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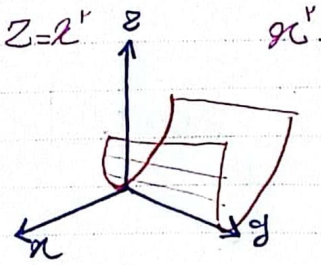
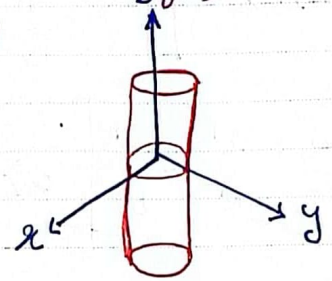
رویه‌های درجه ۲: رویه استوانه‌ای: در رابطه‌ی $F(x,y,z)=c$ اگر حداقل یکی از متغیرها ظاهر نشود آن را رویه استوانه‌ای نامیم.

مثال: رویه‌های زیر را رسم کنید

$z=x^2$



$x^2+y^2=1$

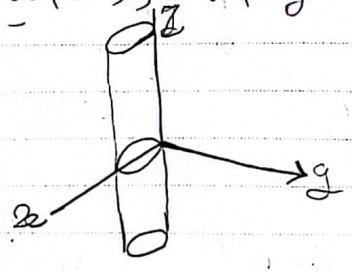


توضیح: برای رسم یک رویه استوانه‌ای به روش زیر عمل می‌کنیم. اگر متغیرهای x,y,z در رابطه ظاهر نشوند ابتدا در صفحه xy یا xz یا yz نمودار داده‌ی را رسم می‌کنیم، سپس از هر نقطه‌ی نمودار یک خط به موازی متغیر سوم (مثلاً z) رسم می‌کنیم مجموعه‌ی همه‌ی این خط‌ها جواب مسئله است.

$x^2-2x+y^2=0$

$(x-1)^2+y^2=1$

رسم کنید $x^2+y^2=2x$

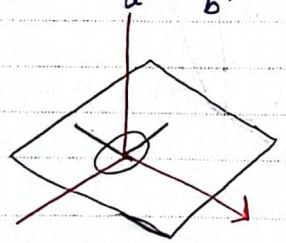


رویه‌های درجه دوم:

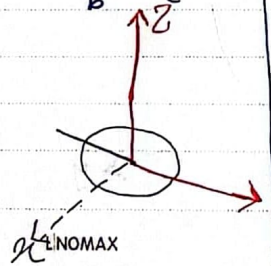
$\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1 \quad a,b,c>0$

بیضی‌گون

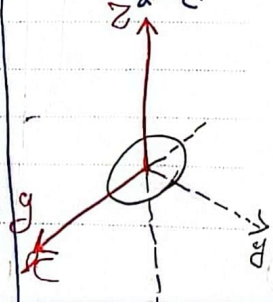
$z=0 \Rightarrow \frac{x^2}{a^2}+\frac{y^2}{b^2}=1$



$x=0 \Rightarrow \frac{y^2}{b^2}+\frac{z^2}{c^2}=1$



$y=0 \Rightarrow \frac{x^2}{a^2}+\frac{z^2}{c^2}=1$



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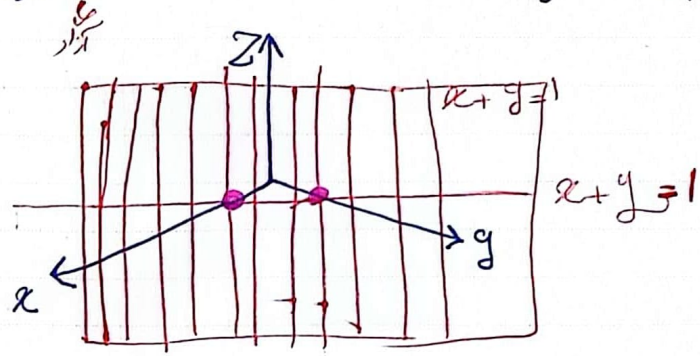
فضای $\mathbb{R}^3$

$x+y=1 \qquad N=i+j$

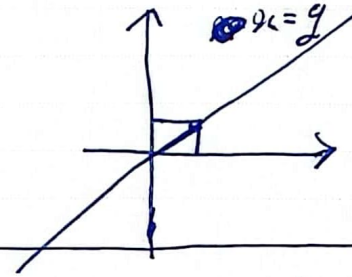
$\{(x,y,z)\in\mathbb{R}^3 \mid x+y=1 \quad z\in\mathbb{R}\} \equiv x+y=1$



فضای $\mathbb{R}^2$

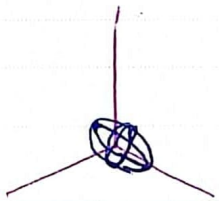
$y=x$

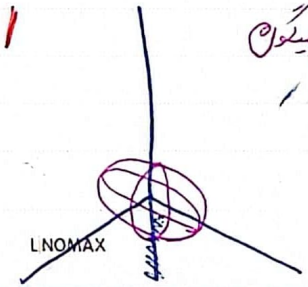
$\{(x,y)\in\mathbb{R}^2 \mid y=x\} \equiv y=x$



$\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$

بیضی‌گون





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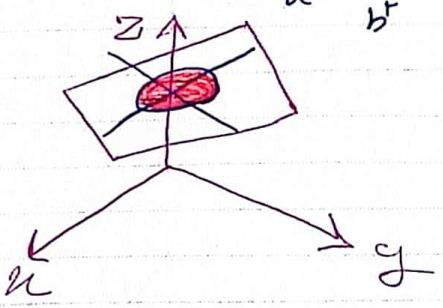
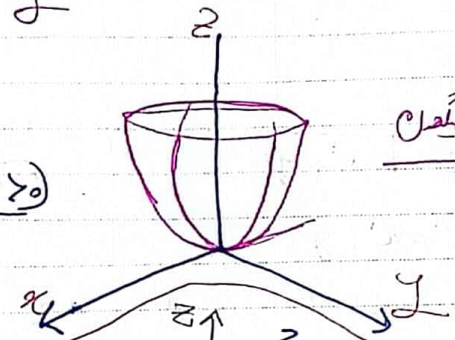
$z = z_0 > 0$ $\quad \frac{x^2}{a^2} + \frac{y^2}{b^2} = \frac{z_0}{c}$



