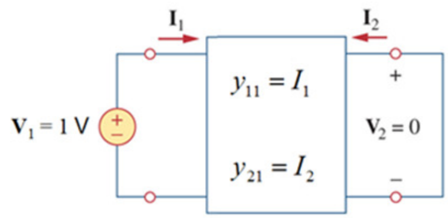


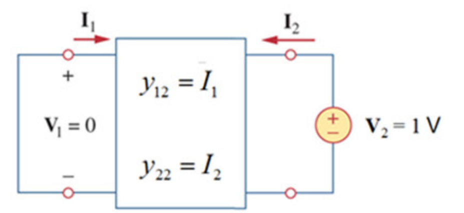
Set V_2 = 1 then solve for I_1 and I_2

Chapter 19 Review

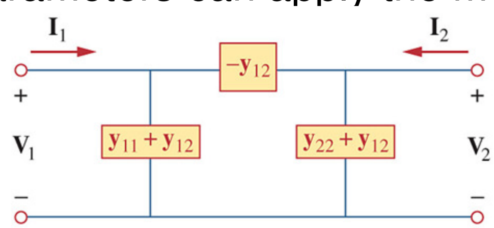
Y-Parameters (Solving Problems)

- To solve Y-parameter problems, can use these techniques
  1. Apply method from previous slide. Apply 1 Volt source at input and output while shorting opposite port

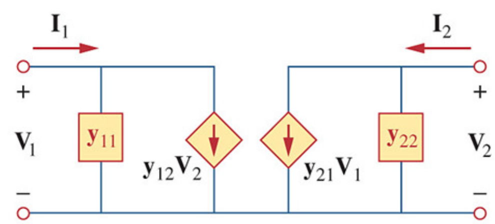




  2. If given Y parameters can apply the model and solve the circuit:



Reciprocal Network



General Network

  3. Make it easy on yourself! Use conversions from Z → Y or Y → Z

$$\begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix} = \left(\frac{1}{\Delta_y}\right)\begin{bmatrix} y_{22} & -y_{12} \\ -y_{21} & y_{11} \end{bmatrix}$$

$$\Delta_y = y_{11}y_{22} - y_{12}y_{21}$$

$$\begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} = \left(\frac{1}{\Delta_z}\right)\begin{bmatrix} z_{22} & -z_{12} \\ -z_{21} & z_{11} \end{bmatrix}$$

$$\Delta_z = z_{11}z_{22} - z_{12}z_{21}$$

Chapter 19 Review

H-Parameters

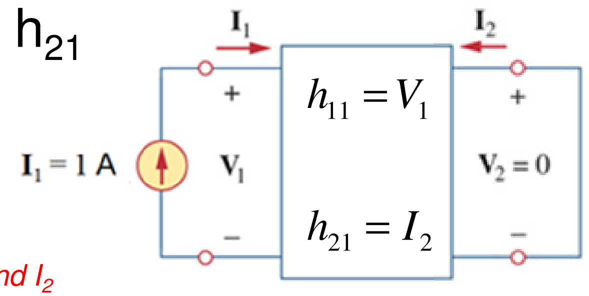
- Parameters (hybrid of z and y):

$$\boxed{\begin{aligned} V_1 &= h_{11}I_1 + h_{12}V_2 \\ I_2 &= h_{21}I_1 + h_{22}V_2 \end{aligned}}$$

- Short circuit the **output** to find h_{11} and h_{21}

$$\begin{aligned} V_1 &= h_{11}I_1 + \cancelto{0}{h_{12}V_2} \\ I_2 &= h_{21}I_1 + \cancelto{0}{h_{22}V_2} \end{aligned} \quad \Rightarrow \quad \begin{aligned} V_1 &= h_{11}I_1 \\ I_2 &= h_{21}I_1 \end{aligned}$$

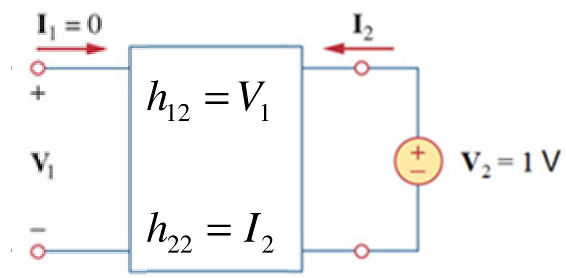
Set I_1 = 1 then solve for V_1 and I_2



- Open circuit the **input** to find h_{21} and h_{22}

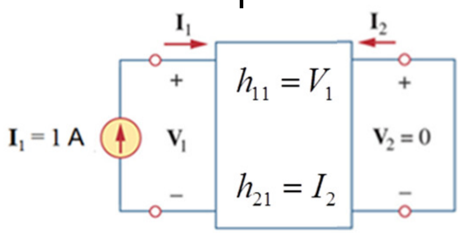
$$\begin{aligned} V_1 &= \cancelto{0}{h_{11}I_1} + h_{12}V_2 \\ I_2 &= \cancelto{0}{h_{21}I_1} + h_{22}V_2 \end{aligned} \quad \Rightarrow \quad \begin{aligned} V_1 &= h_{12}V_2 \\ I_2 &= h_{22}V_2 \end{aligned}$$

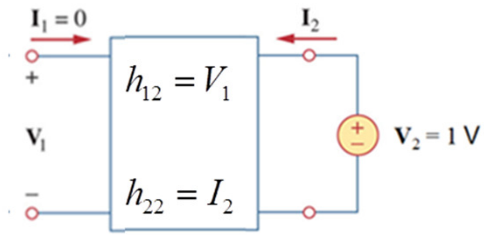
Set V_2 = 1 then solve for V_1 and I_2



Chapter 19 Review
H-Parameters (Solving Problems)

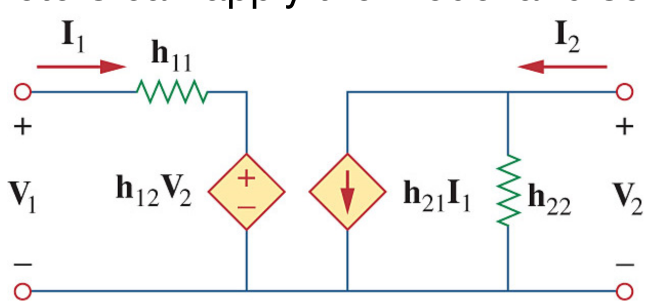
- To solve H-parameter problems, can use these techniques
  1. Apply methods from previous slide.





  2. H parameters can be found by performing a set of tests on the device
     a) Shorting the output and applying a current
     b) Leaving the input open and applying a voltage across the output

  3. If given H parameters can apply the model and solve the circuit:

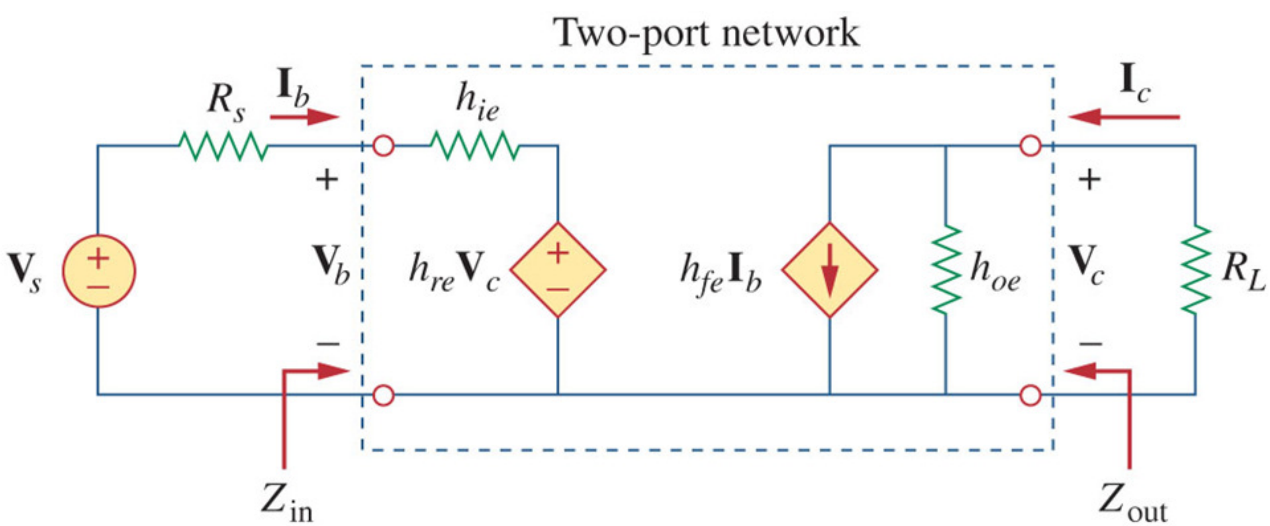


  4. If helpful, use conversion tables

Chapter 19 Review

H-Parameters (Transistor Model)

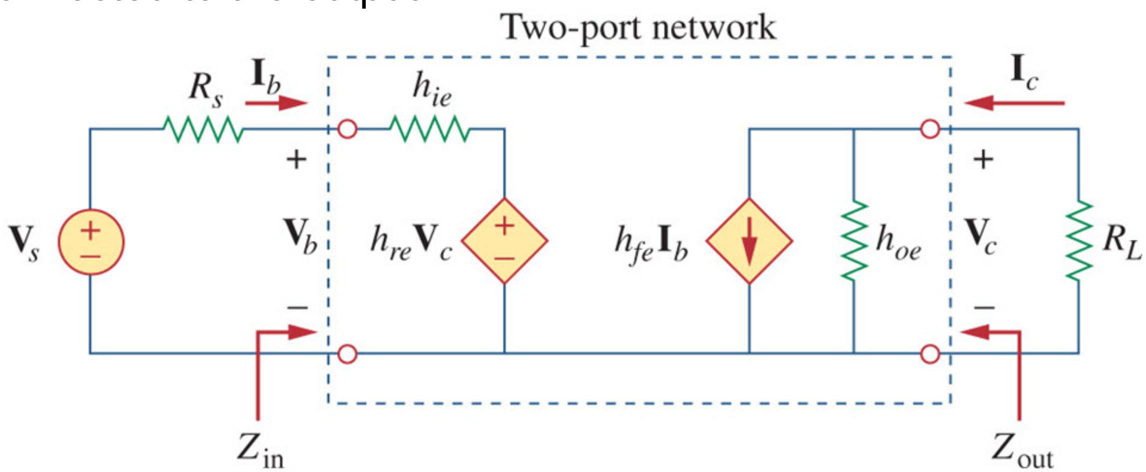
- H parameters are often used in modeling transistors
- Parameters vary depending on biasing conditions
- Spec sheets often use different subscripts:
  - h_{11} → h_{ie} = Base input impedance
  - h_{12} → h_{re} = Reverse voltage feedback ration
  - h_{21} → h_{fe} = Base-collector current gain
  - h_{22} → h_{oe} = Output admittance



Chapter 19 Review

H-Parameters (Transistor Model)

- Equations for calculating input impedance, output impedance, voltage gain, and current gain for simple transistor circuit:
  - V_s and R_s can be the Thevenin equivalent source driving the input.
  - R_L can be the input impedance looking into the load of the circuit connected to the output



Input Impedance

$$Z_{in} = \frac{V_b}{I_b} = h_{ie} - \frac{h_{re}h_{fe}R_L}{1 + h_{oe}R_L}$$

Output Impedance

$$Z_{out} = \left.\frac{V_c}{I_c}\right|_{V_s=0} = \frac{R_s + h_{ie}}{(R_s + h_{ie})h_{oe} - h_{re}h_{fe}}$$

Current Gain

$$A_i = \frac{I_c}{I_b} = \frac{h_{fe}}{1 + h_{oe}R_L}$$

Voltage Gain

$$A_v = \frac{V_c}{V_b} = \frac{-h_{fe}R_L}{h_{ie} + (h_{ie}h_{oe} - h_{re}h_{fe})R_L}$$

Chapter 19 Review

Transmission ("T") Parameters

- Parameters: $\boxed{\begin{aligned} V_1 &= AV_2 - BI_2 \\ I_1 &= CV_2 - DI_2 \end{aligned}}$

- Perform the analysis with the **output** Open Circuited (I_2=0)

$$\begin{aligned} V_1 &= AV_2 - B\cancelto{0}{I_2} \\ I_1 &= CV_2 - D\cancelto{0}{I_2} \end{aligned} \Rightarrow \begin{aligned} V_1 &= AV_2 \\ I_1 &= CV_2 \end{aligned} \Rightarrow \begin{aligned} A &= \frac{V_1}{V_2} \\ C &= \frac{I_1}{V_2} \end{aligned}$$

- Perform the analysis with the **output** Short Circuited(V_2=0)

$$\begin{aligned} V_1 &= A\cancelto{0}{V_2} - BI_2 \\ I_1 &= C\cancelto{0}{V_2} - DI_2 \end{aligned} \Rightarrow \begin{aligned} V_1 &= -BI_2 \\ I_1 &= -DI_2 \end{aligned} \Rightarrow \begin{aligned} B &= -\frac{V_1}{I_2} \\ D &= -\frac{I_1}{I_2} \end{aligned}$$

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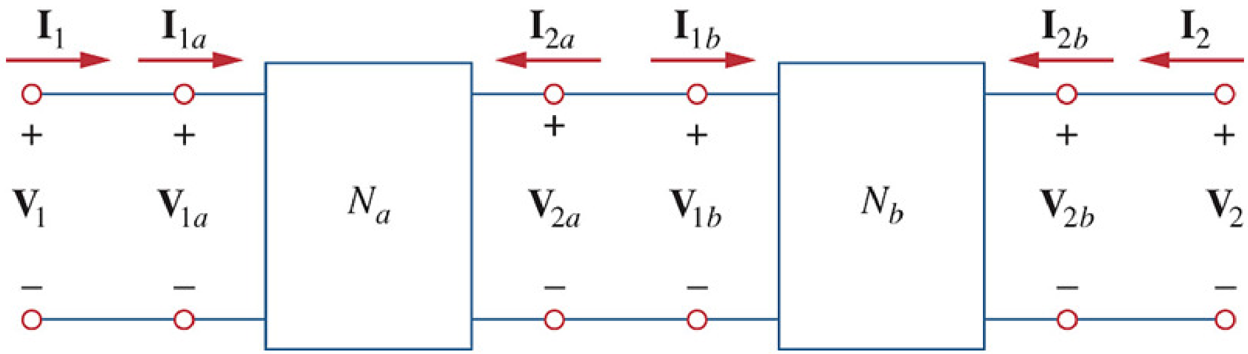
- Primary benefit of "T"-Parameters is their ability to be cascaded.



$$\begin{bmatrix} V_{1a} \\ I_{1a} \end{bmatrix} = \begin{bmatrix} A_a & B_a \\ C_a & D_a \end{bmatrix} \begin{bmatrix} V_{2a} \\ -I_{2a} \end{bmatrix} \qquad \begin{bmatrix} V_{1b} \\ I_{1b} \end{bmatrix} = \begin{bmatrix} A_b & B_b \\ C_b & D_b \end{bmatrix} \begin{bmatrix} V_{2b} \\ -I_{2b} \end{bmatrix}$$

$$-I_{2a} = I_{1b}$$

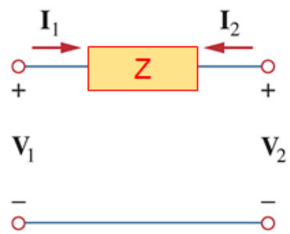
$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A_a & B_a \\ C_a & D_a \end{bmatrix} \begin{bmatrix} A_b & B_b \\ C_b & D_b \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A' & B' \\ C' & D' \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$

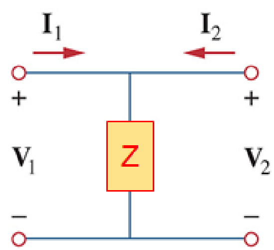
Chapter 19 Review

T - Parameters (Building Block models)

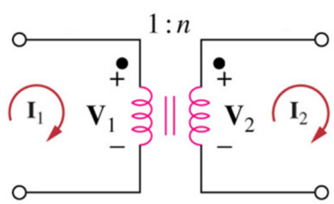
- We can create “building block” models of components by finding their T-parameters and use the cascading property to find the T-parameters for the complete circuit/system.



$$\begin{bmatrix} 1 & Z \\ 0 & 1 \end{bmatrix}$$



$$\begin{bmatrix} 1 & 0 \\ \frac{1}{Z} & 1 \end{bmatrix}$$



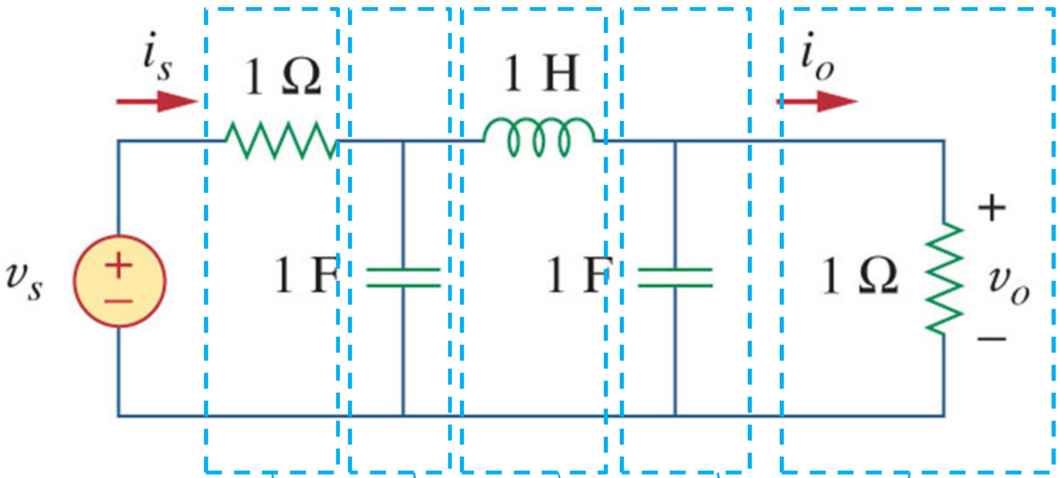
Dots “same”

$$\begin{bmatrix} \frac{1}{n} & 0 \\ 0 & n \end{bmatrix}$$

Dots “opposite”

$$\begin{bmatrix} -\frac{1}{n} & 0 \\ 0 & -n \end{bmatrix}$$

- With “Building Block” approach, circuits can be broke up into discrete components and analyzed using T-parameters



$$\begin{bmatrix} V_s \\ I_s \end{bmatrix} = \underbrace{\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ s & 1 \end{bmatrix}\begin{bmatrix} 1 & s \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ s & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}}_{\begin{bmatrix} A & B \\ C & D \end{bmatrix}}\begin{bmatrix} V_o \\ I_o \end{bmatrix}$$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} \longrightarrow \boxed{A = \left.\frac{V_1}{V_2}\right|_{I_2=0} = \frac{1}{H(s)}}$$

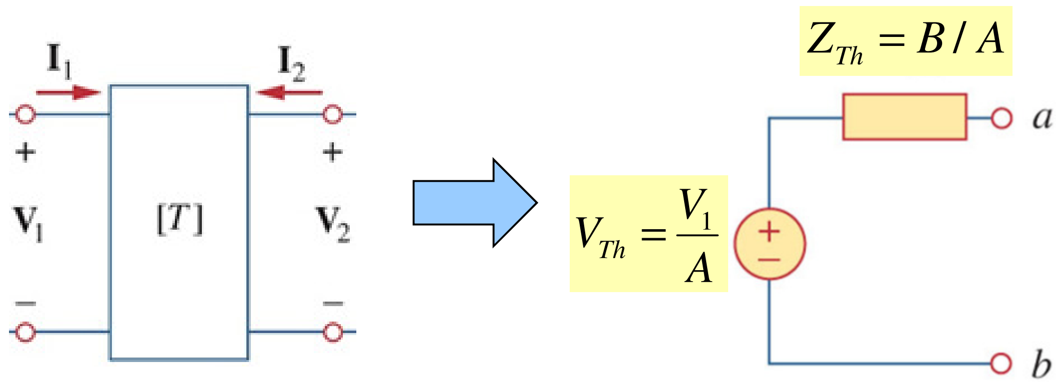
Chapter 19 Review

T - Parameters (Useful Properties)

- The T parameters give us useful properties in the analysis of circuits:
  - Open Circuit Voltage Transfer Function:

$$A = \left.\frac{V_1}{V_2}\right|_{I_2=0} = \frac{1}{H(s)} \qquad H(s) = \frac{1}{A}$$

  - Thevenin Equivalent Circuit (Replace circuit as a source)



Chapter 19 Review

Conversion between Parameters

- Conversion tables exists to convert between parameters

	z		y		h		T	
z	z_{11}	z_{12}	$\frac{y_{22}}{\Delta_y}$	$-\frac{y_{12}}{\Delta_y}$	$\frac{\Delta_h}{h_{22}}$	$\frac{h_{12}}{h_{22}}$	$\frac{A}{C}$	$\frac{\Delta_T}{C}$
	z_{21}	z_{22}	$-\frac{y_{21}}{\Delta_y}$	$\frac{y_{11}}{\Delta_y}$	$-\frac{h_{21}}{h_{22}}$	$\frac{1}{h_{22}}$	$\frac{1}{C}$	$\frac{D}{C}$
y	$\frac{z_{22}}{\Delta_z}$	$-\frac{z_{12}}{\Delta_z}$	y_{11}	y_{12}	$\frac{1}{h_{11}}$	$-\frac{h_{12}}{h_{11}}$	$\frac{D}{B}$	$-\frac{\Delta_T}{B}$
	$-\frac{z_{21}}{\Delta_z}$	$\frac{z_{11}}{\Delta_z}$	y_{21}	y_{22}	$\frac{h_{21}}{h_{11}}$	$\frac{\Delta_h}{h_{11}}$	$-\frac{1}{B}$	$\frac{A}{B}$
h	$\frac{\Delta_z}{z_{22}}$	$\frac{z_{12}}{z_{22}}$	$\frac{1}{y_{11}}$	$-\frac{y_{12}}{y_{11}}$	h_{11}	h_{12}	$\frac{B}{D}$	$\frac{\Delta_T}{D}$
	$-\frac{z_{21}}{z_{22}}$	$\frac{1}{z_{22}}$	$\frac{y_{21}}{y_{11}}$	$\frac{\Delta_y}{y_{11}}$	h_{21}	h_{22}	$-\frac{1}{D}$	$\frac{C}{D}$
T	$\frac{z_{11}}{z_{21}}$	$\frac{\Delta_z}{z_{21}}$	$-\frac{y_{22}}{y_{21}}$	$-\frac{1}{y_{21}}$	$-\frac{\Delta_h}{h_{21}}$	$-\frac{h_{11}}{h_{21}}$	A	B
	$\frac{1}{z_{21}}$	$\frac{z_{22}}{z_{21}}$	$-\frac{\Delta_y}{y_{21}}$	$-\frac{y_{11}}{y_{21}}$	$-\frac{h_{22}}{h_{21}}$	$-\frac{1}{h_{21}}$	C	D

$\Delta_y = y_{11}y_{22} - y_{12}y_{21}$ $\quad \Delta_z = z_{11}z_{22} - z_{12}z_{21}$ $\quad \Delta_h = h_{11}h_{22} - h_{12}h_{21}$ $\quad \Delta_T = AD - BC$

Chapter 19: Two-Port Networks

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19.1 Introduction (1)

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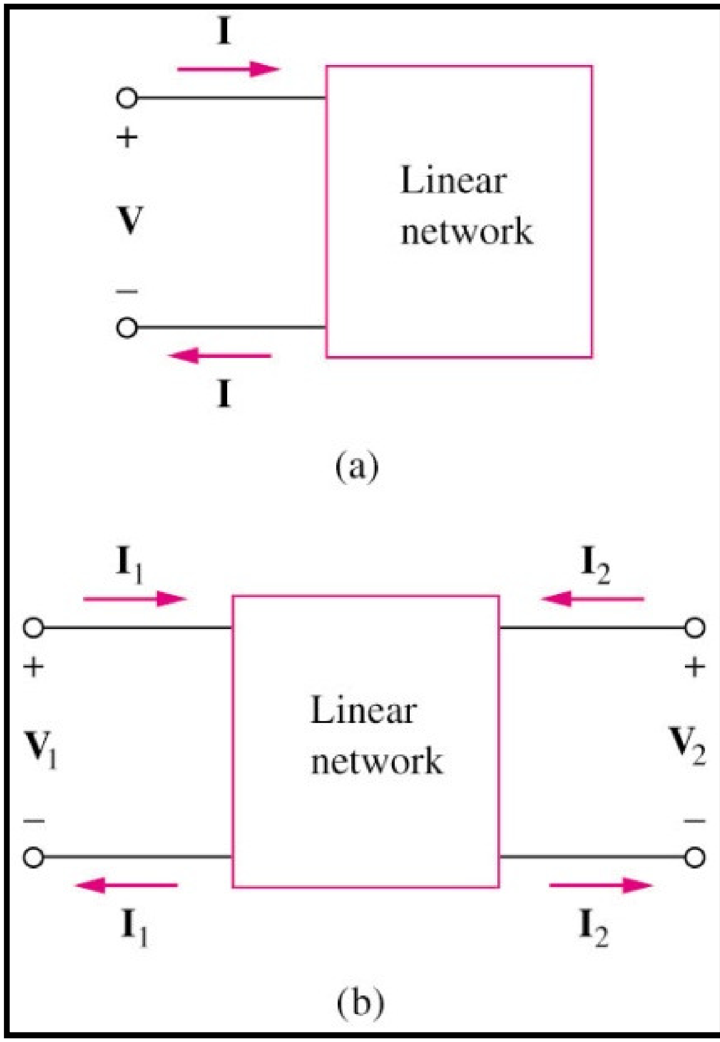
- A port is an access to the network and consists of a pair of terminals; the current entering one terminal leaves through the other terminal so that the net current entering the port equals zero.
- One port networks include two-terminal devices such as resistors, capacitors, and inductors.
- A two-port network has two separate ports for input and output.
- Two port networks include op amps, transistors and transformers.

19.1 Introduction (2)



One port or two terminal circuit

Two port or four terminal circuit

- It is an electrical network with two separate ports for input and output.
- No independent sources.

19.1 Introduction (3)

- Characterizing a two-port network requires that we relate the terminal quantities $\mathbf{V}_1$, $\mathbf{V}_2$, $\mathbf{I}_1$, $\mathbf{I}_2$ out of which two are independent. Six sets of voltage and current parameters will be derived.
- Two port networks are useful in communications, control systems, power systems, and electronics.
- They are used in electronics to model transistors and to facilitate cascaded design.
- Additionally, if we know the parameters of a two-port network it can be treated as a “black box” when embedded within a larger network.

19.2 Impedance Parameters (1)

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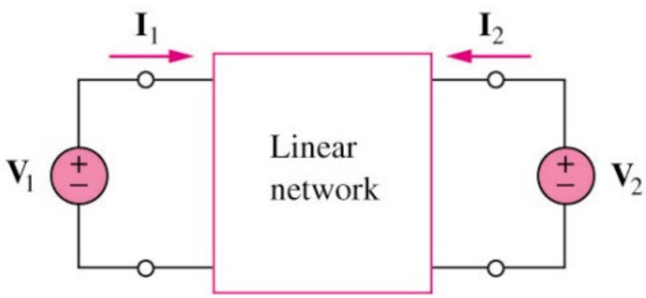
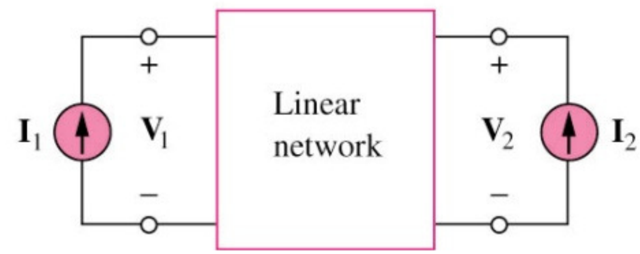
- Often called "**Z-parameters**" since their units are in **ohms** and they represent an impedance relationship between $\mathbf{V}_1$, $\mathbf{V}_2$, $\mathbf{I}_1$, $\mathbf{I}_2$ for the two port network shown below:



Voltage Driven Network

Assume no independent source in the network



Current Driven Network

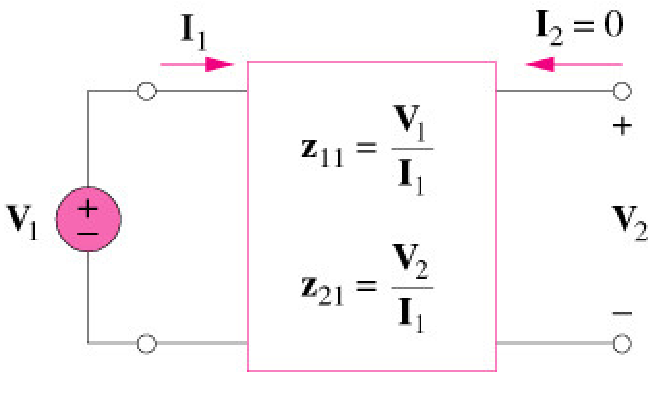
$$\mathrm{V}_1 = \mathrm{z}_{11}\mathrm{I}_1 + \mathrm{z}_{12}\mathrm{I}_2$$
$$\mathrm{V}_2 = \mathrm{z}_{21}\mathrm{I}_1 + \mathrm{z}_{22}\mathrm{I}_2$$

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

- Z-parameters are commonly used in filter synthesis, impedance matching networks design, and power distribution networks analysis.

19.2 Impedance Parameters (2)

The values of parameters can be evaluated by setting I_1=0 or I_2=0 (open circuit)

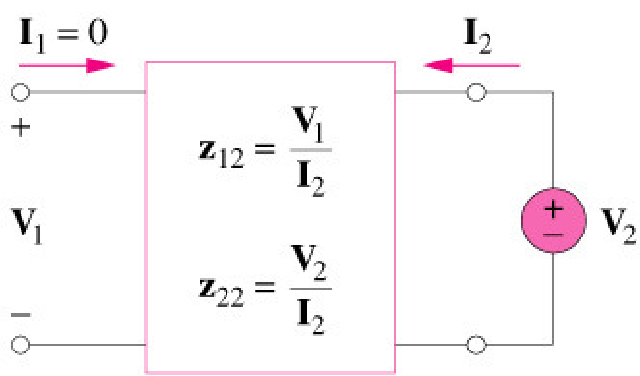


(a)

Setting I_2 = 0

$$z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0} \quad \text{and} \quad z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0}$$

z_{11} = Open-circuit input impedance
z_{21} = Open-circuit transfer impedance from port 2 to port 1



(b)

Setting I_1 = 0

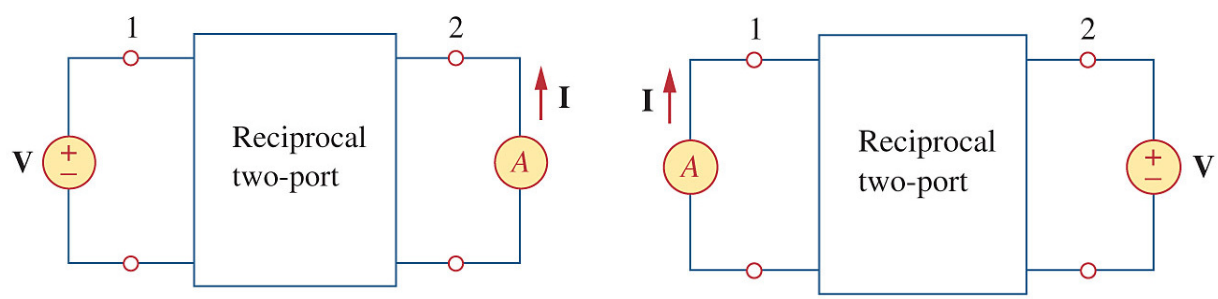
$$z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1=0} \quad \text{and} \quad z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0}$$

z12 = Open-circuit transfer impedance from port 1 to port 2
z22 = Open-circuit output impedance

19.2 Impedance Parameters (3)

Properties of Z-parameters

- Symmetrical networks $z_{11} = z_{22}$
  - Implies a mirror like symmetry
- Reciprocal networks $z_{12} = z_{21}$
  - Any network made up entirely of resistors, capacitors, and inductors must be reciprocal.
  - Linear networks with no dependant sources are reciprocal.
  - Interchanging an ideal voltage source at one port with an ideal ammeter at the other port gives the same ammeter reading.



$$V = z_{12} I \qquad\qquad V = z_{21} I$$

19.2 Impedance Parameters (4)

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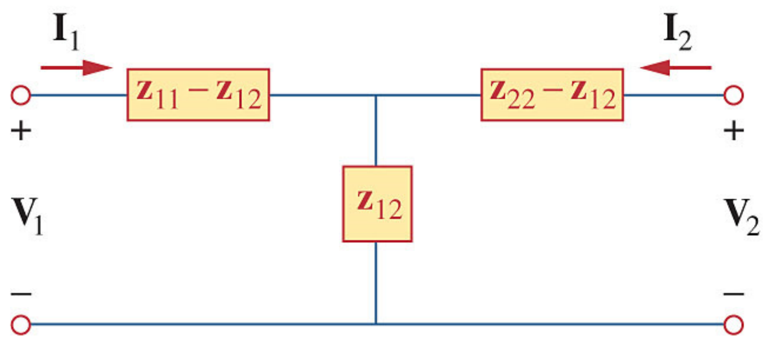
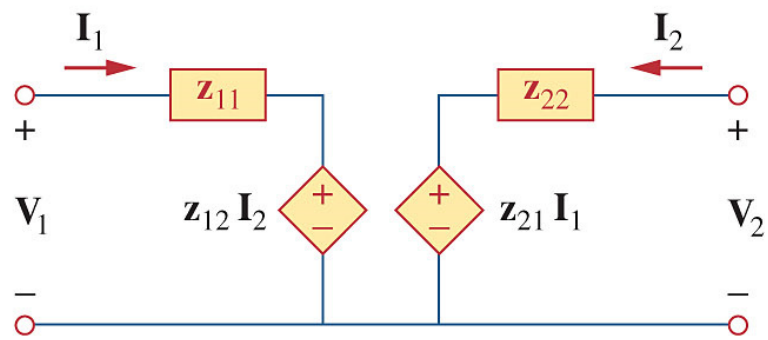


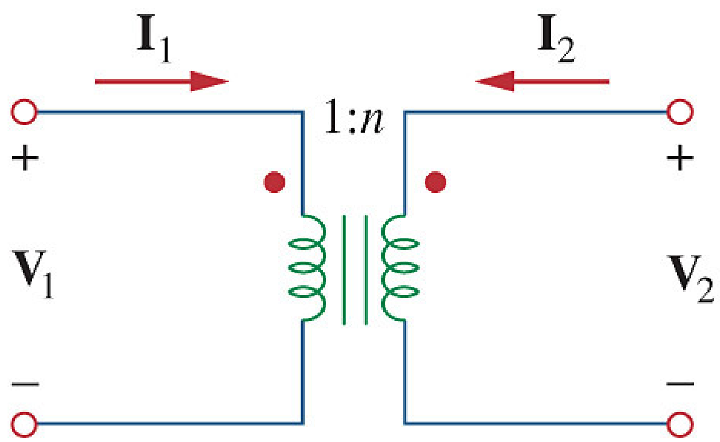
Reciprocal Network



General Network

- A reciprocal network can be replaced by the T-network shown above
- If not reciprocal, the General network is the T-equivalent.

