PLANAR GRAPHS

"PLANARITY" — $O(|V|+|E|)$

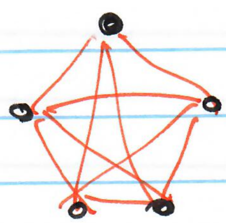
"PLANAR EMBEDDING"

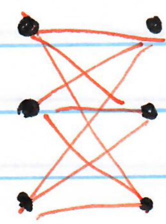
CHARACTERIZE PLANAR GRAPHS

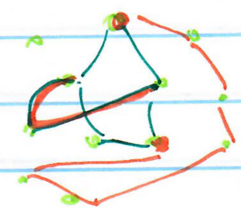
KURATOWSKI'S THM

ONLY TWO NON-PLANAR GRAPHS!

"HOMEOMORPHIC"







K_5

Komplete graph on 5 vertices

$K_{3,3}$

Complete BIPARTITE graph on 3,3 vertices

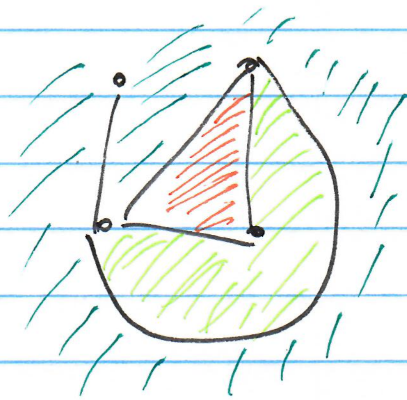
$|V|$

$\binom{|V|}{5}$ — "complete"?

$\binom{|V|}{6}$

$\Theta(|V|^6)$

EULER'S FORMULA



VERTICES

EDGES

REGIONS — FACES

$|V|$

$|E|$

$|F|$

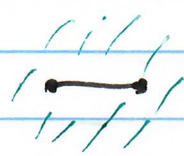
$$\overset{-1}{}\quad \overset{-1}{}$$

$$\boxed{\overset{-1}{|V|} - \overset{-1}{|E|} + |F| = 2}$$

$$4 - 5 + 3 = 2$$

Proof By induction on $|E|$

$|E| = 1$

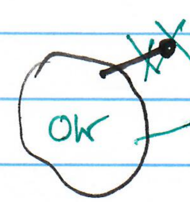


$|V| = 2$
$|E| = 1$
$|F| = 1$

OK

Induction $|E| > 1$

Graph has no cycle $|F| = 1$
$\Rightarrow$ vertex of degree 1



OK

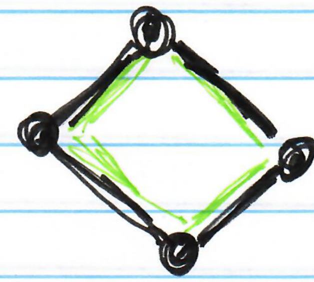
Graph has a cycle

Corollary $\boxed{|E| \le 3|V| - 6}$ for planar graphs

$\boxed{\text{Planar} \Rightarrow |E| \le 3|V| - 6}$

$\Leftarrow$ NO

$\boxed{|V| - |E| + |F| = 2}$



$|E| \Rightarrow 2|E|$ rims

$|F|$ faces $\Rightarrow$ at least $4|F|$ rims at least

$\boxed{2|E| \ge 4|F|}$

$|F| \le \frac{1}{2}|E|$

$|F| = 2 - |V| + |E| \le \frac{2}{3}|E|$

$6 - 3|V| + 3|E| \le 2|E|$

$6 - 3|V| + |E| \le 0$

$|E| \le 3|V| - 6$

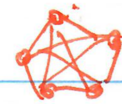
BIPARTITE

$2|V| - 4 \ge |E|$

$P \Rightarrow Q$ $\quad \bar{Q} \Rightarrow \bar{P}$ $\quad Q \Rightarrow P$ NO

Planar $\Rightarrow |E| \le 3|V| - 6$

$\Leftarrow$



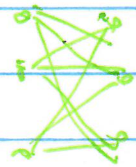
K_5 is not planar

$|V| = 5$

$|E| = 10$

$10 \le 3 \cdot 5 - 6$ NO

$K_{3,3}$



$9 \le 3 \cdot 6 - 6 = 12$

Bipartite/planar $\quad 2|V| - 4 \ge |E|$

$2 \cdot 6 - 4 \ge 9$

$8 < 9$ NO

$\Rightarrow K_{3,3}$ NOT PLANAR

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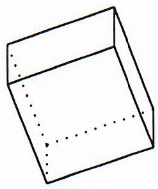
Illinois Institute of Technology
Department of Computer Science

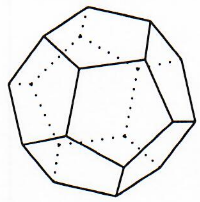
Platonic solids

CS 330 Discrete Structures
Spring Semester, 2019

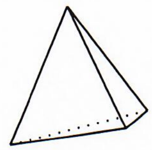
Cube



Dodecahedron



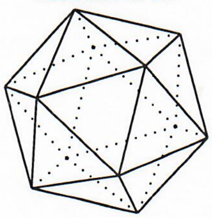
Tetrahedron



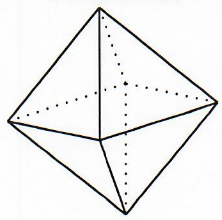
edges make a face — p

edges meet at a point — q

Icosahedron



Octahedron



Tetrahedron

Cube

Octahedron

Planar graph w/ p edges per face vertices of degree q

$$2|E| = q|V| \Rightarrow |V| = \frac{2}{q}|E|$$

$$2|E| = p|F| \Rightarrow |F| = \frac{2}{p}|E|$$

$$|V| - |E| + |F| = 2$$

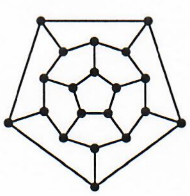
$$\frac{2}{q}|E| - |E| + \frac{2}{p}|E| = 2$$

$$|E|\left(\frac{2}{q} - 1 + \frac{2}{p}\right) = 2$$

$$\Rightarrow \frac{2}{q} - 1 + \frac{2}{p} > 0$$

$$\Rightarrow (p-2)(q-2) < 4$$

Dodecahedron



Icosahedron