40. Let

$p_1(x) = a_0 + a_1x + a_2x^2 + \cdots + a_{n-1}x^{n-1}$ and $p_2(x) = b_0 + b_1x + b_2x^2 + \cdots + b_{n-1}x^{n-1}$

be two different polynomials that pass through the n given points. The polynomial

$p_1(x) - p_2(x) = (a_0 - b_0) + (a_1 - b_1)x + (a_2 - b_2)x^2 + \cdots + (a_{n-1} - b_{n-1})x^{n-1}$

is zero for these n values of x. So, $a_0 = b_0$, $a_1 = b_1$, $a_2 = b_2$, …, $a_{n-1} = b_{n-1}$.

Therefore, there is only one polynomial function of degree $n - 1$ (or less) whose graph passes through n points in the plane with distinct x-coordinates.

42. Choose a fourth-degree polynomial and substitute $x = 1, 2, 3,$ and 4 into $p(x) = a_0 + a_1x + a_2x^2 + a_3x^3 + a_4x^4$. However, when you substitute $x = 3$ into $p(x)$ and equate it to $y = 2$ and $y = 3$ you get the contradictory equations

$$a_0 + 3a_1 + 9a_2 + 27a_3 + 81a_4 = 2$$
$$a_0 + 3a_1 + 9a_2 + 27a_3 + 81a_4 = 3$$

and must conclude that the system containing these two equations will have no solution. Also, y is not a function of x because the x-value of 3 is repeated. By similar reasoning, you cannot choose $p(y) = b_0 + b_1y + b_2y^2 + b_3y^3 + b_4y^4$ because $y = 1$ corresponds to both $x = 1$ and $x = 2$.

Review Exercises for Chapter 1

2. Because the equation cannot be written in the form $a_1x + a_2y = b$, it is *not* linear in the variables x and y.

4. Because the equation is in the form $a_1x + a_2y = b$, it is linear in the variables x and y.

6. Because the equation is in the form $a_1x + a_2y = b$, it is linear in the variables x and y.

8. Choosing x_2 and x_3 as the free variables and letting $x_2 = s$ and $x_3 = t$, you have

$$3x_1 + 2s - 4t = 0$$
$$3x_1 = -2s + 4t$$
$$x_1 = \tfrac{1}{3}(-2s + 4t).$$

10. Row reduce the augmented matrix for this system.

$$\begin{bmatrix} 1 & 1 & -1 \\ 3 & 2 & 0 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 1 & -1 \\ 0 & -1 & 3 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 1 & -1 \\ 0 & 1 & -3 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & -3 \end{bmatrix}$$

Converting back to a linear system, the solution is $x = 2$ and $y = -3$.

12. Rearrange the equations, form the augmented matrix, and row reduce.

$$\begin{bmatrix} 1 & -1 & 3 \\ 4 & -1 & 10 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & -1 & 3 \\ 0 & 3 & -2 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & -1 & 3 \\ 0 & 1 & -\frac{2}{3} \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 & \frac{7}{3} \\ 0 & 1 & -\frac{2}{3} \end{bmatrix}.$$

Converting back to a linear system, you obtain the solution $x = \frac{7}{3}$ and $y = -\frac{2}{3}$.

14. Rearrange the equations, form the augmented matrix, and row reduce.

$$\begin{bmatrix} -5 & 1 & 0 \\ -1 & 1 & 0 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & -1 & 0 \\ -5 & 1 & 0 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & -1 & 0 \\ 0 & -4 & 0 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

Converting back to a linear system, the solution is: $x = 0$ and $y = 0$.

16. Row reduce the augmented matrix for this system.

$$\begin{bmatrix} 40 & 30 & 24 \\ 20 & 15 & -14 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & \frac{3}{4} & \frac{3}{5} \\ 20 & 15 & -14 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & \frac{3}{4} & \frac{3}{5} \\ 0 & 0 & -26 \end{bmatrix}$$

Because the second row corresponds to the false statement $0 = -26$, the system has no solution.

18. Use Gauss-Jordan elimination on the augmented matrix.

$$\begin{bmatrix} \frac{1}{3} & \frac{4}{7} & 3 \\ 2 & 3 & 15 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 & -3 \\ 0 & 1 & 7 \end{bmatrix}$$

So, the solution is: $x = -3$, $y = 7$.

20. Multiplying both equations by 100 and forming the augmented matrix produces

$$\begin{bmatrix} 20 & -10 & 7 \\ 40 & -50 & -1 \end{bmatrix}.$$

Gauss-Jordan elimination yields the following.

$$\begin{bmatrix} 1 & -\frac{1}{2} & \frac{7}{20} \\ 40 & -50 & -1 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & -\frac{1}{2} & \frac{7}{20} \\ 0 & -30 & -15 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & -\frac{1}{2} & \frac{7}{20} \\ 0 & 1 & \frac{1}{2} \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 & \frac{3}{5} \\ 0 & 1 & \frac{1}{2} \end{bmatrix}$$

So, the solution is: $x = \frac{3}{5}$ and $y = \frac{1}{2}$.

22. Because the matrix has 3 rows and 2 columns, it has size 3×2.

24. This matrix corresponds to the system

$$-2x_1 + 3x_2 = 0.$$

Choosing $x_2 = t$ as a free variable, you can describe the solution as $x_1 = \frac{3}{2}t$ and $x_2 = t$, where t is a real number.

26. This matrix corresponds to the system

$$\begin{aligned} x_1 + 2x_2 + 3x_3 &= 0 \\ 0 &= 1. \end{aligned}$$

Because the second equation is not possible, the system has no solution.

28. The matrix satisfies all three conditions in the definition of row-echelon form. Because each column that has a leading 1 (columns 1 and 4) has zeros elsewhere, the matrix is in reduced row-echelon form.

30. The matrix satisfies all three conditions in the definition of row-echelon form. Because each column that has a leading 1 (columns 2 and 3) has zeros elsewhere, the matrix is in reduced row-echelon form.

32. Use Gauss-Jordan elimination on the augmented matrix.

$$\begin{bmatrix} 4 & 2 & 1 & 18 \\ 4 & -2 & -2 & 28 \\ 2 & -3 & 2 & -8 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 & 0 & 5 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & -6 \end{bmatrix}$$

So, the solution is: $x = 5$, $y = 2$, and $z = -6$.

