

Chapter 5 Advanced Encryption Standard



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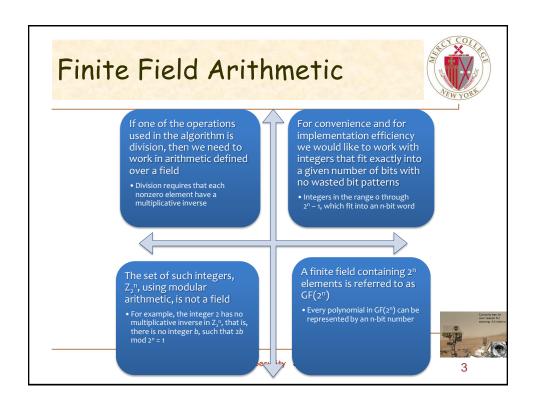
Finite Field Arithmetic

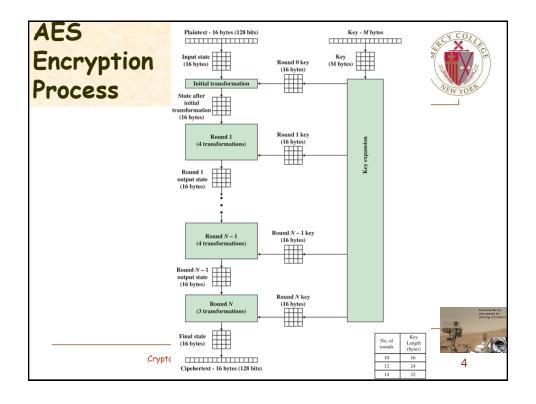


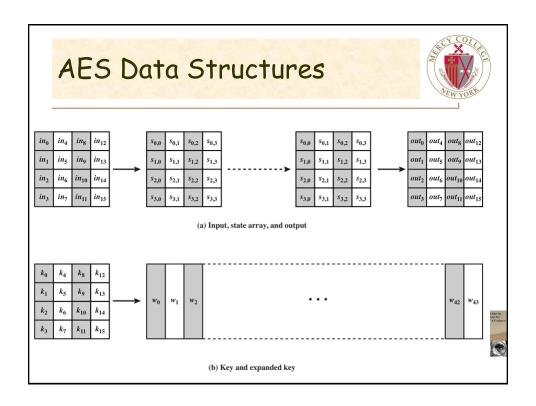
- In the Advanced Encryption Standard (AES) all operations are performed on 8-bit bytes
- The arithmetic operations of addition, multiplication, and division are performed over the finite field GF(28)
- A field is a set in which we can do addition, subtraction, multiplication, and division without leaving the set
- Division is defined with the following rule:
 - $a/b = a(b^{-1})$
- An example of a finite field (one with a finite number of elements) is the set Z_p consisting of all the integers $\{0,1,\ldots,p-1\}$, where p is a prime number and in which arithmetic is carried out modulo p



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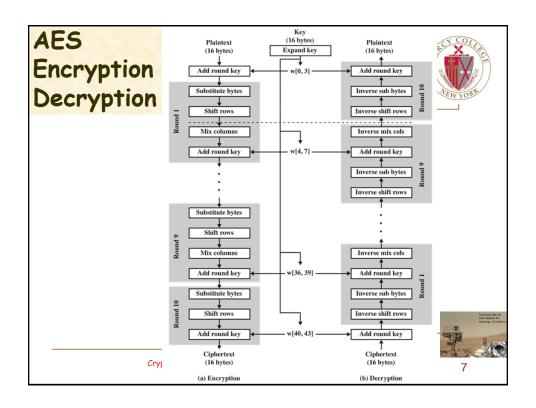
AES Parameters

