$m \leftarrow 1$

for $k \leftarrow 2$ to $\boxed{n}$ do

  if $x_k > x_m$

    $\boxed{m \leftarrow k}$

finds max in $x_1 \cdots x_n$

How many times ?

Best case 0

Worst case n-1

$\boxed{\text{Average ?}}$ — $\boxed{\ln n}$

160 NYC
6 record

$\ln 160 \approx 5.7$

$x_1\ x_2\ x_3 \cdots\ x_n$

n = 3

6 possible inputs

$x_1 < x_2 < x_3$	$\frac{1}{6} \times$	2
$x_1 < x_3 < x_2$	$1/6 \times$	1
$x_2 < x_1 < x_3$		1
$x_2 < x_3 < x_1$		0
$x_3 < x_1 < x_2$		1
$x_3 < x_2 < x_1$	$1/6 \times$	0

5/6 equally likely inputs

$P_3(0) = 2$

$P_3(1) = 3$

$P_3(2) = 1$

Expected value $\rightarrow$ $\boxed{\sum \text{prob} \times \text{cost}}$

$1 \times \$1 = \$1$ = expected value / KNOW

RANDOM: $-\$1\quad -\$1\quad +\$1$

$$\frac{2}{3} \times (-\$1) + \frac{1}{2}(\$1) = -\frac{1}{3}\$1$$

# of input sequences

$P_n(i) =$ # of i assignments w/ n items

$i \geq n$ $= 0$

$$p_n(i) = \frac{P_n(i)}{n!}$$

$n!$ possible input arrangements

$x_1 \; x_2 \; \cdots \; x_{n-1}$ x_n

largest — 1 more time

not largest — no last assignment

$i-1$

$P_{n-1}(i-1)$ = # of arrangements

i assignments

$P_{n-1}(i)$

RECURRENCE RELATION

$$P_n(i) = P_{n-1}(i-1) + \underbrace{(n-1)}_{n-1 \text{ possibilities}} P_{n-1}(i)$$

INITIAL CONDITIONS

$$P_n(0) = (n-1)!$$
$$P_n(n-1) = 1$$

$$p_n(0) = 1/n$$
$$p_n(n-1) = 1/n!$$

$$F_n = F_{n-1} + F_{n-2}$$
$$F_0 = F_1 = 1$$

Polynomials

$$p_n(i) = \frac{P_{n-1}(i-1)}{n!} + \frac{(n-1)\,P_{n-1}(i)}{n!}$$

$$= \frac{1}{n}\underbrace{\frac{P_{n-1}(i-1)}{(n-1)!}}_{p_{n-1}(i-1)} + \frac{n-1}{n}\underbrace{\frac{P_{n-1}(i)}{(n-1)!}}_{p_{n-1}(i)}$$

$$\boxed{p_n(i) = \frac{1}{n}\,p_{n-1}(i-1) + \frac{n-1}{n}\,p_{n-1}(i)}$$ $i \geq 1$

$$\varphi_n(x) = p_n(0)x^0 + p_n(1)x^1 + p_n(2)x^2 + \cdots$$

$$\boxed{\varphi_n(x) = \sum_{i=0}^{\infty} p_n(i)x^i}$$

$$\varphi_n'(x) = \sum_{i=1}^{\infty} i\,p_n(i)\,x^{i-1}$$

$$\boxed{\varphi_n'(1) = \sum_{i=1}^{\infty} i\,p_n(i)}$$

$$\varphi_n(x) = \sum_{i=0}^{\infty} p_n(i)x^i \qquad \text{(def)}$$

$$= \frac{1}{n} + \sum_{i=1}^{\infty} \boxed{p_n(i)}\,x^i$$

$$= \frac{1}{n} + \sum_{i=1}^{\infty}\left[\frac{1}{n}p_{n-1}(i-1) + \frac{n-1}{n}p_{n-1}(i)\right]x^i$$

$$= \frac{1}{n} + \frac{x}{n}\sum_{i=1}^{\infty} p_{n-1}(i-1)\,x^{i-1} + \frac{n-1}{n}\sum_{i=1}^{\infty} p_{n-1}(i)\,x^i$$

$$= \frac{1}{n} + \frac{x}{n}\boxed{\sum_{i=0}^{\infty} p_{n-1}(i)x^i} + \frac{n-1}{n}\Big[\varphi_{n-1}(x) - \underbrace{p_{n-1}(0)}_{1/(n-1)}\Big]$$

$$= \cancel{\frac{1}{n}} + \frac{x}{n}\boxed{\varphi_{n-1}(x)} + \frac{n-1}{n}\varphi_{n-1}(x) - \cancel{\frac{1}{n}}$$

$$\boxed{\varphi_n(x) = \frac{x+n-1}{n}\varphi_{n-1}(x)}$$

Differentiate & set $x=1$

$$\varphi_n'(x) = \frac{d}{dx}\left[\frac{x+n-1}{n}\right]\varphi_{n-1}(x) + \frac{x+n-1}{n}\,\varphi_{n-1}'(x)$$

$$\varphi_n'(x) = \frac{1}{n}\,\varphi_{n-1}(x) + \underline{\frac{x+n-1}{n}\,\varphi_{n-1}'(x)}$$

$$\varphi_n'(1) = \frac{1}{n} + \varphi_{n-1}'(1)$$

$$\boxed{\varphi_n'(1) = \frac{1}{n} + \varphi_{n-1}'(1)}$$

$$= \frac{1}{n} + \frac{1}{n-1} + \varphi_{n-2}'(1)$$

$$= \frac{1}{n} + \frac{1}{n-1} + \frac{1}{n-2} + \varphi_{n-3}'(1)$$

$$\vdots$$

$$= \frac{1}{n} + \frac{1}{n-1} + \frac{1}{n-2} + \cdots \frac{1}{2} + \cancelto{0}{\varphi_1'(1)}$$

$$\varphi_n'(1) = H_n - 1$$

$$\approx \ln n$$

Expectation of a sum = Sum of expectations