

DS501: Graph Data and High Dimensional Data

Prof. Randy Paffenroth
rcpaffenroth@wpi.edu

Worcester Polytechnic Institute

Announcements

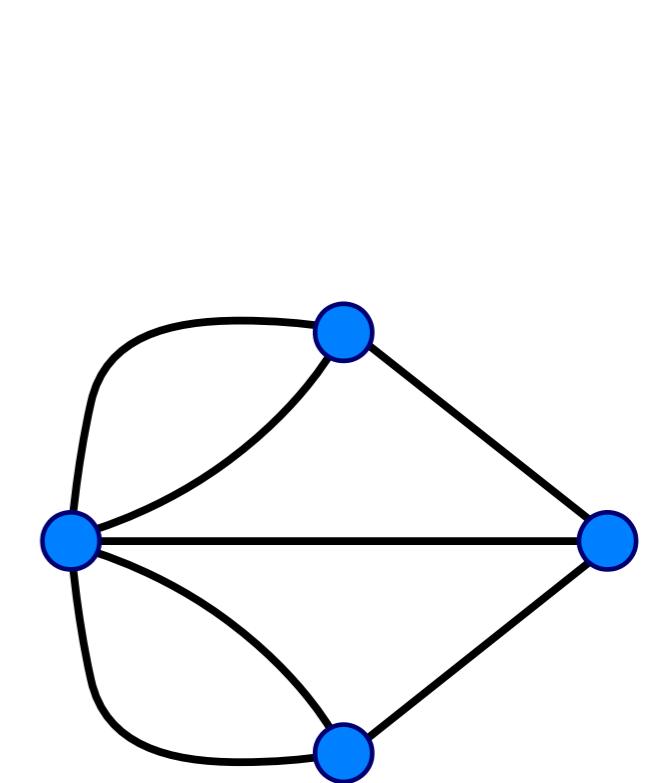
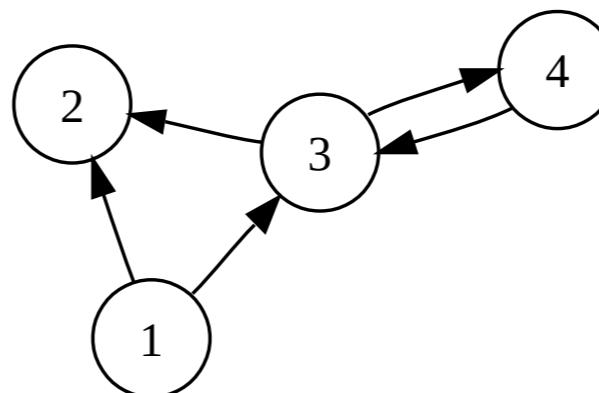
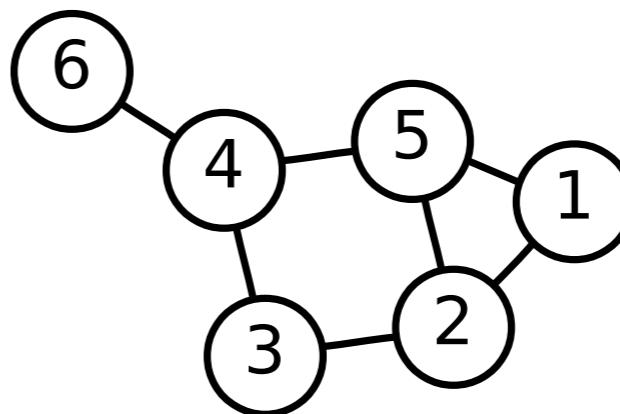
- Case Study 4 is posted!
 - The “Shark Tank”
 - Let's take a look at it.

.

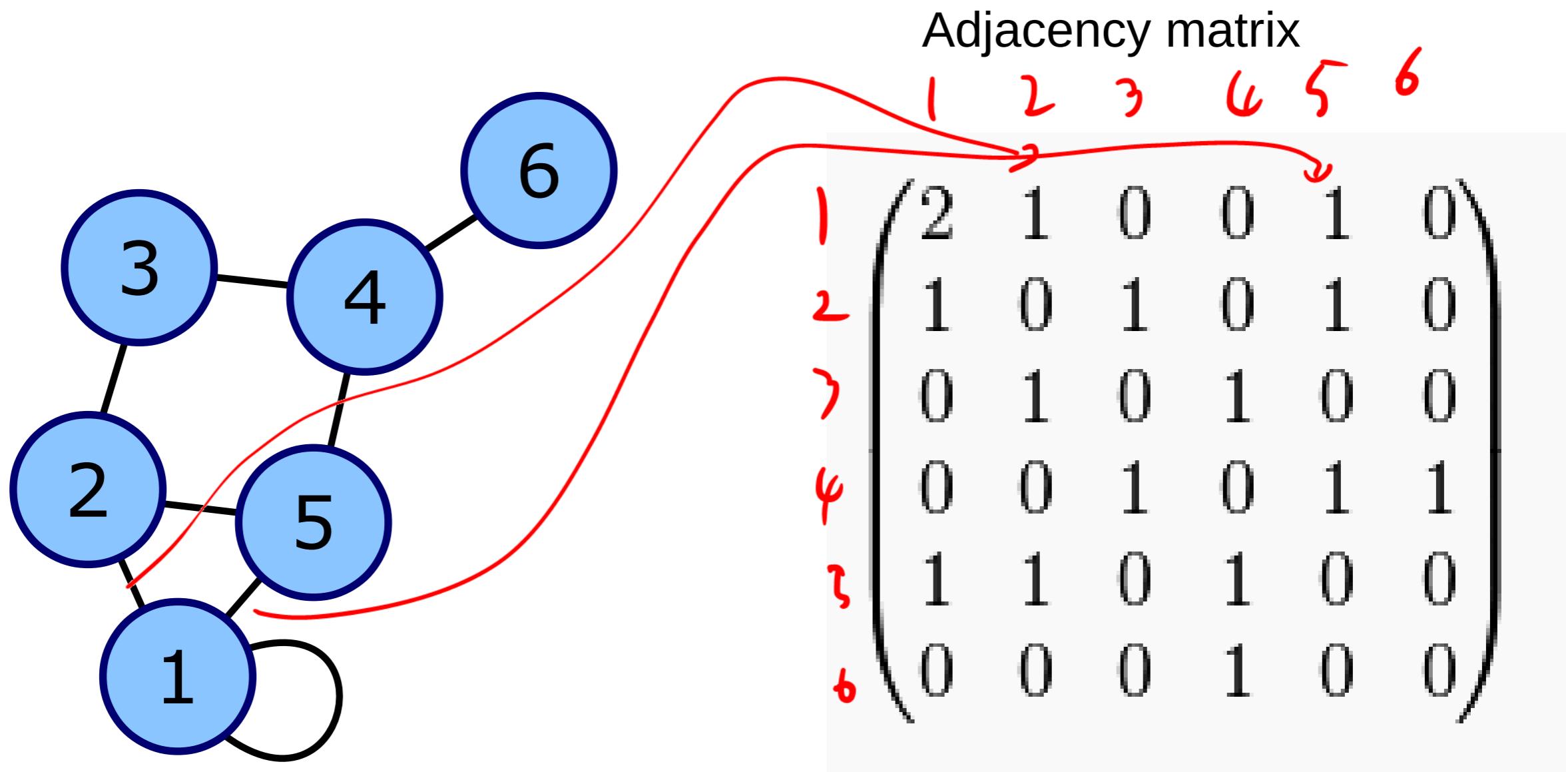
Graph Data

Types of graphs

- Graph types
 - Undirected
 - Directed
 - Mixed
 - Multi-graphs

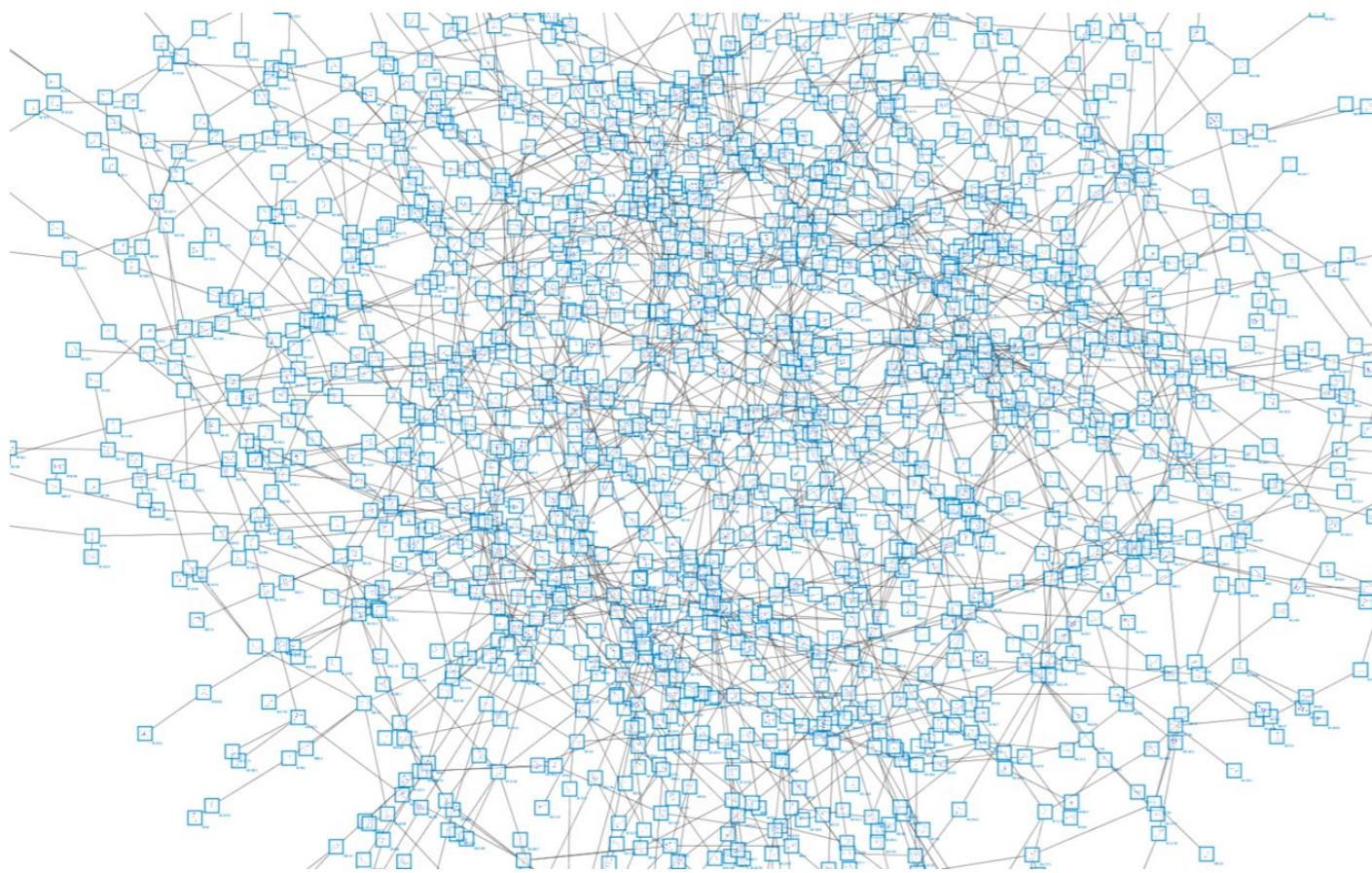


Two views of a graph



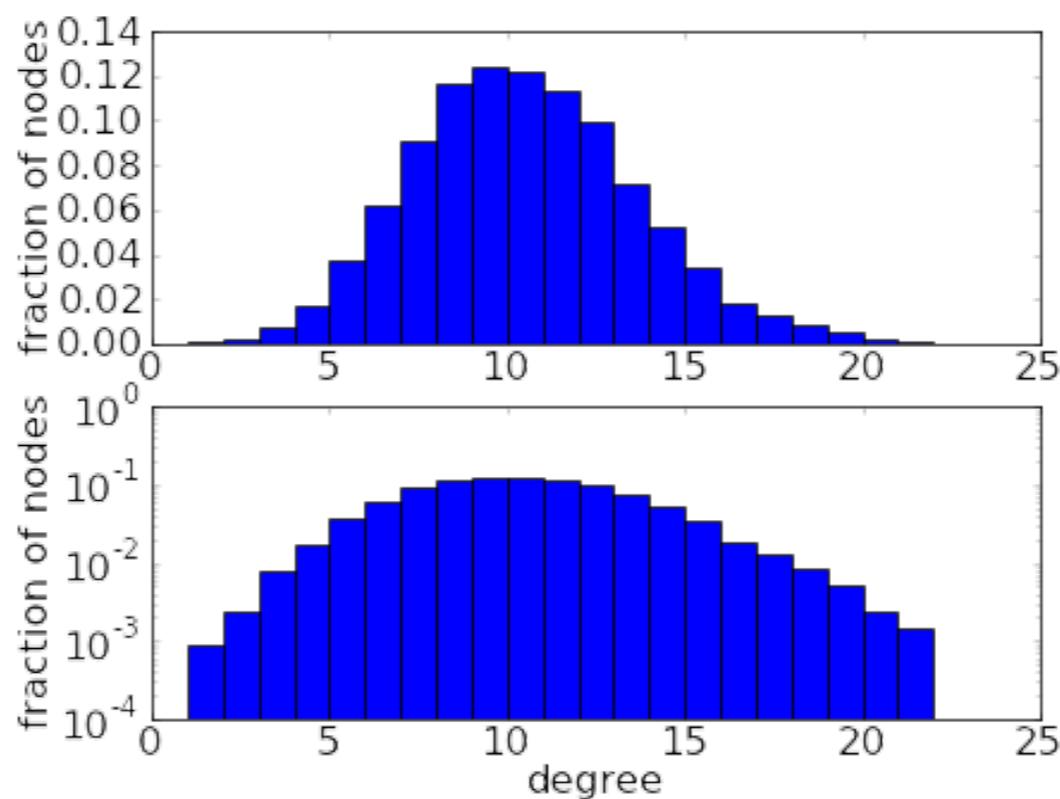
Properties of a Graph

- Degree Distribution
- Structural Properties



Degree Histogram

- Outdegree of a vertex = # outgoing edges
- For each number d , let $n(d) = \# \text{ vertices with outdegree } d$

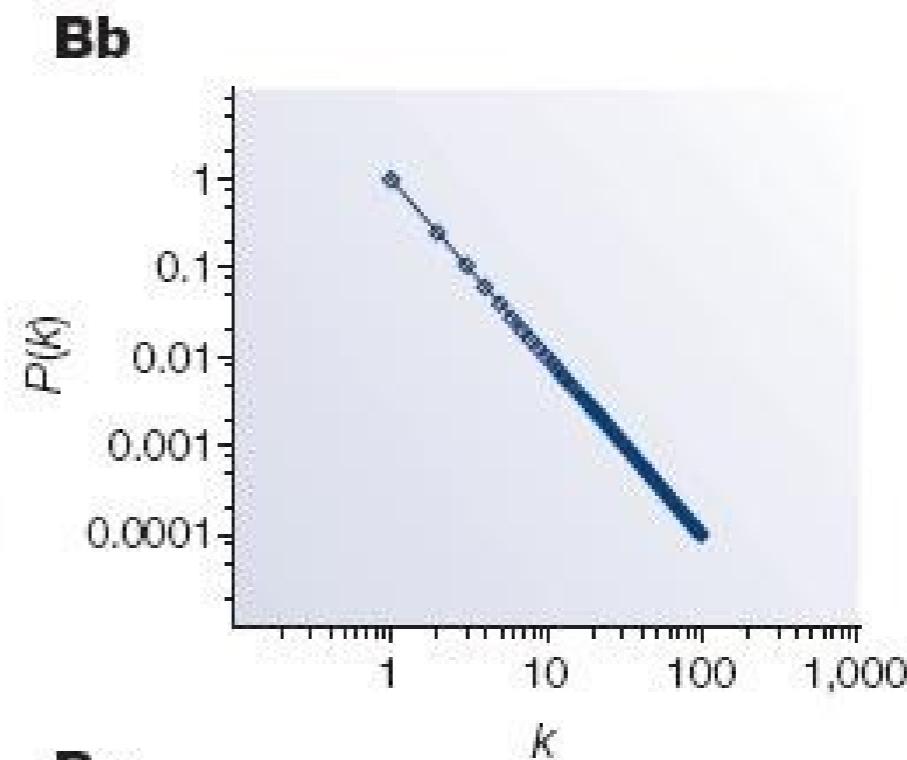
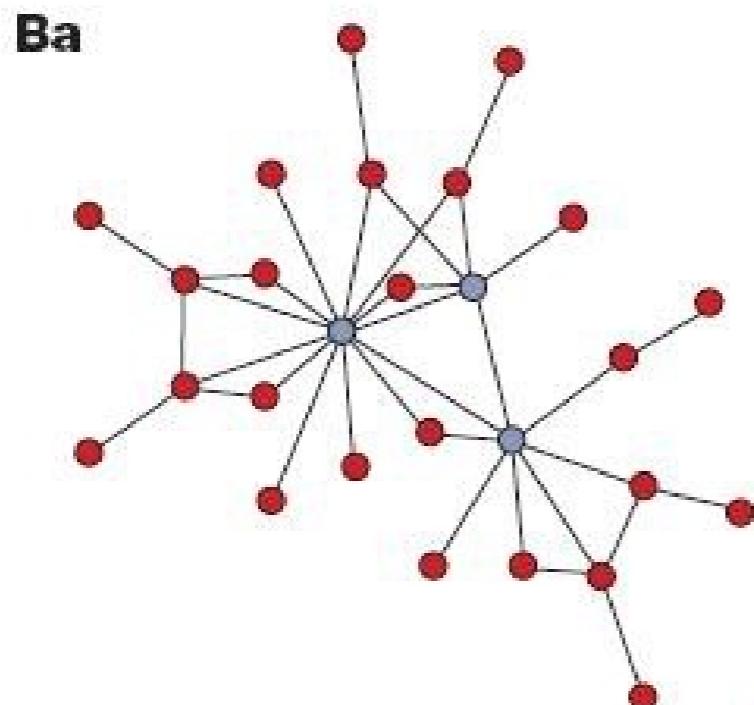


Degree Distribution

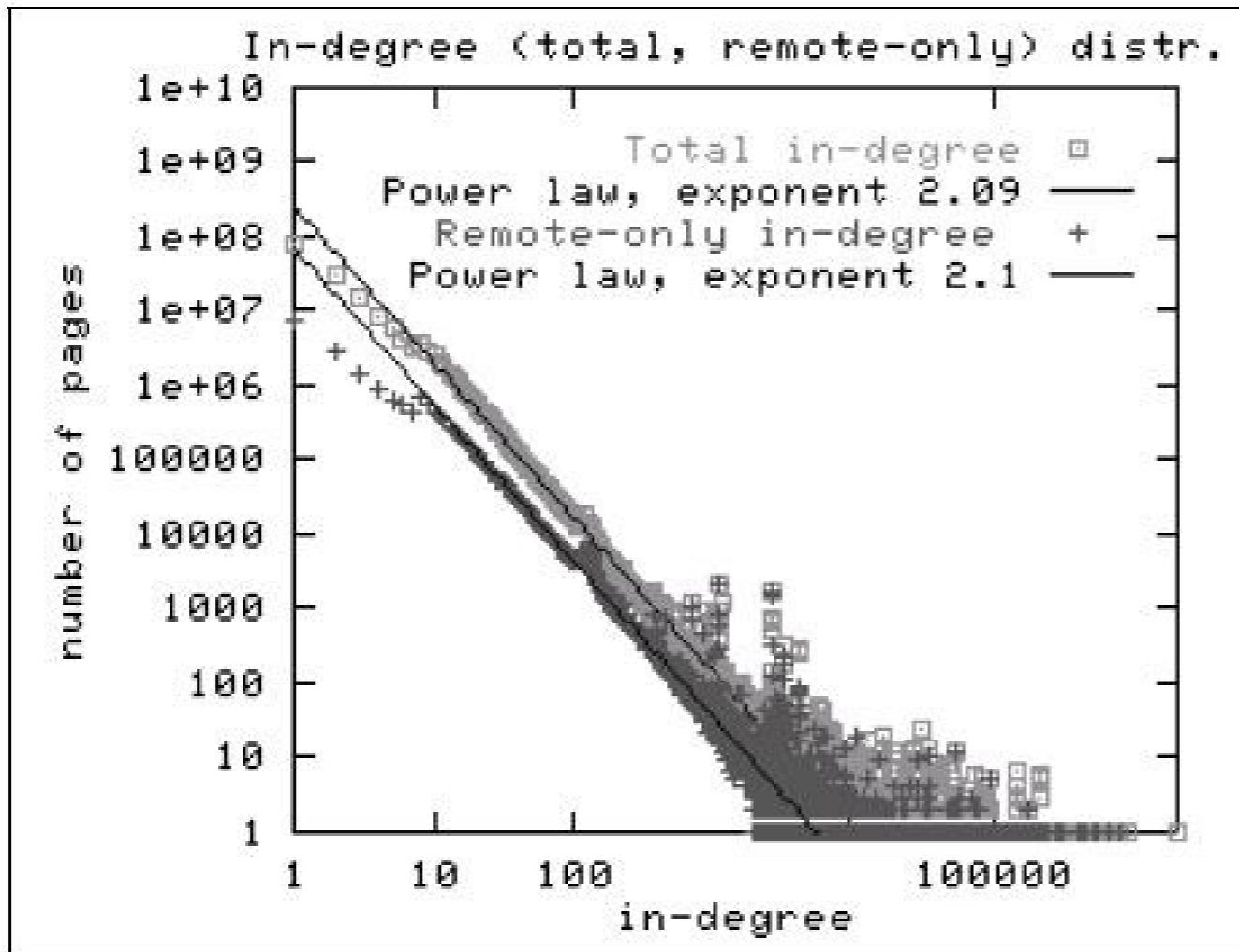
- Are there **large graphs** with “interesting” degree distributions?

Zipf Distribution (power law)

- $n(d) \sim 1/d^x$ for some value $x > 0$
- Human-generated data has Zipf distribution: letters in alphabet, words in vocabulary.
- In log-log scale



Distribution of the Web



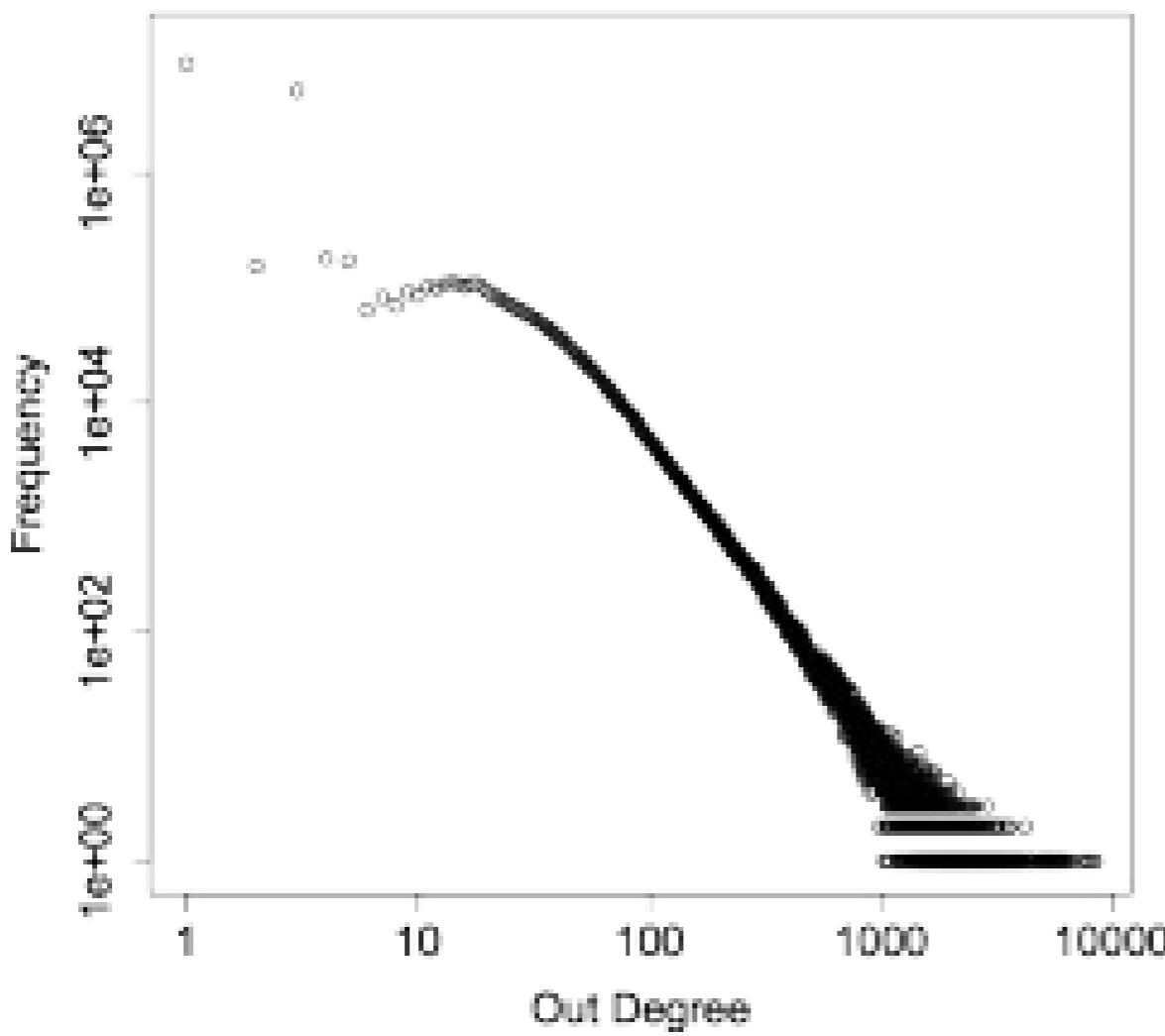
Late 1990's
200M Webpages

Exponential ?
Power Law?

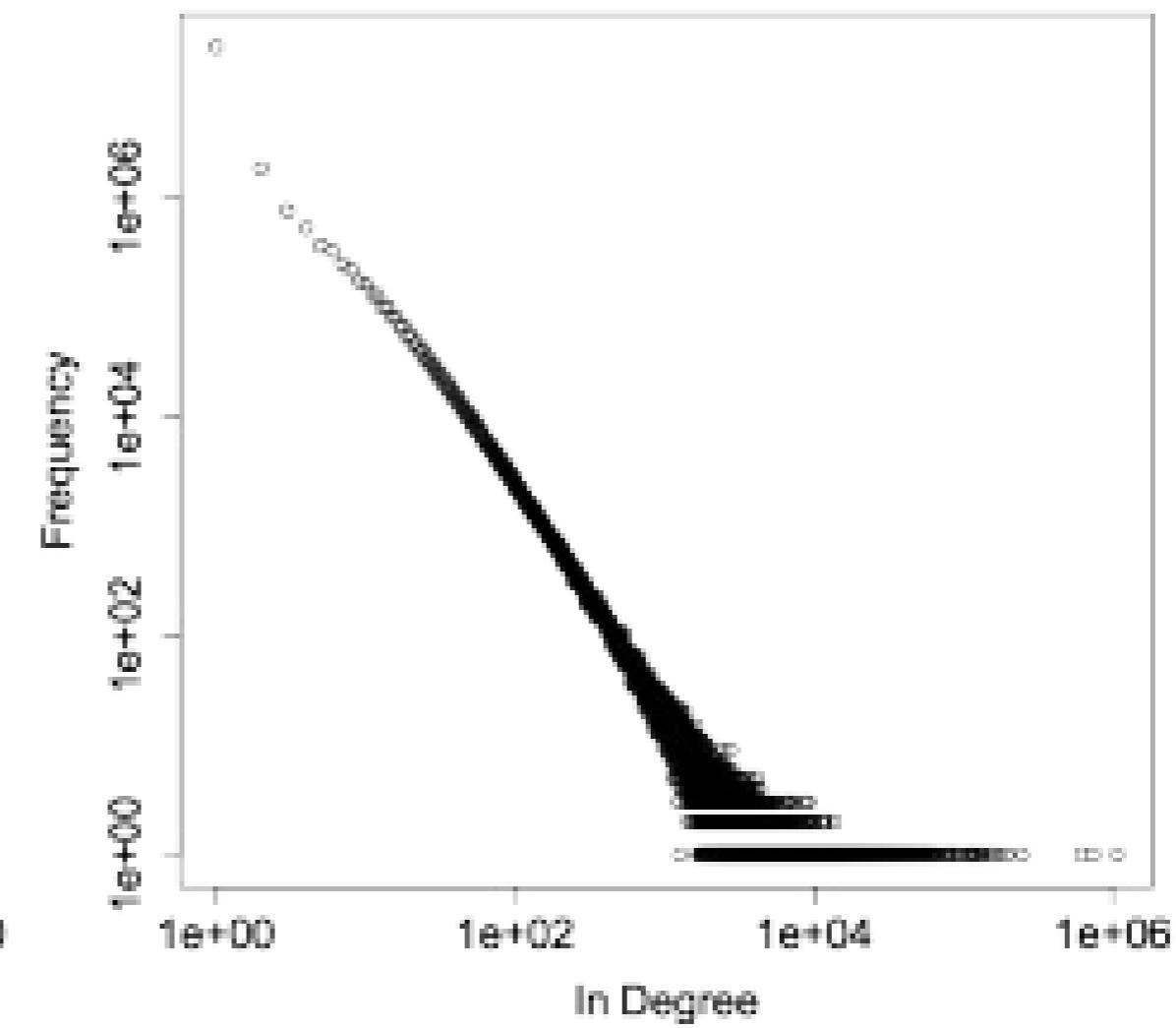
Figure 2: In-degree distribution.

Distribution of Wikipedia

Wikipedia Out-Degree Distribution



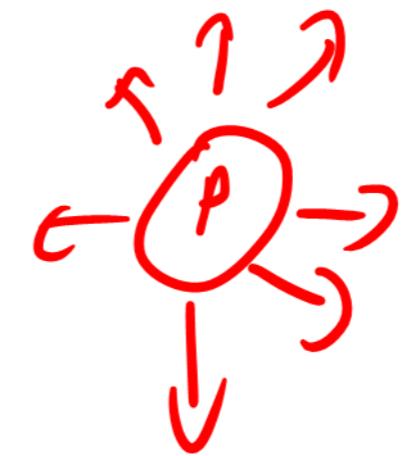
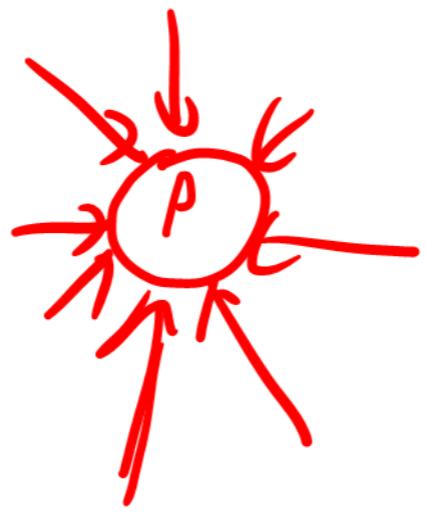
Wikipedia In-Degree Distribution



Evaluating the Nodes

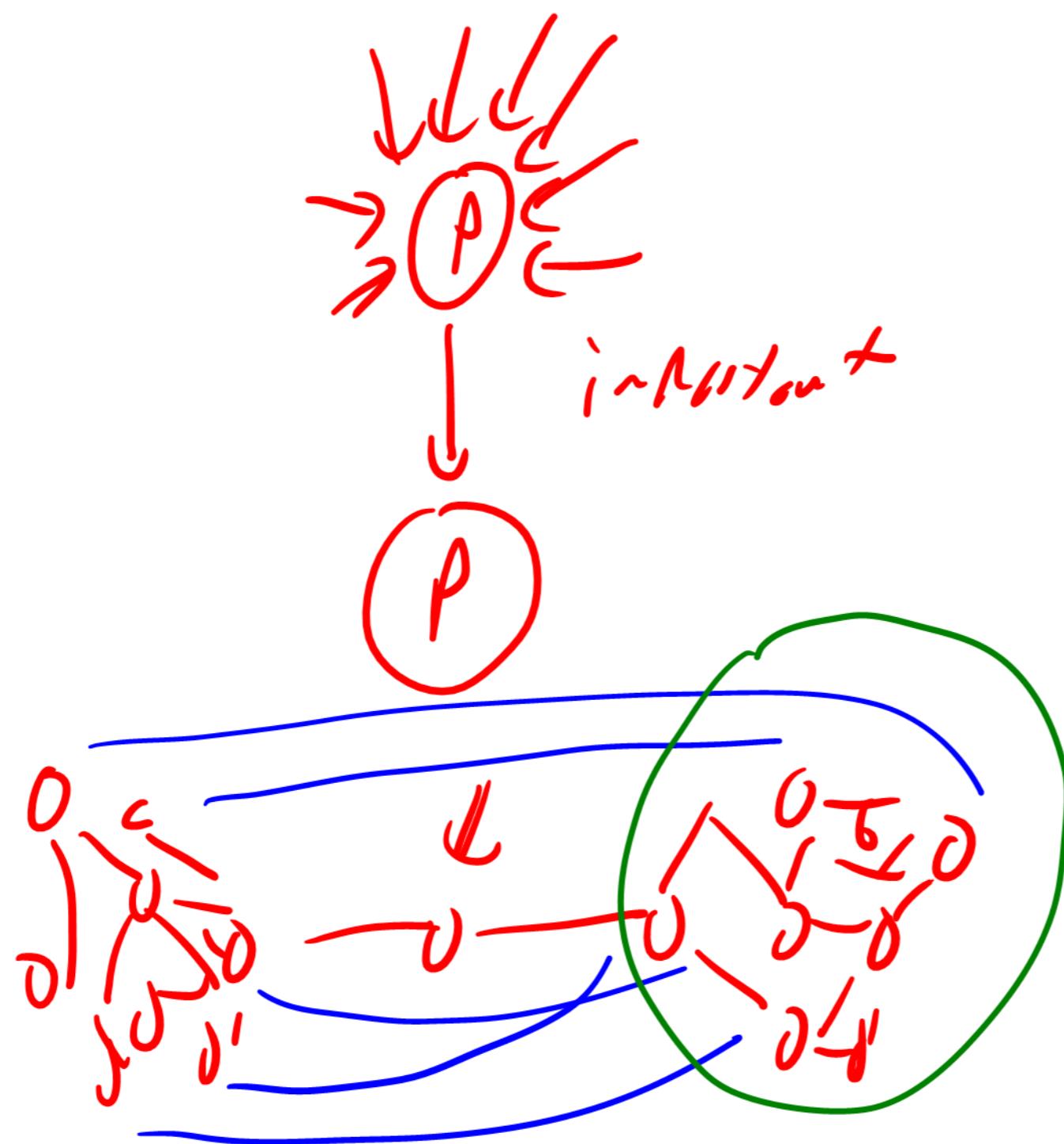
- How would you define the important nodes in a graph?

Following - edges
People - vertices



U.S.