$\frac{z}{c} = \frac{x^2}{a^2} + \frac{y^2}{b^2}$ $(c>0)$



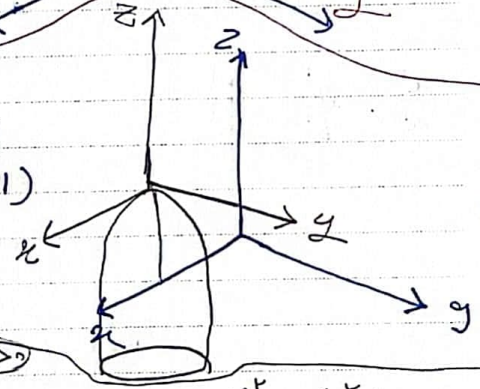
کیلیدی

$x^2 + y^2 - 2x + z = 0$

$(x-1)^2 + y^2 = -z + 1$

$(x-1)^2 + y^2 = -(z+1)$

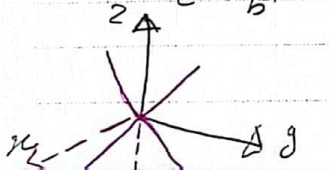
$x^2 + y^2 = z$

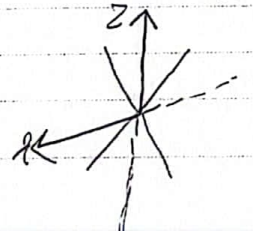


$\frac{z^2}{c^2} = \frac{x^2}{a^2} + \frac{y^2}{b^2}$ $\quad (a,b,c>0)$

$y=0 \rightarrow \frac{z^2}{c^2} = \frac{x^2}{a^2}$

$x=0 \rightarrow \frac{z^2}{c^2} = \frac{y^2}{b^2} \Rightarrow \frac{|z|}{c} = \frac{|y|}{b}$





LINOMAX

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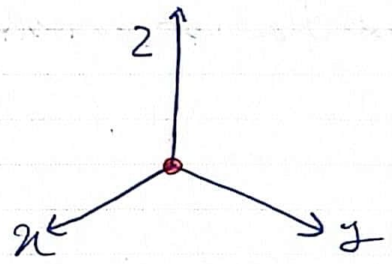
$x \rightarrow -x$
$y = -y$
$z \rightarrow -z$

$a = b = c \Rightarrow$
$x^2 + y^2 + z^2 = a^2$
کره

$\frac{z}{c} = \frac{x^2}{a^2} + \frac{y^2}{b^2}$ $\quad c>0$

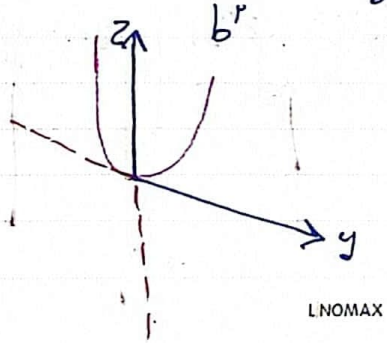
$a, b, c > 0$

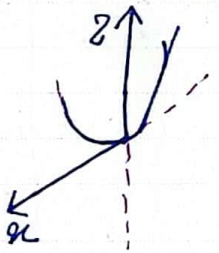
$z = 0 \Rightarrow \frac{x^2}{a^2} + \frac{y^2}{b^2} = 0 \Rightarrow x = y = 0$ $\quad z = z_0$



$x=0 \Rightarrow z = c\frac{y^2}{b^2}$

$y=0 \Rightarrow \frac{z}{c} = \frac{x^2}{a^2}$





LINOMAX

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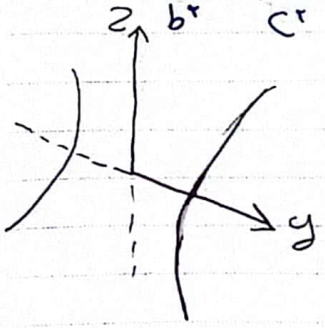
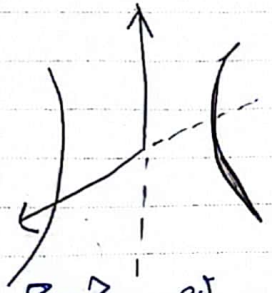
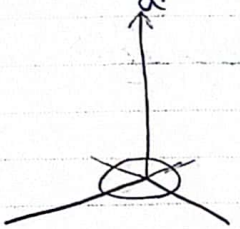
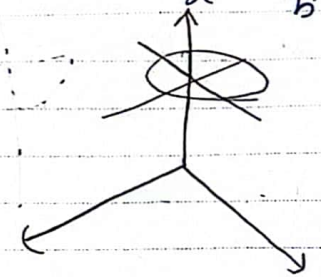
$x=0 \Rightarrow \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1$ $\quad y=0 \Rightarrow \frac{x^2}{a^2} - \frac{z^2}{c^2} = 1$



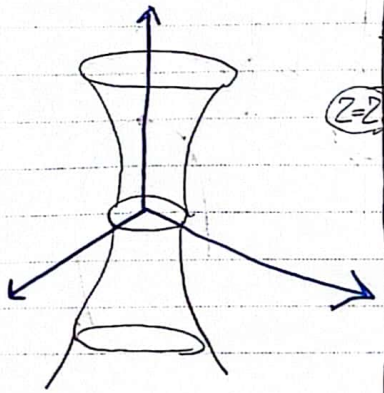


$z=0 \Rightarrow \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ $\quad z=z_0 \Rightarrow \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 + \frac{z_0}{c^2}$