34. Use the Gauss-Jordan elimination on the augmented matrix.

$$\begin{bmatrix} 2 & 1 & 2 & 4 \\ 2 & 2 & 0 & 5 \\ 2 & -1 & 6 & 2 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 & 2 & \frac{3}{2} \\ 0 & 1 & -2 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Choosing $z = t$ as the free variable, you can describe the solution as $x = \frac{3}{2} - 2t$, $y = 1 + 2t$, and $z = t$, where t is any real number.

36. Use Gauss-Jordan elimination on the augmented matrix.

$$\begin{bmatrix} 2 & 0 & 6 & -9 \\ 3 & -2 & 11 & -16 \\ 3 & -1 & 7 & -11 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 & 0 & -\frac{3}{4} \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -\frac{5}{4} \end{bmatrix}$$

So, the solution is: $x = -\frac{3}{4}$, $y = 0$, and $z = -\frac{5}{4}$.

38. Use Gauss-Jordan elimination on the augmented matrix.

$$\begin{bmatrix} 2 & 5 & -19 & 34 \\ 3 & 8 & -31 & 54 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 & 3 & 2 \\ 0 & 1 & -5 & 6 \end{bmatrix}$$

Choosing $x_3 = t$ as the free variable, you can describe the solution as $x_1 = 2 - 3t$, $x_2 = 6 + 5t$, and $x_3 = t$, where t is any real number.

40. Use Gauss-Jordan elimination on the augmented matrix.

$$\begin{bmatrix} 1 & 5 & 3 & 0 & 0 & 14 \\ 0 & 4 & 2 & 5 & 0 & 3 \\ 0 & 0 & 3 & 8 & 6 & 16 \\ 2 & 4 & 0 & 0 & -2 & 0 \\ 2 & 0 & -1 & 0 & 0 & 0 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 2 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 4 \\ 0 & 0 & 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 & 1 & 2 \end{bmatrix}$$

So, the solution is: $x_1 = 2$, $x_2 = 0$, $x_3 = 4$, $x_4 = -1$, and $x_5 = 2$.

42. Using a graphing utility, the augmented matrix reduces to

$$\begin{bmatrix} 1 & 0 & -0.533 & 0 \\ 0 & 1 & 1.733 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

Because $0 \neq 1$, the system has no solution.

44. Using a graphing utility, the augmented matrix reduces to

$$\begin{bmatrix} 1 & 5 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$

The system is inconsistent, so there is no solution.

46. Using a graphing utility, the augmented matrix reduces to

$$\begin{bmatrix} 1 & 0 & 0 & 1.5 & 0 \\ 0 & 1 & 0 & 0.5 & 0 \\ 0 & 0 & 1 & 0.5 & 0 \end{bmatrix}.$$

Choosing $w = t$ as the free variable, you can describe the solution as $x = -1.5t$, $y = -0.5t$, $z = -0.5t$, $w = t$, where t is any real number.

48. Use Gauss-Jordan elimination on the augmented matrix.

$$\begin{bmatrix} 2 & 4 & -7 & 0 \\ 1 & -3 & 9 & 0 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 & \frac{3}{2} & 0 \\ 0 & 1 & -\frac{5}{2} & 0 \end{bmatrix}$$

Letting $x_3 = t$ be the free variable, you have $x_1 = -\frac{3}{2}t$, $x_2 = \frac{5}{2}t$, and $x_3 = t$, where t is any real number.

50. Use Gauss-Jordan elimination on the augmented matrix.

$$\begin{bmatrix} 1 & 3 & 5 & 0 \\ 1 & 4 & \frac{1}{2} & 0 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 & \frac{37}{2} & 0 \\ 0 & 1 & -\frac{9}{2} & 0 \end{bmatrix}$$

Choosing $x_3 = t$ as the free variable, you can describe the solution as $x_1 = -\frac{37}{2}t$, $x_2 = \frac{9}{2}t$, and $x_3 = t$, where t is any real number.

52. Use Gaussian elimination on the augmented matrix.

$$\begin{bmatrix} 1 & -1 & 2 & 0 \\ -1 & 1 & -1 & 0 \\ 1 & k & 1 & 0 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & -1 & 2 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & (k+1) & -1 & 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & -1 & 2 & 0 \\ 0 & (k+1) & -1 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

So, there will be exactly one solution (the trivial solution $x = y = z = 0$) if and only if $k \neq -1$.

54. Form the augmented matrix for the system.

$$\begin{bmatrix} 2 & -1 & 1 & a \\ 1 & 1 & 2 & b \\ 0 & 3 & 3 & c \end{bmatrix}$$

Use Gaussian elimination to reduce the matrix to row-echelon form.

$$\begin{bmatrix} 1 & -\frac{1}{2} & \frac{1}{2} & \frac{a}{2} \\ 1 & 1 & 2 & b \\ 0 & 3 & 3 & c \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} & \frac{a}{2} \\ 0 & \frac{3}{2} & \frac{3}{2} & \frac{2b-a}{2} \\ 0 & 3 & 3 & c \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & -\frac{1}{2} & \frac{1}{2} & \frac{a}{2} \\ 0 & 1 & 1 & \frac{2b-a}{3} \\ 0 & 3 & 3 & c \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & -\frac{1}{2} & \frac{1}{2} & \frac{a}{2} \\ 0 & 1 & 1 & \frac{2b-a}{3} \\ 0 & 0 & 0 & c-2b+a \end{bmatrix}$$

(a) If $c - 2b + a \neq 0$, then the system has no solution.

(b) The system cannot have one solution.

(c) If $c - 2b + a = 0$, then the system has infinitely many solutions

56. Find all possible first rows, where a and b are nonzero real numbers.

$[0\ \ 0\ \ 0]$, $[0\ \ 0\ \ 1]$, $[0\ \ 1\ \ 0]$, $[0\ \ 1\ \ a]$, $[1\ \ 0\ \ 0]$, $[1\ \ a\ \ 0]$, $[1\ \ a\ \ b]$, $[1\ \ 0\ \ a]$

For each of these, examine the possible second rows.