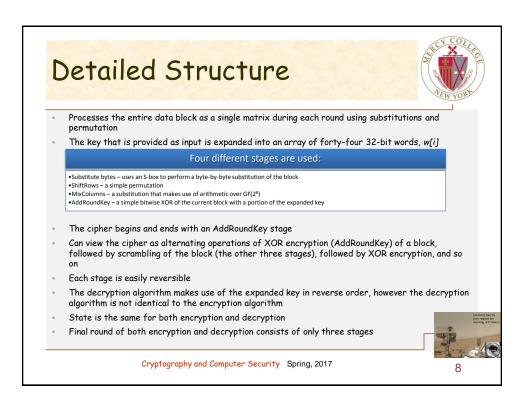


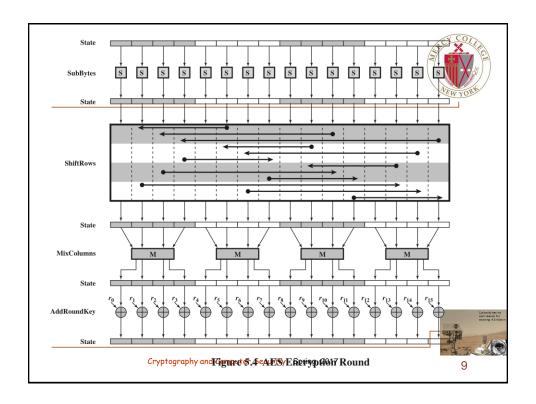
Key Size (words/bytes/bits)	4/16/128	6/24/192	8/32/256	
Plaintext Block Size (words/bytes/bits)	4/16/128	4/16/128	4/16/128	
Number of Rounds	10	12	14	
Round Key Size (words/bytes/bits)	4/16/128	4/16/128	4/16/128	
Expanded Key Size (words/bytes)	44/176	52/208	60/240	

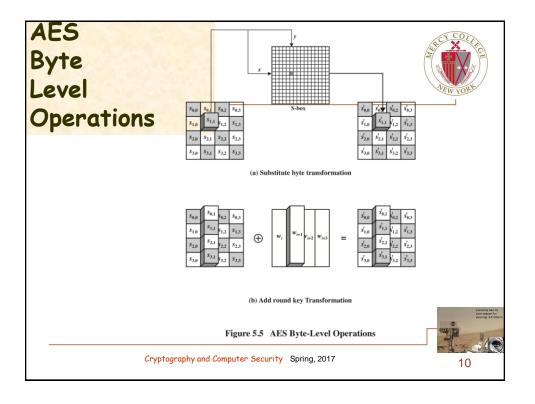


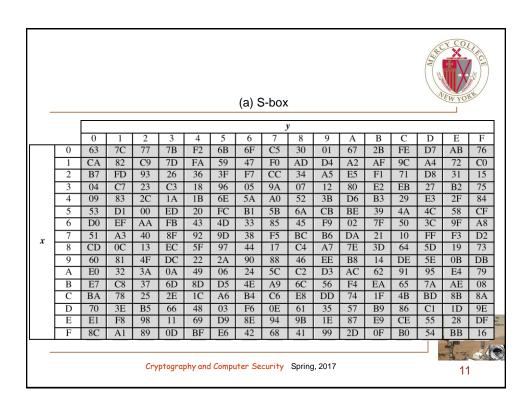
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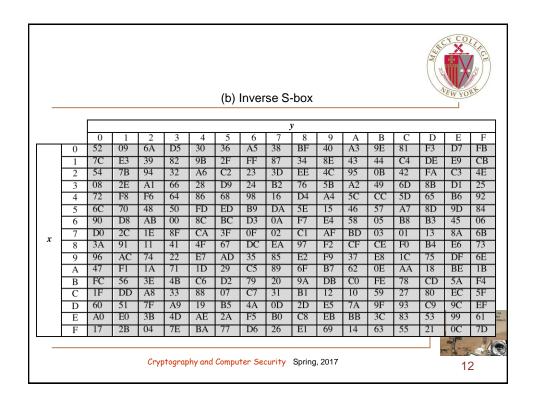


