"Difference eqns"

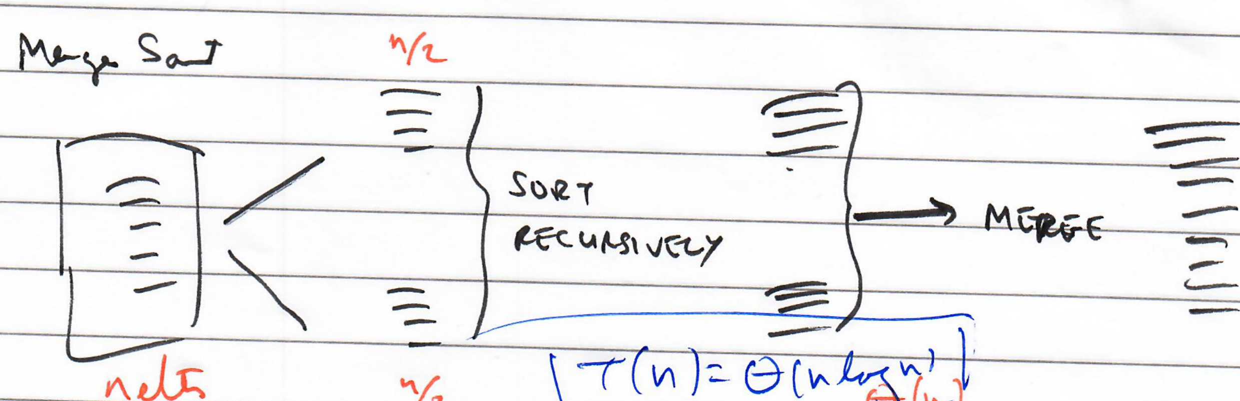
Linear homogeneous Recurrences

E — left shift

polynomial
↓
roots
↓
largest root

Divide + Conquer Recurrences

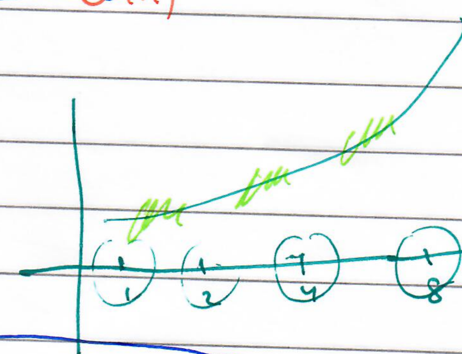
Merge Sort



$\boxed{T(n) = \Theta(n \log n)}$

$$\boxed{\begin{aligned} T(n) &= 2T(n/2) + \cancel{\Theta(n)}\ cn \\ T(1) &= \cancel{O(1)}\ k \end{aligned}}$$

$\boxed{n = 2^i}$



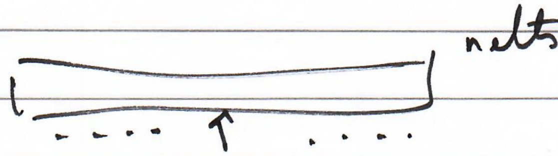
$$\left.\begin{aligned} T(2^i) &= 2T(2^{i-1}) + c2^i \\ T(2^0) &= k \end{aligned}\right\} \quad t_i = T(2^i)$$

$i = \lg n$

$$\begin{aligned} t_i &= \overbrace{2t_{i-1}}^{E-2} + \overbrace{c2^i}^{E-2} \\ t_0 &= k \end{aligned} \quad \Rightarrow \quad \overset{(E-2)^2}{t_i = i2^i}$$

$\Rightarrow T(n) = (\lg n)n$

Binary Search



$O(1)$

$$\boxed{T(n)}^{\,i} = O(1) + \boxed{T(n/2)}_{\,i-1}$$

$$T(1) = O(1)$$

$$\boxed{n = 2^i} \qquad i = \lg n$$

$$t_i = T(2^i) = \Theta(i)$$

$\langle c, c, c, \dots \rangle$

$E - 1$

$$\boxed{t_i = \boxed{O(1)} + \boxed{t_{i-1}}} \qquad E-1$$

$E-1$

$$(E-1)^2 \Rightarrow i \cdot 1^i$$

$$\Theta(i)$$

$$T(n) = \Theta(\lg n)$$

$$T(n) = a\,T(n/b) + \cancel{n}\ \cancel{1}\ n^2$$

$$\boxed{n = b^i} \Rightarrow i = \lg n \qquad t_i = T(b^i)$$

$$\underbrace{t_i = a\,t_{i-1}}_{E-a} + \underbrace{b^i}_{E-b}$$

$(E-a)(E-b)$

$a \neq b$: $\;\square a^i + \square b^i$, $\;\Theta(n)$

$a = b$: $\;\square\, i\, b^i$, $\;(\lg n)(n)$

$$T(n) = \Theta(n \lg n)$$

$$a = b^{\log_b a}$$

$$a^i = \left(b^{\log_b a}\right)^i = \left(b^i\right)^{\log_b a} = \Theta\!\left(n^{\log_b a}\right)$$

$$\frac{a f(n/b)}{f(n)} < 1 \qquad \boxed{T(n) = a T(n/b) + f(n)}$$

$$\frac{a f(n/b)}{f(n)} < 1$$

$$\frac{T(n)}{f(n)} = \frac{a}{f(n)} T(n/b) + 1$$

$$\boxed{\frac{T(n)}{f(n)}} = a \frac{f(n/b)}{f(n)} \boxed{\frac{T(n/b)}{f(n/b)}} + 1$$

n/b in place of n

$$\boxed{\frac{T(n/b)}{f(n/b)}} = \boxed{a \frac{f(n/b^2)}{f(n/b)} \frac{T(n/b^2)}{f(n/b^2)} + 1}$$

$$\frac{T(n)}{f(n)} = a \frac{f(n/b)}{f(n)} \left[a \frac{f(n/b^2)}{f(n/b)} \frac{T(n/b^2)}{f(n/b^2)} + 1 \right] + 1$$

$$= \underbrace{\frac{f(n/b)}{f(n)} a^2 \frac{f(n/b^2)}{f(n/b)}}_{<1} \boxed{\frac{T(n/b^2)}{f(n/b^2)}} + a \frac{f(n/b)}{f(n)} + 1$$

$$\leq a^2 \frac{T(n/b^2)}{f(n/b^2)} + a + 1$$

$$\vdots$$

$$< a^i \frac{T(n/b^i)}{f(n/b^i)} + \underbrace{a^{i-1} + a^{i-2} + \cdots + 1}$$

$i = \log_b n$

a^i

$T(n) = \sqrt{n}\, T(\sqrt{n}) + n$ / $T(n) = \Theta(n \lg \lg n)$