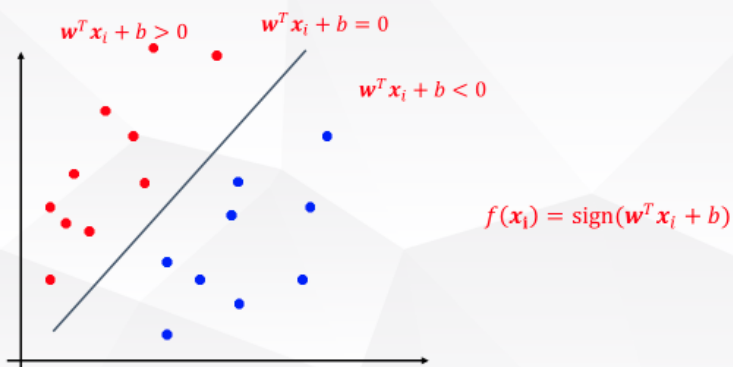


# AIoT Lecture 6 Support Vector Machine (SVM)

## ▼ 1. SVM 介紹

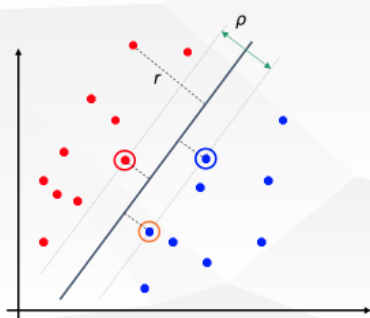
### ➤ Linear Separators

- 二元分類可以看作是在特徵空間中分離類的任務
- 尋找所謂的Hyperplane (比原來特徵平面少一維度的決策面)



P. 1

### ➤ 判別區的決定



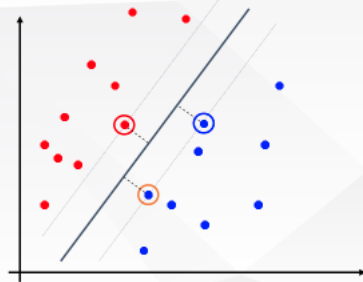
- 從sample  $x_i$  到分隔符的距離為  $r$

$$r = \frac{w^T x_i + b}{\|w\|}$$

- 最接近超平面的 sample 是支援向量
- Margin  $\rho$  of the separator 是支援向量之間的距離

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## ➤ Maximum Margin Classification



➤ **Maximizing the margin** is a good choice

➤ 暗示只有支援向量才重要，其他訓練示例可忽略

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Question: 如何求解

## ▼ 2. 數學式表達

### ➤ Linear SVM: Mathematical Derivations

➤ Let training set  $\{(x_1, y_1), (x_2, y_2), \dots, (x_m, y_m)\}$  and  $y_i \in \{1, -1\}$  be separated by a hyperplane with margin  $\rho$

➤ Then for each training example  $(x_i, y_i)$ , we have

$$y_i = \begin{cases} 1, & w^T x_i + b \geq \rho/2 \\ -1, & w^T x_i + b \leq -\rho/2 \end{cases} \quad \Rightarrow \quad \text{Hyperplane } w^T x_i + b = 0$$

➤ The support vectors lie on

$$w^T x_i + b = \pm \rho/2 \quad \xrightarrow{\text{Rescaling } w} \quad w^T x_i + b = \pm 1$$

➤ The distance from the support vectors to the hyperplane is

$$\frac{2}{\|w\|}$$

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## ➤ Linear SVM: Mathematical

➤ 然後我們可以公式化二次優化問題:

$$\begin{aligned} \max \frac{2}{\|\mathbf{w}\|} \quad & \text{subject to} \quad y_i(\mathbf{w}^T \mathbf{x}_i + b) \geq 1 \\ \min \frac{1}{2} \|\mathbf{w}\|^2 \quad & \text{subject to} \quad y_i(\mathbf{w}^T \mathbf{x}_i + b) \geq 1 \end{aligned}$$