$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1$



هذا لو اردنا رسمها به

$z=0 \Rightarrow \frac{x^2}{a^2} + \frac{y^2}{b^2} = -1$ جواب ندارد

$(z=z_0)$ $\quad \frac{z_0^2}{c^2} - 1 > 0 \Rightarrow z_0^2 > c$ بنابراین $\Rightarrow |z_0| > 0$

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = \frac{z_0^2}{c^2} - 1 > 0$

عدد کاذب

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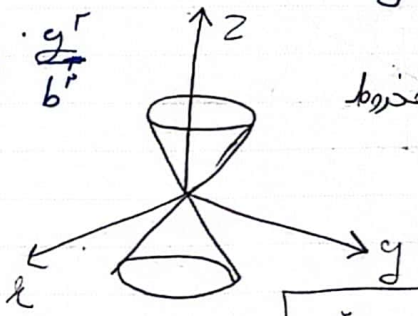
$|z| = |y| \Rightarrow z = \pm |y| \Rightarrow \begin{cases} z = y \\ z = -y \end{cases}$

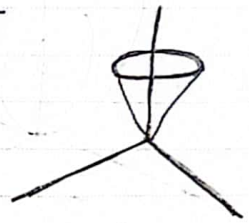
$\frac{z^2}{c^2} = \frac{x^2}{a^2} + \frac{y^2}{b^2}$ مخروط



$z = \sqrt{x^2 + y^2}$

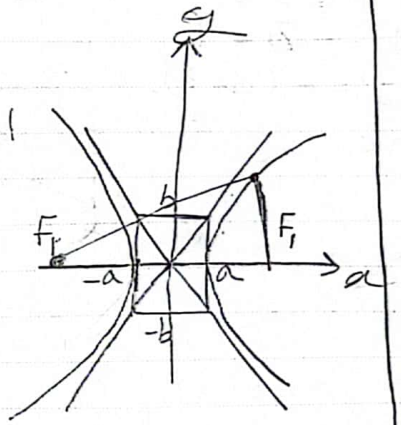
$z^2 = x^2 + y^2$

(z نامنفی است)

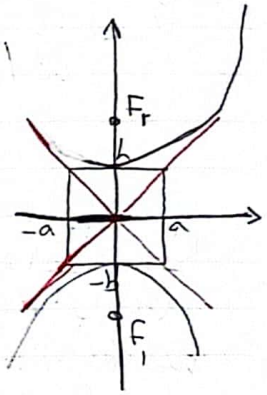


$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1$

$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$



$\frac{y^2}{b^2} - \frac{x^2}{a^2} = 1$



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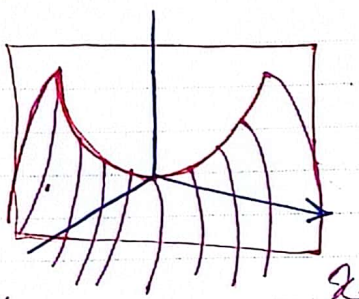
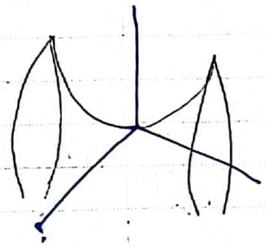
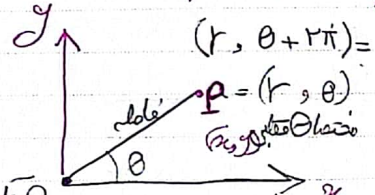


$x=x_0$



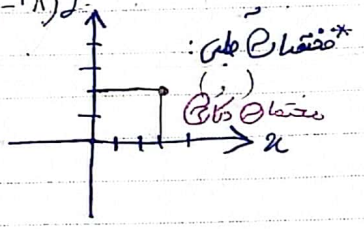
زین اسبی



$P=(r,\theta)$

مختصات قطبی نقطه P

$(r,\theta+2\pi)=(r,\theta-2\pi)$

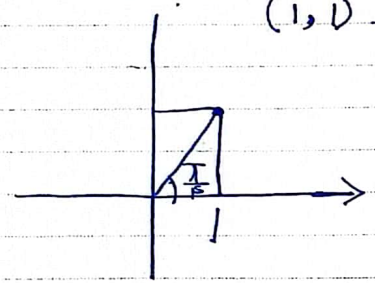


*مختصات قطبی: (r,θ)

مختصات دکارتی

$0<r=\sqrt{x^2+y^2}$

$\theta=0$ زاویه بین شعاع OP و جهت مثبت محور طول‌ها



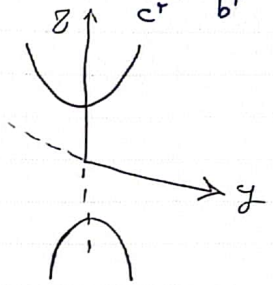
$(1,1)=\left(\sqrt{2},\frac{\pi}{4}\right)=\left(\sqrt{2},2k\pi+\frac{\pi}{4}\right)$

$k\in\mathbb{Z}$

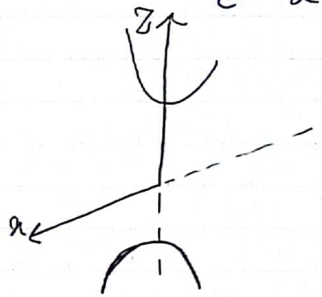
مقدار دادن

$(-r,\theta)=(r,\theta)$ قرینه نقطه‌ی (r,θ) نسبت به قطب

$x=0 \rightarrow \frac{z^2}{c^2}-\frac{y^2}{b^2}=1$



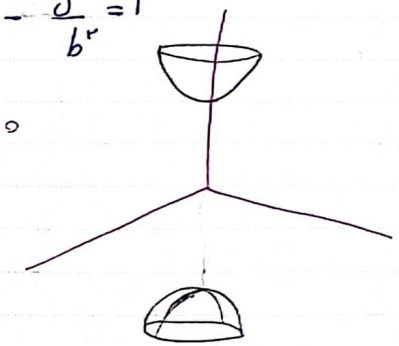
$y=0 \Rightarrow \frac{z^2}{c^2}-\frac{x^2}{a^2}=1$