19.2 Impedance Parameters (5)



- Note: some circuits do not have z-parameter equivalents. (they may have other 2-port equivalents, as we shall see)
- Consider an ideal transformer:

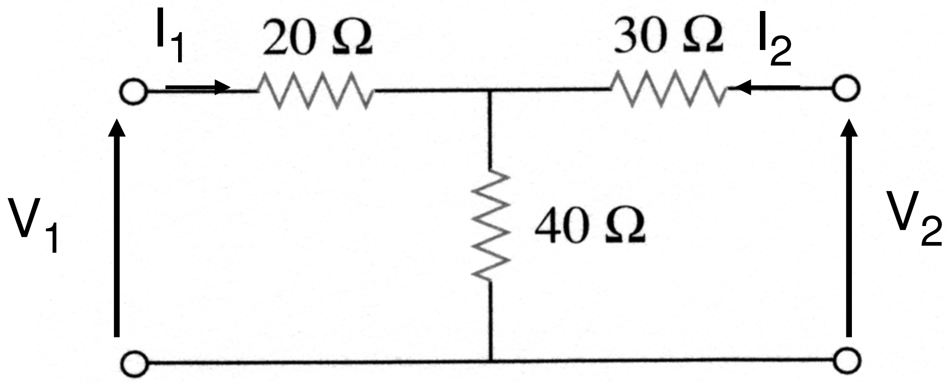
  $V_1 = V_2/n$ and $I_1 = -nI_2$.
- This cannot be expressed by:

$$V_1 = z_{11}I_1 + z_{12}I_2$$
$$V_2 = z_{21}I_1 + z_{22}I_2$$

19.2 Impedance Parameters (6)

Example 19.1

Determine the z-parameters of the following circuit.



$$z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0} \quad \text{and} \quad z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0}$$

$$z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1=0} \quad \text{and} \quad z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0}$$

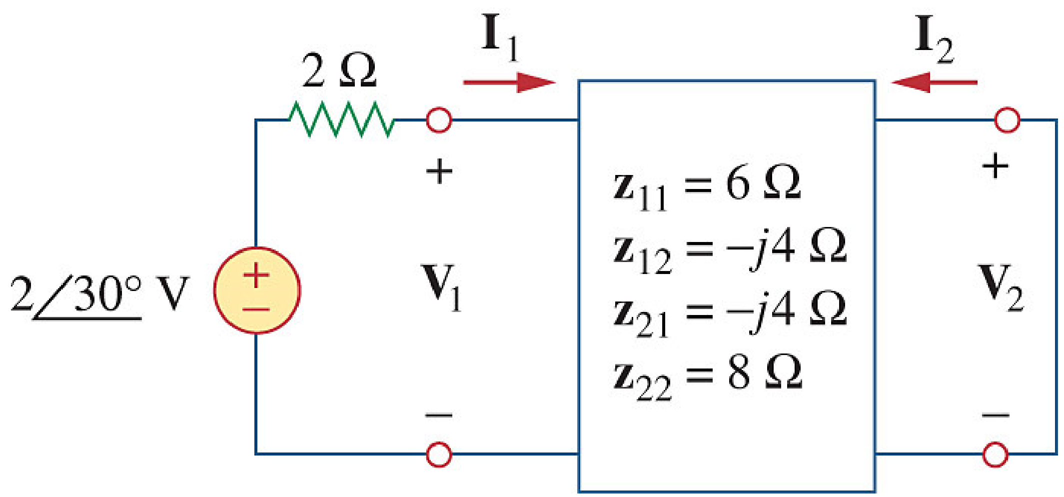
$$z = \begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix} \Omega$$

Answer: $$z = \begin{bmatrix} 60 & 40 \\ 40 & 70 \end{bmatrix} \Omega$$

19.2 Impedance Parameters (7)

Practice Problem 19.2

Determine I_1 and I_2 in the following circuit.

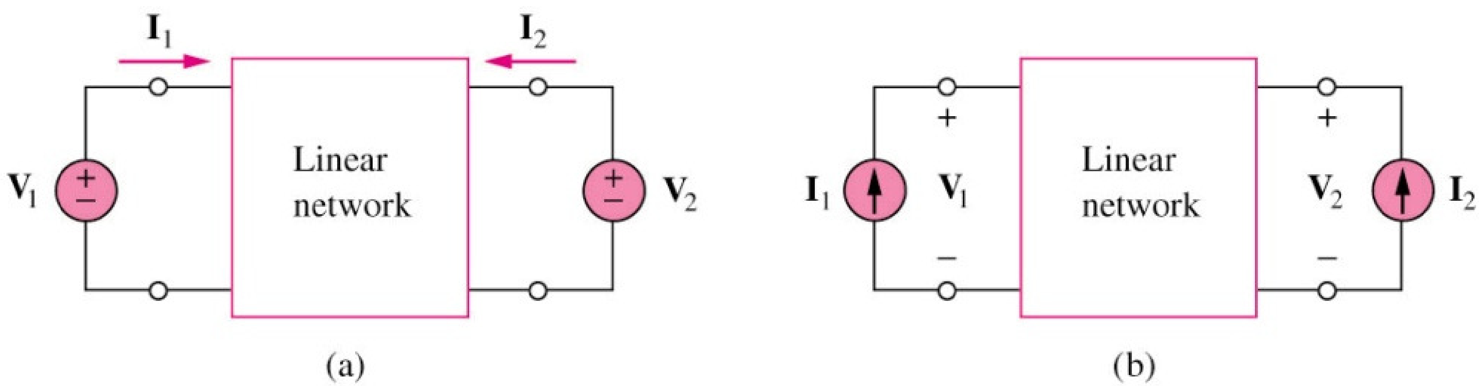


$$V_1 = z_{11}I_1 + z_{12}I_2$$
$$V_2 = z_{21}I_1 + z_{22}I_2$$

Answer:

$$I_1 = 200\angle 30°\ \text{mA}$$
$$I_2 = 100\angle 120°\ \text{mA}$$

19.3 Admittance Parameters (1)



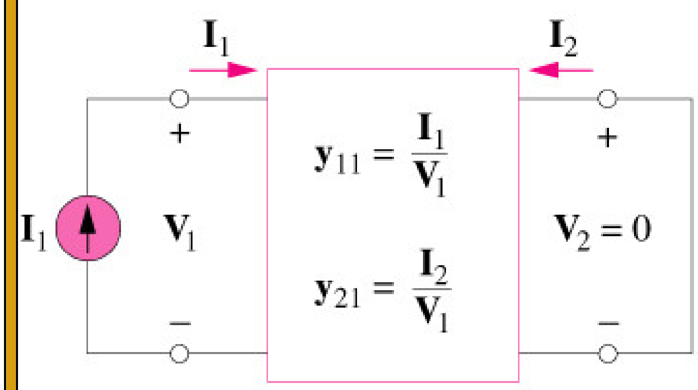
Assume no independent source in the network

$$I_1 = y_{11}V_1 + y_{12}V_2$$
$$I_2 = y_{21}V_1 + y_{22}V_2$$

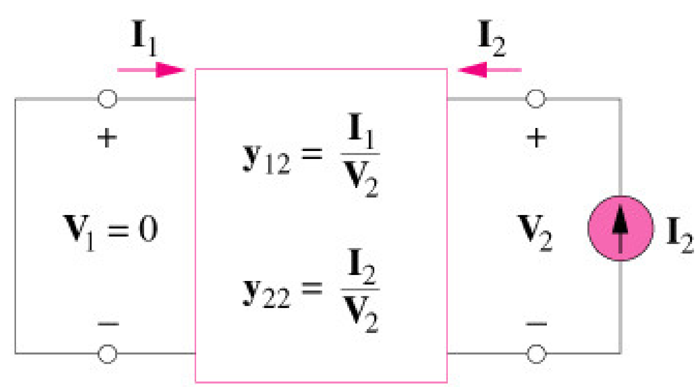
$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = [y] \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

where the **y** terms are called the admittance parameters, or simply y parameters, and they have units of Siemens.

19.3 Admittance Parameters (2)



(a)



(b)

Setting V_2 = 0 (Shorting the output)

$$y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2=0} \quad \text{and} \quad y_{21} = \left.\frac{I_2}{V_1}\right|_{V_2=0}$$

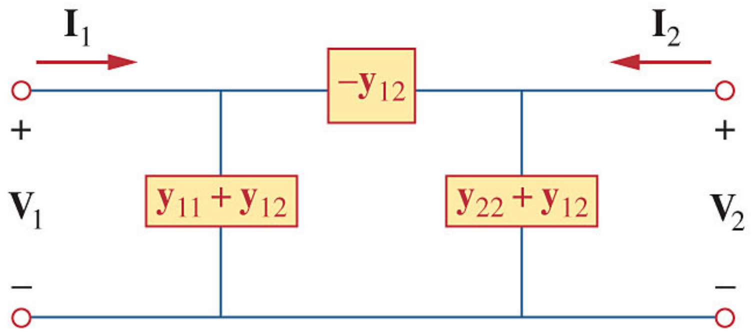
y_{11} = Short-circuit input admittance
y_{21} = Short-circuit transfer admittance from port 1 to port 2

Setting V_1 = 0 (Shorting the input)

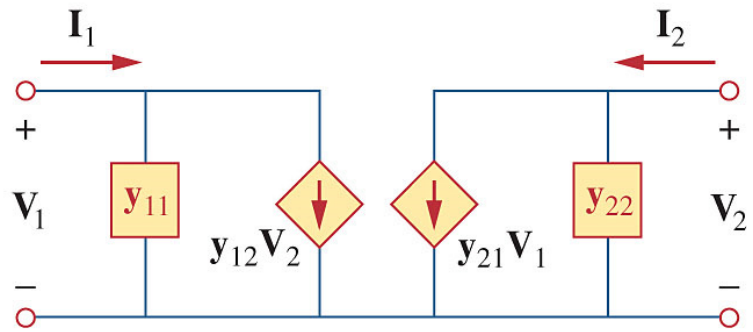
$$y_{12} = \left.\frac{I_1}{V_2}\right|_{V_1=0} \quad \text{and} \quad y_{22} = \left.\frac{I_2}{V_2}\right|_{V_1=0}$$

y_{12} = Short-circuit transfer admittance from port 2 to port 1
y_{22} = Short-circuit output admittance

19.3 Admittance Parameters (3)



Reciprocal
Network



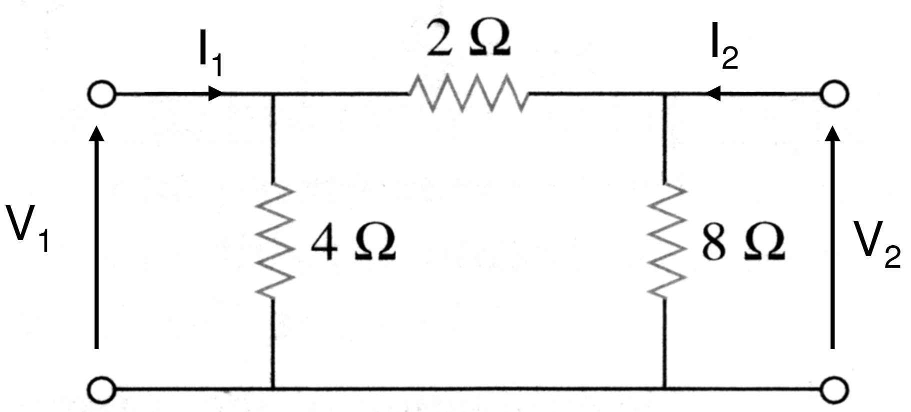
General
Network

- A reciprocal network (y_{12} = y_{21}) can be replaced by the Pi-network in figure (a).
- If not reciprocal, the network in figure (b) is the Pi-equivalent.

19.3 Admittance Parameters (4)

Example 19.3

Determine the y-parameters of the following circuit.



$$y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2=0} \quad \text{and} \quad y_{21} = \left.\frac{I_2}{V_1}\right|_{V_2=0}$$

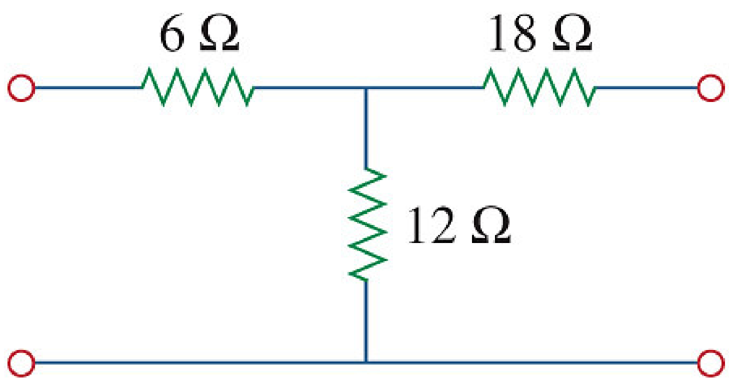
$$y_{12} = \left.\frac{I_1}{V_2}\right|_{V_1=0} \quad \text{and} \quad y_{22} = \left.\frac{I_2}{V_2}\right|_{V_1=0}$$

$$y = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} S$$

Answer: $$y = \begin{bmatrix} 0.75 & -0.5 \\ -0.5 & 0.625 \end{bmatrix} S$$

Practice Problem 19.3

Determine the y-parameters of the following circuit.



$$y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2=0} \quad \text{and} \quad y_{21} = \left.\frac{I_2}{V_1}\right|_{V_2=0}$$

$$y_{12} = \left.\frac{I_1}{V_2}\right|_{V_1=0} \quad \text{and} \quad y_{22} = \left.\frac{I_2}{V_2}\right|_{V_1=0}$$

$$y = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} \mathrm{S}$$

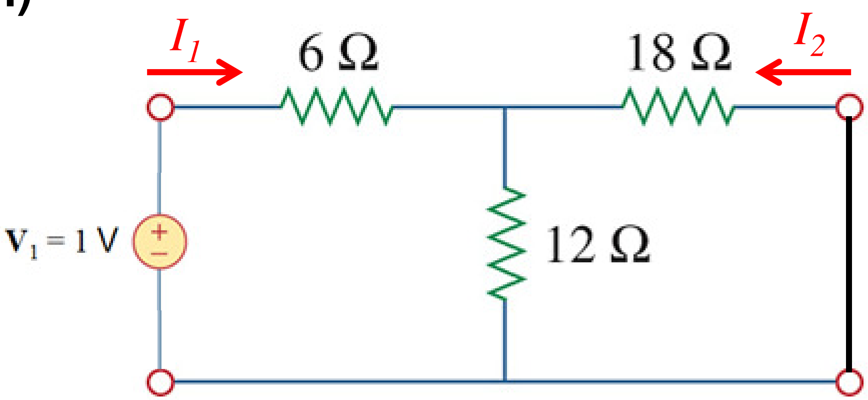
Answer: $y = \begin{bmatrix} 75.77 & -30.3 \\ -30.3 & 45.47 \end{bmatrix} \mathrm{mS}$

19.3 Admittance Parameters (6)

Practice Problem 19.3

Practice Problem 19.3 (Solution)

- Short the output
- Put a 1 volt source at input
- Find I_1 and I_2



$$y_{11} = \frac{I_1}{(1)}\bigg|_{V_2=0} \quad and \quad y_{21} = \frac{I_2}{(1)}\bigg|_{V_2=0}$$

Find Input Impedance

$$Z_{in} = 6 + 12 \| 18 = 13.2$$

$$I_1 = \frac{V_1}{Z_{in}} = \frac{1}{13.2} = 0.07576$$

$$\boxed{y_{11} = 0.07576}$$

Similarly at Output

$$Z_{out} = 18 + 6 \| 12 = 22$$

$$I_2 = \frac{V_2}{Z_{in}} = \frac{1}{22} = 0.04545$$

$$\boxed{y_{22} = 0.04545}$$

Find I_2 from current divider equation

$$I_2 = \frac{-12}{12+18} I_1$$

$$I_2 = (-0.4)0.07576 = -0.0303$$

$$y_{21} = -0.0303$$

$$\boxed{y_{12} = y_{21} = -0.0303}$$

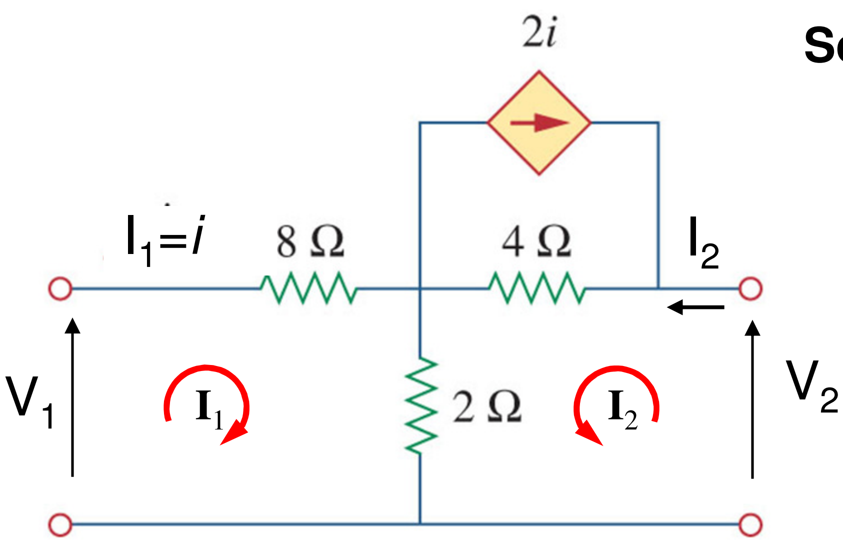
Reciprocal Network

19.3 Admittance Parameters (7)

Example 19.4

Determine the y-parameters of the following circuit.

$$\boxed{\begin{aligned} I_1 &= y_{11}V_1 + y_{12}V_2 \\ I_2 &= y_{21}V_1 + y_{22}V_2 \end{aligned}}$$



Answer: $\boxed{y = \begin{bmatrix} 0.15 & -0.05 \\ -0.25 & 0.25 \end{bmatrix} \text{S}}$

Note: Sometimes two port parameters will fall out directly from mesh equations.

Solution: Apply KVL

Mesh I_1: $V_1 = 8I_1 + 2(I_1 + I_2)$

$$V_1 = 10I_1 + 2I_2$$

Mesh I_2: $V_2 = 4(2i + I_2) + 2(I_1 + I_2)$

$$V_2 = 8I_1 + 4I_2 + 2I_1 + 2I_2$$

$$V_2 = 10I_1 + 6I_2$$

Subtract #1 from #2:

$$V_2 - V_1 = 0 + 4I_2$$

$$I_2 = -0.25V_1 + 0.25V_2$$

Substitute back into #1

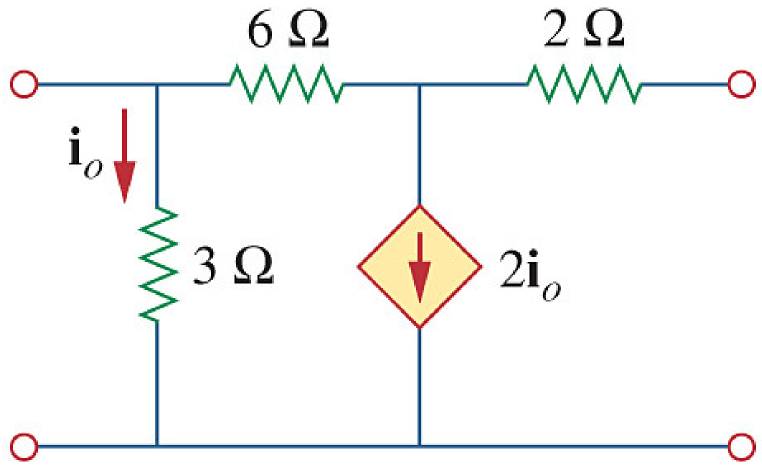
$$V_1 = 10I_1 - 0.5V_1 + 0.5V_2$$

$$10I_1 = 1.5V_1 - 0.5V_2$$

$$I_1 = 0.15V_1 - 0.05V_2$$

Practice Problem 19.4

Determine the y-parameters of the following circuit.



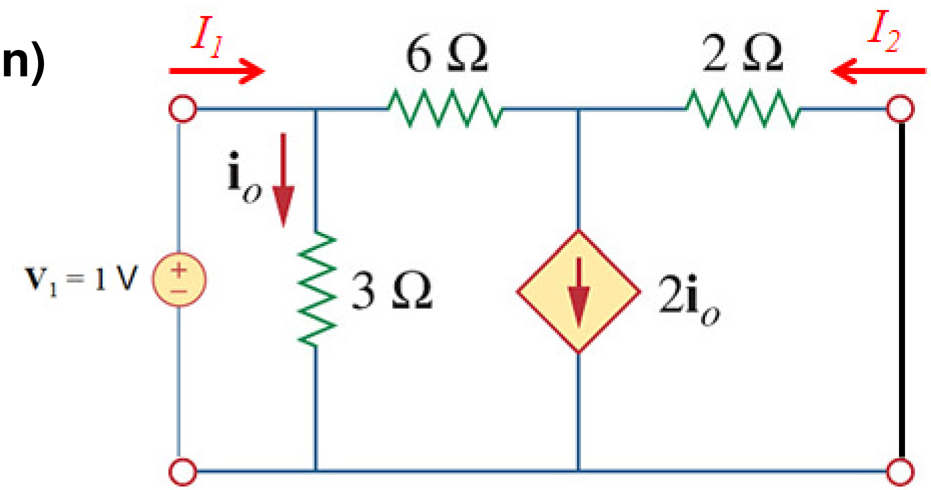
Answer: $$y = \begin{bmatrix} 0.625 & -0.125 \\ 0.375 & 0.125 \end{bmatrix} S$$

19.3 Admittance Parameters (9)

Practice problem 19.4

Practice Problem 19.4 (Solution)

- Short the output
- Put a 1 volt source at input
- Find I_1 and I_2



First find i_o:

$$i_0 = \frac{1}{3}$$

Dependent current source is then 2/3, find I_1 by repetitive source transformations of the dependant current source

$$I_1 = 0.625 \quad \Rightarrow \boxed{y_{11} = 0.625}$$

Next find current across 6 Ω resistor $I_{6\Omega}$:

$$I_{6\Omega} = 0.625 - \frac{1}{3}$$

$$I_2 + I_{6\Omega} = 2i_0$$

$$I_2 = 2i_0 - I_{6\Omega} = \frac{2}{3} - \left(0.625 - \frac{1}{3}\right) = 0.375 \quad \Rightarrow \boxed{y_{12} = 0.375}$$

Z and Y Parameters
Comparison

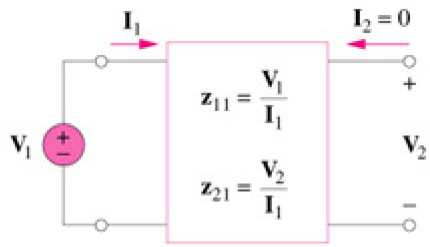
Z-Parameters

$$V_1 = z_{11} I_1 + z_{12} I_2$$
$$V_2 = z_{21} I_1 + z_{22} I_2$$

- **Open** one port (I_1=0 or I_2=0)
- Connect a source to the other port
- Solve to find z-parameters

$$z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0} \quad \text{and} \quad z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0}$$

$$z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1=0} \quad \text{and} \quad z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0}$$



I = 0
Open Circuit

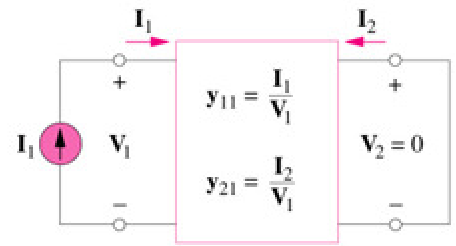
Y-Parameters

$$I_1 = y_{11} V_1 + y_{12} V_2$$
$$I_2 = y_{21} V_1 + y_{22} V_2$$

- **Short** one port (V_1=0 or V_2=0)
- Connect a source to the other port
- Solve to find y-parameters

$$y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2=0} \quad \text{and} \quad y_{21} = \left.\frac{I_2}{V_1}\right|_{V_2=0}$$

$$y_{12} = \left.\frac{I_1}{V_2}\right|_{V_1=0} \quad \text{and} \quad y_{22} = \left.\frac{I_2}{V_2}\right|_{V_1=0}$$



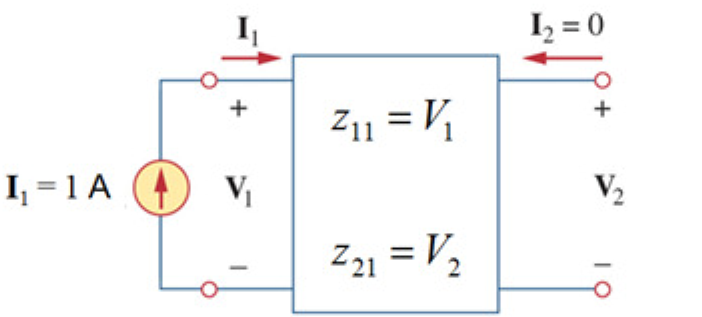
V = 0
Short Circuit

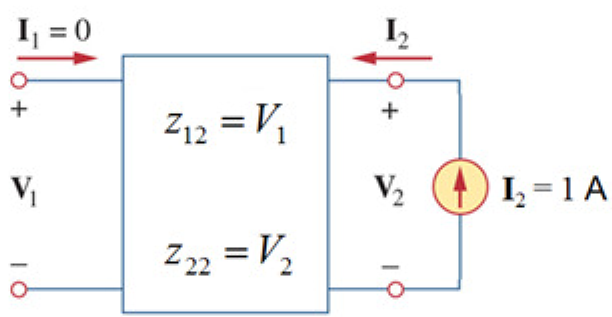
Z and Y parameters

Alternative method (1 Amp / 1 Volt sources)

Z-Parameters

- Open circuit one port
- Put a 1 Amp current source at other port
- Resulting voltages are the z-parameters

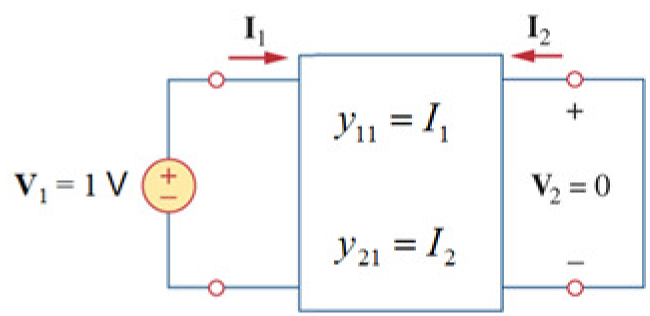


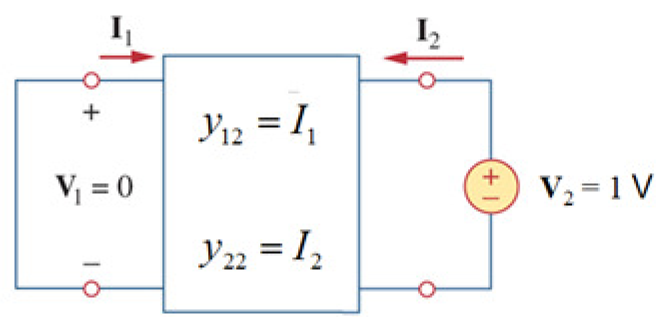


$$V_1 = z_{11}I_1 + z_{12}I_2$$
$$V_2 = z_{21}I_1 + z_{22}I_2$$

Y-Parameters

- Short circuit one port
- Put a 1 Volt voltage source at other port
- Resulting current are the y-parameters

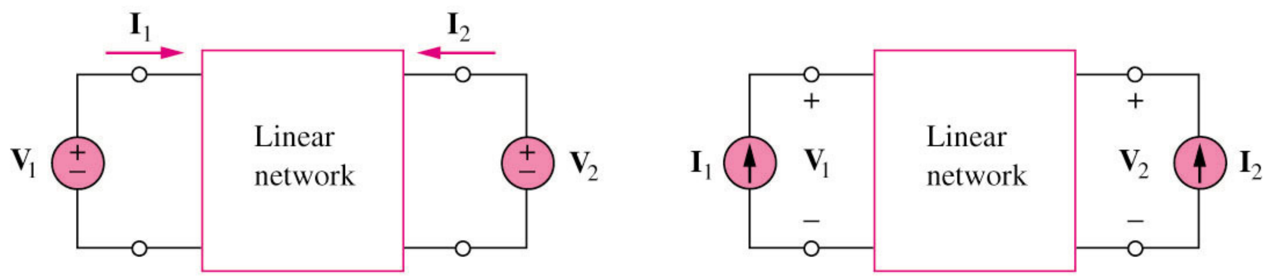




$$I_1 = y_{11}V_1 + y_{12}V_2$$
$$I_2 = y_{21}V_1 + y_{22}V_2$$

19.4 Hybrid Parameters (1)

•The z and y parameters of a two-port network do not always exist. Therefore, there is a need to develop another set of parameters based on making V_1 and I_2 the dependent variables.



Assume no independent source in the network

$$V_1 = h_{11}I_1 + h_{12}V_2$$
$$I_2 = h_{21}I_1 + h_{22}V_2$$

$$\Rightarrow \begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ V_2 \end{bmatrix} = [h] \begin{bmatrix} I_1 \\ V_2 \end{bmatrix}$$

where the **h** terms are called the hybrid parameters, or simply h parameters.

•Hybrid parameters are very useful for describing electronic devices such as transistors because it is much easier to measure the h parameters of these devices than to measure their z or y parameters.

•The ideal transformer can also be described by h parameters.

19.4 Hybrid Parameters (2)

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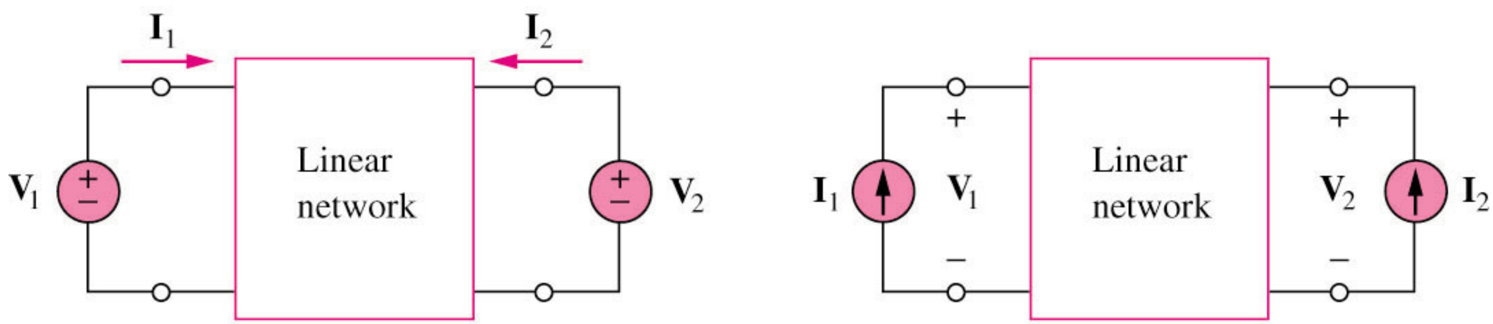


Assume no independent source in the network

$$h_{11} = \left.\frac{V_1}{I_1}\right|_{V_2=0}$$

h_{11} = short-circuit input impedance (Ω)

$$h_{21} = \left.\frac{I_2}{I_1}\right|_{V_2=0}$$

h_{21} = short-circuit forward current gain

$$h_{12} = \left.\frac{V_1}{V_2}\right|_{I_1=0}$$

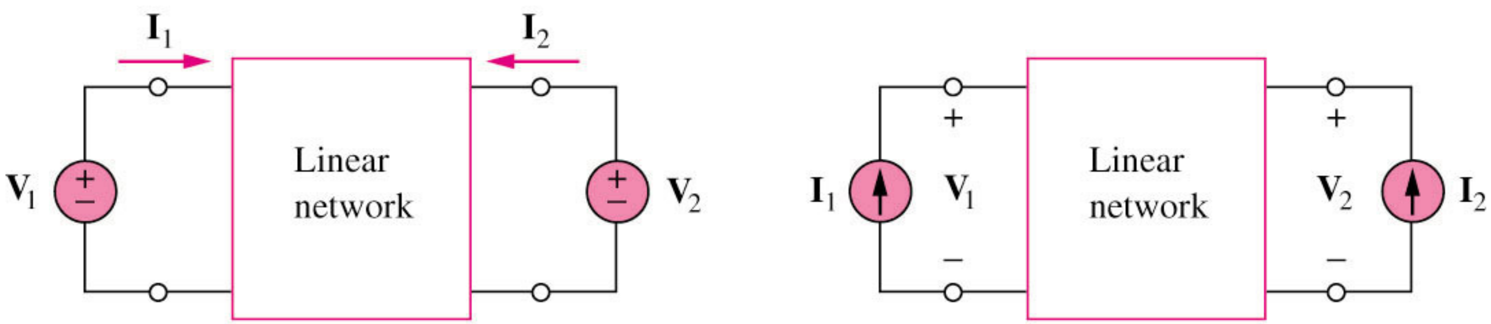
h_{12} = open-circuit reverse voltage-gain

$$h_{22} = \left.\frac{I_2}{V_2}\right|_{I_1=0}$$

h_{22} = open-circuit output admittance (S)

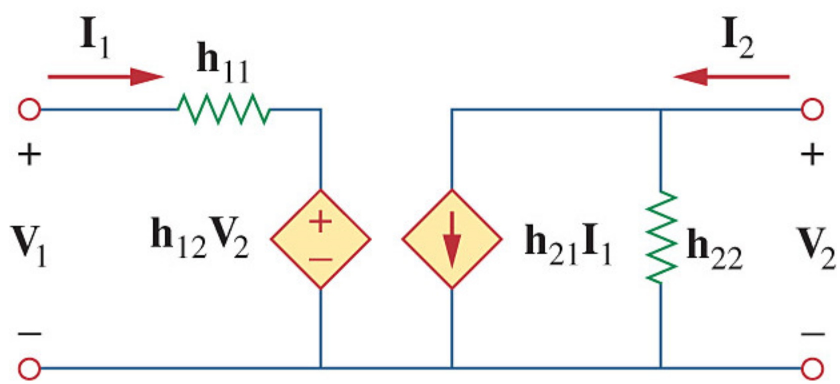
- Note that the h parameters represent an impedance, voltage gain, current gain, and admittance, thereby the term hybrid parameters.
- For reciprocal network, $h_{12} = -h_{21}$

