Introduction to Complexity and Computability: Homework 2

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Homework Problem 1

Show that the following languages are partially decidable:

(a) $ETx = \{\langle M, x\rangle \mid M(x)\downarrow$ and at the end of computation the tape of M is empty$\}$

Use a Universal Turing Machine to simulate M on x. If $M(x)\downarrow$ then clear the tape of M and accept.

(b) $ETE = \{\langle M\rangle \mid (\exists\, x)(M(x)\downarrow$ and at the end of computation the tape of M is empty$)\}$

Run ETx on every possible input string x. Run the first Turing Machine ETx_1 on input x_1 for one step, then the TMs ETx_1 and ETx_2 for one step on input x_1 and x_2 respectively, and so on for every single x. If there is an x that is accepted by ETx, clear the tape of M and ETE accepts.

Homework Problem 2

Show that the language ETx is not decidable.

The proof is by contradiction. Assume that ETx is decidable. Then there is a TM P that decides ETx. Then construct a TM Q to decide $L_{\mathcal{U}}$, the universal language.

Q proceeds as follows on the input $\langle M, x\rangle$:

1. Run P on $\langle M, x\rangle$.
2. If P rejects, reject.
3. If P accepts, simulate M on x until it halts and the tape of M is empty.
4. If M accepts, accept. If M rejects, reject.

If P decides ETx, Q decides $L_{\mathcal{U}}$. But $L_{\mathcal{U}}$ is undecidable, so ETx must also be undecidable.