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix},$$

$$\begin{bmatrix} 0 & 1 & a \\ 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & a \end{bmatrix},$$

$$\begin{bmatrix} 1 & a & 0 \\ 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 1 & a & 0 \\ 0 & 0 & 1 \end{bmatrix}, \begin{bmatrix} 1 & a & b \\ 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 0 & a \\ 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 0 & a \\ 0 & 1 & 0 \end{bmatrix}$$

58. Use Gaussian elimination on the augmented matrix.

$$\begin{bmatrix} (\lambda+2) & -2 & 3 & 0 \\ -2 & (\lambda-1) & 6 & 0 \\ 1 & 2 & \lambda & 0 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 2 & \lambda & 0 \\ 0 & \lambda+3 & 6+2\lambda & 0 \\ 0 & -2\lambda-6 & -\lambda^2-2\lambda+3 & 0 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 2 & \lambda & 0 \\ 0 & \lambda+3 & 6+2\lambda & 0 \\ 0 & 0 & (\lambda^2-2\lambda-15) & 0 \end{bmatrix}$$

So, you need $\lambda^2 - 2\lambda - 15 = (\lambda - 5)(\lambda + 3) = 0$, which implies $\lambda = 5$ or $\lambda = -3$.

60. (a) True. A homogeneous system of linear equations is always consistent, because there is always a trivial solution, *i.e.*, when all variables are equal to zero. See Theorem 1.1 on page 21.

(b) False. Consider, for example, the following system (with three variables and two equations).

$$\begin{aligned} x + y - z &= 2 \\ -2x - 2y + 2z &= 1. \end{aligned}$$

It is easy to see that this system has *no* solution.

62. From the following chart, you obtain a system of equations.

	A	B	C
Mixture X	$\frac{1}{5}$	$\frac{2}{5}$	$\frac{2}{5}$
Mixture Y	0	0	1
Mixture Z	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$
Desired Mixture	$\frac{6}{27}$	$\frac{8}{27}$	$\frac{13}{27}$

$$\left.\begin{aligned} \tfrac{1}{5}x + \tfrac{1}{3}z &= \tfrac{6}{27} \\ \tfrac{2}{5}x + \tfrac{1}{3}z &= \tfrac{8}{27} \end{aligned}\right\} x = \tfrac{10}{27},\ z = \tfrac{12}{27}$$

$$\tfrac{2}{5}x + y + \tfrac{1}{3}z = \tfrac{13}{27} \Rightarrow y = \tfrac{5}{27}$$

To obtain the desired mixture, use 10 gallons of spray X, 5 gallons of spray Y, and 12 gallons of spray Z.

64. $$\frac{3x^2 + 3x - 2}{(x+1)^2(x-1)} = \frac{A}{x+1} + \frac{B}{x-1} + \frac{C}{(x+1)^2}$$

$$\begin{aligned} 3x^2 + 3x - 2 &= A(x+1)(x-1) + B(x+1)^2 + C(x-1) \\ 3x^2 + 3x - 2 &= Ax^2 - A + Bx^2 + 2Bx + B + Cx - C \\ 3x^2 + 3x - 2 &= (A+B)x^2 + (2B+C)x - A + B - C \end{aligned}$$

So,
$$\begin{aligned} A + B \phantom{{}+C} &= 3 \\ 2B + C &= 3 \\ -A + B - C &= -2. \end{aligned}$$

Use Gauss-Jordan elimination to solve the system.

$$\begin{bmatrix} 1 & 1 & 0 & 3 \\ 0 & 2 & 1 & 3 \\ -1 & 1 & -1 & -2 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

The solution is: $A = 2$, $B = 1$, and $C = 1$.

So, $$\frac{3x^2 + 3x - 2}{(x+1)^2(x-1)} = \frac{2}{x+1} + \frac{1}{x-1} + \frac{1}{(x+1)^2}.$$

66. (a) Because there are four points, choose a third-degree polynomial, $p(x) = a_0 + a_1x + a_2x^2 + a_3x^3$.

By substituting the values at each point into this equation, you obtain the system

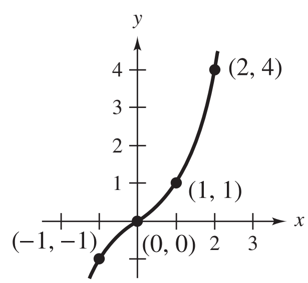
$$\begin{aligned} a_0 - a_1 + a_2 - a_3 &= -1 \\ a_0 &= 0 \\ a_0 + a_1 + a_2 + a_3 &= 1 \\ a_0 + 2a_1 + 4a_2 + 8a_3 &= 4. \end{aligned}$$

Use Gauss-Jordan elimination on the augmented matrix.

$$\begin{bmatrix} 1 & -1 & 1 & -1 & -1 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 2 & 4 & 8 & 4 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & \frac{2}{3} \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & \frac{1}{3} \end{bmatrix}$$

So, $p(x) = \frac{2}{3}x + \frac{1}{3}x^3$.

(b)



68. Substituting the points, (1, 0), (2, 0), (3, 0), and (4, 0) into the polynomial $p(x)$ yields the system

$$\begin{aligned} a_0 + a_1 + a_2 + a_3 &= 0 \\ a_0 + 2a_1 + 4a_2 + 8a_3 &= 0 \\ a_0 + 3a_1 + 9a_2 + 27a_3 &= 0 \\ a_0 + 4a_1 + 16a_2 + 64a_3 &= 0. \end{aligned}$$

Gaussian elimination shows that the only solution is $a_0 = a_1 = a_2 = a_3 = 0$.

70. (a) When $t = 0, s = 160$: $\frac{1}{2}a(0)^2 + v_0(0) + s_0 = 160 \Rightarrow s_0 = 160$

When $t = 1, s = 96$: $\frac{1}{2}a(1)^2 + v_0(1) + s_0 = 96 \Rightarrow \frac{1}{2}a + v_0 + s_0 = 96$

When $t = 2, s = 0$: $\frac{1}{2}a(2)^2 + v_0(2) + s_0 = 0 \Rightarrow 2a + 2v_0 + s_0 = 0$

Use Gaussian elimination to solve the system.

$$\begin{aligned} s_0 &= 160 \\ \tfrac{1}{2}a + v_0 + s_0 &= 96 \\ 2a + 2v_0 + s_0 &= 0 \end{aligned}$$

$$\begin{aligned} a + 2v_0 + 2s_0 &= 192 \\ 2a + 2v_0 + s_0 &= 0 \\ s_0 &= 160 \end{aligned}$$

$$\begin{aligned} a + 2v_0 + 2s_0 &= 192 \\ -2v_0 - 3s_0 &= -384 \qquad (-2)\text{ Eq. } 1 + \text{Eq. } 2 \\ s_0 &= 160 \end{aligned}$$

$$\begin{aligned} a + 2v_0 + 2s_0 &= 192 \\ v_0 + \tfrac{3}{2}s_0 &= 192 \qquad \left(-\tfrac{1}{2}\right)\text{ Eq. } 2 \\ s_0 &= 160 \end{aligned}$$

$$\begin{aligned} s_0 = 160 &\Rightarrow s_0 = 160 \\ v_0 + \tfrac{3}{2}(160) = 192 &\Rightarrow v_0 = -48 \\ a + 2(-48) + 2(160) = 192 &\Rightarrow a = -32 \end{aligned}$$

The position equation is $s = \frac{1}{2}(-32)t^2 - 48t + 160$, or $s = -16t^2 - 48t + 160$.

