# HW3 due 11:30a Mon Oct 24

## 1. Inner product on $\mathbb{C}^n$

Given $x, y \in \mathbb{C}^n$, define the inner product $\langle x, y \rangle$ by the formula

$$\langle x, y \rangle = \sum_{\ell=1}^{n} \overline{x}_\ell \cdot y_\ell$$

where $\overline{x}_\ell \cdot y_\ell$ denotes the complex scalar multiplication between the complex conjugate $\overline{x}_\ell$ of $x_\ell$ and the complex number $y_\ell$.

a. $\langle \cdot, \cdot \rangle$ is a function that takes two complex $n$-vectors as arguments; what is its codomain?

b. Is $\langle \cdot, \cdot \rangle$ linear in its second argument? In other words, does the following equality hold?

$$\forall x, y, z \in \mathbb{C}^n, \zeta \in \mathbb{C} : \langle x, y + \zeta z \rangle = \langle x, y \rangle + \alpha \langle x, z \rangle$$

c. Is $\langle \cdot, \cdot \rangle$ symmetric? In other words, does the following equality hold?

$$\forall x, y \in \mathbb{C}^n : \langle x, y \rangle = \langle y, x \rangle$$

If not, how is $\langle x, y \rangle$ related to $\langle y, x \rangle$?

d. Is $\langle \cdot, \cdot \rangle$ positive definite? In other words, is the following true?

$$\forall x \in \mathbb{C}^n : \langle x, x \rangle \geq 0,\ \langle x, x \rangle = 0 \iff x = 0$$

## 2. Linear functions

Let $(m \times n)$ denote the set

$$(m \times n) = \{(i, j) : i \in \{1, \ldots, m\},\ j \in \{1, \ldots, n\}\}.$$

a. Show that the set $\mathcal{A} = \{A : (m \times n) \to \mathbb{F}\}$ of matrices with $m$ rows and $n$ columns is a vector space over the field $\mathbb{F}$. (You need to define vector addition $+ : \mathcal{A} \times \mathcal{A} \to \mathcal{A}$ and scalar multiplication $\cdot : \mathbb{F} \times \mathcal{A} \to \mathcal{A}$ and show that they satisfy the commutative, associative, distributive, and zero element properties that define a vector space.)

Let $V, W$ be vector spaces over the same field $\mathbb{F}$.

a. Show that the set $\mathcal{L} = \{L : V \to W \mid L \text{ is linear}\}$ of linear maps from $V$ to $W$ is a vector space over the field $\mathbb{F}$ (You need to define vector addition $+ : \mathcal{L} \times \mathcal{L} \to \mathcal{L}$ and scalar multiplication $\cdot : \mathbb{F} \times \mathcal{L} \to \mathcal{L}$ and show that they satisfy the commutative, associative, distributive, and zero element properties that define a vector space.)

# 3. Project control system

**This problem is repeated from HW2 to provide the opportunity for you to revise your project system to address any issues that arose during the self-assessment or feedback from the TAs. If you already addressed each point below in your HW2 submission and dot wish to make any changes, simply include a statement to that effect in your submission for HW3.**

Select a control system for your Project; refer to Canvas/Pages/Project for ideas and requirements.

a. What is the system state? Indicate any parameters (i.e. "states" that don't change in time). ($\geq 3$ dimensions)

b. What are the inputs to the system? Explain the inputs in physical terms, i.e. what physical device or mechanism actuates the input. ($\geq 2$ inputs; create one if needed)

c. What are the outputs from the system? Explain the outputs in physical terms, i.e. what physical device or mechanism measures the output. ($\geq 2$ outputs; create one if needed)

d. Write an ODE control system model for your system's dynamics in the form $\dot{x} = f(x, u),\ y = h(x, u)$. Be sure to specify the domain and codomain of $f, h$.

e. Is the control system linear or nonlinear? Show algebraically or graphically the source of nonlinarity. (must be nonlinear)

f. What disturbances could affect the system's dynamics? Specify what elements of $f$ and/or $h$ the disturbance would affect. ($\geq 1$ disturbance that affects $f$, $\geq 1$ disturbance that affects $h$)

g. Why is your Project system synergistic with your education, research, and/or professional interests?

h. Add your Project system title and a link to $\geq 1$ relevant paper / preprint / technical report in Canvas/Collaboration/Projects; upload the paper .pdf with your hw2 Assigment on Canvas.