

## HW 4

1) Consider the covariance matrix below:

A) Find the eigenvalues and eigenvectors of  $\Sigma$ .

$$(E - yI)v = 0; Ev = Ivy$$

$$\det(E): (1-y)(3-y)=4; 1-4y-y^2=0$$

$$y = \frac{-4 \pm \sqrt{((-4)^2 - 4(1)(1))^{1/2}}}{2(1)} = \frac{-4 \pm \sqrt{(16-4)^{1/2}}}{2}$$

$$= \frac{-4 \pm (3.464/2)}{2} = -4 - 1.732, -4 + 1.732$$

$$= -5.732, -2.286$$

$$\Sigma = \begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix}$$

$$(1-y)o_{11} = -o_{12} : (1+5.732)o_{11} = -o_{12}$$

$$(2+5.732)o_{11} = -3o_{12}$$

$$(1+2.286)o_{21} = -o_{22} : (2+2.286)o_{21} = -3o_{22}$$

$$(o_{11})^2 + (6.732)^2 x (o_{11})^2 = 1$$

$$o_{11} = 1/6.81; o_{12} = (1 - (1/6.81))^{1/2} = .978$$

B) Find the principal components of  $\Sigma$ .

$$((3.286)^2 + 1)(o_{21})^2 = 1, o_{21} = (1/11.8)^{1/2}$$

$$o_{21} = .29, o_{22} = (1 - (.29)^2)^{1/2} = .843$$

$$PCV: .978x_1 + .116x_2; .843x_1 + .29x_2$$

2) Consider the Iris dataset in R.

**R**>attach(iris)

A) Form principal components variables based on the four variables

Sepal.Length, Sepal.Width, Petal.Length, and Petal.Width. List out all

possible principal components.

```
> x=cbind(Sepal.Length,Sepal.Width,Petal.Length,Petal.Width)
> cov(x)
```

```
      Sepal.Length Sepal.Width Petal.Length Petal.Width
Sepal.Length  0.6856935 -0.0424340  1.2743154  0.5162707
Sepal.Width   -0.0424340  0.1899794  -0.3296564 -0.1216394
Petal.Length   1.2743154 -0.3296564  3.1162779  1.2956094
Petal.Width    0.5162707 -0.1216394  1.2956094  0.5810063
```

B) Report the proportion of variance and the cumulative proportion of variance

explained by each principal component.

```
> prcomp(x)
```

```
Standard deviations (1, ..., p=4):
```

```
[1] 2.0562689 0.4926162 0.2796596 0.1543862
```

```
[1] 0.924618723 0.053066483 0.017102610 0.005212184
```

```
Rotation (n x k) = (4 x 4):
```

```
      PC1      PC2      PC3      PC4
Sepal.Length 0.36138659 -0.65658877 0.58202985 0.3154872
Sepal.Width  -0.08452251 -0.73016143 -0.59791083 -0.3197231
Petal.Length 0.85667061 0.17337266 -0.07623608 -0.4798390
Petal.Width  0.35828920 0.07548102 -0.54583143 -0.7536574.
```

```
> summary(prcomp(x))
```

```
Importance of components%:
```

```
      PC1      PC2      PC3      PC4
Standard deviation  2.0563 0.49262 0.2797 0.15439
Proportion of Variance 0.9246 0.05307 0.0171 0.00521
Cumulative Proportion 0.9246 0.97769 0.9948 1.00000
```

C) Produce a scree plot of the principal components and explain how many principal components you would choose.

see attached files

I would choose only the first 2 principal components

since they have the largest difference.

3) Consider the Iris dataset in R.

A) Form linear discriminant variables based on the four variables Sepal.Length, Sepal.Width, Petal.Length, and Petal.Width. List out all possible linear discriminant variables

```
> ldaout<-lda(Species~Sepal.Length+Sepal.Width+Petal.Length+Petal.Width,iris)
> ldaout
Call:
lda(Species ~ Sepal.Length + Sepal.Width + Petal.Length + Petal.Width,
    data = iris)

Prior probabilities of groups:
  setosa versicolor virginica 
0.3333333 0.3333333 0.3333333 

Proportion of trace:
  LD1  LD2 
0.9912 0.0088
```

Group means:

	Sepal.Length	Sepal.Width	Petal.Length	Petal.Width
setosa	5.006	3.428	1.462	0.246
versicolor	5.936	2.770	4.260	1.326
virginica	6.588	2.974	5.552	2.026

Coefficients of linear discriminants:

	LD1	LD2
Sepal.Length	0.8293776	0.02410215
Sepal.Width	1.5344731	2.16452123
Petal.Length	-2.2012117	-0.93192121
Petal.Width	-2.8104603	2.83918785

B) Predict the species of a flower with a sepal length of 5.8, sepal width of 3.1, petal length of 3.8, and petal width of 1.2.

versicolor

versicolor

C) Calculate the first linear discriminant score of a flower with a sepal length of 5.8, sepal width of 3.1, petal length of 3.8, and petal width of 1.2.

$$LD1 = +1.5sw - 2.2pl - 2.8pw$$

D) Using histograms plot first linear discriminant scores by each species group (hint: one histogram for each group).

See second attached.

E) Using histograms plot second discriminant scores by each species group (hint: one histogram for each group).

```
hist(ldaout)
Error in hist.default(ldaout) : 'x' must be numeric
> hist(ldatable)
> ldapred<-predict(ldaout,iris)
> ldaclass<-ldapred$class
> ldatable<-table(ldaclass,iris$Species)
> hist(ldatable)
> ldatables<-table(ldaclass,iris$Species)
> hist(ldatables)
> ldaout<-lda(Species~Sepal.Width)
> ldapred<-predict(ldaout,iris)
> ldaclass<-ldapred$class
> ldablesw<-table(ldaclass,iris$Species)
> hist(ldablesw)
> ldaout<-lda(Species~Petal.Length)
> ldapred<-predict(ldaout,iris)
> ldaclass<-ldapred$class
> ldatablepl<-table(ldaclass,iris$Species)
> hist(ldatablepl)
> ldaout<-lda(Species~Petal.Width)
> ldapred<-predict(ldaout,iris)
> ldaclass<-ldapred$class
> ldatablepw<-table(ldaclass,iris$Species)
> hist(ldatablepw)
```

\*See Second Attached\*

Thanks to: <https://www.youtube.com/watch?v=gu3MMpakTlw>