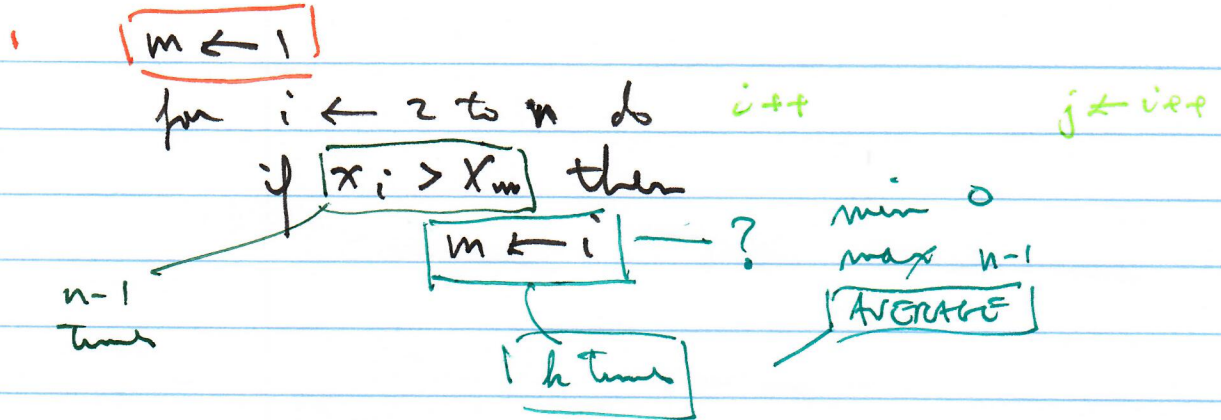


①

$$\max\{x_1 \dots x_n\}$$


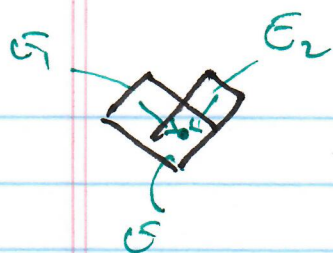
formula in terms of n and k

Rule of Sum

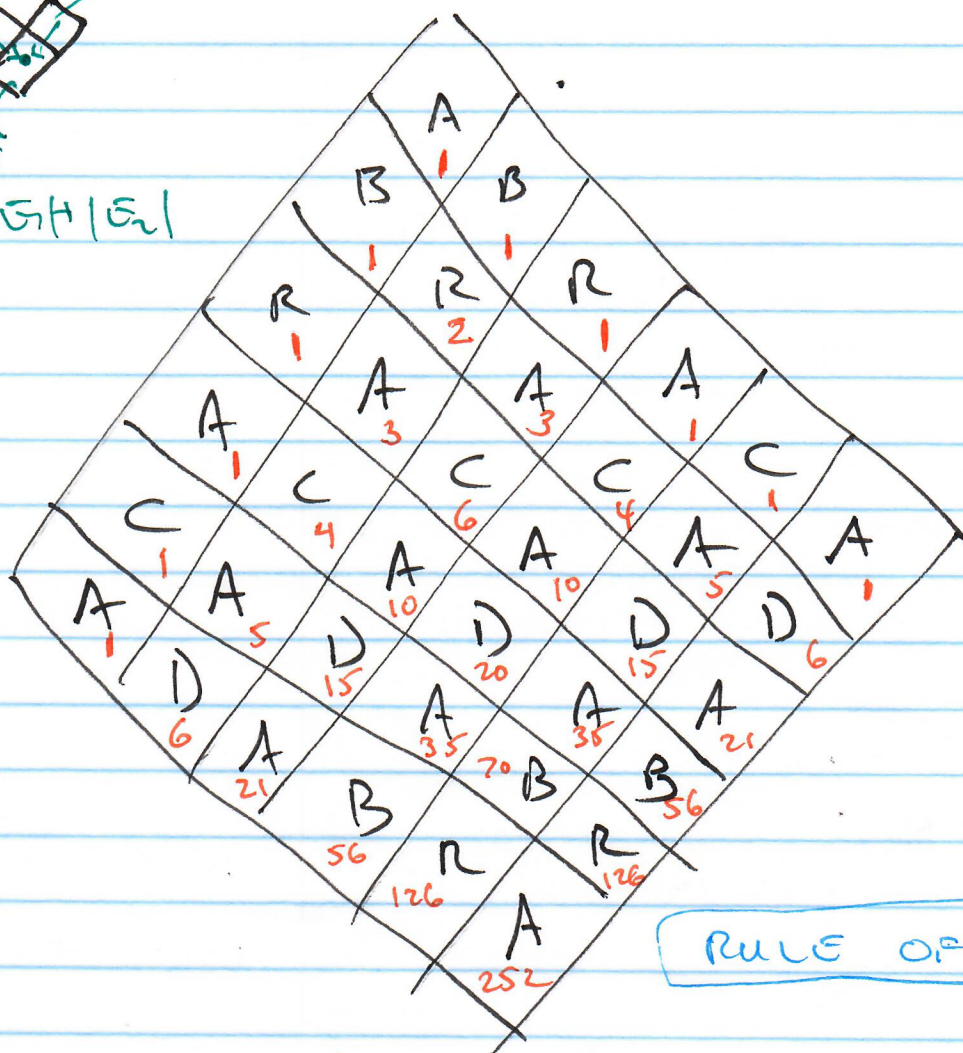
If an event E composed of event E_1 or E_2 and E_1 can happen in $|E_1|$ ways and E_2 can happen in $|E_2|$ ways, then $E = E_1 + E_2$ can happen in $|E_1| + |E_2|$ ways

