$$\boxed{a = \boxed{\overset{X}{E(x)}} = \sum_{\forall x} P_r(x)\,x} = \text{average}$$

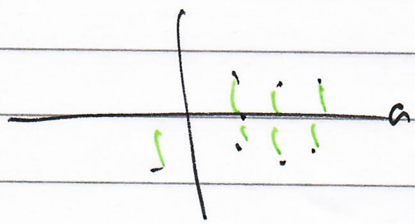
$$E(f(x)) = \sum_{\forall x} P_r(x)\,f(x)$$

Expected value

x − a

$$E(x-a) = \sum_{\forall x} (x-a)\,P_r(x)$$

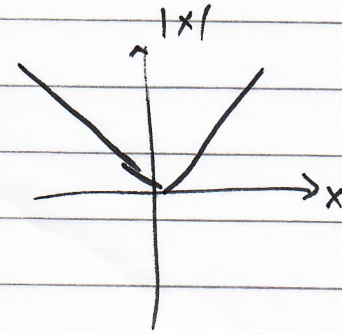
$$= \underbrace{\boxed{\sum_{\forall x} x\,P_r(x)}}_{a} - \cancel{\boxed{\sum_{\forall x} a\,P_r(x)}}$$



$$= a - a = 0$$

UGLY

$$E(|x-a|) = \sum_{\forall x} |x-a|\,P_r(x)$$



$$|x| = \sqrt{x^2}$$

$$E((x-a)^2) = E(x^2 - 2ax + a^2)$$

$$= E(x^2) - E(2ax) + E(a^2)$$

$$= E(x^2) - 2a\,\boxed{E(x)} + a^2$$

$$-2a^2$$

$$= \boxed{E(x^2)} - a^2 = \text{VARIANCE}$$

$$\sqrt{\text{VARIANCE}} = \underset{\sigma}{\text{STD}}\ \text{DEVIATION}$$

average $= 0 \cdot p_n(0) + 1 \cdot p_n(1) + 2 p_n(2) + \cdots$

$x=1$

$$\boxed{\varphi_n(x) = p_n(0)x^0 + p_n(1)x^1 + \cdots}$$

$$= \boxed{\sum_{i=0}^{\infty} p_n(i) x^i}$$

$$= \frac{x+n-1}{n} \varphi_{n-1}(x)$$

average $= \varphi_n'(1) = 0 \cdot p_n(0) + 1 \cdot p_n(1) + 2 p_n(2) + \cdots$

differentiate & set $x=1$

$$\varphi_n'(1) = H_n - 1 \qquad (\approx \ln n)$$

WANT $\quad 0^2 p_n(0) + 1^2 p_n(1) + 2^2 p_n(2) + \cdots = E(x^2)$

$$\sum i^2 p_n(i) x^i$$

$$\varphi_n''(x) = \frac{d}{dx}\left(\sum_{i=1}^{\infty} i\, p_n(i) x^{i-1}\right)$$

$$= \sum_{i=2}^{\infty} i(i-1) p_n(i) x^{i-2}$$

$$\varphi_n''(x) = \sum_{i=2}^{\infty} i^2 p_n(i) x^{i-2} - \sum_{i=2}^{\infty} i\, p_n(i) x^{i-2}$$

$$\varphi_n''(1) = \overbrace{2^2 \cdot p_n(2) + 3^2 p_n(3) + 4^2 p_n(4) + \cdots}^{0^2 p_n(0) + 1^2 p_n(1)} \quad -0^2 p_n(0) - 1^2 p_n(1)$$

$$- \underbrace{\left[\overset{0 \cdot p_n(0) + 1 \cdot p_n(1)}{2 p_n(2) + 3 p_n(3) + 4 p_n(4) + \cdots}\right]}_{\varphi_n'(1)} + 0 p_n(0) + 1 p_n(1)$$

$$\boxed{E(i^2) = \varphi_n''(1) + \varphi_n'(1)}$$

$$\text{VARIANCE} = \overbrace{\varphi_n''(1) + \varphi_n'(1)}^{E(X^2)} - E(\text{avg}^2) \qquad \text{avg}^2$$

$$\boxed{\text{VARIANCE} = \boxed{\varphi_n''(1)} + \boxed{\underbrace{\varphi_n'(1)}_{H_n - 1} - (\varphi_n'(1))^2}}$$ last time

Hmwk $\varphi_n(x) = \frac{x+n-1}{n}\varphi_{n-1}(x)$

given

$$\varphi_n''(1) = \boxed{2S_n} - 2\sum_{i=1}^{\infty} 1/i^2 - 2H_n$$

$\sum_{i=1}^{\infty} 1/i^2$: $O(1)$

$2H_n$: $\approx 2(\ln n)$

$$\int_1^n \frac{\ln x}{x}\,dx \approx \boxed{S_n} = \boxed{\sum_{i=1}^{n} \frac{H_i}{i}}$$

$i, j \quad \frac{1}{ij}$

$S_1 = \frac{H_1}{1} = 1$

$S_2 = \frac{H_1}{1} + \frac{H_2}{2}$

		1	2	3	4	5	n
$1 H_1$ S_1	1	1	1/2	1/3	1/4	1/5	
$\frac{1}{2}H_2$ S_2	2	1/2	1/4	1/6	1/8	1/10	
$\frac{1}{3}H_3$	3	1/3	1/6	1/9	1/12	1/15	
$\frac{1}{4}H_4$	4	1/4	1/8	1/12	1/16	. . .	
	5	.	.	.	.		
	n	.	.	.	.		$1/n^2$

product of all elts

$$\boxed{H_n^2} = \underbrace{\left(1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \dots + \frac{1}{n}\right)}_{1/i}\underbrace{\left(1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \dots + \frac{1}{n}\right)}_{1/j} \rightarrow 1/ij$$

$= \sum$ all fractions

$-$diagonal $\sum \frac{1}{i^2}$

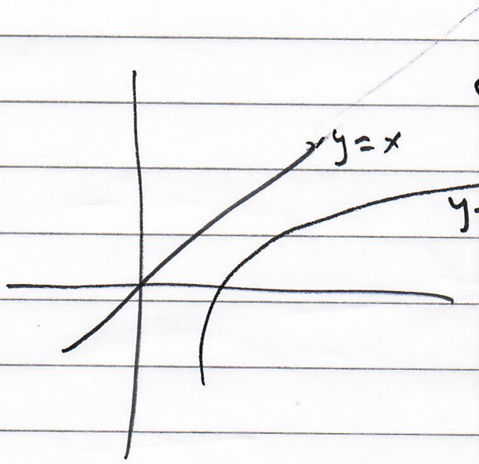
$\boxed{\text{Green}} + \text{Orange} = H_n^2$ $\quad 1/ij$

$\underset{S_n}{\text{Green}} = H_n^2 + \text{diag} - \text{Orange}$

$$S_n = \frac{1}{2}\left(H_n^2 + \sum 1/i^2\right)$$

$$\varphi_n''(1) = 2S_n - 2\sum 1/i^2 - 2H_n$$

VARIANCE = H_n − constant



σ = STD DEV = $\sqrt{H_n - \text{constant}}$

$\approx \sqrt{\ln n}$

AVG $\approx \ln n$