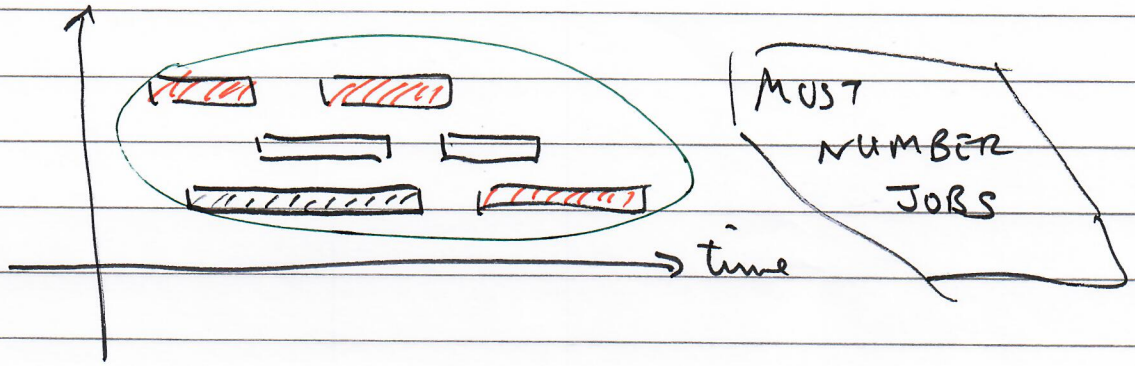
Activity Selection - Scheduling Problem



Maximize # of jobs scheduled

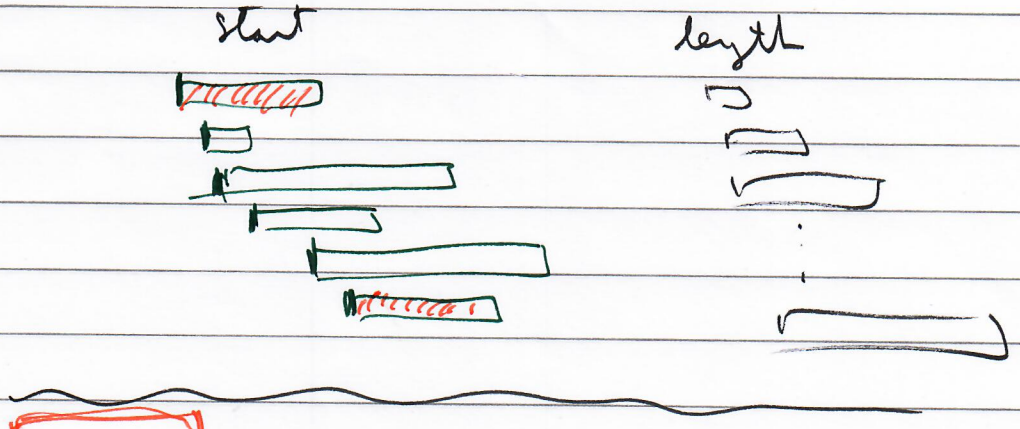
{A, B, C, D, E, F}

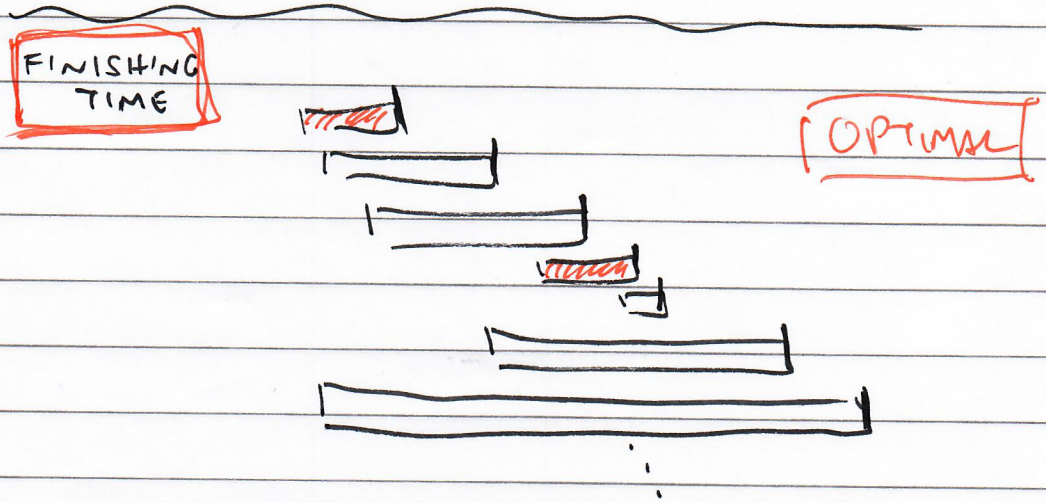
2^6

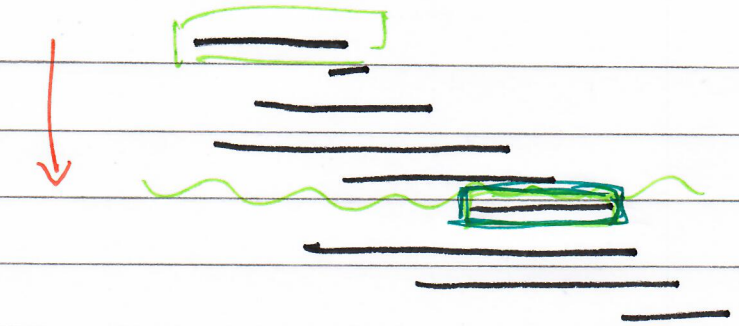
n jobs 2^n subsets

$$\binom{n}{0} + \binom{n}{1} + \binom{n}{2} + \cdots + \binom{n}{n} = 2^n - 1$$

GREED

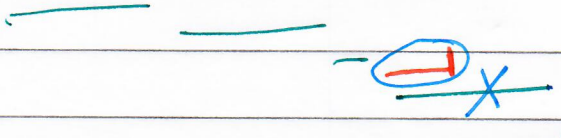


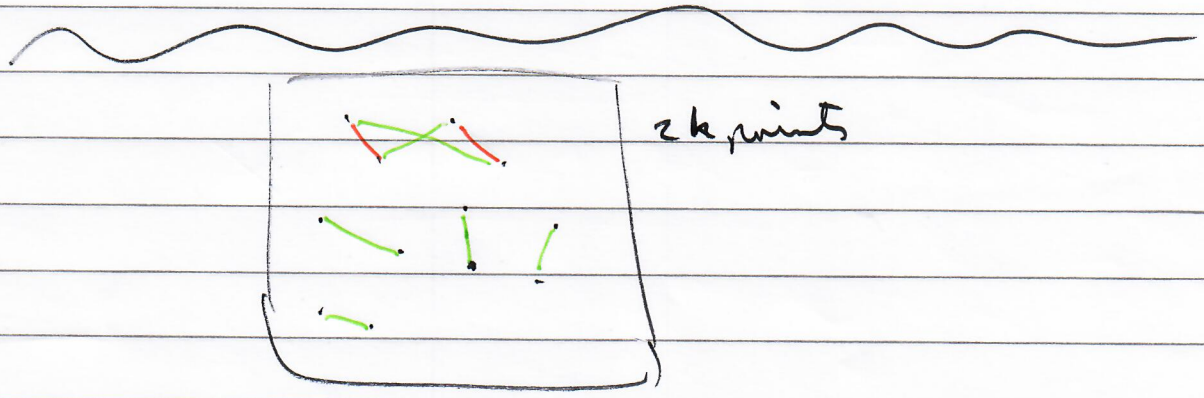




SUPPOSE NOT

FIRST MISTAKE





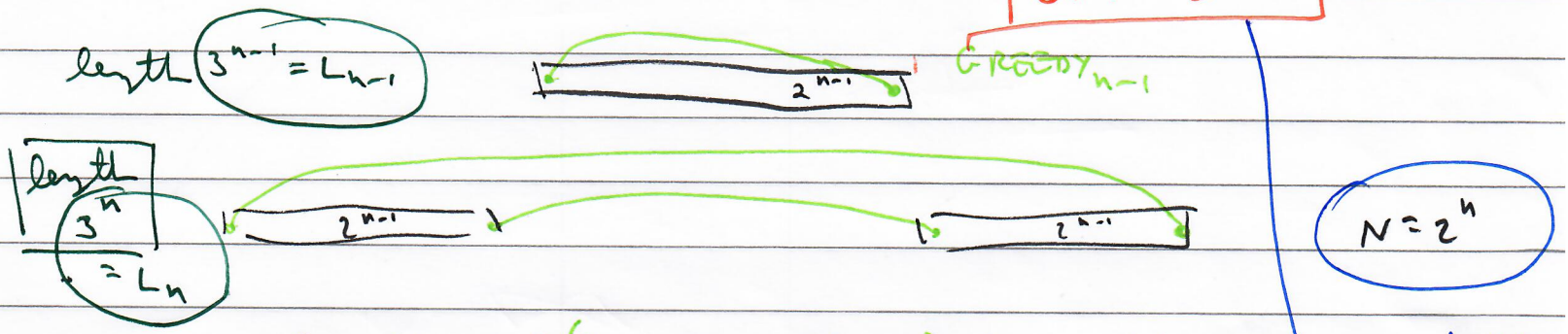
length = 1 — GREEDY = OPT = 1

length = 3 — OPT = 2, GREEDY = 4

length = 9 — OPT = 4, GREEDY = 14

length = 27 — OPT = 2^{n-1}

length $3^{n-1} = L_{n-1}$ — GREEDY$_{n-1}$



$N = 2^n$

$n = \lg N$

