Coherent Groupoids

Let $\mathbb{G} = G_1 \rightrightarrows G_0$ be a gpd.

Define $\Delta^1 : G_1 \to G_0 \times G_0$,

$$\begin{tikzcd}
G_2 \arrow[r] \arrow[d] \arrow[dr, phantom, "\lrcorner", very near start] & G_1 \arrow[d, "\Delta^1"] \\
G_1 \arrow[r, "\Delta^1"'] & G_0 \times G_0
\end{tikzcd}$$

$$\Delta^2 : G_2 \to G_1 \times G_1$$

Let $\mathbb{G}^I = G_2 \rightrightarrows G_1$ path gpd,

etc. for higher path gpd.s:

$$\mathbb{G} \leftleftarrows \mathbb{G}^I \leftleftarrows (\mathbb{G}^I)^I \leftleftarrows \cdots$$

Note G is a 1-gpd $\Leftrightarrow$ $\Delta^2 : G_2 \rightarrowtail G_1 \times G_1$

and similarly for n-gpd.s.

Define An n-gpd $\mathbb{G} = G_0 \leftleftarrows G_1 \leftleftarrows \cdots$ is coherent if G_0 is cpt & f.all $n > 0$, $\Delta^{n+1} : G_{n+1} \to G_n \times G_n$ is cpt.

Def. An n-gpd

$$G_0 \leftleftarrows G_1 \leftleftarrows G_2 \leftleftarrows \cdots$$

is <u>coherent</u> if G_0 is compact & f. all $(n \geq 0)$ $\Delta^{n+1} : G_{n+1} \to G_n \times G_n$ is cpt.

<u>In part.</u>, a 1-gpd $\mathbb{G} = G_1 \rightrightarrows G_0$ is <u>coh.</u> iff:

$$G_0 \text{ is cpt}$$
$$G_1 \to G_0 \times G_0 \text{ is cpt}$$
$$G_2 \to G_1 \times G_1 \text{ is cpt.}$$

It follows that a coh. 1-gpd has a "cpt cover":

$$\begin{tikzcd}
P_2 \arrow[r, two heads] \arrow[dr] & \cdot \arrow[r, two heads] \arrow[d] \arrow[dr, phantom, "\lrcorner", very near start] & G_2 \arrow[d] \\
& P_1 \times P_1 \arrow[r, two heads] & G_1 \times G_1
\end{tikzcd}
\qquad
\begin{tikzcd}
P_2 \arrow[d, shift left] \arrow[d, shift right] \\
P_1
\end{tikzcd}$$

$$\begin{tikzcd}
P_1 \arrow[r, two heads] \arrow[dr] & \cdot \arrow[r, two heads] \arrow[d] \arrow[dr, phantom, "\lrcorner", very near start] & G_1 \arrow[d] \\
& P_0 \times P_0 \arrow[r, two heads] & G_0 \times G_0
\end{tikzcd}
\qquad
\begin{tikzcd}
P_1 \arrow[d, shift left] \arrow[d, shift right] \\
P_0
\end{tikzcd}$$

This is a <u>pseudo-groupoid</u> in $\mathbb{H}$.