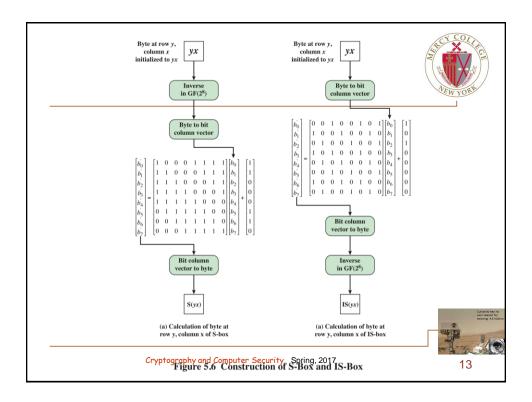












S-Box Rationale

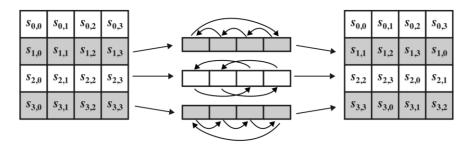


- The S-box is designed to be resistant to known cryptanalytic attacks
- The Rijndael developers sought a design that has a low correlation between input bits and output bits and the property that the output is not a linear mathematical function of the input
- The nonlinearity is due to the use of the multiplicative inverse

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Shift Row Transformation





(a) Shift row transformation

Figure 5.7 AES Row and Column Operations



(Figure can be found on page 144 in textbook) Spring, 2017

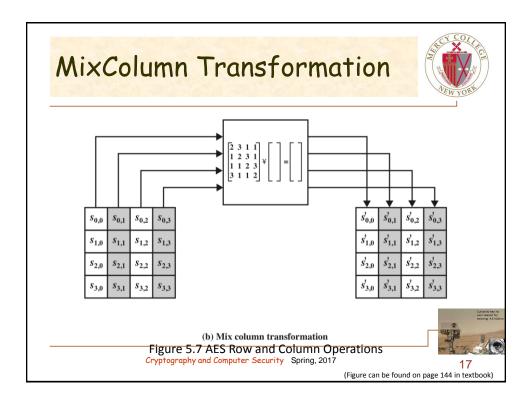
15

Shift Row Rationale



- More substantial than it may first appear
- The State, as well as the cipher input and output, is treated as an array of four 4-byte columns
- On encryption, the first 4 bytes of the plaintext are copied to the first column of State, and so on
- The round key is applied to State column by column
 - Thus, a row shift moves an individual byte from one column to another, which is a linear distance of a multiple of 4 bytes
- Transformation ensures that the 4 bytes of one column are spread out to four different columns

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Mix Columns Rationale



- Coefficients of a matrix based on a linear code with maximal distance between code words ensures a good mixing among the bytes of each column
- The mix column transformation combined with the shift row transformation ensures that after a few rounds all output bits depend on all input bits



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AddRoundKey Transformation



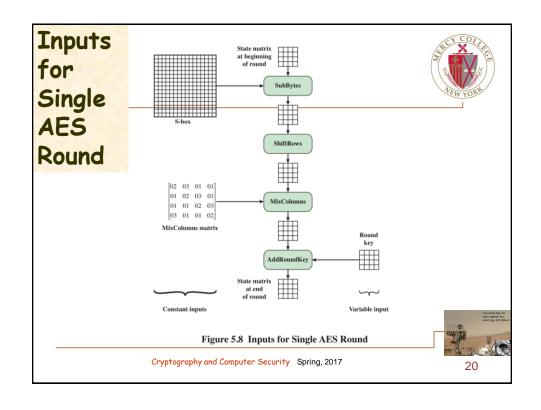
- The 128 bits of State are bitwise XORed with the 128 bits of the round key
- Operation is viewed as a columnwise operation between the 4 bytes of a State column and one word of the round key
 - Can also be viewed as a byte-level operation

Rationale:

Is as simple as possible and affects every bit of State

The complexity of the round key expansion plus the complexity of the other stages of AES ensure security