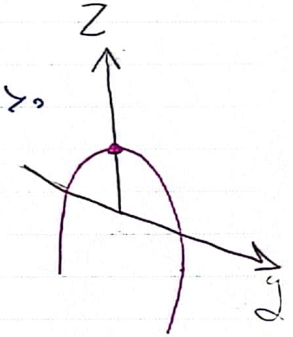


$\frac{z^2}{c^2}-\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$

هذلولیوار دوپارچه

$a,b,c>0$





$\frac{z}{c}=\frac{x^2}{a^2}-\frac{y^2}{b^2}$

$y=0 \Rightarrow \frac{z}{c}=\frac{x^2}{a^2}$

$x=x_0 \Rightarrow \frac{z}{c}=\frac{x_0^2}{a}-\frac{y^2}{b^2}$

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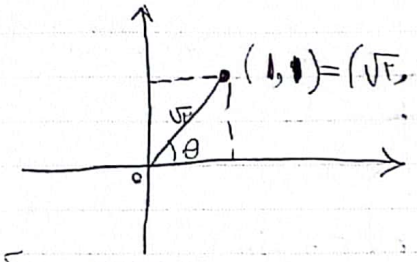
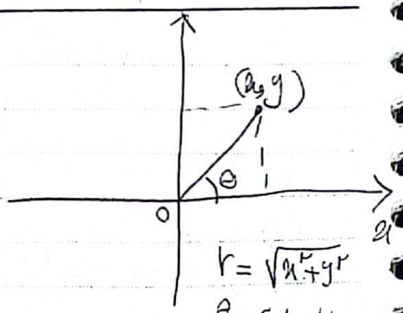
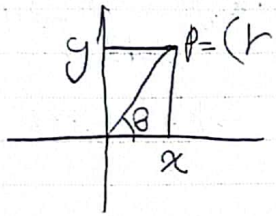


$$\begin{cases} r=\sqrt{x^2+y^2}=\sqrt{2} \\ \theta=\frac{\pi}{4} \end{cases}$$

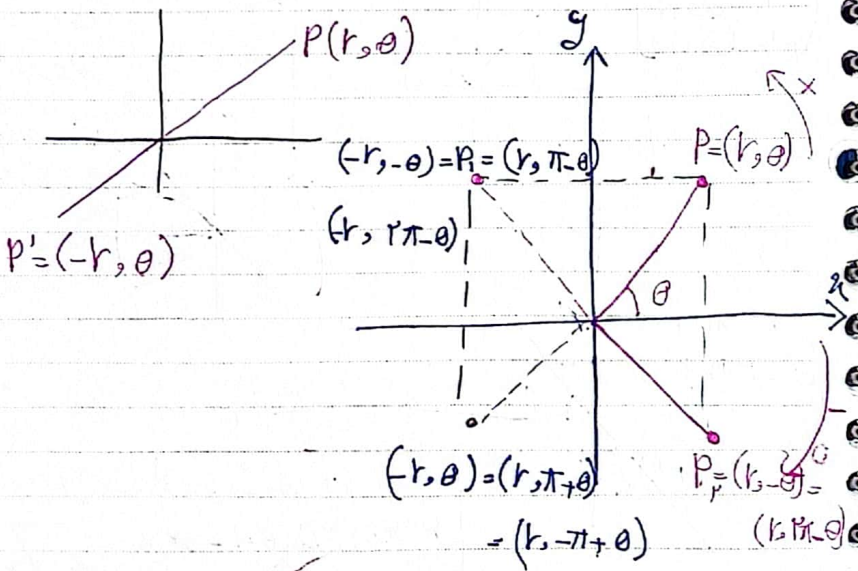


$r=\sqrt{x^2+y^2}$

$\theta=$ زاویه با بیت نسبت محور xها



θ بر حسب θ های $\begin{cases} x=r\cos\theta \\ y=r\sin\theta \end{cases}$



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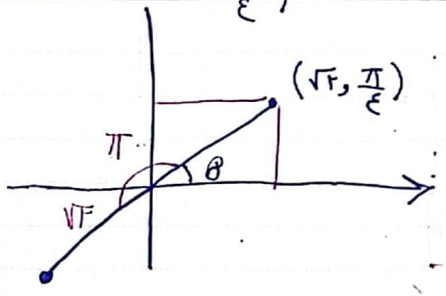
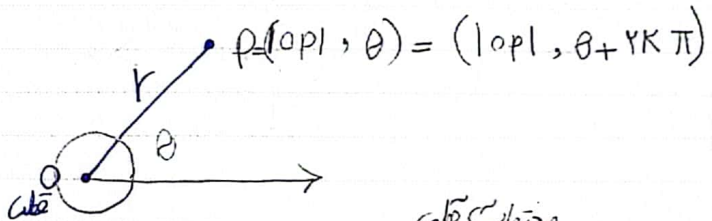
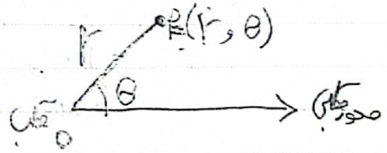
$\left(-\sqrt{2},\frac{\pi}{4}\right)$