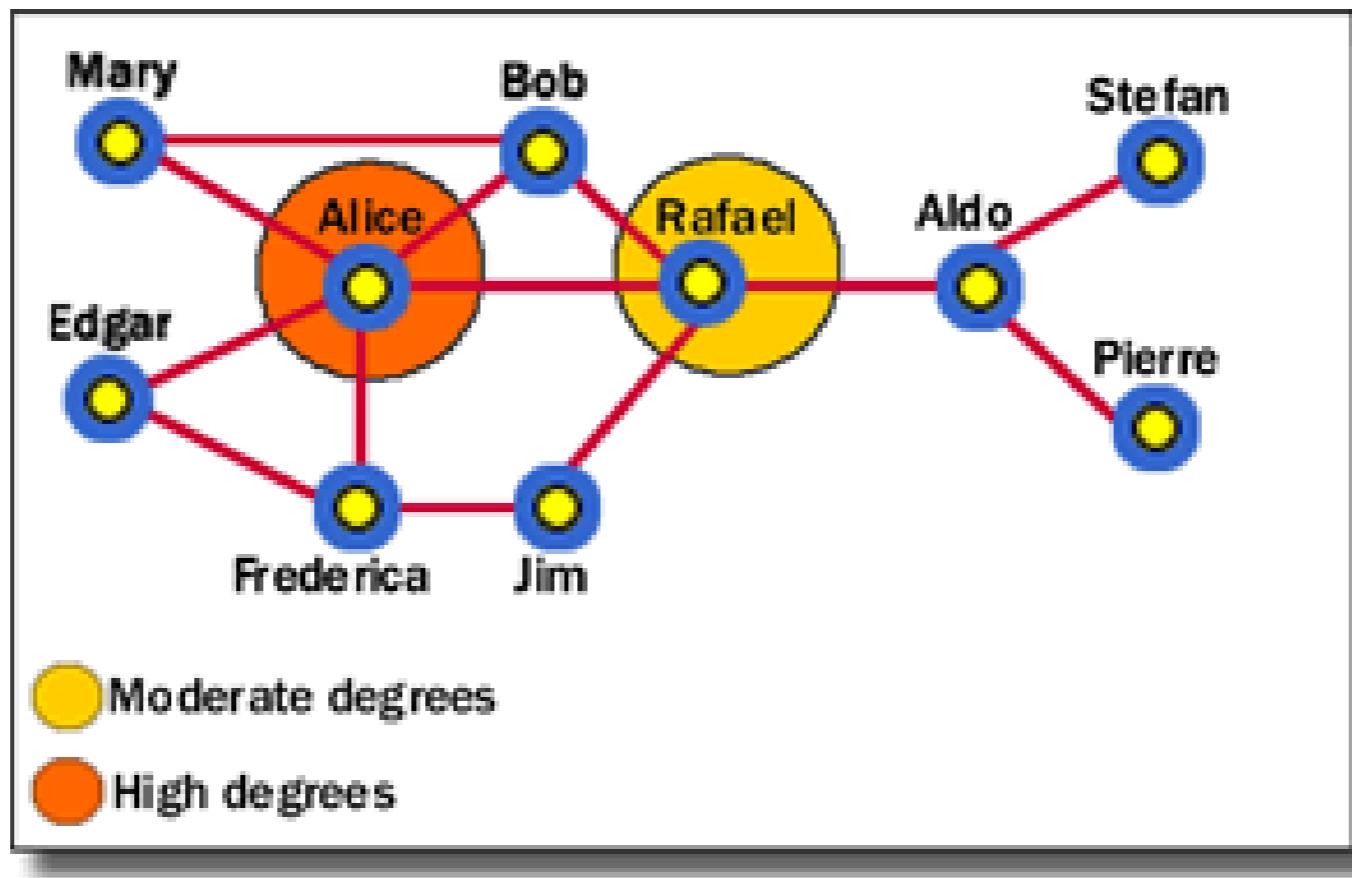


Evaluating the Nodes

- Degree Centrality
- Betweenness : Number paths
- Eigenvector Centrality (i.e, PageRank)

Node: Degree Centrality

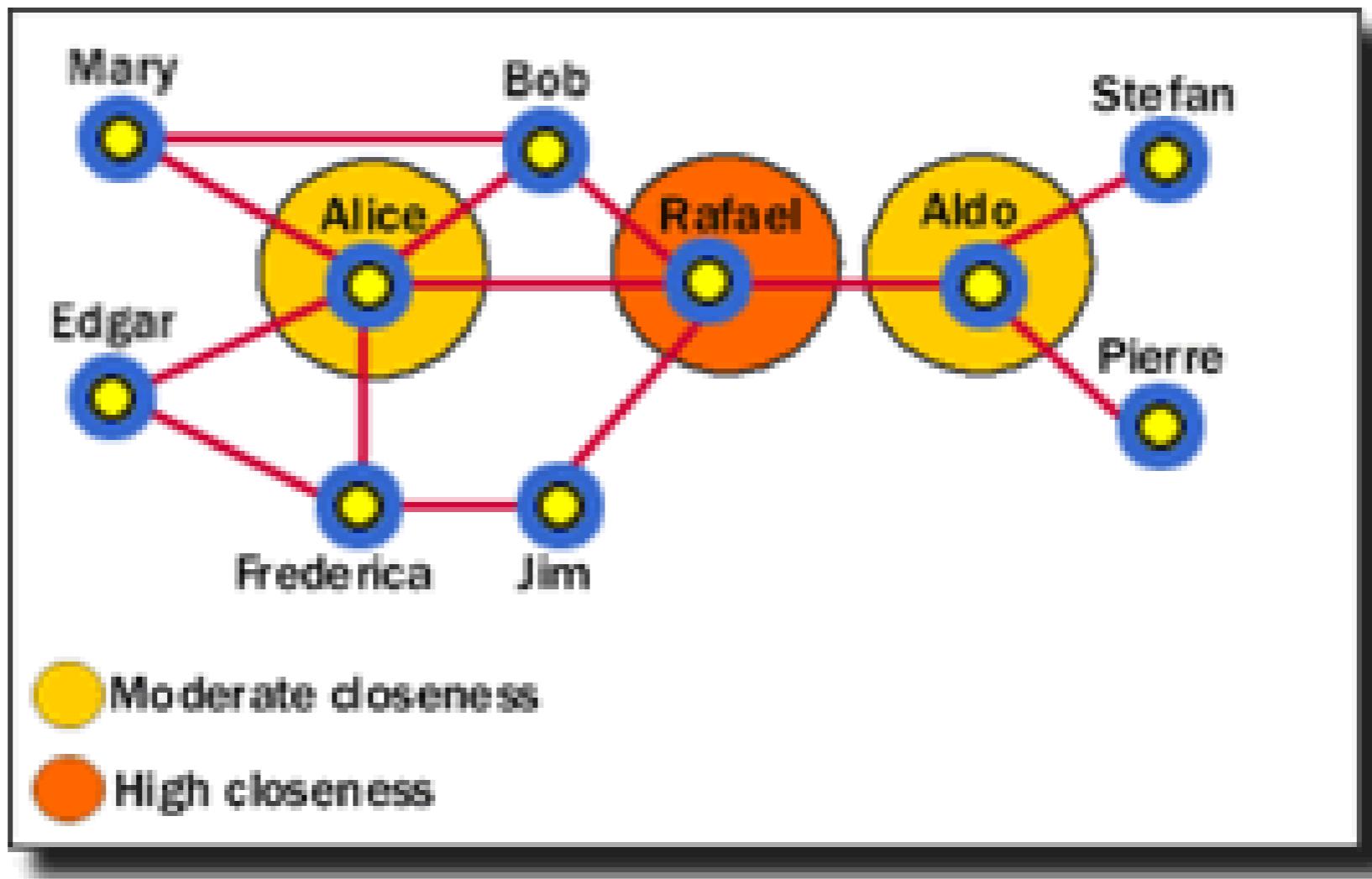
- **Degree Centrality:** number of edges



Pros: Is generally an active player in the network
Cons: ignore the whole graph

Node: Closeness Centrality

- **Closeness** of a vertex: the inverse of the average length of all its shortest paths



Pros: Is close to other entities

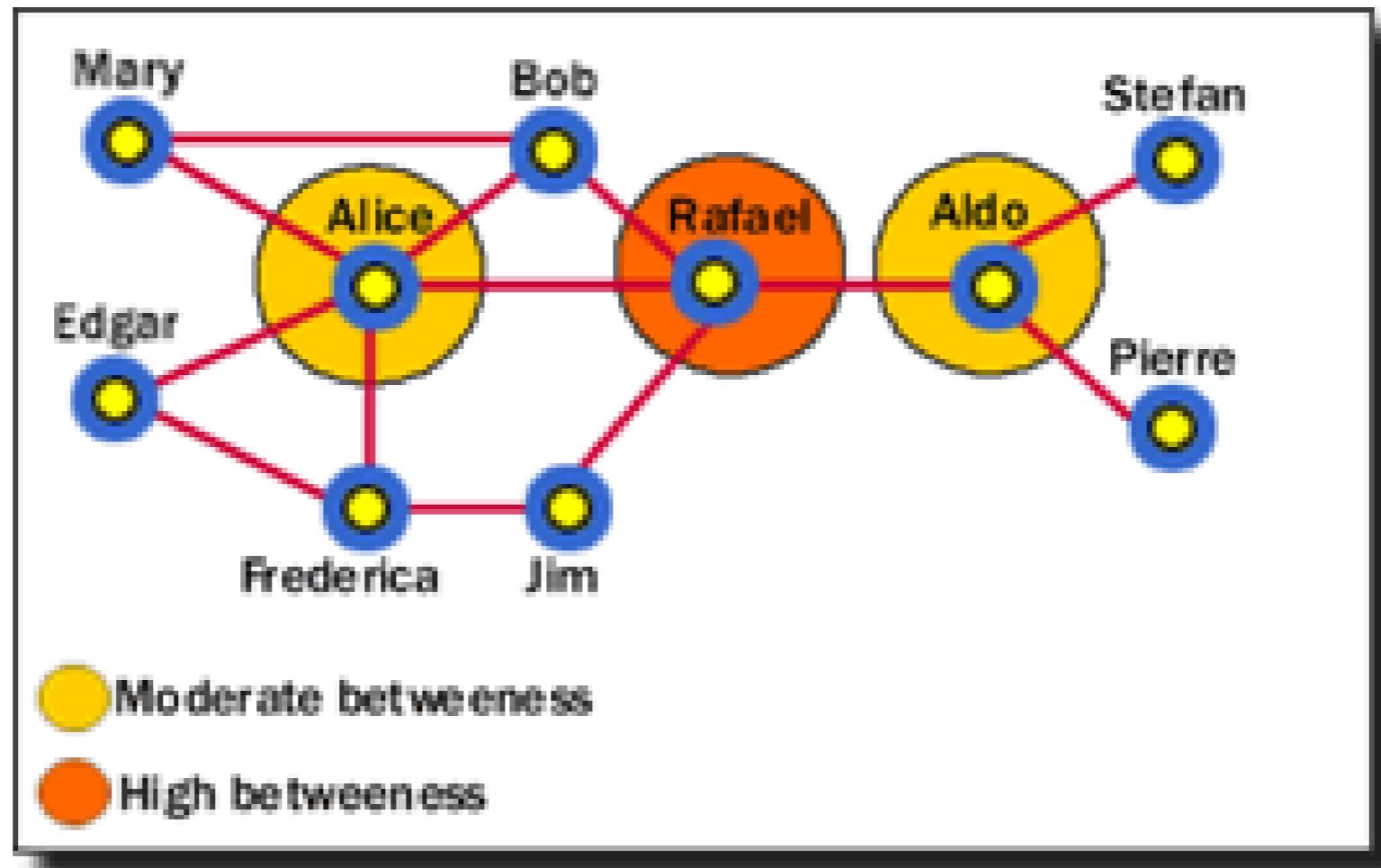
Cons: ignore the importance of different nodes

Node: Betweenness Centrality

- **Betweenness** of a vertex v : the fraction of all shortest paths that pass through v .

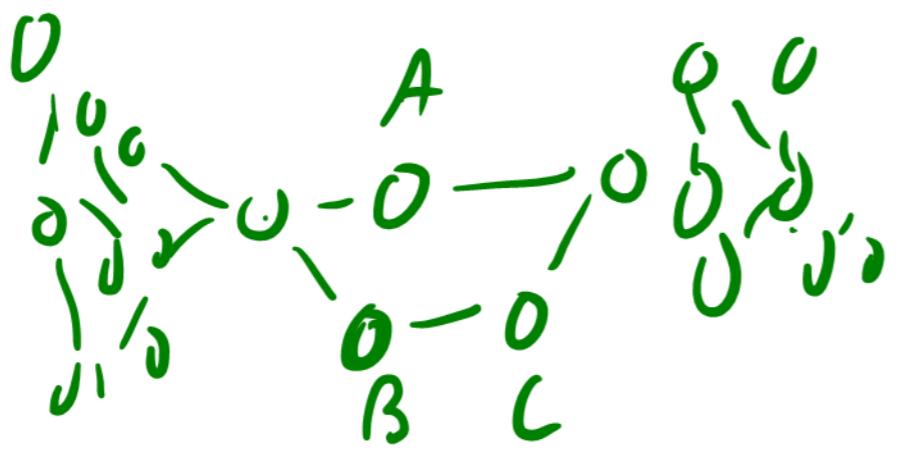
High

$\frac{9}{10}$
 $\frac{1}{10}$
 $\frac{1}{10}$
 $\frac{1}{10}$
 $\frac{1}{10}$
 $\frac{1}{10}$



Pros: represents a single point of failure, has a greater amount of influence over what happens in a network.

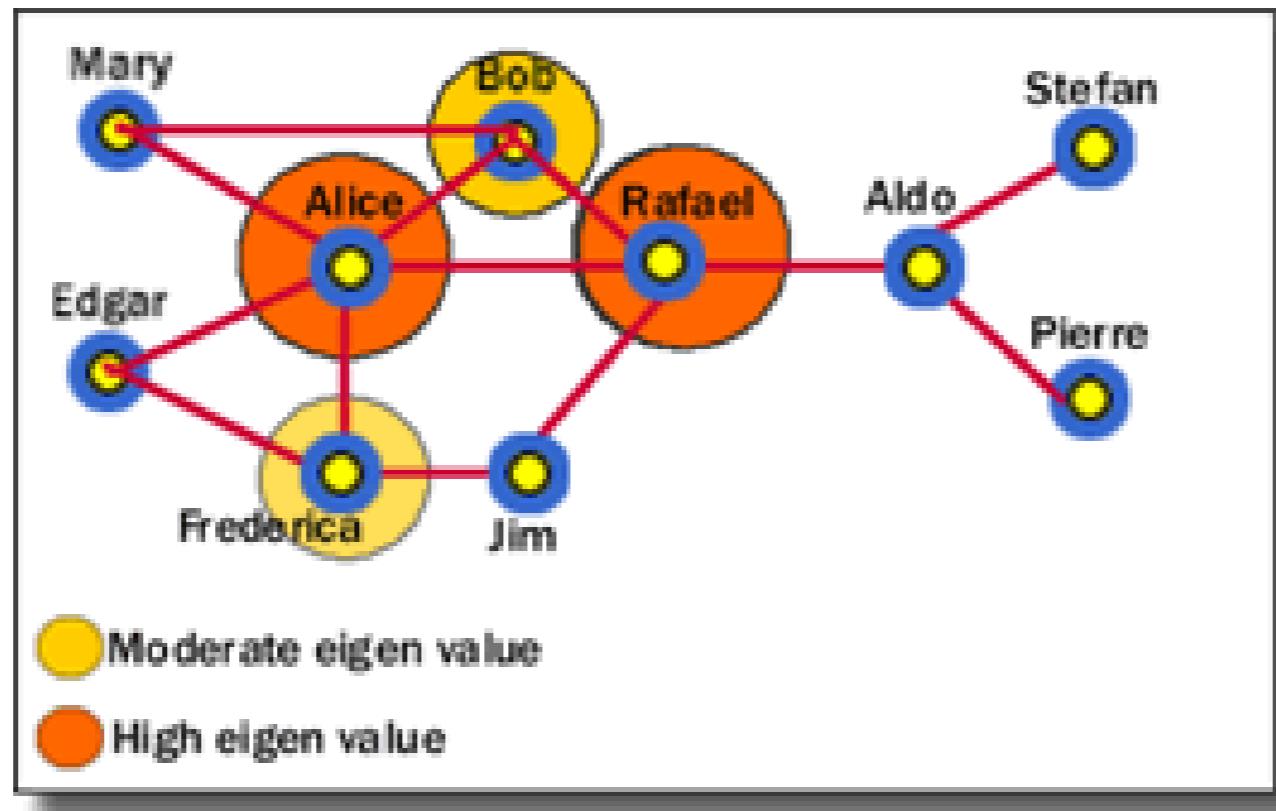
Cons: ignore the importance of different nodes



which has higher
betweenness
centrality?

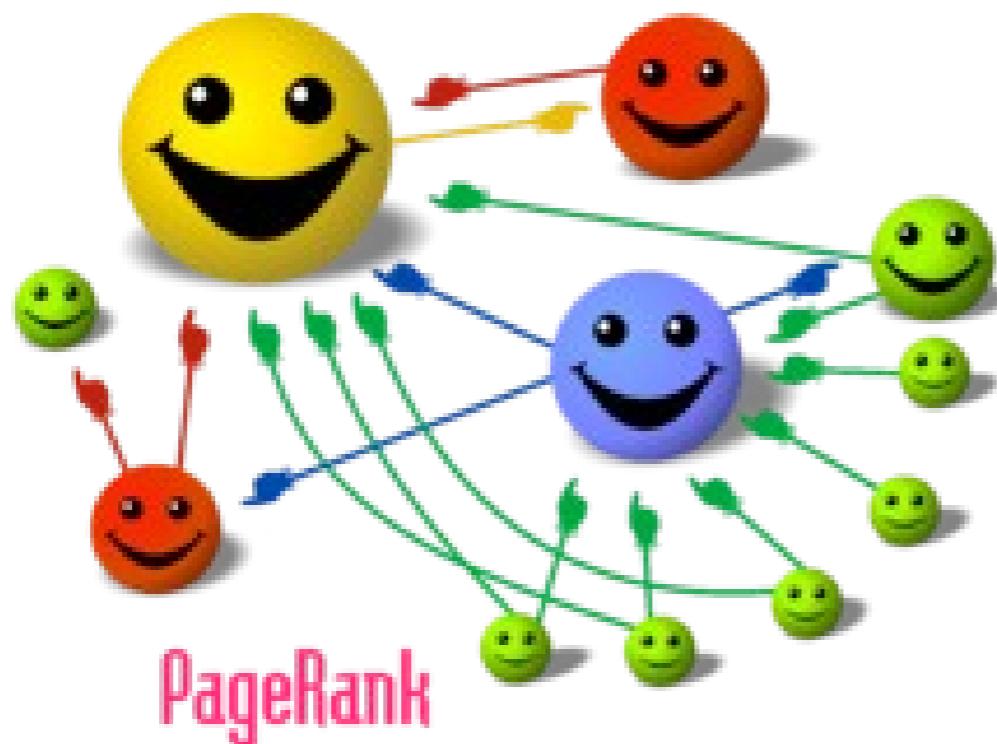
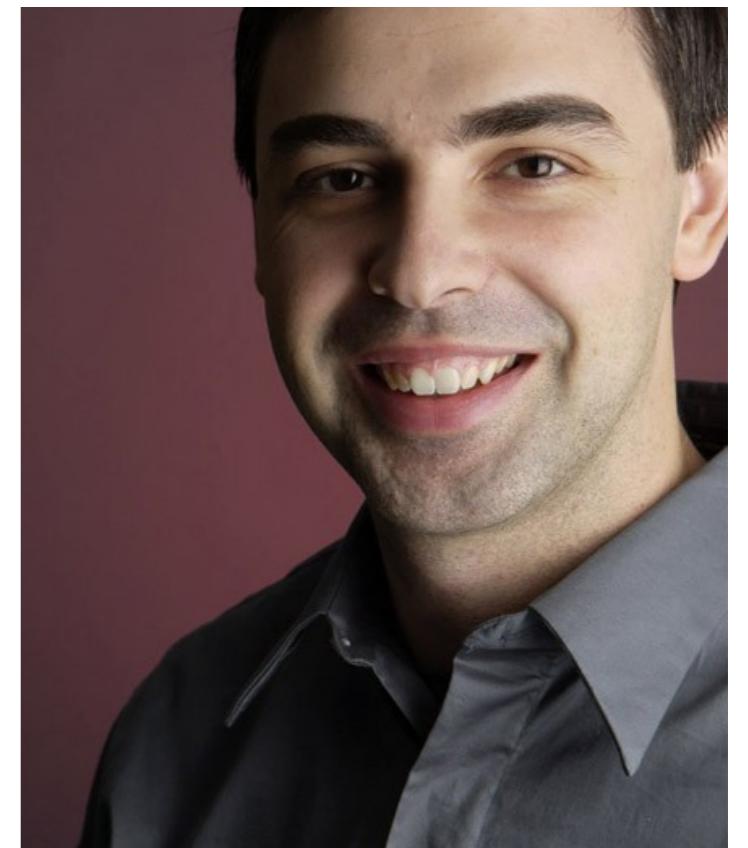
Node: Eigenvalue (PageRank)

- Eigenvalue measures how **close** an entity is to other highly **close** entities within a network



PageRank

- **WebPage:** ranking pages
- **Larry Page:** author of the paper



The PageRank Citation Ranking: Bringing Order to the Web

January 29, 1998

Abstract

The importance of a Web page is an inherently subjective matter, which depends on the readers interests, knowledge and attitudes. But there is still much that can be said objectively about the relative importance of Web pages. This paper describes PageRank, a method for rating Web pages objectively and mechanically, effectively measuring the human interest and attention devoted to them.

We compare PageRank to an idealized random Web surfer. We show how to efficiently compute PageRank for large numbers of pages. And, we show how to apply PageRank to search and to user navigation.

1 Introduction and Motivation

The World Wide Web creates many new challenges for information retrieval. It is very large and heterogeneous. Current estimates are that there are over 150 million web pages with a doubling life of less than one year. More importantly, the web pages are extremely diverse, ranging from "What is Joe having for lunch today?" to journals about information retrieval. In addition to these major challenges, search engines on the Web must also contend with inexperienced users and pages engineered to manipulate search engine ranking functions.

However, unlike "flat" document collections, the World Wide Web is hypertext and provides considerable auxiliary information on top of the text of the web pages, such as link structure and

Idea of PageRank

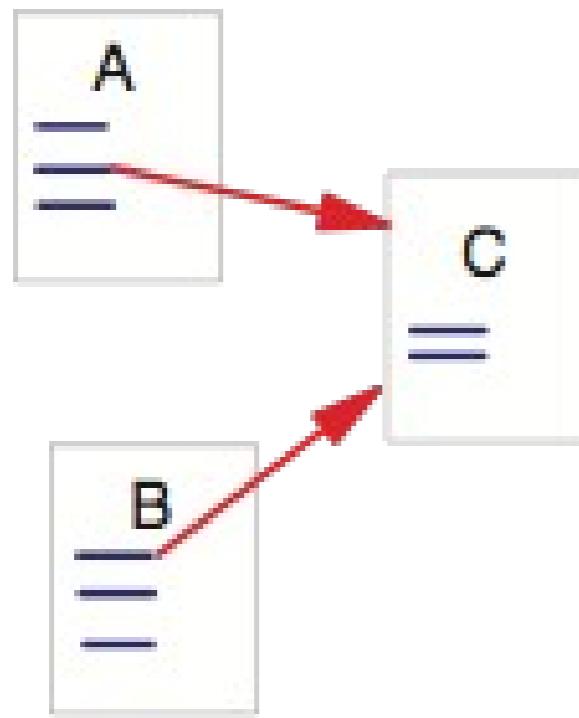


Figure 1: A and B are Backlinks of C

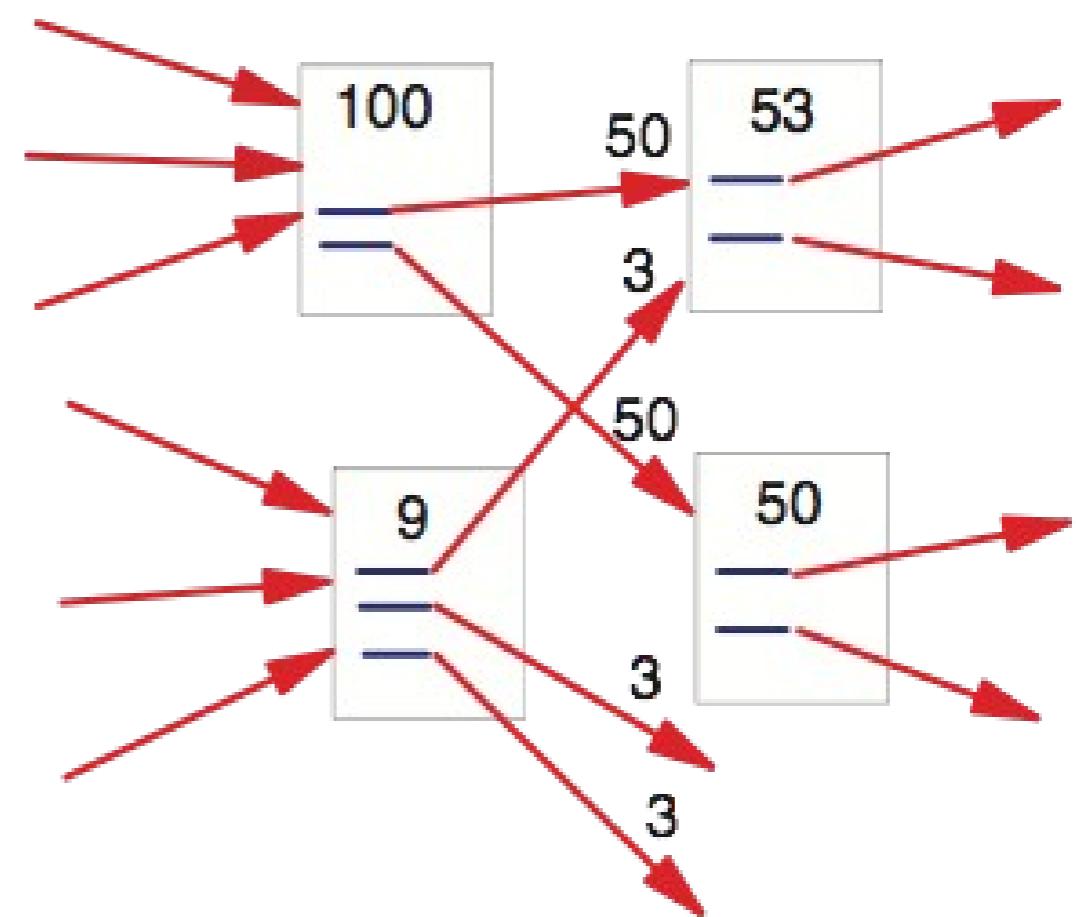
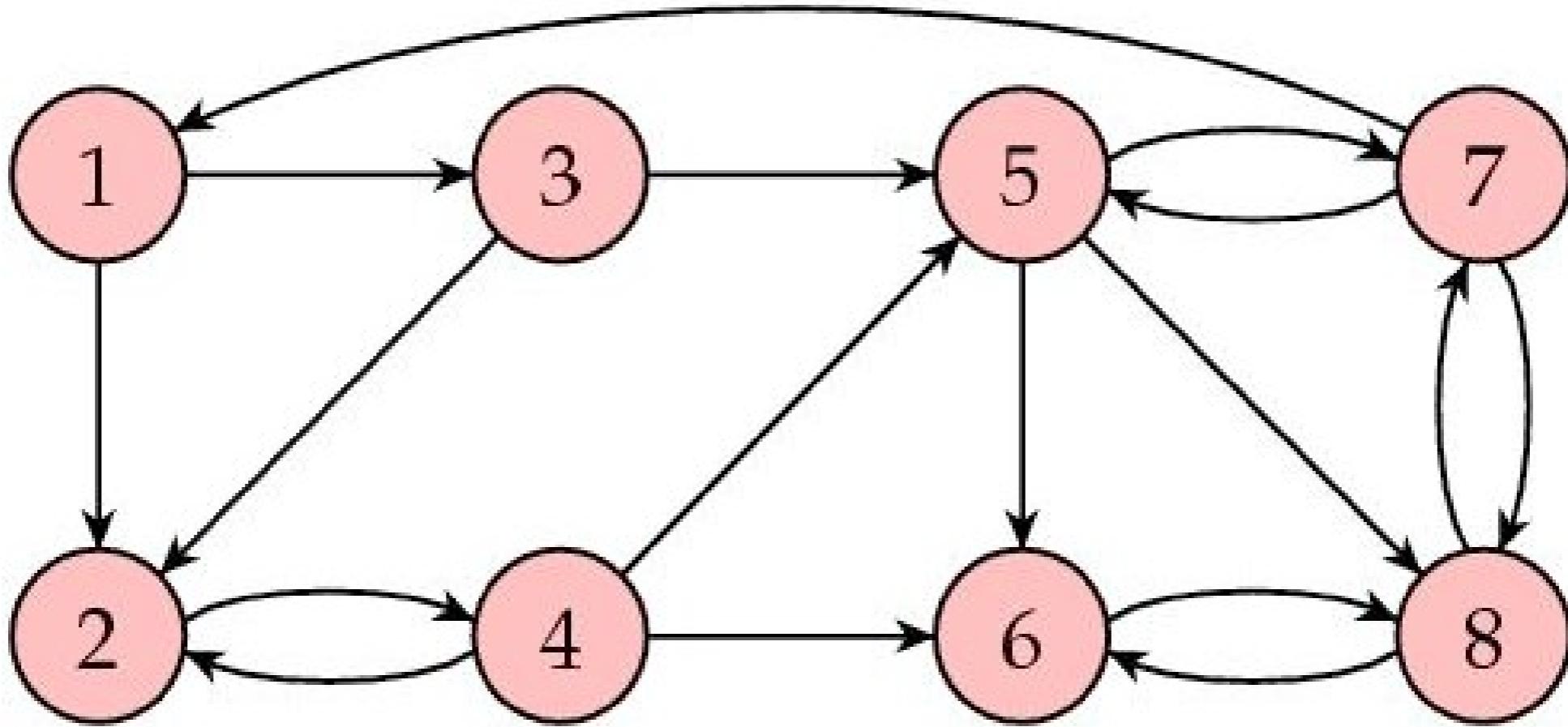


Figure 2: Simplified PageRank Calculation

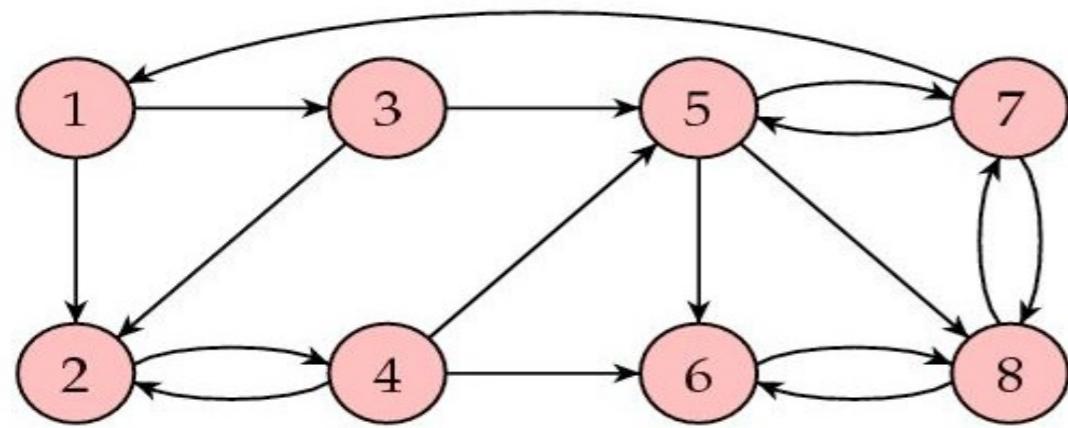


The corresponding matrix is

$$H = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 1/3 & 0 \\ 1/2 & 0 & 1/2 & 1/3 & 0 & 0 & 0 & 0 \\ 1/2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1/2 & 1/3 & 0 & 0 & 1/3 & 0 \\ 0 & 0 & 0 & 1/3 & 1/3 & 0 & 0 & 1/2 \\ 0 & 0 & 0 & 0 & 1/3 & 0 & 0 & 1/2 \\ 0 & 0 & 0 & 0 & 1/3 & 1 & 1/3 & 0 \end{bmatrix}$$

Probabilistic Interpretation of H

Random Surfing



The corresponding matrix is

$$H = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 1/3 & 0 \\ 1/2 & 0 & 1/2 & 1/3 & 0 & 0 & 0 & 0 \\ 1/2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1/2 & 1/3 & 0 & 0 & 1/3 & 0 \\ 0 & 0 & 0 & 1/3 & 1/3 & 0 & 0 & 1/2 \\ 0 & 0 & 0 & 0 & 1/3 & 0 & 0 & 1/2 \\ 0 & 0 & 0 & 0 & 1/3 & 1 & 1/3 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Random Walk

$$H(HI)$$

$$HI = \begin{bmatrix} 0 \\ 1/2 \\ 1/2 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

How to Compute!?

Power method

$$I_{k+1} = H I_k$$

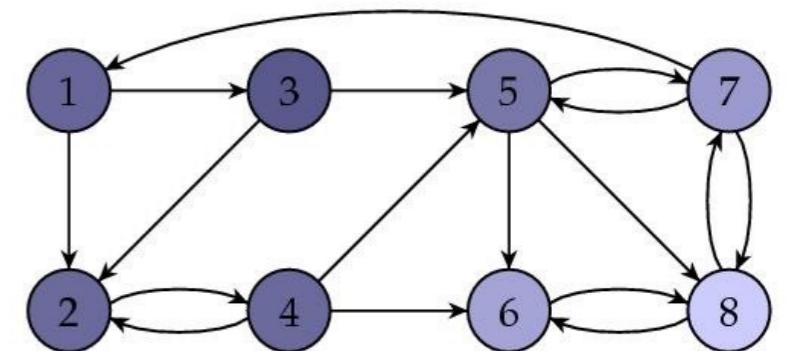
$$I_k = H^k I_0$$

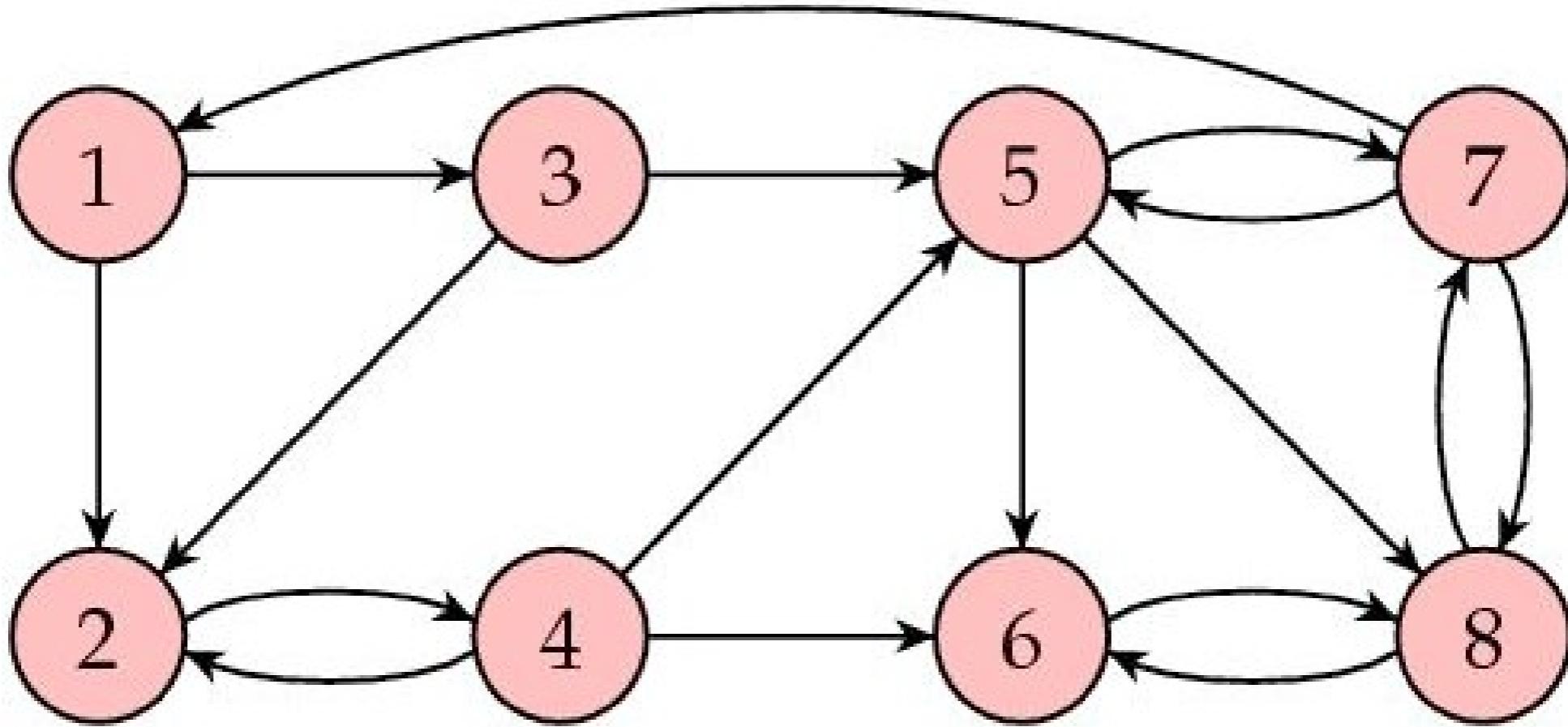
How to Compute!?

