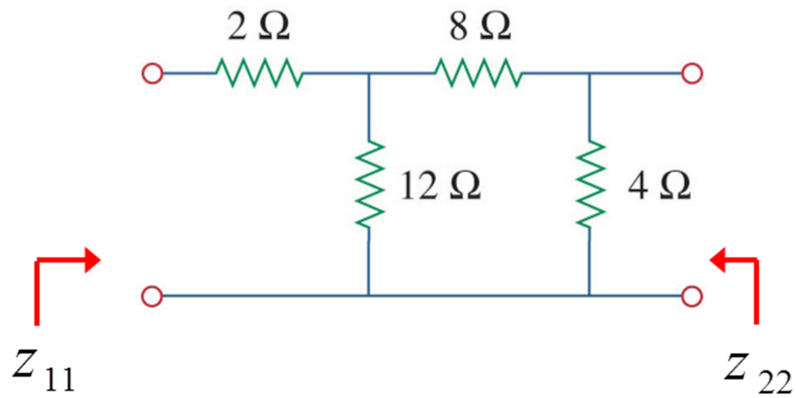
IUPUI ECE 202 Spring 2015:
Homework #10 (SOLUTIONS) Name:

1. (Prob. 19.1 in text) Obtain the z parameters for the network in the figure below:



First find z_{11} which is the open circuit input impedance:

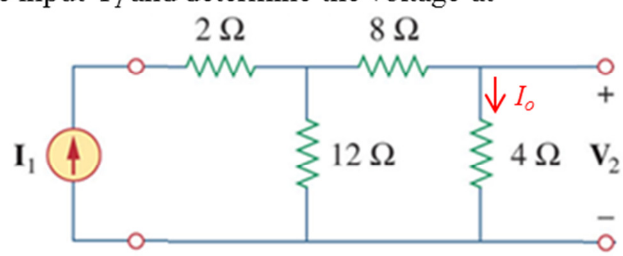
$$z_{11} = 2 + 12 \parallel (8+4) = 2 + \frac{12(12)}{12+12} = 8 \quad \Omega$$

Likewise find z_{22} which is the open circuit output impedance:

$$z_{22} = 4 \parallel (8+12) = \frac{4(20)}{4+20} = 3.333 \quad \Omega$$

To find z_{21} apply a current source to the input I_1 and determine the voltage at the output and V_2:

$$z_{21} = \left. \frac{V_2}{I_1} \right|_{I_2=0}$$



Use a current divider equation to find the current in the branch with the 8 Ω and 4 Ω resistors.

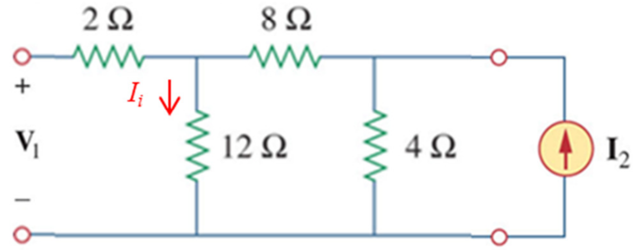
$$I_0 = \frac{12}{12+(4+8)} I_1 = \frac{1}{2} I_1$$

$$V_2 = 4I_0 = 4\left(\frac{1}{2} I_1\right) = 2I_1$$

$$z_{21} = \left. \frac{V_2}{I_1} \right|_{I_2=0} = \frac{2I_1}{I_1} = 2 \quad \Omega$$

To find z_{12} apply a current source to the output I_2 and determine the voltage at the input and V_1:

$$z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1=0}$$



Use a current divider equation to find the current in the branch with the 8 Ω and 12 Ω resistors.

$$I_i = \frac{4}{4+(8+12)} I_2 = \frac{1}{6} I_2$$

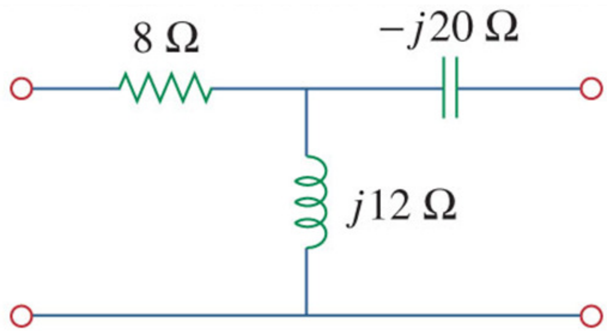
$$V_2 = 12 I_i = 12\left(\frac{1}{6} I_2\right) = 2 I_2$$

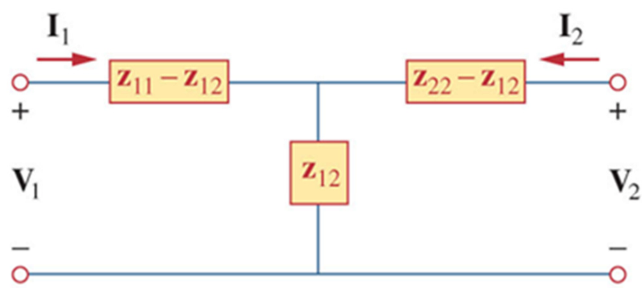
$$z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1=0} = \frac{2I_2}{I_2} = 2 \ \ \Omega$$

Note: we could have also used the fact that since the circuit contains all R's, L's, and C's and has no dependant sources that the circuit is reciprocal and thus $z_{12} = z_{21}$.