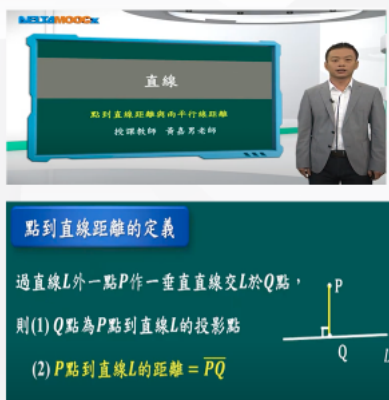
- 需要優化受線性約束的二次函數
- 二次優化問題是一類眾所周知的數學規劃問題

P. 9

### ▼ 3. 高中線性方程求解

## ➤ 回憶高中數學 點到線距離

➤ <https://www.youtube.com/watch?v=XKOgeVGHY74>



學習重點：點到直線距離公式

已知平面上一點  $P(x_0, y_0)$  與一直線  $L: ax + by + c = 0$

若點  $P$  到直線  $L$  的距離為  $d$ ；

則  $d = d(P, L) = \frac{|ax_0 + by_0 + c|}{\sqrt{a^2 + b^2}}$

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## ➤ 兩平行線距離1

### 學習重點：兩平行線距離公式

平面坐標系中，

已知兩平行直線 $L_1: ax + by + c_1 = 0$

與 $L_2: ax + by + c_2 = 0$

則兩平行直線的距離為 $\frac{|c_1 - c_2|}{\sqrt{a^2 + b^2}}$

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## ➤ 兩平行線距離2

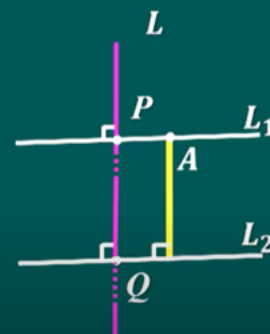
試求兩平行線  $L_1: 5x - 12y = 15$  的距離？  
 $L_2: 5x - 12y = 2$

<解>：

在直線 $L_1$ 上任取一點 $A(3, 0)$

且直線 $L_2: 5x - 12y - 2 = 0$

$$\text{則 } d(A, L_2) = \frac{|3 \times 5 - 12 \times 0 - 2|}{\sqrt{5^2 + 12^2}} = \frac{13}{13} = 1$$



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## ➤ 兩種公式總結

1. 平面坐標系中，已知直線  $L: ax + by + c = 0$  與點  $P(x_0, y_0)$ ，則  $P$  點到直線  $L$  的距離為

$$d(P, L) = \frac{|ax_0 + by_0 + c|}{\sqrt{a^2 + b^2}}$$

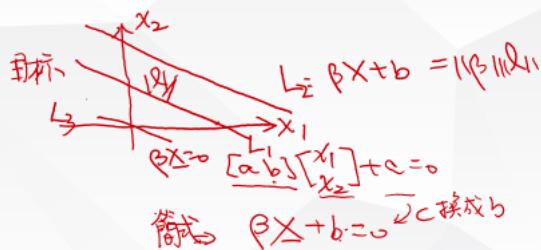
2. 平面坐標系中，已知兩平行直線  $L_1: ax + by + c_1 = 0$  與  $L_2: ax + by + c_2 = 0$ ，則兩平行直線的距離為

$$\frac{|c_1 - c_2|}{\sqrt{a^2 + b^2}}$$

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## ➤ 內積推導

用 vector 內積來推導：



$L_1: \beta X + b = 0$   
 $L_2: \beta X + b = \|\beta\| \|L_1\|$   
 $\beta X = -[a \ b] \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + c = 0$   
 簡式  $\beta X + b = 0$  (c 換成 b)

要證明  $\Rightarrow$  當  $L_1, L_2$  距離是  $\|L_1\|$ ，且  $L_1: \beta X + b = 0$   
 則  $L_2: \beta X + b = \pm \|\beta\| \|L_1\|$