Power method

$$I^{k+1} = HI^k$$

I^0	I^1	I^2	I^3	I^4	\dots	I^{60}	I^{61}
1	0	0	0	0.0278	...	0.06	0.06
0	0.5	0.25	0.1667	0.0833	...	0.0675	0.0675
0	0.5	0	0	0	...	0.03	0.03
0	0	0.5	0.25	0.1667	...	0.0675	0.0675
0	0	0.25	0.1667	0.1111	...	0.0975	0.0975
0	0	0	0.25	0.1806	...	0.2025	0.2025
0	0	0	0.0833	0.0972	...	0.18	0.18
0	0	0	0.0833	0.3333	...	0.295	0.295





The corresponding matrix is

$$H = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 1/3 & 0 \\ 1/2 & 0 & 1/2 & 1/3 & 0 & 0 & 0 & 0 \\ 1/2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1/2 & 1/3 & 0 & 0 & 1/3 & 0 \\ 0 & 0 & 0 & 1/3 & 1/3 & 0 & 0 & 1/2 \\ 0 & 0 & 0 & 0 & 1/3 & 0 & 0 & 1/2 \\ 0 & 0 & 0 & 0 & 1/3 & 1 & 1/3 & 0 \end{bmatrix}$$

with stationary vector $I =$

$$\begin{bmatrix} 0.0600 \\ 0.0675 \\ 0.0300 \\ 0.0675 \\ 0.0975 \\ 0.2025 \\ 0.1800 \\ 0.2950 \end{bmatrix}$$

Does it always converge!?

Problem with this solution

- Does the sequence I^k always converge?
- Is the vector to which it converges independent of the initial vector I^0 ?
- Do the importance rankings contain the information that we want?



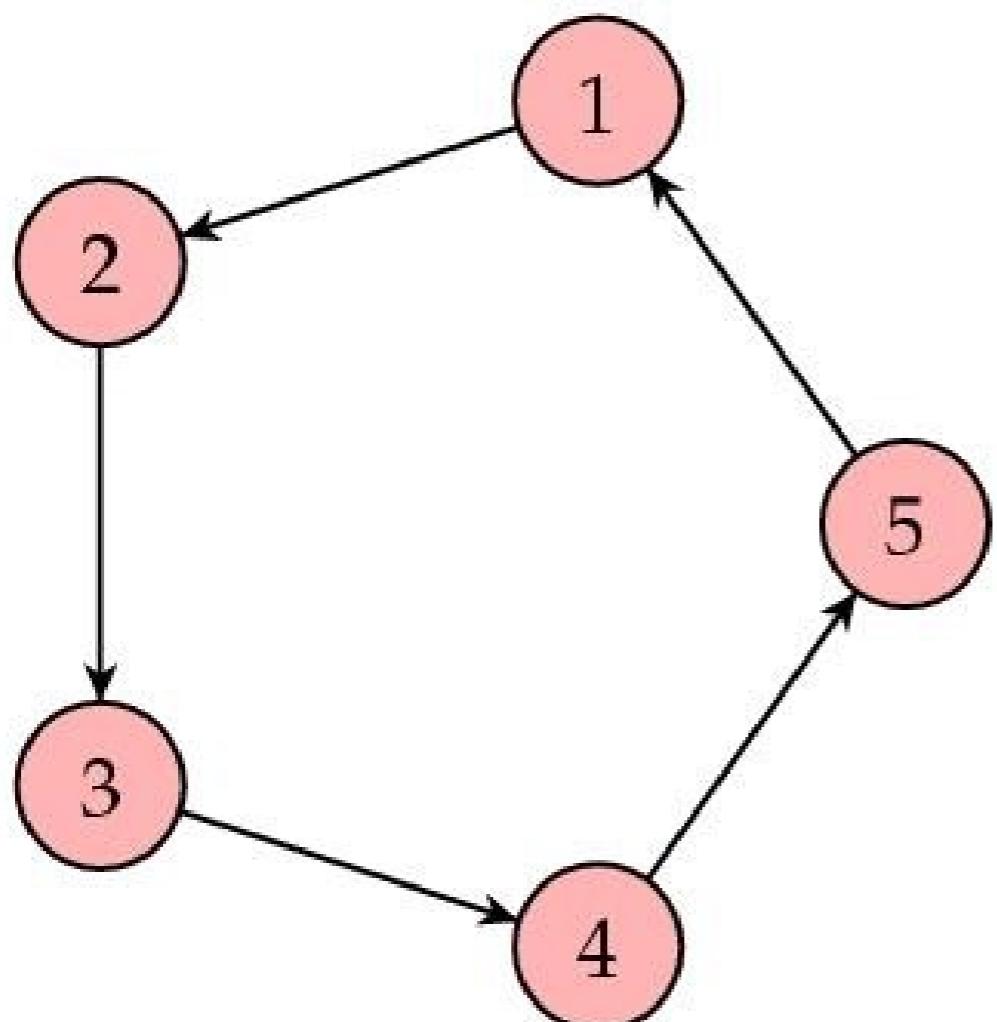
with matrix

$$H = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}$$

Here is one way in which our algorithm could proceed:

I^0	I^1	I^2	$I^3 = I$
1	0	0	0
0	1	0	0

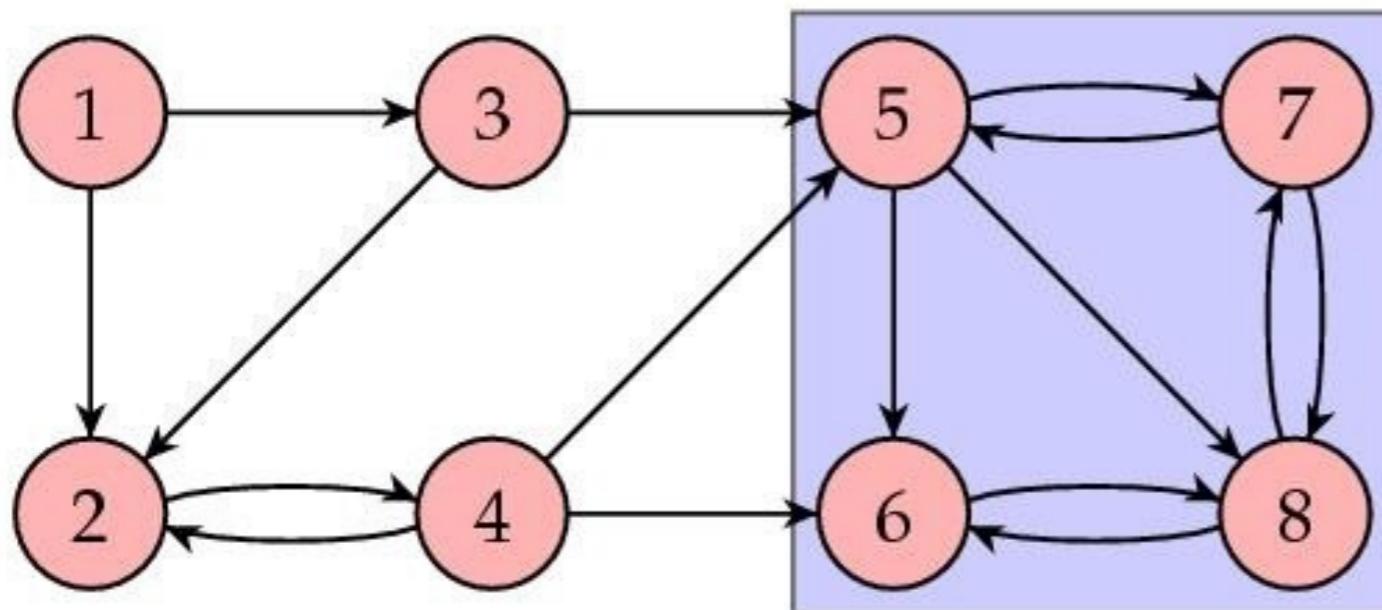
Problem 2



$$H = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

I^0	I^1	I^2	I^3	I^4	I^5
1	0	0	0	0	1
0	1	0	0	0	0
0	0	1	0	0	0
0	0	0	1	0	0
0	0	0	0	1	0

Problem 3: No Outlinks



$$H = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1/2 & 0 & 1/2 & 1/3 & 0 & 0 & 0 & 0 \\ 1/2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1/2 & 1/3 & 0 & 0 & 1/2 & 0 \\ 0 & 0 & 0 & 1/3 & 1/3 & 0 & 0 & 1/2 \\ 0 & 0 & 0 & 0 & 1/3 & 0 & 0 & 1/2 \\ 0 & 0 & 0 & 0 & 1/3 & 1 & 1/2 & 0 \end{bmatrix}$$

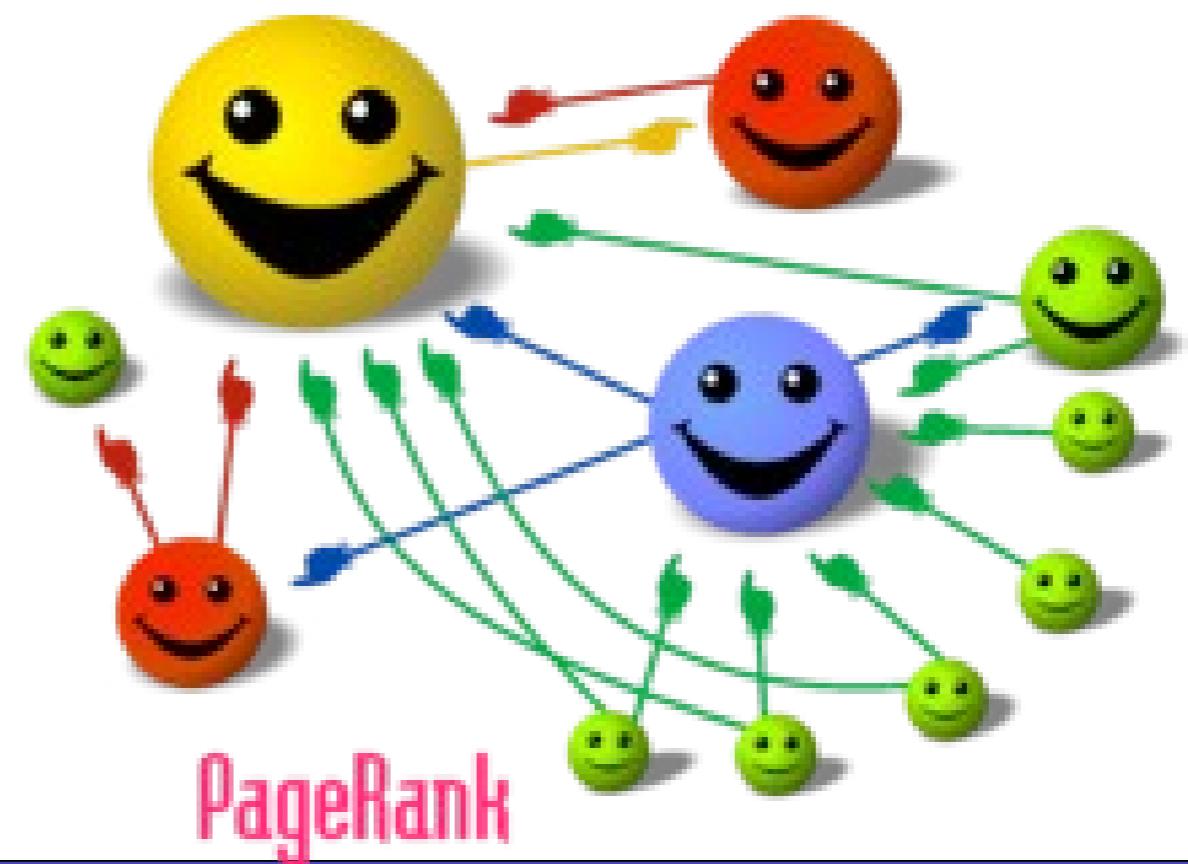
Better Solution

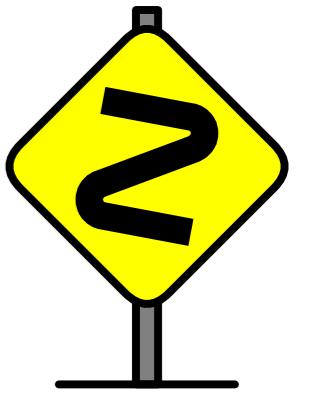
$$G = \alpha H + (1 - \alpha)O$$

$$O = \begin{bmatrix} 1 & 1 & \cdots & 1 \\ 1 & 1 & \cdots & 1 \\ \vdots & \vdots & \ddots & \vdots \\ 1 & 1 & \cdots & 1 \end{bmatrix}$$

Teleportation!

Adding (very unlikely) outlinks for every node





Graph Laplacian

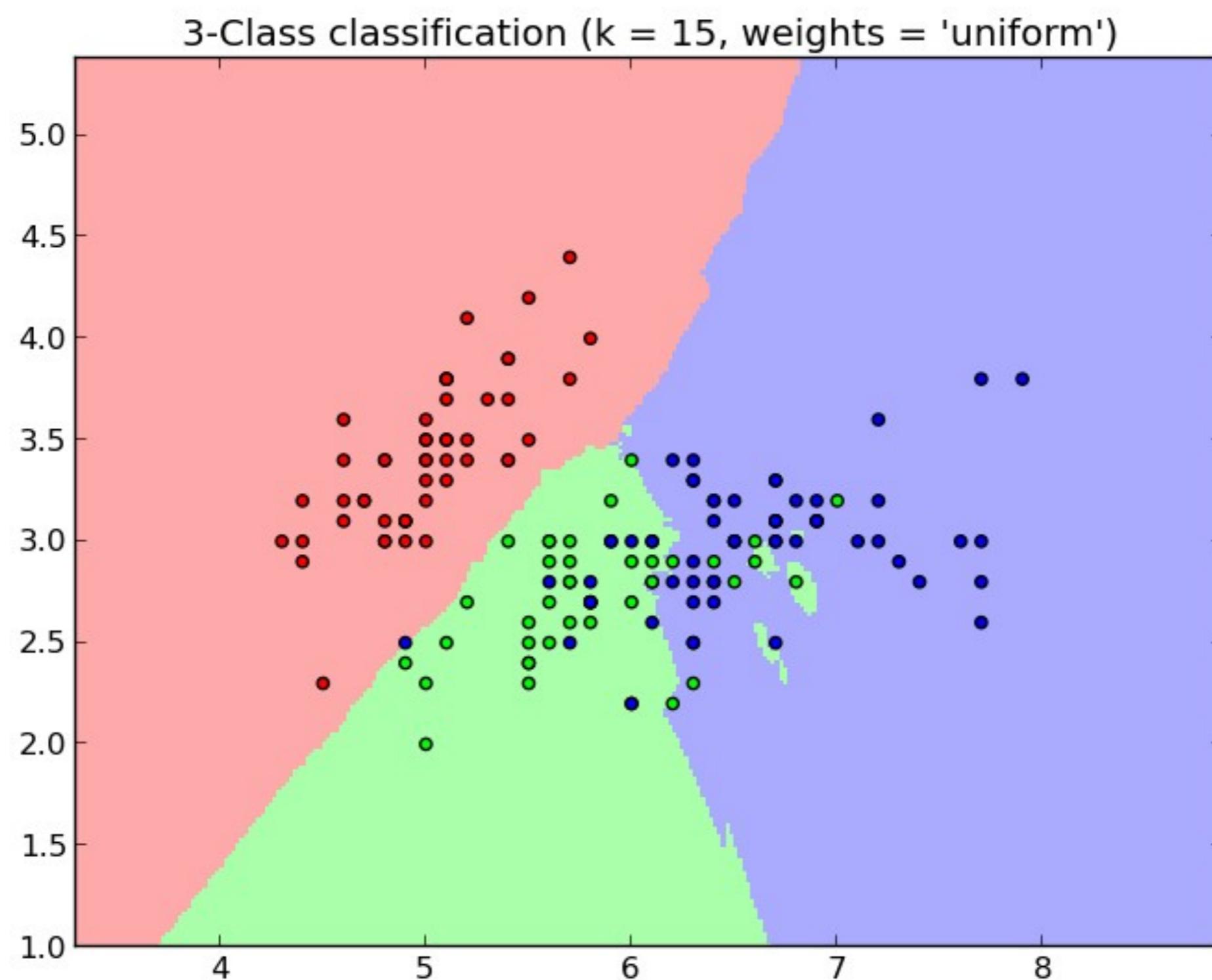
Labeled graph	Degree matrix	Adjacency matrix	Laplacian matrix
	$\begin{pmatrix} 2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$	$\begin{pmatrix} 0 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{pmatrix}$	$\begin{pmatrix} 2 & -1 & 0 & 0 & -1 & 0 \\ -1 & 3 & -1 & 0 & -1 & 0 \\ 0 & -1 & 2 & -1 & 0 & 0 \\ 0 & 0 & -1 & 3 & -1 & -1 \\ -1 & -1 & 0 & -1 & 3 & 0 \\ 0 & 0 & 0 & -1 & 0 & 1 \end{pmatrix}$

<https://www.youtube.com/watch?v=yDltiyH1Hxw>

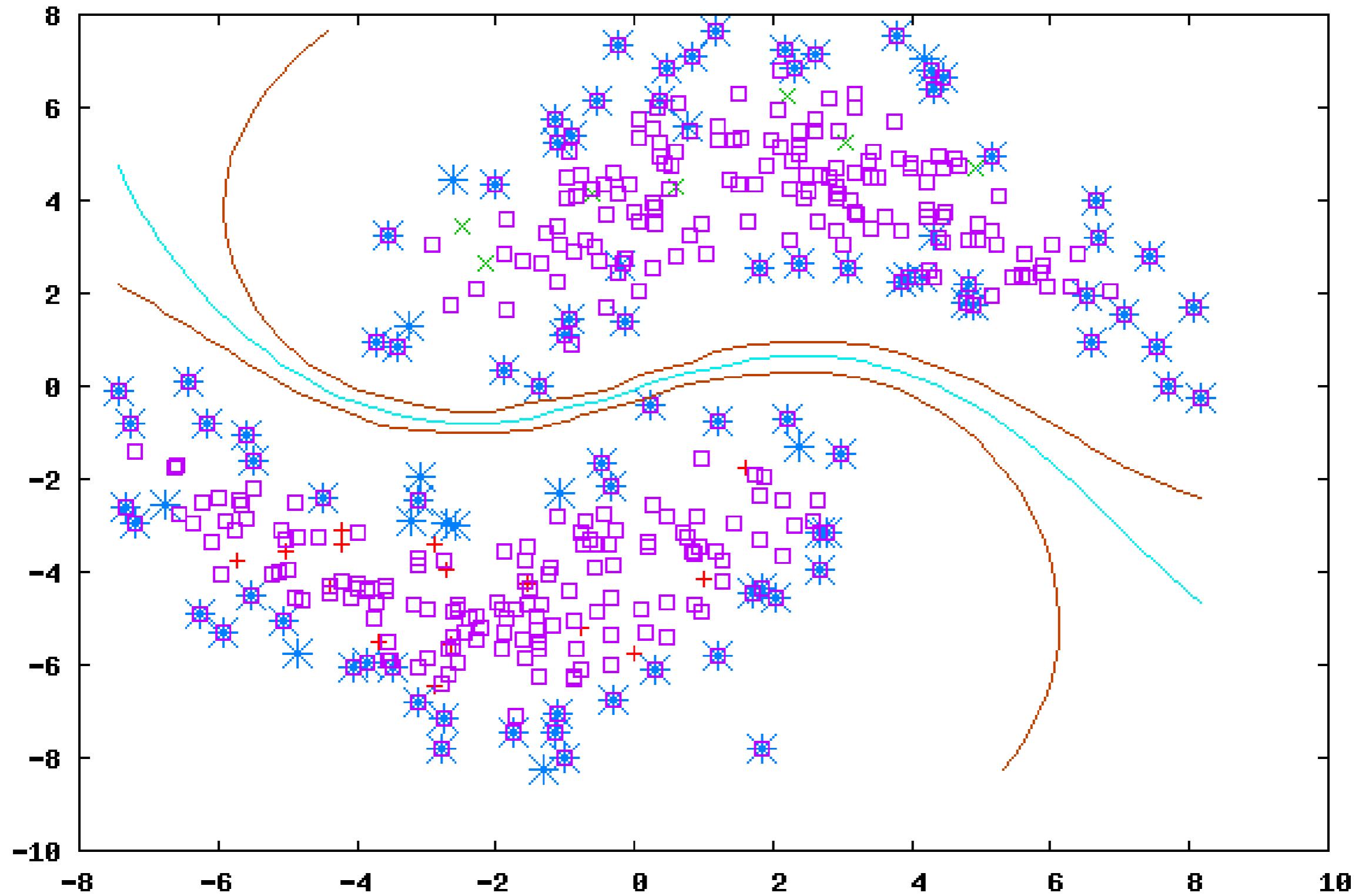
10:45

High Dimensional Data

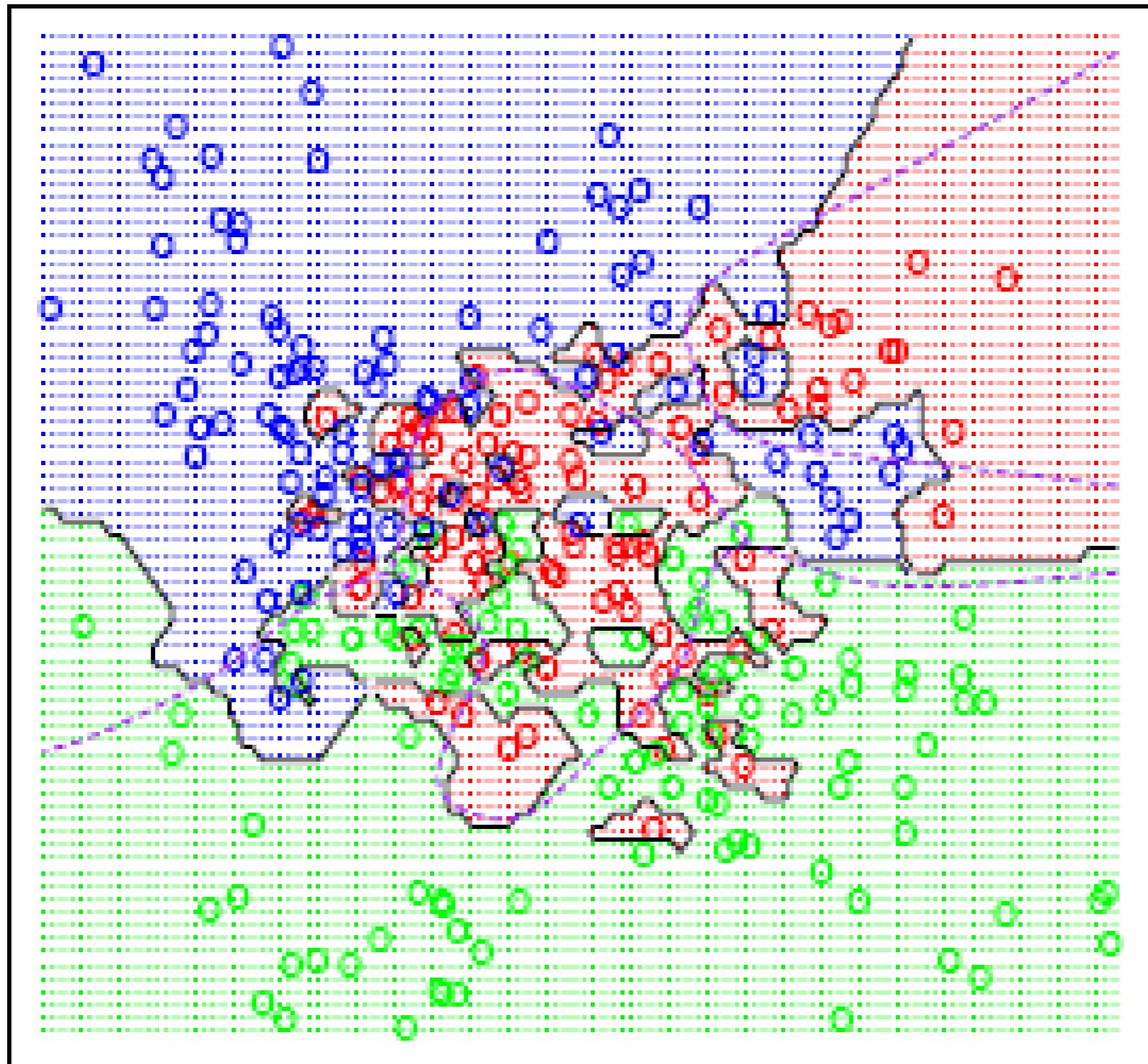
“Perfect World” for Data Analysis



“Perfect World” for Data Analysis



“Perfect World” for Data Analysis



Real World Data Analysis

The screenshot shows a Google search results page. The top bar includes links for Home, Images, Maps, News, Videos, Gmail, Docs, Sheets, Slides, and Drive. Below the search bar, there's a 'Translate' button and a dropdown menu. The main content area displays a snippet from the Wikipedia article on Data Science, which discusses its interdisciplinary nature and applications in various fields like business, engineering, and technology. It also mentions the growth of data science as a discipline and its impact on society.

The screenshot shows a Facebook profile for Mark Zuckerberg. The profile header features a large photo of him smiling. Below it, sections include 'Education and Work' (Employer: Facebook, College: Harvard University), 'Family' (Mother: Karen Zuckerberg, Father: Edward Zuckerberg, Sister: Randi Zuckerberg, Sister: Donna Zuckerberg, Sister: Arielle Zuckerberg), 'High School' (Ardsley High School), 'Philosophy' (quote: "All children are artists. The problem is how to remain an artist once he grows up."), and 'Favorite Quotes'. On the right side, there are ads for 'Police Auctions' and 'SF Bucket List'.

First Name	Last Name	Address	City	Age
Mickey	Mouse	123 Fantasy Way	Anaheim	73
Bat	Man	321 Cavern Ave	Gotham	54
Wonder	Woman	987 Truth Way	Paradise	39
Donald	Duck	555 Quack Street	Mallard	65
Bugs	Bunny	567 Carrot Street	Rascal	58
Wiley	Coyote	999 Acme Way	Canyon	61
Cat	Woman	234 Purrfect Street	Hairball	32
Tweety	Bird	543	Itotltaw	28

High Dimension?

P ∈ Big N

First Name	Last Name	Address	City	Age
Mickey	Mouse	123 Fantasy Way	Anaheim	73
Bat	Man	321 Cavern Ave	Gotham	54
Wonder	Woman	987 Truth Way	Paradise	39
Donald	Duck	555 Quack Street	Mallard	65
Bugs	Bunny	567 Carrot Street	Rascal	58
Wiley	Coyote	999 Acme Way	Canyon	61
Cat	Woman	234 Purrfect Street	Hairball	32
Tweety	Bird	543	Itotltaw	28

N
P
Big
Smile

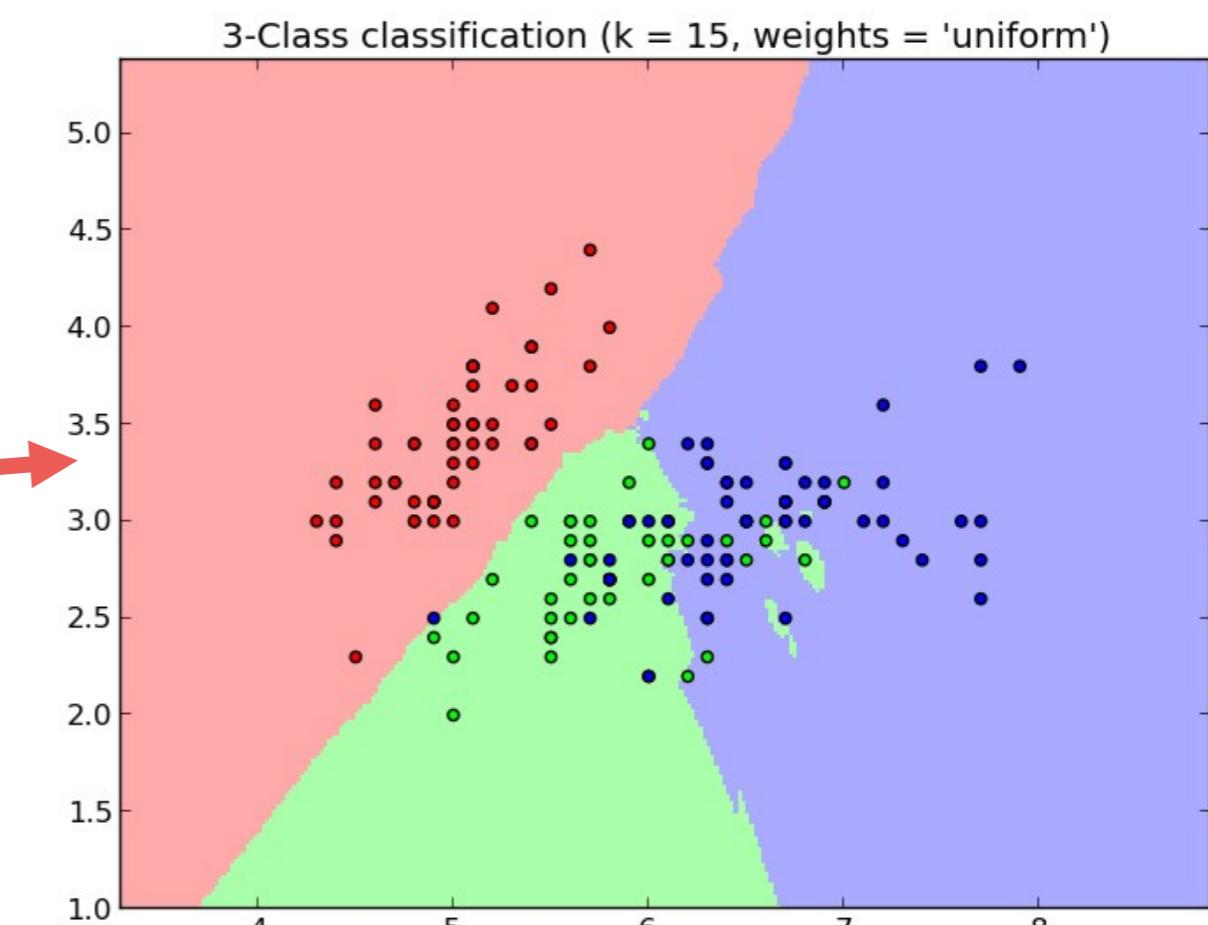
Issue is not ρ "big".

Issue is " ρ is big relative to n ".

For a given ρ How much
 $\sim \rho_0$ you need?

“Perfect World” for Data Analysis

Volume (cm ³)	Pressure (bar)
3.701502	-0.366814
7.858678	0.132824
11.6975	0.907633
15.46395	1.790605
19.18672	2.770453
23.00292	3.734302
26.83077	4.753389
30.75468	5.746156
34.73695	6.742996



Less than Perfect

First Name	Last Name	Address	City	Age
Mickey	Mouse	123 Fantasy Way	Anaheim	73
Bat	Man	321 Cavern Ave	Gotham	54
Wonder	Woman	987 Truth Way	Paradise	39
Donald	Duck	555 Quack Street	Mallard	65
Bugs	Bunny	567 Carrot Street	Rascal	58
Wiley	Coyote	999 Acme Way	Canyon	61
Cat	Woman	234 Purrfect Street	Hairball	32
Tweety	Bird	543	Itotltaw	28

5 dimensional

High Dimensional Data Classification

Anchor



Joshua Tree



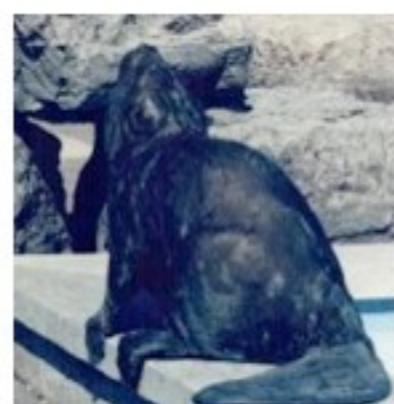
Beaver



Lotus



Water Lily



Predictors are Pixels!

High Dimensional Data

Other Examples?

here's

Voice Recognition weather

text Processing sports

Handwriting stocks

Brain mapping

PIGS

**What is really happening in
High Dimensional Space?**

<https://www.youtube.com/watch?v=C6kn6nXMWF0>

Outline

- What are spheres and cubes in high dimensions?
- What does distance mean in high dimensions?
- What does random mean in high dimensions?
- What does data mean in high dimensions?

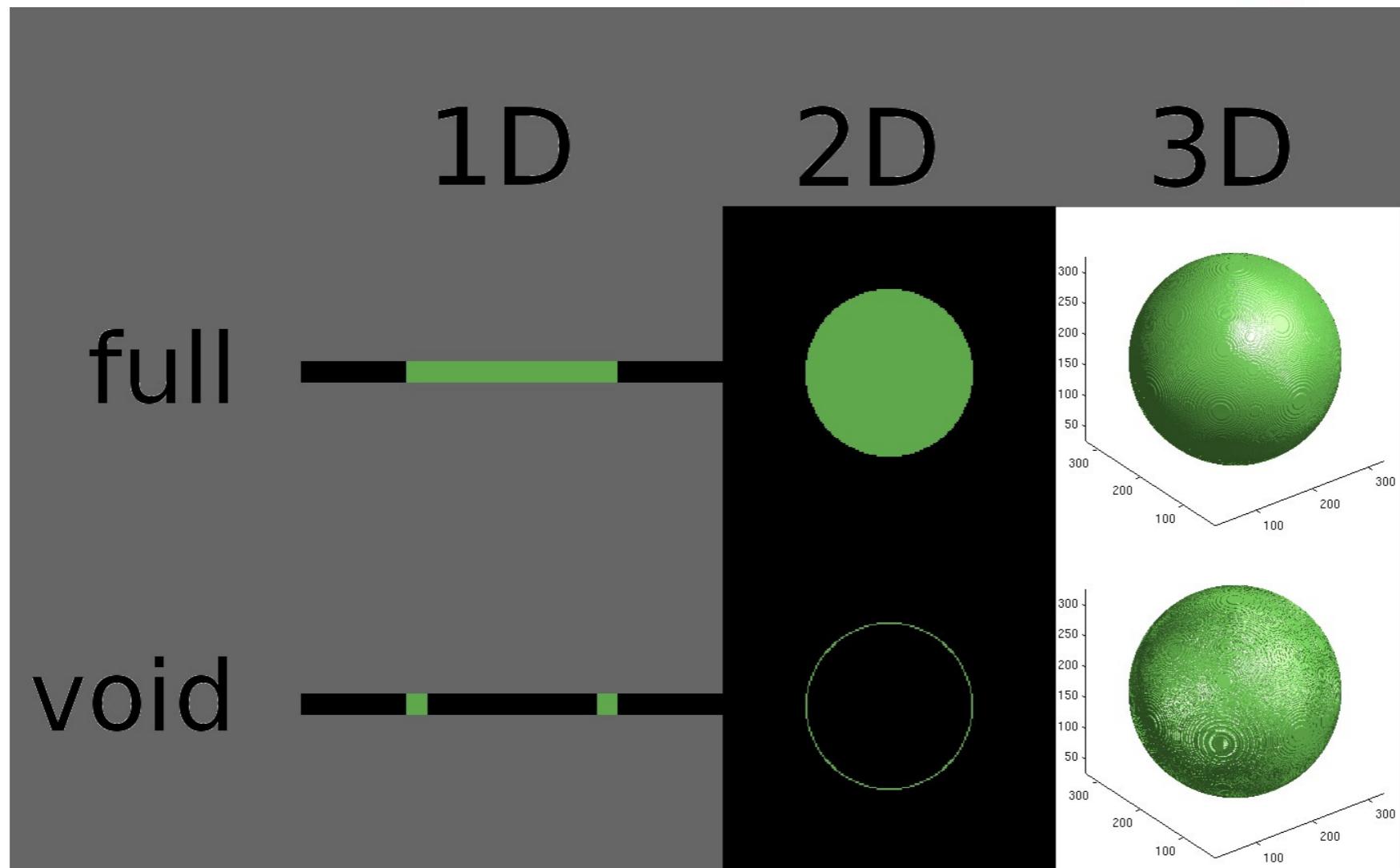
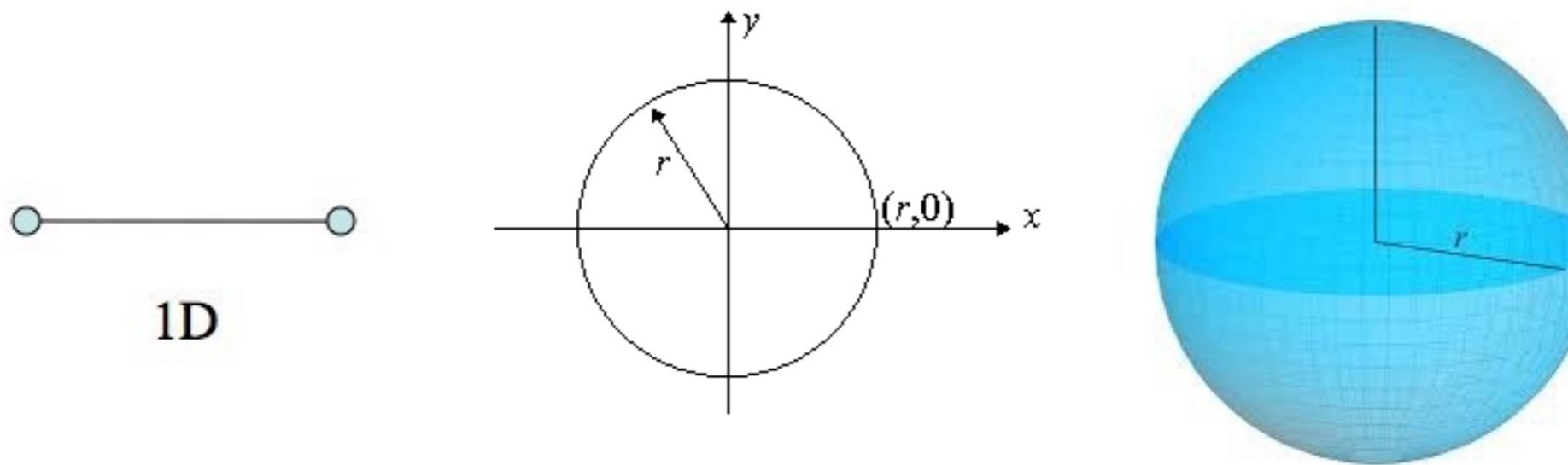
Eigenface #16

Who is this
guy?



