Project Euler: Problem 1

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1 Multiples of 3 and 5

If we list all the natural numbers below 10 that are multiples of 3 or 5, we get 3, 5, 6 and 9. The sum of these multiples is 23. Find the sum of all the multiples of 3 or 5 below 1000.

2 Solution

Well, the brute force algorithm is easy. Just a for loop up to 1000 summing up if the modulus of 3 or 5 is 0. The answer is 233168.

But, Wikipedia has nicely provided me with a much more clever solution. We can easily figure out how many integers below 1000 (max 999) are divisible by 3 or 5, 999/3 = 333 and 999/5 = 199. Then, it's easy to get the sums of these numbers by using the formula,

$$1 + 2 + 3 + 4 + 5 + \dots = \sum_{i=1}^{n} i = \frac{n(n+1)}{2}.$$
 (1)

Then, we've got

$$sum = 3 * \frac{999(999 + 1)}{2} + 5 * \frac{199(199 + 1)}{2}$$
 (2)

But, that double counts the numbers that are divisible by both 3 and 5 (*i.e.* divisible by 15. So, we just correct for that

$$sum = 3 * \frac{999(999+1)}{2} + 5 * \frac{199(199+1)}{2} - 15 * \frac{66(66+1)}{2} = 233168.$$
 (3)