19.4 Hybrid Parameters (3)



Assume no independent source in the network

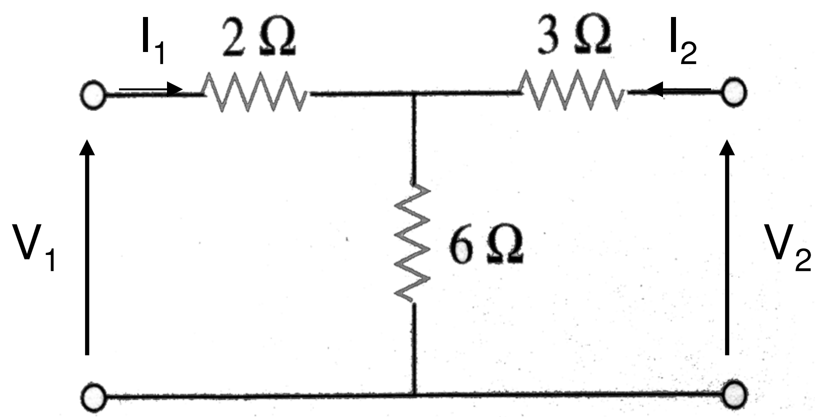
Hybrid model of a two-port network:



19.4 Hybrid Parameters (4)

Example 19.5:

Determine the h-parameters of the following circuit.



$$V_1 = h_{11} I_1 + h_{12} V_2$$
$$I_2 = h_{21} I_1 + h_{22} V_2$$

$$h_{11} = \left.\frac{V_1}{I_1}\right|_{V_2=0} \quad \text{and} \quad h_{21} = \left.\frac{I_2}{I_1}\right|_{V_2=0}$$

$$h_{12} = \left.\frac{V_1}{V_2}\right|_{I_1=0} \quad \text{and} \quad h_{22} = \left.\frac{I_2}{V_2}\right|_{I_1=0}$$

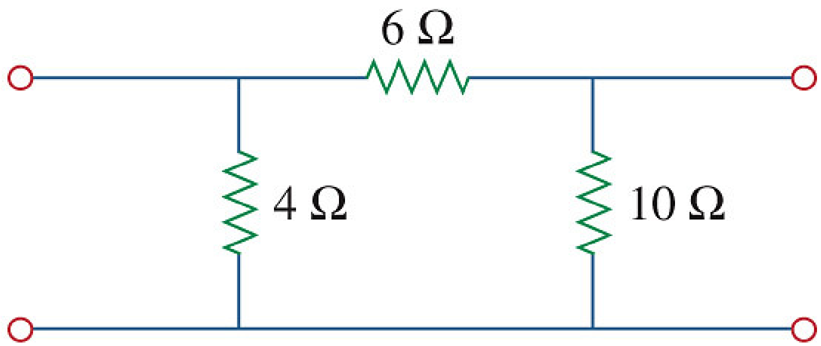
$$h = \begin{bmatrix} h_{11}\Omega & h_{12} \\ h_{21} & h_{22}S \end{bmatrix}$$

Answer: $h = \begin{bmatrix} 4\Omega & \frac{2}{3} \\ -\frac{2}{3} & \frac{1}{9}S \end{bmatrix}$

19.4 Hybrid Parameters (5)

Practice Problem 19.5:

Determine the h-parameters of the following circuit.



$$h_{11} = \left.\frac{V_1}{I_1}\right|_{V_2=0} \quad \text{and} \quad h_{21} = \left.\frac{I_2}{I_1}\right|_{V_2=0}$$

$$h_{12} = \left.\frac{V_1}{V_2}\right|_{I_1=0} \quad \text{and} \quad h_{22} = \left.\frac{I_2}{V_2}\right|_{I_1=0}$$

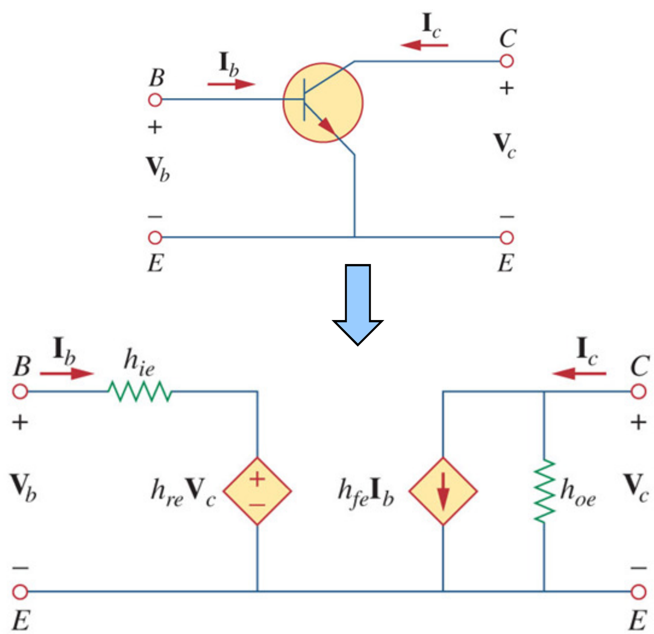
$$h = \begin{bmatrix} h_{11}\Omega & h_{12} \\ h_{21} & h_{22}S \end{bmatrix}$$

Answer: $h = \begin{bmatrix} 2.4\Omega & 0.4 \\ -0.4 & 0.2S \end{bmatrix}$

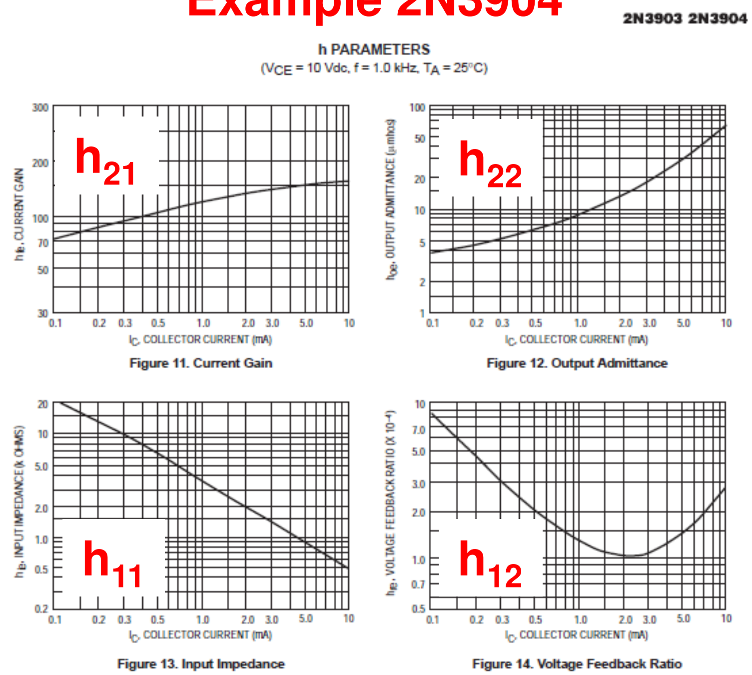
19.9.1 Transistor Circuits (1)

Hybrid Parameters

- H-parameters are often used to model transistor circuits
- The h-parameters vary depending on biasing conditions
- Parameters are given different subscripts:
  - h_{11} → h_{ie} = Base input impedance
  - h_{12} → h_{re} = Reverse voltage feedback ration
  - h_{21} → h_{fe} = Base-collector current gain
  - h_{22} → h_{oe} = Output admittance



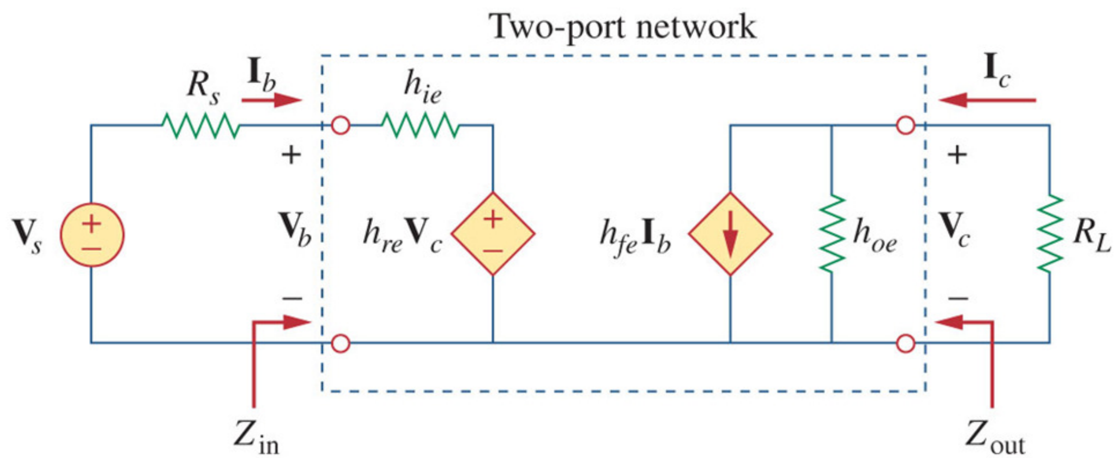
Example 2N3904



19.9.1 Transistor Circuits (2)

Hybrid Parameters

- H parameters are often found in manufacturers spec sheets
- Provide ability to calculate the exact voltage gain, input impedance, and output impedance of the transistor.



Input Impedance

$$\boxed{Z_{in} = \frac{V_b}{I_b} = h_{ie} - \frac{h_{re} h_{fe} R_L}{1 + h_{oe} R_L}}$$

Output Impedance

$$\boxed{Z_{out} = \left.\frac{V_c}{I_c}\right|_{V_s = 0} = \frac{R_s + h_{ie}}{(R_s + h_{ie})h_{oe} - h_{re} h_{fe}}}$$

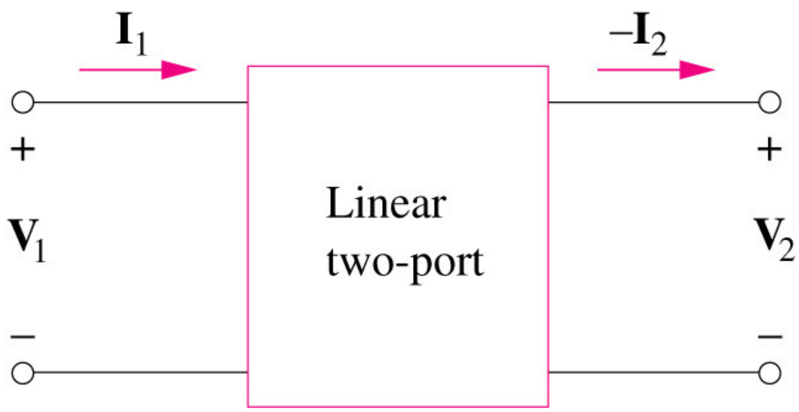
Current Gain

$$\boxed{A_i = \frac{I_c}{I_b} = \frac{h_{fe}}{1 + h_{oe} R_L}}$$

Voltage Gain

$$\boxed{A_v = \frac{V_c}{V_b} = \frac{-h_{fe} R_L}{h_{ie} + (h_{ie} h_{oe} - h_{re} h_{fe}) R_L}}$$

19.5 Transmission Parameters (1)



Assume no independent source in the network

$$V_1 = AV_2 - BI_2$$
$$I_1 = CV_2 - DI_2$$

$$\Rightarrow \begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix} = [T] \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$

where the **T** terms are called the transmission parameters, or simply T or ABCD parameters.

•Note that $-I_2$ is used since the current is considered to be leaving the network. It is logical to think of I_2 as leaving the two-port; this is customary convention in the power industry.

19.5 Transmission Parameters (2)

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- These two-port transmission parameters provide a measure of how a circuit transmits voltage and current form a source to a load.
- They are useful in the analysis of transmission lines and are therefore called transmission parameters.
- They are also known as ABCD parameters and are used in the design of telephone systems, microwave networks, and radars.

$$A = \left.\frac{V_1}{V_2}\right|_{I_2=0}$$

A=open-circuit voltage ratio

$$B = -\left.\frac{V_1}{I_2}\right|_{V_2=0}$$

B= negative short-circuit transfer impedance (Ω)

$$C = \left.\frac{I_1}{V_2}\right|_{I_2=0}$$

C= open-circuit transfer admittance (S)

$$D = -\left.\frac{I_1}{I_2}\right|_{V_2=0}$$

D=negative short-circuit current ratio

19.5 Transmission Parameters (3)

Solving for Transmission Parameters

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- To find the transmission parameters, analyze the circuit as follows:
- Perform the analysis with the output Open Circuited (I_2=0)

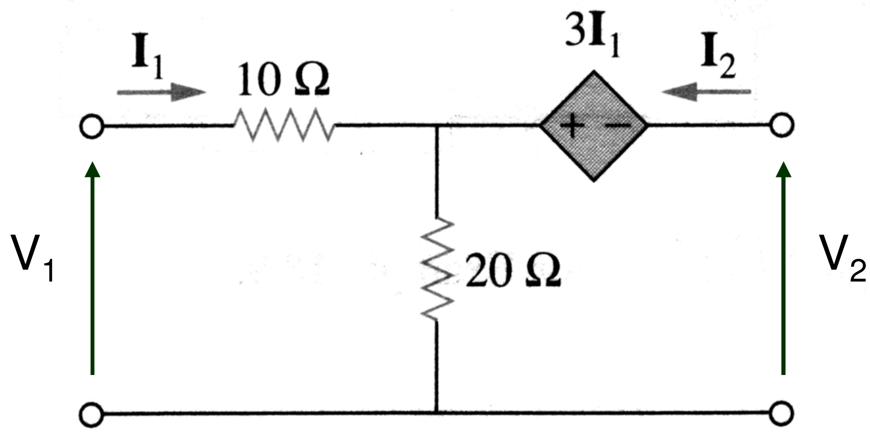
$$\begin{aligned} \mathrm{V}_1 &= \mathrm{AV}_2 - \cancel{\mathrm{BI}_2}^{\,0} \\ \mathrm{I}_1 &= \mathrm{CV}_2 - \cancel{\mathrm{DI}_2}^{\,0} \end{aligned} \Rightarrow \begin{aligned} V_1 &= AV_2 \\ I_1 &= CV_2 \end{aligned} \Rightarrow \begin{aligned} A &= \frac{V_1}{V_2} \\ C &= \frac{I_1}{V_2} \end{aligned}$$

- Perform the analysis with the output Short Circuited(V_2=0)

$$\begin{aligned} \mathrm{V}_1 &= \cancel{\mathrm{AV}_2}^{\,0} - \mathrm{BI}_2 \\ \mathrm{I}_1 &= \cancel{\mathrm{CV}_2}^{\,0} - \mathrm{DI}_2 \end{aligned} \Rightarrow \begin{aligned} V_1 &= -BI_2 \\ I_1 &= -DI_2 \end{aligned} \Rightarrow \begin{aligned} B &= -\frac{V_1}{I_2} \\ D &= -\frac{I_1}{I_2} \end{aligned}$$

Example 19.8

Determine the T-parameters of the following circuit.



$$V_1 = AV_2 - BI_2$$
$$I_1 = CV_2 - DI_2$$

Apply KVL

$$V_1 = 10I_1 + 20(I_1 + I_2)$$
$$V_2 = -3I_1 + 20(I_1 + I_2)$$

$$V_1 = \frac{30}{17}V_2 - \frac{260}{17}I_2$$
$$I_1 = \frac{1}{17}V_2 - \frac{20}{17}I_2$$

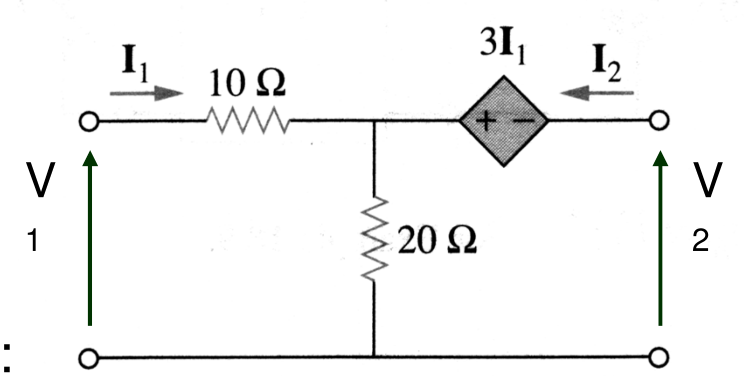
Answer: $T = \begin{bmatrix} 1.765 & 15.294\Omega \\ 0.059\text{S} & 1.176 \end{bmatrix}$

19.5 Transmission Parameters (5)

Example 19.8

From KVL:

$$V_1 = 10I_1 + 20(I_1 + I_2) = 30I_1 + 20I_2$$
$$V_2 = -3I_1 + 20(I_1 + I_2) = 17I_1 + 20I_2$$



If we "open circuit" the output we get:

$$V_1 = 30I_1 + 20\cancelto{0}{I_2} \qquad V_1 = 30I_1$$
$$V_2 = 17I_1 + 20\cancelto{0}{I_2} \qquad V_2 = 17I_1$$

$$A = \frac{V_1}{V_2} = \frac{30I_1}{17I_1} = \frac{30}{17} = 1.765$$
$$C = \tfrac{1}{17} = 0.0588$$

If we "short circuit" the output we get:

$$V_1 = 30I_1 + 20I_2 \qquad V_1 = 30I_1 + 20I_2$$
$$\cancelto{0}{V_2} = 17I_1 + 20I_2 \qquad 0 = 17I_1 + 20I_2$$

$$V_1 = 30\left(\tfrac{-20}{17}\right)I_2 + 20I_2$$
$$I_1 = \tfrac{-20}{17} I_2$$

$$B = -\frac{V_1}{I_2} = -\frac{\left(30\left(\frac{-20}{17}\right) + 20\right)I_2}{I_2} = 15.29$$
$$D = -\frac{I_1}{I_2} = \frac{20}{17} = 1.176$$

19.5 Transmission Parameters (6)

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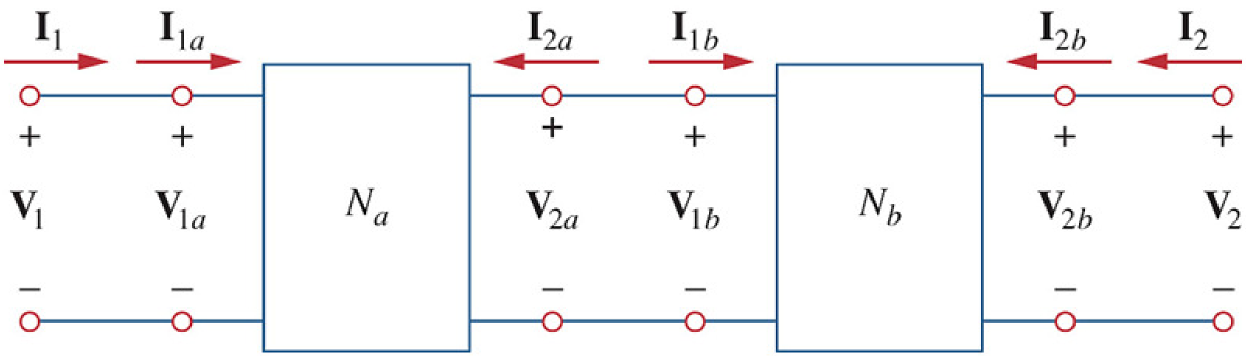
- Transmission Parameters can be cascaded with the result found through simple matrix multiplication



$$\begin{bmatrix} V_{1a} \\ I_{1a} \end{bmatrix} = \begin{bmatrix} A_a & B_a \\ C_a & D_a \end{bmatrix} \begin{bmatrix} V_{2a} \\ -I_{2a} \end{bmatrix} \qquad \begin{bmatrix} V_{1b} \\ I_{1b} \end{bmatrix} = \begin{bmatrix} A_b & B_b \\ C_b & D_b \end{bmatrix} \begin{bmatrix} V_{2b} \\ -I_{2b} \end{bmatrix}$$

$$-I_{2a} = I_{1b}$$

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A_a & B_a \\ C_a & D_a \end{bmatrix} \begin{bmatrix} A_b & B_b \\ C_b & D_b \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$

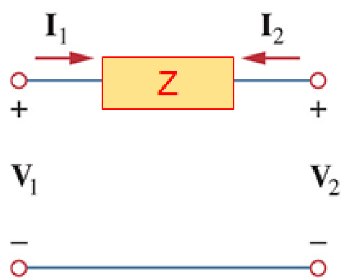
$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A' & B' \\ C' & D' \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$

19.5 Transmission Parameters (7)

Properties: Building Block Circuits

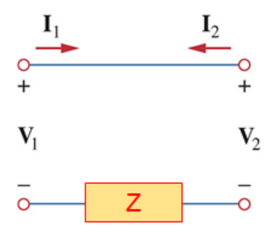
Consider the following simple circuits

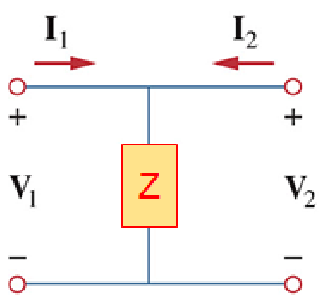
We can find their T Parameters to be:



$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} 1 & Z \\ 0 & 1 \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$

Note, following is equivalent





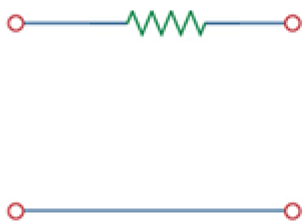
$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ \frac{1}{Z} & 1 \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$

19.5 Transmission Parameters (8)

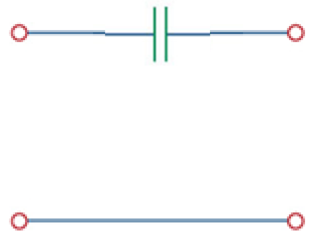
Properties: Building Block Circuits

- We can use this to construct the following “building block T parameters” to find the T parameters for any ladder type circuit.

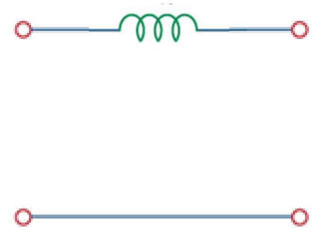
$$\begin{bmatrix} 1 & R \\ 0 & 1 \end{bmatrix}$$



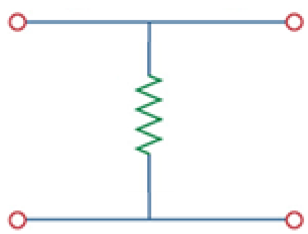
$$\begin{bmatrix} 1 & \frac{1}{sC} \\ 0 & 1 \end{bmatrix}$$



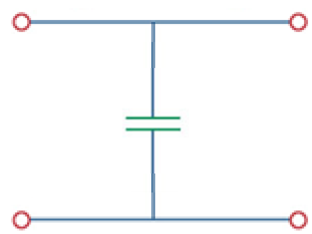
$$\begin{bmatrix} 1 & sL \\ 0 & 1 \end{bmatrix}$$



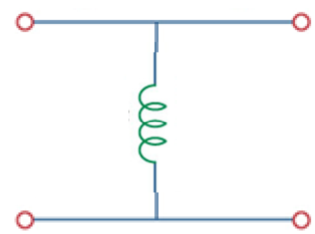
$$\begin{bmatrix} 1 & 0 \\ \frac{1}{R} & 1 \end{bmatrix}$$



$$\begin{bmatrix} 1 & 0 \\ sC & 1 \end{bmatrix}$$



$$\begin{bmatrix} 1 & 0 \\ \frac{1}{sL} & 1 \end{bmatrix}$$



19.5 Transmission Parameters (9)

Properties: Transfer function / Thevenin Equivalent

- The “A” parameter can be used to provide the inverse of the voltage Transfer Function H(s).

$$A = \left.\frac{V_1}{V_2}\right|_{I_2=0} = \frac{1}{H(s)}$$

- Parameters “A” and “B” can be used to find a relationship between the Open Circuit Voltage (V_2) and the Short Circuit Current ($-I_2$).
- We can us this to find the parameters for the Thevenin Equivalent Circuit.

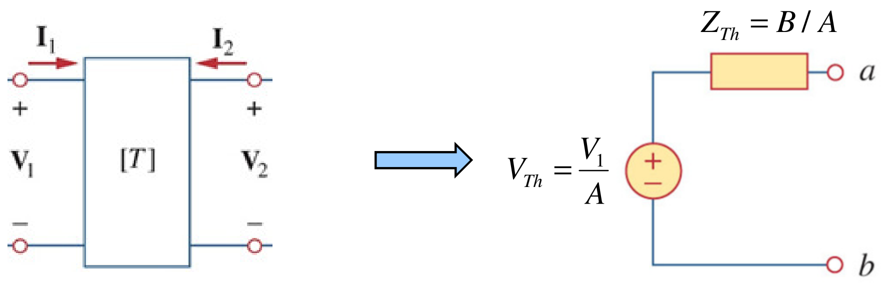
$$A = \left.\frac{V_1}{V_2}\right|_{I_2=0} = \frac{V_1}{V_{oc}}$$

$$V_{Th} = V_{oc} = \frac{V_1}{A}$$

$$B = -\left.\frac{V_1}{I_2}\right|_{V_2=0} = \frac{V_1}{I_{sc}}$$

$$I_N = I_{sc} = \frac{V_1}{B}$$

$$Z_{Th} = \frac{V_{oc}}{I_{sc}} = \frac{B}{A}$$

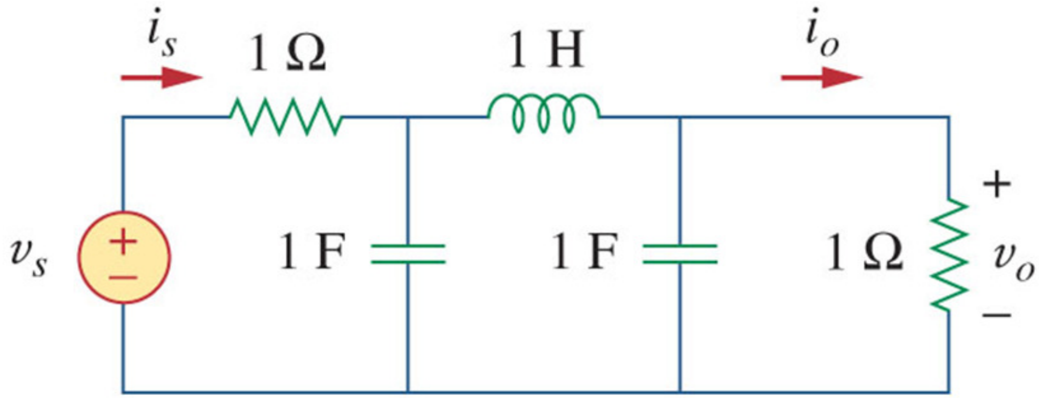


19.5 Transmission Parameters (10)

Transfer Function - Example

Problem 16.80(a)

Find the transfer function $V_o(s)/V_s(s)$ for the following circuit



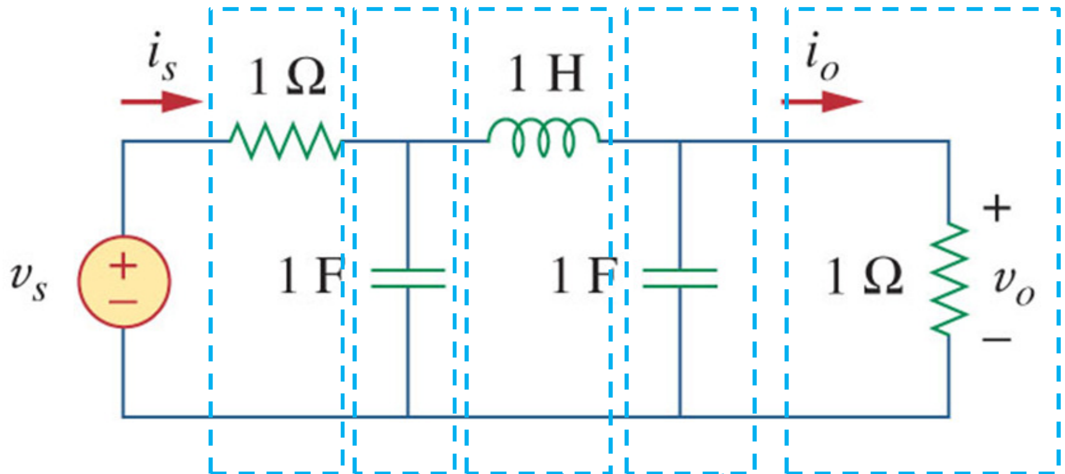
Answer: $$H(s) = \frac{1}{s^3 + 2s^2 + 3s + 2}$$

19.5 Transmission Parameters (11)

Transfer Function - Example

Problem 16.80(a) Solution:

a) Break up the circuit into a series of cascaded series and shunt components
b) Find the composite “T” parameters for the circuit
c) Use the relationship between the parameter “A” and the Transfer function



$$\underbrace{\begin{bmatrix} V_s \\ I_s \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ s & 1 \end{bmatrix} \begin{bmatrix} 1 & s \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ s & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}}_{\begin{bmatrix} A & B \\ C & D \end{bmatrix}} \begin{bmatrix} V_o \\ I_o \end{bmatrix}$$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} \rightarrow \boxed{A = \left.\frac{V_1}{V_2}\right|_{I_2=0} = \frac{1}{H(s)}}$$

19.5 Transmission Parameters (12)

Transfer Function - Example

Finding the combined T-matrix

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ s & 1 \end{bmatrix}\begin{bmatrix} 1 & s \\ 0 & 1 \end{bmatrix}\underbrace{\begin{bmatrix} 1 & 0 \\ s & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}}$$

$$\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ s & 1 \end{bmatrix}\underbrace{\begin{bmatrix} 1 & s \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ (s+1) & 1 \end{bmatrix}}$$

$$\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}\underbrace{\begin{bmatrix} 1 & 0 \\ s & 1 \end{bmatrix}\begin{bmatrix} 1+s(s+1) & s \\ (s+1) & 1 \end{bmatrix}}$$

$$\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1+s(s+1) & s \\ s+s^2(s+1)+(s+1) & s^2 \end{bmatrix}$$

$$\underbrace{\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} s^2+s+1 & s \\ s^3+s^2+2s+1 & s^2 \end{bmatrix}}$$

$$\begin{bmatrix} s^3+2s^2+3s+2 & s+s^2 \\ s^3+s^2+2s+1 & s^2 \end{bmatrix}$$

The transfer function can be found directly from the Transmission Parameter "A" !

$$A = \left.\frac{V_1}{V_2}\right|_{I_2=0} = \frac{1}{H(s)}$$

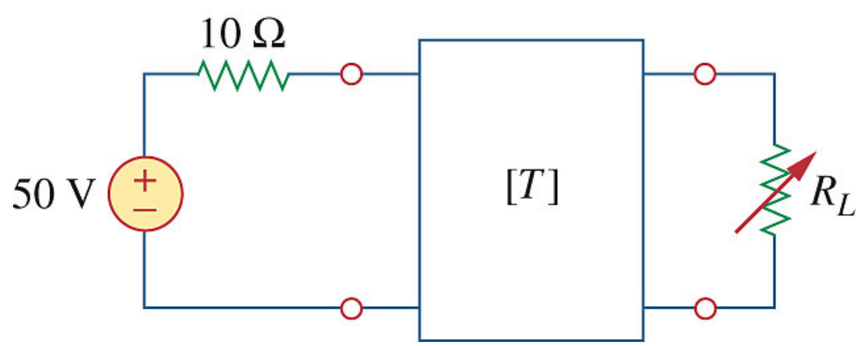
$$H(s) = \frac{1}{s^3+2s^2+3s+2}$$

19.5 Transmission Parameters (13)

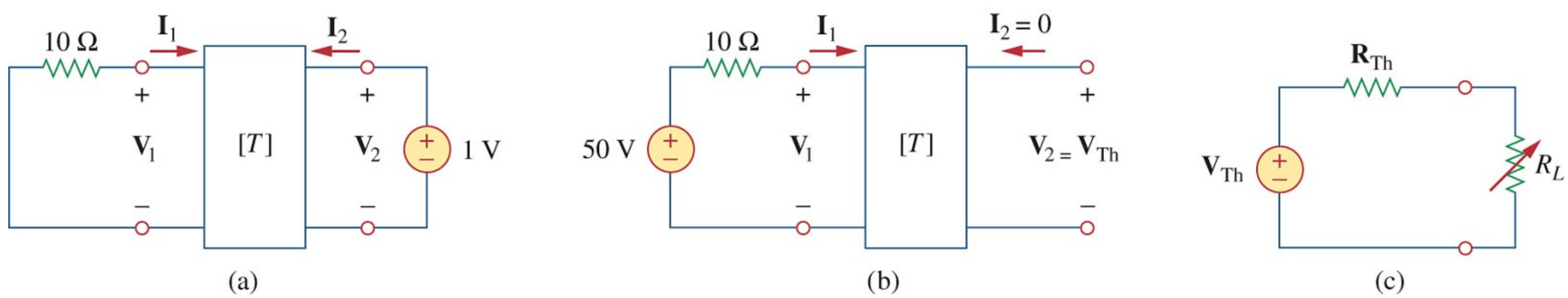
Example 19.9

The ABCD parameters of the two-port network at right are

$$T = \begin{bmatrix} 4 & 20\ \Omega \\ 0.1S & 2 \end{bmatrix}$$



The output port is connected to a variable load for maximum power transfer. Find R_L and the maximum power transferred.

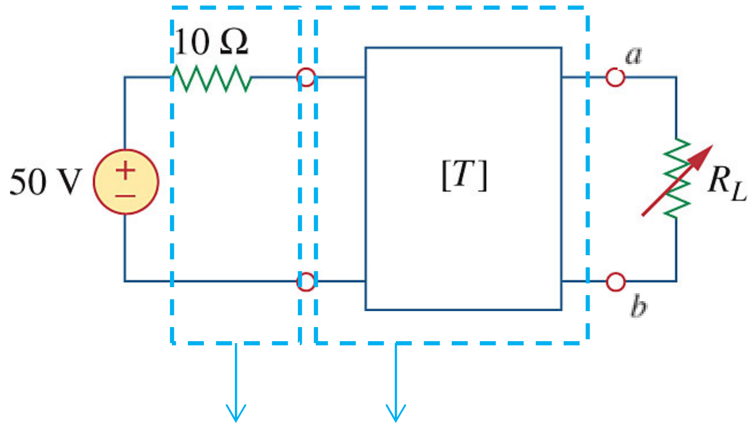


Answer: V_{TH} = 10V V; R_L = 8Ω; Pm = 3.125W.