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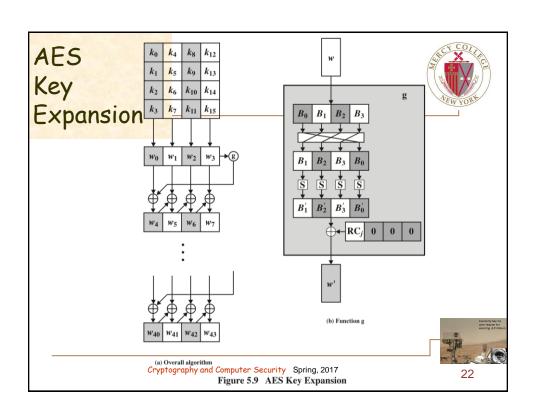


AES Key Expansion



- Takes as input a four-word (16 byte) key and produces a linear array of 44 words (176) bytes
 - This is sufficient to provide a four-word round key for the initial AddRoundKey stage and each of the 10 rounds of the cipher
- Key is copied into the first four words of the expanded key
 - The remainder of the expanded key is filled in four words at a time
- Each added word w[i] depends on the immediately preceding word, w[i-1], and the word four positions back, w[i-4]
 - In three out of four cases a simple XOR is used
 - For a word whose position in the w array is a multiple of 4, a more complex function is used

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Key Expansion Rationale



- The Rijndael developers designed the expansion key algorithm to be resistant to known cryptanalytic attacks
- Inclusion of a rounddependent round constant eliminates the symmetry between the ways in which round keys are generated in different rounds

The specific criteria that were used are:

- Knowledge of a part of the cipher key or round key does not enable calculation of many other round-key hits.
- An invertible transformation
- Speed on a wide range of processors
- Usage of round constants to eliminate symmetries
- Diffusion of cipher key differences into the round keys
- Enough nonlinearity to prohibit the full determination of round key differences from cipher key differences only
- Simplicity of description

7-34

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AES
Example
Key
Expansion

(Table is located on page 15

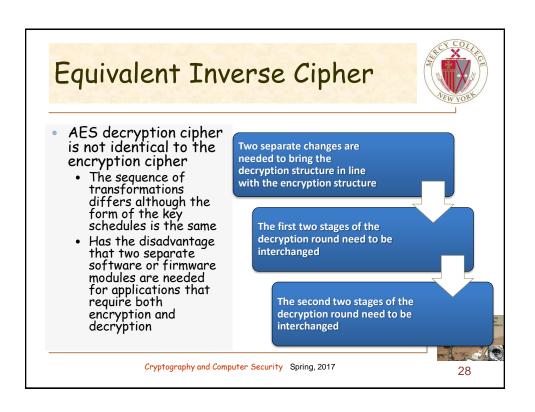
in textbook)