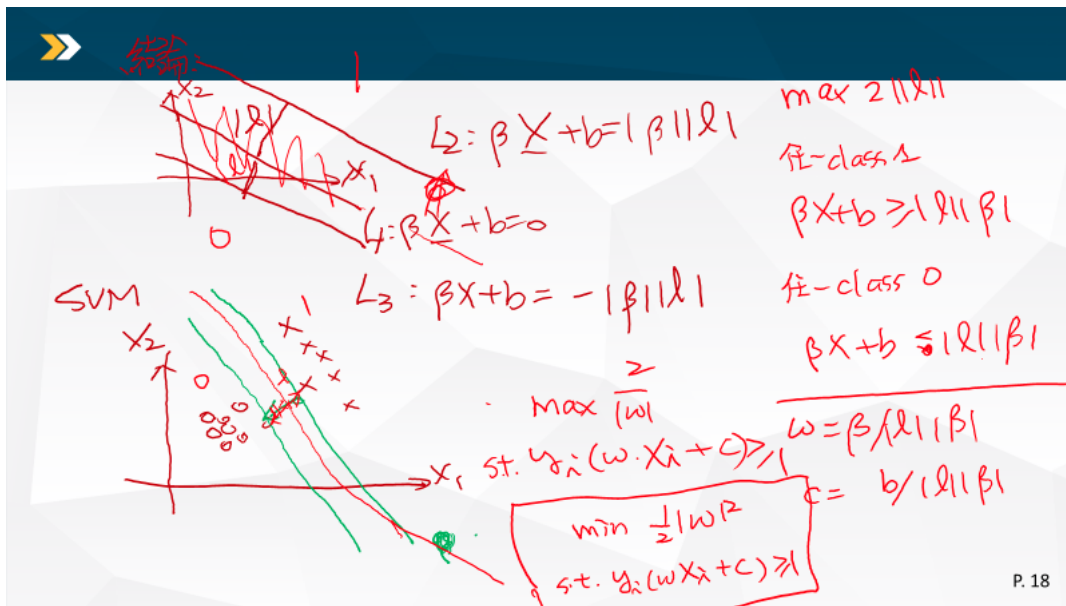
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$$L+: \beta X + b = \|\beta\| \|L\|$$