19.5 Transmission Parameters (14)

Solution: Example 19.9

a) Cascade the Series Resistor with the network
b) Find the composite "T" parameters for the circuit
c) Use the relationships to find V_{Th} and Z_{Th}

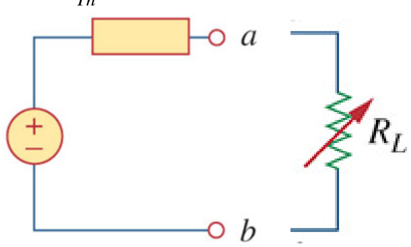


$$[T'] = \begin{bmatrix} 1 & 10 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 4 & 20 \\ 0.1 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 40 \\ 0.1 & 2 \end{bmatrix}$$

Find the Thevenin Equivalent Circuit for the source

$Z_{Th} = 40/5 = 8$

$V_{Th} = \frac{50}{5} = 10$



For Max Power Transfer

$$\boxed{R_L = Z_{Th} = 8\ \Omega}$$

$$P_{max} = I^2 R_L$$

$$P_{max} = \left(\frac{V_{Th}}{R_L + Z_{Th}}\right)^2 R_L$$

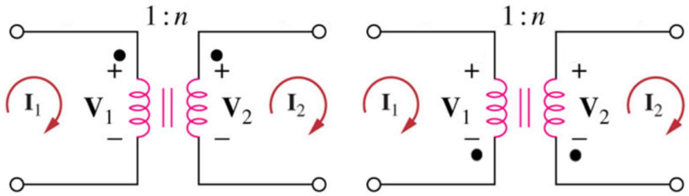
$$\boxed{P_{max} = \left(\frac{10}{16}\right)^2 8 = 3.125\ \text{W}}$$

19.5 Transmission Parameters (15)

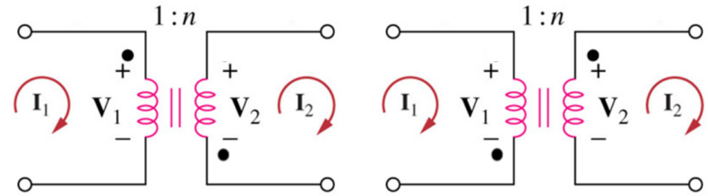
Properties: Building Block Circuits – Ideal Transformer

- We can also use these “building blocks” to model ideal transformers. Remember from Chapter 13

Dots “Same” → “+”



Dots “Different” → “-”



$$V_2 = nV_1$$
$$I_2 = \frac{I_1}{n}$$

$$\Downarrow$$

$$\begin{bmatrix} \frac{1}{n} & 0 \\ 0 & n \end{bmatrix}$$

T - parameters

$$V_1 = AV_2 - BI_2$$
$$I_1 = CV_2 - DI_2$$

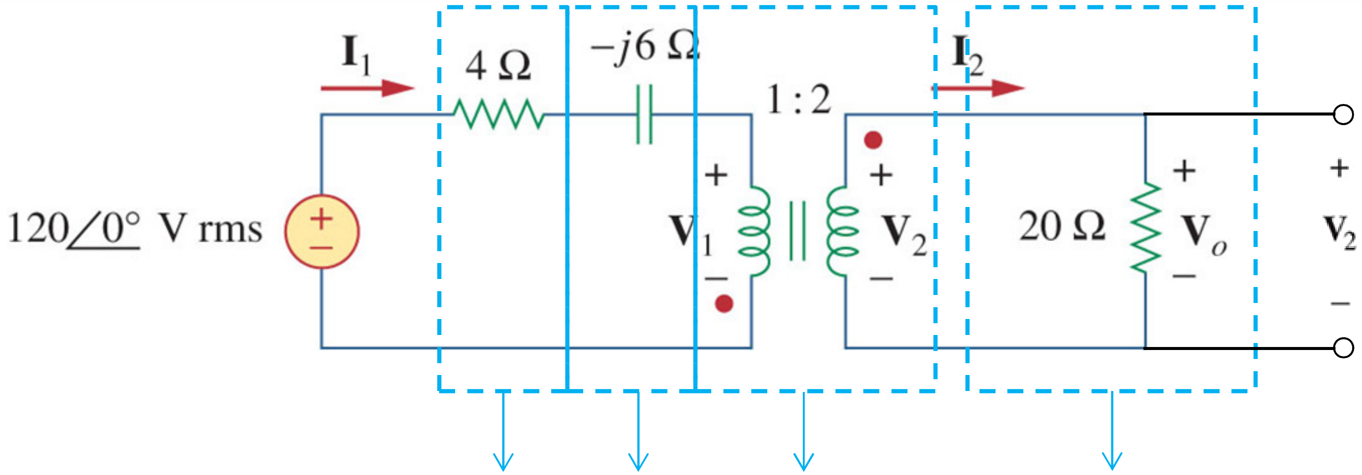
$$V_2 = -nV_1$$
$$I_2 = -\frac{I_1}{n}$$

$$\Downarrow$$

$$\begin{bmatrix} -\frac{1}{n} & 0 \\ 0 & -n \end{bmatrix}$$

19.5 Transmission Parameters (16)

Example 13.8 Revisited



Cascaded T parameters

$$\underbrace{\begin{bmatrix} 1 & 4 \\ 0 & 1 \end{bmatrix}}_{a} \underbrace{\begin{bmatrix} 1 & -j6 \\ 0 & 1 \end{bmatrix}}_{b} \underbrace{\begin{bmatrix} -\frac{1}{2} & 0 \\ 0 & -2 \end{bmatrix}}_{c} \underbrace{\begin{bmatrix} 1 & 0 \\ 0.05 & 1 \end{bmatrix}}_{d}$$

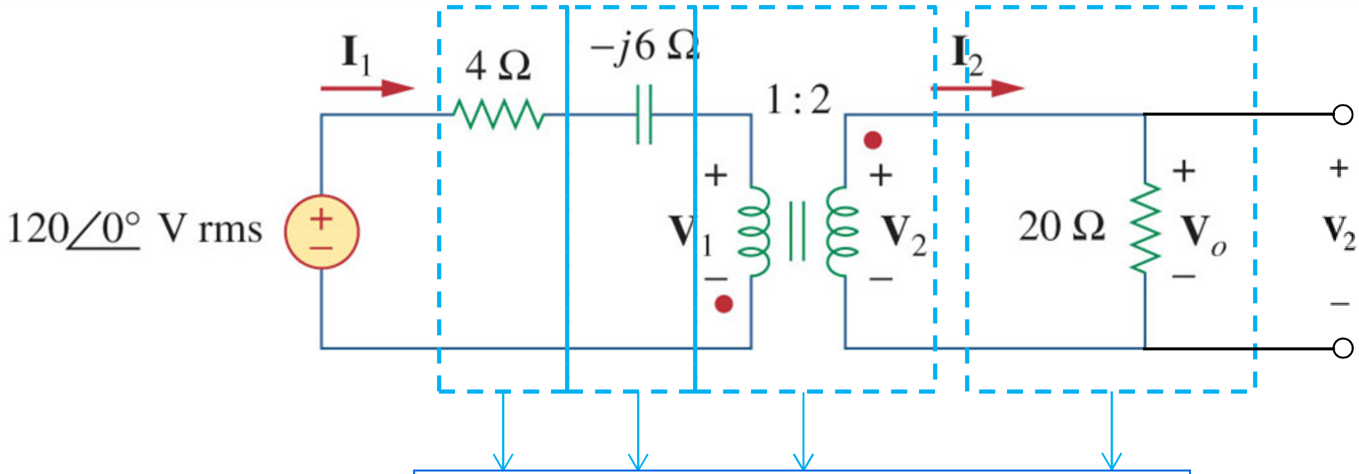
Using MATLAB:

```
>> T=a*b*c*d

T =

  -0.9000 + 0.6000i  -8.0000 +12.0000i
  -0.1000 + 0.0000i  -2.0000 + 0.0000i
```

$$V_0 = V_2 = \frac{1}{A}V_1 = \left(\frac{1}{\boxed{-0.9+0.6j}}\right)(120\angle 0°) = \boxed{110.94\angle -146.31° \text{V}}$$ ✓

19.5 Transmission Parameters (17)

Example 13.8 Revisited

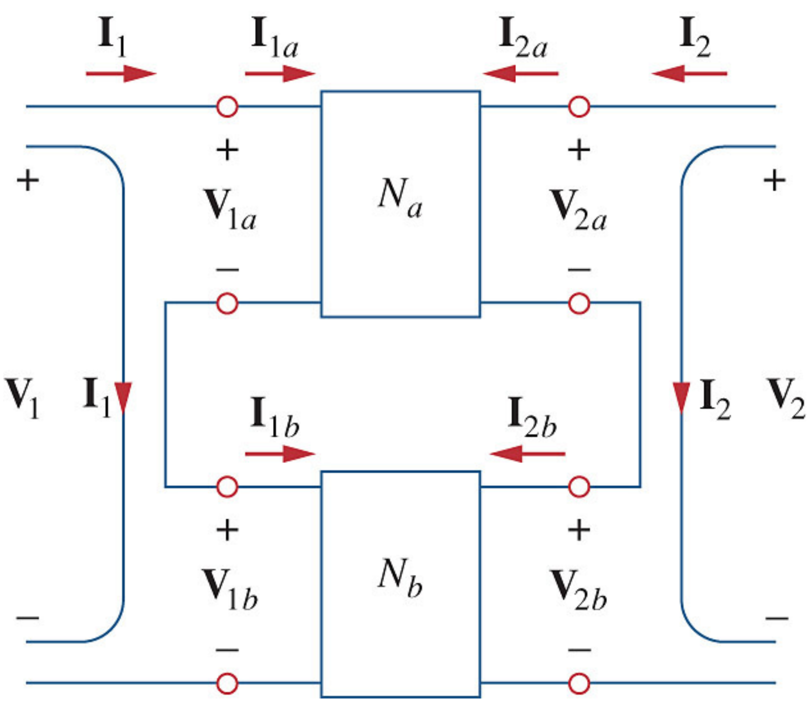



```
>> T=a*b*c*d

T =

  -0.9000 + 0.6000i  -8.0000 +12.0000i
  -0.1000 + 0.0000i  -2.0000 + 0.0000i
```

$$I_1 = CV_2 = \boxed{(-0.1)}(110.94\angle -146.31°) = \boxed{11.09\angle 33.69°\ \text{A}}$$

$$I_2 = \frac{V_2}{20} = \frac{(110.94\angle -146.31°)}{20} = \boxed{5.55\angle -146.31°\ \text{A}}$$

19.7 Interconnection of Networks (1)

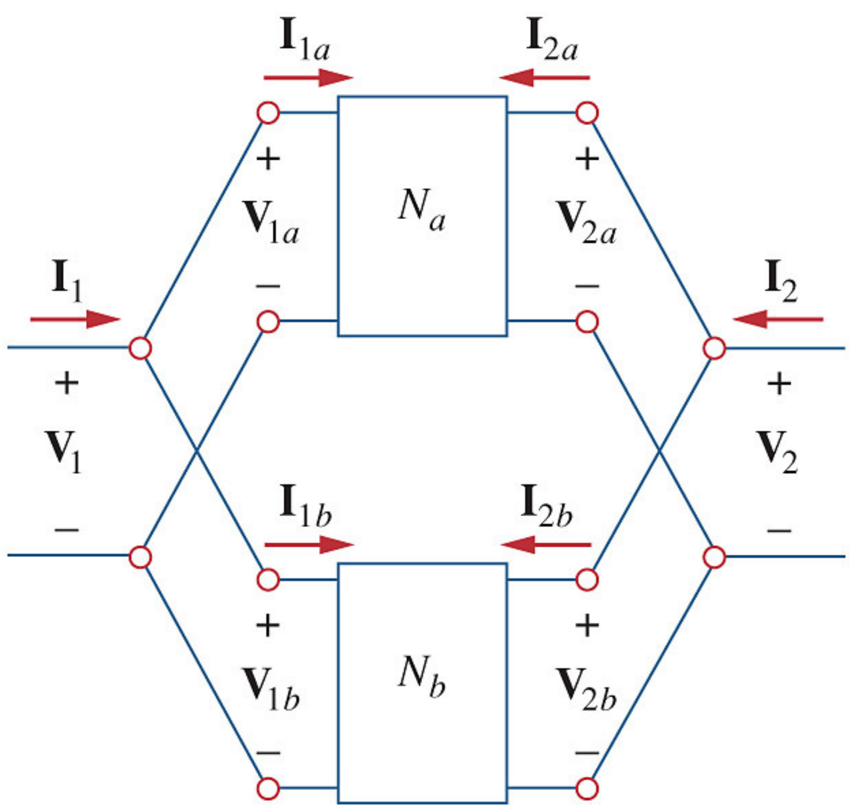


Series Connection of two-port networks:

For Impedances; ADD matrices.

$Z = Z_a + Z_b$

19.7 Interconnection of Networks (2)



Parallel Connection of two-port networks:

For Admittances; ADD matrices.

$Y = Y_a + Y_b$

19.6 Relationships Between Networks

- Use this table to convert between two port parameters

	z		y		h		T	
z	z_{11}	z_{12}	$\frac{y_{22}}{\Delta_y}$	$-\frac{y_{12}}{\Delta_y}$	$\frac{\Delta_h}{h_{22}}$	$\frac{h_{12}}{h_{22}}$	$\frac{A}{C}$	$\frac{\Delta_T}{C}$
	z_{21}	z_{22}	$-\frac{y_{21}}{\Delta_y}$	$\frac{y_{11}}{\Delta_y}$	$-\frac{h_{21}}{h_{22}}$	$\frac{1}{h_{22}}$	$\frac{1}{C}$	$\frac{D}{C}$
y	$\frac{z_{22}}{\Delta_z}$	$-\frac{z_{12}}{\Delta_z}$	y_{11}	y_{12}	$\frac{1}{h_{11}}$	$-\frac{h_{12}}{h_{11}}$	$\frac{D}{B}$	$-\frac{\Delta_T}{B}$
	$-\frac{z_{21}}{\Delta_z}$	$\frac{z_{11}}{\Delta_z}$	y_{21}	y_{22}	$\frac{h_{21}}{h_{11}}$	$\frac{\Delta_h}{h_{11}}$	$-\frac{1}{B}$	$\frac{A}{B}$
h	$\frac{\Delta_z}{z_{22}}$	$\frac{z_{12}}{z_{22}}$	$\frac{1}{y_{11}}$	$-\frac{y_{12}}{y_{11}}$	h_{11}	h_{12}	$\frac{B}{D}$	$\frac{\Delta_T}{D}$
	$-\frac{z_{21}}{z_{22}}$	$\frac{1}{z_{22}}$	$\frac{y_{21}}{y_{11}}$	$\frac{\Delta_y}{y_{11}}$	h_{21}	h_{22}	$-\frac{1}{D}$	$\frac{C}{D}$
T	$\frac{z_{11}}{z_{21}}$	$\frac{\Delta_z}{z_{21}}$	$-\frac{y_{22}}{y_{21}}$	$-\frac{1}{y_{21}}$	$-\frac{\Delta_h}{h_{21}}$	$-\frac{h_{11}}{h_{21}}$	A	B
	$\frac{1}{z_{21}}$	$\frac{z_{22}}{z_{21}}$	$-\frac{\Delta_y}{y_{21}}$	$-\frac{y_{11}}{y_{21}}$	$-\frac{h_{22}}{h_{21}}$	$-\frac{1}{h_{21}}$	C	D

$\Delta_y = y_{11}y_{22} - y_{12}y_{21}$ $\quad \Delta_z = z_{11}z_{22} - z_{12}z_{21}$ $\quad \Delta_h = h_{11}h_{22} - h_{12}h_{21}$ $\quad \Delta_T = AD - BC$

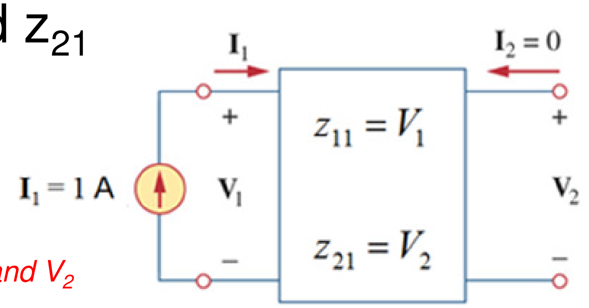
Chapter 19 Review

Z-Parameters

- Parameters: $\boxed{\begin{aligned} V_1 &= z_{11}I_1 + z_{12}I_2 \\ V_2 &= z_{21}I_1 + z_{22}I_2 \end{aligned}}$

- Open circuit the **output** to find z_{11} and z_{21}

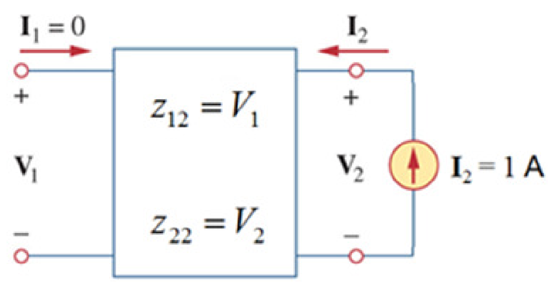
$$\begin{aligned} V_1 &= z_{11}I_1 + \cancel{z_{12}I_2}^{\,0} \\ V_2 &= z_{21}I_1 + \cancel{z_{22}I_2}^{\,0} \end{aligned} \quad \Rightarrow \quad \begin{aligned} V_1 &= z_{11}I_1 \\ V_2 &= z_{21}I_1 \end{aligned}$$



Set I_1 = 1 then solve for V_1 and V_2

- Open circuit the **input** to find z_{21} and z_{22}

$$\begin{aligned} V_1 &= \cancel{z_{11}I_1}^{\,0} + z_{12}I_2 \\ V_2 &= \cancel{z_{21}I_1}^{\,0} + z_{22}I_2 \end{aligned} \quad \Rightarrow \quad \begin{aligned} V_1 &= z_{12}I_2 \\ V_2 &= z_{22}I_2 \end{aligned}$$

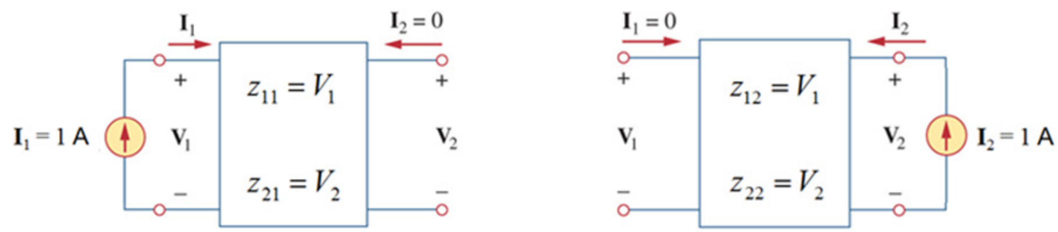


Set I_2 = 1 then solve for V_1 and V_2

Chapter 19 Review

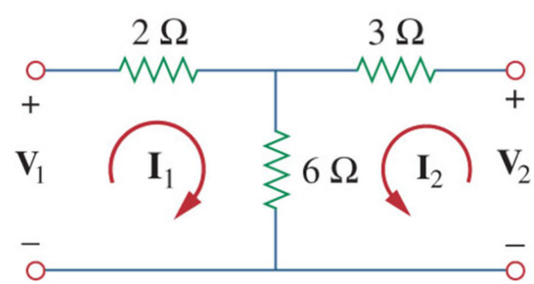
Z-Parameters (Given a circuit, find Z-parameters)

- Solving problems to find z-parameters:
  1. Refer to definition, apply 1 amp source at input and output with opposite port left open (see previous slide)



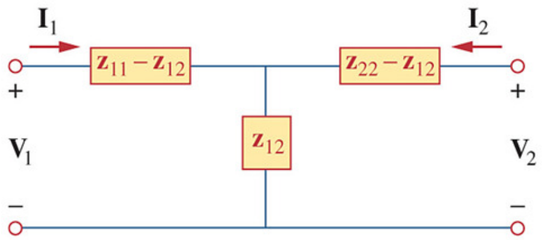
  2. Sometimes, KVL (mesh current equations) will cause z-parameters to fall right out! :

$$V_1 = 2I_1 + 6(I_1 + I_2) = 8I_1 + 6I_2$$
$$V_2 = 6(I_1 + I_2) + 3I_2 = 6I_1 + 9I_2$$
$$\Downarrow$$
$$z = \begin{bmatrix} 8 & 6 \\ 6 & 9 \end{bmatrix} \Omega$$

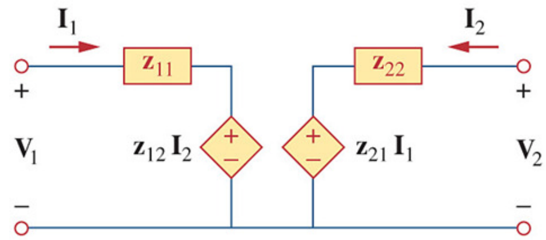


This mesh defined in counter clockwise direction for convenience

- If given, z-parameters can use following techniques to find other circuit parameters (V_1, V_2, I_1, I_2, etc.):
  1. Apply the model and solve the circuit:



Reciprocal Network



General Network

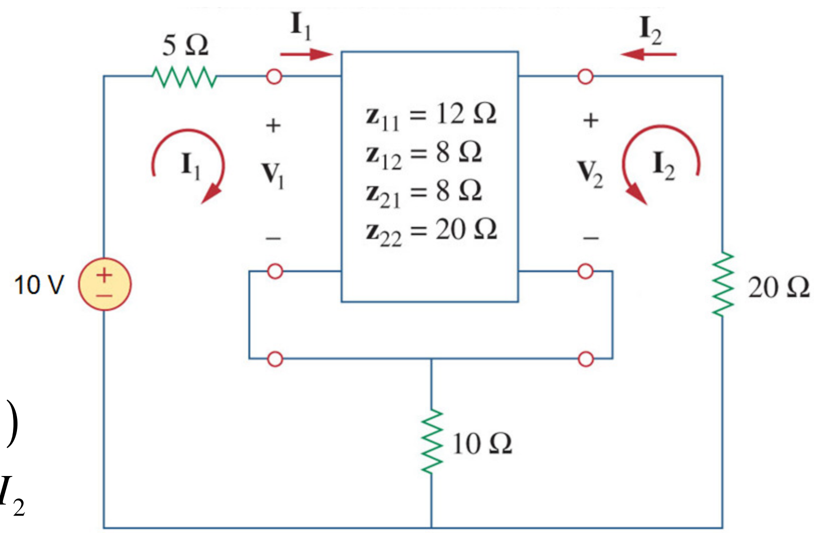
  2. Substitute the defining equations into your analysis:

Mesh Analysis

$$10 = 5I_1 + V_1 + 10(I_1 + I_2)$$
$$0 = V_2 + 10(I_1 + I_2) + 20I_2$$

Substitute for V_1 and V_2

$$10 = 5I_1 + (12I_1 + 8I_2) + 10(I_1 + I_2)$$
$$0 = (8I_1 + 20I_2) + 10(I_1 + I_2) + 20I_2$$



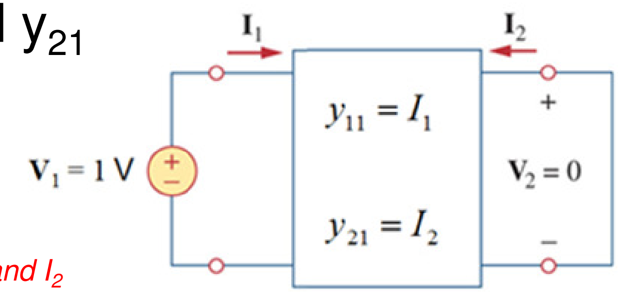
Chapter 19 Review

Y-Parameters

- Parameters: $$\boxed{\begin{aligned} I_1 &= y_{11}V_1 + y_{12}V_2 \\ I_2 &= y_{21}V_1 + y_{22}V_2 \end{aligned}}$$

- Short circuit the **output** to find y_{11} and y_{21}

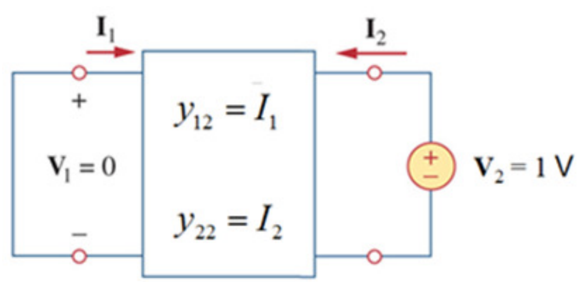
$$I_1 = y_{11}V_1 + \cancelto{0}{y_{12}V_2} \qquad I_2 = y_{21}V_1 + \cancelto{0}{y_{22}V_2} \quad \Rightarrow \quad I_1 = y_{11}V_1 \qquad I_2 = y_{21}V_1$$



Set $V_1 = 1$ then solve for I_1 and I_2

- Short circuit the **input** to find y_{21} and y_{22}

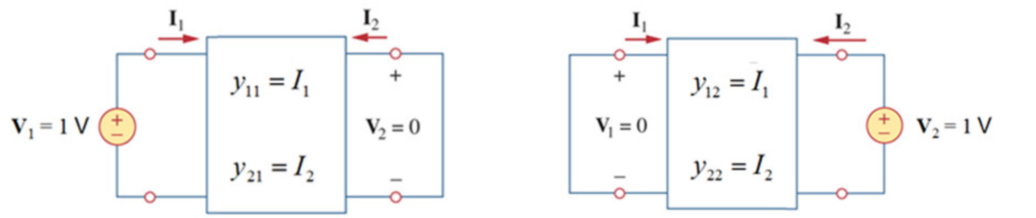
$$I_1 = \cancelto{0}{y_{11}V_1} + y_{12}V_2 \qquad I_2 = \cancelto{0}{y_{21}V_1} + y_{22}V_2 \quad \Rightarrow \quad I_1 = y_{12}V_2 \qquad I_2 = y_{22}V_2$$



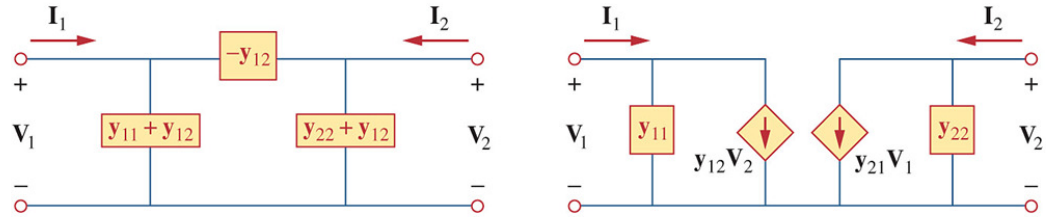
Set $V_2 = 1$ then solve for I_1 and I_2

Chapter 19 Review
Y-Parameters (Solving Problems)

- To solve Y-parameter problems, can use these techniques
  1. Apply method from previous slide. Apply 1 Volt source at input and output while shorting opposite port



  2. If given Y parameters can apply the model and solve the circuit:



Reciprocal Network

General Network

  3. Make it easy on yourself! Use conversions from Z → Y or Y → Z

$$\begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix} = \left(\frac{1}{\Delta_y}\right)\begin{bmatrix} y_{22} & -y_{12} \\ -y_{21} & y_{11} \end{bmatrix}$$

$$\Delta_y = y_{11}y_{22} - y_{12}y_{21}$$

$$\begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} = \left(\frac{1}{\Delta_z}\right)\begin{bmatrix} z_{22} & -z_{12} \\ -z_{21} & z_{11} \end{bmatrix}$$

$$\Delta_z = z_{11}z_{22} - z_{12}z_{21}$$

Chapter 19 Review

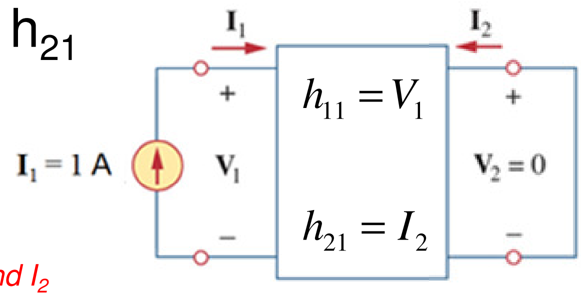
H-Parameters

- Parameters (hybrid of z and y): $\boxed{\begin{aligned} V_1 &= h_{11}I_1 + h_{12}V_2 \\ I_2 &= h_{21}I_1 + h_{22}V_2 \end{aligned}}$

- Short circuit the **output** to find h_{11} and h_{21}

$$\begin{aligned} V_1 &= h_{11}I_1 + \cancelto{0}{h_{12}V_2} \\ I_2 &= h_{21}I_1 + \cancelto{0}{h_{22}V_2} \end{aligned} \quad \Rightarrow \quad \begin{aligned} V_1 &= h_{11}I_1 \\ I_2 &= h_{21}I_1 \end{aligned}$$

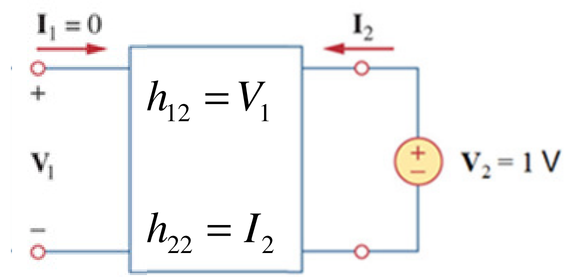
Set $I_1 = 1$ then solve for V_1 and I_2



- Open circuit the **input** to find h_{21} and h_{22}

$$\begin{aligned} V_1 &= \cancelto{0}{h_{11}I_1} + h_{12}V_2 \\ I_2 &= \cancelto{0}{h_{21}I_1} + h_{22}V_2 \end{aligned} \quad \Rightarrow \quad \begin{aligned} V_1 &= h_{12}V_2 \\ I_2 &= h_{22}V_2 \end{aligned}$$

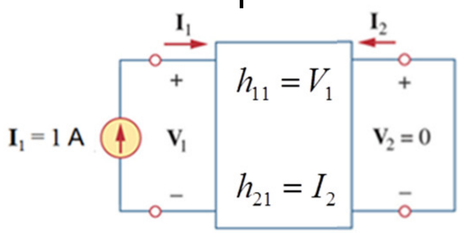
Set $V_2 = 1$ then solve for V_1 and I_2

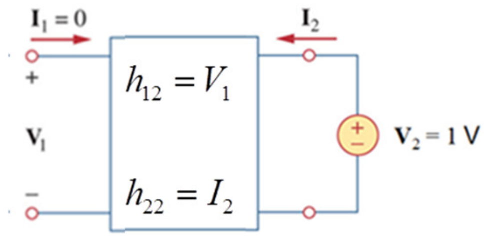


Chapter 19 Review

H-Parameters (Solving Problems)

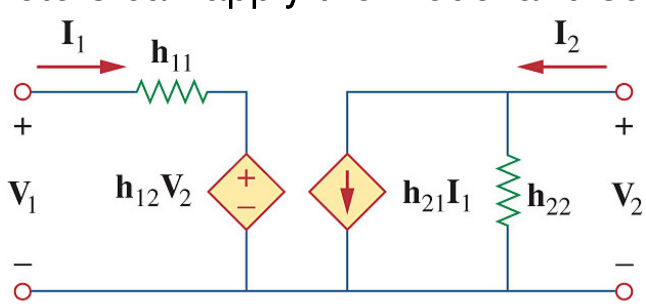
- To solve H-parameter problems, can use these techniques
  1. Apply methods from previous slide.