Key Words	Auxiliary Function	
w0 = 0f 15 71 c9	RotWord(w3)= 7f 67 98 af = x1	V C
w1 = 47 d9 e8 59	SubWord(x1)= d2 85 46 79 = y1	-
w2 = 0c b7 ad d6	Rcon(1)= 01 00 00 00	
w3 = af 7f 67 98	y1 Rcon(1) = d3 85 46 79 = z1	*
$w4 = w0 \oplus z1 = dc \ 90 \ 37 \ b0$	RotWord(w7)= 81 15 a7 38 = x2	HIREY
w5 = w4 ⊕ w1 = 9b 49 df e9	SubWord(x4)= 0c 59 5c 07 = y2	A III
$w6 = w5 \oplus w2 = 97 \text{ fe } 72 \text{ 3f}$	Rcon(2)= 02 00 00 00	WI/
w7 = w6 ⊕ w3 = 38 81 15 a7	$y2 \oplus Rcon(2) = 0e 59 5c 07 = z2$	EW
w8 = w4 \oplus z2 = d2 c9 6b b7	RotWord(w11)= ff d3 c6 e6 = x3	EW)
$w9 = w8 \oplus w5 = 49 \ 80 \ b4 \ 5e$	SubWord(x2)= 16 66 b4 8e = y3	
w10 = w9 ⊕ w6 = de 7e c6 61	Rcon(3)= 04 00 00 00	
$w11 = w10 \oplus w7 = e6 \text{ ff d3 c6}$	y3 Rcon(3) = 12 66 b4 8e = z3	
$w12 = w8 \oplus z3 = c0$ af df 39	RotWord(w15)= ae 7e c0 b1 = x4	
w13 = w12 \oplus w9 = 89 2f 6b 67	SubWord(x3) = e4 f3 ba c8 = y4	
$w14 = w13 \oplus w10 = 57 51 ad 06$	Rcon(4)= 08 00 00 00	
w15 = w14 \oplus w11 = b1 ae 7e c0	y4 Rcon(4) = ec f3 ba c8 = 4	
w16 = w12 ⊕ z4 = 2c 5c 65 f1	RotWord(w19)= 8c dd 50 43 = x5	
$w17 = w16 \oplus w13 = a5 73 0e 96$	SubWord(x4)= 64 c1 53 la = y5	
w18 = w17 \oplus w14 = f2 22 a3 90	Rcon(5)= 10 00 00 00	
w19 = w18 \oplus w15 = 43 8c dd 50	y5 ① Rcon(5)= 74 cl 53 la = z5	
w20 = w16 \oplus z5 = 58 9d 36 eb	RotWord(w23)= 40 46 bd 4c = x6	
$w21 = w20 \oplus w17 = fd ee 38 7d$	SubWord(x5)= 09 5a 7a 29 = y6	
w22 = w21 \oplus w18 = 0f cc 9b ed	Rcon(6)= 20 00 00 00	
w23 = w22 \oplus w19 = 4c 40 46 bd	y6 Rcon(6) = 29 5a 7a 29 = z6	
$w24 = w20 \oplus z6 = 71 c7 4c c2$	RotWord(w27)= a5 a9 ef cf = x7	
$w25 = w24 \oplus w21 = 8c 29 74 bf$	SubWord(x6)= 06 d3 df 8a = y7	
$w26 = w25 \oplus w22 = 83 \text{ e5 ef } 52$	Rcon(7)= 40 00 00 00	
$w27 = w26 \oplus w23 = cf a5 a9 ef$	y7 Rcon(7)= 46 d3 df 8a = z7	
$w28 = w24 \oplus z7 = 37 14 93 48$	RotWord(w31)= 7d a1 4a f7 = x8	
$w29 = w28 \oplus w25 = bb \ 3d \ e7 \ f7$	SubWord($x7$)= ff 32 d6 68 = y8	
$w30 = w29 \oplus w26 = 38 d8 08 a5$	Rcon(8)= 80 00 00 00	
w31 = w30 ⊕ w27 = f7 7d al 4a	y8 Rcon(8) = 7f 32 d6 68 = z8	
$w32 = w28 \oplus z8 = 48 \ 26 \ 45 \ 20$	RotWord(w35)= be 0b 38 3c = x9	
$w33 = w32 \oplus w29 = f3 \ 1b \ a2 \ d7$	SubWord(x8) = ae 2b 07 eb = y9	
$w34 = w33 \oplus w30 = cb c3 aa 72$	Rcon(9)= 1B 00 00 00	
$w35 = w34 \oplus w32 = 3c be 0b 38$	y9 ⊕ Rcon(9)= b5 2b 07 eb = z9	
w36 = w32 \oplus z9 = fd 0d 42 cb	RotWord(w39)= 6b 41 56 f9 = x10	
w37 = w36 ⊕ w33 = 0e 16 e0 1c	SubWord(x9)= 7f 83 b1 99 = y10	10,00
w38 = w37 \oplus w34 = c5 d5 4a 6e	Rcon(10)= 36 00 00 00	2
w39 = w38 \oplus w35 = f9 6b 41 56	y10 ⊕ Rcon(10)= 49 83 b1 99 = z10	
$w40 = w36 \oplus z10 = b4 \ 8e \ f3 \ 52$		7
w41 = w40 ⊕ w37 = ba 98 13 4e		-11
1 42 = w41 ⊕ w38 = 7f 4d 59 20		
w43 = w42 \oplus w39 = 86 26 18 76		