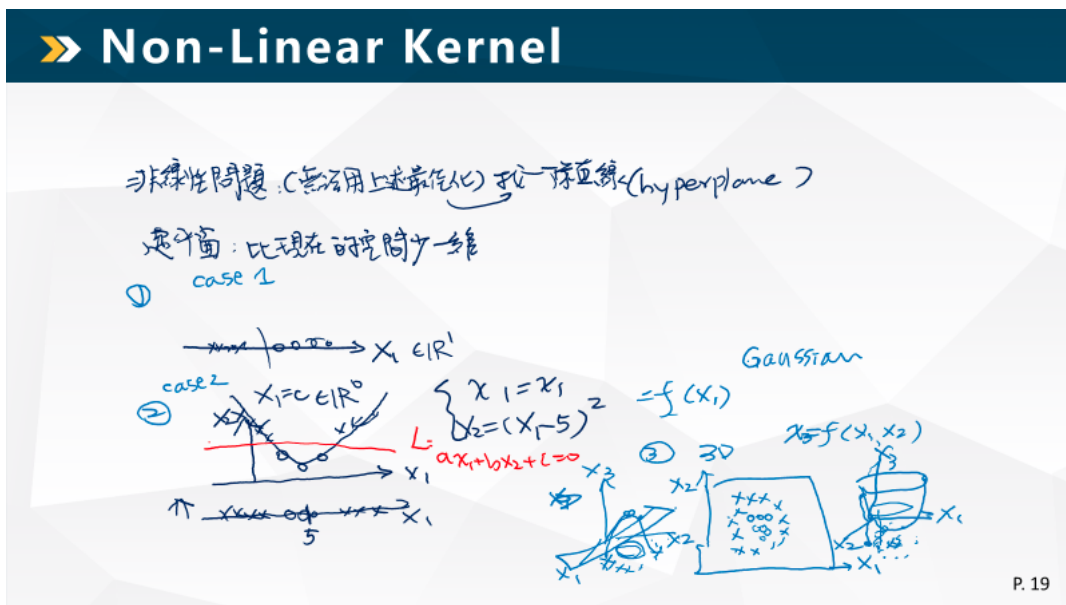
$$L1: \beta X + b = 0$$

$$L-: \beta X + b = -\|\beta\| \|L\|$$

$$\text{公式與兩條線距離對應} \|L\| = \frac{|c_1 - c_2|}{\sqrt{a^2 + b^2}} = \frac{\|\beta\| \|L\| - 0}{\|\beta\|}$$



#### ▼ 4. 非線性Kernel



LinearSVC (線性)  
kernel='linear' (線性)  
kernel='poly' (非線性)  
kernel='rbf' (非線性)

- <https://www.geeksforgeeks.org/major-kernel-functions-in-support-vector-machine-svm/>
- <https://notes.andywu.tw/2020/白話文講解支持向量機二-非線性svm/>

#### ▼ 5. 實作範例觀摩與畫圖工具

- <https://medium.com/jameslearningnote/資料分析-機器學習-第3-4講-支援向量機-support-vector-machine-介紹-9c6c6925856b>

- <https://ithelp.ithome.com.tw/articles/10270447>

Countour Plot in Matplotlib

## ➤ Countour Plot in Matplotlib

➤ 資料視覺化之 contour (matplotlib.pyplot) 教學與用法

Seachaos

<https://tree.rocks/python-matplotlib-plt-contour-or-contourf-tutorial-with-sklearn-e4497f76280>



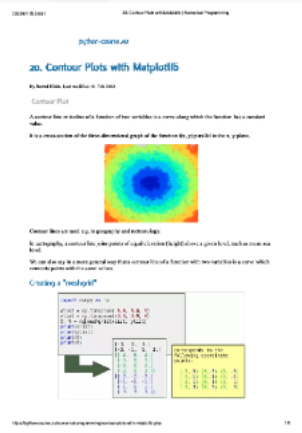
P. 4

## ➤ Contourf Plot in Matplotlib

➤ Contour Plots with Matplotlib

python-course.eu

<https://python-course.eu/numerical-programming/contour-plots-with-matplotlib.php>



P. 5

### ▼ Code example 1 : 線性分兩類的

```
# Topic : ML 分類 2 class (線性)
* follow CRISP-DM

## Step 1: Load data, import library
```

```

import numpy as np

# step 1: Load data, import library

# =====for deep learning =====

import torch
import torch.nn as nn

#===== for ML =====
import pandas as pd
import numpy as np
import matplotlib.pyplot as plt
from sklearn import datasets

# define function
def plotResult(X,y,w1=2, w2=2,b=2):
    plt.scatter(X[y==0,0],X[y==0,1])
    plt.scatter(X[y==1,0],X[y==1,1])
    xm=np.array([X.min(),X.max()])
    ym=(w1*xm+b)/(-w2)
    plt.plot(xm,ym,'r')
    print("w1=",w1,"w2=",w2,"b=",b)
    plt.show()

def getParameters(model):
    w,b =model.parameters()
    w1=w[0][0].item()
    w2=w[0][1].item()
    b=b[0].item()
    return w1,w2,b

# test 1 generate data =====
n_samples=200
centers=[[-0.5,0.5],[0.5,-0.5]]
X,y=datasets.make_blobs(n_samples=n_samples,centers=centers,cluster_std=0.4,random_state=3)
#print(X[0],type(X[0]),X[0].shape)
plotResult(X,y)

# test 2 read from 400pts datasets
# data=pd.read_csv("400pts.csv")
# X=data.iloc[:, :-1].values.reshape(-1,2)
# y=data.iloc[:, -1].values
# print(type(X))

print(type(X),X[:5])
print(type(y),y[:5])

# 練習
# plt.scatter(X[y==0,0],X[y==0,1])
# # plt.show()
# plt.scatter(X[y==1,0],X[y==1,1])
# plt.show()

## Step 2: Preprocessing

#step 2 : Preprocessing for pytorch
tensor_X=torch.FloatTensor(X)
tensor_y=torch.FloatTensor(y.reshape(200,1))
print(tensor_X.size())

# Step 3: Build Model

# step 3:
from sklearn.linear_model import LogisticRegression as LR
model=LR()
model.fit(X,y)
print(model.coef_,model.intercept_)
[[w1,w2]]=model.coef_
[b]=model.intercept_
print('Logistic regression')
plotResult(X,y,w1=w1, w2=w2,b=b)

# step 3:
# from sklearn.svm import SVC
# model2=SVC(kernel="linear")
# print(model2.get_params())
# model2.fit(X,y)
# print(model2.coef_,model.intercept_)
# [[w1,w2]]=model2.coef_
# [b]=model2.intercept_

