

# papers on enriched higher category theory

2021-12-09

## Papers on enriched $\infty$ -category theory

I created this repo so that I have somewhere to point people when they ask “what’s up with enriched higher category theory? I mean what’s up with it?”

### Papers of the now

Five hand-picked papers that represent what I know about the subject right now. I’ve probably missed some important ones — let me know or come and edit the list yourself!

- 1312.3178 — *Enriched  $\infty$ -categories via non-symmetric  $\infty$ -operads*, David Gepner and Rune Haugseng, 2013 (v1); 2015 (Adv. Math.); 2019 (v4).

The major (in 2021) definition of enriched  $\infty$ -categories in use. Establishes properties of the category of presentable  $\mathcal{V}$ -categories (such as presentability when  $\mathcal{V}$  is presentable). First construction of *adjoint enrichments* using a generators-and-relations model.

- 1805.07635 — *Yoneda lemma for enriched infinity categories*, V. Hinich, 2019 (v1); 2020 (Adv. Math.); 2021 (v8).

Another attack on enriched  $\mathcal{V}$ -categories that focuses on the corepresenting planar operad (as opposed to the *generalised* planar operad prioritised by Gepner-Haugseng). Makes the adjoint enrichment a central theme and uses it to define enriched presheaves as a left  $\mathcal{V}$ -module and prove a Yoneda lemma.

- 2009.02428 — *An equivalence between enriched  $\infty$ -categories and  $\infty$ -categories with weak action*, Hadrian Heine, 2020.

Upgrades either Hinich’s or Gepner-Haugseng’s construction of adjoint enrichments to an equivalence between the categories of enriched  $\mathcal{V}$ -categories and left  $\mathcal{V}$ -modules (I haven’t checked which). This should make the theory of adjoint enrichments really usable, since in practice this seems to be the main way of defining  $\mathcal{V}$ -enrichments.

- 1902.08881 — *The operad corepresenting enrichment*, A. W. Macpherson, 2019 (v1); 2021 (HHA).

Equivalence of Hinich’s planar operad  $\mathbf{Ass}_X$  and Gepner-Haugsgeng’s simplicial multicategory  $\mathcal{O}_X$  for  $X$  an  $\infty$ -groupoid, and hence equivalence of the corresponding  $\infty$ -categories of enriched  $\infty$ -categories. This does *not* address the important question of comparing their constructions of adjoint enrichments — still open as far as I know!

- 2008.11323 — *Enriched infinity categories I: enriched presheaves*, John D. Berman, 2020.

Establishes fundamental properties of enriched presheaves over a presentable  $\mathcal{V}$  as a  $\mathcal{V}$ -module: that they form a presentable  $\mathcal{V}$ -module generated as such by representables, that limits and colimits are pointwise, the tensor identities

$$\mathrm{PSh}^{\mathcal{V}}(C, M) \cong \mathrm{PSh}^{\mathcal{V}}(C, \mathcal{V}) \otimes_{\mathcal{V}} M$$

and that  $\mathrm{PSh}^{\mathcal{V}}(C)$  is dual to  $\mathrm{PSh}^{\mathcal{V}}(C^{\mathrm{op}})$  (all in the symmetric monoidal category of presentable  $\mathcal{V}$ -modules).

## Other papers that might be important

Leaving room here for other papers that don’t fit into the above list (which will be kept to 7 items at most).