open problems in enriched higher category theory

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Open problems in enriched ∞-category theory

Or more precisely, problems that are solved at best in a paper I haven't read.

1. A construction of the corepresenting operad $\mathtt{Ass}_X$ that is actually feasible to work with (I have a good idea of how to do this, but haven't found time to write it down yet.)
2. Show that Heine's construction of adjoint enrichments specialises to both Gepner-Haugseng and Hinich's construction on each object (probably one of these is shown in the paper and the other is open).
3. Show that for presentably symmetric monoidal $\mathcal{V}$ and presentable $\mathcal{V}$-module Y, Heine's equivalence exchanges Hinich's $\mathcal{V}$-module of functors $\mathrm{Fun}_{\mathcal{V}}(X, Y)$ with the internal Hom in $\mathcal{V}\mathbf{Cat}$ from X to the dual $\mathcal{V}$-enrichment of Y. (This might follow from results of Heine and Berman.)
4. Show that the autoequivalence group of $\mathcal{V}\mathbf{Cat}$ fits into an exact sequence

   $$1 \to \mathrm{Aut}(\mathcal{V}, \otimes) \to \mathrm{Aut}(\mathcal{V}\mathbf{Cat}) \to H \to 1$$

   where

   $$H = \begin{cases} C_2 & \text{if } (\mathcal{V}, \otimes) \simeq (\mathcal{V}, \otimes^{\mathrm{op}}) \\ 1 & \text{otherwise.} \end{cases}$$

   The space of splittings of the sequence is equivalent to the space of involutive structures on $\mathcal{V}$ (which can be thought of as inducing "opposite" autoequivalences on $\mathcal{V}\mathbf{Cat}$). (I thought I had proved this, but there was a gap in my argument that has yet to be filled.)