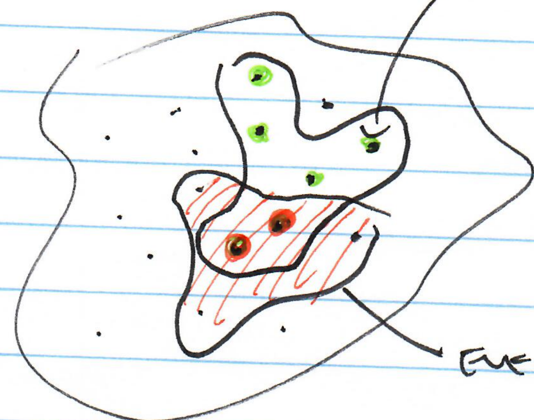
CONDITIONAL PROBABILITY



$\frac{|F \cap E|}{|E|}$ $\frac{|E \cap F|}{|F|}$

$P(E|F)$ = prob of event E given we know event F happened

$$P(E|F) = \frac{Pr(E \cap F)}{Pr(F)} \quad \frac{\frac{|E \cap F|}{|\Omega|}}{\frac{|F|}{|\Omega|}}$$

$$P(F|E) = \frac{Pr(F \cap E)}{Pr(E)} \Rightarrow Pr(F \cap E) = Pr(E)\,Pr(F|E)$$

BAYES LAW

$$\boxed{Pr(E|F) = \frac{Pr(E)\,Pr(F|E)}{Pr(F)}}$$

BAYES' THEOREM

$$\boxed{Pr(E|F) = \frac{Pr(E)\,Pr(F|E)}{Pr(F)}}$$

Data

Pr(breast cancer in a woman in her 40's) = $\frac{40}{10000}$

Pr(positive mammogram all women) = $\frac{1028}{10000}$

Pr(positive mammograph among cancer patients) = $\frac{32}{40}$

Pr(cancer in patient | positive mammogram)

$$= \frac{Pr(\text{cancer in 40's})\,Pr(\text{positive M.} \mid \text{breast cancer})}{Pr(\text{positive M})}$$

$$= \frac{\frac{40}{10000} \times \frac{32}{40}}{\frac{1028}{10000}} \approx 0.03$$

1 2 3

$$Pr(\text{gray}) = \underset{0}{Pr(\text{gray in Urn 1})} \times Pr(\text{Urn 1}) \quad 1/3$$

$$+ \underset{1}{Pr(\text{gray} \mid \text{Urn 2})}\, Pr(\text{Urn 2})$$

$$+ \underset{1/2}{Pr(\text{gray} \mid \text{Urn 3})}\, Pr(\text{Urn 3})$$

$$= 1/2$$

$$\Pr(\text{Urn } 1 \mid \text{gray}) = \frac{\Pr(\text{Urn } 1)\Pr(\text{gray} \mid \text{urn } 1)}{\Pr(\text{gray})}$$

$$= \frac{1/3 \times 0}{1/2} = 0$$

$$\Pr(\text{Urn } 2 \mid \text{gray}) = \frac{\Pr(\text{Urn } 2)\Pr(\text{gray} \mid \text{Urn } 2)}{\Pr(\text{gray})}$$

$$= \frac{1/3 \times 1}{1/2} = 2/3$$

$$\Pr(\text{Urn } 3 \mid \text{gray}) = \frac{\Pr(\text{Urn } 3)\Pr(\text{gray} \mid \text{Urn } 3)}{\Pr(\text{gray})}$$

$$= \frac{1/3 \times 1/2}{1/2} = \frac{1}{3}$$

$$\Pr(\text{Urn } 1 \mid 2\text{ gray}) = \frac{\Pr(\text{Urn } 1)\Pr(2\text{ gray balls} \mid \text{Urn } 1)}{\Pr(2\text{ gray balls})}$$

$$= \frac{1/3 \times 0}{\Pr(2\text{ gray balls})} = 0$$

$\Pr(2\text{ gray balls})$

$$= \underbrace{\boxed{\begin{aligned}&\Pr(\text{Urn } 1)\\ +&\Pr(\text{Urn } 2)\\ +&\Pr(\text{Urn } 3)\end{aligned}}}_{1/3}\ \begin{aligned}&\Pr(2\text{ gray from Urn } 1) \rightarrow 0\\ &\Pr(2\text{ gray from Urn } 2) \rightarrow 1\\ &\Pr(2\text{ gray from Urn } 3) \rightarrow 1/4\end{aligned}$$

$$= \frac{1}{3} \times 0 + \frac{1}{3} \times 1 + \frac{1}{3} \times \frac{1}{4}$$

$$= 5/12$$

$$\Pr(\text{Urn } 2 \mid 2\text{ gray balls}) = \frac{1/3 \times 1}{5/12} = \frac{4}{5}$$

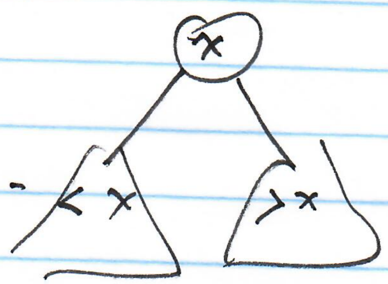
$$\Pr(\text{Urn } 3 \mid 2\text{ gray balls}) = \frac{1/3 \times 1/4}{5/12} = \frac{1}{5}$$

Exercise n times?

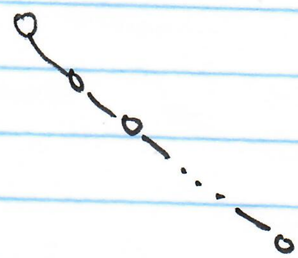
Balanced SEARCH Trees

LEXIOCOGRAPHIC TREE

DYNAMIC



height $O(\log n)$

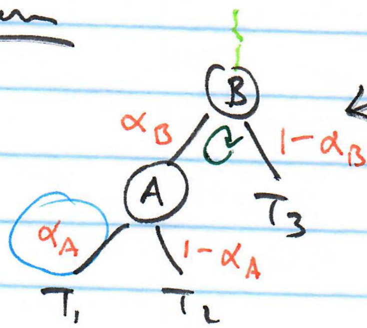


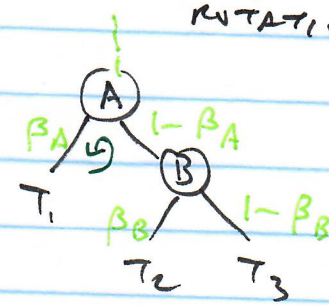
Rotation

"rebalance"

SINGLE ROTATION

?





$\alpha_A = \Pr(\text{left} \mid \text{got to } A)$

$\alpha_B = \Pr(\text{left} \mid \text{got to } B)$

$\beta_A = \Pr(\text{left} \mid \text{got } A)$

$\beta_B = \Pr(\text{left} \mid \text{got } B)$

express β's in terms of α's

$$\alpha_A = \Pr(<A \mid <B) = \frac{\overbrace{\Pr(<A)}^{\beta_A}\;\overbrace{\Pr(<B \mid <A)}^{1}}{\underbrace{\Pr(<B)}_{\beta_A + (1-\beta_A)\beta_B}}$$

$$\boxed{\alpha_B = \beta_A + (1-\beta_A)\beta_B}$$

$$\boxed{\alpha_A = \frac{\beta_A}{\beta_A + (1-\beta_A)\beta_B}}$$