  2. H parameters can be found by performing a set of tests on the device
     a) Shorting the output and applying a current
     b) Leaving the input open and applying a voltage across the output

  3. If given H parameters can apply the model and solve the circuit:

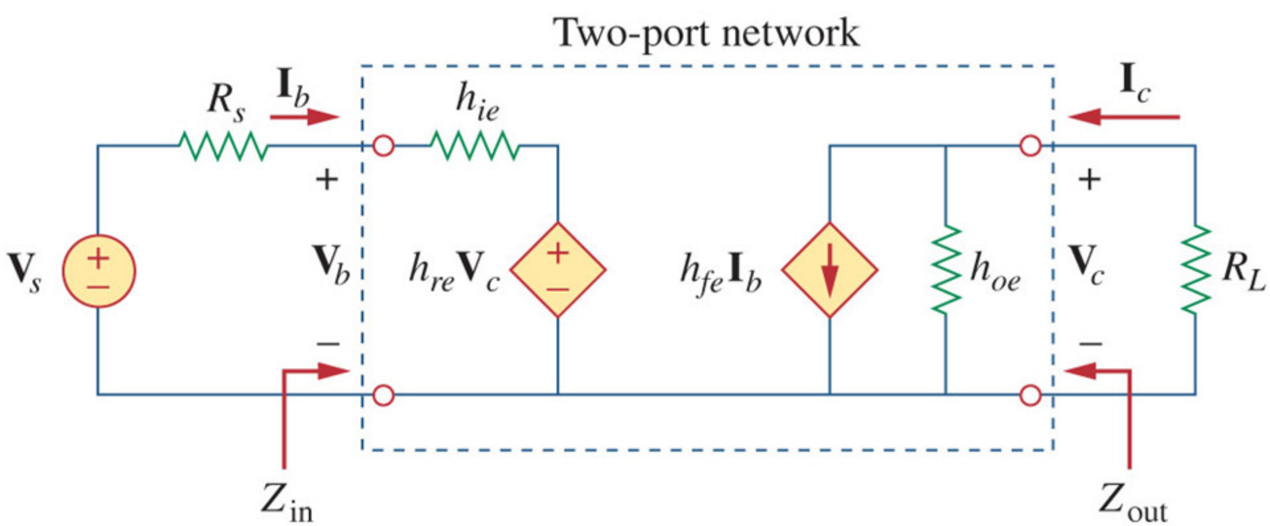


  4. If helpful, use conversion tables

Chapter 19 Review

H-Parameters (Transistor Model)

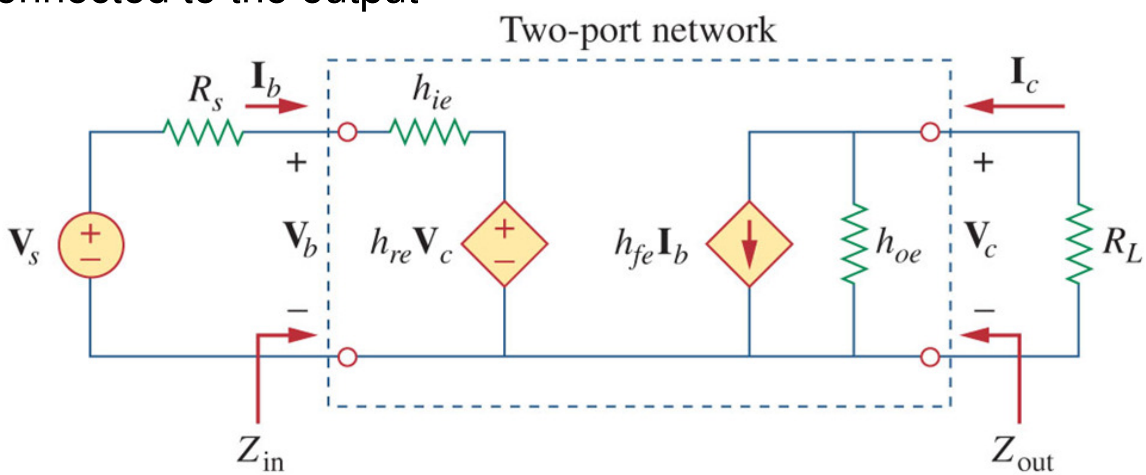
- H parameters are often used in modeling transistors
- Parameters vary depending on biasing conditions
- Spec sheets often use different subscripts:
  - h_{11} → h_{ie} = Base input impedance
  - h_{12} → h_{re} = Reverse voltage feedback ration
  - h_{21} → h_{fe} = Base-collector current gain
  - h_{22} → h_{oe} = Output admittance



Chapter 19 Review

H-Parameters (Transistor Model)

- Equations for calculating input impedance, output impedance, voltage gain, and current gain for simple transistor circuit:
  - V_s and R_s can be the Thevenin equivalent source driving the input.
  - R_L can be the input impedance looking into the load of the circuit connected to the output



Input Impedance

$$Z_{in} = \frac{V_b}{I_b} = h_{ie} - \frac{h_{re}h_{fe}R_L}{1 + h_{oe}R_L}$$

Output Impedance

$$Z_{out} = \left.\frac{V_c}{I_c}\right|_{V_s=0} = \frac{R_s + h_{ie}}{(R_s + h_{ie})h_{oe} - h_{re}h_{fe}}$$

Current Gain

$$A_i = \frac{I_c}{I_b} = \frac{h_{fe}}{1 + h_{oe}R_L}$$

Voltage Gain

$$A_v = \frac{V_c}{V_b} = \frac{-h_{fe}R_L}{h_{ie} + (h_{ie}h_{oe} - h_{re}h_{fe})R_L}$$

Chapter 19 Review

Transmission ("T") Parameters

- Parameters: $$\boxed{\begin{aligned} V_1 &= AV_2 - BI_2 \\ I_1 &= CV_2 - DI_2 \end{aligned}}$$

- Perform the analysis with the **output** Open Circuited (I_2=0)

$$\begin{aligned} V_1 &= AV_2 - B\cancel{I_2}^{0} \\ I_1 &= CV_2 - D\cancel{I_2}^{0} \end{aligned} \Rightarrow \begin{aligned} V_1 &= AV_2 \\ I_1 &= CV_2 \end{aligned} \Rightarrow \begin{aligned} A &= \frac{V_1}{V_2} \\ C &= \frac{I_1}{V_2} \end{aligned}$$

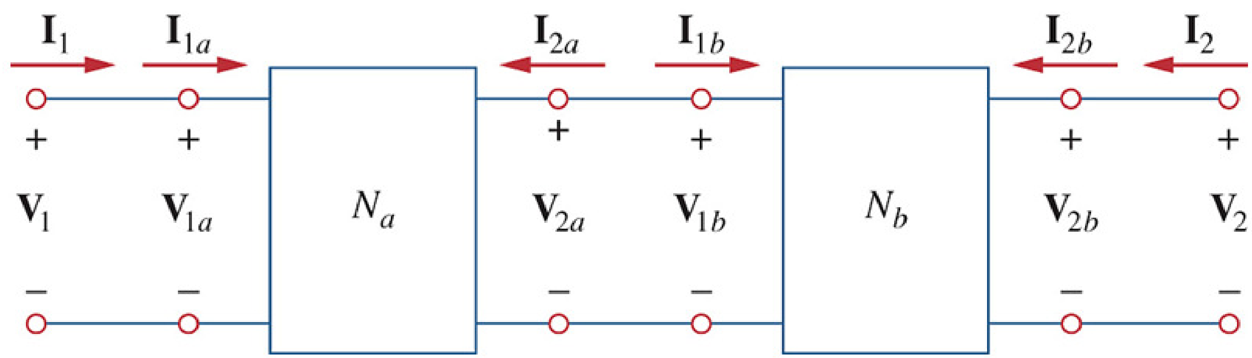
- Perform the analysis with the **output** Short Circuited(V_2=0)

$$\begin{aligned} V_1 &= A\cancel{V_2}^{0} - BI_2 \\ I_1 &= C\cancel{V_2}^{0} - DI_2 \end{aligned} \Rightarrow \begin{aligned} V_1 &= -BI_2 \\ I_1 &= -DI_2 \end{aligned} \Rightarrow \begin{aligned} B &= -\frac{V_1}{I_2} \\ D &= -\frac{I_1}{I_2} \end{aligned}$$

Chapter 19 Review

Transmission (“T”) Parameters (Cascading)

- Primary benefit of “T”-Parameters is their ability to be cascaded.



$$\begin{bmatrix} V_{1a} \\ I_{1a} \end{bmatrix} = \begin{bmatrix} A_a & B_a \\ C_a & D_a \end{bmatrix} \begin{bmatrix} V_{2a} \\ -I_{2a} \end{bmatrix} \qquad \begin{bmatrix} V_{1b} \\ I_{1b} \end{bmatrix} = \begin{bmatrix} A_b & B_b \\ C_b & D_b \end{bmatrix} \begin{bmatrix} V_{2b} \\ -I_{2b} \end{bmatrix}$$

$$\boxed{-I_{2a} = I_{1b}}$$

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A_a & B_a \\ C_a & D_a \end{bmatrix} \begin{bmatrix} A_b & B_b \\ C_b & D_b \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$

$$\boxed{\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A' & B' \\ C' & D' \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}}$$

Chapter 19 Review

T - Parameters (Building Block models)

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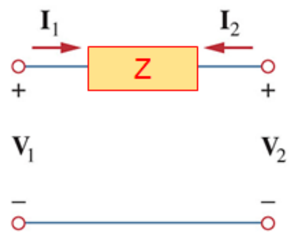
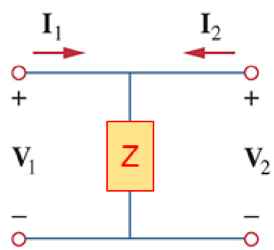
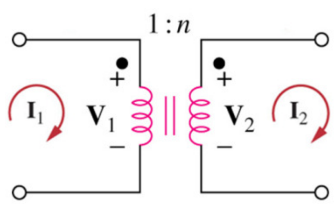
- We can create "building block" models of components by finding their T-parameters and use the cascading property to find the T-parameters for the complete circuit/system.



$$\begin{bmatrix} 1 & Z \\ 0 & 1 \end{bmatrix}$$



$$\begin{bmatrix} 1 & 0 \\ \frac{1}{Z} & 1 \end{bmatrix}$$



Dots "same"

$$\begin{bmatrix} \frac{1}{n} & 0 \\ 0 & n \end{bmatrix}$$

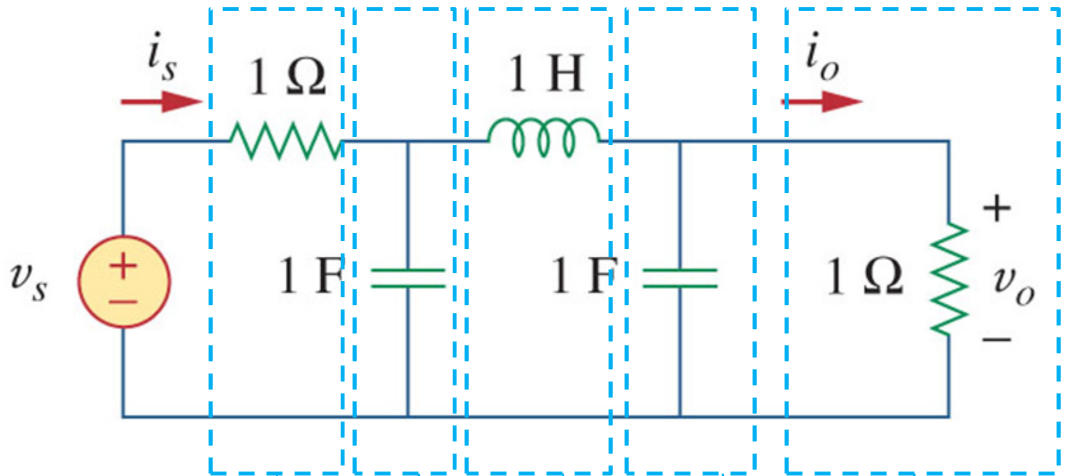
Dots "opposite"

$$\begin{bmatrix} -\frac{1}{n} & 0 \\ 0 & -n \end{bmatrix}$$

Chapter 19 Review

T - Parameters (Building Block models)

- With "Building Block" approach, circuits can be broke up into discrete components and analyzed using T-parameters



$$\begin{bmatrix} V_s \\ I_s \end{bmatrix} = \underbrace{\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ s & 1 \end{bmatrix}\begin{bmatrix} 1 & s \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ s & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}}_{\begin{bmatrix} A & B \\ C & D \end{bmatrix}}\begin{bmatrix} V_o \\ I_o \end{bmatrix}$$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} \longrightarrow \boxed{A = \left.\frac{V_1}{V_2}\right|_{I_2=0} = \frac{1}{H(s)}}$$

Chapter 19 Review

T - Parameters (Useful Properties)

- The T parameters give us useful properties in the analysis of circuits:
  - Open Circuit Voltage Transfer Function:

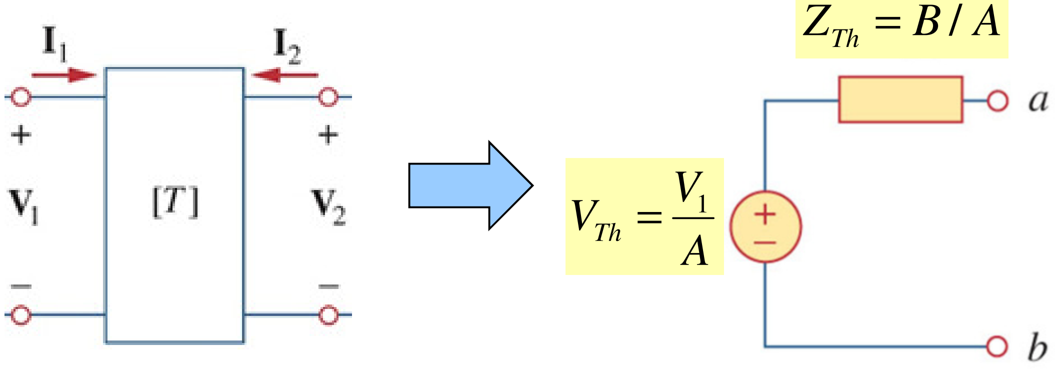
$$A = \left.\frac{V_1}{V_2}\right|_{I_2=0} = \frac{1}{H(s)}$$

$$H(s) = \frac{1}{A}$$

  - Thevenin Equivalent Circuit (Replace circuit as a source)

$$Z_{Th} = B / A$$

$$V_{Th} = \frac{V_1}{A}$$



Chapter 19 Review

Conversion between Parameters

- Conversion tables exists to convert between parameters

	z		y		h		T	
z	z_{11}	z_{12}	$\frac{y_{22}}{\Delta_y}$	$-\frac{y_{12}}{\Delta_y}$	$\frac{\Delta_h}{h_{22}}$	$\frac{h_{12}}{h_{22}}$	$\frac{A}{C}$	$\frac{\Delta_T}{C}$
	z_{21}	z_{22}	$-\frac{y_{21}}{\Delta_y}$	$\frac{y_{11}}{\Delta_y}$	$-\frac{h_{21}}{h_{22}}$	$\frac{1}{h_{22}}$	$\frac{1}{C}$	$\frac{D}{C}$
y	$\frac{z_{22}}{\Delta_z}$	$-\frac{z_{12}}{\Delta_z}$	y_{11}	y_{12}	$\frac{1}{h_{11}}$	$-\frac{h_{12}}{h_{11}}$	$\frac{D}{B}$	$-\frac{\Delta_T}{B}$
	$-\frac{z_{21}}{\Delta_z}$	$\frac{z_{11}}{\Delta_z}$	y_{21}	y_{22}	$\frac{h_{21}}{h_{11}}$	$\frac{\Delta_h}{h_{11}}$	$-\frac{1}{B}$	$\frac{A}{B}$
h	$\frac{\Delta_z}{z_{22}}$	$\frac{z_{12}}{z_{22}}$	$\frac{1}{y_{11}}$	$-\frac{y_{12}}{y_{11}}$	h_{11}	h_{12}	$\frac{B}{D}$	$\frac{\Delta_T}{D}$
	$-\frac{z_{21}}{z_{22}}$	$\frac{1}{z_{22}}$	$\frac{y_{21}}{y_{11}}$	$\frac{\Delta_y}{y_{11}}$	h_{21}	h_{22}	$-\frac{1}{D}$	$\frac{C}{D}$
T	$\frac{z_{11}}{z_{21}}$	$\frac{\Delta_z}{z_{21}}$	$-\frac{y_{22}}{y_{21}}$	$-\frac{1}{y_{21}}$	$-\frac{\Delta_h}{h_{21}}$	$-\frac{h_{11}}{h_{21}}$	A	B
	$\frac{1}{z_{21}}$	$\frac{z_{22}}{z_{21}}$	$-\frac{\Delta_y}{y_{21}}$	$-\frac{y_{11}}{y_{21}}$	$-\frac{h_{22}}{h_{21}}$	$-\frac{1}{h_{21}}$	C	D

$\Delta_y = y_{11}y_{22} - y_{12}y_{21}$ $\quad \Delta_z = z_{11}z_{22} - z_{12}z_{21}$ $\quad \Delta_h = h_{11}h_{22} - h_{12}h_{21}$ $\quad \Delta_T = AD - BC$

Chapter 19: Two-Port Networks

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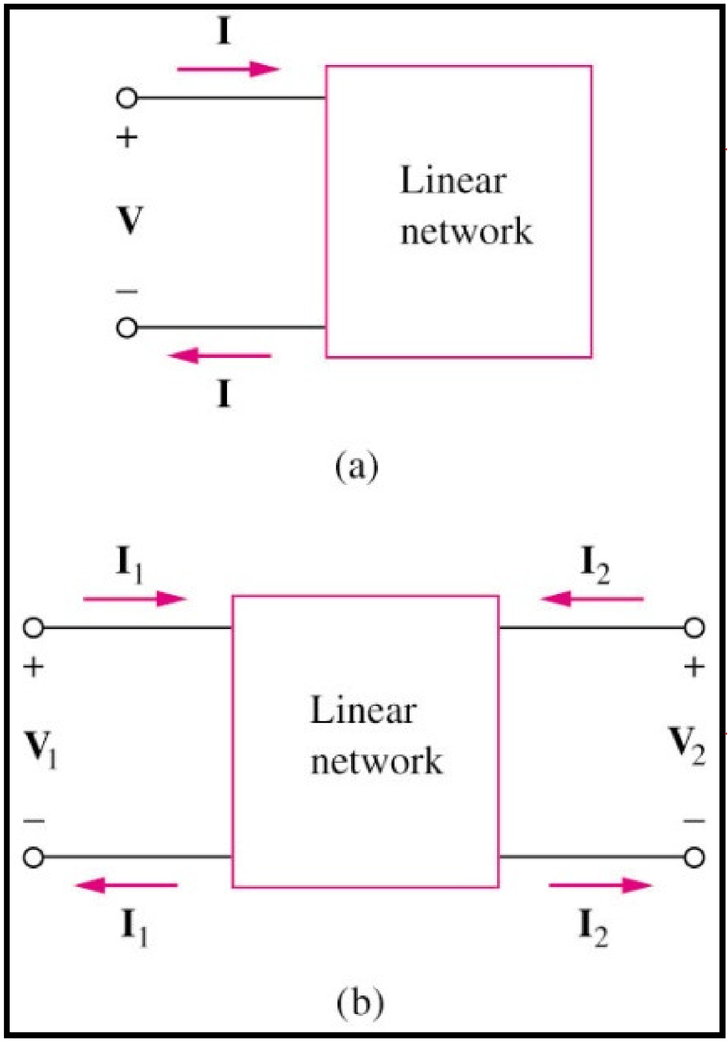
19.1 Introduction (1)

- A port is an access to the network and consists of a pair of terminals; the current entering one terminal leaves through the other terminal so that the net current entering the port equals zero.
- One port networks include two-terminal devices such as resistors, capacitors, and inductors.
- A two-port network has two separate ports for input and output.
- Two port networks include op amps, transistors and transformers.

19.1 Introduction (2)



One port or two terminal circuit

Two port or four terminal circuit

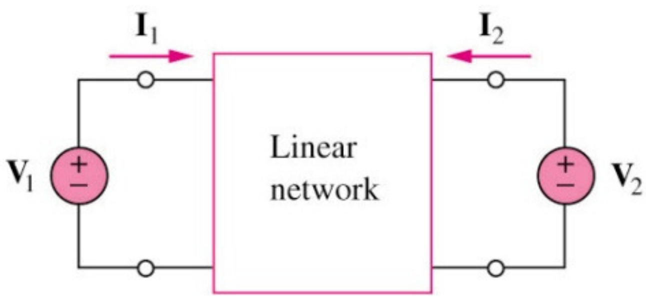
- **It is an electrical network with two separate ports for input and output.**
- **No independent sources.**

19.1 Introduction (3)

- Characterizing a two-port network requires that we relate the terminal quantities $\mathbf{V}_1$, $\mathbf{V}_2$, $\mathbf{I}_1$, $\mathbf{I}_2$ out of which two are independent. Six sets of voltage and current parameters will be derived.
- Two port networks are useful in communications, control systems, power systems, and electronics.
- They are used in electronics to model transistors and to facilitate cascaded design.
- Additionally, if we know the parameters of a two-port network it can be treated as a “black box” when embedded within a larger network.

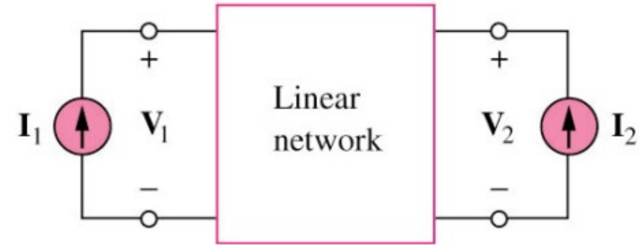
19.2 Impedance Parameters (1)

- Often called "**Z-parameters**" since their units are in **ohms** and they represent an impedance relationship between $\mathbf{V}_1$, $\mathbf{V}_2$, $\mathbf{I}_1$, $\mathbf{I}_2$ for the two port network shown below:



Voltage Driven Network

Assume no independent source in the network



Current Driven Network

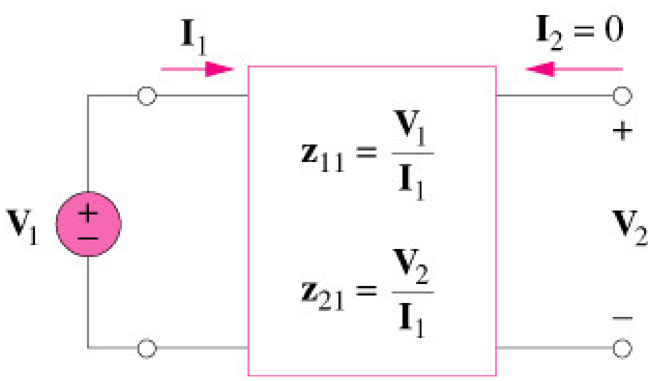
$$\mathrm{V_1 = z_{11}I_1 + z_{12}I_2}$$
$$\mathrm{V_2 = z_{21}I_1 + z_{22}I_2}$$

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

- Z-parameters are commonly used in filter synthesis, impedance matching networks design, and power distribution networks analysis.

19.2 Impedance Parameters (2)

The values of parameters can be evaluated by setting I_1=0 or I_2=0 (open circuit)

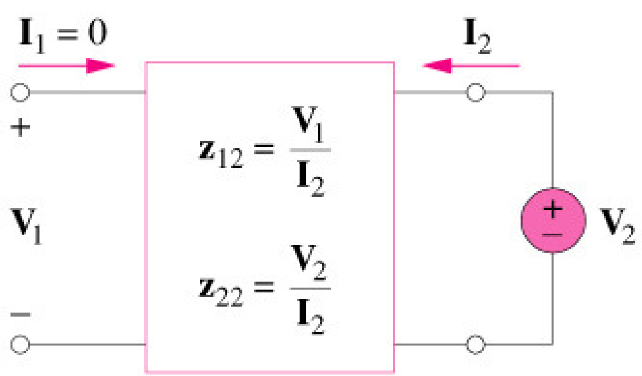


(a)

Setting I_2 = 0

$$z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0} \quad \text{and} \quad z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0}$$

z_{11} = Open-circuit input impedance
z_{21} = Open-circuit transfer impedance from port 2 to port 1



(b)

Setting I_1 = 0

$$z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1=0} \quad \text{and} \quad z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0}$$

z12 = Open-circuit transfer impedance from port 1 to port 2
z22 = Open-circuit output impedance

19.2 Impedance Parameters (3)

Properties of Z-parameters

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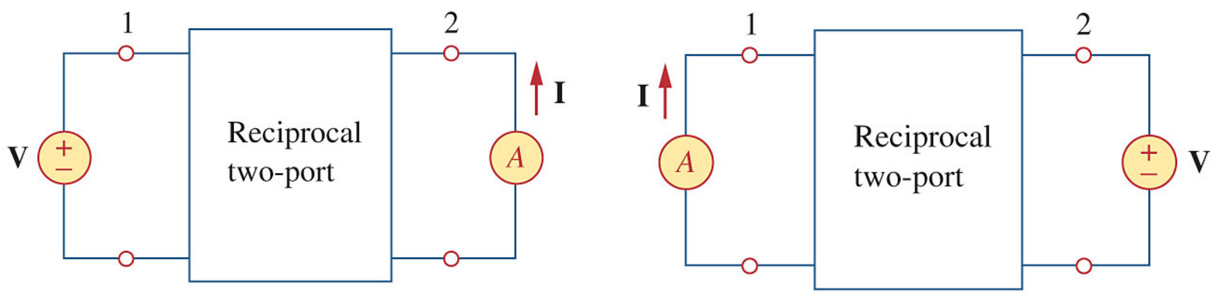
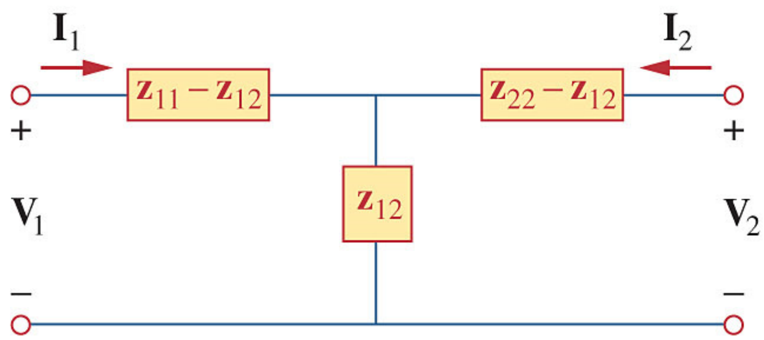
- Symmetrical networks $z_{11} = z_{22}$
  - Implies a mirror like symmetry
- Reciprocal networks $z_{12} = z_{21}$
  - Any network made up entirely of resistors, capacitors, and inductors must be reciprocal.
  - Linear networks with no dependant sources are reciprocal.
  - Interchanging an ideal voltage source at one port with an ideal ammeter at the other port gives the same ammeter reading.

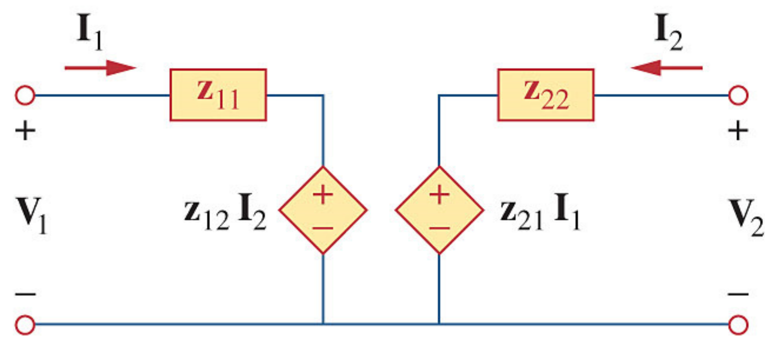


$$V = z_{12} I \qquad\qquad V = z_{21} I$$

19.2 Impedance Parameters (4)



Reciprocal Network



General Network

- A reciprocal network can be replaced by the T-network shown above
- If not reciprocal, the General network is the T-equivalent.

19.2 Impedance Parameters (5)

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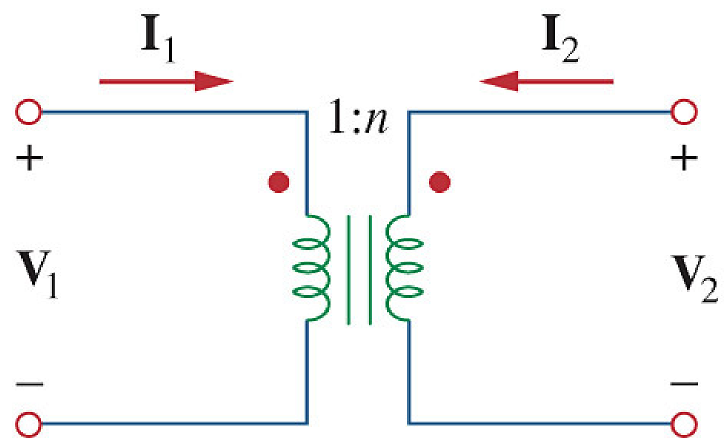


•Note: some circuits do not have z-parameter equivalents. (they may have other 2-port equivalents, as we shall see)

•Consider an ideal transformer:

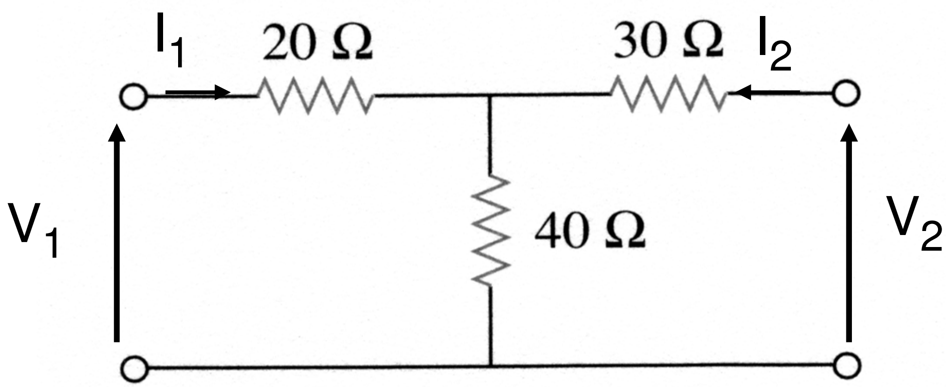
$$V_1 = V_2/n \quad \text{and} \quad I_1 = -nI_2.$$

•This cannot be expressed by:

$$V_1 = z_{11}I_1 + z_{12}I_2$$
$$V_2 = z_{21}I_1 + z_{22}I_2$$

Example 19.1

Determine the z-parameters of the following circuit.



$$z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0} \quad \text{and} \quad z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0}$$

$$z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1=0} \quad \text{and} \quad z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0}$$

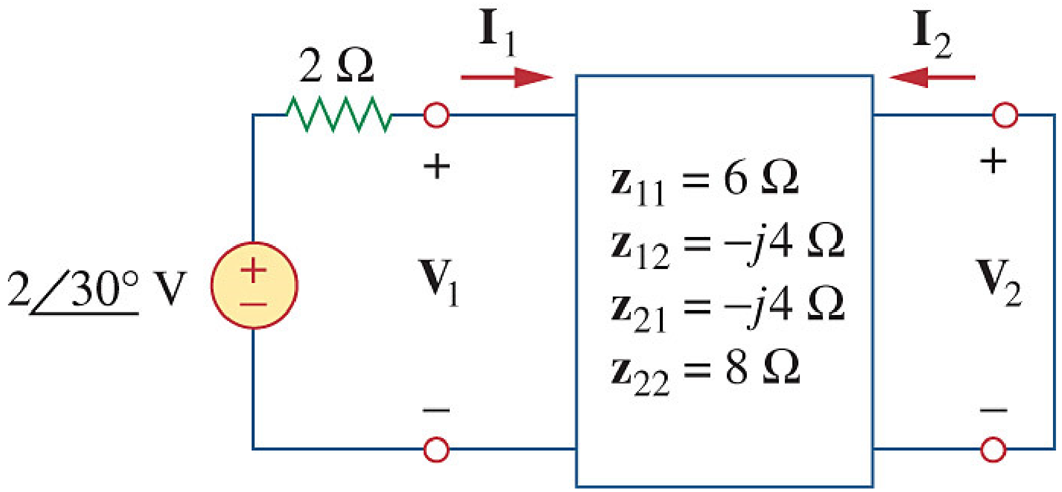
$$z = \begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix} \Omega$$

Answer: $$z = \begin{bmatrix} 60 & 40 \\ 40 & 70 \end{bmatrix} \Omega$$

19.2 Impedance Parameters (7)

Practice Problem 19.2

Determine I_1 and I_2 in the following circuit.

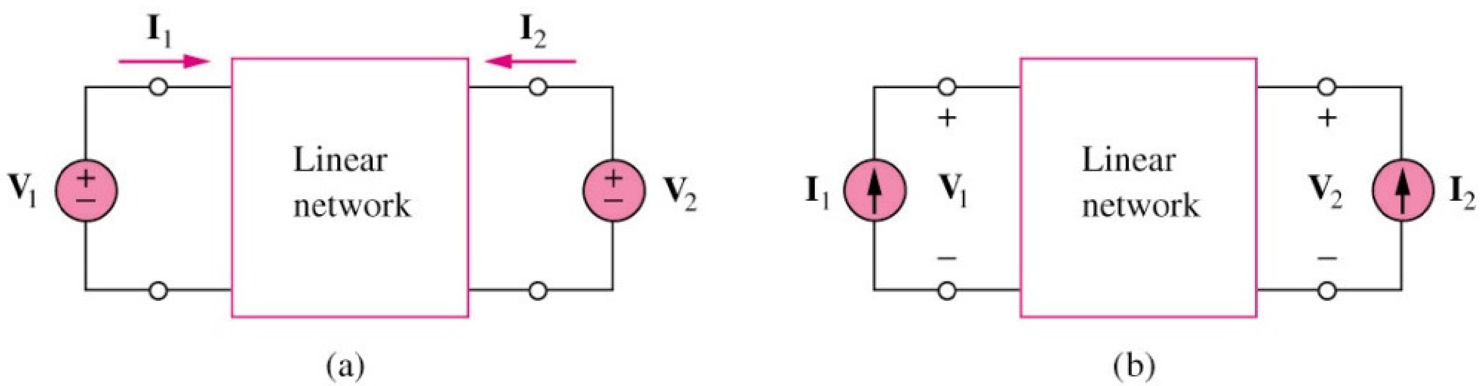


$$V_1 = z_{11}I_1 + z_{12}I_2$$
$$V_2 = z_{21}I_1 + z_{22}I_2$$

Answer:

$$I_1 = 200\angle 30°\ \text{mA}$$
$$I_2 = 100\angle 120°\ \text{mA}$$

19.3 Admittance Parameters (1)



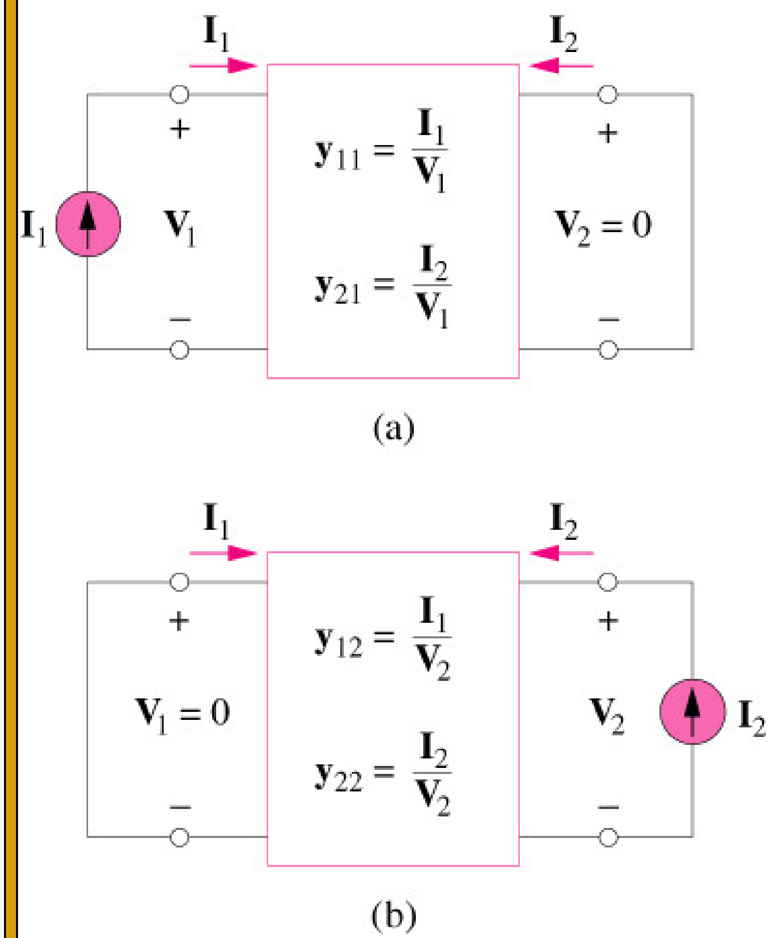
Assume no independent source in the network

$$I_1 = y_{11}V_1 + y_{12}V_2$$
$$I_2 = y_{21}V_1 + y_{22}V_2$$

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = [y] \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

where the **y** terms are called the admittance parameters, or simply y parameters, and they have units of Siemens.

19.3 Admittance Parameters (2)



Setting $V_2 = 0$ (Shorting the output)

$$y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2=0} \quad \text{and} \quad y_{21} = \left.\frac{I_2}{V_1}\right|_{V_2=0}$$

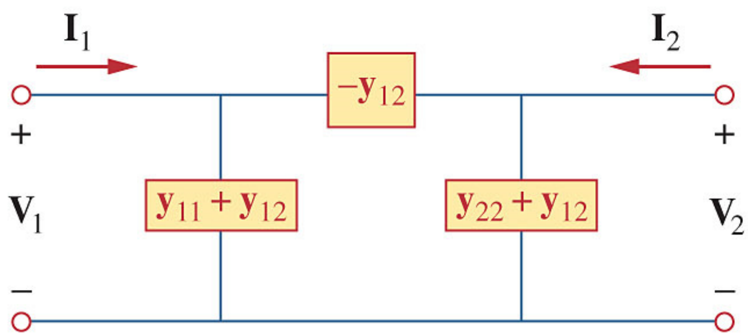
y_{11} = Short-circuit input admittance
y_{21} = Short-circuit transfer admittance from port 1 to port 2

Setting $V_1 = 0$ (Shorting the input)

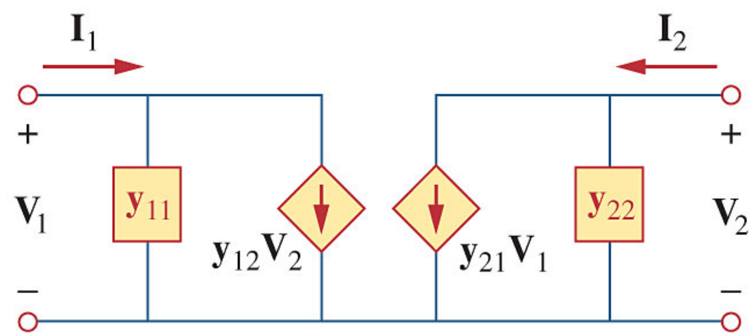
$$y_{12} = \left.\frac{I_1}{V_2}\right|_{V_1=0} \quad \text{and} \quad y_{22} = \left.\frac{I_2}{V_2}\right|_{V_1=0}$$

y_{12} = Short-circuit transfer admittance from port 2 to port 1
y_{22} = Short-circuit output admittance

19.3 Admittance Parameters (3)



Reciprocal
Network



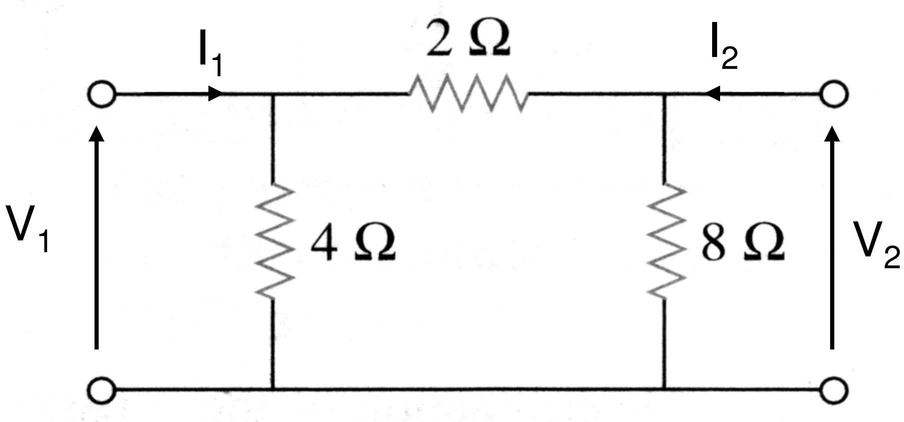
General
Network

•A reciprocal network ($y_{12} = y_{21}$) can be replaced by the Pi-network in figure (a).
•If not reciprocal, the network in figure (b) is the Pi-equivalent.