(b) When $t = 1, s = 134$: $\frac{1}{2}a(1)^2 + v_0(1) + s_0 = 134 \Rightarrow a + 2v_0 + 2s_0 = 268$

When $t = 2, s = 86$: $\frac{1}{2}a(2)^2 + v_0(2) + s_0 = 86 \Rightarrow 2a + 2v_0 + s_0 = 86$

When $t = 3, s = 6$: $\frac{1}{2}a(3)^2 + v_0(3) + s_0 = 6 \Rightarrow 9a + 6v_0 + 2s_0 = 12$

Use Gaussian elimination to solve the system.

$$\begin{aligned} a + 2v_0 + 2s_0 &= 268 \\ 2a + 2v_0 + s_0 &= 86 \\ 9a + 6v_0 + 2s_0 &= 12 \end{aligned}$$

$$\begin{aligned} a + 2v_0 + 2s_0 &= 268 \\ -2v_0 - 3s_0 &= -450 \qquad (-2)\text{Eq.1} + \text{Eq.2} \\ -12v_0 - 16s_0 &= -2400 \qquad (-9)\text{Eq.1} + \text{Eq.3} \end{aligned}$$

$$\begin{aligned} a + 2v_0 + 2s_0 &= 268 \\ -2v_0 - 3s_0 &= -450 \\ 3v_0 + 4s_0 &= 600 \qquad \left(-\tfrac{1}{4}\right)\text{Eq.3} \end{aligned}$$

$$\begin{aligned} a + 2v_0 + 2s_0 &= 268 \\ -2v_0 - 3s_0 &= -450 \\ -s_0 &= -150 \qquad 3\text{Eq.2} + 2\text{Eq.3} \end{aligned}$$

$$\begin{aligned} -s_0 = -150 &\Rightarrow s_0 = 150 \\ -2v_0 - 3(150) = -450 &\Rightarrow v_0 = 0 \\ a + 2(0) + 2(150) = 268 &\Rightarrow a = -32 \end{aligned}$$

The position equation is $s = \frac{1}{2}(-32)t^2 + (0)t + 150$, or $s = -16t^2 + 150$.

(c) When $t = 1, s = 184$: $\frac{1}{2}a(1)^2 + v_0(1) + s_0 = 134 \Rightarrow a + 2v_0 + 2s_0 = 368$

When $t = 2, s = 116$: $\frac{1}{2}a(2)^2 + v_0(2) + s_0 = 116 \Rightarrow 2a + 2v_0 + s_0 = 116$

When $t = 3, s = 16$: $\frac{1}{2}a(3)^2 + v_0(3) + s_0 = 16 \Rightarrow 9a + 6v_0 + 2s_0 = 32$

Use Gaussian elimination to solve the system.

$$\begin{aligned} a + 2v_0 + 2s_0 &= 368 \\ 2a + 2v_0 + s_0 &= 116 \\ 9a + 6v_0 + 2s_0 &= 32 \end{aligned}$$

$$\begin{aligned} a + 2v_0 + 2s_0 &= 368 \\ -2v_0 - 3s_0 &= -620 & \quad (-2)\text{ Eq. } 1 + \text{Eq. } 2 \\ -12v_0 - 16s_0 &= -3280 & \quad (-9)\text{ Eq. } 1 + \text{Eq. } 3 \end{aligned}$$

$$\begin{aligned} a + 2v_0 + 2s_0 &= 368 \\ v_0 + \tfrac{3}{2}s_0 &= 310 & \quad \left(-\tfrac{1}{2}\right)\text{ Eq. } 2 \\ -12v_0 - 16s_0 &= -3280 \end{aligned}$$

$$\begin{aligned} a + 2v_0 + 2s_0 &= 368 \\ v_0 + \tfrac{3}{2}s_0 &= 310 \\ 2s_0 &= 440 & \quad 12\text{ Eq. } 2 + \text{Eq. } 3 \end{aligned}$$

$$\begin{aligned} 2s_0 = 440 &\Rightarrow s_0 = 220 \\ -2v_0 - 3(220) = -620 &\Rightarrow v_0 = -20 \\ a + 2(-20) + 2(220) = 368 &\Rightarrow a = -32 \end{aligned}$$

The position equation is $s = -\frac{1}{2}(-32)t^2 + (-20)t + 220$, or $s = -16t^2 - 20t + 220$.

72. Applying Kirchoff's first law to either junction produces

$I_1 + I_3 = I_2$ and applying the second law to the two paths produces

$R_1I_1 + R_2I_2 = 3I_1 + 4I_2 = 3$

$R_2I_2 + R_3I_3 = 4I_2 + 2I_3 = 2.$

Rearrange these equations, form the augmented matrix, and use Gauss-Jordan elimination.

$$\begin{bmatrix} 1 & -1 & 1 & 0 \\ 3 & 4 & 0 & 3 \\ 0 & 4 & 2 & 2 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 & 0 & \frac{5}{13} \\ 0 & 1 & 0 & \frac{6}{13} \\ 0 & 0 & 1 & \frac{1}{13} \end{bmatrix}$$

So, the solution is $I_1 = \frac{5}{13}$, $I_2 = \frac{6}{13}$, and $I_3 = \frac{1}{13}$.