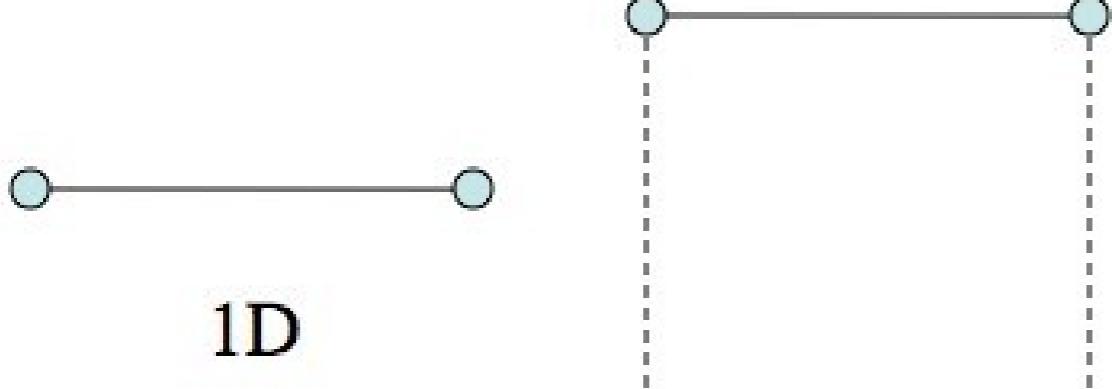
Let's start simple: What are spheres?

Sphere

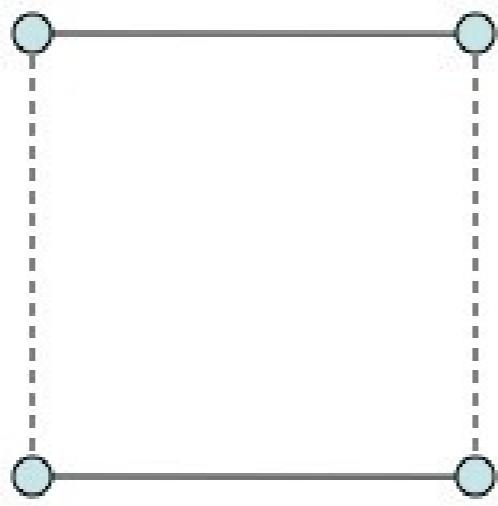


Let's start simple: What are cubes?

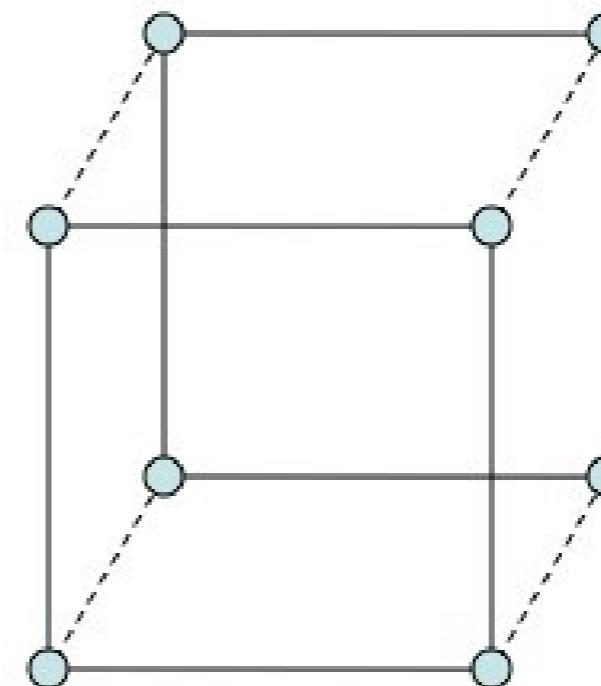
Line - Square - Cube - HyperCube



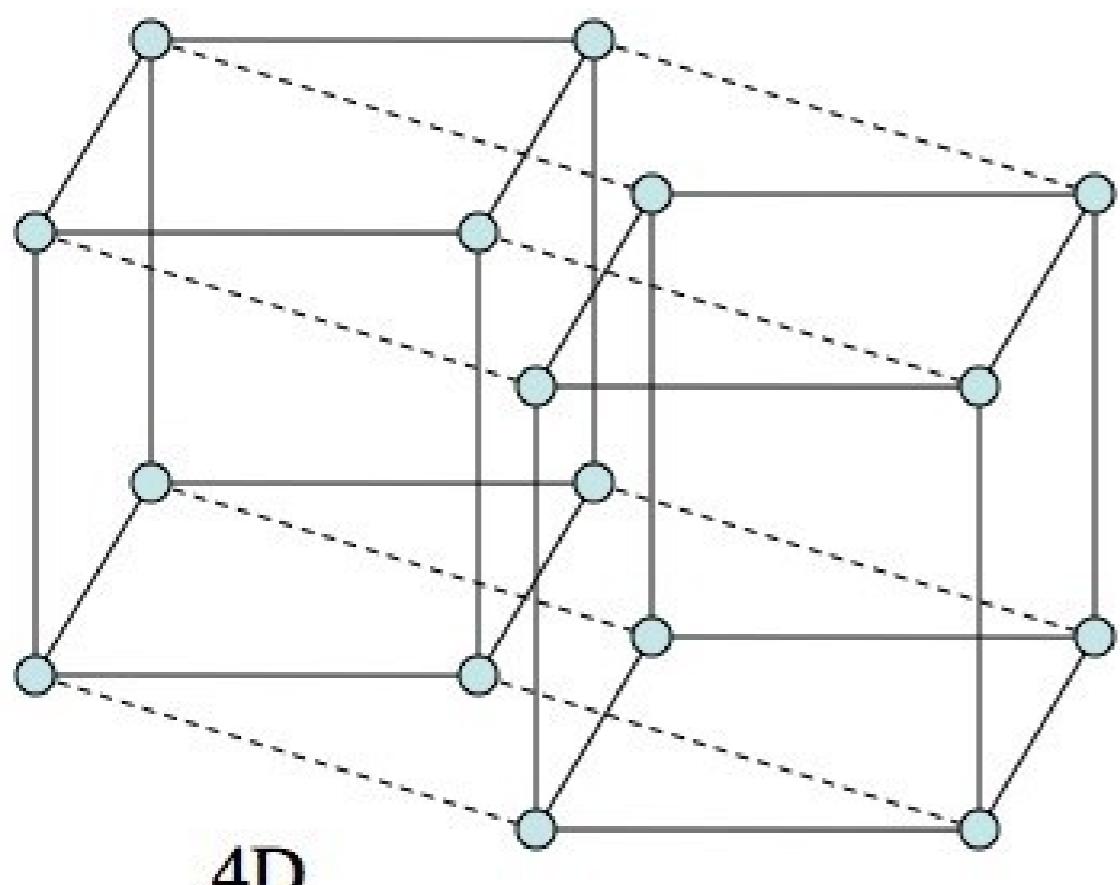
1D



2D



3D

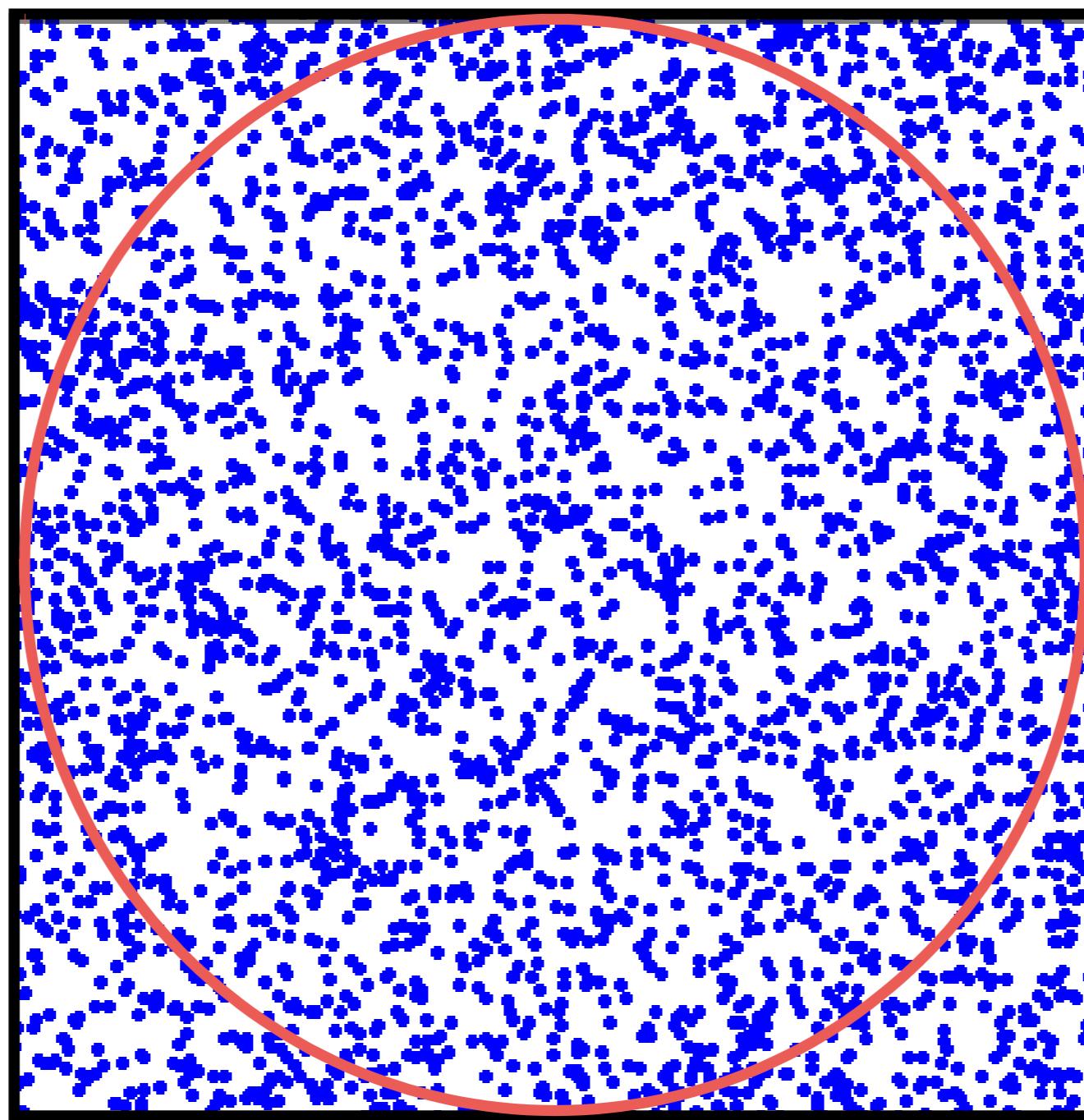


4D

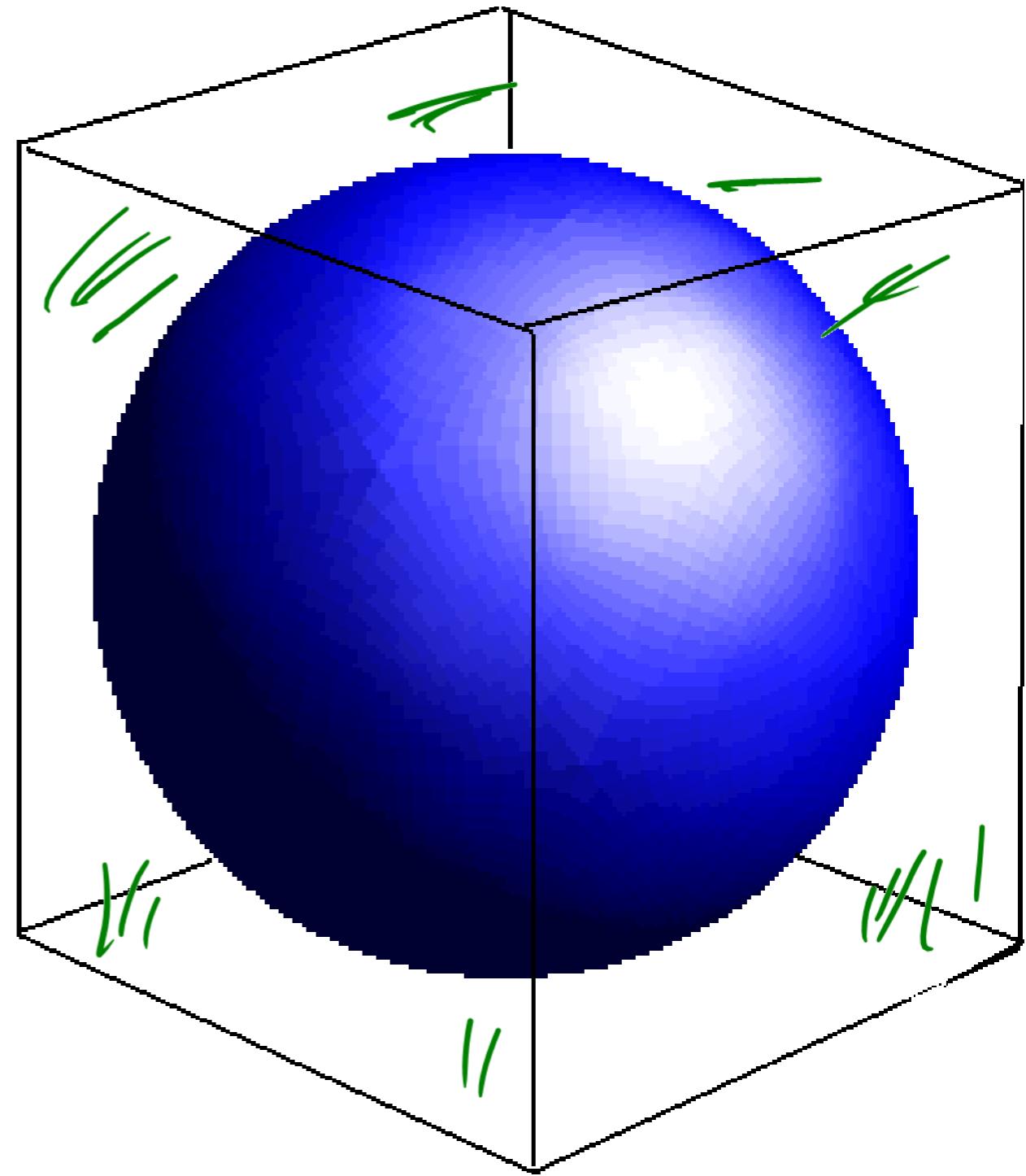
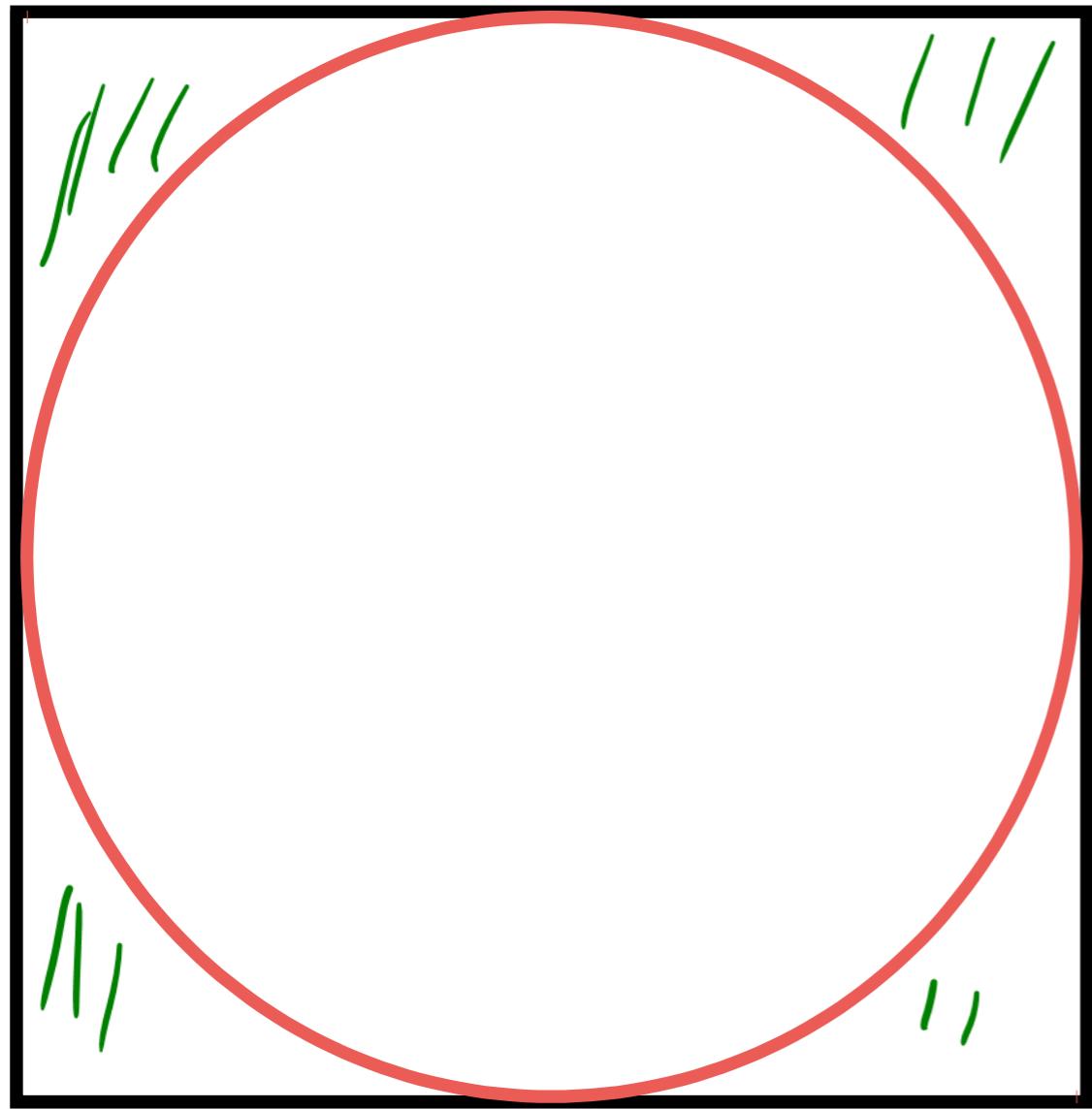
“Cube”

How do spheres and cubes relate in
high-dimensions?

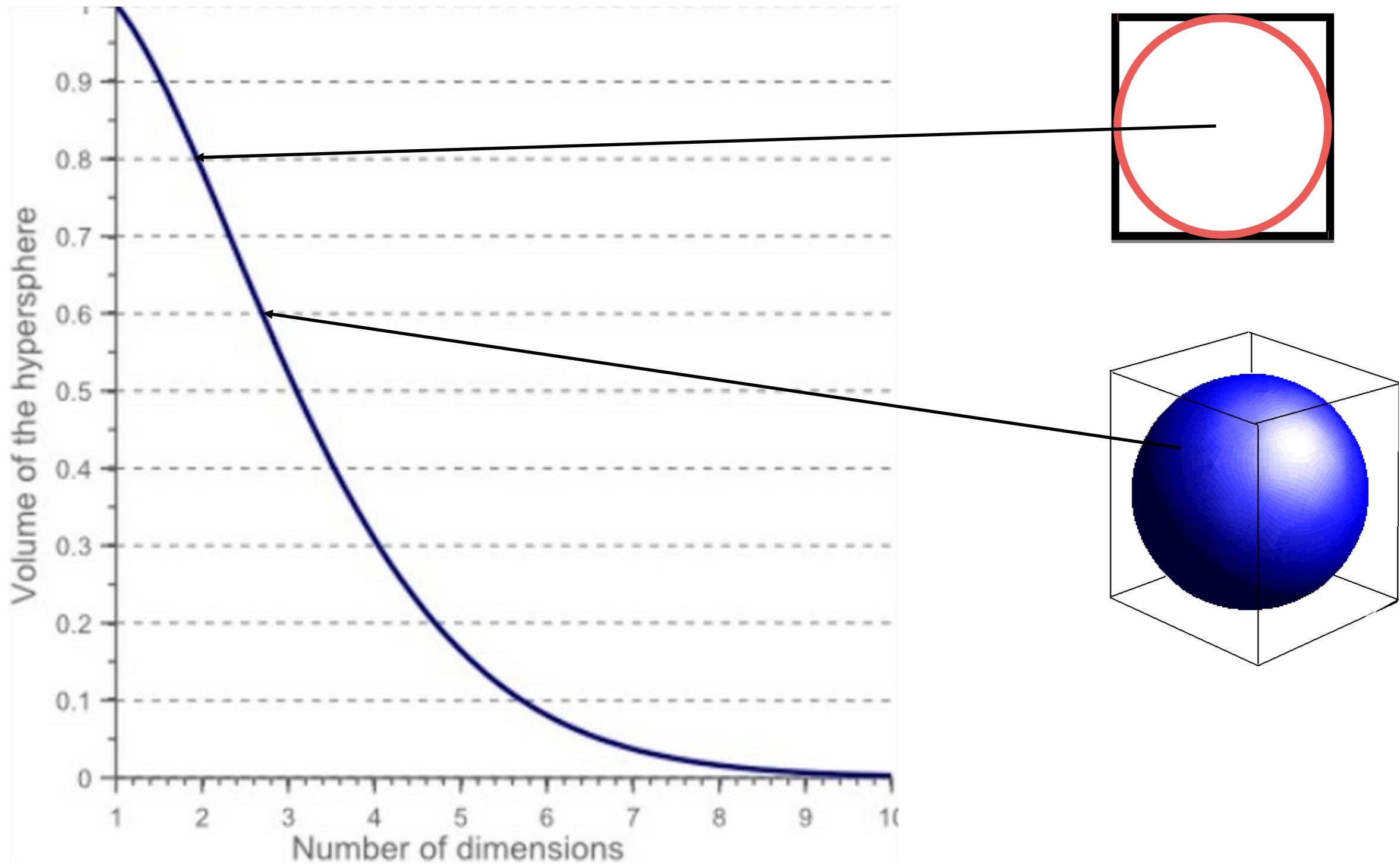
Question #1: Uniform Distribution (Center vs Corner)



Center vs Corner



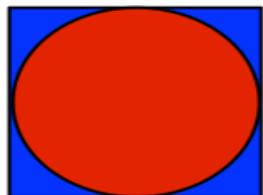
Volume of the hypersphere



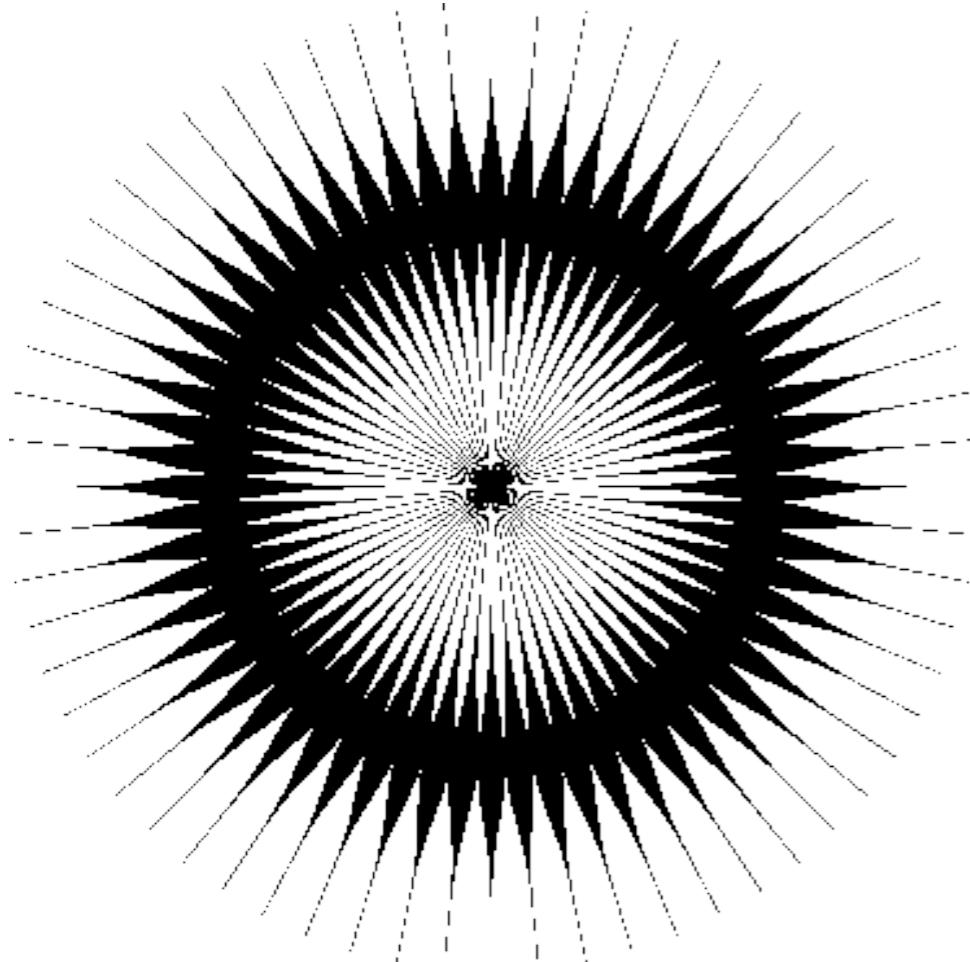
High Dimensional Cube

- 10^d points cover $[0, 1]^d$ at a distance 10^{-1} .

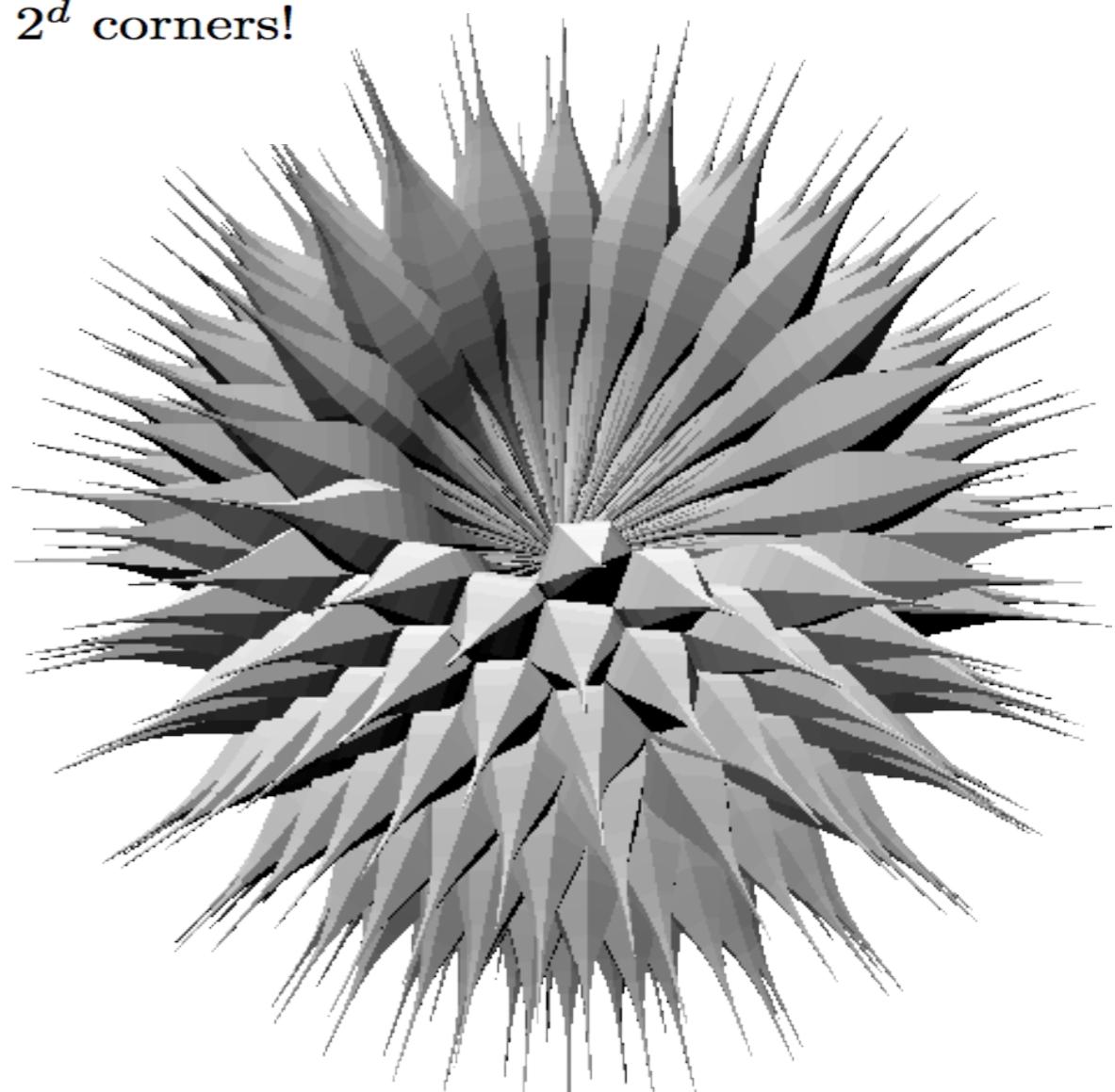
$$\lim_{d \rightarrow \infty} \frac{\text{volume sphere of radius } r}{\text{volume } [0, r]^d} = 0$$



: nearly all points are in the 2^d corners!