	Start of round	4.61	After	After	D 1 V	1
	Start of round	After SubBytes	ShiftRows	MixColumns	Round Key	0.50
	01 89 fe 76	Subbytes	Silitrows	MIXCOILIIIIS	Of 47 Oc af	CYCOLI
AES Example	23 ab dc 54				15 d9 b7 7f	ST ST
ALS Example	45 cd ba 32				71 e8 ad 67	
•	67 ef 98 10				c9 59 d6 98	HEACT
	0e ce f2 d9	ab 8b 89 35	ab 8b 89 35	b9 94 57 75	dc 9b 97 38	
	36 72 6b 2b	05 40 7f f1	40 7f fl 05	e4 8e 16 51	90 49 fe 81	
	34 25 17 55	18 3f f0 fc	f0 fc 18 3f	47 20 9a 3f	37 df 72 15	VENDRY
	ae b6 4e 88 65 0f c0 4d	e4 4e 2f c4 4d 76 ba e3	c4 e4 4e 2f 4d 76 ba e3	c5 d6 f5 3b 8e 22 db 12	b0 e9 3f a7 d2 49 de e6	VEW YORK
	74 c7 e8 d0	92 c6 9b 70	c6 9b 70 92	b2 f2 dc 92	c9 80 7e ff	
	70 ff e8 2a	51 16 9b e5	9b e5 51 16	df 80 f7 c1	6b b4 c6 d3	1
	75 3f ca 9c	9d 75 74 de	de 9d 75 74	2d c5 le 52	b7 5e 61 c6]
	5c 6b 05 f4	4a 7f 6b bf	4a 7f 6b bf	bl cl Ob cc	c0 89 57 b1	1
	7b 72 a2 6d	21 40 3a 3c	40 3a 3c 21	ba f3 8b 07	af 2f 51 ae	
	b4 34 31 12 9a 9b 7f 94	8d 18 c7 c9 b8 14 d2 22	c7 c9 8d 18 22 b8 14 d2	f9 1f 6a c3 1d 19 24 5c	df 6b ad 7e 39 67 06 c0	
	71 48 5c 7d	a3 52 4a ff	a3 52 4a ff	d4 11 fe 0f	2c a5 f2 43	1
	15 dc da a9	59 86 57 d3	86 57 d3 59	3b 44 06 73	5c 73 22 8c	1
	26 74 c7 bd	f7 92 c6 7a	c6 7a f7 92	cb ab 62 37	65 0e a3 dd	1
	24 7e 22 9c	36 f3 93 de	de 36 f3 93	19 b7 07 ec	f1 96 90 50]
	f8 b4 0c 4c	41 8d fe 29	41 8d fe 29	2a 47 c4 48	58 fd 0f 4c	1
	67 37 24 ff ae a5 c1 ea	85 9a 36 16 e4 06 78 87	9a 36 16 85 78 87 e4 06	83 e8 18 ba 84 18 27 23	9d ee cc 40 36 38 9b 46	1
	e8 21 97 bc	9b fd 88 65	65 9b fd 88	eb 10 0a f3	eb 7d ed bd	
	72 ba cb 04	40 f4 1f f2	40 f4 1f f2	7b 05 42 4a	71 8c 83 cf	1
	le 06 d4 fa	72 6f 48 2d	6f 48 2d 72	le d0 20 40	c7 29 e5 a5	1
	b2 20 bc 65	37 b7 65 4d	65 4d 37 b7	94 83 18 52	4c 74 ef a9	1
	00 6d e7 4e	63 3c 94 2f	2f 63 3c 94	94 c4 43 fb	c2 bf 52 ef	4
	0a 89 cl 85 d9 f9 c5 e5	67 a7 78 97 35 99 a6 d9	67 a7 78 97 99 a6 d9 35	ec 1a c0 80 0c 50 53 c7	37 bb 38 f7 14 3d d8 7d	1
	d8 f7 f7 fb	61 68 68 Of	68 Of 61 68	3b d7 00 ef	93 e7 08 a1	1
	56 7b 11 14	b1 21 82 fa	fa b1 21 82	b7 22 72 e0	48 f7 a5 4a	1
	db al f8 77	b9 32 41 f5	b9 32 41 f5	bl la 44 17	48 f3 cb 3c	1
	18 6d 8b ba	ad 3c 3d f4	3c 3d f4 ad	3d 2f ec b6	26 1b c3 be	
	a8 30 08 4e ff d5 d7 aa	c2 04 30 2f 16 03 0e ac	30 2f c2 04 ac 16 03 0e	0a 6b 2f 42 9f 68 f3 b1	45 a2 aa 0b 20 d7 72 38	
	ff d5 d7 aa f9 e9 8f 2b	99 1e 73 f1	99 1e 73 f1	31 30 3a c2	fd 0e c5 f9	1
	1b 34 2f 08	af 18 15 30	18 15 30 af	ac 71 8c c4	0d 16 d5 6b	
	4f c9 85 49	84 dd 97 3b	97 3b 84 dd	46 65 48 eb	42 e0 4a 41	1
	bf bf 81 89	08 08 0c a7	a7 08 08 0c	6a 1c 31 62	cb 1c 6e 56	Curiosity has its own reason for
	cc 3e ff 3b	4b b2 16 e2	4b b2 16 e2	4b 86 8a 36	b4 ba 7f 86	existing -A EinScein
	a1 67 59 af 04 85 02 aa	32 85 cb 79 f2 97 77 ac	85 cb 79 32 77 ac f2 97	b1 cb 27 5a fb f2 f2 af	8e 98 4d 26 f3 13 59 18	SERVING
	al 00 5f 34	32 63 cf 18	18 32 63 cf	cc 5a 5b cf	52 4e 20 76	
	ff 08 69 64				10 20 70	
	0b 53 34 14					
Cryptogra	84 bf ab 8f					25
1	4a 7c 43 b9					