Project Solutions for Chapter 1

1 Graphing Linear Equations

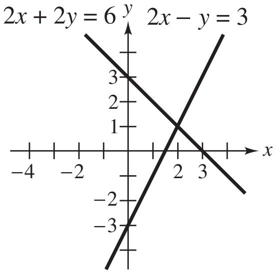
1. $\begin{bmatrix} 2 & -1 & 3 \\ a & b & 6 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & -\frac{1}{2} & \frac{3}{2} \\ 0 & b + \frac{1}{2}a & 6 - \frac{3}{2}a \end{bmatrix}$

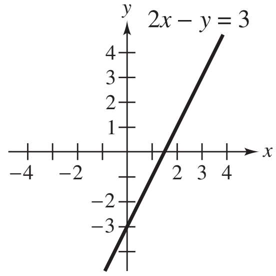
(a) Unique solution if $b + \frac{1}{2}a \neq 0$. For instance, $a = b = 2$.

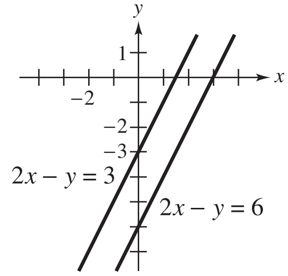
(b) Infinite number of solutions if $b + \frac{1}{2}a = 6 - \frac{3}{2}a = 0 \Rightarrow a = 4$ and $b = -2$.

(c) No solution if $b + \frac{1}{2}a = 0$ and $6 - \frac{3}{2}a \neq 0 \Rightarrow a \neq 4$ and $b = -\frac{1}{2}a$. For instance, $a = 2, b = -1$.

(d)







(a) $\begin{aligned} 2x - y &= 3 \\ 2x + 2y &= 6 \end{aligned}$

(b) $\begin{aligned} 2x - y &= 3 \\ 4x - 2y &= 6 \end{aligned}$

(c) $\begin{aligned} 2x - y &= 3 \\ 2x - y &= 6 \end{aligned}$

(The answers are not unique.)

2. (a) $\begin{aligned} x + y + z &= 0 \\ x + y + z &= 0 \\ x - y - z &= 0 \end{aligned}$

(b) $\begin{aligned} x + y + z &= 0 \\ y + z &= 1 \\ z &= 2 \end{aligned}$

(c) $\begin{aligned} x + y + z &= 0 \\ x + y + z &= 1 \\ x - y - z &= 0 \end{aligned}$

(The answers are not unique.)

There are other configurations, such as three mutually parallel planes or three planes that intersect pairwise in lines.

2 Underdetermined and Overdetermined Systems of Equations

1. Yes, $x + y = 2$ is a consistent underdetermined system.

2. Yes,

$$\begin{aligned} x + y &= 2 \\ 2x + 2y &= 4 \\ 3x + 3y &= 6 \end{aligned}$$

is a consistent, overdetermined system.

3. Yes,

$$\begin{aligned} x + y + z &= 1 \\ x + y + z &= 2 \end{aligned}$$

is an inconsistent underdetermined system.

4. Yes,

$$\begin{aligned} x + y &= 1 \\ x + y &= 2 \\ x + y &= 3 \end{aligned}$$

is an inconsistent underdetermined system.

5. In general, a linear system with more equations than variables would probably be inconsistent. Here is an intuitive reason: Each variable represents a degree of freedom, while each equation gives a condition that in general reduces number of degrees of freedom by one. If there are more equations (conditions) than variables (degrees of freedom), then there are too many conditions for the system to be consistent. So you expect such a system to be inconsistent in general. But, as Exercise 2 shows, this is not always true.

6. In general, a linear system with more variables than equations would probably be consistent. As in Exercise 5, the intuitive explanation is as follows. Each variable represents a degree of freedom, and each equation represents a condition that takes away one degree of freedom. If there are more variables than equations, in general, you would expect a solution. But, as Exercise 3 shows, this is not always true.

C H A P T E R 2
Matrices

CHAPTER 2
Matrices

Section 2.1 Operations with Matrices

2. $x = 13, y = 12$

4. $x + 2 = 2x + 6 \qquad 2y = 18$
$-4 = x \qquad y = 9$

$2x = -8 \qquad y + 2 = 11$
$x = -4 \qquad y = 9$

6. (a) $A + B = \begin{bmatrix} 6 & -1 \\ 2 & 4 \\ -3 & 5 \end{bmatrix} + \begin{bmatrix} 1 & 4 \\ -1 & 5 \\ 1 & 10 \end{bmatrix} = \begin{bmatrix} 6+1 & -1+4 \\ 2+(-1) & 4+5 \\ -3+1 & 5+10 \end{bmatrix} = \begin{bmatrix} 7 & 3 \\ 1 & 9 \\ -2 & 15 \end{bmatrix}$

(b) $A - B = \begin{bmatrix} 6 & -1 \\ 2 & 4 \\ -3 & 5 \end{bmatrix} - \begin{bmatrix} 1 & 4 \\ -1 & 5 \\ 1 & 10 \end{bmatrix} = \begin{bmatrix} 6-1 & -1-4 \\ 2-(-1) & 4-5 \\ -3-1 & 5-10 \end{bmatrix} = \begin{bmatrix} 5 & -5 \\ 3 & -1 \\ -4 & -5 \end{bmatrix}$

(c) $2A = 2\begin{bmatrix} 6 & -1 \\ 2 & 4 \\ -3 & 5 \end{bmatrix} = \begin{bmatrix} 2(6) & 2(-1) \\ 2(2) & 2(4) \\ 2(-3) & 2(5) \end{bmatrix} = \begin{bmatrix} 12 & -2 \\ 4 & 8 \\ -6 & 10 \end{bmatrix}$

(d) $2A - B = \begin{bmatrix} 12 & -2 \\ 4 & 8 \\ -6 & 10 \end{bmatrix} - \begin{bmatrix} 1 & 4 \\ -1 & 5 \\ 1 & 10 \end{bmatrix} = \begin{bmatrix} 12-1 & -2-4 \\ 4-(-1) & 8-5 \\ -6-1 & 10-10 \end{bmatrix} = \begin{bmatrix} 11 & -6 \\ 5 & 3 \\ -7 & 0 \end{bmatrix}$

(e) $B + \frac{1}{2}A = \begin{bmatrix} 1 & 4 \\ -1 & 5 \\ 1 & 10 \end{bmatrix} + \frac{1}{2}\begin{bmatrix} 6 & -1 \\ 2 & 4 \\ -3 & 5 \end{bmatrix} = \begin{bmatrix} 1 & 4 \\ -1 & 5 \\ 1 & 10 \end{bmatrix} + \begin{bmatrix} 3 & -\frac{1}{2} \\ 1 & 2 \\ -\frac{3}{2} & \frac{5}{2} \end{bmatrix} = \begin{bmatrix} 4 & \frac{7}{2} \\ 0 & 7 \\ -\frac{1}{2} & \frac{25}{2} \end{bmatrix}$