19.3 Admittance Parameters (4)

Example 19.3

Determine the y-parameters of the following circuit.



$$y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2=0} \quad \text{and} \quad y_{21} = \left.\frac{I_2}{V_1}\right|_{V_2=0}$$

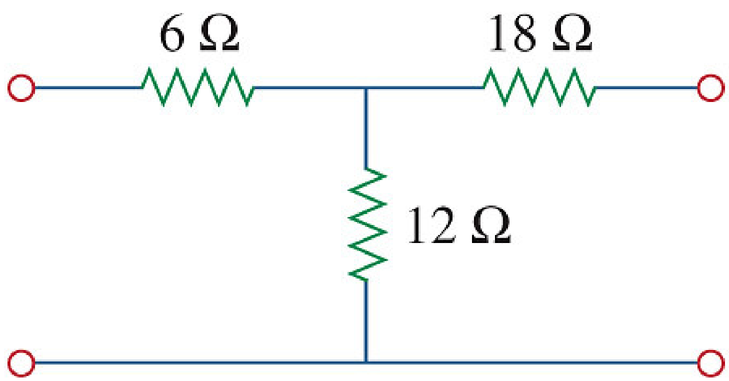
$$y_{12} = \left.\frac{I_1}{V_2}\right|_{V_1=0} \quad \text{and} \quad y_{22} = \left.\frac{I_2}{V_2}\right|_{V_1=0}$$

$$y = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} S$$

Answer: $y = \begin{bmatrix} 0.75 & -0.5 \\ -0.5 & 0.625 \end{bmatrix} S$

Practice Problem 19.3

Determine the y-parameters of the following circuit.



$$y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2=0} \quad \text{and} \quad y_{21} = \left.\frac{I_2}{V_1}\right|_{V_2=0}$$

$$y_{12} = \left.\frac{I_1}{V_2}\right|_{V_1=0} \quad \text{and} \quad y_{22} = \left.\frac{I_2}{V_2}\right|_{V_1=0}$$

$$\Downarrow$$

$$y = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} S$$

Answer: $y = \begin{bmatrix} 75.77 & -30.3 \\ -30.3 & 45.47 \end{bmatrix} \text{mS}$

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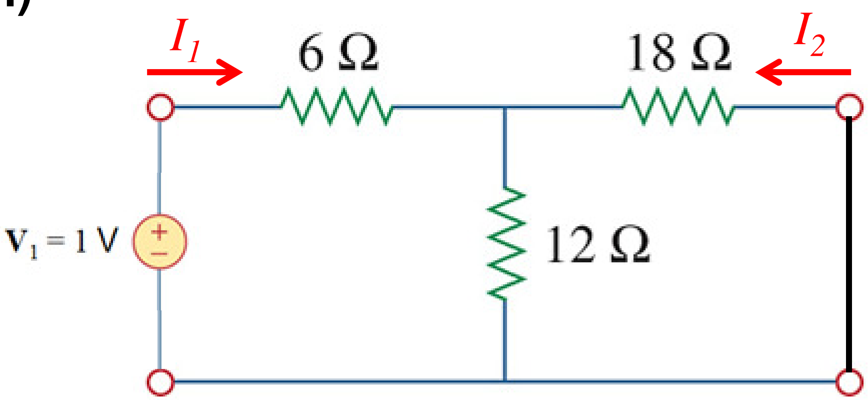
Practice Problem 19.3 (Solution)

- Short the output
- Put a 1 volt source at input
- Find I_1 and I_2



$$\boxed{y_{11} = \frac{I_1}{(1)}\Bigg|_{V_2=0} \quad and \quad y_{21} = \frac{I_2}{(1)}\Bigg|_{V_2=0}}$$

Find Input Impedance

$$Z_{in} = 6 + 12 \parallel 18 = 13.2$$

$$I_1 = \frac{V_1}{Z_{in}} = \frac{1}{13.2} = 0.07576$$

$$\boxed{y_{11} = 0.07576}$$

Similarly at Output

$$Z_{out} = 18 + 6 \parallel 12 = 22$$

$$I_2 = \frac{V_2}{Z_{in}} = \frac{1}{22} = 0.04545$$

$$\boxed{y_{22} = 0.04545}$$

Find I_2 from current divider equation

$$I_2 = \frac{-12}{12+18} I_1$$

$$I_2 = (-0.4)0.07576 = -0.0303$$

$$y_{21} = -0.0303$$

$$\boxed{y_{12} = y_{21} = -0.0303}$$

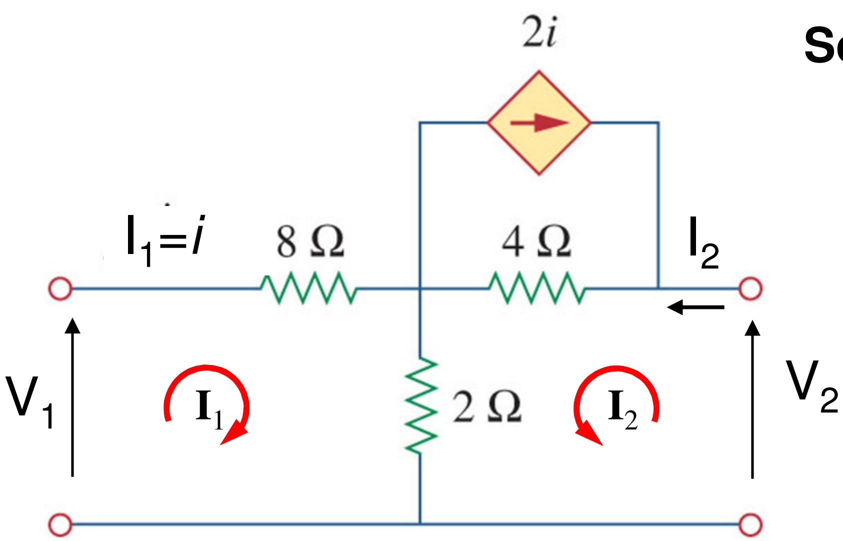
Reciprocal Network

19.3 Admittance Parameters (7)

Example 19.4

Determine the y-parameters of the following circuit.

$$I_1 = y_{11}V_1 + y_{12}V_2$$
$$I_2 = y_{21}V_1 + y_{22}V_2$$



Solution: Apply KVL

Mesh I_1: $V_1 = 8I_1 + 2(I_1 + I_2)$

$V_1 = 10I_1 + 2I_2$

Mesh I_2: $V_2 = 4(2i + I_2) + 2(I_1 + I_2)$

$V_2 = 8I_1 + 4I_2 + 2I_1 + 2I_2$

$V_2 = 10I_1 + 6I_2$

Subtract #1 from #2:

$V_2 - V_1 = 0 + 4I_2$

$I_2 = -0.25V_1 + 0.25V_2$

Substitute back into #1

$V_1 = 10I_1 - 0.5V_1 + 0.5V_2$

$10I_1 = 1.5V_1 - 0.5V_2$

$I_1 = 0.15V_1 - 0.05V_2$

Answer: $y = \begin{bmatrix} 0.15 & -0.05 \\ -0.25 & 0.25 \end{bmatrix} S$

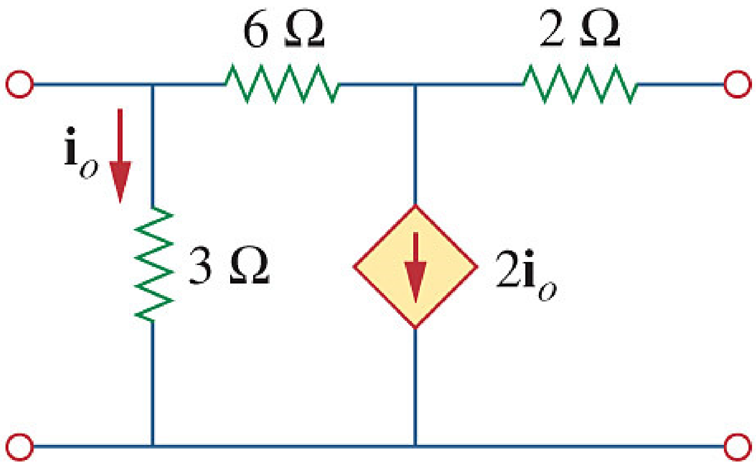
Note: Sometimes two port parameters will fall out directly from mesh equations.

19.3 Admittance Parameters (8)

Practice problem 19.4

Practice Problem 19.4

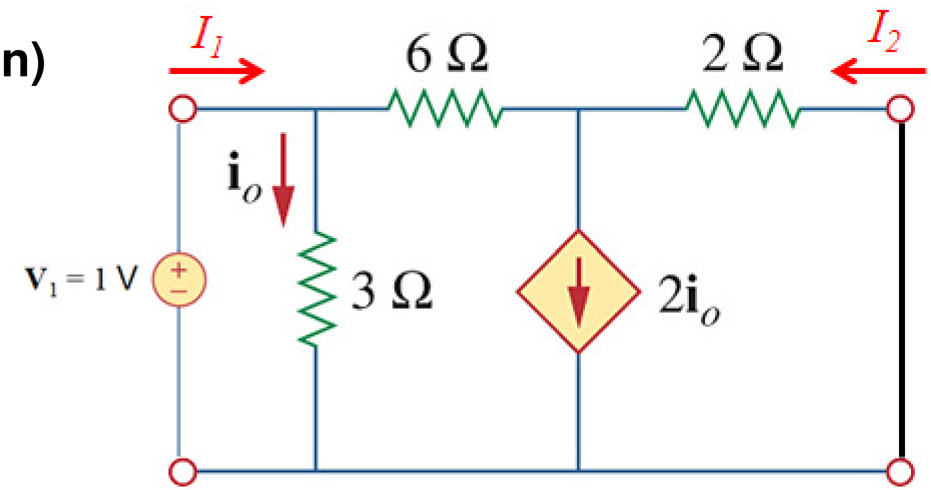
Determine the y-parameters of the following circuit.



Answer: $y = \begin{bmatrix} 0.625 & -0.125 \\ 0.375 & 0.125 \end{bmatrix} S$

Practice Problem 19.4 (Solution)

- Short the output
- Put a 1 volt source at input
- Find I_1 and I_2



First find i_o:

$$i_0 = \frac{1}{3}$$

Dependent current source is then 2/3, find I_1 by repetitive source transformations of the dependant current source

$$I_1 = 0.625 \quad \Rightarrow \boxed{y_{11} = 0.625}$$

Next find current across 6 Ω resistor $I_{6\Omega}$:

$$I_{6\Omega} = 0.625 - \frac{1}{3}$$

$$I_2 + I_{6\Omega} = 2i_0$$

$$I_2 = 2i_0 - I_{6\Omega} = \frac{2}{3} - \left(0.625 - \frac{1}{3}\right) = 0.375 \quad \Rightarrow \boxed{y_{12} = 0.375}$$

Z and Y Parameters
Comparison

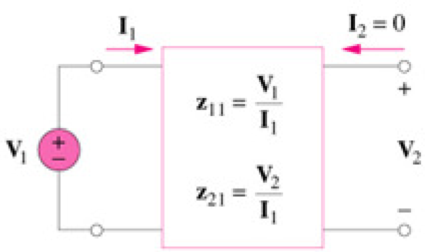
Z-Parameters

$$V_1 = z_{11}I_1 + z_{12}I_2$$
$$V_2 = z_{21}I_1 + z_{22}I_2$$

- **Open** one port (I_1=0 or I_2=0)
- Connect a source to the other port
- Solve to find z-parameters

$$z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0} \quad \text{and} \quad z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0}$$

$$z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1=0} \quad \text{and} \quad z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0}$$



I = 0
Open Circuit

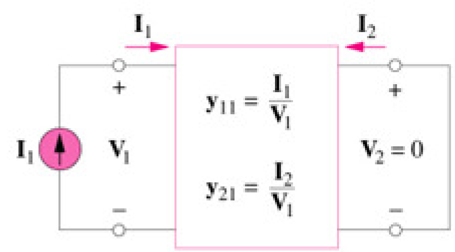
Y-Parameters

$$I_1 = y_{11}V_1 + y_{12}V_2$$
$$I_2 = y_{21}V_1 + y_{22}V_2$$

- **Short** one port (V_1=0 or V_2=0)
- Connect a source to the other port
- Solve to find y-parameters

$$y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2=0} \quad \text{and} \quad y_{21} = \left.\frac{I_2}{V_1}\right|_{V_2=0}$$

$$y_{12} = \left.\frac{I_1}{V_2}\right|_{V_1=0} \quad \text{and} \quad y_{22} = \left.\frac{I_2}{V_2}\right|_{V_1=0}$$



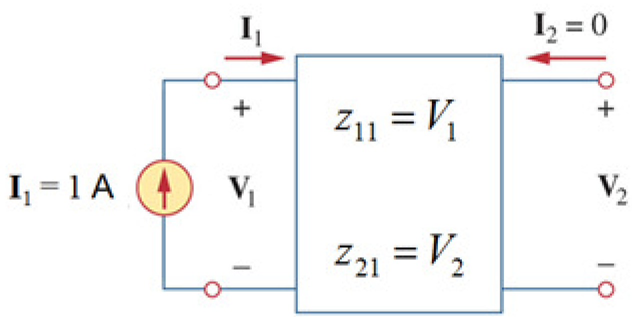
V = 0
Short Circuit

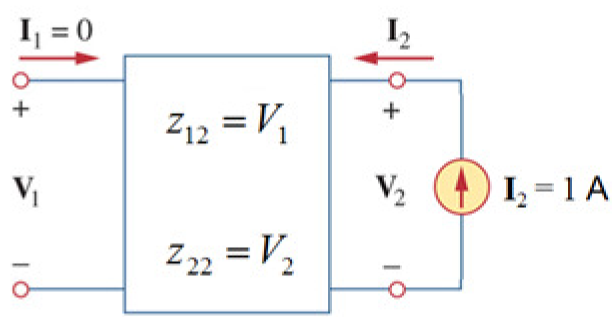
Z and Y parameters

Alternative method (1 Amp / 1 Volt sources)

Z-Parameters

- Open circuit one port
- Put a 1 Amp current source at other port
- Resulting voltages are the z-parameters

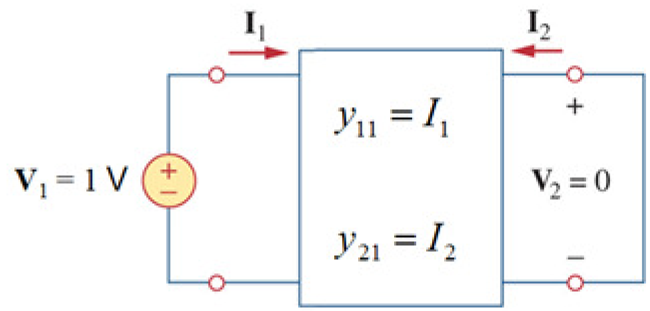


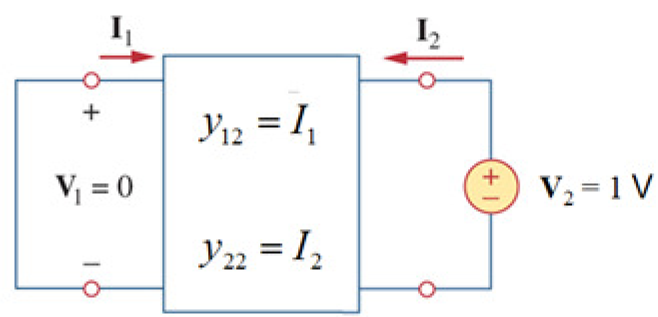


$$V_1 = z_{11} I_1 + z_{12} I_2$$
$$V_2 = z_{21} I_1 + z_{22} I_2$$

Y-Parameters

- Short circuit one port
- Put a 1 Volt voltage source at other port
- Resulting current are the y-parameters

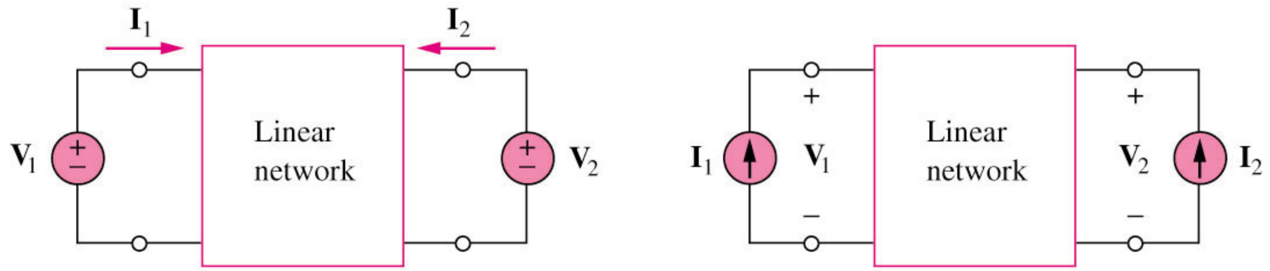




$$I_1 = y_{11} V_1 + y_{12} V_2$$
$$I_2 = y_{21} V_1 + y_{22} V_2$$

19.4 Hybrid Parameters (1)

•The z and y parameters of a two-port network do not always exist. Therefore, there is a need to develop another set of parameters based on making V_1 and I_2 the dependent variables.



Assume no independent source in the network

$$V_1 = h_{11}I_1 + h_{12}V_2$$
$$I_2 = h_{21}I_1 + h_{22}V_2$$

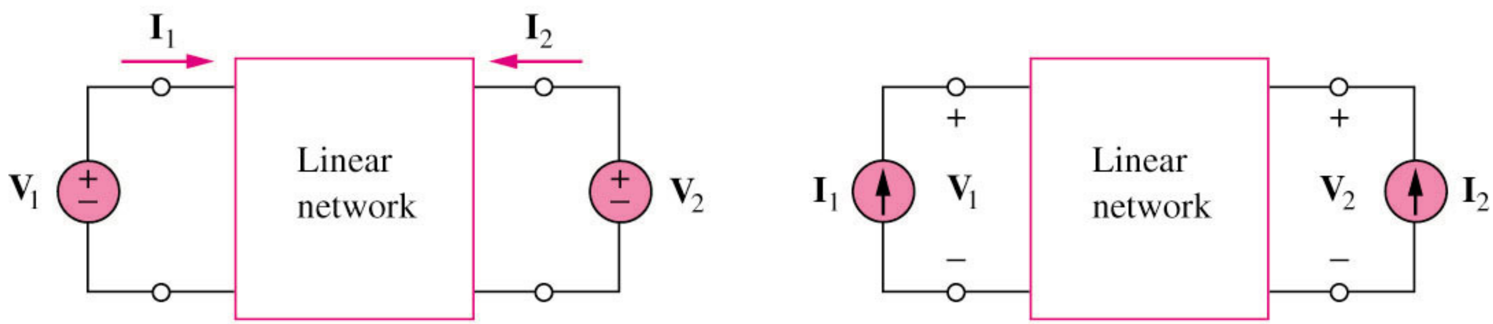
$$\begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ V_2 \end{bmatrix} = [h] \begin{bmatrix} I_1 \\ V_2 \end{bmatrix}$$

where the **h** terms are called the hybrid parameters, or simply h parameters.

•Hybrid parameters are very useful for describing electronic devices such as transistors because it is much easier to measure the h parameters of these devices than to measure their z or y parameters.

•The ideal transformer can also be described by h parameters.

19.4 Hybrid Parameters (2)



Assume no independent source in the network

$$h_{11} = \left.\frac{V_1}{I_1}\right|_{V_2=0}$$

h_{11} = short-circuit input impedance (Ω)

$$h_{21} = \left.\frac{I_2}{I_1}\right|_{V_2=0}$$

h_{21} = short-circuit forward current gain

$$h_{12} = \left.\frac{V_1}{V_2}\right|_{I_1=0}$$

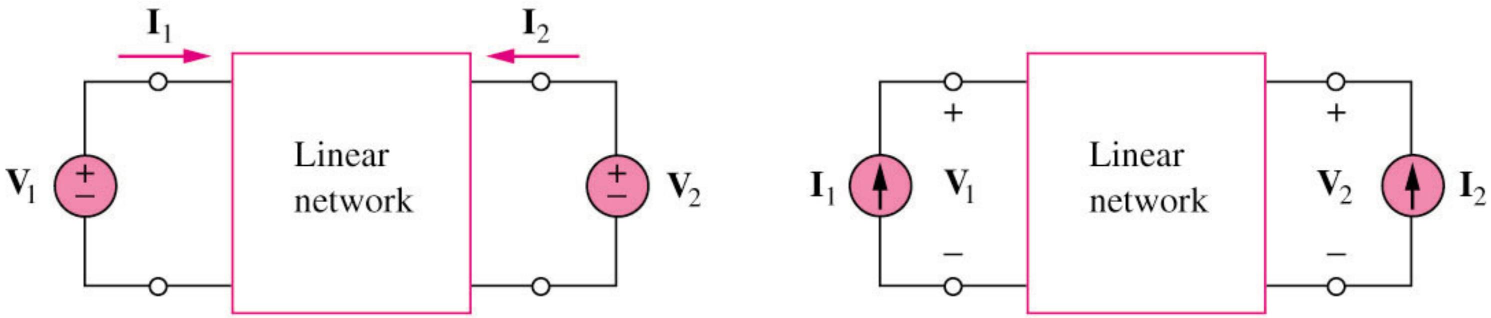
h_{12} = open-circuit reverse voltage-gain

$$h_{22} = \left.\frac{I_2}{V_2}\right|_{I_1=0}$$

h_{22} = open-circuit output admittance (S)

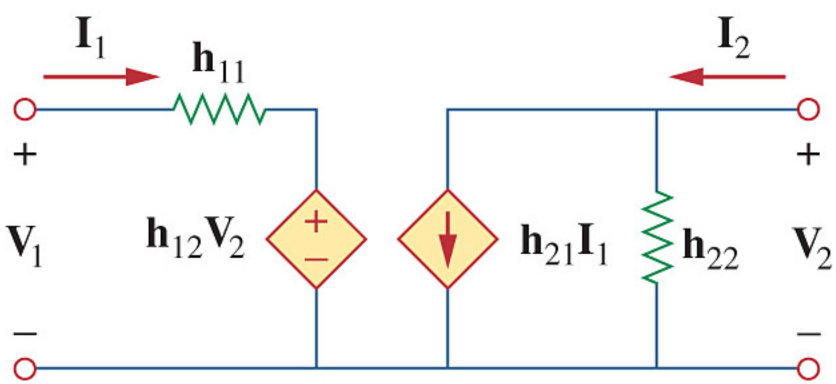
- Note that the h parameters represent an impedance, voltage gain, current gain, and admittance, thereby the term hybrid parameters.
- For reciprocal network, $h_{12} = -h_{21}$

19.4 Hybrid Parameters (3)



Assume no independent source in the network

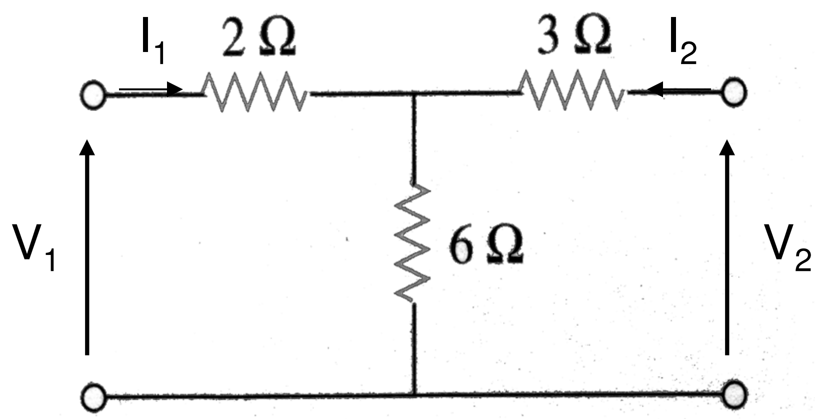
Hybrid model of a two-port network:



19.4 Hybrid Parameters (4)

Example 19.5:

Determine the h-parameters of the following circuit.



$$V_1 = h_{11}I_1 + h_{12}V_2$$
$$I_2 = h_{21}I_1 + h_{22}V_2$$

$$h_{11} = \left.\frac{V_1}{I_1}\right|_{V_2=0} \quad \text{and} \quad h_{21} = \left.\frac{I_2}{I_1}\right|_{V_2=0}$$

$$h_{12} = \left.\frac{V_1}{V_2}\right|_{I_1=0} \quad \text{and} \quad h_{22} = \left.\frac{I_2}{V_2}\right|_{I_1=0}$$

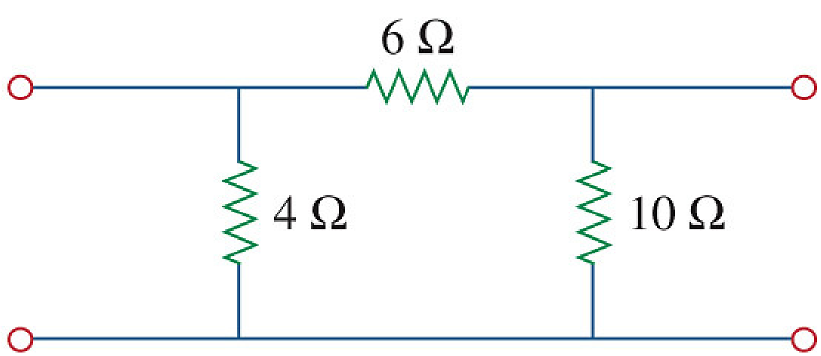
$$h = \begin{bmatrix} h_{11}\Omega & h_{12} \\ h_{21} & h_{22}S \end{bmatrix}$$

Answer: $h = \begin{bmatrix} 4\Omega & \frac{2}{3} \\ -\frac{2}{3} & \frac{1}{9}S \end{bmatrix}$

19.4 Hybrid Parameters (5)

Practice Problem 19.5:

Determine the h-parameters of the following circuit.



$$h_{11} = \left.\frac{V_1}{I_1}\right|_{V_2=0} \quad \text{and} \quad h_{21} = \left.\frac{I_2}{I_1}\right|_{V_2=0}$$

$$h_{12} = \left.\frac{V_1}{V_2}\right|_{I_1=0} \quad \text{and} \quad h_{22} = \left.\frac{I_2}{V_2}\right|_{I_1=0}$$

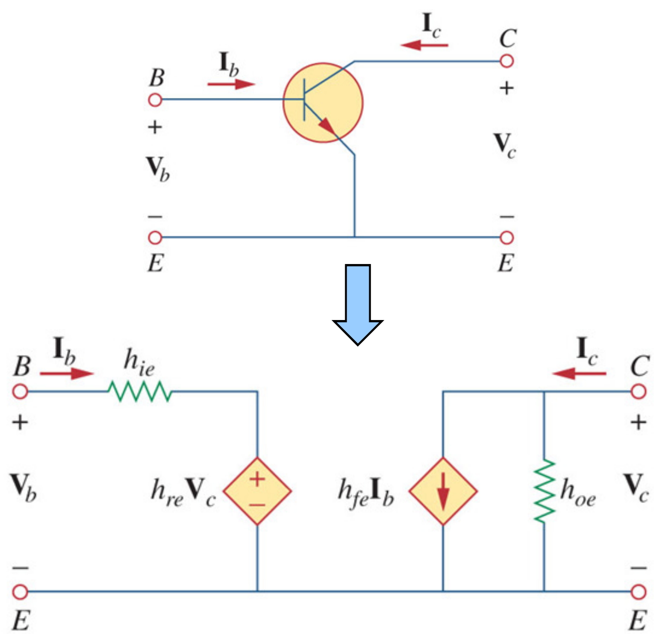
$$h = \begin{bmatrix} h_{11}\Omega & h_{12} \\ h_{21} & h_{22}S \end{bmatrix}$$

Answer: $h = \begin{bmatrix} 2.4\Omega & 0.4 \\ -0.4 & 0.2S \end{bmatrix}$

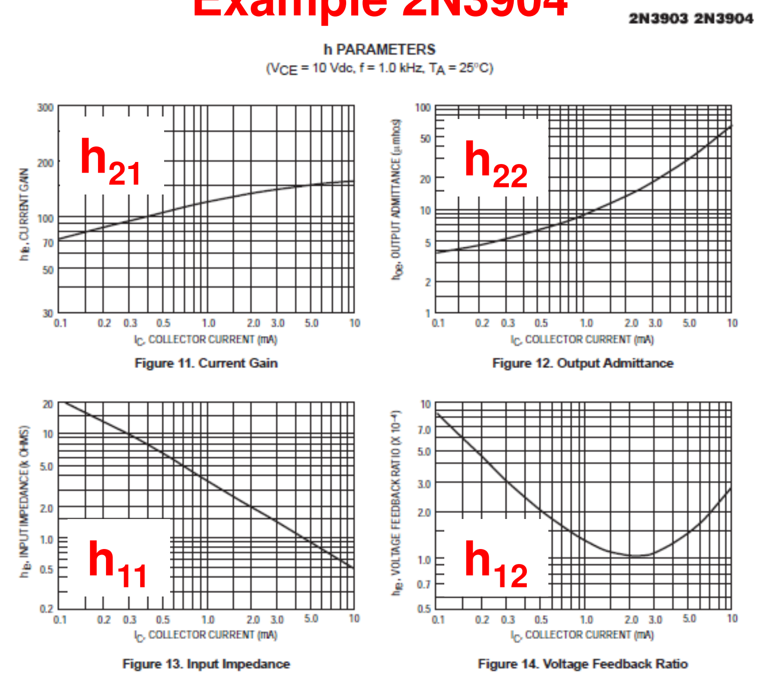
19.9.1 Transistor Circuits (1)

Hybrid Parameters

- H-parameters are often used to model transistor circuits
- The h-parameters vary depending on biasing conditions
- Parameters are given different subscripts:
  - h_{11} → h_{ie} = Base input impedance
  - h_{12} → h_{re} = Reverse voltage feedback ration
  - h_{21} → h_{fe} = Base-collector current gain
  - h_{22} → h_{oe} = Output admittance



Example 2N3904



19.9.1 Transistor Circuits (2)

Hybrid Parameters

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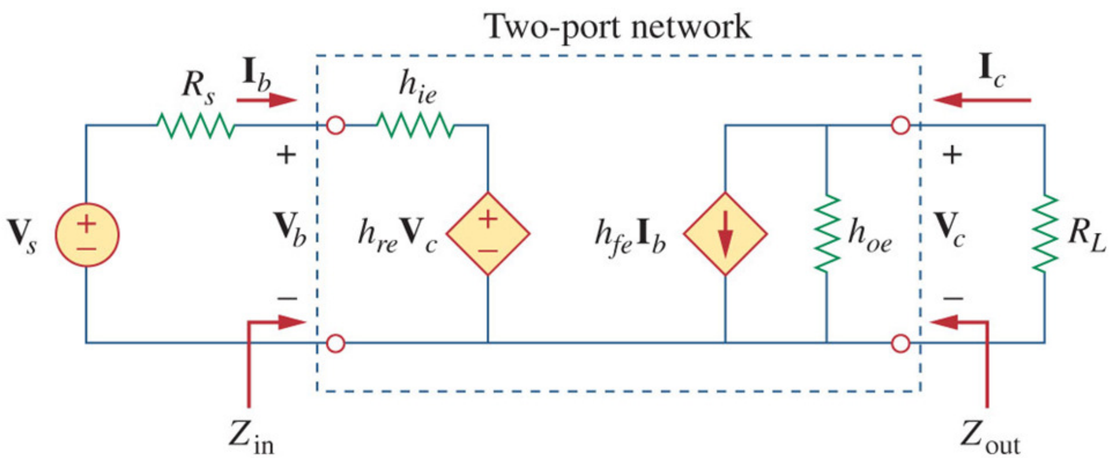
- H parameters are often found in manufacturers spec sheets
- Provide ability to calculate the exact voltage gain, input impedance, and output impedance of the transistor.



Input Impedance

$$\boxed{Z_{in} = \frac{V_b}{I_b} = h_{ie} - \frac{h_{re}h_{fe}R_L}{1 + h_{oe}R_L}}$$

Output Impedance

$$\boxed{Z_{out} = \left.\frac{V_c}{I_c}\right|_{V_s=0} = \frac{R_s + h_{ie}}{(R_s + h_{ie})h_{oe} - h_{re}h_{fe}}}$$

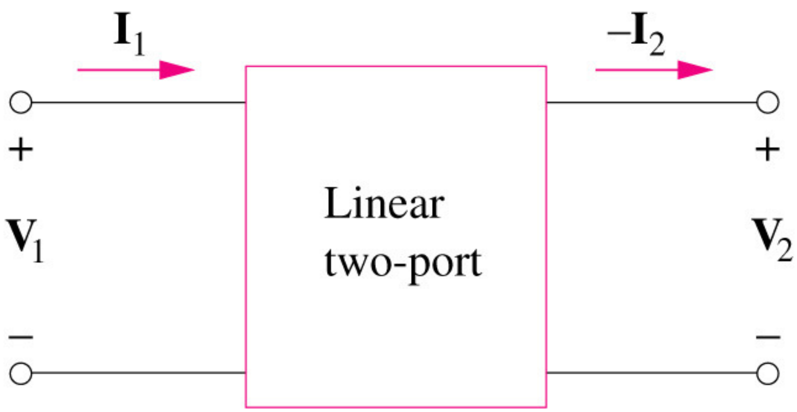
Current Gain

$$\boxed{A_i = \frac{I_c}{I_b} = \frac{h_{fe}}{1 + h_{oe}R_L}}$$

Voltage Gain

$$\boxed{A_v = \frac{V_c}{V_b} = \frac{-h_{fe}R_L}{h_{ie} + (h_{ie}h_{oe} - h_{re}h_{fe})R_L}}$$

19.5 Transmission Parameters (1)



Assume no independent source in the network

$$V_1 = AV_2 - BI_2$$
$$I_1 = CV_2 - DI_2$$

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix} = [T] \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$

where the **T** terms are called the transmission parameters, or simply T or ABCD parameters.

