Math Induction

$$1+2+3+\cdots+n = \frac{n(n+1)}{2}$$

truth for $n \Rightarrow$ truth $n+1$

Harmonic numbers $H_1, H_2, H_3, \ldots$

$$H_{n+1} = H_n + \frac{1}{n+1}$$

$$H_1 = 1$$

$$\vdots$$

$$H_n = 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \cdots + \frac{1}{n}$$

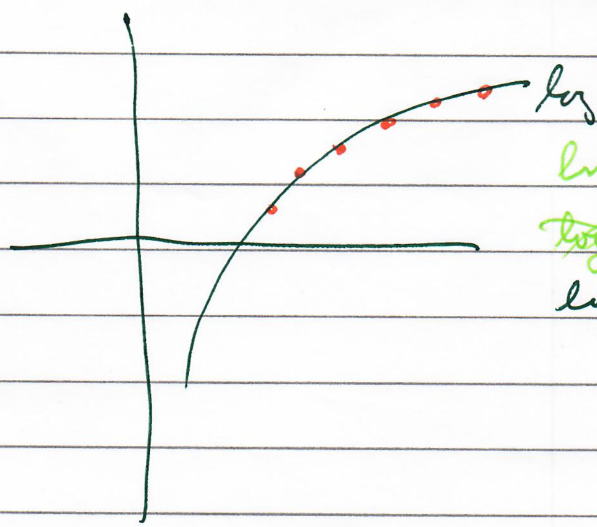
$2n\, H_n$

$$\boxed{\underbrace{1+n \geq H_{2^n} \geq 1 + \frac{n}{2}}}$$

$H_k \approx$ log behavior growth

$H_k = \Theta(\log k)$

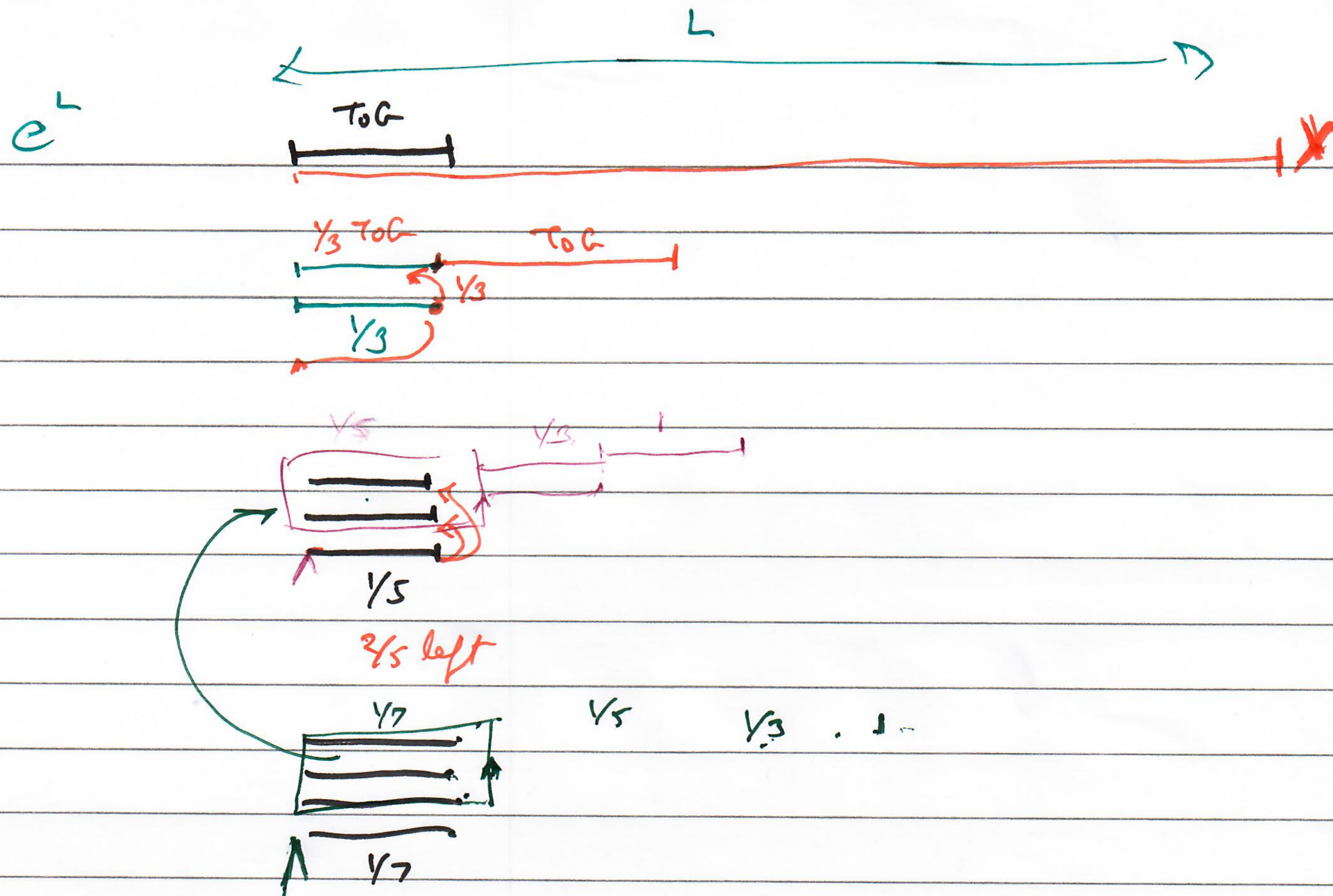
H_k grows proportionately to $\log k$



$\ln = \log_e$

~~$\log = \log_{10}$~~

$\log_2$ BINARY LOG. lg



$\vdots$

D_{n-1} Distance w/ n cars

$D_{n-1} + \frac{1}{2n-1}$

$$\boxed{\begin{aligned} D_1 &= 1 \\ D_n &= D_{n-1} + \frac{1}{2n-1} \end{aligned}}$$

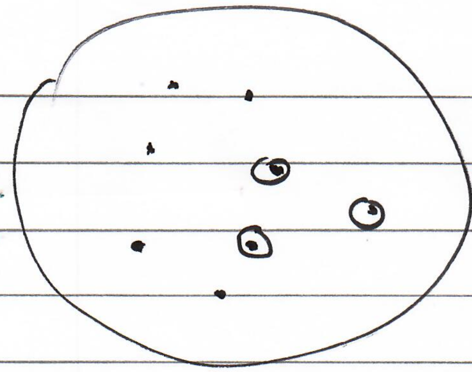
ln	base e
lg	base 2
log	

HARMONIC NUMBERS

$$D_n \approx \frac{\ln n}{2}$$

n people

≥ 1 black hat



Thm If there are k people w/ black hats those k leave room at k^{th} time.

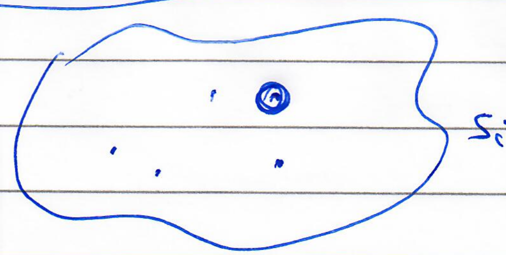
BASE
$$\begin{cases} k=1 & \text{OK} \\ k=2 & \text{OK} \\ k=3 & \end{cases}$$

Statement $S_k \Rightarrow S_{k+1}$

S_i = Everybody knows that at least i people have black hats

Claim S_i is true at i^{th} chime