8. (a) $A + B = \begin{bmatrix} 3 & 2 & -1 \\ 2 & 4 & 5 \\ 0 & 1 & 2 \end{bmatrix} + \begin{bmatrix} 0 & 2 & 1 \\ 5 & 4 & 2 \\ 2 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 3+0 & 2+2 & -1+1 \\ 2+5 & 4+4 & 5+2 \\ 0+2 & 1+1 & 2+0 \end{bmatrix} = \begin{bmatrix} 3 & 4 & 0 \\ 7 & 8 & 7 \\ 2 & 2 & 2 \end{bmatrix}$

(b) $A - B = \begin{bmatrix} 3 & 2 & -1 \\ 2 & 4 & 5 \\ 0 & 1 & 2 \end{bmatrix} - \begin{bmatrix} 0 & 2 & 1 \\ 5 & 4 & 2 \\ 2 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 3-0 & 2-2 & -1-1 \\ 2-5 & 4-4 & 5-2 \\ 0-2 & 1-1 & 2-0 \end{bmatrix} = \begin{bmatrix} 3 & 0 & -2 \\ -3 & 0 & 3 \\ -2 & 0 & 2 \end{bmatrix}$

(c) $2A = 2\begin{bmatrix} 3 & 2 & -1 \\ 2 & 4 & 5 \\ 0 & 1 & 2 \end{bmatrix} = \begin{bmatrix} 2(3) & 2(2) & 2(-1) \\ 2(2) & 2(4) & 2(5) \\ 2(0) & 2(1) & 2(2) \end{bmatrix} = \begin{bmatrix} 6 & 4 & -2 \\ 4 & 8 & 10 \\ 0 & 2 & 4 \end{bmatrix}$

(d) $2A - B = 2\begin{bmatrix} 3 & 2 & -1 \\ 2 & 4 & 5 \\ 0 & 1 & 2 \end{bmatrix} - \begin{bmatrix} 0 & 2 & 1 \\ 5 & 4 & 2 \\ 2 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 6 & 4 & -2 \\ 4 & 8 & 10 \\ 0 & 2 & 4 \end{bmatrix} - \begin{bmatrix} 0 & 2 & 1 \\ 5 & 4 & 2 \\ 2 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 6 & 2 & -3 \\ -1 & 4 & 8 \\ -2 & 1 & 4 \end{bmatrix}$

(e) $B + \frac{1}{2}A = \begin{bmatrix} 0 & 2 & 1 \\ 5 & 4 & 2 \\ 2 & 1 & 0 \end{bmatrix} + \frac{1}{2}\begin{bmatrix} 3 & 2 & -1 \\ 2 & 4 & 5 \\ 0 & 1 & 2 \end{bmatrix} = \begin{bmatrix} 0 & 2 & 1 \\ 5 & 4 & 2 \\ 2 & 1 & 0 \end{bmatrix} + \begin{bmatrix} \frac{3}{2} & 1 & -\frac{1}{2} \\ 1 & 2 & \frac{5}{2} \\ 0 & \frac{1}{2} & 1 \end{bmatrix} = \begin{bmatrix} \frac{3}{2} & 3 & \frac{1}{2} \\ 6 & 6 & \frac{9}{2} \\ 2 & \frac{3}{2} & 1 \end{bmatrix}$

10. (a) $A + B$ is not possible. A and B have different sizes.

(b) $A - B$ is not possible. A and B have different sizes.

(c) $2A = 2\begin{bmatrix} 3 \\ 2 \\ -1 \end{bmatrix} = \begin{bmatrix} 6 \\ 4 \\ -2 \end{bmatrix}$

(d) $2A - B$ is not possible. A and B have different sizes.

(e) $B + \frac{1}{2}A$ is not possible. A and B have different sizes.

12. (a) $c_{23} = 5a_{23} + 2b_{23} = 5(2) + 2(11) = 32$

(b) $c_{32} = 5a_{32} + 2b_{32} = 5(1) + 2(4) = 13$

14. Simplifying the right side of the equation produces

$$\begin{bmatrix} w & x \\ y & x \end{bmatrix} = \begin{bmatrix} -4 + 2y & 3 + 2w \\ 2 + 2z & -1 + 2x \end{bmatrix}.$$

By setting corresponding entries equal to each other, you obtain four equations.

$$\begin{aligned} w &= -4 + 2y \\ x &= 3 + 2w \\ y &= 2 + 2z \\ x &= -1 + 2x \end{aligned} \Rightarrow \begin{cases} -2y + w = -4 \\ x - 2w = 3 \\ y - 2z = 2 \\ x = 1 \end{cases}$$

The solution to this linear system is: $x = 1$, $y = \frac{3}{2}$, $z = -\frac{1}{4}$, and $w = -1$.

16. (a) $AB = \begin{bmatrix} 2 & -2 \\ -1 & 4 \end{bmatrix}\begin{bmatrix} 4 & 1 \\ 2 & -2 \end{bmatrix} = \begin{bmatrix} 2(4) + (-2)(2) & 2(1) + (-2)(-2) \\ -1(4) + 4(2) & -1(1) + 4(-2) \end{bmatrix} = \begin{bmatrix} 4 & 6 \\ 4 & -9 \end{bmatrix}$

(b) $BA = \begin{bmatrix} 4 & 1 \\ 2 & -2 \end{bmatrix}\begin{bmatrix} 2 & -2 \\ -1 & 4 \end{bmatrix} = \begin{bmatrix} 4(2) + 1(-1) & 4(-2) + 1(4) \\ 2(2) + (-2)(-1) & 2(-2) + (-2)(4) \end{bmatrix} = \begin{bmatrix} 7 & -4 \\ 6 & -12 \end{bmatrix}$