$\left(\sqrt{2},\frac{\pi}{4}+\pi\right)=\left(-\sqrt{2},\frac{\pi}{4}\right)$

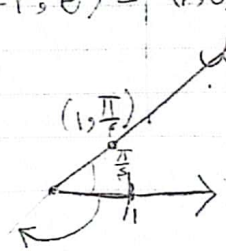




$r=0$ فاصله نقطه P از قطب

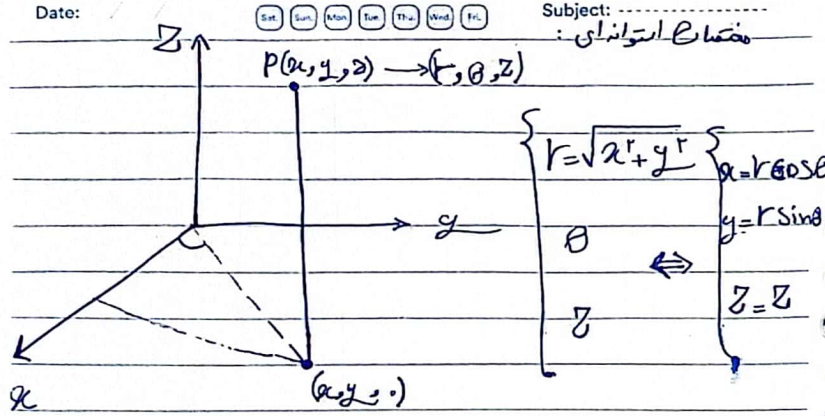
$\theta=$ زاویه شعاع OP با محور قطبی

$(-r,\theta)=(r,\theta)$ قرینه نقطه نسبت به قطب $=(r,\theta+\pi)$



LINOMAX

Date: Sat Sun Mon Tue Thu Wed Fri Subject: مختصات استوانه‌ای:

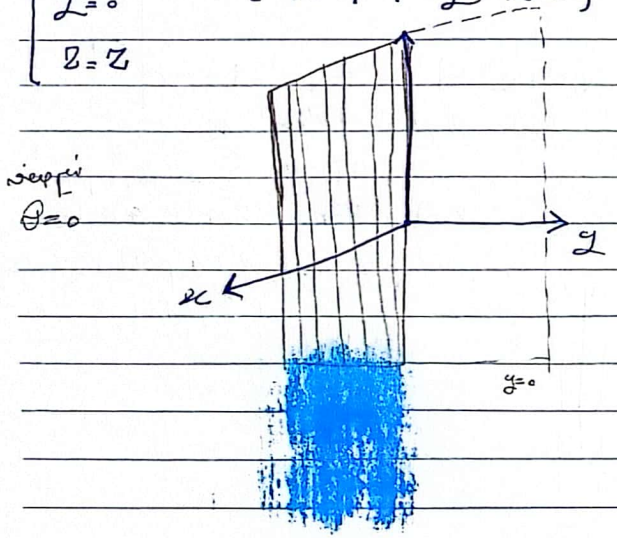


$$\begin{cases} r=\sqrt{x^2+y^2} \\ \theta \\ z \end{cases} \Leftrightarrow \begin{cases} x=r\cos\theta \\ y=r\sin\theta \\ z=z \end{cases}$$

$\theta=0$

$$\begin{cases} x=r>0 \\ y=0 \\ z=z \end{cases}$$

$\{(x,y,z) \mid x>0, y=0, z\in\mathbb{R}\}$

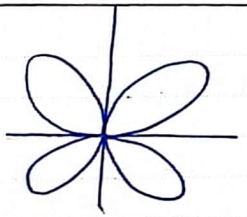


نیم‌صفحه $\theta=0$

Subject
Date

رسم توابع قطبی:

$r=\sin 2\theta$



$$\begin{cases} \theta \to -\theta \\ r \to r \end{cases} \Rightarrow r=\sin(2(-\theta)) = -\sin 2\theta$$

$$\begin{cases} \theta \to 2\pi-\theta \\ r \to r \end{cases} \Rightarrow r=\sin 2(2\pi-\theta) = \sin(4\pi-2\theta) = \sin 2\theta$$

تبدیل مختصات قطبی به کارتزی و بالعکس

$x^2+y^2=1 \Rightarrow r^2\cos^2\theta+r^2\sin^2\theta=1$

$x=r\cos\theta$

$y=r\sin\theta$

$\Rightarrow r^2=1 \rightarrow r=1$

$r=a\pm b\cos\theta$

$r=a\pm b\sin\theta$

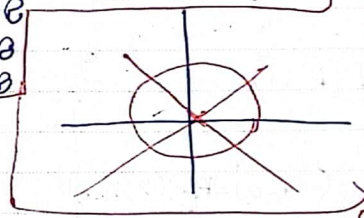
$r=\sin n\theta$ $n\in\mathbb{N}$ به صورت کلی