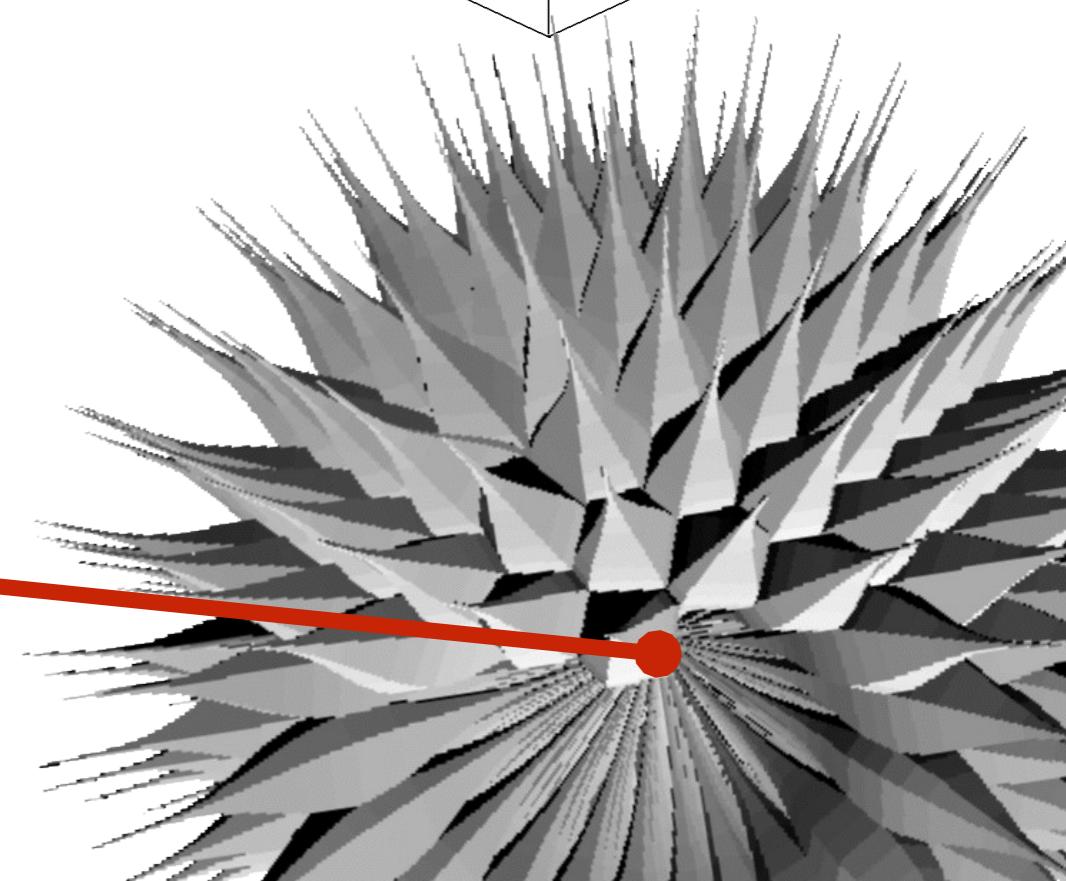
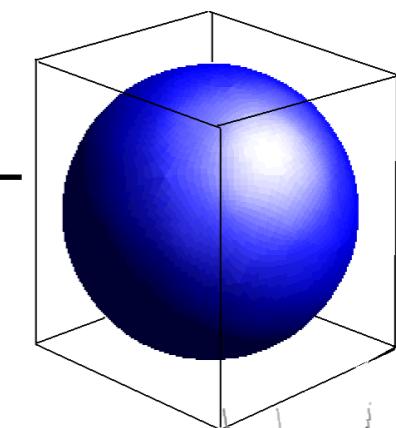
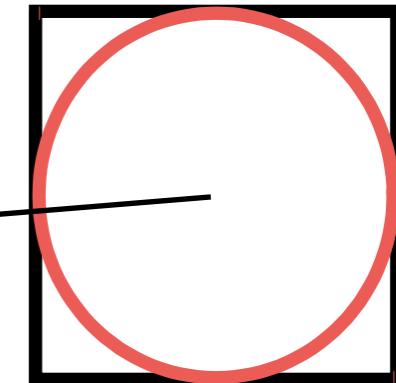
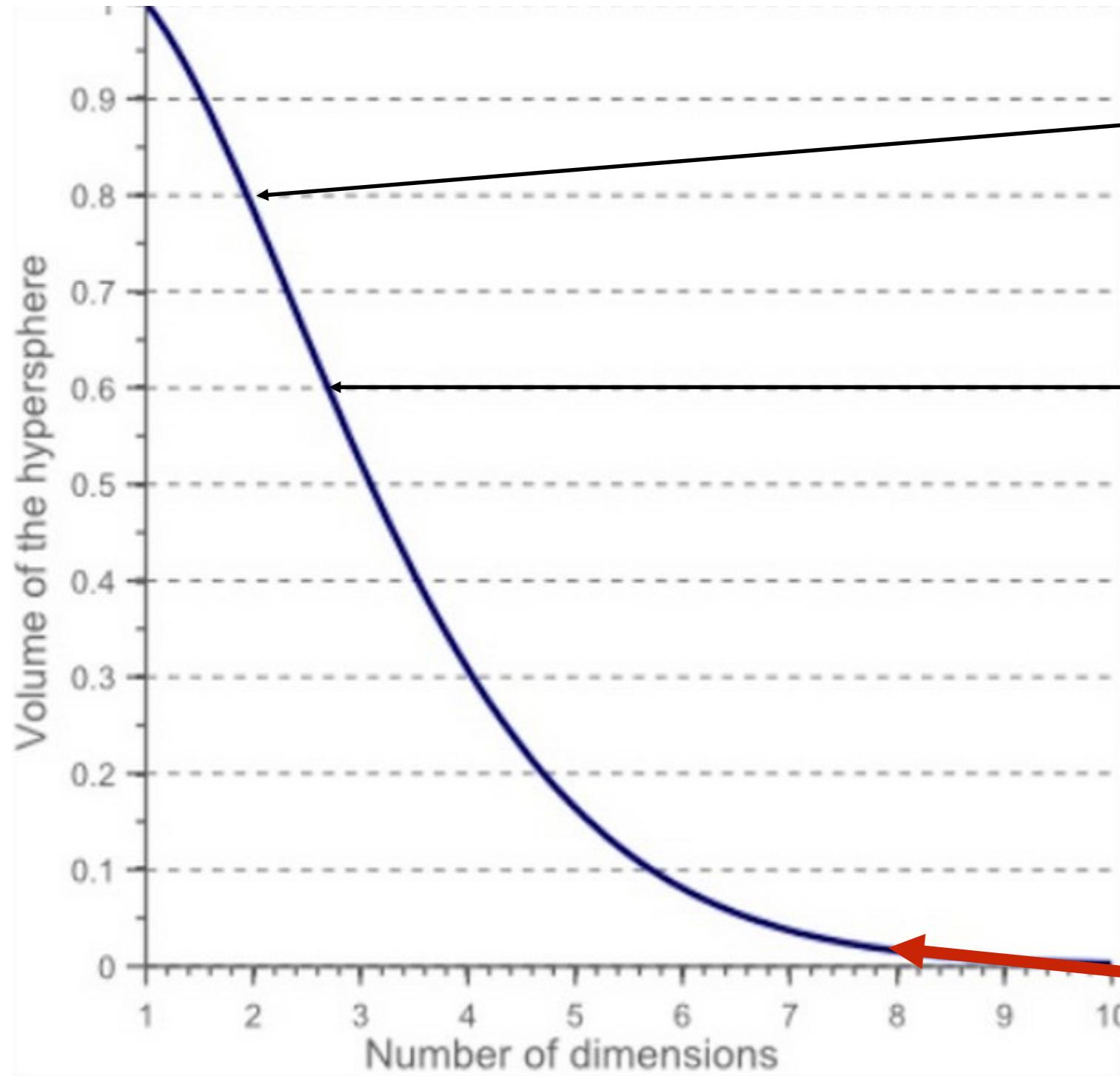


6 dimensional cube
projection



8 dimensional cube
projection

Volume of the hypersphere

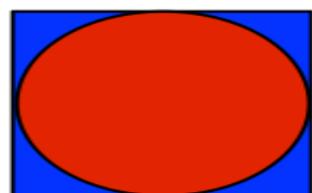


In High Dimensional Space:

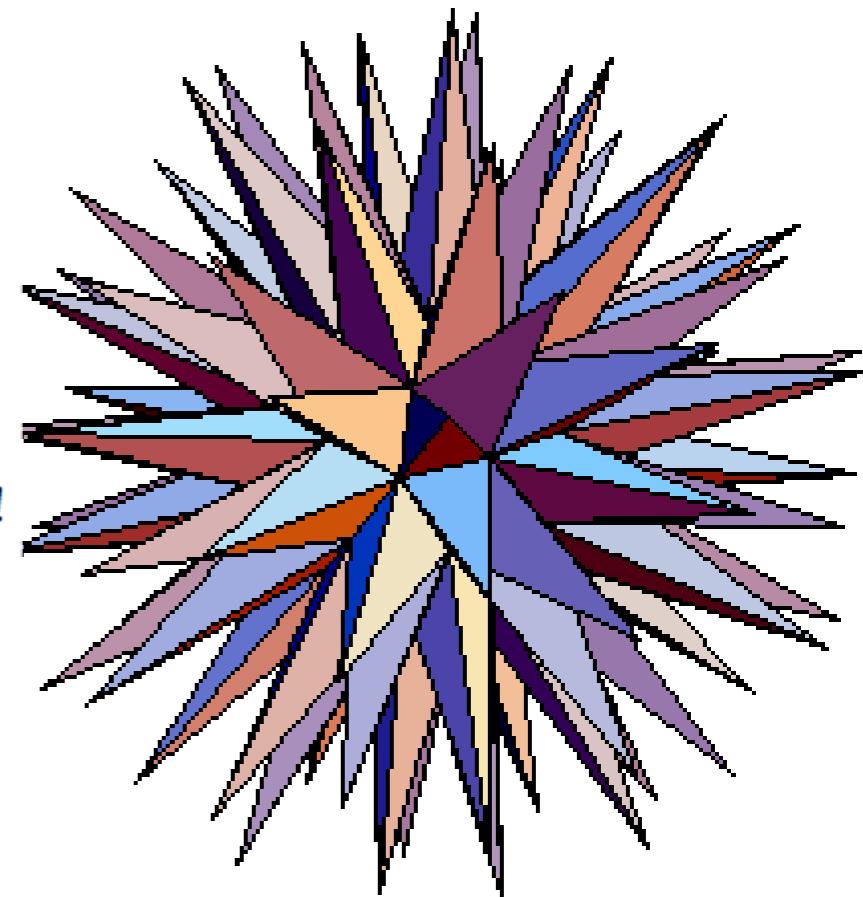
Rule # 1

- Center v.s. Corners :
 - Corners win!

$$\lim_{n \rightarrow \infty} \frac{\text{volume sphere of radius } r}{\text{volume } [0, r]^d} = 0$$

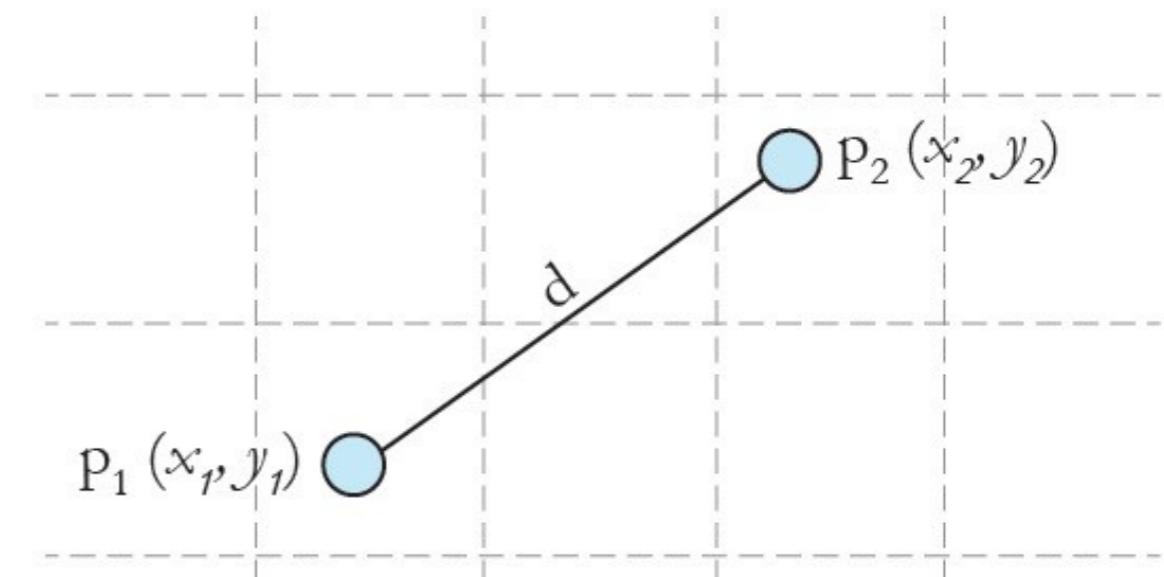
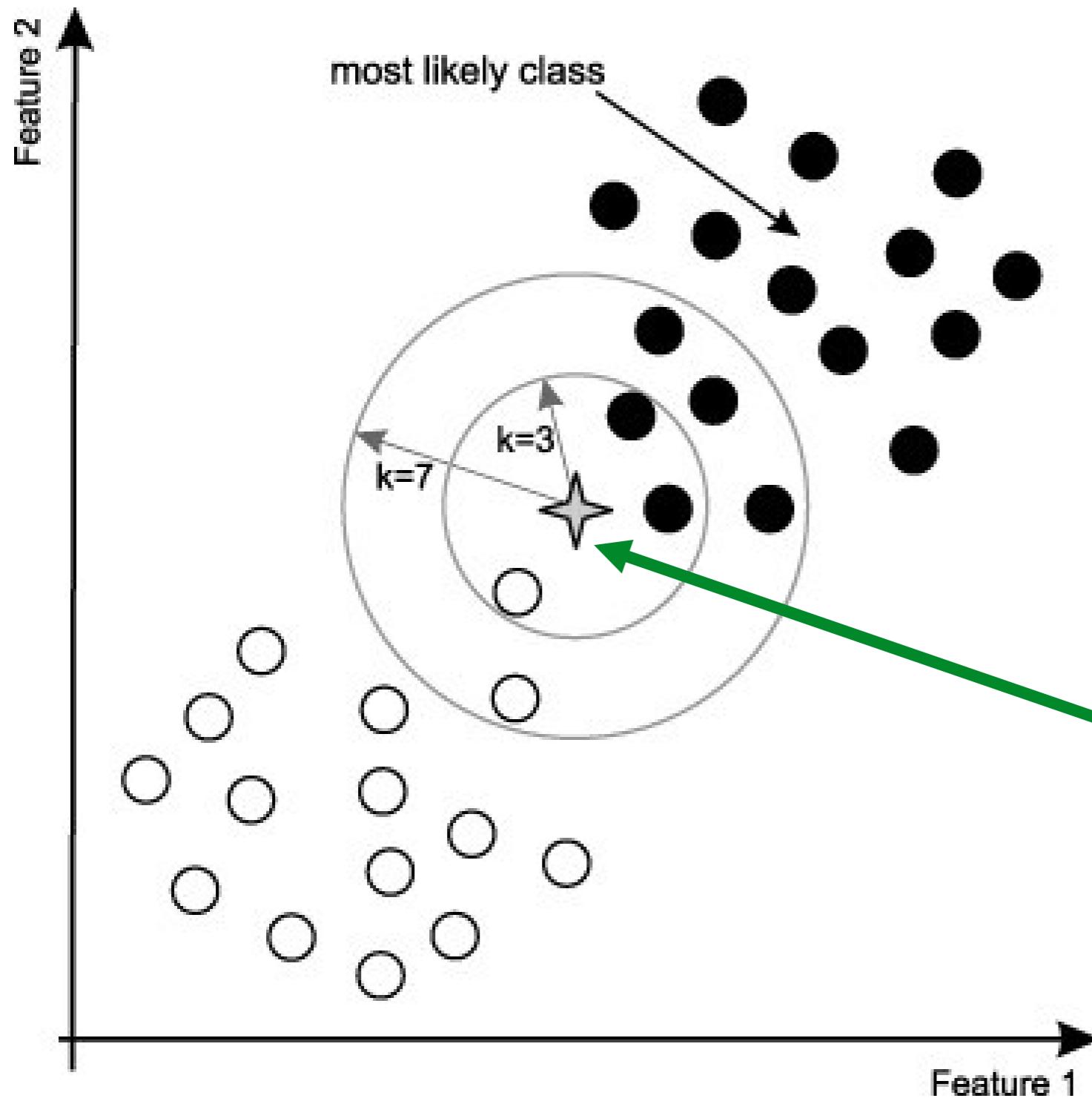


: nearly all points are in the 2^d corners!

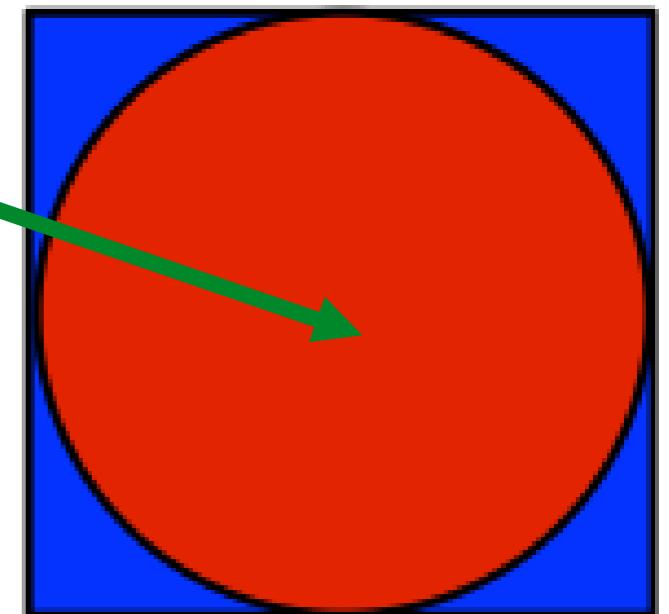


What does distance mean in high-dimensions?

Question #2: Euclidean Distance



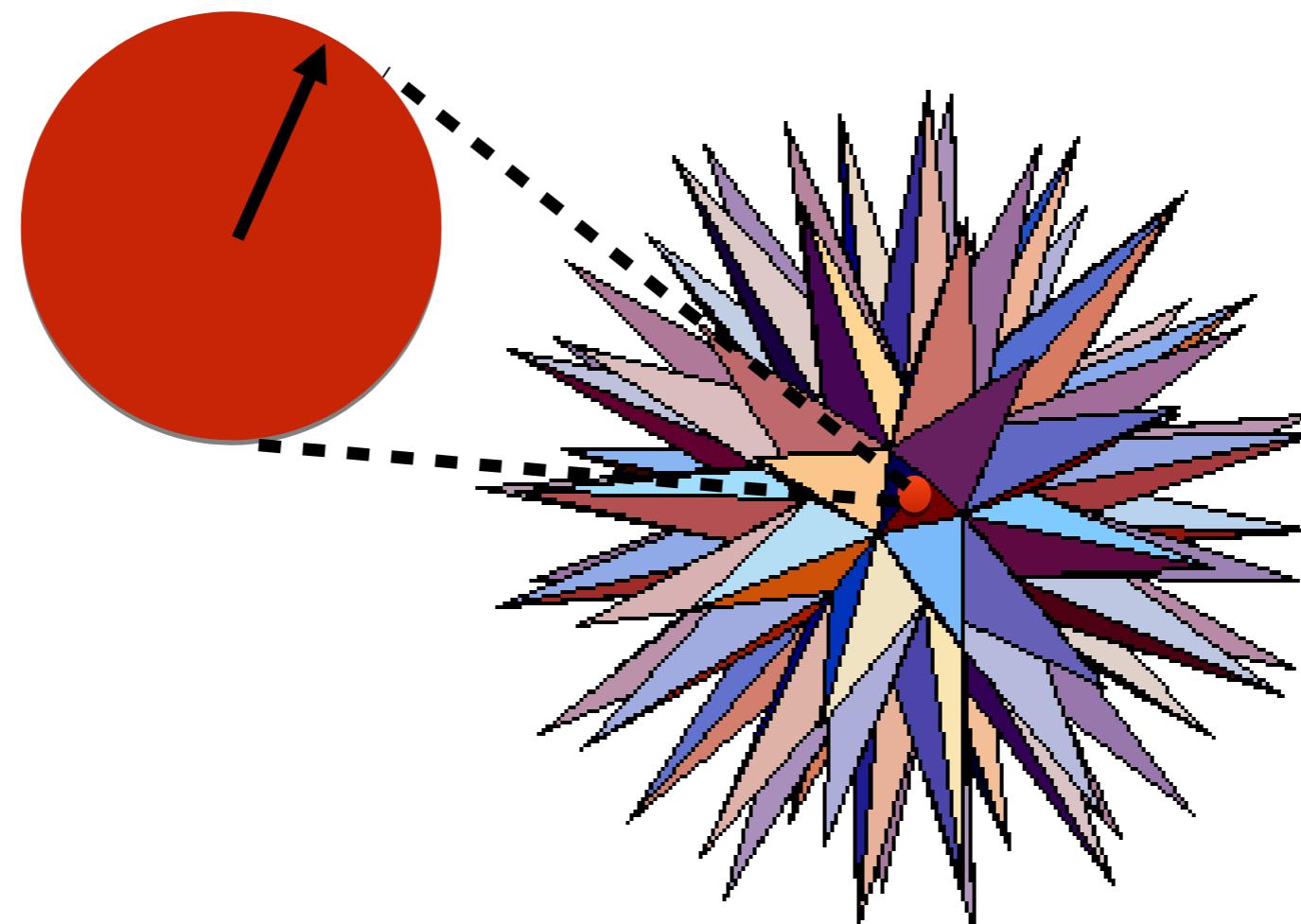
$$\text{Euclidean distance } (d) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$



In High Dimensional Space:

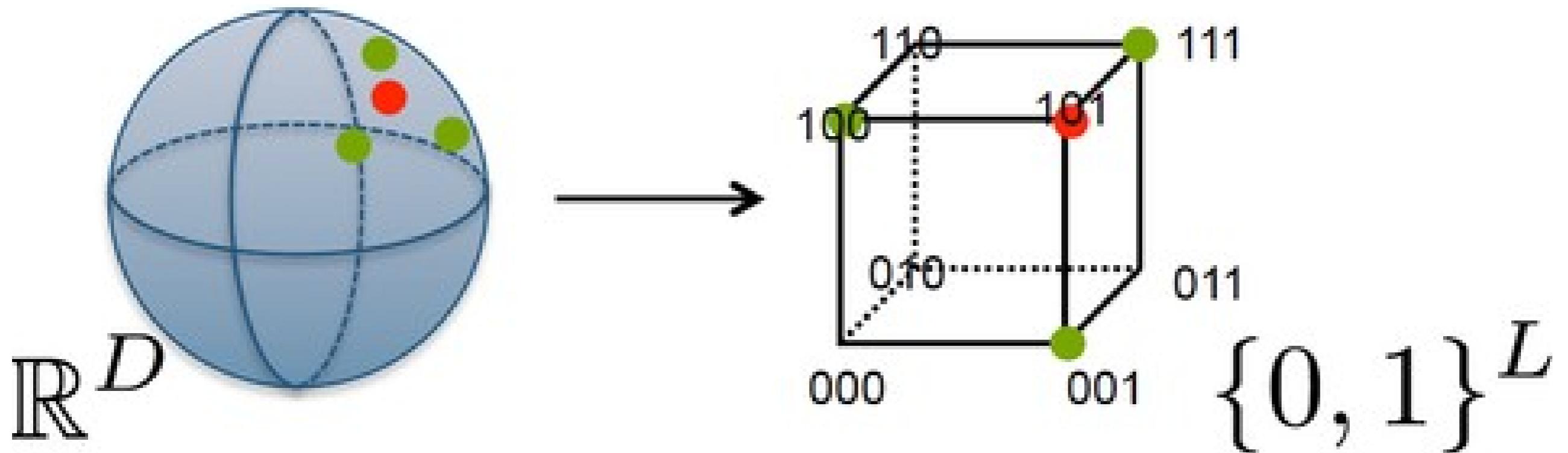
Rule # 2

- Don't Use Euclidean Distance

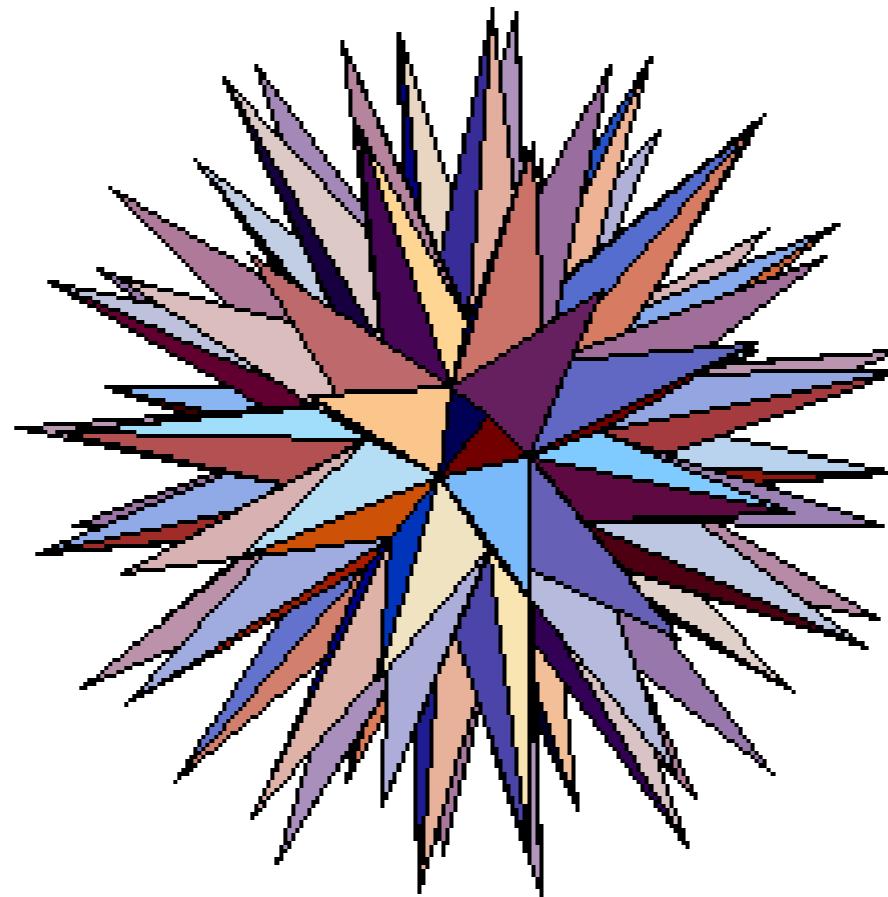
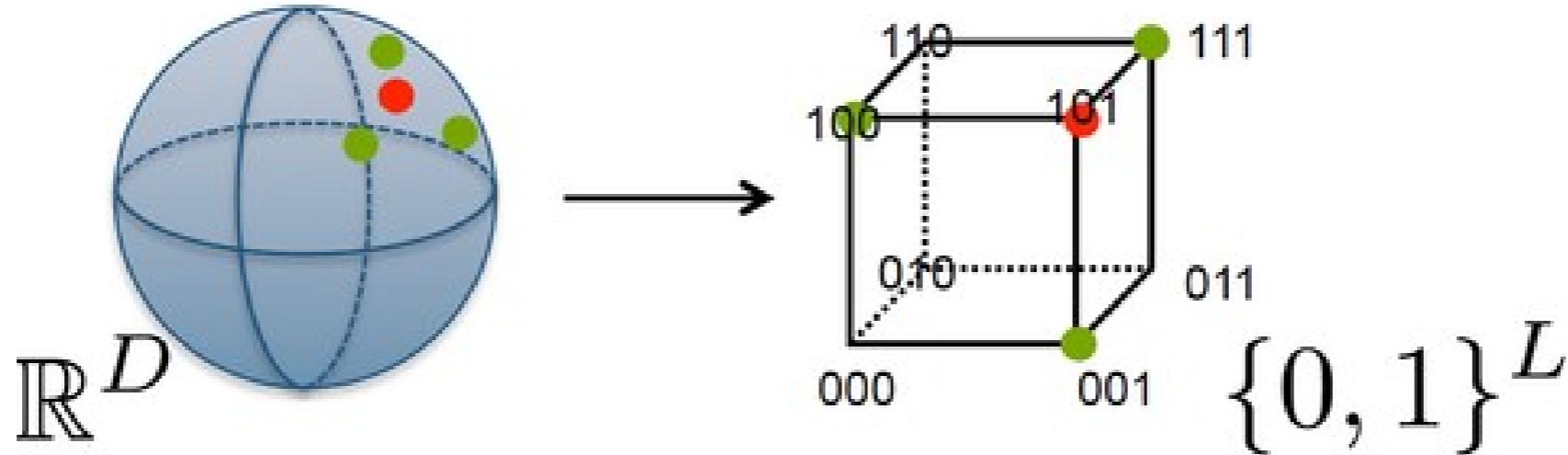


all your “neighbors” are far away

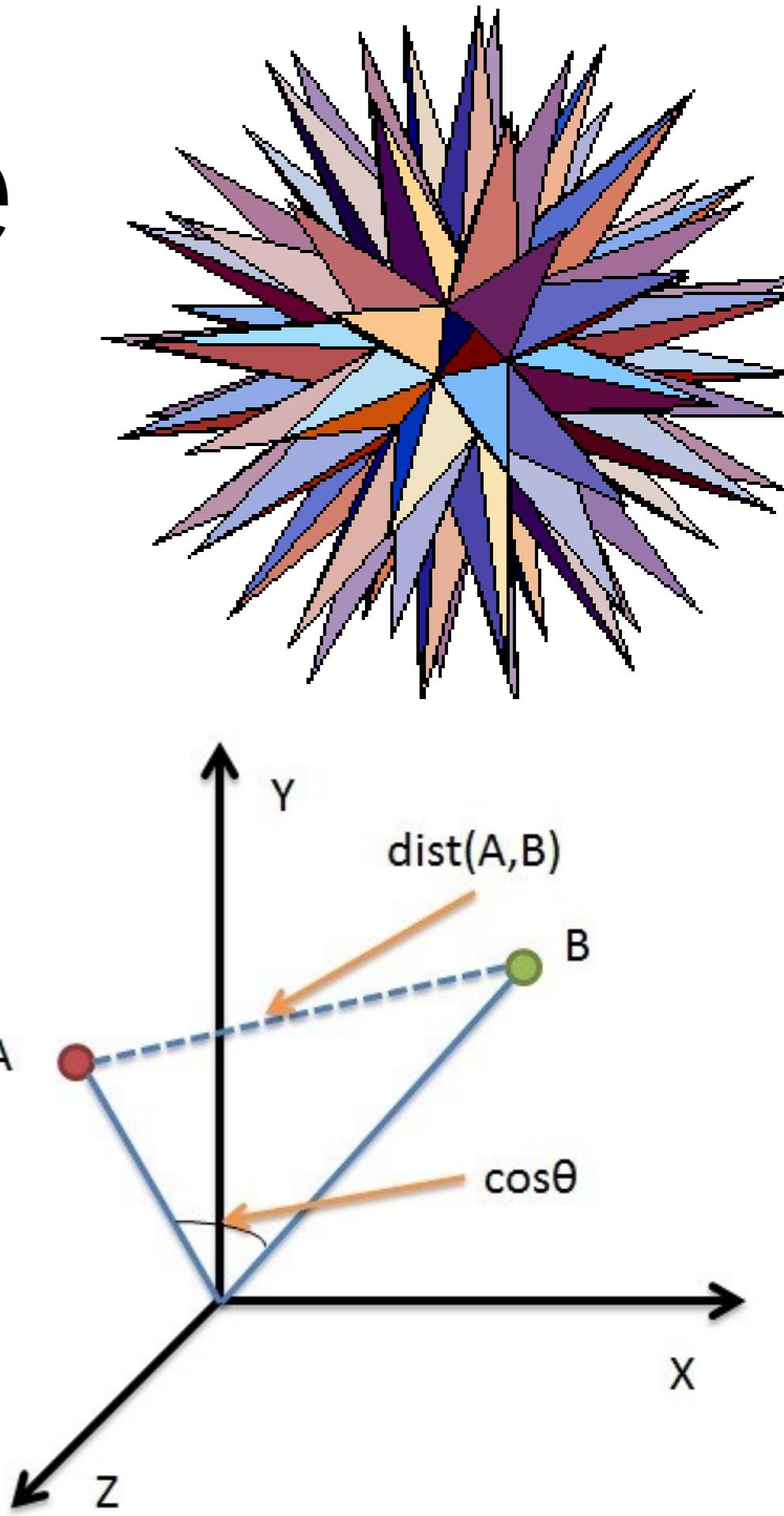
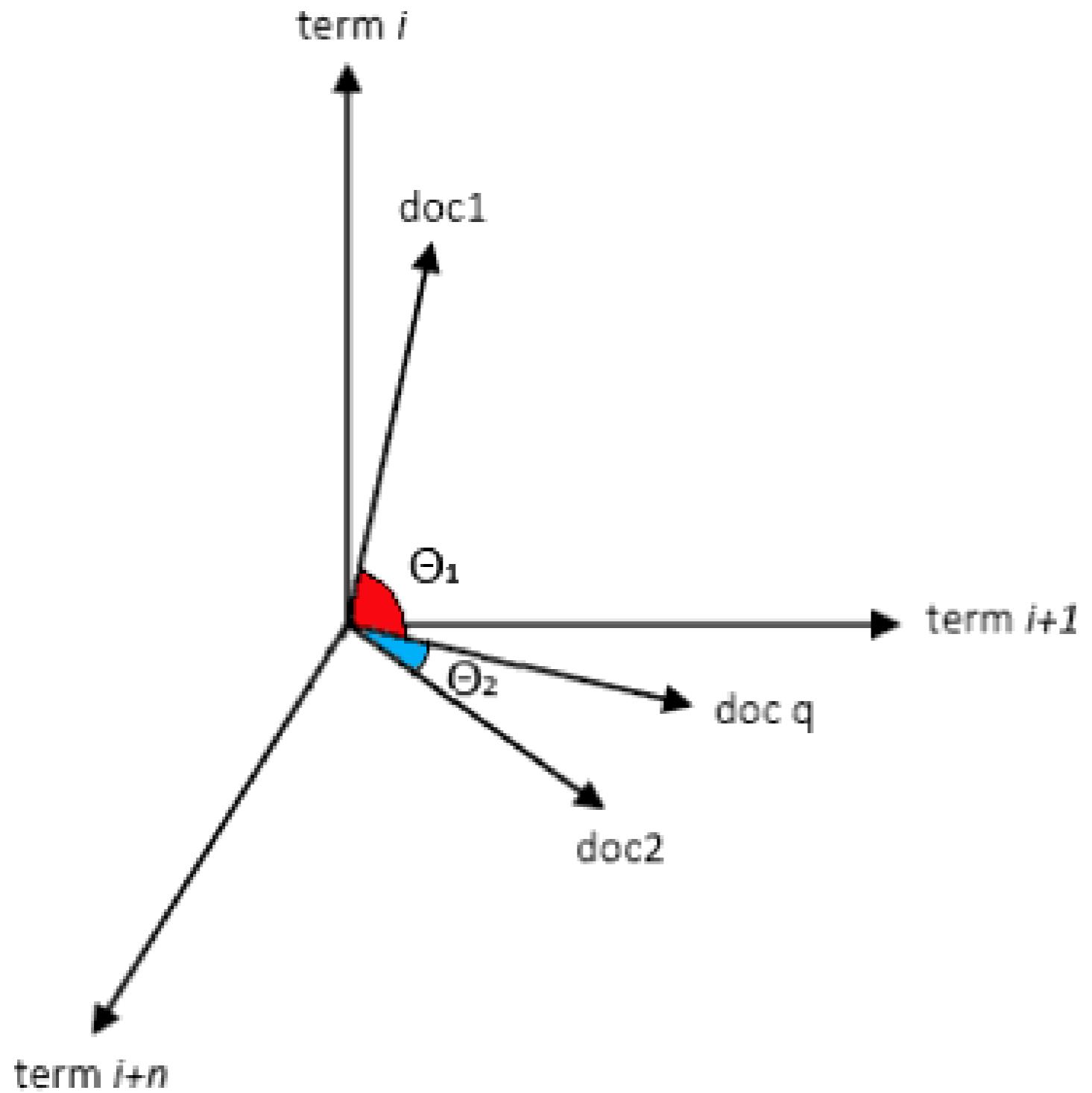
All “neighbors” are
“equally” far away