18. (a) $AB = \begin{bmatrix} 1 & -1 & 7 \\ 2 & -1 & 8 \\ 3 & 1 & -1 \end{bmatrix}\begin{bmatrix} 1 & 1 & 2 \\ 2 & 1 & 1 \\ 1 & -3 & 2 \end{bmatrix} = \begin{bmatrix} 1(1) + (-1)(2) + 7(1) & 1(1) + (-1)(1) + 7(-3) & 1(2) + (-1)(1) + 7(2) \\ 2(1) + (-1)(2) + 8(1) & 2(1) + (-1)(1) + 8(-3) & 2(2) + (-1)(1) + 8(2) \\ 3(1) + 1(2) + (-1)(1) & 3(1) + 1(1) + (-1)(-3) & 3(2) + 1(1) + (-1)(2) \end{bmatrix} = \begin{bmatrix} 6 & -21 & 15 \\ 8 & -23 & 19 \\ 4 & 7 & 5 \end{bmatrix}$

(b) $BA = \begin{bmatrix} 1 & 1 & 2 \\ 2 & 1 & 1 \\ 1 & -3 & 2 \end{bmatrix}\begin{bmatrix} 1 & -1 & 7 \\ 2 & -1 & 8 \\ 3 & 1 & -1 \end{bmatrix} = \begin{bmatrix} 1(1) + 1(2) + 2(3) & 1(-1) + 1(-1)1 + 2(1) & 1(7) + 1(8) + 2(-1) \\ 2(1) + 1(2) + 1(3) & 2(-1) + 1(-1) + 1(1) & 2(7) + 1(8) + 1(-1) \\ 1(1) + (-3)(2) + 2(3) & 1(-1) + (-3)(-1) + 2(1) & 1(7) + (-3)(8) + 2(-1) \end{bmatrix} = \begin{bmatrix} 9 & 0 & 13 \\ 7 & -2 & 21 \\ 1 & 4 & -19 \end{bmatrix}$

20. (a) $AB = \begin{bmatrix} 3 & 2 & 1 \\ -3 & 0 & 4 \\ 4 & -2 & -4 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 2 & -1 \\ 1 & -2 \end{bmatrix} = \begin{bmatrix} 3(1) + 2(2) + 1(1) & 3(2) + 2(-1) + 1(-2) \\ -3(1) + 0(2) + 4(1) & -3(2) + 0(-1) + 4(-2) \\ 4(1) + (-2)(2) + (-4)(1) & 4(2) + (-2)(-1) + (-4)(-2) \end{bmatrix} = \begin{bmatrix} 8 & 2 \\ 1 & -14 \\ -4 & 18 \end{bmatrix}$

(b) BA is not defined because B is 3×2 and A is 3×3.

22. (a) $AB = \begin{bmatrix} -1 \\ 2 \\ -2 \\ 1 \end{bmatrix}\begin{bmatrix} 2 & 1 & 3 & 2 \end{bmatrix} = \begin{bmatrix} -1(2) & -1(1) & -1(3) & -1(2) \\ 2(2) & 2(1) & 2(3) & 2(2) \\ -2(2) & -2(1) & -2(3) & -2(2) \\ 1(2) & 1(1) & 1(3) & 1(2) \end{bmatrix} = \begin{bmatrix} -2 & -1 & -3 & -2 \\ 4 & 2 & 6 & 4 \\ -4 & -2 & -6 & -4 \\ 2 & 1 & 3 & 2 \end{bmatrix}$

(b) $BA = \begin{bmatrix} 2 & 1 & 3 & 2 \end{bmatrix}\begin{bmatrix} -1 \\ 2 \\ -2 \\ 1 \end{bmatrix} = \begin{bmatrix} 2(-1) + 1(2) + 3(-2) + 2(1) \end{bmatrix} = \begin{bmatrix} -4 \end{bmatrix}$

24. (a) AB is not defined because A is 2×2 and B is 3×2.

(b) $BA = \begin{bmatrix} 2 & 1 \\ 1 & 3 \\ 2 & -1 \end{bmatrix}\begin{bmatrix} 2 & -3 \\ 5 & 2 \end{bmatrix} = \begin{bmatrix} 2(2) + 1(5) & 2(-3) + 1(2) \\ 1(2) + 3(5) & 1(-3) + 3(2) \\ 2(2) + (-1)(5) & 2(-3) + (-1)(2) \end{bmatrix} = \begin{bmatrix} 9 & -4 \\ 17 & 3 \\ -1 & -8 \end{bmatrix}$

26. (a) $AB = \begin{bmatrix} 2 & 1 & 2 \\ 3 & -1 & -2 \\ -2 & 1 & -2 \end{bmatrix} \begin{bmatrix} 4 & 0 & 1 & 3 \\ -1 & 2 & -3 & -1 \\ -2 & 1 & 4 & 3 \end{bmatrix}$

$$= \begin{bmatrix} 2(4)+1(-1)+2(-2) & 2(0)+1(2)+2(1) & 2(1)+1(-3)+2(4) & 2(3)+1(-1)+2(3) \\ 3(4)+(-1)(-1)+(-2)(-2) & 3(0)+(-1)(2)+(-2)(1) & 3(1)+(-1)(-3)+(-2)(4) & 3(3)+(-1)(-1)+(-2)(3) \\ -2(4)+1(-1)+(-2)(-2) & -2(0)+1(2)+(-2)(1) & -2(1)+1(-3)+(-2)(4) & -2(3)+1(-1)+(-2)(3) \end{bmatrix}$$

$$= \begin{bmatrix} 3 & 4 & 7 & 11 \\ 17 & -4 & -2 & 4 \\ -5 & 0 & -13 & -13 \end{bmatrix}$$

(b) BA is not defined because B is 3×4 and A is 3×3.

28. (a) AB is not defined because A is 2×5 and B is 2×2.

(b) $BA = \begin{bmatrix} 1 & 6 \\ 4 & 2 \end{bmatrix} \begin{bmatrix} 1 & 0 & 3 & -2 & 4 \\ 6 & 13 & 8 & -17 & 20 \end{bmatrix}$

$$= \begin{bmatrix} 1(1)+6(6) & 1(0)+6(13) & 1(3)+6(8) & 1(-2)+6(-17) & 1(4)+6(20) \\ 4(1)+2(6) & 4(0)+2(13) & 4(3)+2(8) & 4(-2)+2(-17) & 4(4)+2(20) \end{bmatrix}$$

$$= \begin{bmatrix} 37 & 78 & 51 & -104 & 124 \\ 16 & 26 & 28 & -42 & 56 \end{bmatrix}$$

30. $C + E$ is not defined because C and E have different sizes.

32. $-4A$ is defined and has size 3×4 because A has size 3×4.

34. BE is defined. Because B has size 3×4 and E has size 4×3, the size of BE is 3×3.

36. $2D + C$ is defined and has size 4×2 because $2D$ and C have size 4×2.