•Note that $-I_2$ is used since the current is considered to be leaving the network. It is logical to think of I_2 as leaving the two-port; this is customary convention in the power industry.

19.5 Transmission Parameters (2)

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- These two-port transmission parameters provide a measure of how a circuit transmits voltage and current form a source to a load.
- They are useful in the analysis of transmission lines and are therefore called transmission parameters.
- They are also known as ABCD parameters and are used in the design of telephone systems, microwave networks, and radars.

$$A = \left.\frac{V_1}{V_2}\right|_{I_2=0}$$

A=open-circuit voltage ratio

$$C = \left.\frac{I_1}{V_2}\right|_{I_2=0}$$

C= open-circuit transfer admittance (S)

$$B = -\left.\frac{V_1}{I_2}\right|_{V_2=0}$$

B= negative short-circuit transfer impedance (Ω)

$$D = -\left.\frac{I_1}{I_2}\right|_{V_2=0}$$

D=negative short-circuit current ratio

19.5 Transmission Parameters (3)

Solving for Transmission Parameters

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- To find the transmission parameters, analyze the circuit as follows:
- Perform the analysis with the output Open Circuited (I_2=0)

$$\begin{aligned} V_1 &= AV_2 - B\cancel{I_2}^{\,0} \\ I_1 &= CV_2 - D\cancel{I_2}^{\,0} \end{aligned} \Rightarrow \begin{aligned} V_1 &= AV_2 \\ I_1 &= CV_2 \end{aligned} \Rightarrow \begin{aligned} A &= \frac{V_1}{V_2} \\ C &= \frac{I_1}{V_2} \end{aligned}$$

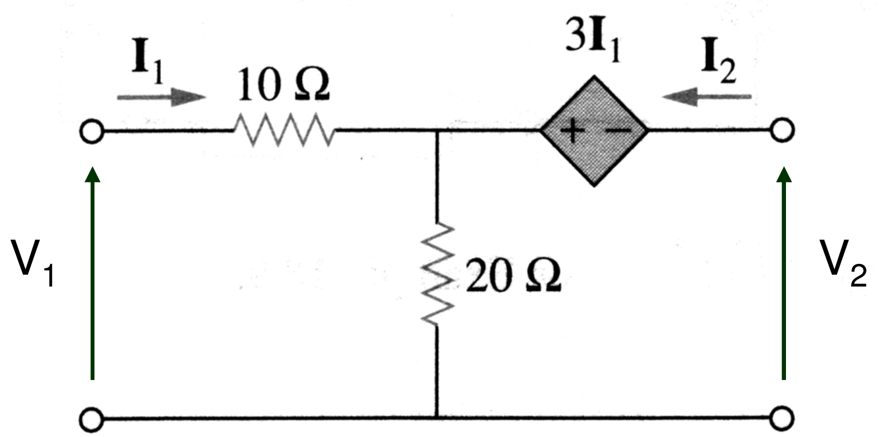
- Perform the analysis with the output Short Circuited(V_2=0)

$$\begin{aligned} V_1 &= A\cancel{V_2}^{\,0} - BI_2 \\ I_1 &= C\cancel{V_2}^{\,0} - DI_2 \end{aligned} \Rightarrow \begin{aligned} V_1 &= -BI_2 \\ I_1 &= -DI_2 \end{aligned} \Rightarrow \begin{aligned} B &= -\frac{V_1}{I_2} \\ D &= -\frac{I_1}{I_2} \end{aligned}$$

19.5 Transmission Parameters (4)

Example 19.8

Determine the T-parameters of the following circuit.



$$V_1 = AV_2 - BI_2$$
$$I_1 = CV_2 - DI_2$$

Apply KVL

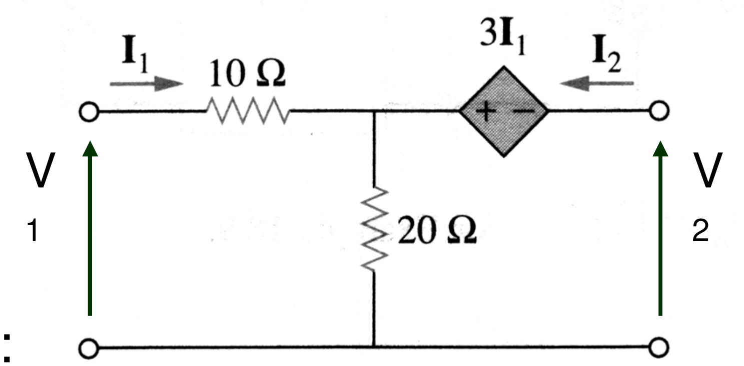
$$V_1 = 10I_1 + 20(I_1 + I_2)$$
$$V_2 = -3I_1 + 20(I_1 + I_2)$$

$$V_1 = \frac{30}{17}V_2 - \frac{260}{17}I_2$$
$$I_1 = \frac{1}{17}V_2 - \frac{20}{17}I_2$$

Answer: $$T = \begin{bmatrix} 1.765 & 15.294\Omega \\ 0.059S & 1.176 \end{bmatrix}$$

19.5 Transmission Parameters (5)

Example 19.8



From KVL:

$$V_1 = 10I_1 + 20(I_1 + I_2) = 30I_1 + 20I_2$$
$$V_2 = -3I_1 + 20(I_1 + I_2) = 17I_1 + 20I_2$$

If we "open circuit" the output we get:

$$V_1 = 30I_1 + 20\cancel{I_2}^{\,0} \qquad V_1 = 30I_1$$
$$V_2 = 17I_1 + 20\cancel{I_2}^{\,0} \qquad V_2 = 17I_1$$

$$A = \frac{V_1}{V_2} = \frac{30I_1}{17I_1} = \frac{30}{17} = 1.765$$
$$C = \tfrac{1}{17} = 0.0588$$

If we "short circuit" the output we get:

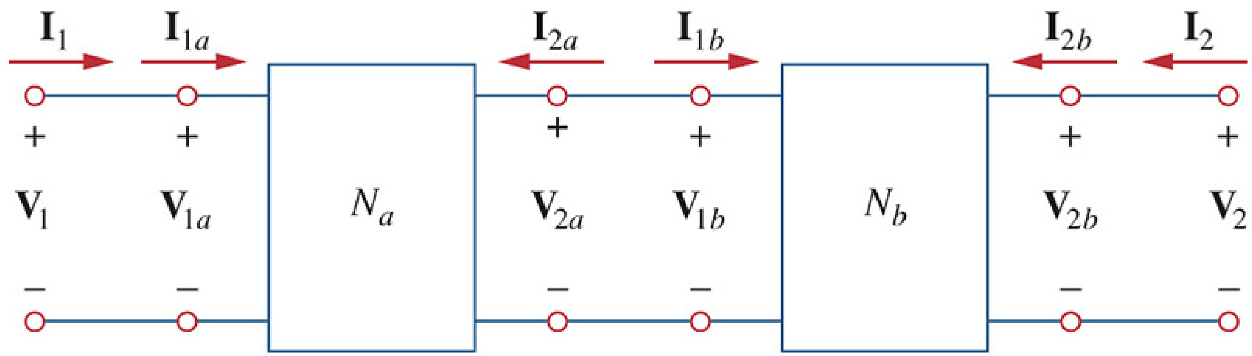
$$V_1 = 30I_1 + 20I_2 \qquad V_1 = 30I_1 + 20I_2$$
$$\cancel{V_2}^{\,0} = 17I_1 + 20I_2 \qquad 0 = 17I_1 + 20I_2$$

$$V_1 = 30\left(\tfrac{-20}{17}\right)I_2 + 20I_2$$
$$I_1 = \tfrac{-20}{17} I_2$$

$$B = -\frac{V_1}{I_2} = -\frac{\left(30\left(\frac{-20}{17}\right) + 20\right)I_2}{I_2} = 15.29$$
$$D = -\frac{I_1}{I_2} = \frac{20}{17} = 1.176$$

19.5 Transmission Parameters (6)

- Transmission Parameters can be cascaded with the result found through simple matrix multiplication



$$\begin{bmatrix} V_{1a} \\ I_{1a} \end{bmatrix} = \begin{bmatrix} A_a & B_a \\ C_a & D_a \end{bmatrix} \begin{bmatrix} V_{2a} \\ -I_{2a} \end{bmatrix} \qquad \begin{bmatrix} V_{1b} \\ I_{1b} \end{bmatrix} = \begin{bmatrix} A_b & B_b \\ C_b & D_b \end{bmatrix} \begin{bmatrix} V_{2b} \\ -I_{2b} \end{bmatrix}$$

$$-I_{2a} = I_{1b}$$

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A_a & B_a \\ C_a & D_a \end{bmatrix} \begin{bmatrix} A_b & B_b \\ C_b & D_b \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$

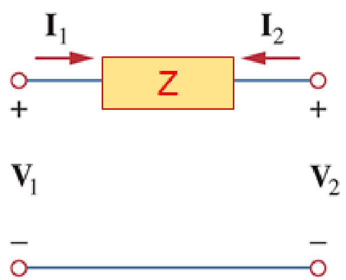
$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A' & B' \\ C' & D' \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$

19.5 Transmission Parameters (7)

Properties: Building Block Circuits

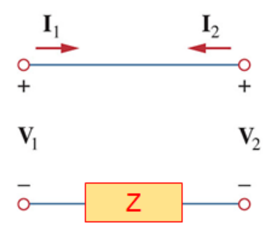
Consider the following simple circuits

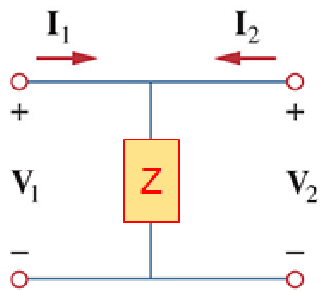
We can find their T Parameters to be:



$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} 1 & Z \\ 0 & 1 \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$

Note, following is equivalent





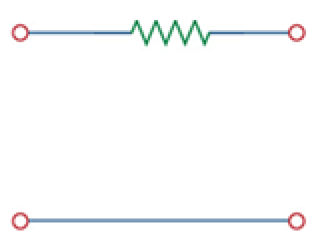
$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ \frac{1}{Z} & 1 \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$

19.5 Transmission Parameters (8)

Properties: Building Block Circuits

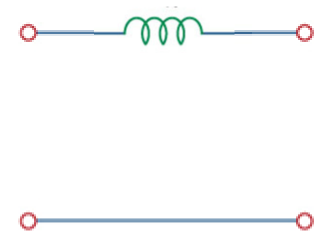
- We can use this to construct the following "building block T parameters" to find the T parameters for any ladder type circuit.

$$\begin{bmatrix} 1 & R \\ 0 & 1 \end{bmatrix}$$

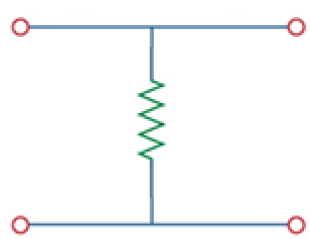


$$\begin{bmatrix} 1 & \frac{1}{sC} \\ 0 & 1 \end{bmatrix}$$

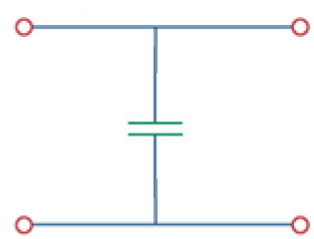
$$\begin{bmatrix} 1 & sL \\ 0 & 1 \end{bmatrix}$$



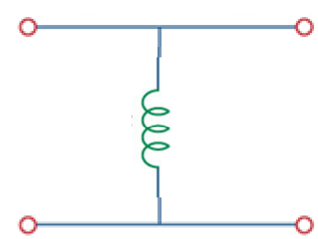
$$\begin{bmatrix} 1 & 0 \\ \frac{1}{R} & 1 \end{bmatrix}$$



$$\begin{bmatrix} 1 & 0 \\ sC & 1 \end{bmatrix}$$



$$\begin{bmatrix} 1 & 0 \\ \frac{1}{sL} & 1 \end{bmatrix}$$



19.5 Transmission Parameters (9)

Properties: Transfer function / Thevenin Equivalent

- The "A" parameter can be used to provide the inverse of the voltage Transfer Function H(s).

$$\boxed{A = \left.\frac{V_1}{V_2}\right|_{I_2=0} = \frac{1}{H(s)}}$$

- Parameters "A" and "B" can be used to find a relationship between the Open Circuit Voltage (V_2) and the Short Circuit Current ($-I_2$).
- We can us this to find the parameters for the Thevenin Equivalent Circuit.

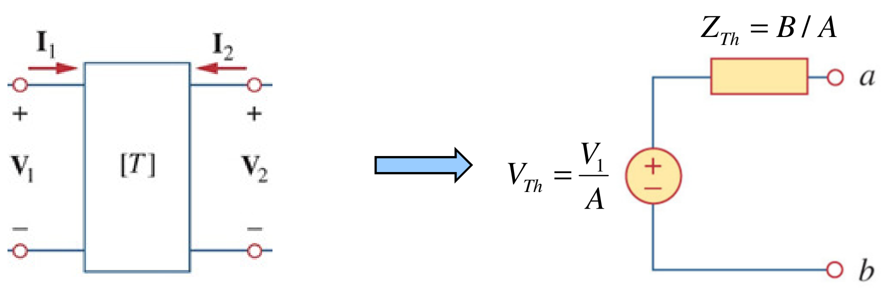
$$\boxed{A = \left.\frac{V_1}{V_2}\right|_{I_2=0} = \frac{V_1}{V_{oc}}}$$

$$\boxed{V_{Th} = V_{oc} = \frac{V_1}{A}}$$

$$\boxed{Z_{Th} = \frac{V_{oc}}{I_{sc}} = \frac{B}{A}}$$

$$\boxed{B = -\left.\frac{V_1}{I_2}\right|_{V_2=0} = \frac{V_1}{I_{sc}}}$$

$$\boxed{I_N = I_{sc} = \frac{V_1}{B}}$$

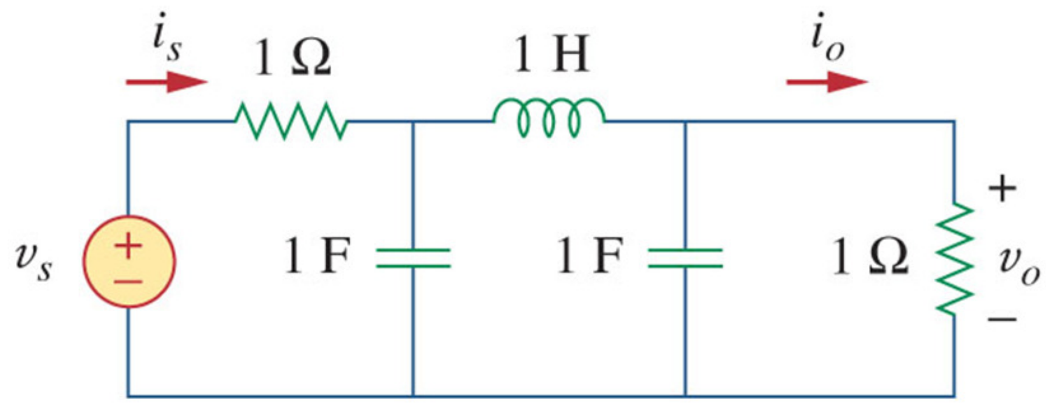


19.5 Transmission Parameters (10)

Transfer Function - Example

Problem 16.80(a)

Find the transfer function $V_o(s)/V_s(s)$ for the following circuit



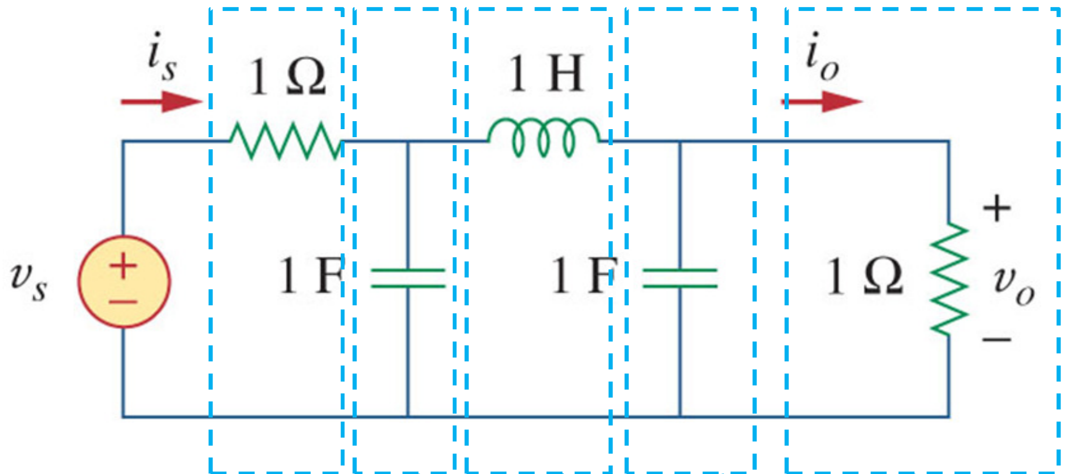
Answer: $$H(s) = \frac{1}{s^3 + 2s^2 + 3s + 2}$$

19.5 Transmission Parameters (11)

Transfer Function - Example

Problem 16.80(a) Solution:

a) Break up the circuit into a series of cascaded series and shunt components
b) Find the composite "T" parameters for the circuit
c) Use the relationship between the parameter "A" and the Transfer function



$$\begin{bmatrix} V_s \\ I_s \end{bmatrix} = \underbrace{\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ s & 1 \end{bmatrix}\begin{bmatrix} 1 & s \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ s & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}}_{\begin{bmatrix} A & B \\ C & D \end{bmatrix}}\begin{bmatrix} V_o \\ I_o \end{bmatrix}$$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} \rightarrow \boxed{A = \left.\frac{V_1}{V_2}\right|_{I_2=0} = \frac{1}{H(s)}}$$

19.5 Transmission Parameters (12)

Transfer Function - Example

Finding the combined T-matrix

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ s & 1 \end{bmatrix}\begin{bmatrix} 1 & s \\ 0 & 1 \end{bmatrix}\underbrace{\begin{bmatrix} 1 & 0 \\ s & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}}$$

$$\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ s & 1 \end{bmatrix}\underbrace{\begin{bmatrix} 1 & s \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ (s+1) & 1 \end{bmatrix}}$$

$$\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}\underbrace{\begin{bmatrix} 1 & 0 \\ s & 1 \end{bmatrix}\begin{bmatrix} 1+s(s+1) & s \\ (s+1) & 1 \end{bmatrix}}$$

$$\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1+s(s+1) & s \\ s+s^2(s+1)+(s+1) & s^2 \end{bmatrix}$$

$$\underbrace{\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} s^2+s+1 & s \\ s^3+s^2+2s+1 & s^2 \end{bmatrix}}$$

$$\begin{bmatrix} s^3+2s^2+3s+2 & s+s^2 \\ s^3+s^2+2s+1 & s^2 \end{bmatrix}$$

The transfer function can be found directly from the Transmission Parameter “A” !

$$\boxed{A = \left.\frac{V_1}{V_2}\right|_{I_2=0} = \frac{1}{H(s)}}$$

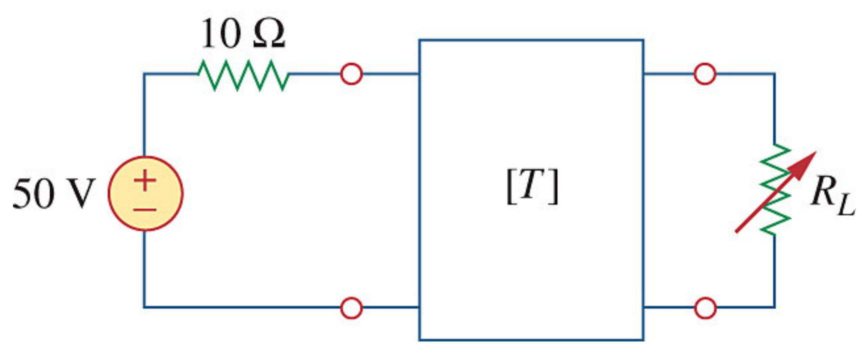
$$\boxed{H(s) = \frac{1}{s^3+2s^2+3s+2}}$$

19.5 Transmission Parameters (13)

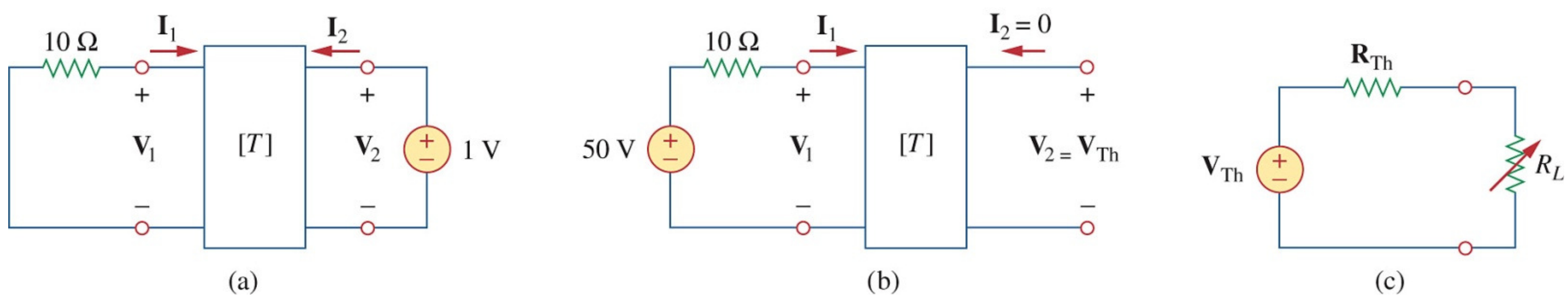
Example 19.9

The ABCD parameters of the two-port network at right are

$$T = \begin{bmatrix} 4 & 20\ \Omega \\ 0.1S & 2 \end{bmatrix}$$



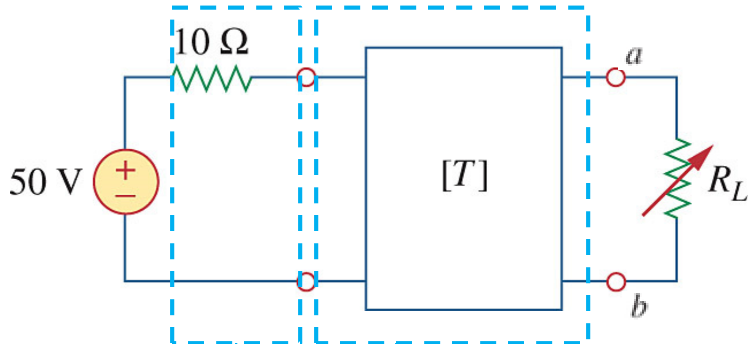
The output port is connected to a variable load for maximum power transfer. Find R_L and the maximum power transferred.



Answer: V_{TH} = 10V V; R_L = 8Ω; Pm = 3.125W.

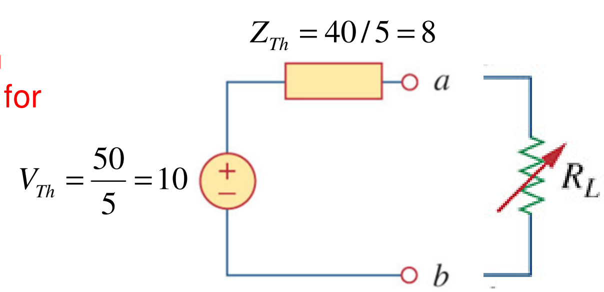
Solution: Example 19.9

a) Cascade the Series Resistor with the network
b) Find the composite "T" parameters for the circuit
c) Use the relationships to find V_{Th} and Z_{Th}



$$[T'] = \begin{bmatrix} 1 & 10 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 4 & 20 \\ 0.1 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 40 \\ 0.1 & 2 \end{bmatrix}$$

Find the Thevenin Equivalent Circuit for the source



For Max Power Transfer

$$\boxed{R_L = Z_{Th} = 8\ \Omega}$$

$$P_{\max} = I^2 R_L$$

$$P_{\max} = \left(\frac{V_{Th}}{R_L + Z_{Th}} \right)^2 R_L$$

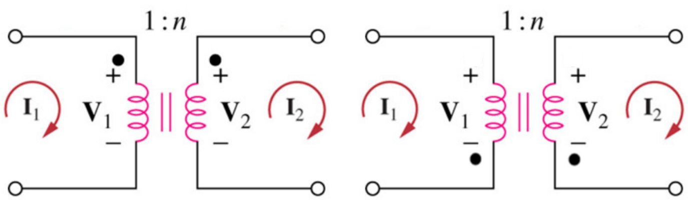
$$\boxed{P_{\max} = \left(\frac{10}{16} \right)^2 8 = 3.125\,\mathrm{W}}$$

19.5 Transmission Parameters (15)

Properties: Building Block Circuits – Ideal Transformer

- We can also use these "building blocks" to model ideal transformers. Remember from Chapter 13

Dots "Same" → "+"

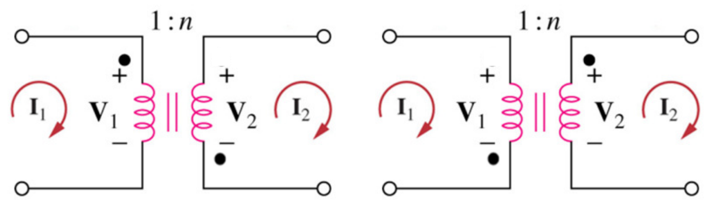


$$V_2 = nV_1$$

$$I_2 = \frac{I_1}{n}$$

$$\begin{bmatrix} \frac{1}{n} & 0 \\ 0 & n \end{bmatrix}$$

Dots "Different" → "-"



$$V_2 = -nV_1$$

$$I_2 = -\frac{I_1}{n}$$

$$\begin{bmatrix} -\frac{1}{n} & 0 \\ 0 & -n \end{bmatrix}$$

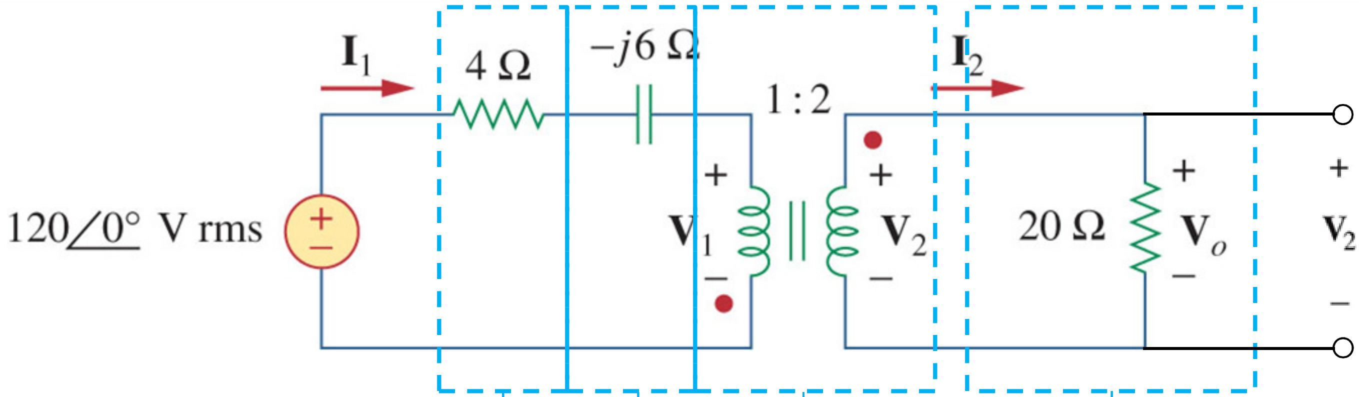
T - parameters

$$V_1 = AV_2 - BI_2$$

$$I_1 = CV_2 - DI_2$$

19.5 Transmission Parameters (16)

Example 13.8 Revisited



Cascaded T parameters

$$\begin{bmatrix}1 & 4\\0 & 1\end{bmatrix}\begin{bmatrix}1 & -j6\\0 & 1\end{bmatrix}\begin{bmatrix}-\frac{1}{2} & 0\\0 & -2\end{bmatrix}\quad\begin{bmatrix}1 & 0\\0.05 & 1\end{bmatrix}$$

a b c d

Using MATLAB:

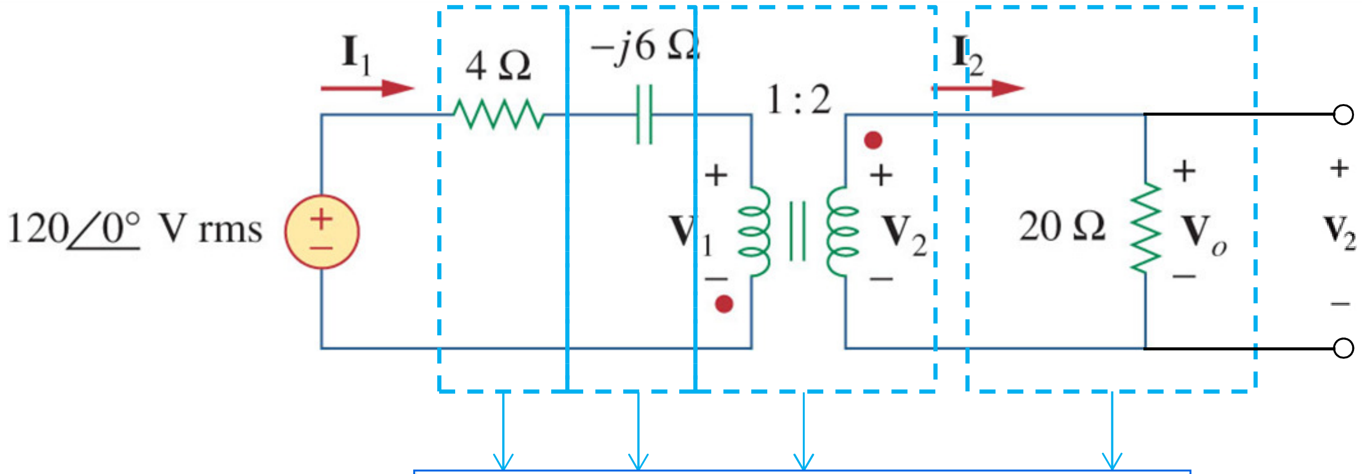
```
>> T=a*b*c*d

T =

  -0.9000 + 0.6000i  -8.0000 +12.0000i
  -0.1000 + 0.0000i  -2.0000 + 0.0000i
```

$$V_0 = V_2 = \frac{1}{A}V_1 = \left(\frac{1}{\boxed{-0.9+0.6j}}\right)(120\angle 0^\circ) = \boxed{110.94\angle -146.31^\circ \text{V}}$$

✓

19.5 Transmission Parameters (17)

Example 13.8 Revisited

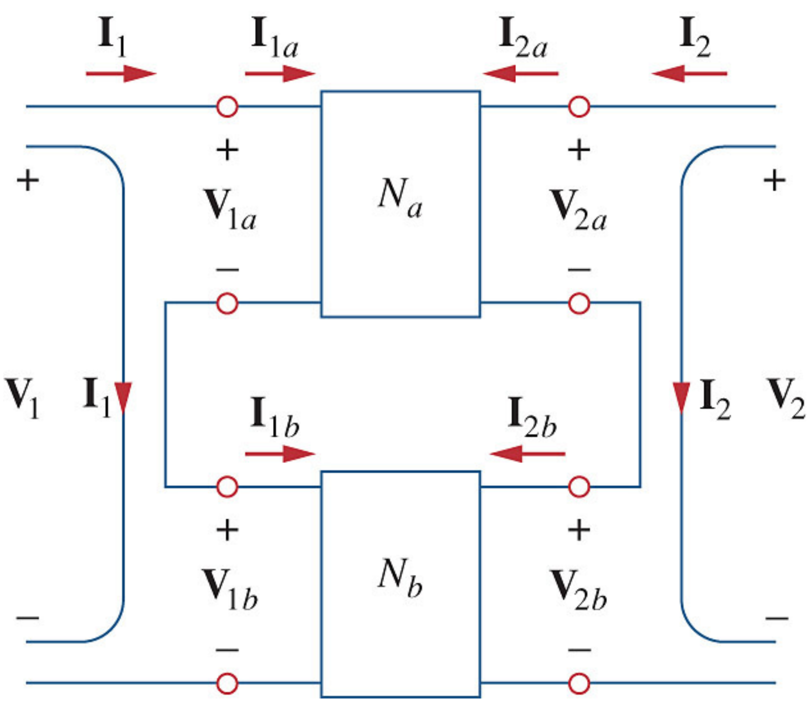



```
>> T=a*b*c*d

T =

   -0.9000 + 0.6000i  -8.0000 +12.0000i
   -0.1000 + 0.0000i  -2.0000 + 0.0000i
```

$$I_1 = CV_2 = \boxed{(-0.1)}(110.94\angle -146.31°) = \boxed{11.09\angle 33.69° \text{ A}}$$

$$I_2 = \frac{V_2}{20} = \frac{(110.94\angle -146.31°)}{20} = \boxed{5.55\angle -146.31° \text{ A}}$$

19.7 Interconnection of Networks (1)



Series Connection of two-port networks:

For Impedances; ADD matrices.

$Z = Z_a + Z_b$

19.7 Interconnection of Networks (2)

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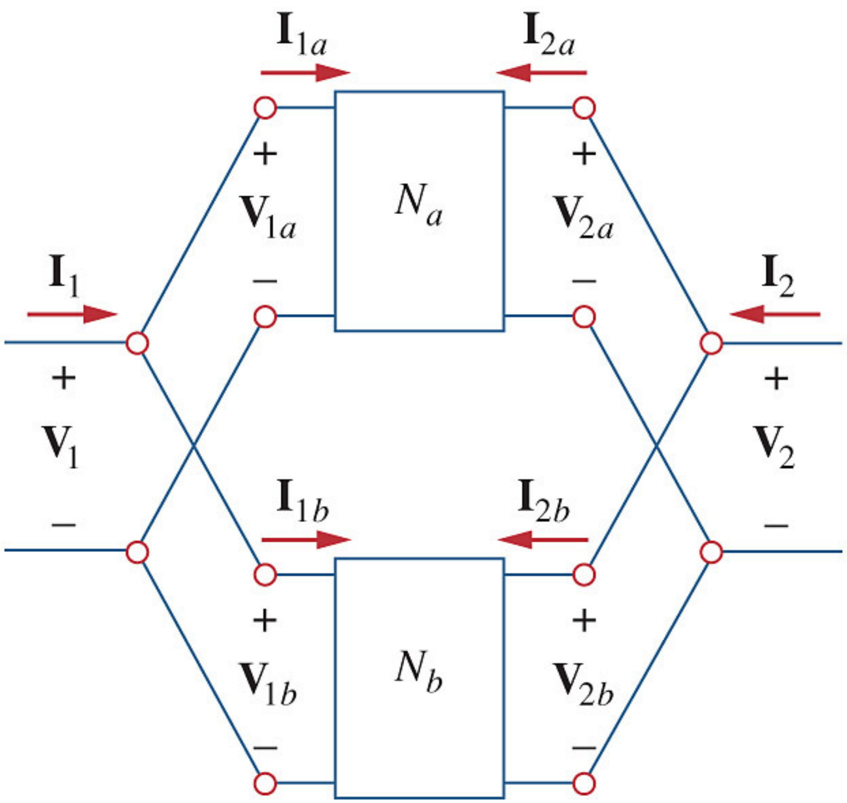


Parallel Connection of two-port networks:

For Admittances; ADD matrices.