Pf By induction



Analyze Algorithms — Worst Case

Euclid's Algorithm for G.C.D

$u \geq v$ $\quad$ $r_0 = u$

$r_1 = v$

remainder

$$\boxed{r_{i+1} = r_{i-1} \boxed{\bmod} r_i}$$

$\gcd(21, 13) = \gcd(13, 8)$

$$\gcd(u,v) = \begin{cases} u & v=0 \\ \gcd(v, u \bmod v) \end{cases}$$

$21 \bmod 13 \longrightarrow 8$

WORST CASE?

$$r_{i-1} = q\, r_i + r_{i+1}$$

$q \geq 1$ $\quad$ $r_0 \geq r_1$

$$r_{i-1} \geq r_i + r_{i+1}$$

$i=1$ $\quad$ $r_0 \geq r_1 + r_2 \geq r_2 + r_3 + r_2 = 2r_2 + r_3$

$i=2$ $\quad$ $r_1 \geq r_2 + r_3$ $\quad$ $\geq 2(r_3 + r_4) + r_3$

$i=3$ $\quad$ $r_2 \geq r_3 + r_4$ $\quad$ $= 3r_3 + 2r_4$

$\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad$ $\geq 3(r_4 + r_5) + 2r_4$

$i=4$ $\quad$ $r_3 \geq r_4 + r_5$ $\quad$ $\geq 5r_4 + 3r_5$

$$r_0 \geq 5 r_4 + 3 r_5$$

$$\vdots$$

$$\geq F_h r_h + F_{h-1} r_{h-1}$$

$$\geq \cdots\cdots$$

0, 1, 1, 2, 3, 5, 8, 13, 21 ...

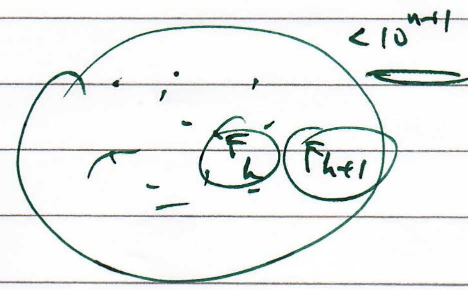
$$\boxed{\begin{aligned} F_{h+1} &= F_h + F_{h-1} \\ F_0 &= 0 \\ F_1 &= 1 \end{aligned}}$$

FIBONACCI NUMBERS

n-digit

P.I.D.O.O.M.A.

$10^{n+1} >$



$$10^{n+1} > F_h \approx \frac{1}{\sqrt{5}} \left(\frac{1+\sqrt{5}}{2} \right)^h$$

$$\Longrightarrow h \approx 4.785\, n$$