Introduction to Complexity and Computability: Homework 1

Andrew McIsaac

October 14, 2021

Homework Problem 1

A single-tape Turing machine M can be converted to a TM M' by modifying the transition function so that any single transition performed by M is performed in two transitions by M'.

The first transition changes the state and tape symbol to the desired tape symbol and the intermediate state consists of a tuple with the desired state and desired head position. The second transition then changes state and head position based on the state and head position stored in the state of the intermediate transition.

The set of states of M', Q', is augmented, so that $Q' = Q \cup Q \times \{L, R\}$

E.g. For M, a transition $\delta(q, a) = (q', a', L)$ is done in M' with two steps:

$$\begin{aligned} \delta(q, a) &= ((q', L), a') \\ \delta((q', L), a') &= (q', L) \end{aligned}$$

Homework Problem 2

A single-tape TM M can be converted to a TM M' where the tape is one-way infinite with only right (R) and RESTART head movements by creating M' where the head movements of M are replicated in the following way.

For R movements from M, M' exactly replicates the movement of M.

For L movements from M, M' first marks the current tape symbol, then performs a RESTART movement. The head then proceeds right, entirely replicating the input tape one step to the right, with the first symbol being replaced by a placeholder symbol. This continues until the marked symbol is reached, where the tape symbol to the left of the original marked symbol is marked in its place, and the rest of the input then continues to be copied, unmarking the originally marked symbol. Finally, the tape is RESTARTed, and moves right until the marked symbol is reached.

This marked symbol can perform exactly the same transitions as the unmarked symbol, and is overwritten by either the unmarked symbol or a new symbol depending on the next transition (or is kept marked if the next transition is a left transition, restarting the process described above). The head

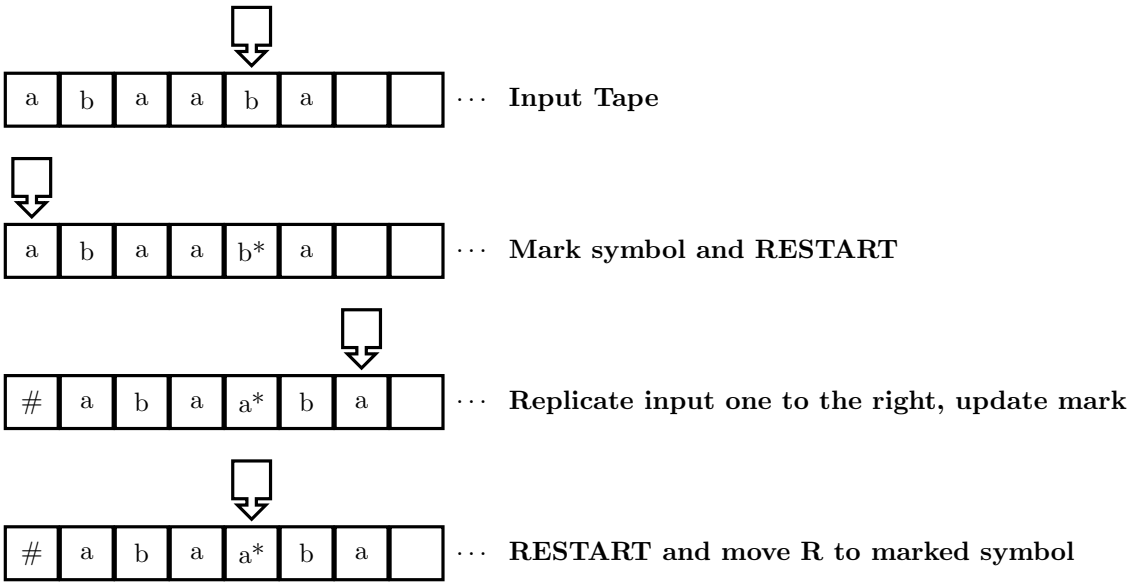

Figure 1: TM M' simulating effective left movement of TM M. The result shows the entire original input tape replicated on the same tape but one cell to the right, with the head position in its original place.

is effectively one position left of its original position, with the entire original tape shifted one position to the right. Fig. 1 shows an example diagram of this notion.