$$\text{GREEDY}_n = 2(\text{GREEDY}_{n-1} - L_{n-1}) + L_{n-1} + L_n$$

$$= 2(\text{GREEDY}_{n-1} - 3^{n-1}) + 3^{n-1} + \underbrace{3^n}_{3 \cdot 3^{n-1}}$$

$$\underbrace{\text{GREEDY}_n}_{(c=2)} = \underbrace{2\,\text{GREEDY}_{n-1}}_{(c=2)} + \underbrace{\frac{2 \cdot 3^n}{3}}_{(c=3)}$$

$$\Rightarrow \boxed{\text{GREEDY}_n = u\,2^n + v\,3^n = 2 \cdot 3^{n-1} - 2^{n-1}}$$

"BADNESS RATIO"

$$\frac{\text{GREEDY}_n}{\text{OPT}_n} = \frac{2 \cdot 3^{n-1} - 2^{n-1}}{2^{n-1}} = 2\left(\frac{3}{2}\right)^{n-1} - 1$$

$$= \frac{2}{3} 2 \left(\frac{3}{2}\right)^{\lg N} - 1$$

$$= \frac{4}{3}\left(2^{\lg 3/2}\right)^{\lg N} - 1 = \frac{4}{3}\left(2^{\lg N}\right)^{\lg 3/2} - 1$$

$$\boxed{\frac{\text{GREEDY}_n}{\text{OPT}_n} = \frac{4}{3} N^{\lg 3/2} - 1}$$

N points