Introduction to Categorical Logic

80-514/814

Suggested Topics for Student Projects

Updated: March 1, 2023

Come talk to me for more information about any of these topics. And feel free to suggest others!

1. Lawvere Duality
    - Adamek, Lawvere, Rosicky: On the duality between varieties and algebraic theories, Algebra Universalis, 2003.
    - Adamek, Rosicky, Vitale: Algebraic theories, Cambridge University Press, 2010.
2. Gabriel-Ulmer duality
    - Makkai, Pitts, Some results on locally finitely presentable categories, Transactions of the AMS 1987.
    - Adamek, Rosicky, Vitale: Algebraic theories, Cambridge University Press, 2010.
3. H-Sets are regular / coherent / topos:
    - van Oosten, Basic Category Theory, 2002.
    - M.P. Fourman and D.S. Scott. Sheaves and logic. In: Applications of sheaves, edited by Fourman, Mulvey, Scott, Lecture notes in mathematics, 753, Springer, 1979, pp. 302–401.

- Topoi and categories of fuzzy sets, Lawrence Neff Stout, Fuzzy Sets and Systems, February 1984, pp.169–184

4. Stone-type duality for distributive lattices and Heyting algebras

   - P.T. Johnstone, Stone Spaces, Cambridge University Press, 1982.
   - M. Makkai and G. Reyes, Completeness results for intuitionistic and modal logic in a categorical setting, Annals of Pure and Applied Logic, Volume 72, Issue 1, 10 March 1995, Pages 25–101

5. Exact categories, Quotients, Setoids, Exact completions

   - Carboni and Vitali, Regular and exact completions, Journal of Pure and Applied Algebra, March 1998, pp. 79–116.
   - Carboni and Rosolini, Locally cartesian closed exact completions, Journal of Pure and Applied Algebra, December 2000, pp. 103–116.

6. Hyperdoctrines

   - F.W. Lawvere, Equality in hyperdoctrines and comprehension schema as an adjoint functor, Proceedings of the AMS Symposium on Pure Mathematics XVII (1970), 1–14.
   - Pitts CS Logic handbook article.
   - R.A.G. Seely, Hyperdoctrines, natural deduction, and the Beck condition, Zeitschrift für math. Logik und Grundlagen der Math., Band 29, 505–542 (1983).

7. Bi-Heyting logic

   - F.W. Lawvere, Intrinsic Co-Heyting Boundaries and the Leibniz Rule in Certain Toposes, in A. Carboni, M. Pedicchio, G. Rosolini (eds.), Category Theory - Como 1990, LNM 1488 Springer Heidelberg 1991.
   - Gonzalo E. Reyes, Houman Zolfaghari, Bi-Heyting Algebras, Toposes and Modalities, J. Phi. Logic 25 (1996) pp. 25–43.
   - Kripke models of bi-Heyting logic (CMU MS thesis by J. Winkler).

8. Joyal's embedding theorem, completeness

- M. Makkai and G. Reyes, Completeness results for intuitionistic and modal logic in a categorical setting, Annals of Pure and Applied Logic, Volume 72, Issue 1, 10 March 1995, Pages 25–101
- My Fischbachau notes.

9. Set-valued completeness for regular theories, classical completeness for Boolean theories:

   - P.T. Johnstone, Sketches of an Elephant, section D1.5.
   - Deligne's Theorem, M&M.

10. Lambda-calculus and CCCs

    - D.S. Scott. Relating theories of the λ-calculus. In R. Hindley and J. Seldin, editors, To H.B. Curry: Essays in Combinatory Logic, Lambda Calculus and Formalisms, pp. 403–450. Academic Press, 1980.
    - D.S. Scott, Lambda Calculus: Some Models, Some Philosophy, Studies in Logic and the Foundations of Mathematics, Volume 101, 1980, pp. 223–265
    - S. Awodey, Topological representation of the λ-calculus, Math. Struct. in Comp. Science, 2000.
    - λ-calculus with sums $A + B$.
    - λ-theory of a tiny object/interval/tangent vector.
    - Kripke models of some λ-theories: Scott reflexive object, etc.
    - Untyped λ-calculus as a λ-theory (Scott, Lambek-Scott).
    - Equilogical spaces (Scott's paper).

11. Dependent type theory and LCCC (locally cartesian closed categories):

    - R. A. G. Seely. Locally cartesian closed categories and type theory. Math. Proc. Camb. Phil. Soc., 95:33-48, 1984.
    - S. Awodey and F. Rabe, Kripke Semantics for Martin-Löf's Extensional Type Theory, Log. Methods Comput. Sci., 2011.
    - S. Awodey, N. Gambino, S. Hazratpour, Kripke-Joyal forcing for type theory and uniform fibrations, arXiv:2110.14576.

12. Modal Logic

    - S. Awodey and K. Kishida, Topology and Modality: The Topological Interpretation of First-Order Modal Logic, Review of Symbolic Logic, 2008.
    - Modal Logic Project notes (just ask).
    - Modal propositional logic: McKinsey-Tarski topological completeness.
    - Gödel translation of IPL into ModalPL.

13. Some more topics:

    - Equilogical spaces are regular LCCC, but not exact (Scott's paper)
    - Equational theory of Heyting algebras
    - Linear logic (Shulman's paper)
    - Sheaves for a Grothendieck topology
    - HOL and elementary toposes
    - Diaconescu cover of a presheaf topos for a Kripke model of IFOL
    - Kripke completeness of IFOL
    - Algebraic models of IFOL
    - Gluing for IHOL (Lambek-Scott), HA^{ω}.
    - Algebras and coalgebras for endofunctors.