

# Introduction to Categorical Logic

80-514/814

## Suggested Topics for Student Projects

Updated: March 29, 2023

Come talk to me for more information about any of these topics. And feel free to suggest others!

### 1. Lawvere Duality

- Adamek, Lawvere, Rosicky: On the duality between varieties and algebraic theories, *Algebra Universalis*, 2003.
- Adamek, Rosicky, Vitale: *Algebraic theories*, Cambridge University Press, 2010.

### 2. Gabriel-Ulmer duality

- Makkai, Pitts, Some results on locally finitely presentable categories, *Transactions of the AMS* 1987.
- Adamek, Rosicky, Vitale: *Algebraic theories*, Cambridge University Press, 2010.

### 3. Stone-type dualities for commutative rings, distributive lattices, Heyting algebras, etc.

- P.T. Johnstone, *Stone Spaces*, Cambridge University Press, 1982.
- M. Makkai and G. Reyes, Completeness results for intuitionistic and modal logic in a categorical setting, *Annals of Pure and Applied Logic*, Volume 72, Issue 1, 10 March 1995, Pages 25–101.

#### 4. Bi-Heyting logic

- F.W. Lawvere, Intrinsic Co-Heyting Boundaries and the Leibniz Rule in Certain Toposes, in A. Carboni, M. Pedicchio, G. Rosolini (eds.), *Category Theory - Como 1990*, LNM 1488 Springer Heidelberg 1991.
- Gonzalo E. Reyes, Houman Zolfaghari, Bi-Heyting Algebras, Toposes and Modalities, *J. Phi. Logic* 25 (1996) pp. 25–43.
- Kripke models of bi-Heyting logic (CMU MS thesis by J. Winkler).

#### 5. Joyal's embedding theorem, completeness

- M. Makkai and G. Reyes, Completeness results for intuitionistic and modal logic in a categorical setting, *Annals of Pure and Applied Logic*, Volume 72, Issue 1, 10 March 1995, Pages 25–101.
- Topological completeness of IPC.
- My Fischbachau notes.

#### 6. Modal Logic

- Kripke semantics for classical modal logic: various references, just ask (also a CMU MS thesis by H.-C. Kotzsch).
- S. Awodey and K. Kishida, *Topology and Modality: The Topological Interpretation of First-Order Modal Logic*, *Review of Symbolic Logic*, 2008.
- What is intuitionistic S4 modal logic?
- Modal propositional logic: McKinsey-Tarski topological completeness.
- Gödel translation of IPC into classical modal PC.

#### 7. Lambda-calculus and CCCs

- D.S. Scott. Relating theories of the  $\lambda$ -calculus. In R. Hindley and J. Seldin, editors, *To H.B. Curry: Essays in Combinatory Logic, Lambda Calculus and Formalisms*, pp. 403–450. Academic Press, 1980.

- D.S. Scott, Lambda Calculus: Some Models, Some Philosophy, Studies in Logic and the Foundations of Mathematics, Volume 101, 1980, pp. 223–265
- S. Awodey, Topological representation of the  $\lambda$ -calculus, Math. Struct. in Comp. Science, 2000.
- $\lambda$ -calculus with sums  $A + B$ .
- $\lambda$ -theory of a tiny object/interval/tangent vector.
- Kripke models of some  $\lambda$ -theories: Scott reflexive object, etc.
- Untyped  $\lambda$ -calculus as a  $\lambda$ -theory (Scott, Lambek-Scott).
- Equilogical spaces (Scott’s paper).
- Modal type theory (my NASLLI talk, other).

8. Some more topics:

- Kripke and/or topological counter-models and correspondences
- Free Heyting algebras
- Linear logic (Shulman’s paper)
- Lawvere’s paper “Adjointness in foundations”, Reprints in Theory and Applications of Categories, No. 16, 2006, pp. 1–16.
- Do a formalization in Lean with this project: <https://lean-catlogic.github.io>