```



```

# print('SVC')
# plotResult(X,y,w1=w1, w2=w2,b=b)

<img src= "https://lh6.googleusercontent.com/pgnVbPBuXCiR5Ri2t0R9xIL4rpuTXWwZ5euvGhegi8DH8H0Lbcu19DwX86f4C50hexXsOI9V6iIG9mUcAGBw1f" data-bbox="144 144 886 174"/>

#step 3: build model

class MyLogRegNN(nn.Module):
    def __init__(self,inSize,outSize):
        super().__init__()
        self.linear=nn.Linear(inSize,outSize)
    def forward(self,x):
        x=self.linear(x)
        y_hat=torch.sigmoid(x)
        return y_hat
    def predict(self,x):
        pred=self.forward(x)
        if pred >=0.5:
            return 1
        return 0

model=MyLogRegNN(2,1)
print(model)

a=[1,2,3,4,5,6]
a_it=iter(a)
for i in range(6):
    print(next(a_it))

#step 4 training
criterion=nn.BCELoss()
optimizer = torch.optim.SGD(model.parameters(),lr=0.8)
epochs=100
losses=[]
for e in range(epochs):
    #compute loss
    y_hat=model(tensor_X)
    loss_=criterion(y_hat, tensor_y).detach().numpy()
    loss=criterion(y_hat, tensor_y)

    losses.append(loss_)

    # 3 steps for Gradient update
    optimizer.zero_grad()
    loss.backward()
    optimizer.step()

plt.plot(range(epochs), losses)

# plot results

w1,w2,b =getParameters(model)
plotResult(X,y,w1,w2,b)

```

## ▼ Code example 2: 非線性分兩類

```

Topic: Classification (ML+DL)
Step 1: load data, import library
[ ]
# step 1: Load data, import library
# import torch
# import torch.nn as nn

import numpy as np
import matplotlib.pyplot as plt
import pandas as pd
from sklearn import datasets

# define function

# def getParameters(model):
#     w,b =model.parameters()
#     w1=w[0][0].item()