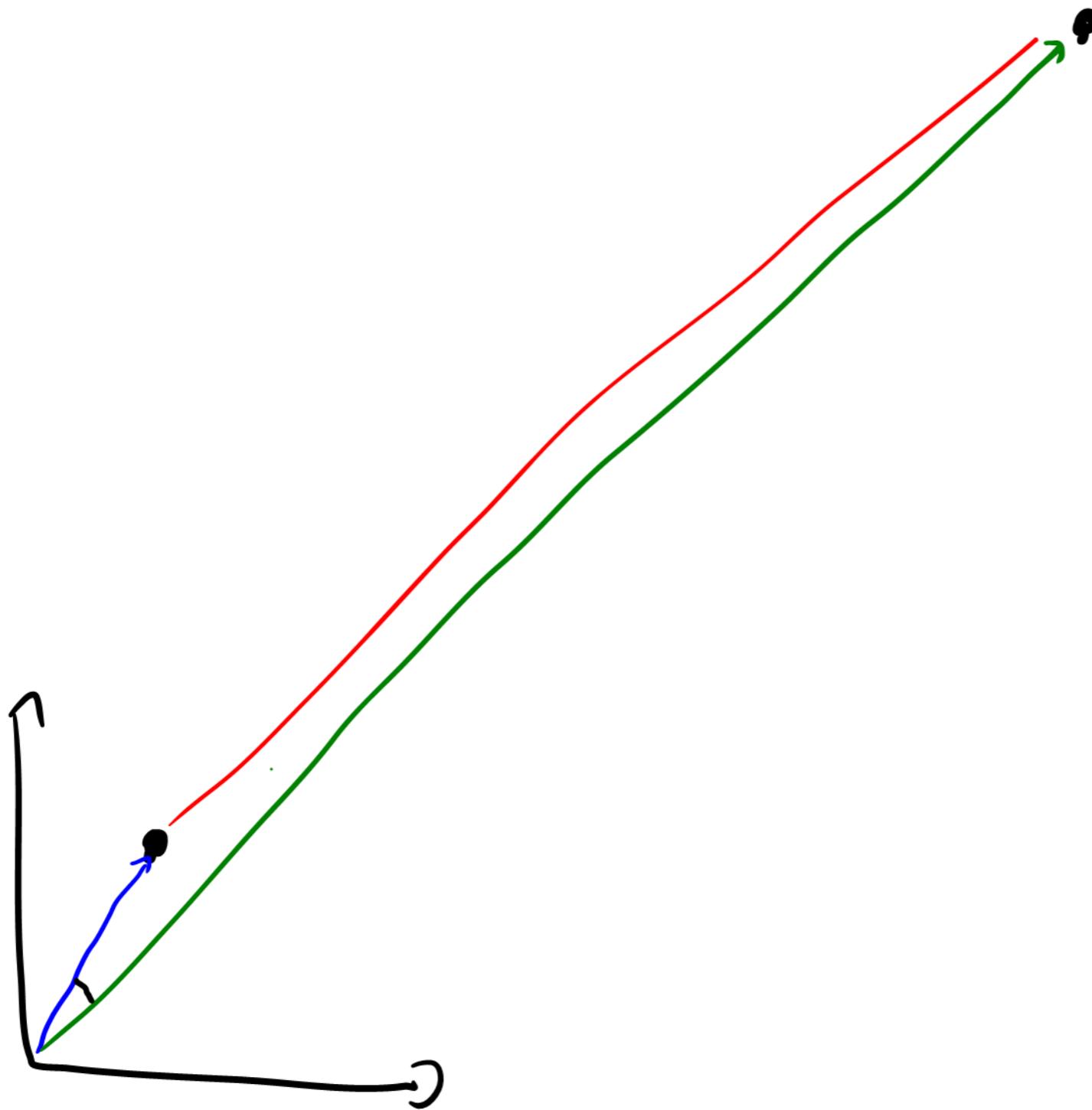


All “neighbors” are many
very far away



Cosine Distance

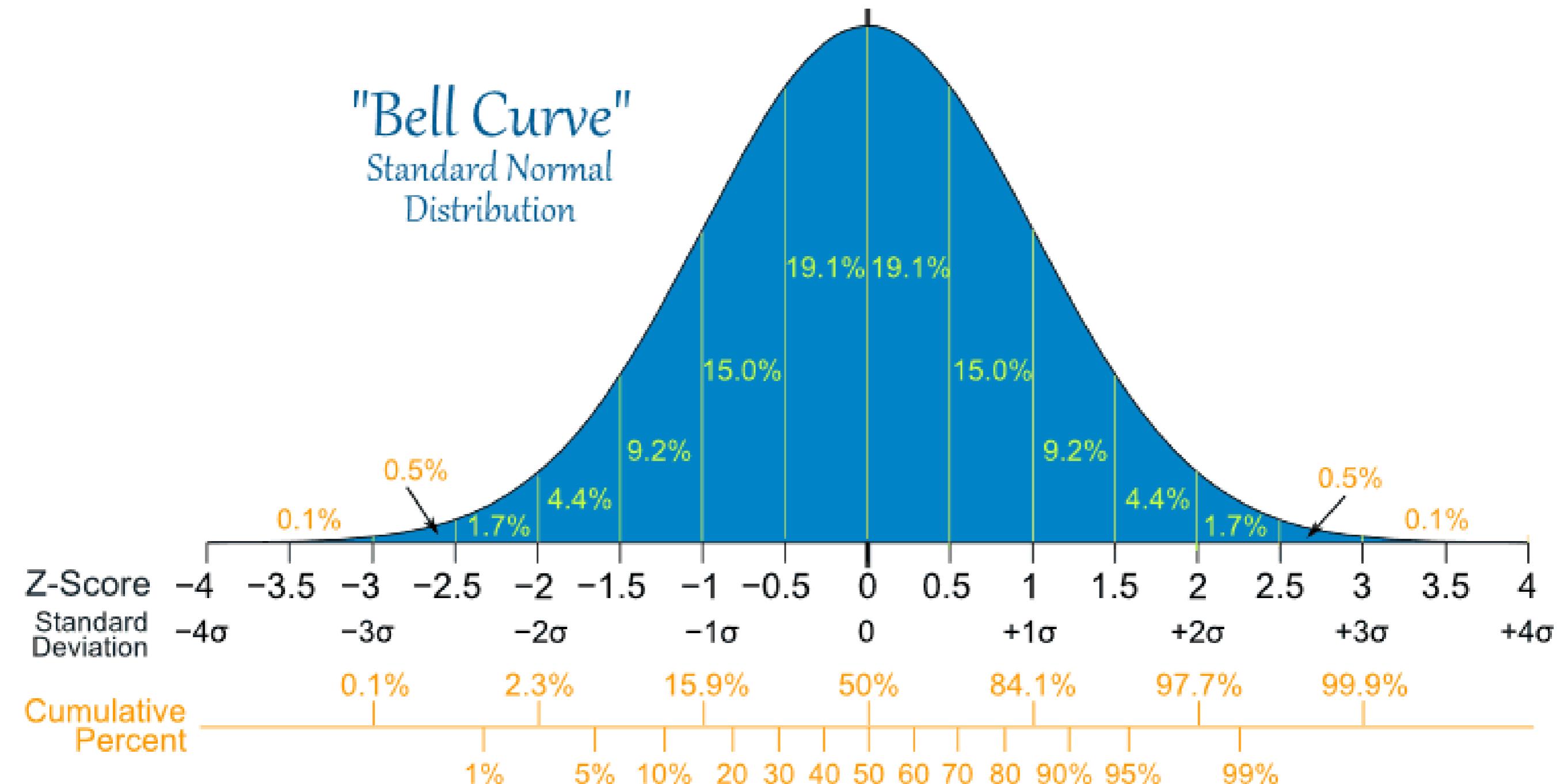


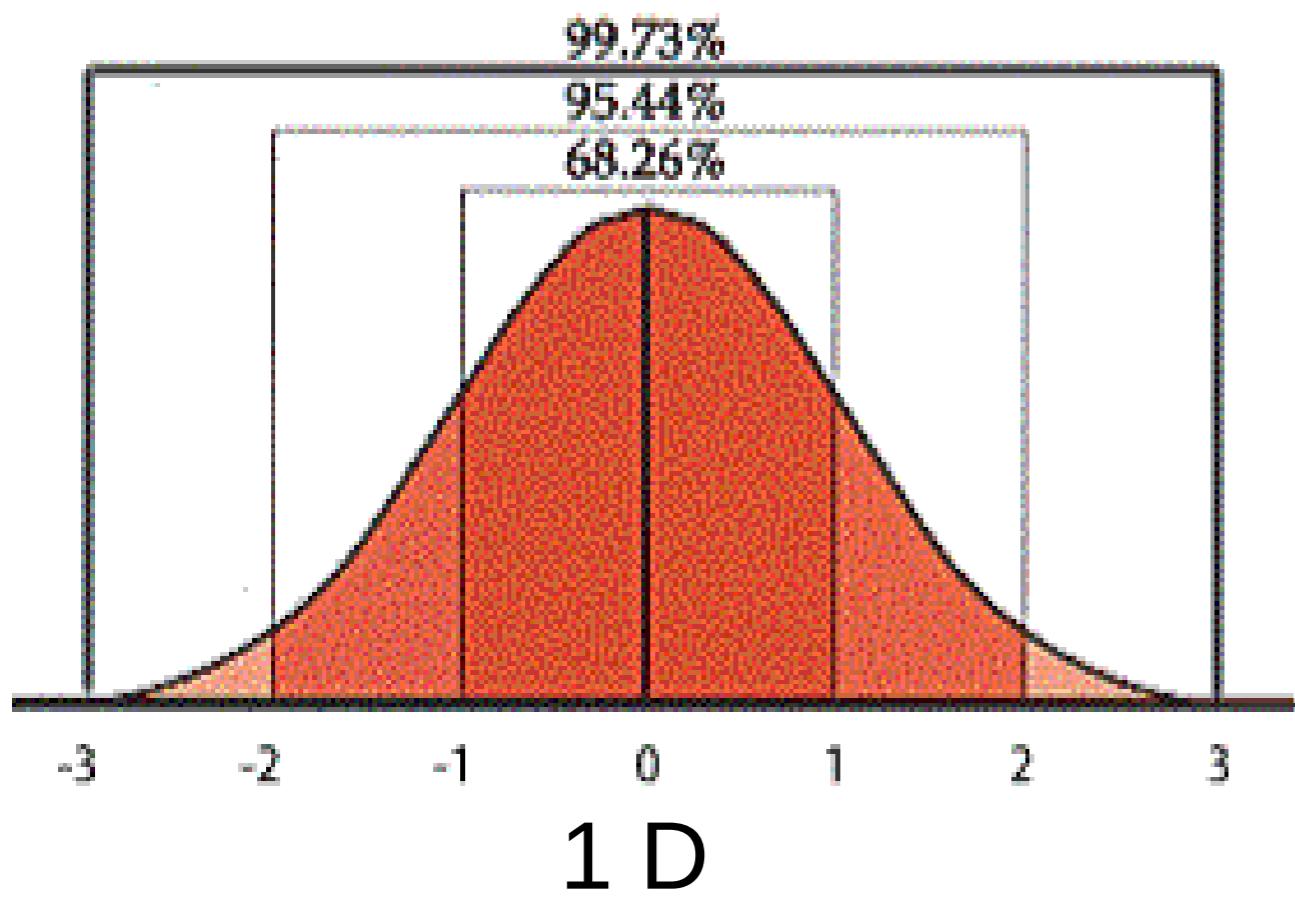


What does random mean in high-dimensions?

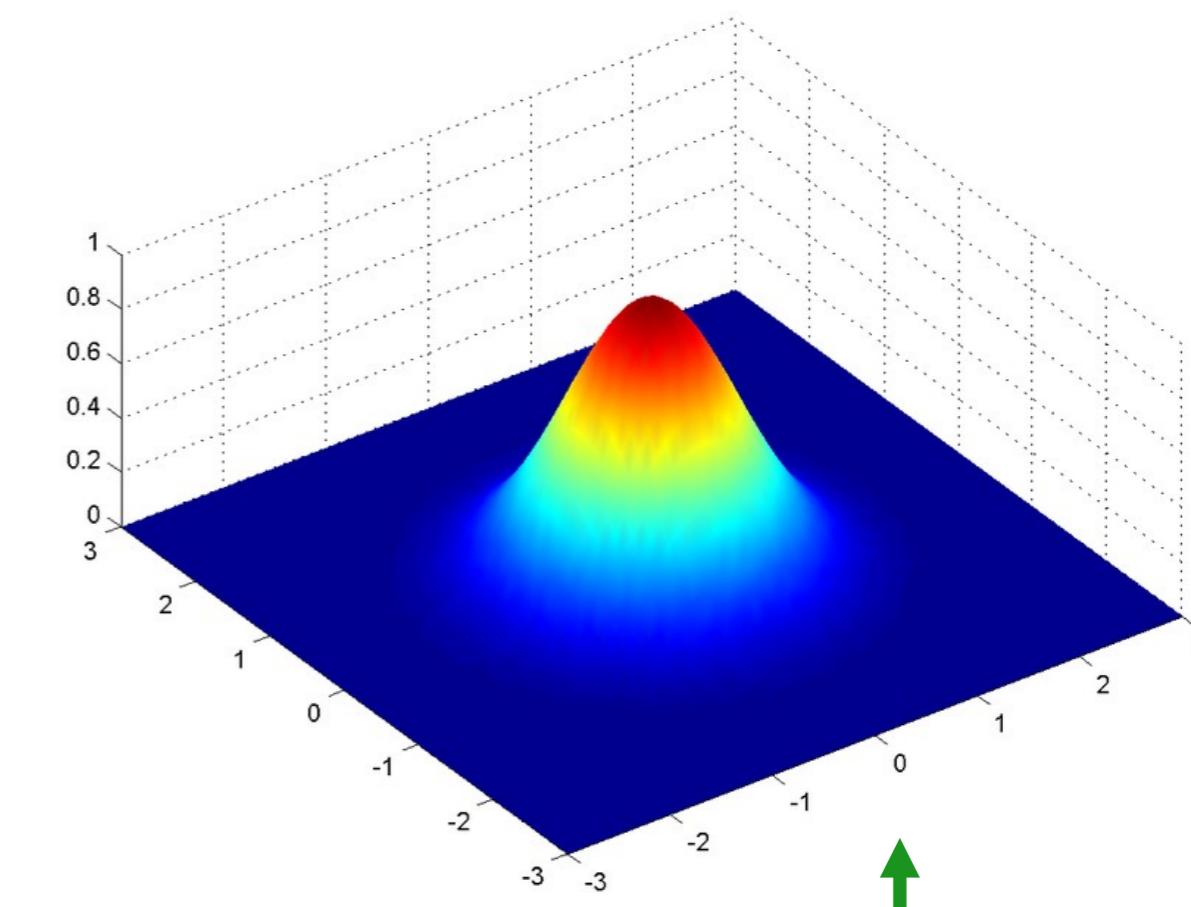
Question #3:

Gaussian Distribution (Center v.s. Surface)

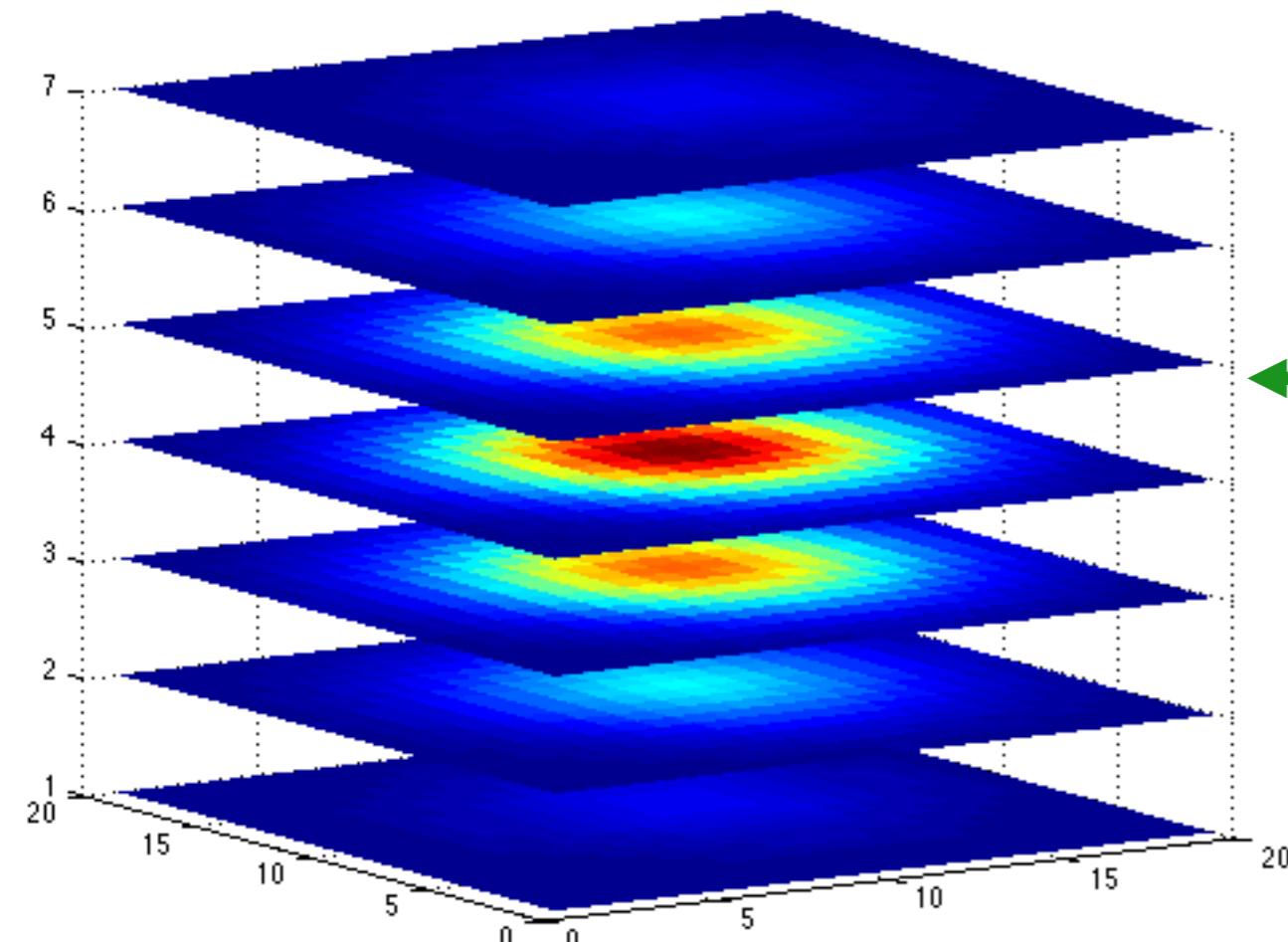




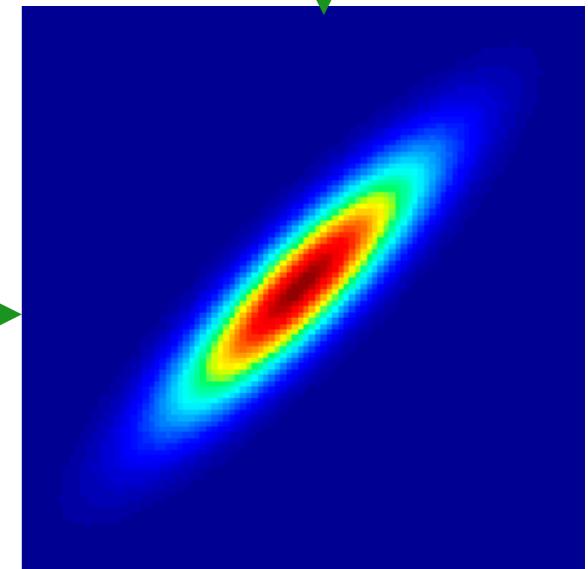
1 D



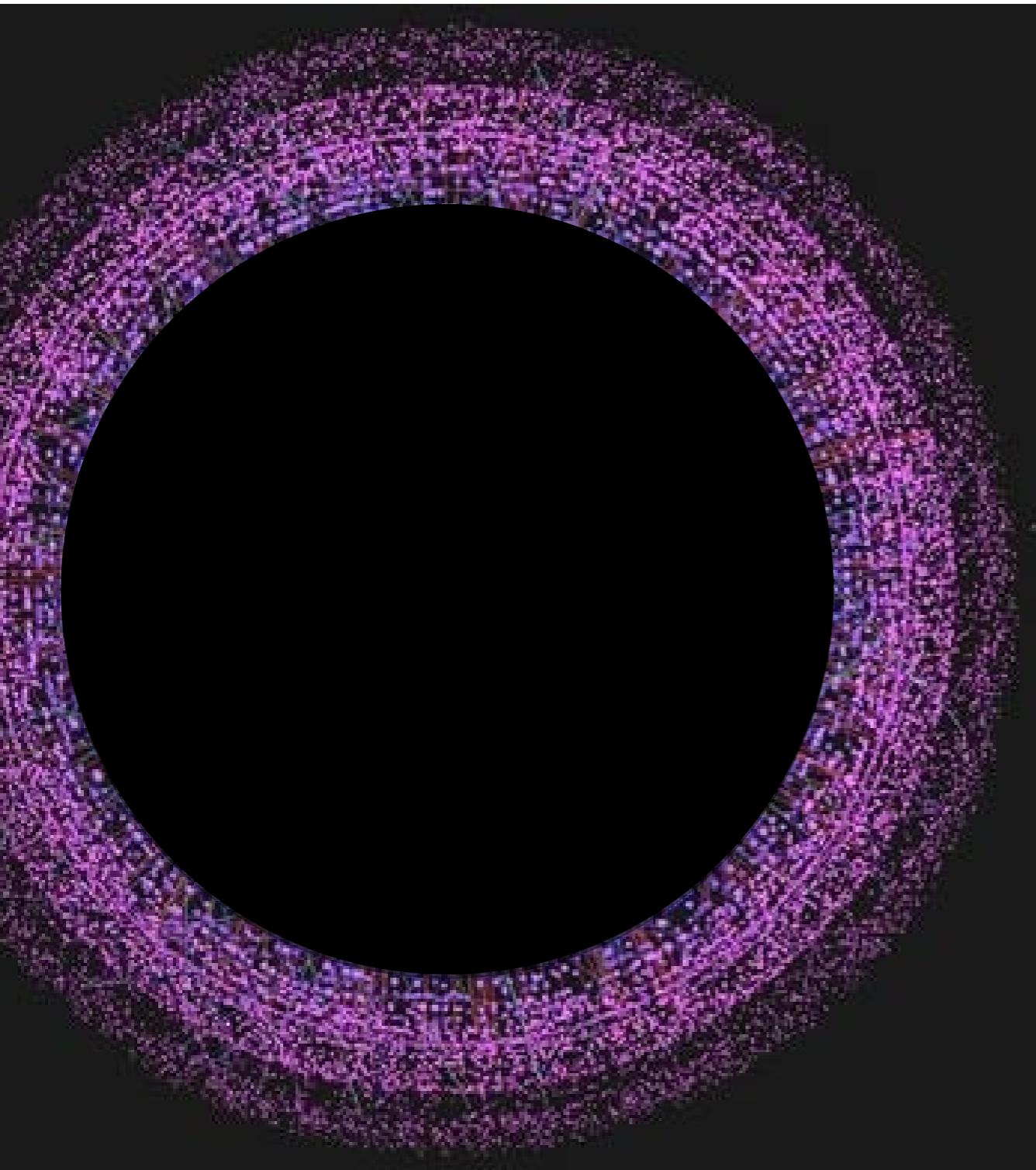
2 D



3 D



High Dimensional Gaussian Distribution



Question #3: Gaussian Distribution (Center v.s. Surface)

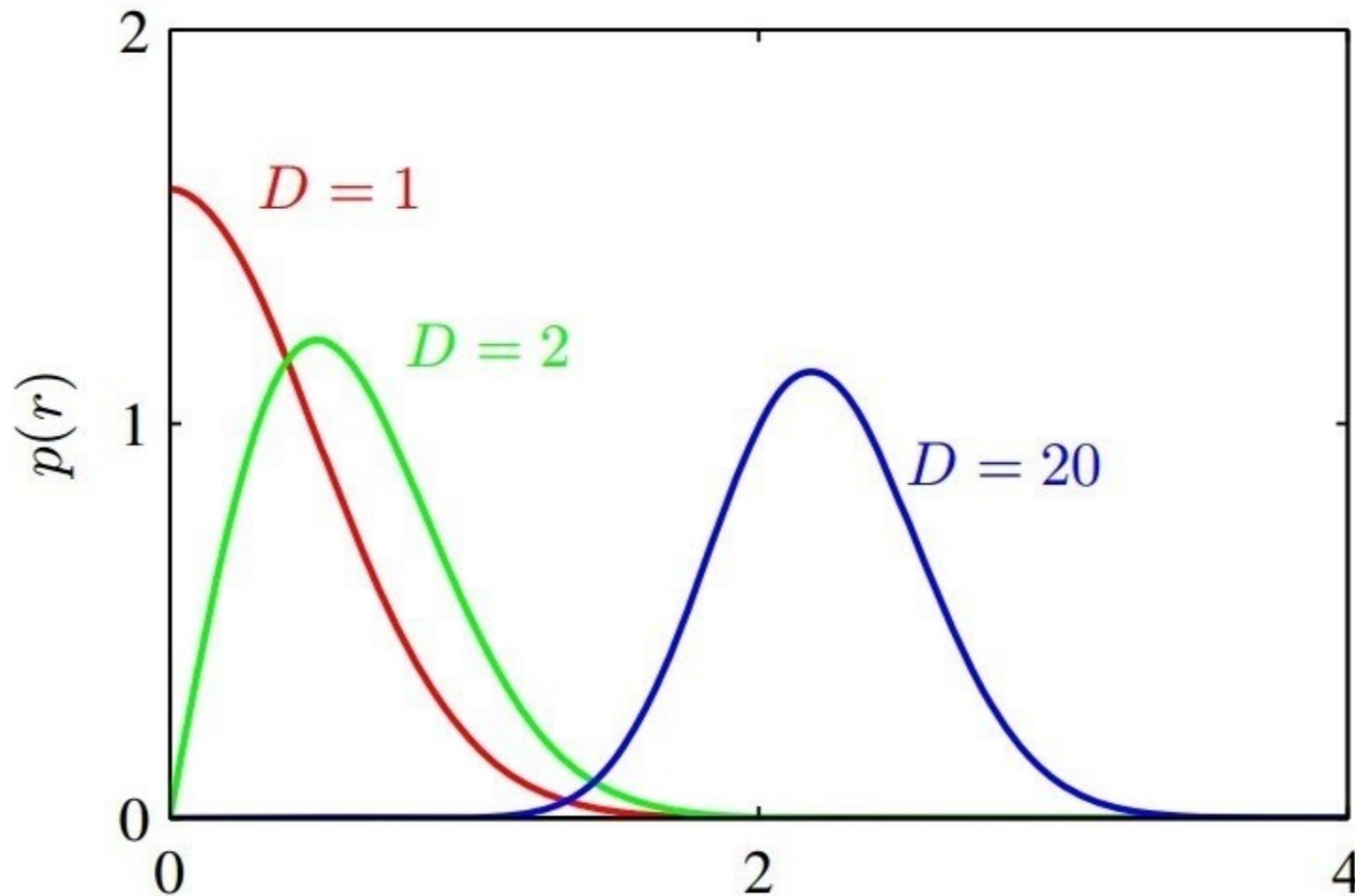
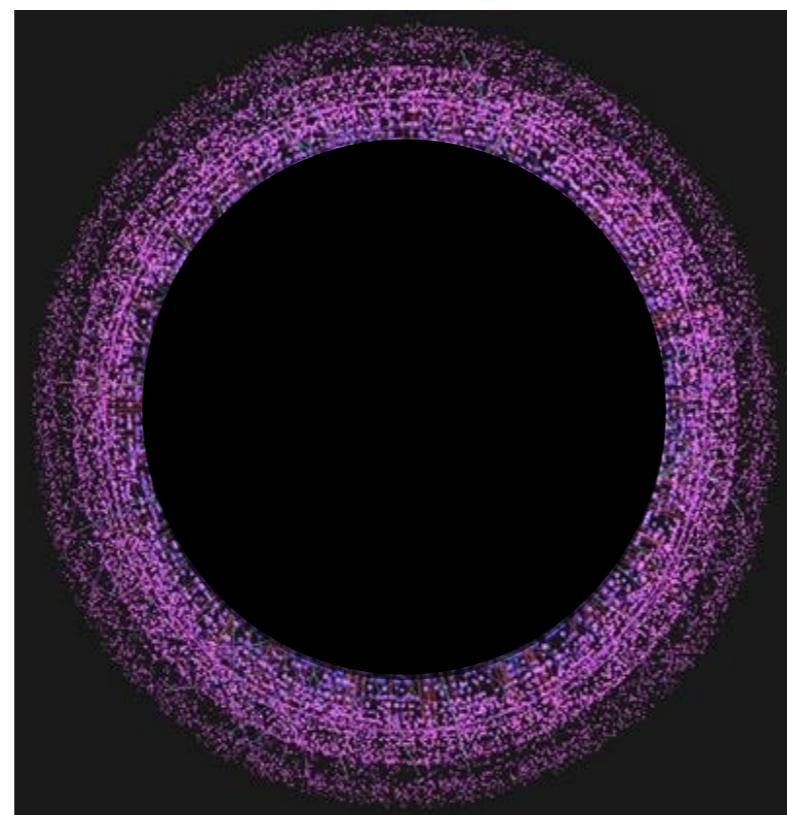


Figure 1.23 Plot of the probability density with respect to radius r of a Gaussian distribution for various values of the dimensionality D . In a high-dimensional space, most of the probability mass of a Gaussian is located within a thin shell at a specific radius.

In High Dimensional Space:

Rule # 3

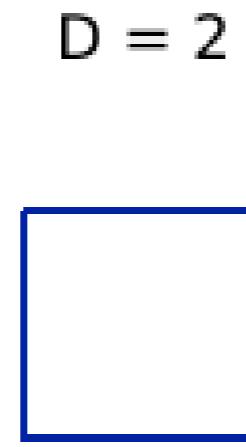
- All data points are on the surface
 - By chance, data will never reach the inner space of the ball.



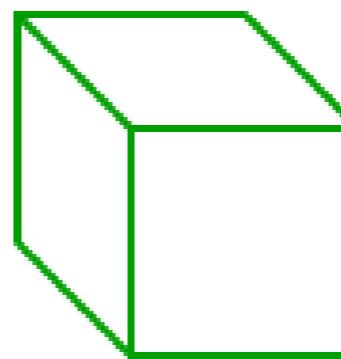
What does data mean in high-dimensions?

Question # 4: do we have enough data

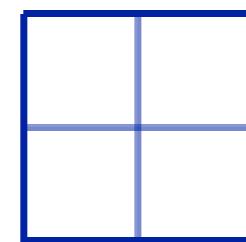
$D = 1$
—



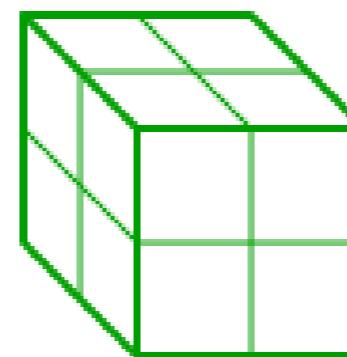
$D = 3$



$r = 2$
—
 $N = 2$

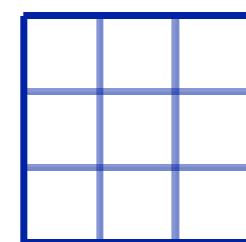


$N = 4$

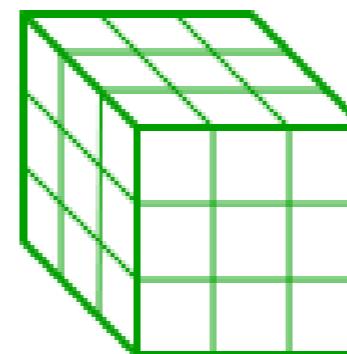


$N = 8$

$r = 3$
—
 $N = 3$



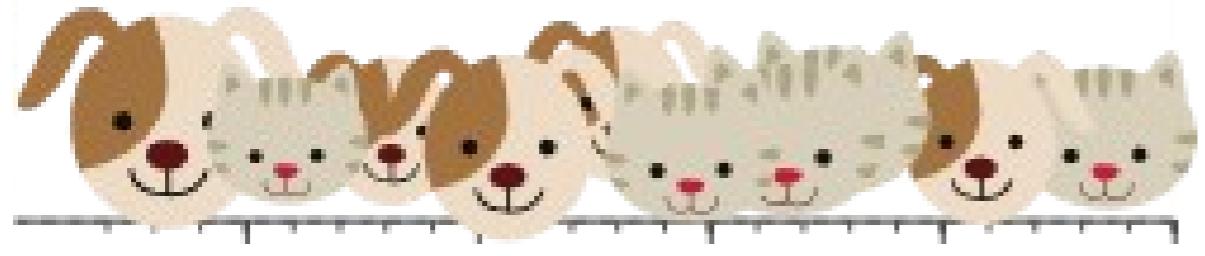
$N = 9$



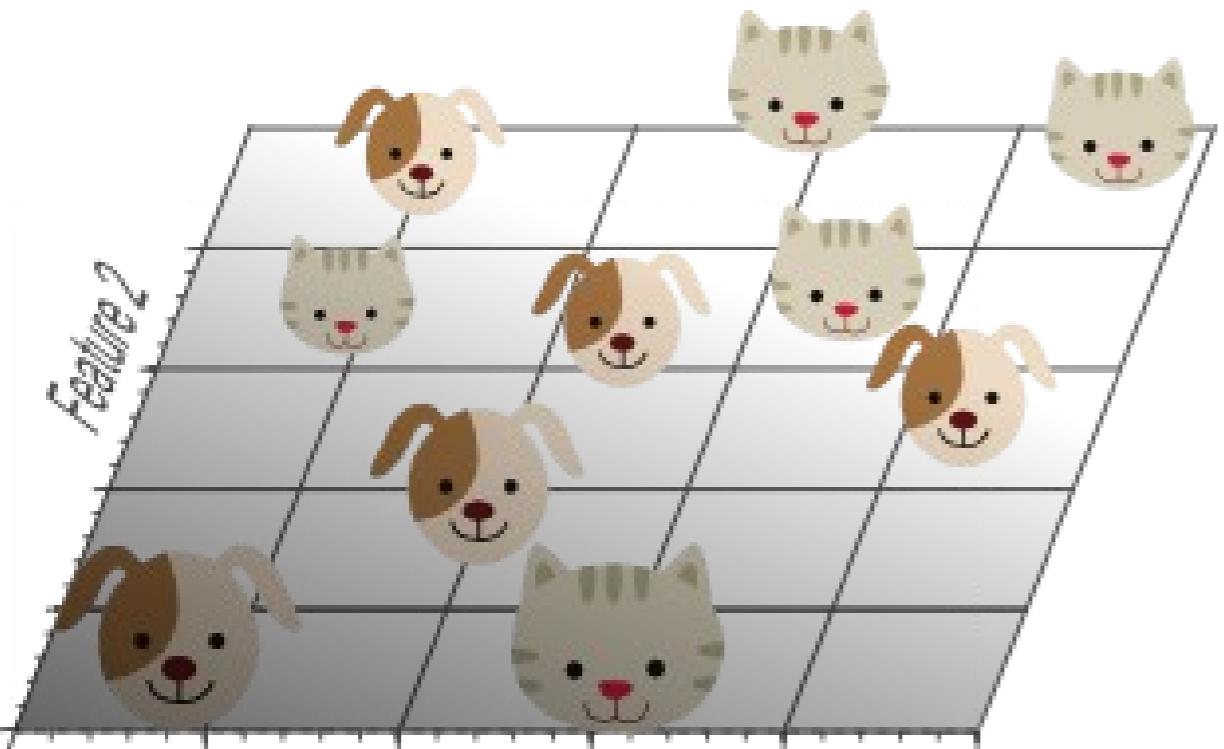
$N = 27$

Number of
“Rooms”

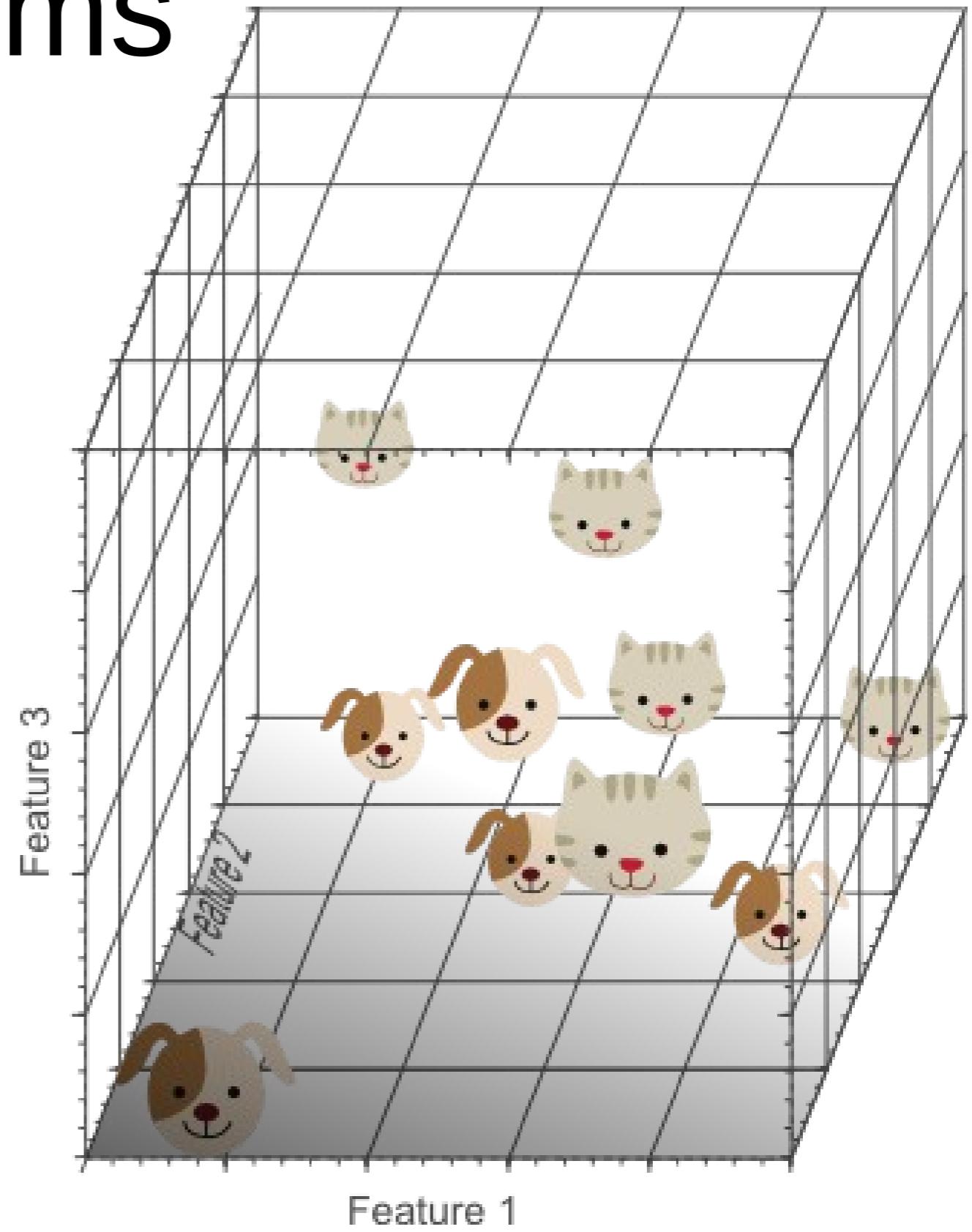
You need more data to fill the rooms



Feature 1



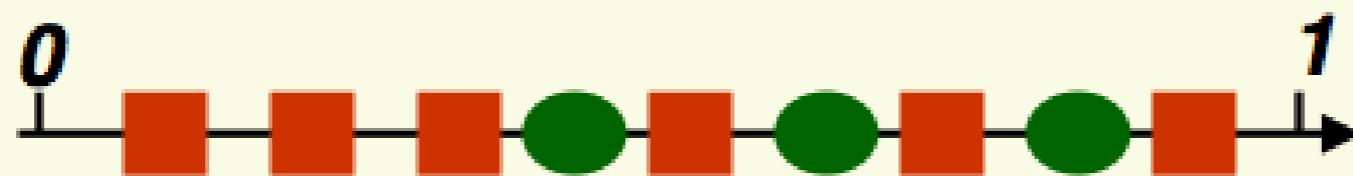
Feature 1



Feature 1

Data become sparser

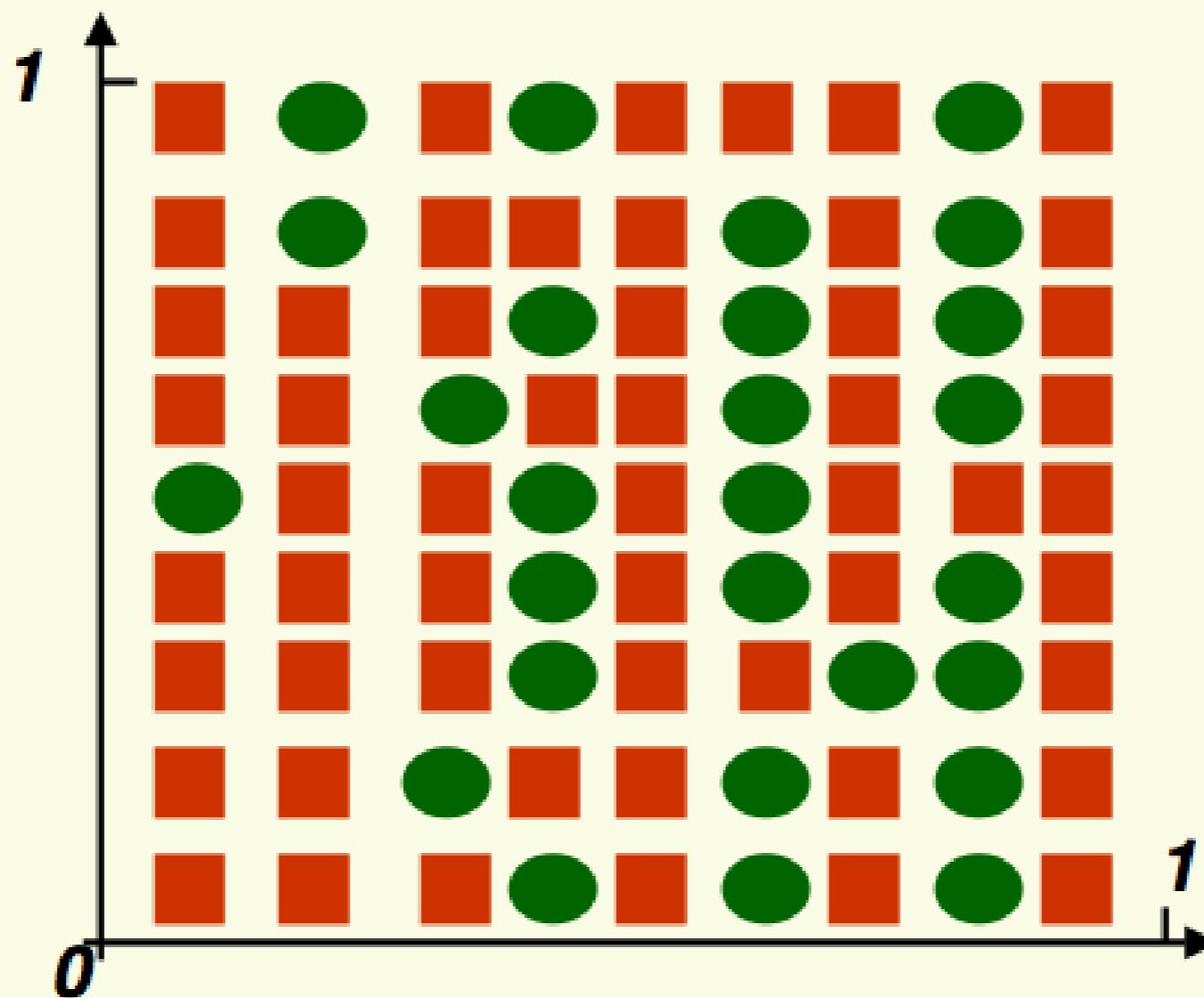
- Suppose we want to use the nearest neighbor approach with $k = 1$ (**1NN**)
- Suppose we start with only one feature



- This feature is not discriminative, i.e. it does not separate the classes well
- We decide to use 2 features. For the 1NN method to work well, need a lot of samples, i.e. samples have to be dense
- To maintain the same density as in 1D (9 samples per unit length), how many samples do we need?