$r=\cos n\theta$

$r^2=r\cos 2\theta$

$r^2=r\sin 2\theta$

$x^2+y^2=r^2$



$(x^2-y^2)-(x^2+y^2)^2=0$

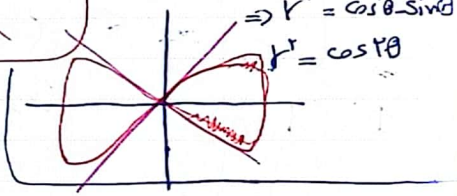
$\Rightarrow x=r\cos\theta$

$y=r\sin\theta$

$\Rightarrow r^2\cos^2\theta-r^2\sin^2\theta=r^4$

$\Rightarrow r^2=\cos^2\theta-\sin^2\theta$

$r^2=\cos 2\theta$

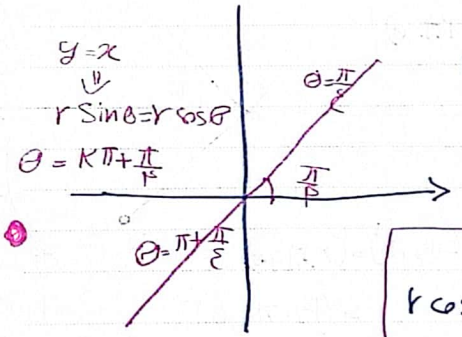


$y=x$

$\Downarrow$

$r\sin\theta=r\cos\theta$

$\theta=K\pi+\frac{\pi}{4}$



نیم‌خط $\theta=\frac{5\pi}{4}$

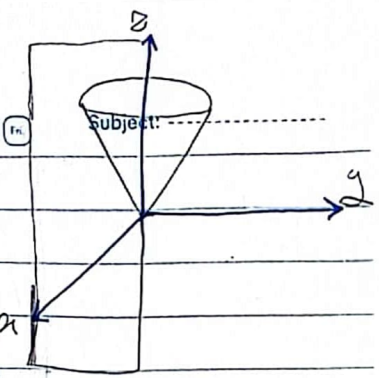
معادله‌ی خط $y=-x+1$ را بیابید:

$r\cos\theta+r\sin\theta=1 \rightarrow r=\frac{1}{\cos\theta+\sin\theta}$

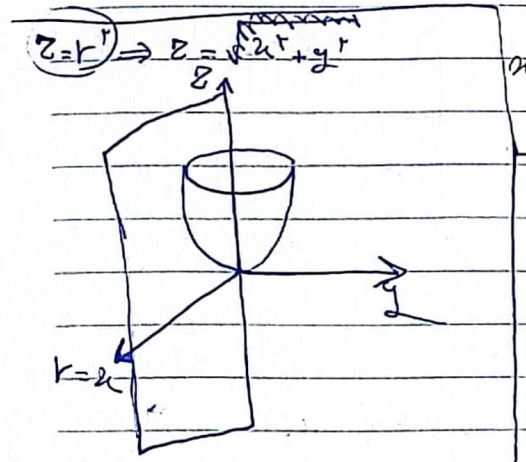
LNOMAX

Date: / / Sat Sun Mon Tue Thu Wed Fri Subject:

$z=r \Rightarrow z=\sqrt{x^2+y^2}$ مخروط

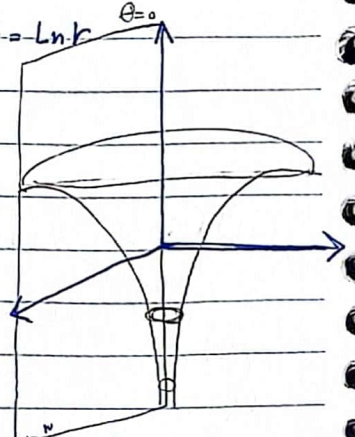


$z=r^2 \Rightarrow z=\sqrt{x^2+y^2}$



$\theta=0$

$z=-\ln r$

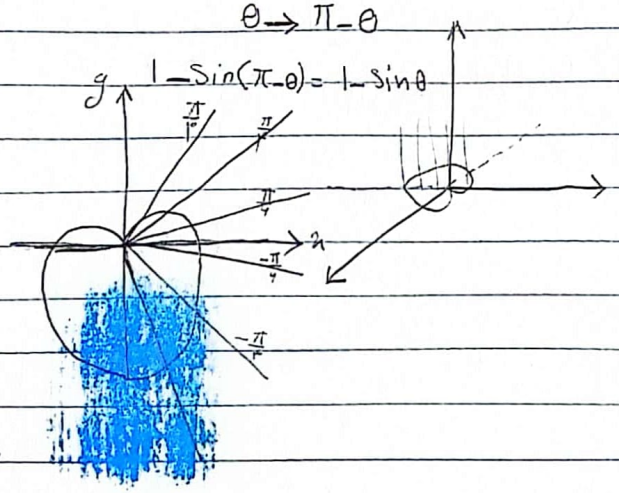


r	θ
1	0
0.5	$\frac{\pi}{6}$
0.3	$\frac{\pi}{4}$
0.1	$\frac{\pi}{3}$
0	$\frac{\pi}{2}$
0.5	$-\frac{\pi}{6}$
1.7	$-\frac{\pi}{4}$
1.9	$-\frac{\pi}{3}$
2	$-\frac{\pi}{2}$

نمودار $r=1-\sin\theta$ را در $\mathbb{R}^2$ رسم کنید

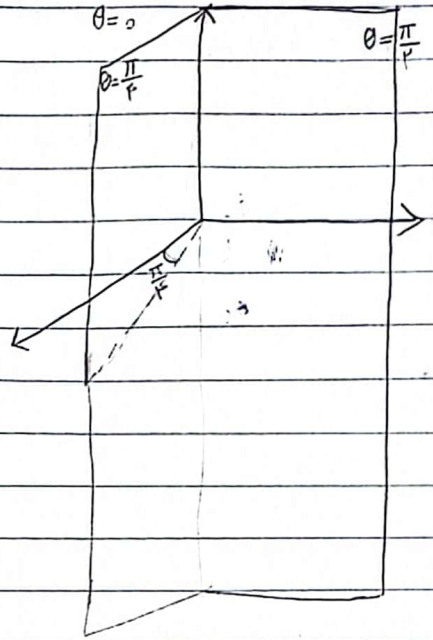
$\theta \to \pi-\theta$

$1-\sin(\pi-\theta)=1-\sin\theta$

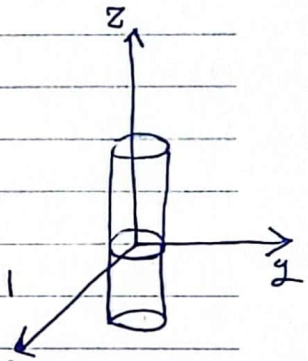


$\theta=\frac{\pi}{2}$

Date: / / Sat Sun Mon Tue Thu Wed Fri Subject:



$\begin{cases} \theta=0 \\ y=r \geq 0 \\ z=z \end{cases}$



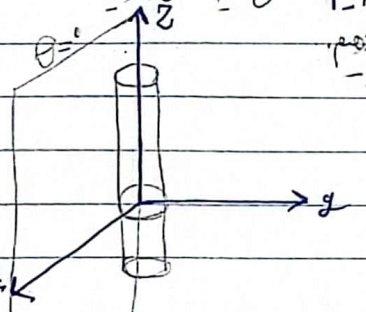
$r=1 \Rightarrow \sqrt{x^2+y^2}=1 \Rightarrow x^2+y^2=1$

استوانه‌ای با سطح مقطع دایره به شعاع ۱

رسم رویه‌های به صورت $F(r,z)=0$

$z=\ln r$ $\quad z=r^2$ $\quad z=r$ $\quad r=1$

سوال: z و r را داریم θ را نداریم چگونه رسم کنیم؟ روش ترسیم: شکل را در صفحه‌ای به $\theta=$ ثابت رسم می‌کنیم و سپس آن را حول محور z ها دوران می‌دهیم.



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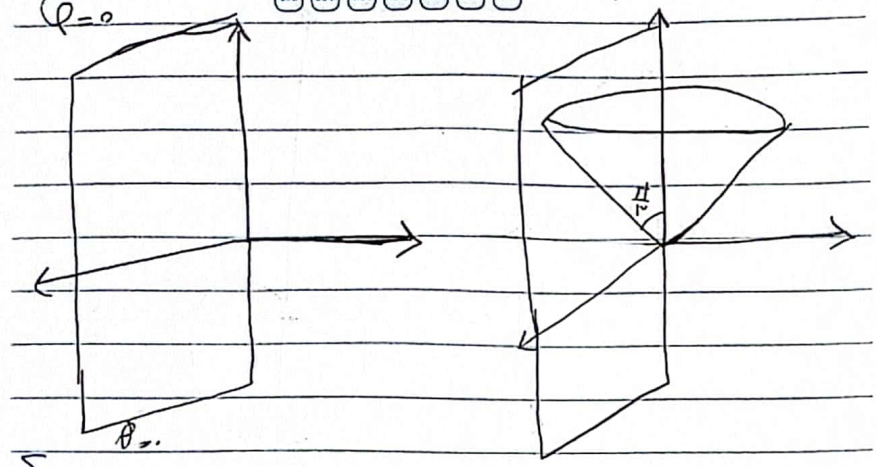
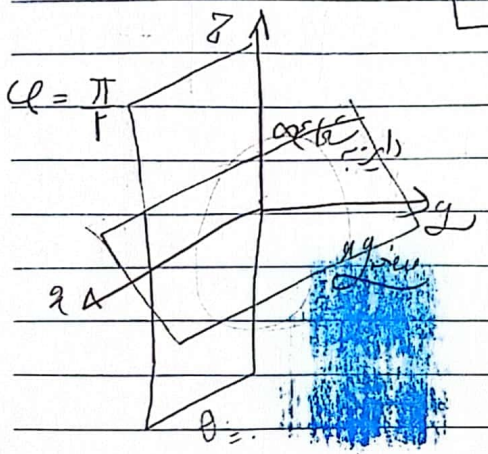


$$\begin{cases} x=\rho\sin\varphi\cos\theta \\ y=\rho\sin\varphi\sin\theta \\ z=\rho\cos\varphi \end{cases} \xrightarrow{\varphi=\frac{\pi}{2}} \begin{cases} x=\rho\frac{\sqrt{2}}{2}\cos\theta \\ y=\rho\frac{\sqrt{2}}{2}\sin\theta \\ z=\frac{1}{2}\rho \Rightarrow z^2=\frac{1}{4}\rho^2 \end{cases}$$

$$x^2+y^2=\rho^2\frac{1}{2}=2\left(\frac{1}{4}\rho^2\right)=2z^2$$

$$\rho^2=x^2+y^2 \quad |\quad z\geq 0$$



$\sin\varphi=\frac{r}{\rho}\Rightarrow r=\rho\sin\varphi$

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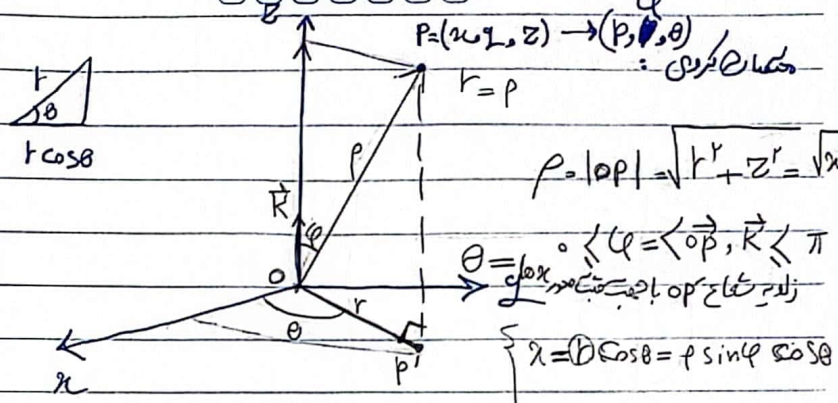


محاسبات کروی:

$$\rho=|op|=\sqrt{r^2+z^2}=\sqrt{x^2+y^2+z^2}$$

$0\leq\varphi=\langle\overrightarrow{op},\vec{k}\rangle\leq\pi$

$\theta=$ زاویه شعاع op با جهت مثبت محور xها

$$\begin{cases} x=r\cos\theta=\rho\sin\varphi\cos\theta \\ y=r\sin\theta=\rho\sin\varphi\sin\theta \\ z=\rho\cos\varphi \end{cases}$$

ترسیم رویه $F(\rho,\varphi)=0$ یا $F(\rho,\varphi)$ را رسم می‌کنیم و سپس شکل را حول محور zها دوران می‌دهیم.