38. As a system of linear equations, $A\mathbf{x} = \mathbf{0}$ is

$$\begin{aligned} x_1 + 2x_2 + x_3 + 3x_4 &= 0 \\ x_1 - x_2 \qquad\quad + x_4 &= 0. \\ x_2 - x_3 + 2x_4 &= 0 \end{aligned}$$

Use Gauss-Jordan elimination on the augmented matrix for this system.

$$\begin{bmatrix} 1 & 2 & 1 & 3 & 0 \\ 1 & -1 & 0 & 1 & 0 \\ 0 & 1 & -1 & 2 & 0 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 & 0 & 2 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & -1 & 0 \end{bmatrix}$$

Choosing $x_4 = t$, the solution is $x_1 = -2t$, $x_2 = -t$, $x_3 = t$, and $x_4 = t$, where t is any real number.

40. In matrix form $A\mathbf{x} = \mathbf{b}$, the system is

$$\begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 5 \\ 10 \end{bmatrix}.$$

Use Gauss-Jordan elimination on the augmented matrix.

$$\begin{bmatrix} 2 & 3 & 5 \\ 1 & 4 & 10 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & 3 \end{bmatrix}$$

So, the solution is $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} -2 \\ 3 \end{bmatrix}$.

42. In matrix form $A\mathbf{x} = \mathbf{b}$, the system is

$$\begin{bmatrix} -4 & 9 \\ 1 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} -13 \\ 12 \end{bmatrix}.$$

Use Gauss-Jordan elimination on the augmented matrix.

$$\begin{bmatrix} -4 & 9 & -13 \\ 1 & -3 & 12 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 & -23 \\ 0 & 1 & -\frac{35}{3} \end{bmatrix}$$

So, the solution is $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} -23 \\ -\frac{35}{3} \end{bmatrix}$.

44. In matrix form $A\mathbf{x} = \mathbf{b}$, the system is

$$\begin{bmatrix} 1 & 1 & -3 \\ -1 & 2 & 0 \\ 1 & -1 & 1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -1 \\ 1 \\ 2 \end{bmatrix}.$$

Use Gauss-Jordan elimination on the augmented matrix.

$$\begin{bmatrix} 1 & 1 & -3 & -1 \\ -1 & 2 & 0 & 1 \\ 1 & -1 & 1 & 2 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & \frac{3}{2} \\ 0 & 0 & 1 & \frac{3}{2} \end{bmatrix}$$

So, the solution is $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 2 \\ \frac{3}{2} \\ \frac{3}{2} \end{bmatrix}$.

46. In matrix form $A\mathbf{x} = \mathbf{b}$, the system is

$$\begin{bmatrix} 1 & -1 & 4 \\ 1 & 3 & 0 \\ 0 & -6 & 5 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 17 \\ -11 \\ 40 \end{bmatrix}.$$

Use Gauss-Jordan elimination on the augmented matrix.

$$\begin{bmatrix} 1 & -1 & 4 & 17 \\ 1 & 3 & 0 & -11 \\ 0 & -6 & 5 & 40 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 & 0 & 4 \\ 0 & 1 & 0 & -5 \\ 0 & 0 & 1 & 2 \end{bmatrix}$$

So, the solution is $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 4 \\ -5 \\ 2 \end{bmatrix}$.

48. In matrix form $A\mathbf{x} = \mathbf{b}$, the system is

$$\begin{bmatrix} 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ -1 & 1 & -1 & 1 & -1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 5 \end{bmatrix}.$$

Use Gauss-Jordan elimination on the augmented matrix.

$$\begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 \\ -1 & 1 & -1 & 1 & -1 & 5 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & -1 \\ 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & -1 \\ 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & -1 \end{bmatrix}$$

So, the solution is $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} -1 \\ 1 \\ -1 \\ 1 \\ -1 \end{bmatrix}$.

50. The augmented matrix row reduces as follows.

$$\begin{bmatrix} 1 & 2 & 4 & 1 \\ -1 & 0 & 2 & 3 \\ 0 & 1 & 3 & 2 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 & -2 & -3 \\ 0 & 1 & 3 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

There are an infinite number of solutions. For example, $x_3 = 0,\ x_2 = 2,\ x_1 = -3$.

So, $\mathbf{b} = \begin{bmatrix} 1 \\ 3 \\ 2 \end{bmatrix} = -3\begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix} + 2\begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix} + 0\begin{bmatrix} 4 \\ 2 \\ 3 \end{bmatrix}$.

52. The augmented matrix row reduces as follows.

$$\begin{bmatrix} -3 & 5 & -22 \\ 3 & 4 & 4 \\ 4 & -8 & 32 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & -3 & 10 \\ 0 & 9 & -18 \\ 0 & -4 & 8 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & -3 & 10 \\ 0 & 1 & -2 \\ 0 & 1 & -2 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 & 4 \\ 0 & 1 & -2 \\ 0 & 0 & 0 \end{bmatrix}.$$

So,

$$\mathbf{b} = \begin{bmatrix} -22 \\ 4 \\ 32 \end{bmatrix} = 4\begin{bmatrix} -3 \\ 3 \\ 4 \end{bmatrix} + (-2)\begin{bmatrix} 5 \\ 4 \\ -8 \end{bmatrix}.$$

54. Expanding the left side of the equation produces

$$\begin{bmatrix} 2 & -1 \\ 3 & -2 \end{bmatrix}A = \begin{bmatrix} 2 & -1 \\ 3 & -2 \end{bmatrix}\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$$

$$= \begin{bmatrix} 2a_{11} - a_{21} & 2a_{12} - a_{22} \\ 3a_{11} - 2a_{21} & 3a_{12} - 2a_{22} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

and you obtain the system

$$\begin{aligned} 2a_{11} \quad - a_{21} \quad &= 1 \\ 2a_{12} \quad - a_{22} &= 0 \\ 3a_{11} \quad - 2a_{21} \quad &= 0 \\ 3a_{12} \quad - 2a_{22} &= 1. \end{aligned}$$

Solving by Gauss-Jordan elimination yields

$a_{11} = 2,\ a_{12} = -1,\ a_{21} = 3,$ and $a_{22} = -2$.

So, you have $A = \begin{bmatrix} 2 & -1 \\ 3 & -2 \end{bmatrix}$.