$Y = Y_a + Y_b$

19.6 Relationships Between Networks

- Use this table to convert between two port parameters

	z		y		h		T	
z	z_{11}	z_{12}	$\frac{y_{22}}{\Delta_y}$	$-\frac{y_{12}}{\Delta_y}$	$\frac{\Delta_h}{h_{22}}$	$\frac{h_{12}}{h_{22}}$	$\frac{A}{C}$	$\frac{\Delta_T}{C}$
	z_{21}	z_{22}	$-\frac{y_{21}}{\Delta_y}$	$\frac{y_{11}}{\Delta_y}$	$-\frac{h_{21}}{h_{22}}$	$\frac{1}{h_{22}}$	$\frac{1}{C}$	$\frac{D}{C}$
y	$\frac{z_{22}}{\Delta_z}$	$-\frac{z_{12}}{\Delta_z}$	y_{11}	y_{12}	$\frac{1}{h_{11}}$	$-\frac{h_{12}}{h_{11}}$	$\frac{D}{B}$	$-\frac{\Delta_T}{B}$
	$-\frac{z_{21}}{\Delta_z}$	$\frac{z_{11}}{\Delta_z}$	y_{21}	y_{22}	$\frac{h_{21}}{h_{11}}$	$\frac{\Delta_h}{h_{11}}$	$-\frac{1}{B}$	$\frac{A}{B}$
h	$\frac{\Delta_z}{z_{22}}$	$\frac{z_{12}}{z_{22}}$	$\frac{1}{y_{11}}$	$-\frac{y_{12}}{y_{11}}$	h_{11}	h_{12}	$\frac{B}{D}$	$\frac{\Delta_T}{D}$
	$-\frac{z_{21}}{z_{22}}$	$\frac{1}{z_{22}}$	$\frac{y_{21}}{y_{11}}$	$\frac{\Delta_y}{y_{11}}$	h_{21}	h_{22}	$-\frac{1}{D}$	$\frac{C}{D}$
T	$\frac{z_{11}}{z_{21}}$	$\frac{\Delta_z}{z_{21}}$	$-\frac{y_{22}}{y_{21}}$	$-\frac{1}{y_{21}}$	$-\frac{\Delta_h}{h_{21}}$	$-\frac{h_{11}}{h_{21}}$	A	B
	$\frac{1}{z_{21}}$	$\frac{z_{22}}{z_{21}}$	$-\frac{\Delta_y}{y_{21}}$	$-\frac{y_{11}}{y_{21}}$	$-\frac{h_{22}}{h_{21}}$	$-\frac{1}{h_{21}}$	C	D

$\Delta_y = y_{11}y_{22} - y_{12}y_{21}$ $\quad \Delta_z = z_{11}z_{22} - z_{12}z_{21}$ $\quad \Delta_h = h_{11}h_{22} - h_{12}h_{21}$ $\quad \Delta_T = AD - BC$

Chapter 19 Review

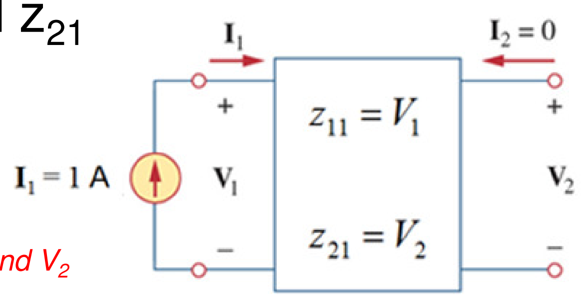
Z-Parameters

- Parameters: $\boxed{\begin{aligned} V_1 &= z_{11}I_1 + z_{12}I_2 \\ V_2 &= z_{21}I_1 + z_{22}I_2 \end{aligned}}$

- Open circuit the **output** to find z_{11} and z_{21}

$$V_1 = z_{11}I_1 + \cancel{z_{12}I_2}^{0} \qquad V_1 = z_{11}I_1$$
$$V_2 = z_{21}I_1 + \cancel{z_{22}I_2}^{0} \qquad V_2 = z_{21}I_1$$

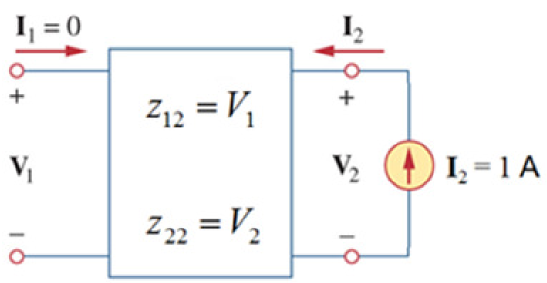
Set I_1 = 1 then solve for V_1 and V_2



- Open circuit the **input** to find z_{21} and z_{22}

$$V_1 = \cancel{z_{11}I_1}^{0} + z_{12}I_2 \qquad V_1 = z_{12}I_2$$
$$V_2 = \cancel{z_{21}I_1}^{0} + z_{22}I_2 \qquad V_2 = z_{22}I_2$$

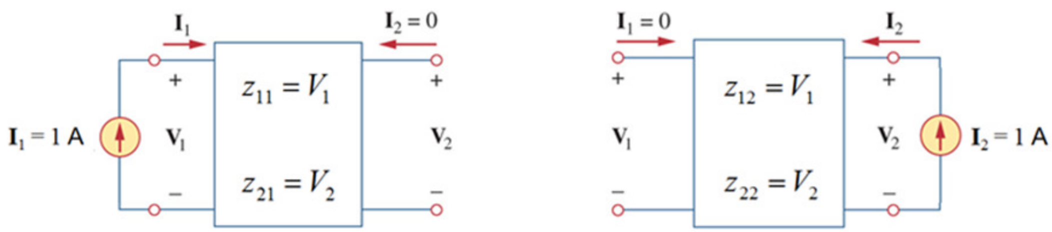
Set I_2 = 1 then solve for V_1 and V_2



Chapter 19 Review

Z-Parameters (Given a circuit, find Z-parameters)

- Solving problems to find z-parameters:
  1. Refer to definition, apply 1 amp source at input and output with opposite port left open (see previous slide)

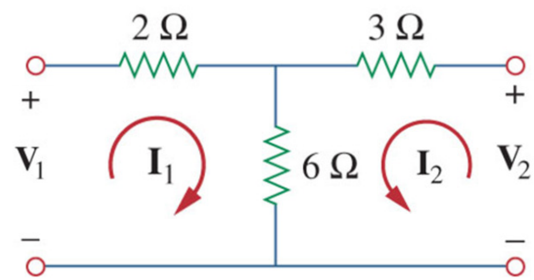


  2. Sometimes, KVL (mesh current equations) will cause z-parameters to fall right out! :

$$V_1 = 2I_1 + 6(I_1 + I_2) = 8I_1 + 6I_2$$
$$V_2 = 6(I_1 + I_2) + 3I_2 = 6I_1 + 9I_2$$

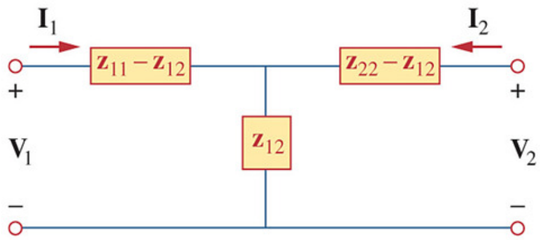
$$\Downarrow$$

$$z = \begin{bmatrix} 8 & 6 \\ 6 & 9 \end{bmatrix} \Omega$$

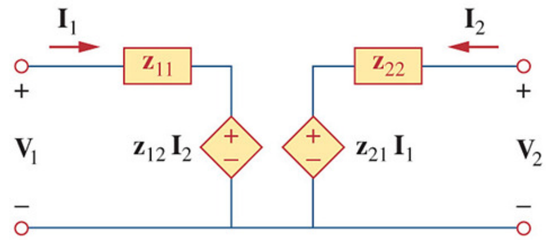


This mesh defined in counter clockwise direction for convenience

- If given, z-parameters can use following techniques to find other circuit parameters (V_1, V_2, I_1, I_2, etc.):
  1. Apply the model and solve the circuit:

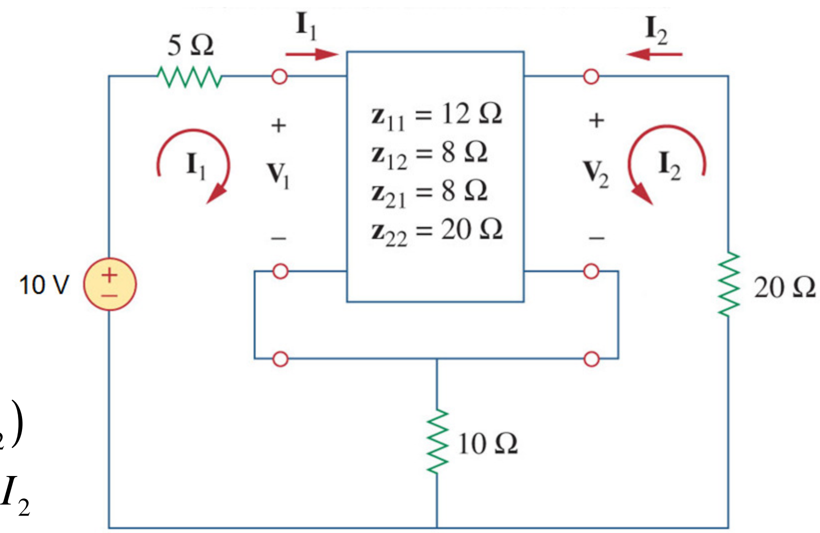


Reciprocal Network



General Network

  2. Substitute the defining equations into your analysis:



Mesh Analysis

$$10 = 5I_1 + V_1 + 10(I_1 + I_2)$$
$$0 = V_2 + 10(I_1 + I_2) + 20I_2$$

Substitute for V_1 and V_2

$$10 = 5I_1 + (12I_1 + 8I_2) + 10(I_1 + I_2)$$
$$0 = (8I_1 + 20I_2) + 10(I_1 + I_2) + 20I_2$$

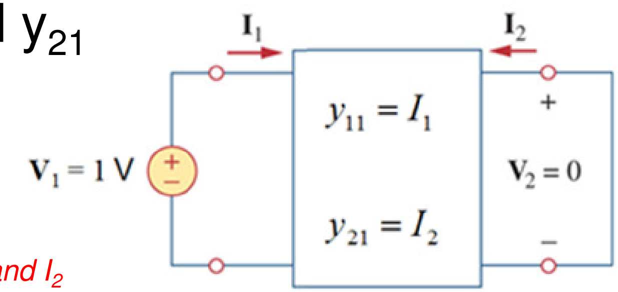
Chapter 19 Review
Y-Parameters

- Parameters:

$$\boxed{\begin{aligned} I_1 &= y_{11}V_1 + y_{12}V_2 \\ I_2 &= y_{21}V_1 + y_{22}V_2 \end{aligned}}$$

- Short circuit the **output** to find y_{11} and y_{21}

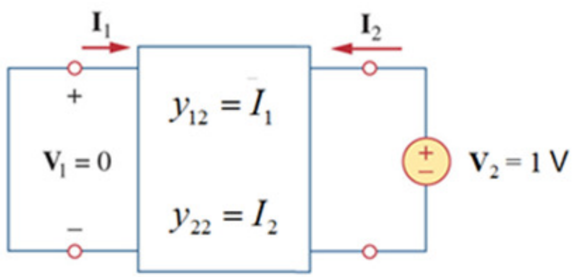
$$I_1 = y_{11}V_1 + \cancel{y_{12}V_2}^{\,0} \qquad I_2 = y_{21}V_1 + \cancel{y_{22}V_2}^{\,0} \quad \Rightarrow \quad I_1 = y_{11}V_1 \qquad I_2 = y_{21}V_1$$



Set V_1 = 1 then solve for I_1 and I_2

- Short circuit the **input** to find y_{21} and y_{22}

$$I_1 = \cancel{y_{11}V_1}^{\,0} + y_{12}V_2 \qquad I_2 = \cancel{y_{21}V_1}^{\,0} + y_{22}V_2 \quad \Rightarrow \quad I_1 = y_{12}V_2 \qquad I_2 = y_{22}V_2$$

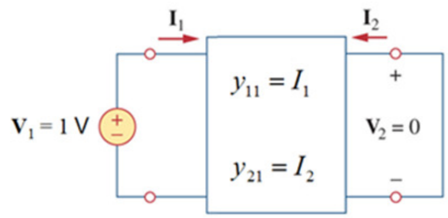


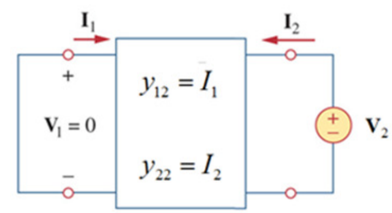
Set V_2 = 1 then solve for I_1 and I_2

Chapter 19 Review

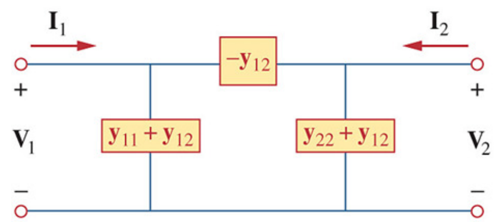
Y-Parameters (Solving Problems)

- To solve Y-parameter problems, can use these techniques
  1. Apply method from previous slide. Apply 1 Volt source at input and output while shorting opposite port

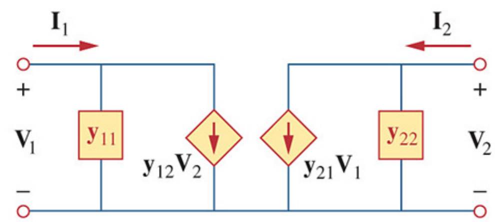




  2. If given Y parameters can apply the model and solve the circuit:



Reciprocal Network



General Network

  3. Make it easy on yourself! Use conversions from Z → Y or Y → Z

$$\begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix} = \left(\frac{1}{\Delta_y}\right)\begin{bmatrix} y_{22} & -y_{12} \\ -y_{21} & y_{11} \end{bmatrix}$$

$$\Delta_y = y_{11}y_{22} - y_{12}y_{21}$$

$$\begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} = \left(\frac{1}{\Delta_z}\right)\begin{bmatrix} z_{22} & -z_{12} \\ -z_{21} & z_{11} \end{bmatrix}$$

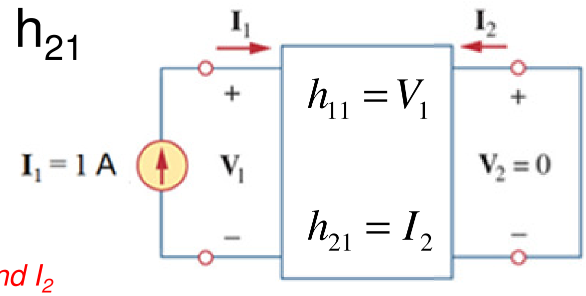
$$\Delta_z = z_{11}z_{22} - z_{12}z_{21}$$

- Parameters (hybrid of z and y):

$$\boxed{\begin{aligned} V_1 &= h_{11}I_1 + h_{12}V_2 \\ I_2 &= h_{21}I_1 + h_{22}V_2 \end{aligned}}$$

- Short circuit the **output** to find h_{11} and h_{21}

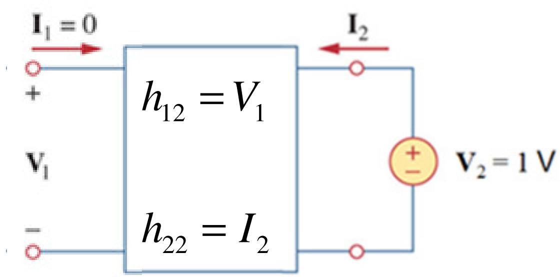
$$\begin{aligned} V_1 &= h_{11}I_1 + \cancelto{0}{h_{12}V_2} \\ I_2 &= h_{21}I_1 + \cancelto{0}{h_{22}V_2} \end{aligned} \quad \Rightarrow \quad \begin{aligned} V_1 &= h_{11}I_1 \\ I_2 &= h_{21}I_1 \end{aligned}$$



Set $I_1 = 1$ then solve for V_1 and I_2

- Open circuit the **input** to find h_{21} and h_{22}

$$\begin{aligned} V_1 &= \cancelto{0}{h_{11}I_1} + h_{12}V_2 \\ I_2 &= \cancelto{0}{h_{21}I_1} + h_{22}V_2 \end{aligned} \quad \Rightarrow \quad \begin{aligned} V_1 &= h_{12}V_2 \\ I_2 &= h_{22}V_2 \end{aligned}$$

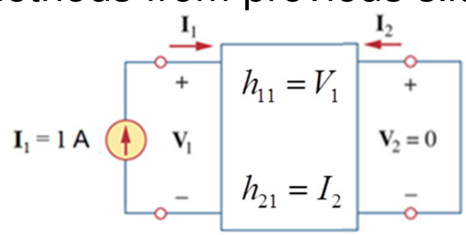


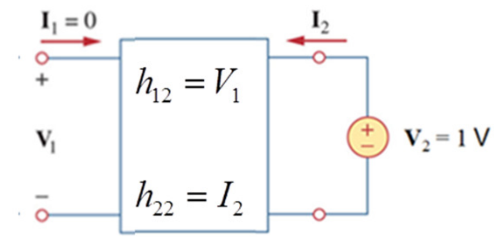
Set $V_2 = 1$ then solve for V_1 and I_2

Chapter 19 Review

H-Parameters (Solving Problems)

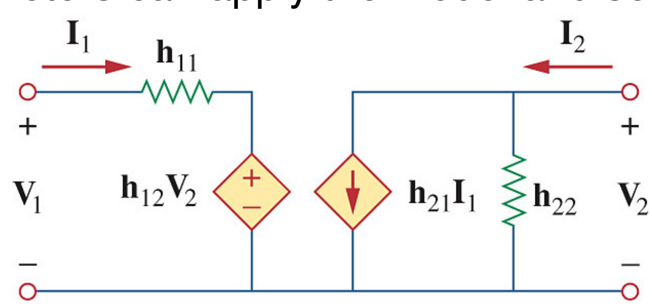
- To solve H-parameter problems, can use these techniques
  1. Apply methods from previous slide.





  2. H parameters can be found by performing a set of tests on the device
     a) Shorting the output and applying a current
     b) Leaving the input open and applying a voltage across the output

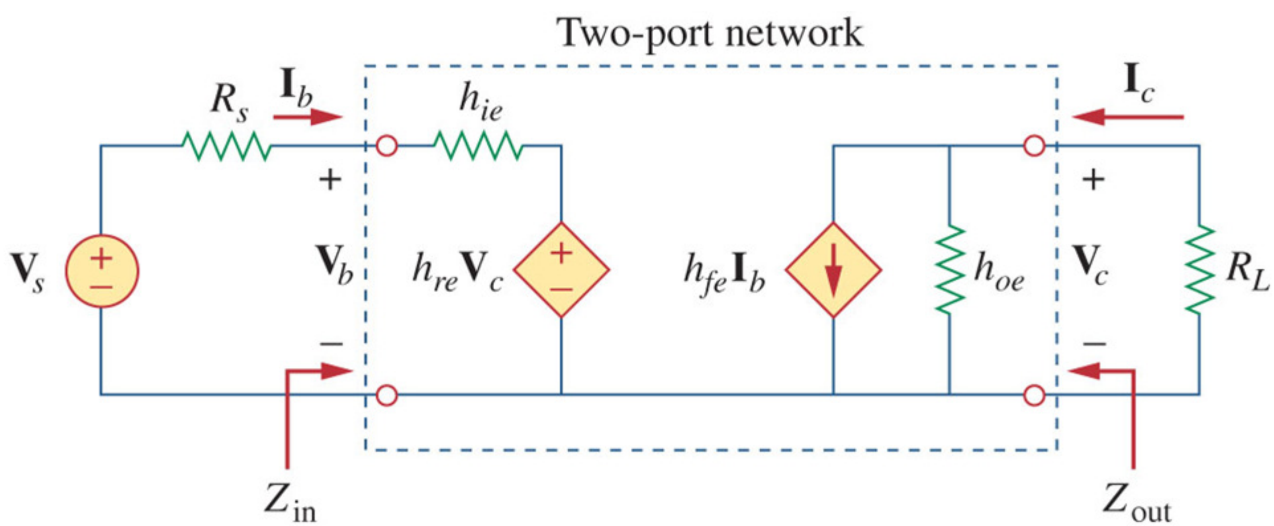
  3. If given H parameters can apply the model and solve the circuit:



  4. If helpful, use conversion tables

Chapter 19 Review
H-Parameters (Transistor Model)

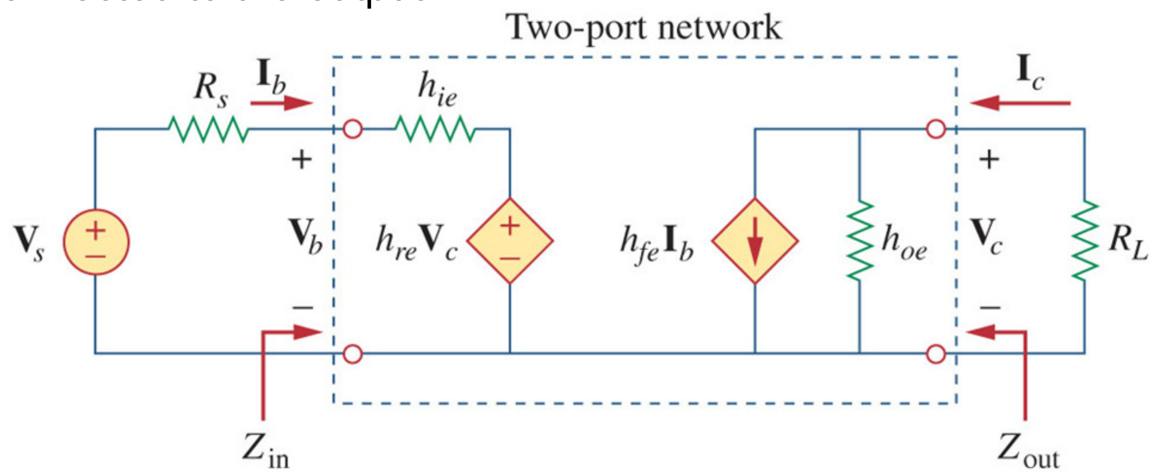
- H parameters are often used in modeling transistors
- Parameters vary depending on biasing conditions
- Spec sheets often use different subscripts:
  - h_{11} → h_{ie} = Base input impedance
  - h_{12} → h_{re} = Reverse voltage feedback ration
  - h_{21} → h_{fe} = Base-collector current gain
  - h_{22} → h_{oe} = Output admittance



Chapter 19 Review

H-Parameters (Transistor Model)

- Equations for calculating input impedance, output impedance, voltage gain, and current gain for simple transistor circuit:
  - V_s and R_s can be the Thevenin equivalent source driving the input.
  - R_L can be the input impedance looking into the load of the circuit connected to the output



Input Impedance

$$Z_{in} = \frac{V_b}{I_b} = h_{ie} - \frac{h_{re}h_{fe}R_L}{1+h_{oe}R_L}$$

Output Impedance

$$Z_{out} = \left.\frac{V_c}{I_c}\right|_{V_s=0} = \frac{R_s + h_{ie}}{(R_s + h_{ie})h_{oe} - h_{re}h_{fe}}$$

Current Gain

$$A_i = \frac{I_c}{I_b} = \frac{h_{fe}}{1+h_{oe}R_L}$$

Voltage Gain

$$A_v = \frac{V_c}{V_b} = \frac{-h_{fe}R_L}{h_{ie} + (h_{ie}h_{oe} - h_{re}h_{fe})R_L}$$

Chapter 19 Review

Transmission ("T") Parameters

- Parameters: $$\boxed{\begin{aligned} V_1 &= AV_2 - BI_2 \\ I_1 &= CV_2 - DI_2 \end{aligned}}$$

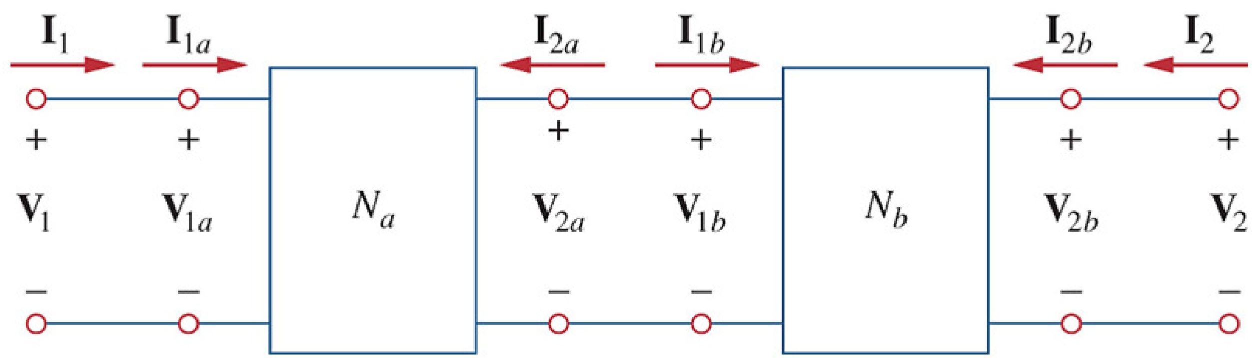
- Perform the analysis with the **output** Open Circuited (I_2=0)

$$\begin{aligned} V_1 &= AV_2 - B\cancel{I_2}^{\,0} \\ I_1 &= CV_2 - D\cancel{I_2}^{\,0} \end{aligned} \Rightarrow \begin{aligned} V_1 &= AV_2 \\ I_1 &= CV_2 \end{aligned} \Rightarrow \begin{aligned} A &= \frac{V_1}{V_2} \\ C &= \frac{I_1}{V_2} \end{aligned}$$

- Perform the analysis with the **output** Short Circuited(V_2=0)

$$\begin{aligned} V_1 &= A\cancel{V_2}^{\,0} - BI_2 \\ I_1 &= C\cancel{V_2}^{\,0} - DI_2 \end{aligned} \Rightarrow \begin{aligned} V_1 &= -BI_2 \\ I_1 &= -DI_2 \end{aligned} \Rightarrow \begin{aligned} B &= -\frac{V_1}{I_2} \\ D &= -\frac{I_1}{I_2} \end{aligned}$$

- Primary benefit of "T"-Parameters is their ability to be cascaded.



$$\begin{bmatrix} V_{1a} \\ I_{1a} \end{bmatrix} = \begin{bmatrix} A_a & B_a \\ C_a & D_a \end{bmatrix} \begin{bmatrix} V_{2a} \\ -I_{2a} \end{bmatrix} \qquad \begin{bmatrix} V_{1b} \\ I_{1b} \end{bmatrix} = \begin{bmatrix} A_b & B_b \\ C_b & D_b \end{bmatrix} \begin{bmatrix} V_{2b} \\ -I_{2b} \end{bmatrix}$$

$$-I_{2a} = I_{1b}$$

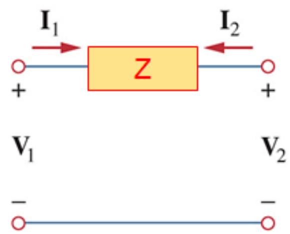
$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A_a & B_a \\ C_a & D_a \end{bmatrix} \begin{bmatrix} A_b & B_b \\ C_b & D_b \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A' & B' \\ C' & D' \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$

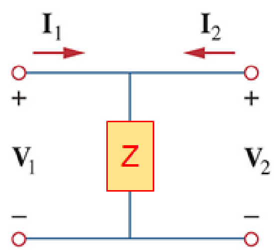
Chapter 19 Review

T - Parameters (Building Block models)

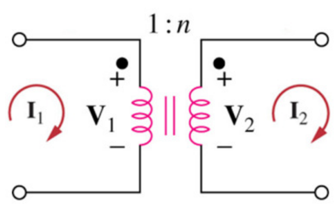
- We can create “building block” models of components by finding their T-parameters and use the cascading property to find the T-parameters for the complete circuit/system.



$$\begin{bmatrix} 1 & Z \\ 0 & 1 \end{bmatrix}$$



$$\begin{bmatrix} 1 & 0 \\ \frac{1}{Z} & 1 \end{bmatrix}$$



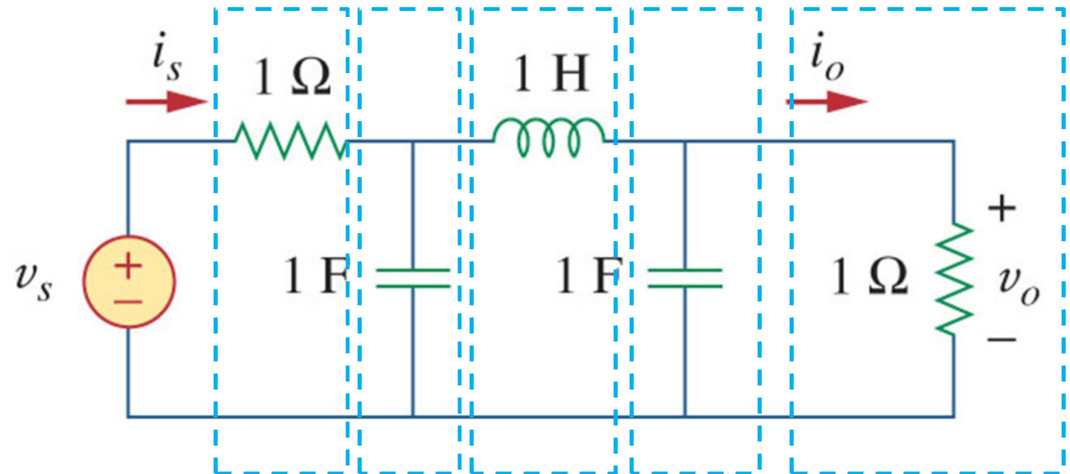
Dots “same”

$$\begin{bmatrix} \frac{1}{n} & 0 \\ 0 & n \end{bmatrix}$$

Dots “opposite”

$$\begin{bmatrix} -\frac{1}{n} & 0 \\ 0 & -n \end{bmatrix}$$

- With “Building Block” approach, circuits can be broke up into discrete components and analyzed using T-parameters



$$\begin{bmatrix} V_s \\ I_s \end{bmatrix} = \underbrace{\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ s & 1 \end{bmatrix}\begin{bmatrix} 1 & s \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ s & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}}_{\begin{bmatrix} A & B \\ C & D \end{bmatrix}}\begin{bmatrix} V_o \\ I_o \end{bmatrix}$$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} \longrightarrow \boxed{A = \left.\frac{V_1}{V_2}\right|_{I_2=0} = \frac{1}{H(s)}}$$

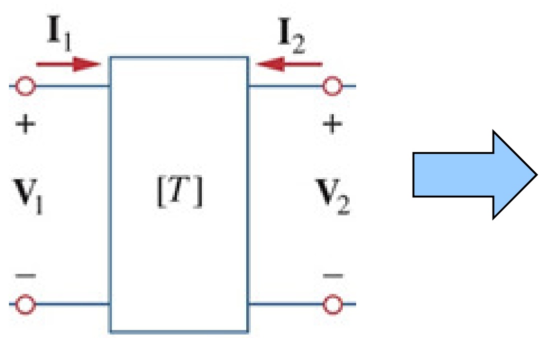
Chapter 19 Review

T - Parameters (Useful Properties)

- The T parameters give us useful properties in the analysis of circuits:
  - Open Circuit Voltage Transfer Function:

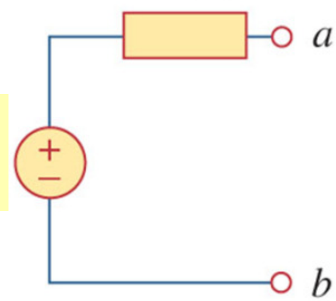
$$A = \left.\frac{V_1}{V_2}\right|_{I_2=0} = \frac{1}{H(s)} \qquad H(s) = \frac{1}{A}$$

  - Thevenin Equivalent Circuit (Replace circuit as a source)



$$Z_{Th} = B / A$$

$$V_{Th} = \frac{V_1}{A}$$



Chapter 19 Review

Conversion between Parameters

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- Conversion tables exists to convert between parameters

	z		**y**		**h**		**T**	
z	z_{11}	z_{12}	$\frac{y_{22}}{\Delta_y}$	$-\frac{y_{12}}{\Delta_y}$	$\frac{\Delta_h}{h_{22}}$	$\frac{h_{12}}{h_{22}}$	$\frac{A}{C}$	$\frac{\Delta_T}{C}$
	z_{21}	z_{22}	$-\frac{y_{21}}{\Delta_y}$	$\frac{y_{11}}{\Delta_y}$	$-\frac{h_{21}}{h_{22}}$	$\frac{1}{h_{22}}$	$\frac{1}{C}$	$\frac{D}{C}$
y	$\frac{z_{22}}{\Delta_z}$	$-\frac{z_{12}}{\Delta_z}$	y_{11}	y_{12}	$\frac{1}{h_{11}}$	$-\frac{h_{12}}{h_{11}}$	$\frac{D}{B}$	$-\frac{\Delta_T}{B}$
	$-\frac{z_{21}}{\Delta_z}$	$\frac{z_{11}}{\Delta_z}$	y_{21}	y_{22}	$\frac{h_{21}}{h_{11}}$	$\frac{\Delta_h}{h_{11}}$	$-\frac{1}{B}$	$\frac{A}{B}$
h	$\frac{\Delta_z}{z_{22}}$	$\frac{z_{12}}{z_{22}}$	$\frac{1}{y_{11}}$	$-\frac{y_{12}}{y_{11}}$	h_{11}	h_{12}	$\frac{B}{D}$	$\frac{\Delta_T}{D}$
	$-\frac{z_{21}}{z_{22}}$	$\frac{1}{z_{22}}$	$\frac{y_{21}}{y_{11}}$	$\frac{\Delta_y}{y_{11}}$	h_{21}	h_{22}	$-\frac{1}{D}$	$\frac{C}{D}$
T	$\frac{z_{11}}{z_{21}}$	$\frac{\Delta_z}{z_{21}}$	$-\frac{y_{22}}{y_{21}}$	$-\frac{1}{y_{21}}$	$-\frac{\Delta_h}{h_{21}}$	$-\frac{h_{11}}{h_{21}}$	A	B
	$\frac{1}{z_{21}}$	$\frac{z_{22}}{z_{21}}$	$-\frac{\Delta_y}{y_{21}}$	$-\frac{y_{11}}{y_{21}}$	$-\frac{h_{22}}{h_{21}}$	$-\frac{1}{h_{21}}$	C	D

$\Delta_y = y_{11}y_{22} - y_{12}y_{21}$ $\quad \Delta_z = z_{11}z_{22} - z_{12}z_{21}$ $\quad \Delta_h = h_{11}h_{22} - h_{12}h_{21}$ $\quad \Delta_T = AD - BC$