Rule of Product

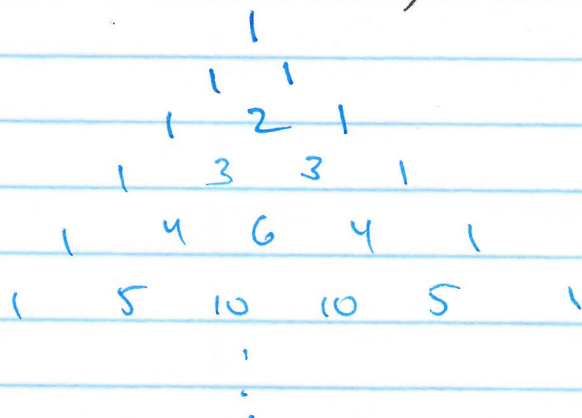
If an event E composed of events E_1 and E_2 and E_1 can happen in $|E_1|$ ways, ~~that~~ E_2 can happen in $|E_2|$ ways then $E = E_1 \cap E_2$ can happen in $|E_1| \times |E_2|$ ways



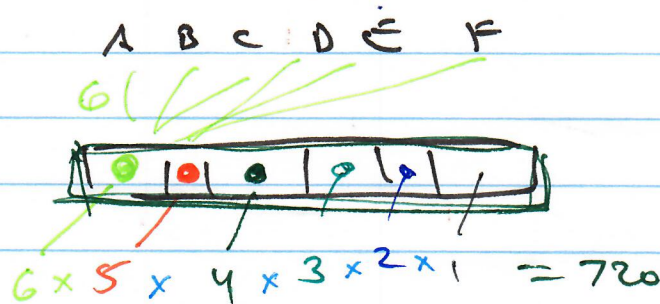
$$|E| = |G| + |H| + |E_2|$$



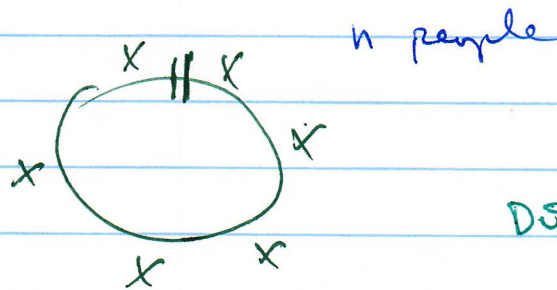
RULE OF SUM



Rule of Product

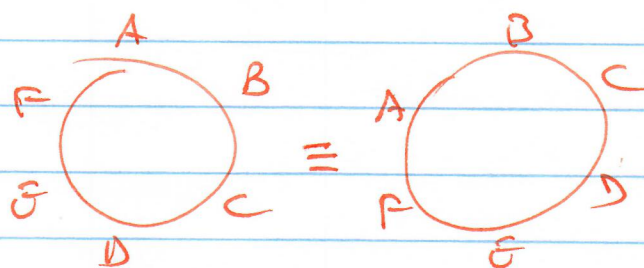


$$n(n-1)(n-2)\dots 1 = n! \\ (n \text{ factorial})$$



DEFINITION

$$0! = 1$$



$$n! = n \times (n-1)! \\ = 1 \quad n=0$$

ways to arrange n students in row = # arrangements in a circle \times open circle

$n!$ = \times n ways

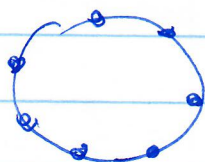
B_1, B_2

$B_1, B_2, B_3, \dots, B_n$

$$\frac{n!}{n}$$

How many?

$$(n-1)!$$



of arrangements on a circle

$$(n-1)!$$

ways

$$\frac{(n-1)!}{2}$$

up or down

2

$$\# \text{ ways arrangement} = \begin{cases} 1 & n = 0, 1, 2 \\ \frac{(n-1)!}{2} & n \geq 3 \end{cases}$$

Rule of Sum

$$E = E_1 + E_2$$

$$|E| = |E_1| + |E_2|$$

$$E_1 \cap E_2 = \emptyset$$

$$|E| = |E_1| + |E_2| - |E_1 \cap E_2|$$

$$E_1 \quad E_2 \quad E_3$$

$$|E| = |E_1| + |E_2| + |E_3|$$

$$- |E_1 \cap E_2| - |E_1 \cap E_3| - |E_2 \cap E_3|$$

$$+ |E_1 \cap E_2 \cap E_3|$$

$$|E| = |E_1| + |E_2| + |E_3| + |E_4|$$

$$- |E_1 \cap E_2| - |E_1 \cap E_3| - |E_1 \cap E_4| - |E_2 \cap E_3| - |E_3 \cap E_4| - |E_2 \cap E_4|$$

$$+ |E_1 \cap E_2 \cap E_3| + |E_2 \cap E_3 \cap E_4| + |E_1 \cap E_3 \cap E_4| + |E_1 \cap E_2 \cap E_4|$$

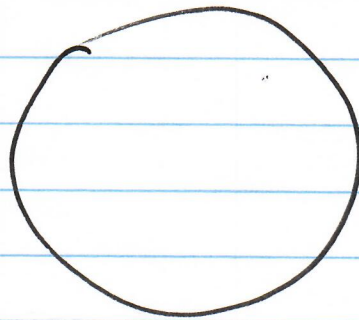
$$- |E_1 \cap E_2 \cap E_3 \cap E_4|$$

Principle
of Inclusion
Exclusion


 $n!$

$D_n =$ derangements of n items

$2n$ people
 M_1, M_2, \dots, M_n
 w_1, w_2, \dots, w_n



Ménage

Google