$$[z] = \begin{bmatrix} 8 & 2 \\ 2 & 3.333 \end{bmatrix} \ \Omega$$

2. (Prob. 19.3 from Text) Find the z parameters of the circuit below:



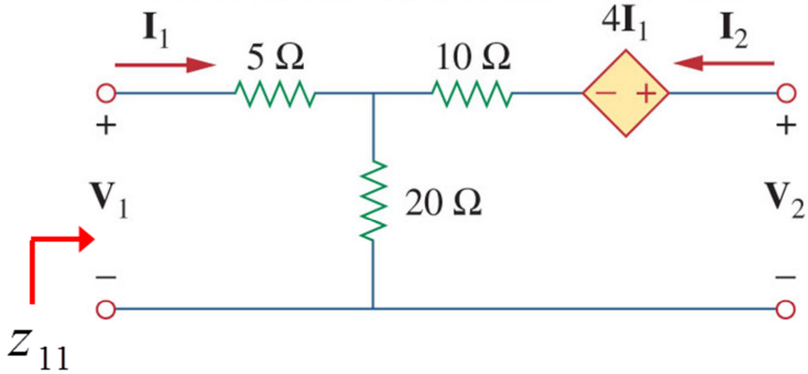


$$z_{12} = z_{21} = j12\ \Omega$$

$$z_{11} - z_{12} = 8 \Rightarrow z_{11} = 8 + z_{12} = 8 + j12\ \Omega$$

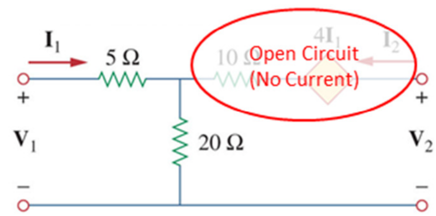
$$z_{22} - z_{12} = -j20 \Rightarrow z_{22} = -j20 + z_{12} = -j20 + j12 = -j8\ \Omega$$

3. (Prob. 19.6 from Text) Compute the z parameters of the circuit below:



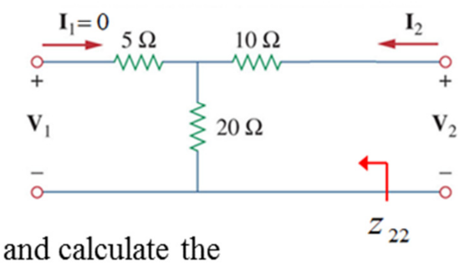
To find z_{11} note that the 10 Ω resistor and dependent source contribute nothing since they are part of an open branch.

$$z_{11} = 5 + 20 = 25 \;\; \Omega$$



To find z_{22} note that since I_1 is zero, the dependant source is 4(0) = 0 (short circuit) so the circuit can be replaced as follows if the input is open:

$$z_{22} = 10 + 20 = 30 \;\; \Omega$$

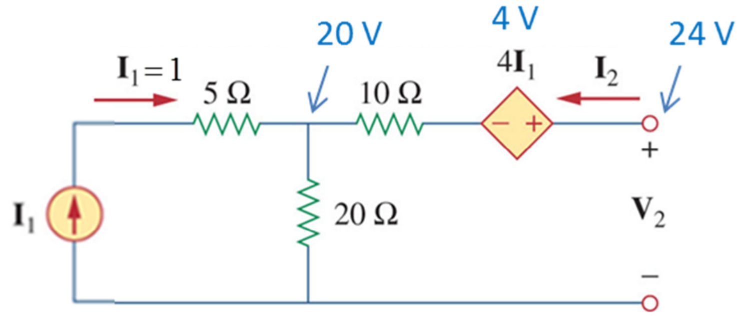


To find z_{21} we can place a 1 A current source at the input and calculate the output voltage V_2:

$$I_1 = 1$$

Voltage across the 20 Ω is just:

$$V_{20\,\Omega} = 20 I_1 = 20$$



There is no current in the open branch so there is no voltage drop across the 10 Ω resistor, however the dependant source will raise the voltage at the output by 4 volts (see figure)

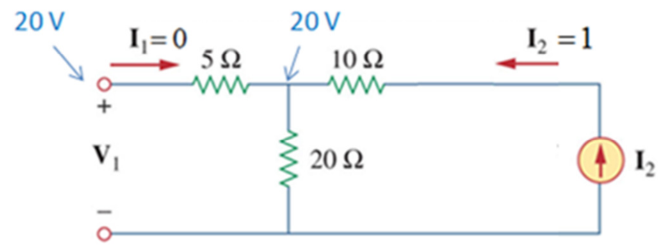
$$V_2 = 20 + 4I_1 = 24$$

$$z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0} = \frac{24}{1} = 24 \; \Omega$$

To find z_{12} we can place a 1 A current source at the output and calculate the input voltage V_1:

Voltage across the 20 Ω is just:

$$V_{20\,\Omega} = 20I_1 = 20$$



There is no current in the open branch so there is no voltage drop across the 5 Ω resistor, therefore the input voltage is the same as the voltage across the 20 Ω resistor.

$$V_2 = V_{20\,\Omega} = 20$$

$$z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1=0} = \frac{20}{1} = 20\ \Omega$$

$$[z] = \begin{bmatrix} 25 & 20 \\ 24 & 30 \end{bmatrix}\ \Omega$$