	Round		Number of Bits that Differ	X COLL
Avalanche		0123456789abcdeffedcba9876543210 0023456789abcdeffedcba9876543210	1	HIGH X
Effect	0	0e3634aece7225b6f26b174ed92b5588 0f3634aece7225b6f26b174ed92b5588	1	VEWYORK
211000	1	657470750fc7ff3fc0e8e8ca4dd02a9c c4a9ad090fc7ff3fc0e8e8ca4dd02a9c	20	
in AES:	2	5c7bb49a6b72349b05a2317ff46d1294 fe2ae569f7ee8bb8c1f5a2bb37ef53d5	58	
Change in	3	7115262448dc747e5cdac7227da9bd9c ec093dfb7c45343d689017507d485e62	59	
Change in	4	f867aee8b437a5210c24c1974cffeabc 43efdb697244df808e8d9364ee0ae6f5	61	
Plaintext	5	721eb200ba06206dcbd4bce704fa654e 7b28a5d5ed643287e006c099bb375302	68	
	6	0ad9d85689f9f77bc1c5f71185e5fb14 3bc2d8b6798d8ac4fe36a1d891ac181a	64	
	7	db18a8ffa16d30d5f88b08d777ba4eaa 9fb8b5452023c70280e5c4bb9e555a4b	67	
	8	f91b4fbfe934c9bf8f2f85812b084989 20264e1126b219aef7feb3f9b2d6de40	65	
	9	cca104a13e678500ff59025f3bafaa34 b56a0341b2290ba7dfdfbddcd8578205	61	Curiosity has resident of the control of the contro
	10	ff0b844a0853bf7c6934ab4364148fb9 612b89398d0600cde116227ce72433f0	58	20
				26