```

```

# w2=w[0][1].item()
# b=b[0].item()
# return w1,w2,b

# generate data =====
n_samples=500
X,y=datasets.make_circles(n_samples=n_samples,noise=0.2,random_state=3,factor=0.2)
#print(X[0],type(X[0]),X[0].shape)

[ ]

def plotBoundary(X,y,model):
    row_condition= (y==0)

    plt.scatter(X[row_condition,0],X[y==0,1])
    plt.scatter(X[y==1,0],X[y==1,1])

    x_span=np.linspace(min(X[:,0])-0.25,max(X[:,0])+0.25)
    y_span=np.linspace(min(X[:,1])-0.25,max(X[:,1])+0.25)
    xx,yy=np.meshgrid(x_span,y_span)
    print(x_span.shape,xx.shape)
    grid=np.c_[xx.ravel(),yy.ravel()]

    z=model.predict_proba(grid)
    print(type(z),z.shape)
    plt.contour(xx,yy,z)
    plt.show()

[ ]
plt.scatter(X[y==0,0],X[y==0,1])
plt.scatter(X[y==1,0],X[y==1,1])
plt.show()

Step 2 : Preprocessing
[ ]
#step 2 : Preprocessing for ML
print(type(X),X.shape)
print(type(y),y.shape)
print(y[:5])

#step 2 : Preprocessing for DL
# tensor_X=torch.FloatTensor(X)
# tensor_y=torch.FloatTensor(y.reshape(n_samples,1))
# print(tensor_X.size())
# print(tensor_y.size())
<class 'numpy.ndarray'> (500, 2)
<class 'numpy.ndarray'> (500,)
[0 1 0 0 0]

Step 3: Build Model
参考blog

[ ]
def plotBoundary(X,y,model):
    row_condition= (y==0)

    plt.scatter(X[row_condition,0],X[y==0,1])
    plt.scatter(X[y==1,0],X[y==1,1])

    x_span=np.linspace(min(X[:,0])-0.25,max(X[:,0])+0.25)
    y_span=np.linspace(min(X[:,1])-0.25,max(X[:,1])+0.25)
    xx,yy=np.meshgrid(x_span,y_span)
    print(x_span.shape,xx.shape)
    grid=np.c_[xx.flatten(),yy.flatten()]

    z=model.predict_proba(grid)
    z0=z[:,0].reshape(xx.shape)
    print(type(z0),z0.shape)
    plt.contour(xx,yy,z0,alpha=0.5)
    plt.show()

[ ]
# step 3: Build model for ML
# Fitting Kernel SVM to the Training set
from sklearn.svm import SVC
model = SVC(kernel='rbf',random_state = 0,probability=True)

model.fit(X, y)

y_hat=model.predict_proba(X)
print(y_hat[:5])

plotBoundary(X,y,model)

# plt.scatter(X[y==0,0],X[y==0,1])

```

```

# plt.scatter(X[y==1,0],X[y==1,1])

# x_span=np.linspace(min(X[:,0])-0.25,max(X[:,0])+0.25)
# y_span=np.linspace(min(X[:,1])-0.25,max(X[:,1])+0.25)
# xx,yy=np.meshgrid(x_span,y_span)
# print(x_span.shape, 'xx.shape=', xx.shape)
# grid=np.c_[xx.ravel(),yy.ravel()]
# print(grid.shape)
# z=model.predict_proba(grid)
# print('z=',z)
# print(type(z),z.shape)
# plt.contour(xx,yy,z)
# # plt.show()

[ ]

#step 3: build model for DL

# class MyDNN(nn.Module):
#     def __init__(self, inSize, h1Size, h2Size, outSize):
#         super().__init__()
#         self.linear=nn.Linear(inSize, h1Size)
#         self.linear2=nn.Linear(h1Size, h2Size)
#         self.linear3=nn.Linear(h2Size, outSize)
#     def forward(self, x):
#         x=self.linear(x)
#         x=torch.sigmoid(x)
#         x=self.linear2(x)
#         x=torch.sigmoid(x)
#         x=self.linear3(x)
#         x=torch.sigmoid(x)
#         return x
#     def predict(self, x):
#         pred=self.forward(x)
#         if pred >=0.5:
#             return 1
#         return 0

# model=MyDNN(2,10,4,1)
# print(model)

[ ]

[ ]
#step 4 training
# criterion=nn.BCELoss()
# optimizer = torch.optim.Adam(model.parameters(), lr=0.01)
# epochs=5000
# losses=[]
# for e in range(epochs):
#     #compute loss
#     if e%1000 ==0: print("",end="")
#     y_hat=model(tensor_X)
#     loss=criterion(y_hat, tensor_y)
#     losses.append(loss)

#     # 3 steps for Gradient update
#     optimizer.zero_grad()
#     loss.backward()
#     optimizer.step()
# plt.plot(range(epochs), losses)

[ ]
# plot results
plotBoundary(X,y,model)

[ ]

```