$\theta=0$ صفحه در F ترسیم

$\rho=2$

$\varphi=0$ $\quad \rho=1-\sin\varphi$

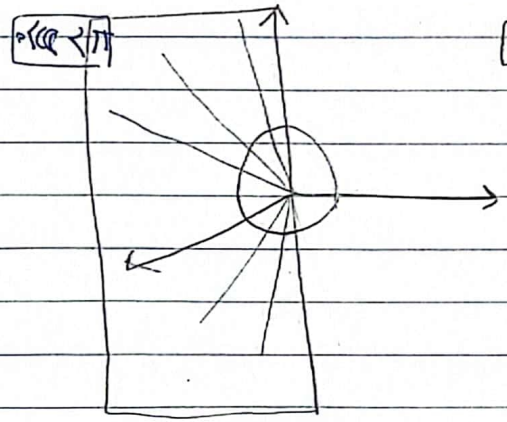
$\varphi=\frac{\pi}{2}$

$\varphi=\frac{3\pi}{4}$

$\varphi=\pi$

$\sqrt{x^2+y^2+z^2}=2\Rightarrow x^2+y^2+z^2=4$

$\rho=2$



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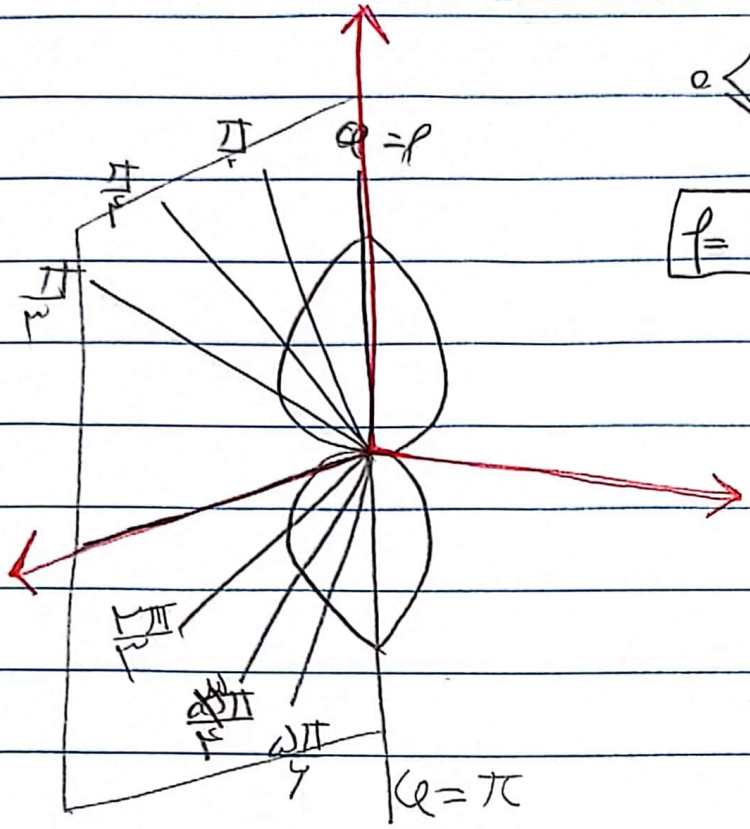
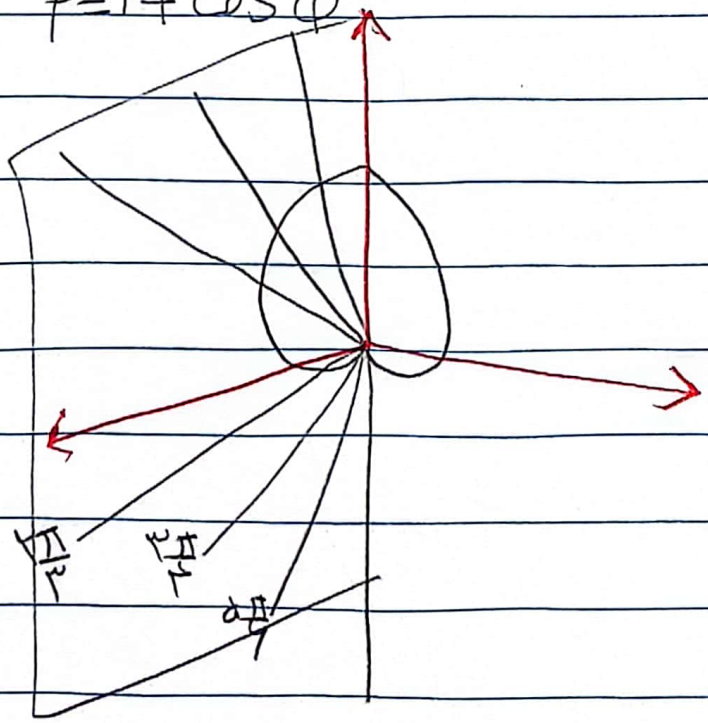
$0 \leq \varphi \leq \pi$

$r = 1 - \sin\varphi$



$r = 1 + \cos\varphi$



$\theta = 0$

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۲- $x^2+y^2+z^2=4$ ، $z=\sqrt{x^2+y^2}$.

۳- داخل مخروط $x^2+y^2=1$ ، $x^2+y^2+z^2=4$.

۴- خارج مخروط $x^2+y^2=1$ ، داخل کره $x^2+y^2+z^2=4$.

۵- $z=x^2+y^2$ ، $z=\sqrt{x^2+y^2}$

۶- $x^2+y^2=2x$ ، $x^2+y^2+z^2=4$

۷- $z=\sqrt{x^2+y^2}$ ، $x^2+y^2+z^2=2z$

۸- $z=x^2+y^2$ ، $z=2$