Avalanche	Round		Number of Bits that Differ	COLLE
/ Wararrerre		0123456789abcdeffedcba9876543210 0123456789abcdeffedcba9876543210	0	
Effect	0	0e3634aece7225b6f26b174ed92b5588 0f3634aece7225b6f26b174ed92b5588	1	VYORK
in AES:	1	657470750fc7ff3fc0e8e8ca4dd02a9c c5a9ad090ec7ff3fc1e8e8ca4cd02a9c	22	
	2	5c7bb49a6b72349b05a2317ff46d1294 90905fa9563356d15f3760f3b8259985	58	
Change	3	7115262448dc747e5cdac7227da9bd9c 18aeb7aa794b3b66629448d575c7cebf	67	
in Key	4	f867aee8b437a5210c24c1974cffeabc f81015f993c978a876ae017cb49e7eec	63	
III KCy	5	721eb200ba06206dcbd4bce704fa654e 5955c91b4e769f3cb4a94768e98d5267	81	
	6	0ad9d85689f9f77bc1c5f71185e5fb14 dc60a24d137662181e45b8d3726b2920	70	
	7	db18a8ffa16d30d5f88b08d777ba4eaa fe8343b8f88bef66cab7e977d005a03c	74	
	8	f91b4fbfe934c9bf8f2f85812b084989 da7dad581d1725c5b72fa0f9d9d1366a	67	Curiosity has its own reason for
	9	cca104a13e678500ff59025f3bafaa34 0ccb4c66bbfd912f4b511d72996345e0	59	waisting -A tindbeln
	10	ff0b844a0853bf7c6934ab4364148fb9 fc8923ee501a7d207ab670686839996b	53	27



Interchanging InvShiftRows and InvSubBytes



- InvShiftRows affects the sequence of bytes in State but does not alter byte contents and does not depend on byte contents to perform its transformation
- InvSubBytes affects the contents of bytes in State but does not alter byte sequence and does not depend on byte sequence to perform its transformation

Thus, these two operations commute and can be interchanged



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29

Interchanging AddRoundKey and InvMixColumns



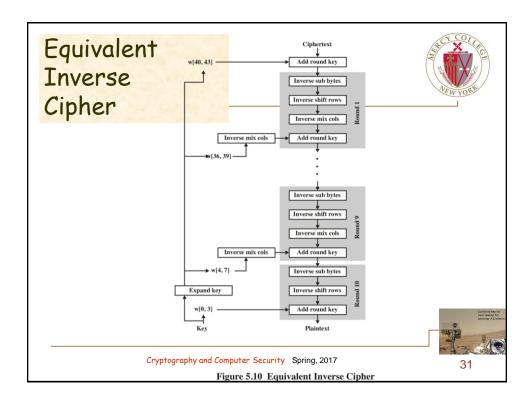
The transformations AddRoundKey and InvMixColumns do not alter the sequence of bytes in State

key as a
sequence of
words, then
both
AddRoundKey
and
InvMixColumns
operate on State
one column at a
time

If we view the

These two operations are linear with respect to the column input

3U



Implementation Aspects



- AES can be implemented very efficiently on an 8-bit processor
- AddRoundKey is a bytewise XOR operation
- ShiftRows is a simple byte-shifting operation
- SubBytes operates at the byte level and only requires a table of 256 bytes
- MixColumns requires matrix multiplication in the field GF(28), which means that all operations are carried out on bytes

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Implementation Aspects



- Can efficiently implement on a 32-bit processor
 - Redefine steps to use 32-bit words
 - Can precompute 4 tables of 256-words
 - Then each column in each round can be computed using 4 table lookups + 4 XORs
 - At a cost of 4Kb to store tables
- Designers believe this very efficient implementation was a key factor in its selection as the AES cipher



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33

Summary



- Finite field arithmetic
- AES structure
 - General structure
 - Detailed structure
- AES key expansion
 - Key expansion algorithm
 - Rationale

- AES transformation functions
 - Substitute bytes
 - ShiftRows
 - MixColumns
 - AddRoundKey
- AES implementation
 - Equivalent inverse cipher
 - Implementation aspects



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