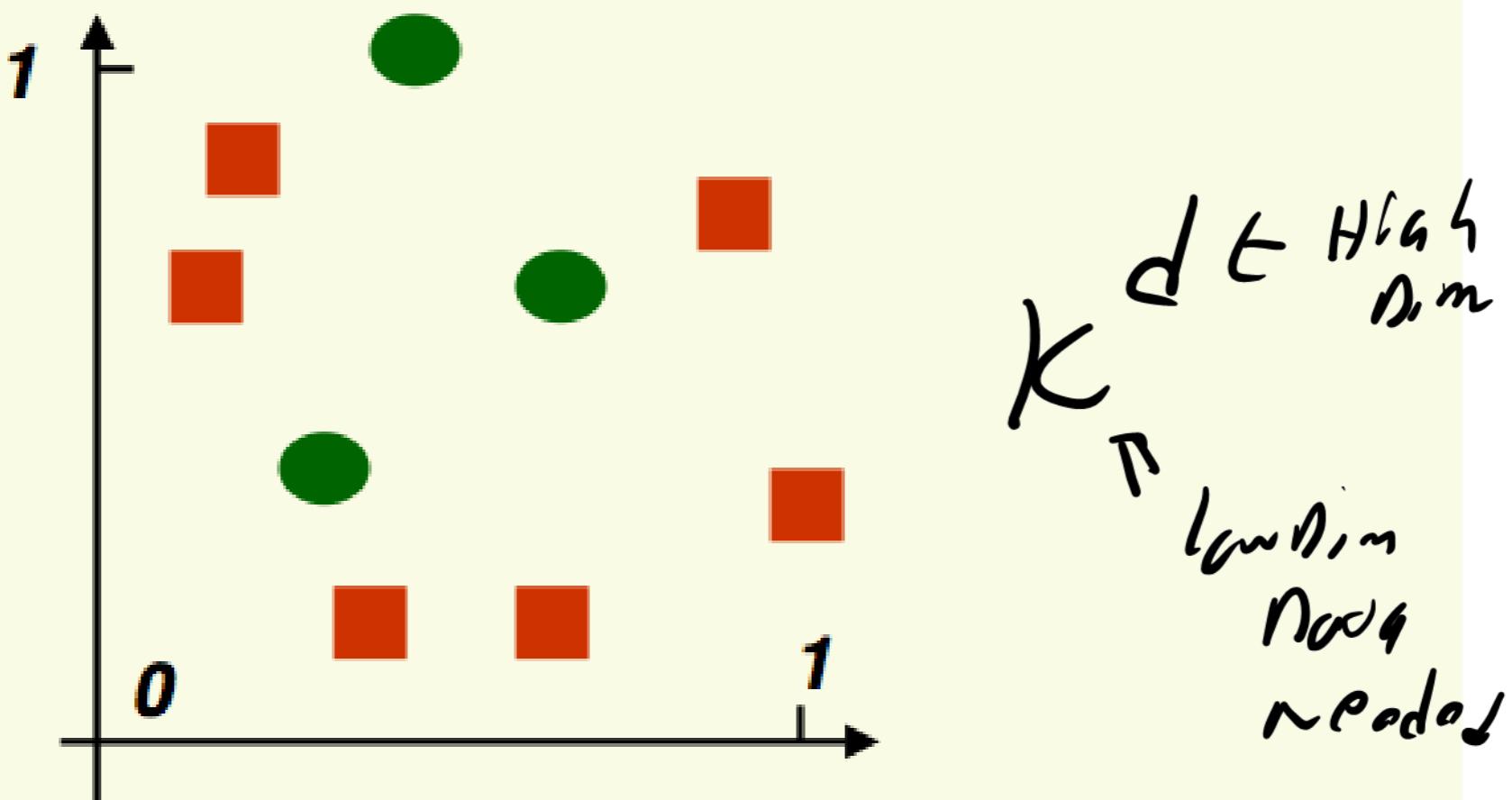
Data become Sparser: Number of Samples

- We need 9^2 samples to maintain the same density as in $1D$



Curse of Dimensionality: Number of Samples

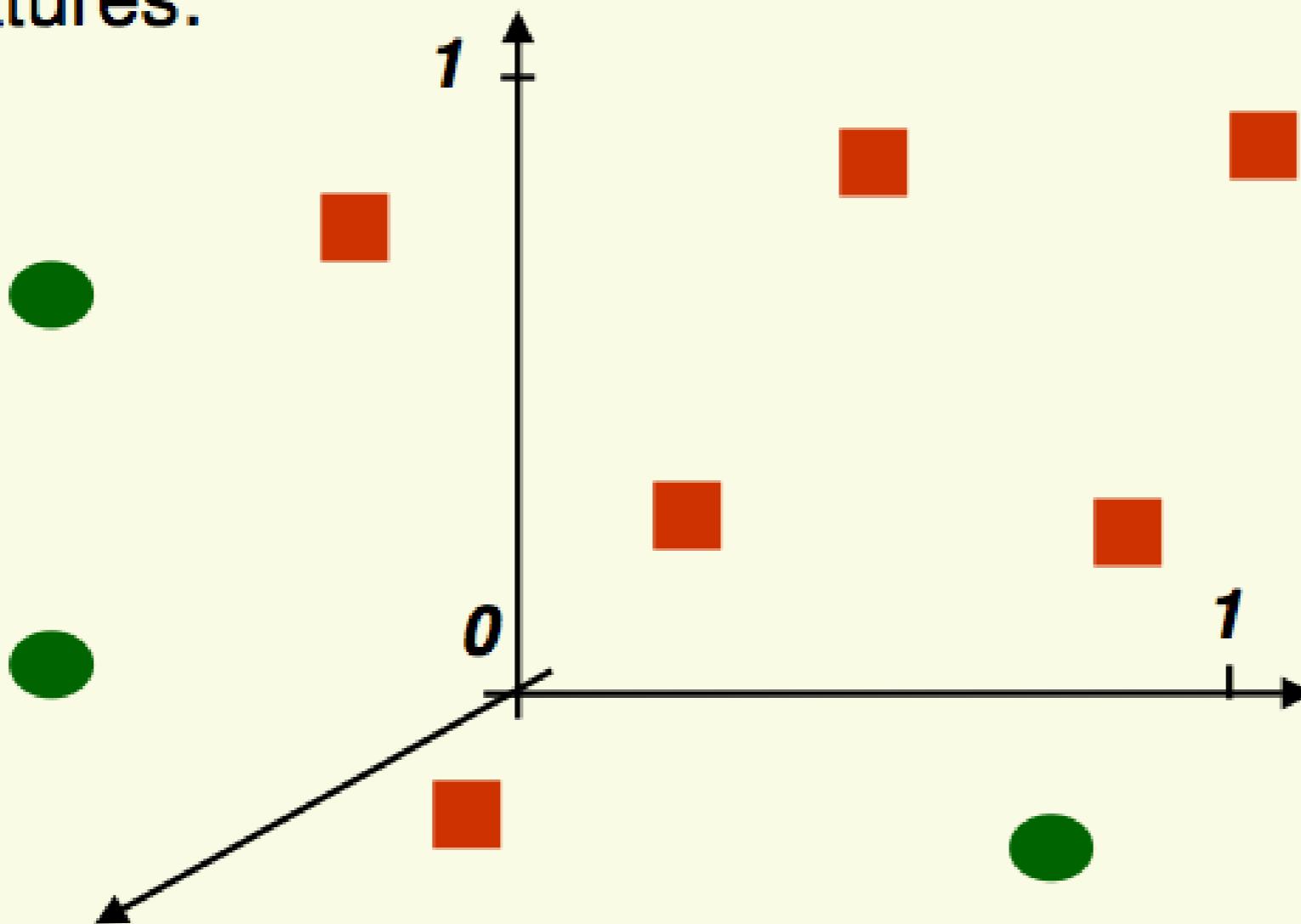
- Of course, when we go from 1 feature to 2, no one gives us more samples, we still have 9



- This is way too sparse for 1NN to work well

Curse of Dimensionality: Number of Samples

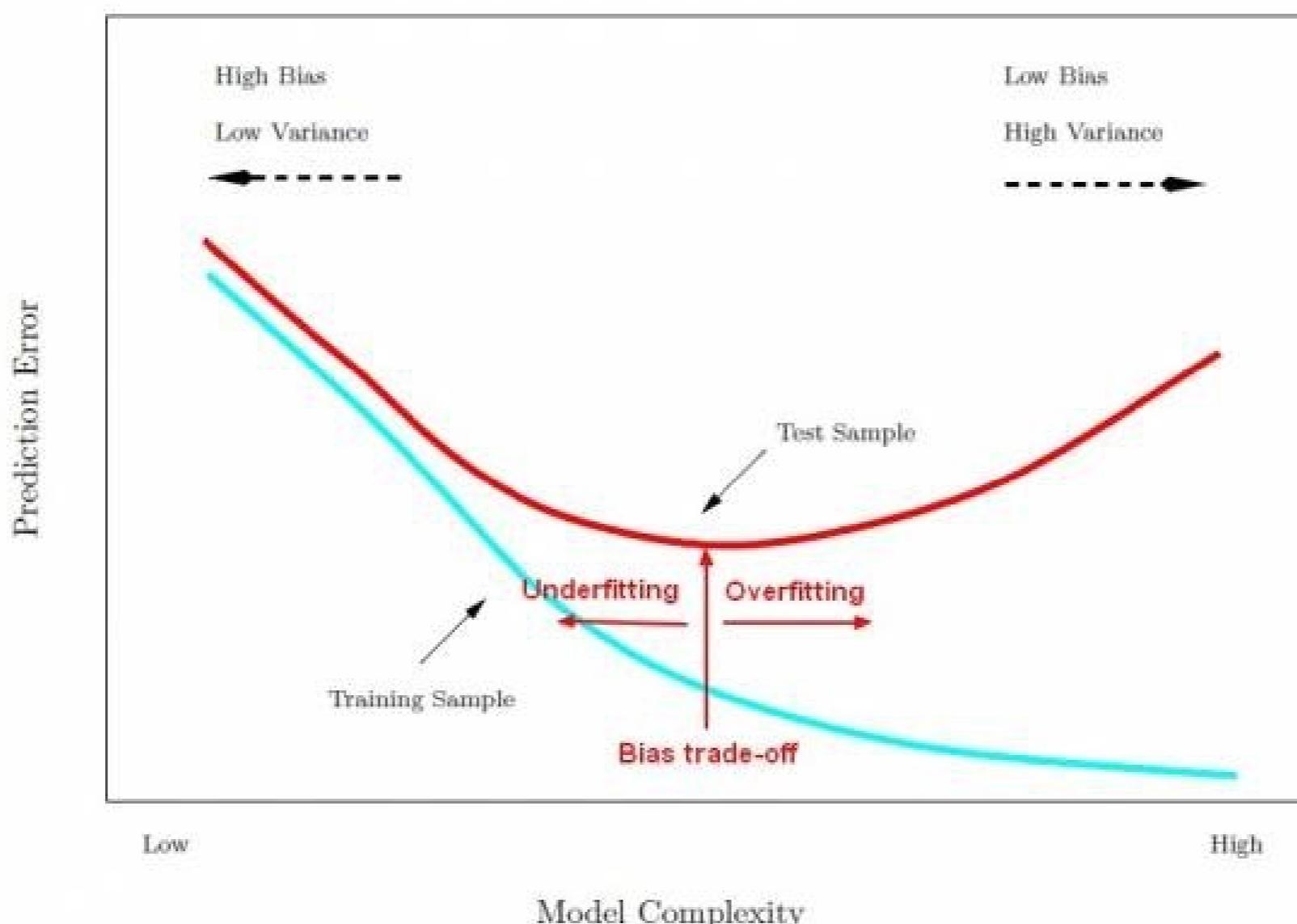
- Things go from bad to worse if we decide to use 3 features:



- If 9 was dense enough in 1D, in 3D we need $9^3=729$ samples!

Rule #4: need **MORE** data to train your model

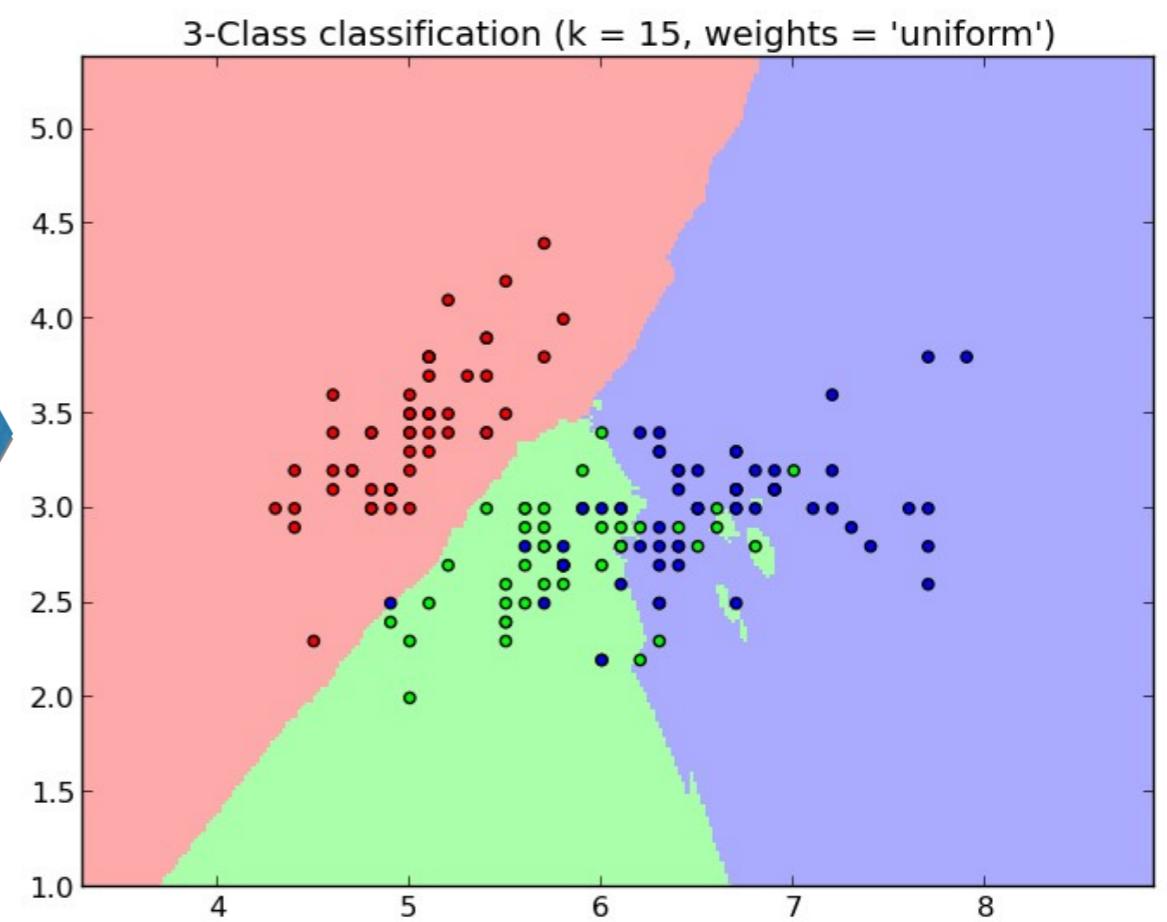
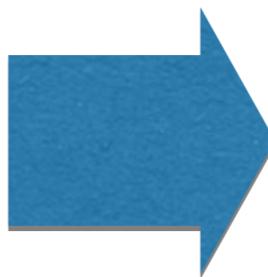
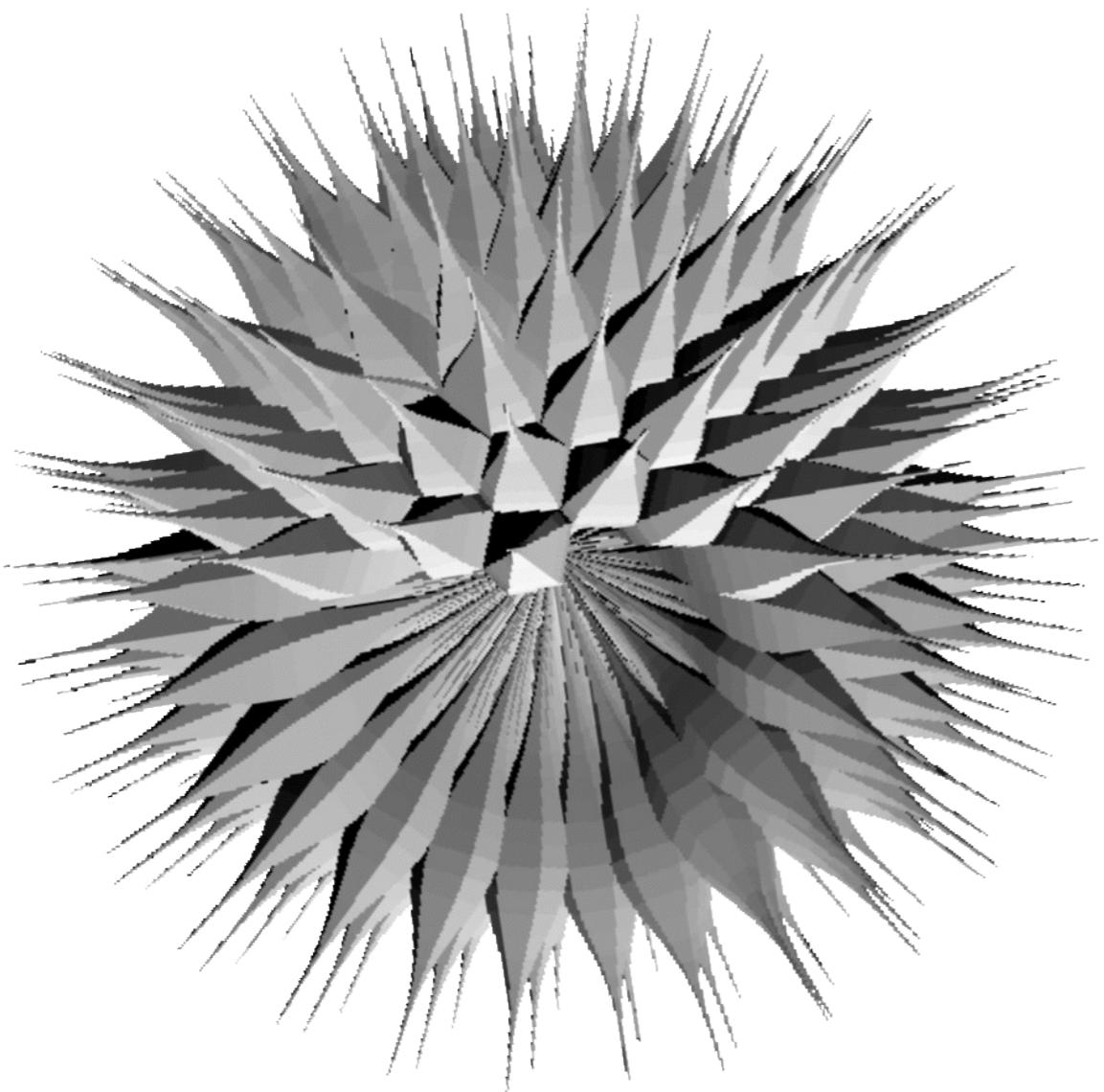
- models tend to **overfit** with a “small” amount of data



Curse of Dimensionality: Number of Samples

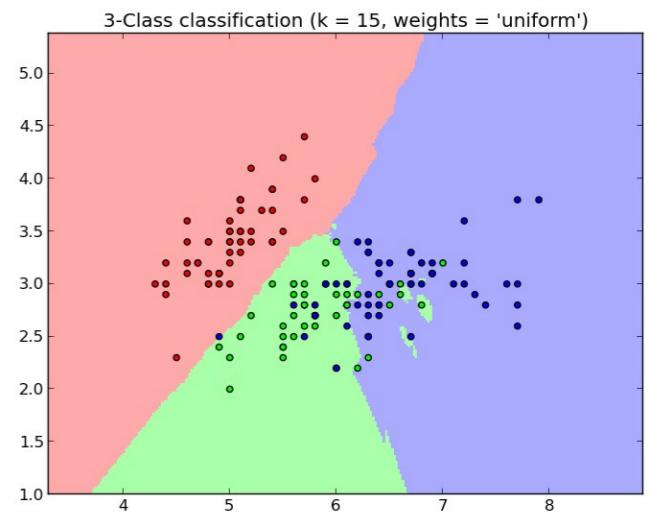
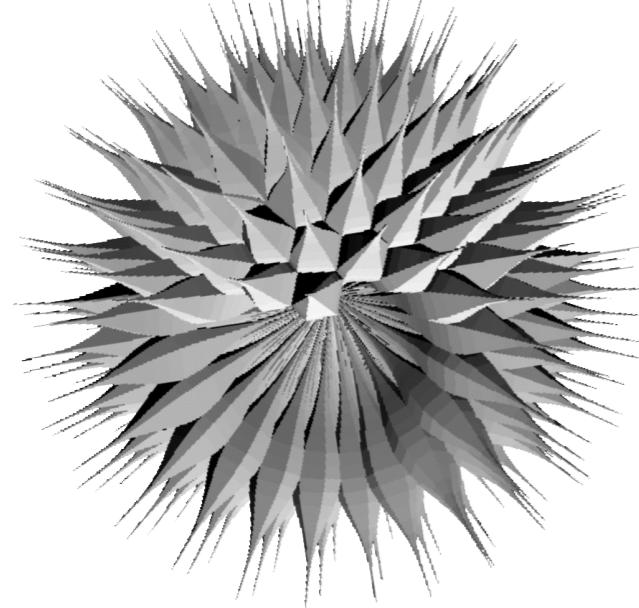
- In general, if n samples is dense enough in $1D$
- Then in d dimensions we need n^d samples!
- And n^d grows really really fast as a function of d
- Common pitfall:
 - If we can't solve a problem with a few features, adding more features seems like a good idea
 - However the number of samples usually stays the same
 - The method with more features is likely to perform worse instead of expected better

**Now you know what is really happening
in High Dimensional Space**



Real world

Perfect world



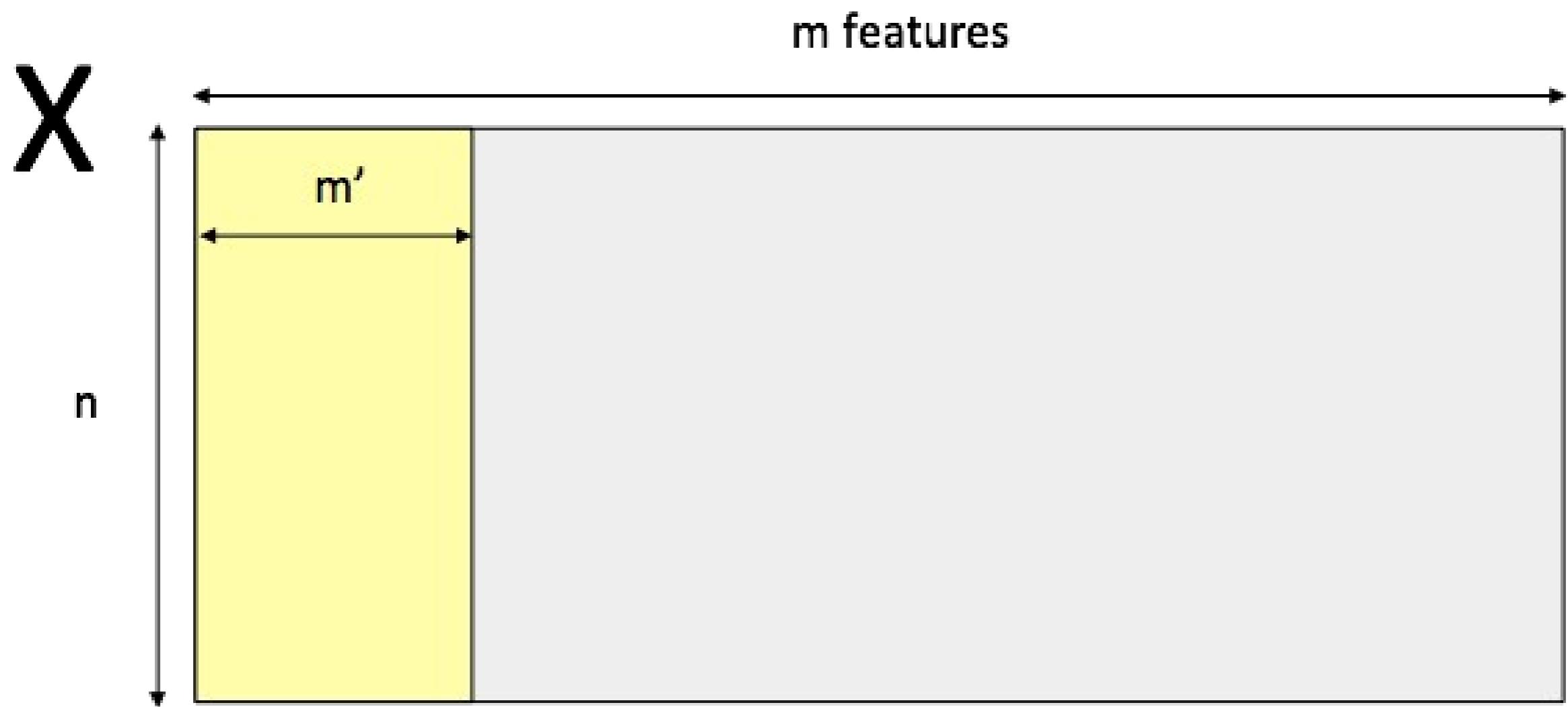
■ Feature Selection

- Filtering approach
- Wrapper approach
- Embedded methods

■ Dimensionality Reduction

- Principal Components Analysis (PCA)
- Nonlinear PCA (Kernel PCA, CatPCA)
- Multi-Dimensional Scaling (MDS)
- Homogeneity Analysis

Feature Selection



Feature Selection

- **Filtering approach:**

ranks features or feature subsets independently of the predictor (classifier).

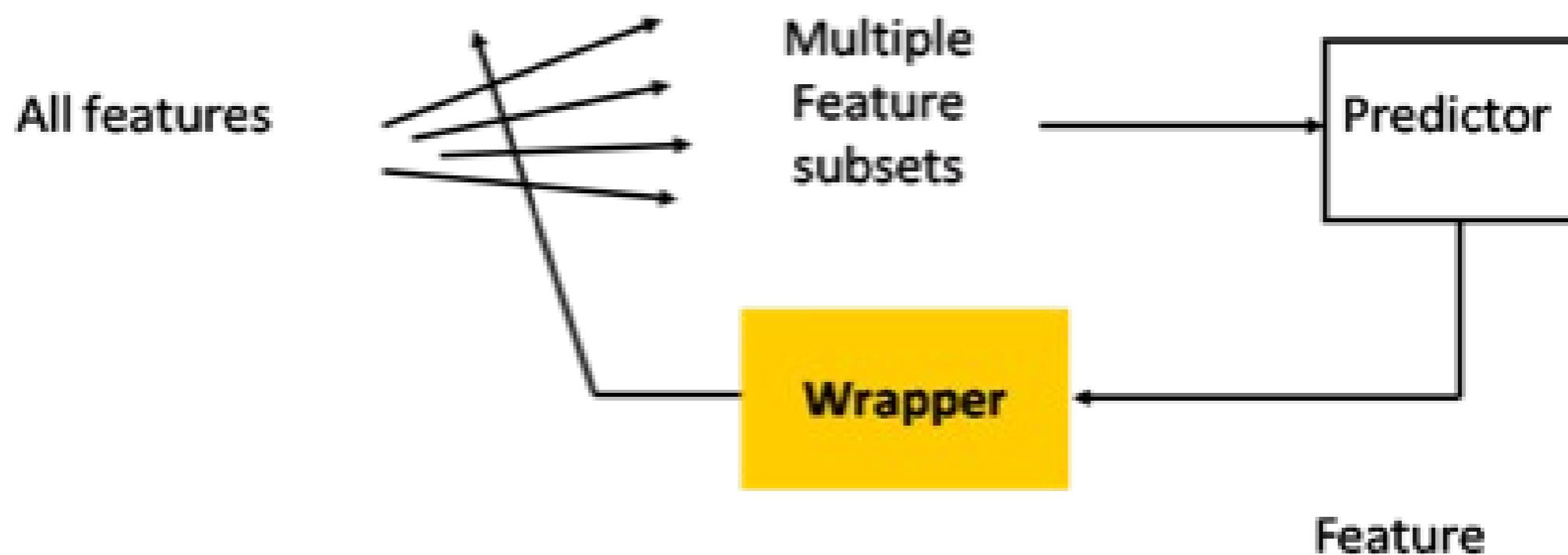
- ...using univariate methods: consider **one** variable at a time
- ...using multivariate methods: consider **more than one** variables at a time

- **Wrapper approach:**

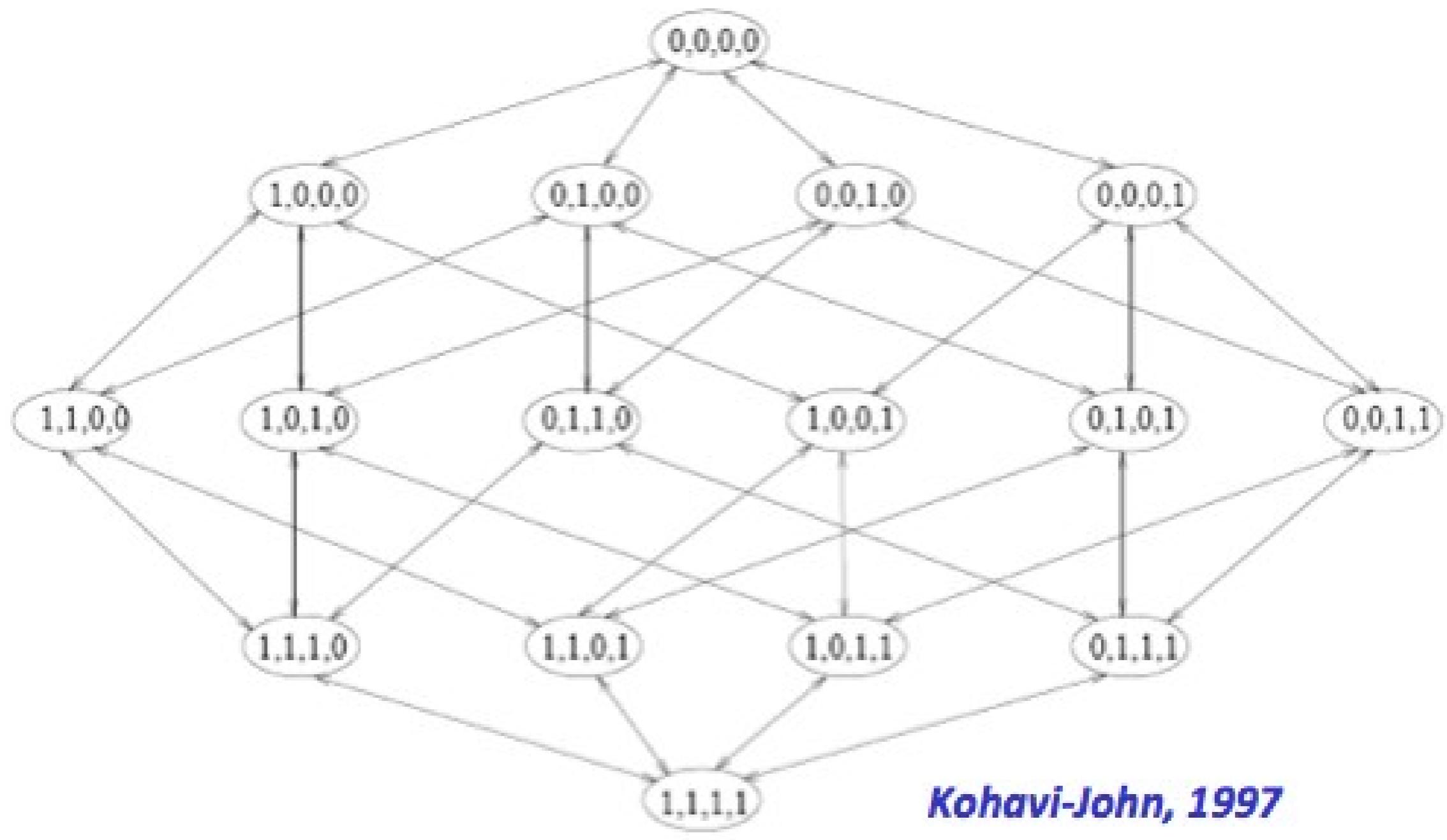
uses a classifier to assess (many) features or feature subsets.

Feature selection: Filter vs. Wrapper methods

- Main goal: rank subsets of useful features



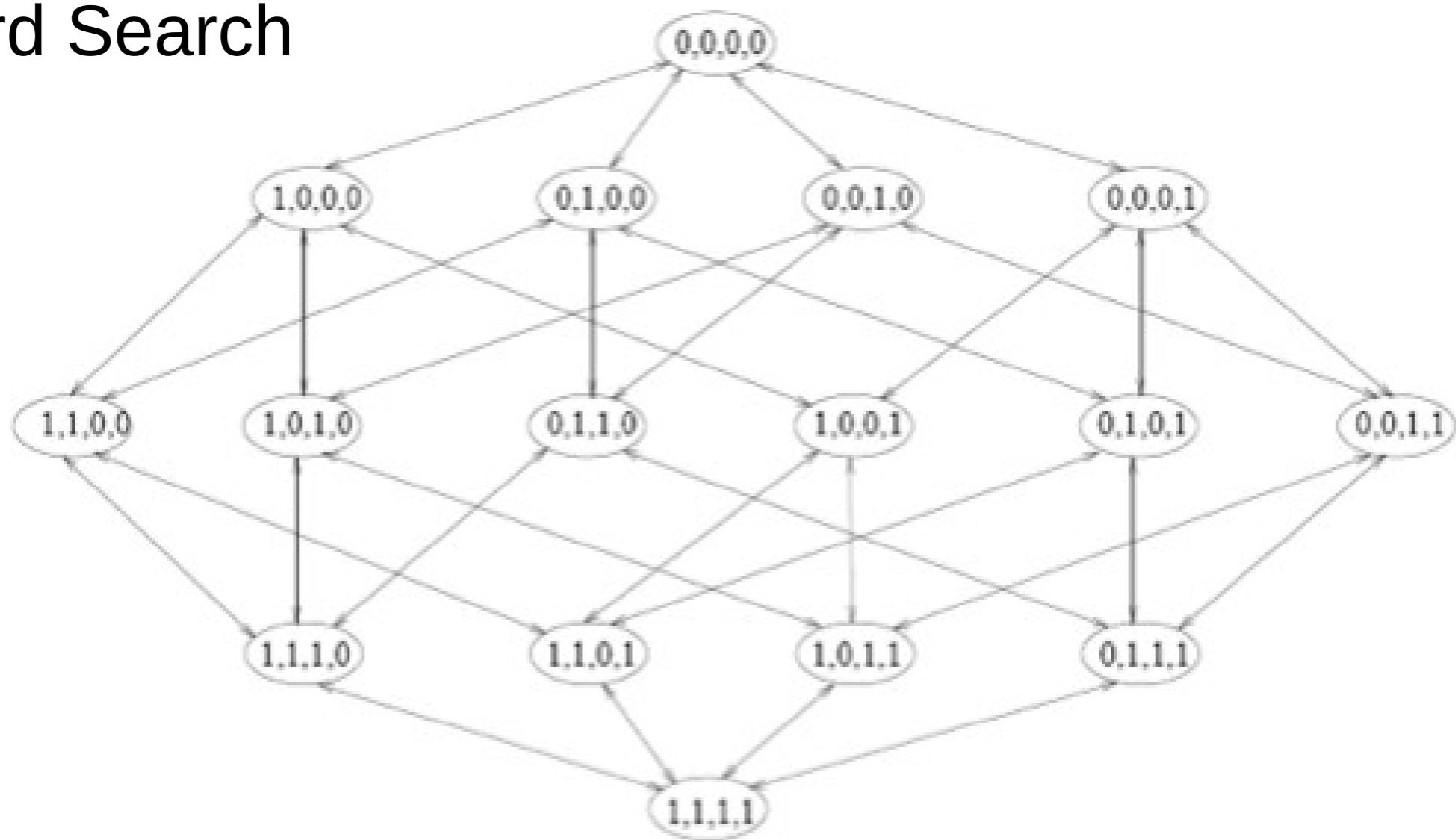
Feature Selection



N features, 2^N possible feature subsets!

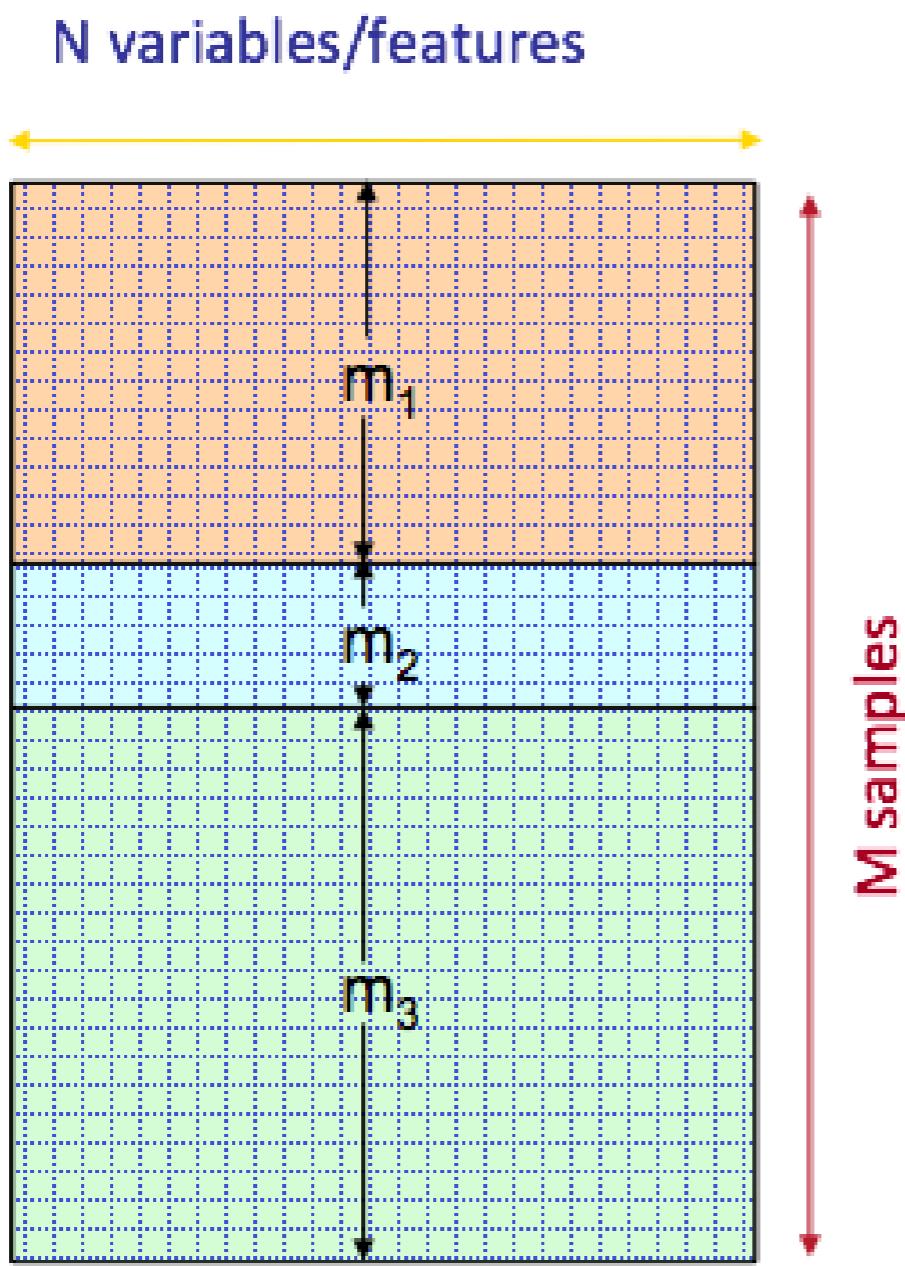
Search Strategies

Forward Search



Backward Search

Wrapper methods



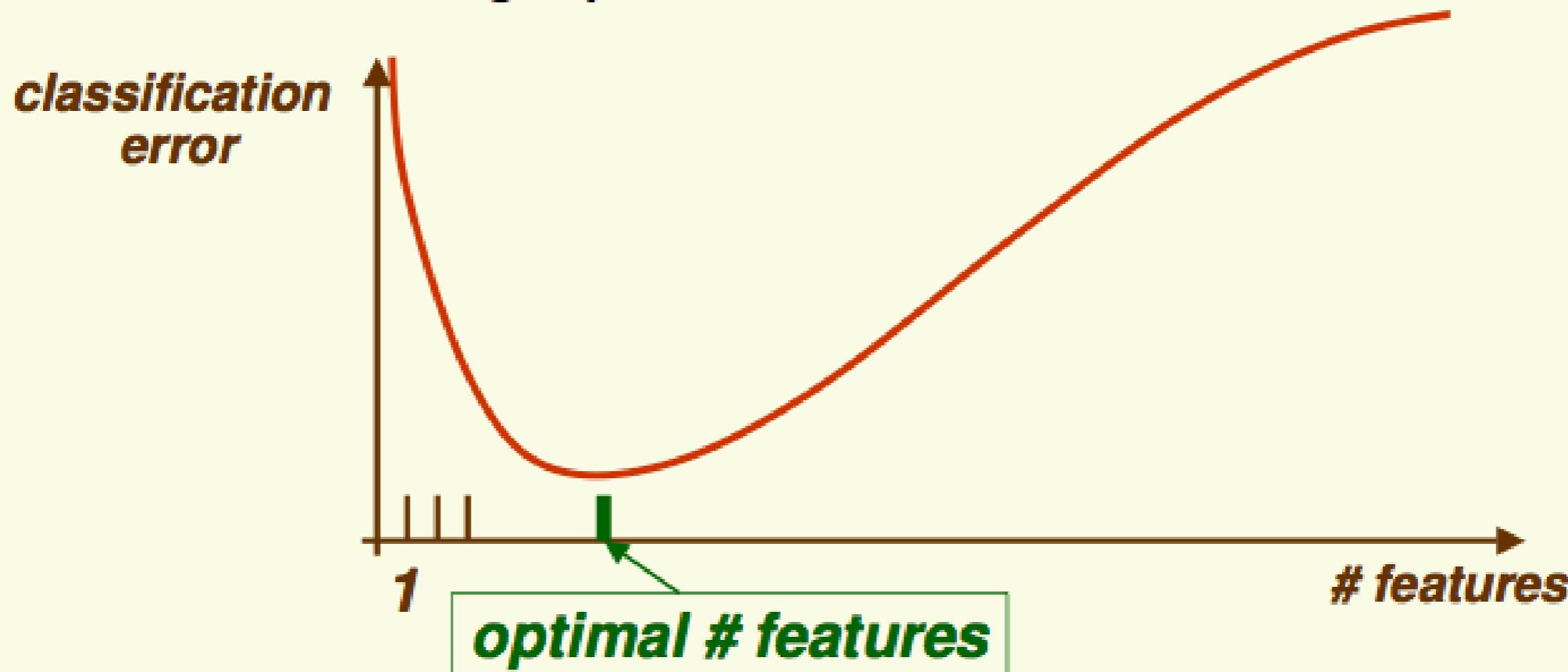
Split data into 3 sets:
training, **validation**, and **test set**.

- 1) For each feature subset, train predictor on **training data**.
- 2) Select the feature subset, which performs best on **validation data**.
 - Repeat and average if you want to reduce variance (cross-validation).
- 3) Test on **test data**.

Danger of over-fitting with intensive search!

Over-fitting: Number of Samples

- For a fixed number of samples, as we add features, the graph of classification error:



- Thus for each fixed sample size n , there is the optimal number of features to use

Dimensionality Reduction

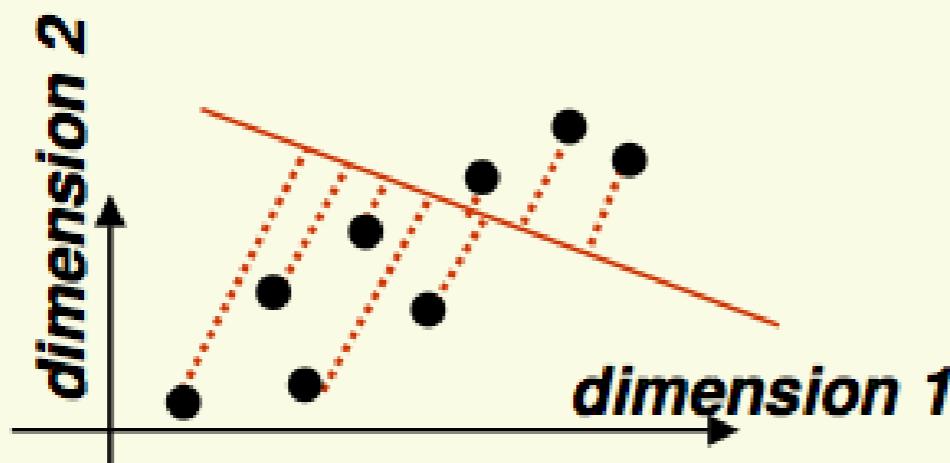
- High dimensionality is challenging and redundant
- It is natural to try to reduce dimensionality
- Reduce dimensionality by feature combination:
combine old features \mathbf{x} to create new features \mathbf{y}

$$\mathbf{x} = \begin{bmatrix} \mathbf{x}_1 \\ \mathbf{x}_2 \\ \vdots \\ \mathbf{x}_d \end{bmatrix} \rightarrow f\left(\begin{bmatrix} \mathbf{x}_1 \\ \mathbf{x}_2 \\ \vdots \\ \mathbf{x}_d \end{bmatrix}\right) = \begin{bmatrix} \mathbf{y}_1 \\ \vdots \\ \mathbf{y}_k \end{bmatrix} = \mathbf{y} \quad \text{with } k < d$$

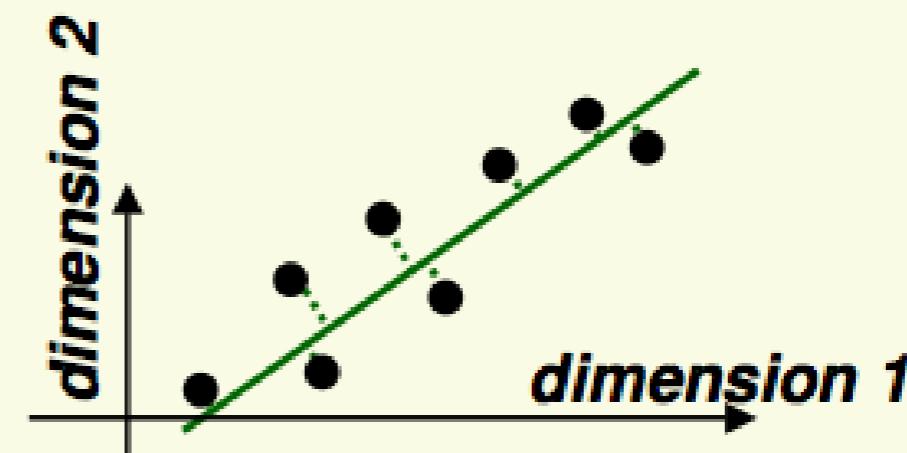
- For example,
- $$\mathbf{x} = \begin{bmatrix} \mathbf{x}_1 \\ \mathbf{x}_2 \\ \mathbf{x}_3 \\ \mathbf{x}_4 \end{bmatrix} \rightarrow \begin{bmatrix} \mathbf{x}_1 + \mathbf{x}_2 \\ \mathbf{x}_3 + \mathbf{x}_4 \end{bmatrix} = \mathbf{y}$$
- Ideally, the new vector \mathbf{y} should retain from \mathbf{x} all information important for classification

PCA: Principle Component Analysis

- **Main idea:** seek most accurate data representation in a lower dimensional space
- Example in 2-D
 - Project data to 1-D subspace (a line) which minimize the projection error



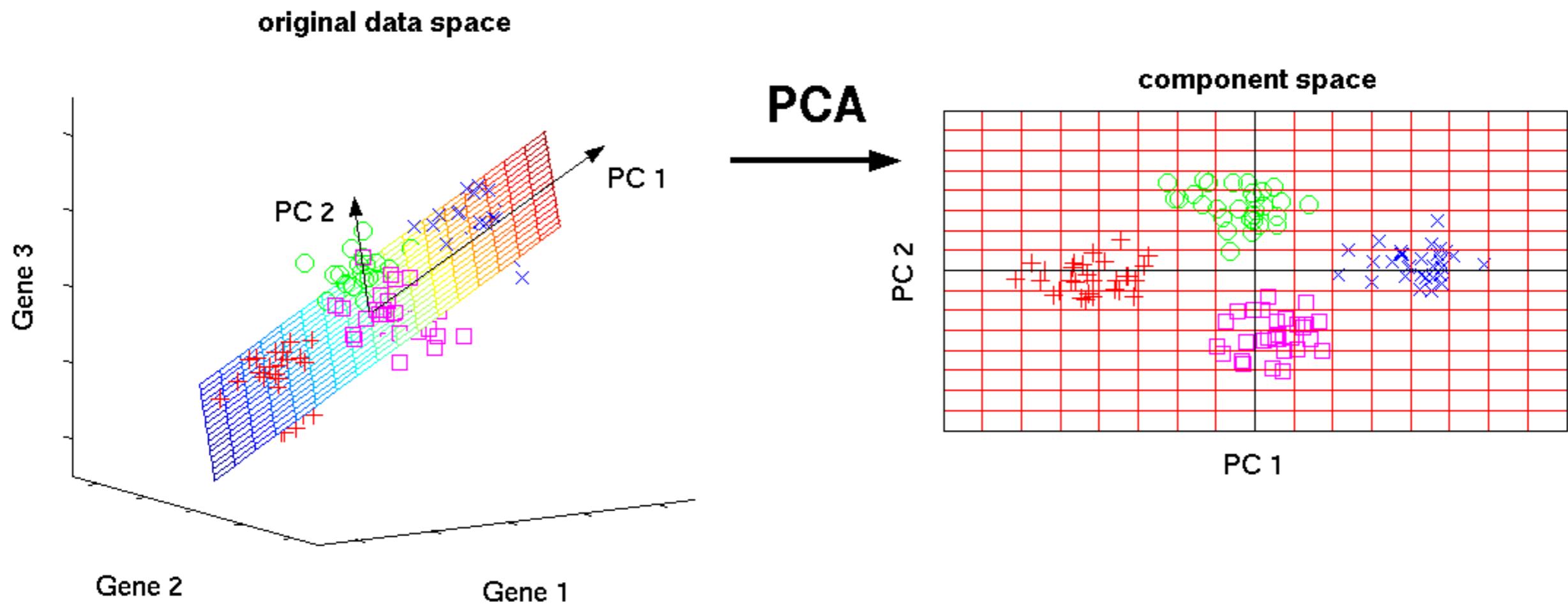
*large projection errors,
bad line to project to*



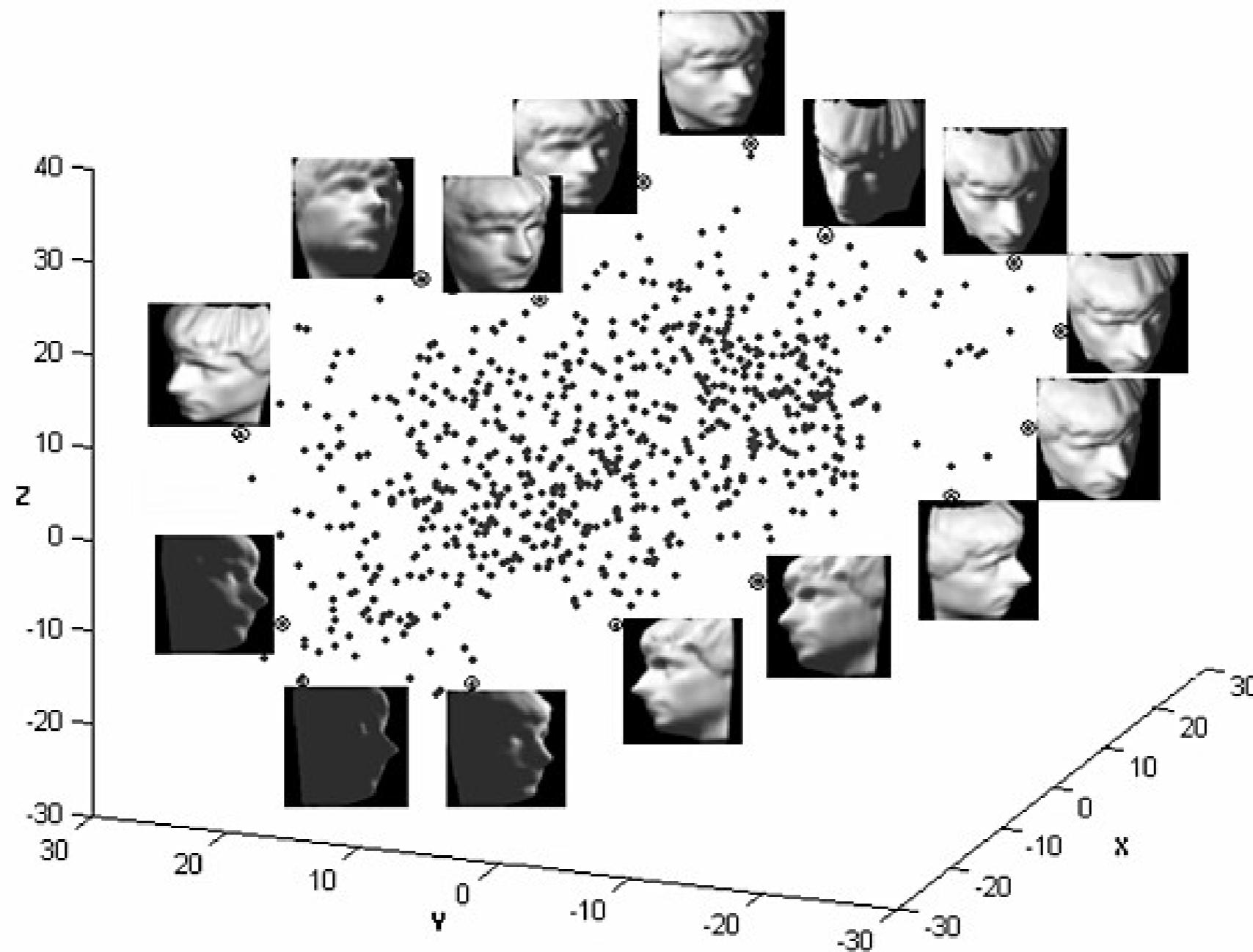
*small projection errors,
good line to project to*

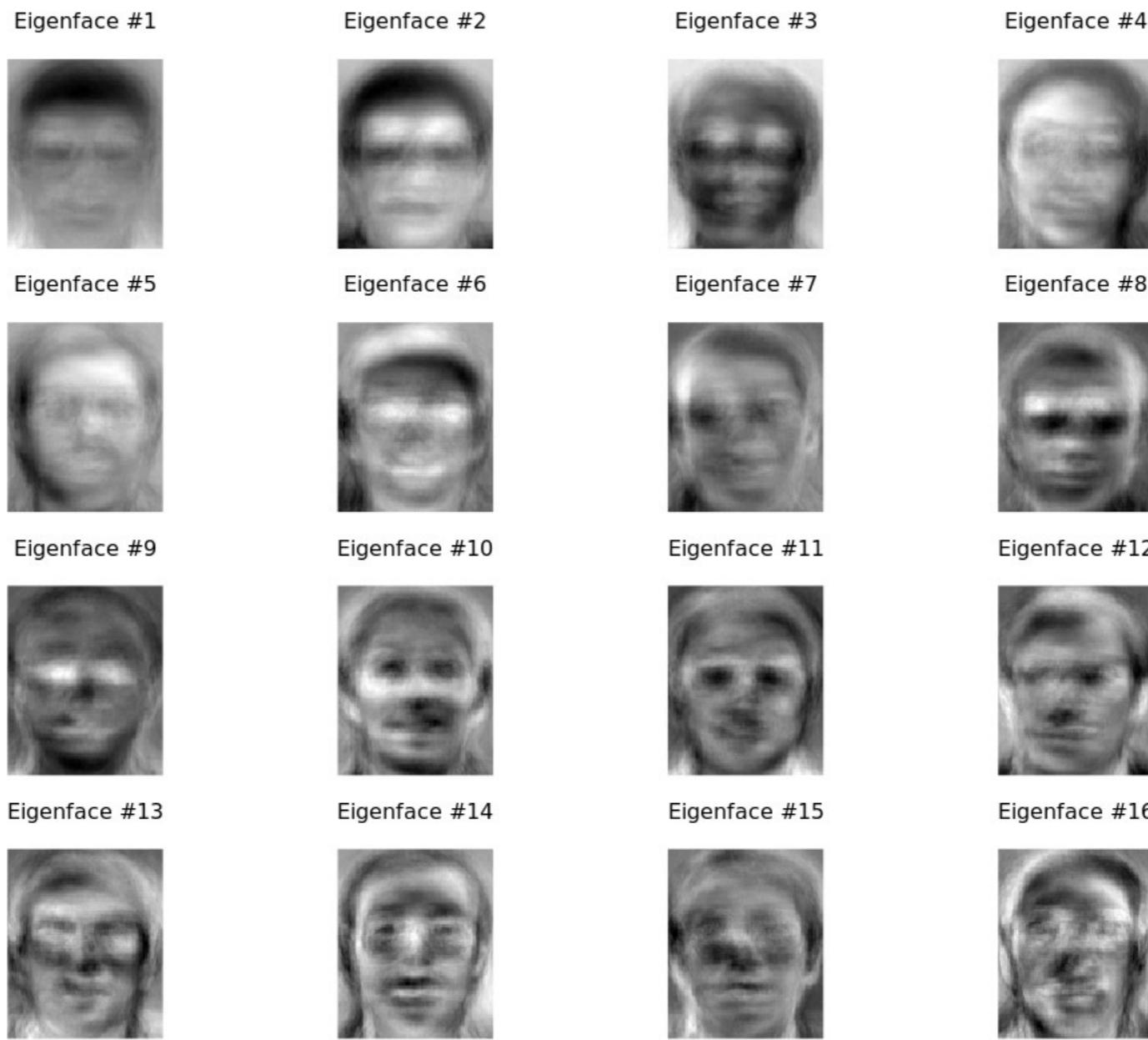
- Notice that the the good line to use for projection lies in the direction of largest variance

PCA in 3D



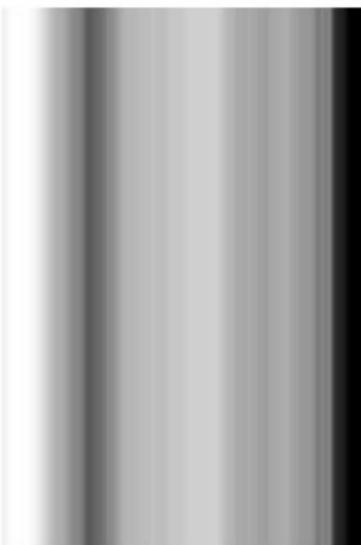
Neat application: Eigenfaces!





http://bytefish.de/static/images/blog/eigenfaces/subplot_eigenfaces.png

PCs # 0



PCs # 10



PCs # 20



PCs # 30



PCs # 40



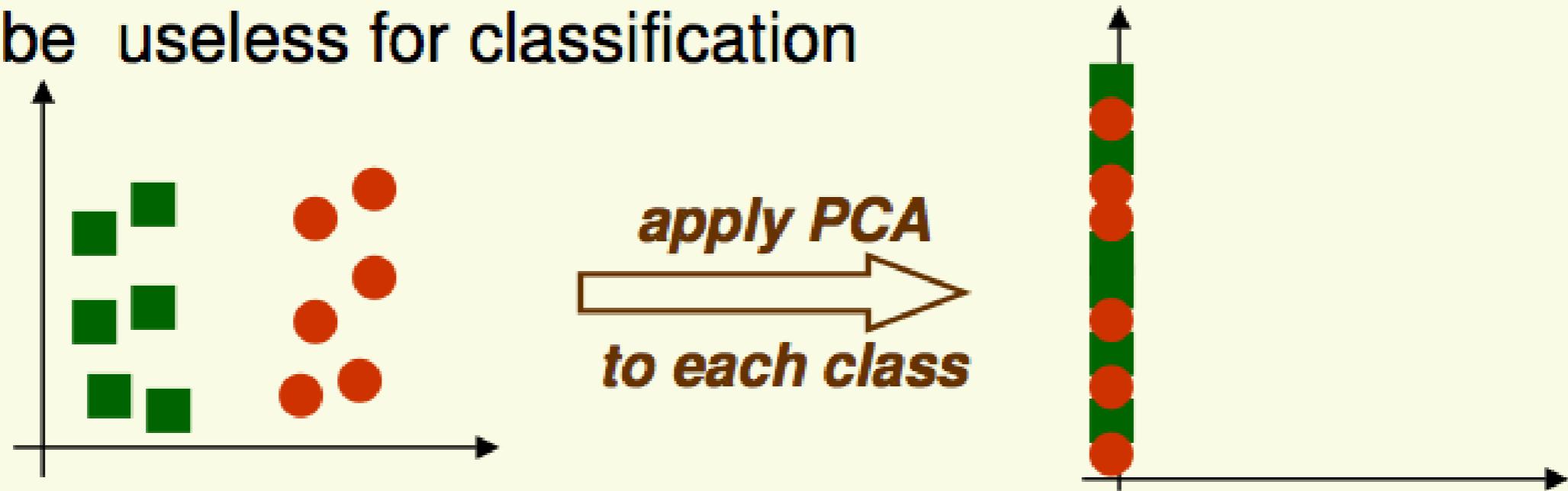
PCs # 50



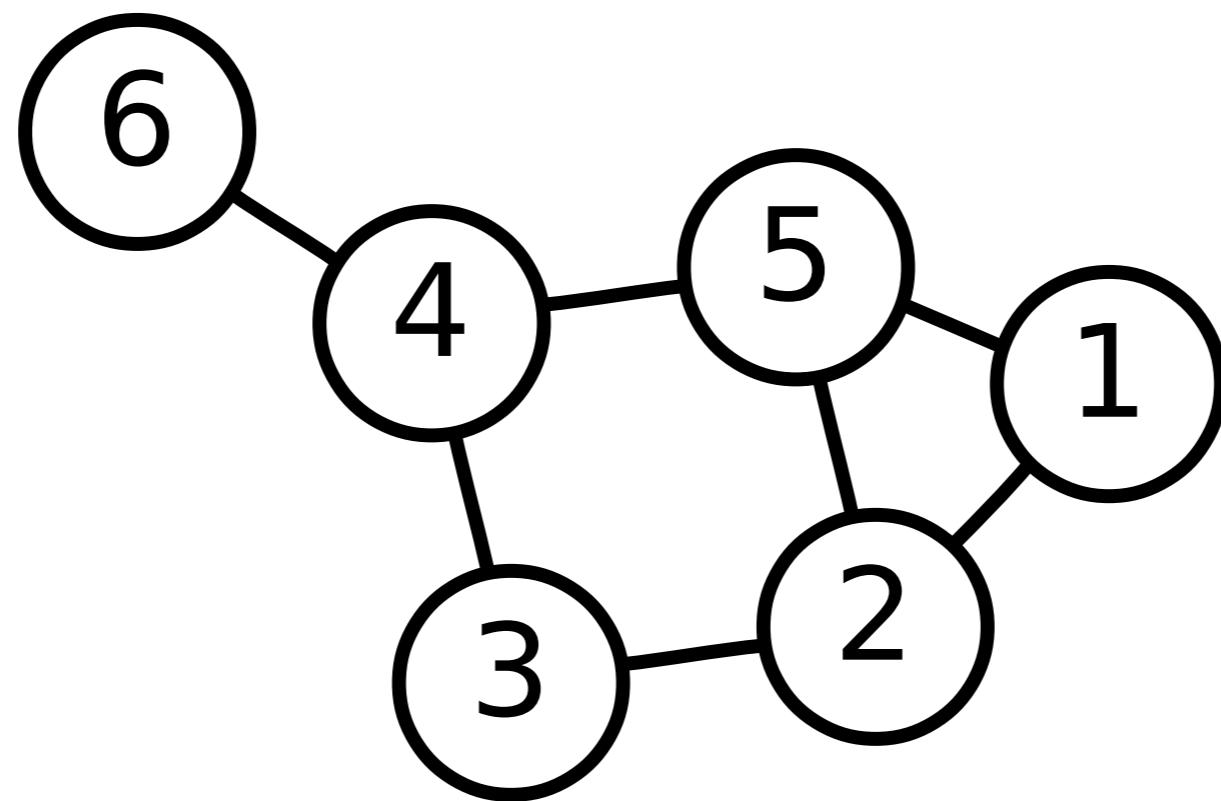
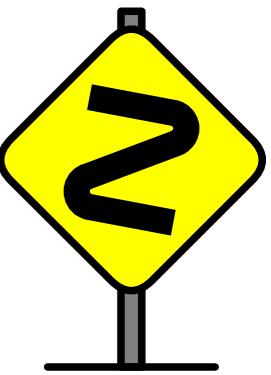
http://3.bp.blogspot.com/-eTWWNr-Cyel/Ti1I2h9ldnI/AAAAAAAEE0/fUF2hl-hTMQ/s1600/pca_shakira.png

Drawbacks of PCA

- PCA was designed for accurate *data representation*, not for **data classification**
 - Preserves as much variance in data as possible
 - If directions of maximum variance is important for classification, will work
- However the directions of maximum variance may be useless for classification



Distance preserving: Johnson-Lindenstrauss



Do not worry about your difficulties in Mathematics. I can assure you mine are still greater.



- Albert Einstein

Read more at <http://www.brainyquote.com/quotes/quotes/a/alberteins125370.html#65g20AeMDxm86kRt.99>

Theorem: Johnson-Lindenstrauss

Given $0 < \epsilon < 1$, a set X of m points in \mathbb{R}^N , and a number $n > \frac{8 \log(m)}{\epsilon^2}$, there is a linear map $f : \mathbb{R}^N \rightarrow \mathbb{R}^n$ such that

$$(1 - \epsilon) \|u - v\|^2 \leq \|f(u) - f(v)\|^2 \leq (1 + \epsilon) \|u - v\|^2$$

for all $u, v \in X$



"Fear has big eyes" by Robbie Grubbs from Houston - What?????. Licensed under Creative Commons Attribution-Share Alike 2.0 via Wikimedia Commons - http://commons.wikimedia.org/wiki/File:Fear_has_big_eyes.jpg#mediaviewer/File:Fear_has_big_eyes.jpg



WPI