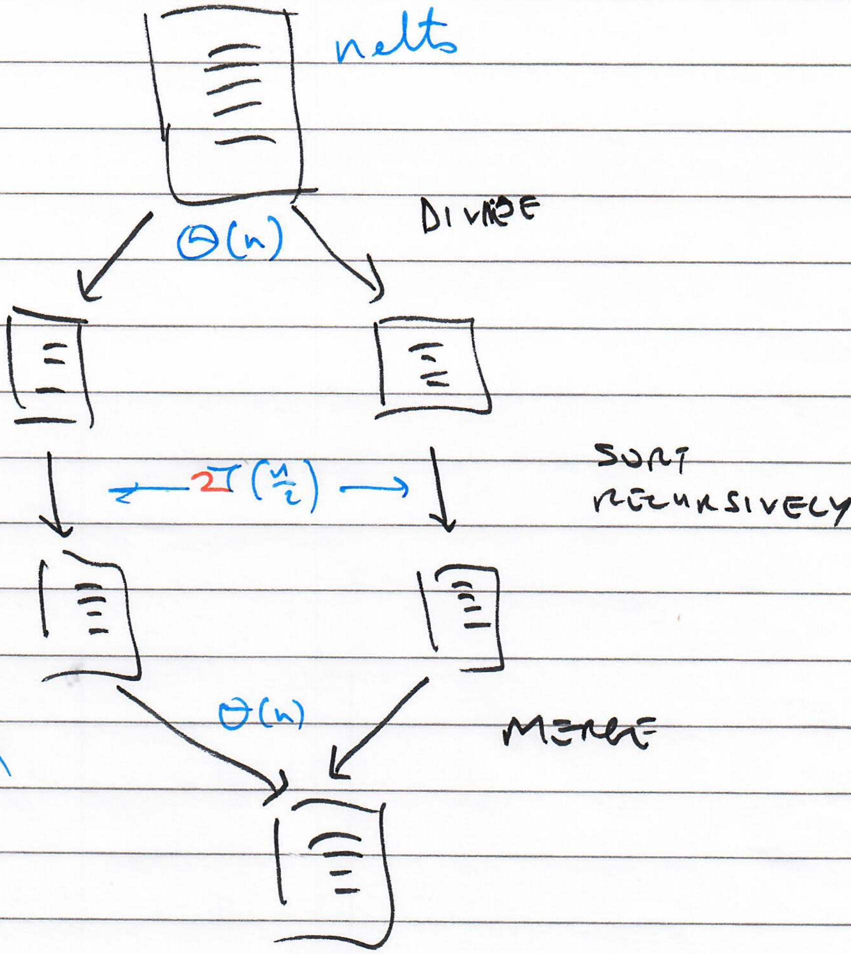
DIVIDE & CONQUER

Merge Sort

$T(n)$

$= 2T(n/2) + \Theta(n)$



$$\boxed{n = 2^i} \qquad T(n) = 2T(n/2) + \Theta(n)$$

$$T(2^i) = 2T(2^{i-1}) + \Theta(2^i) \qquad t_i = T(2^i)$$

$$\Rightarrow \quad \underbrace{t_i = 2t_{i-1}}_{(E-2)} + \Theta(2^i)$$

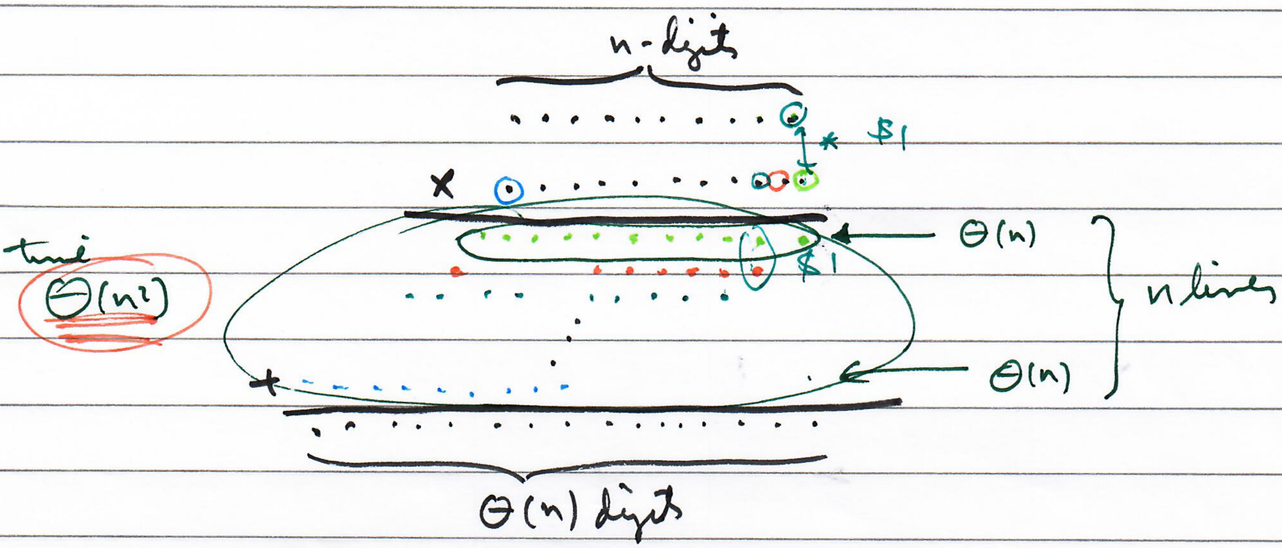
$$(E-2) \Rightarrow (E-2)^2$$

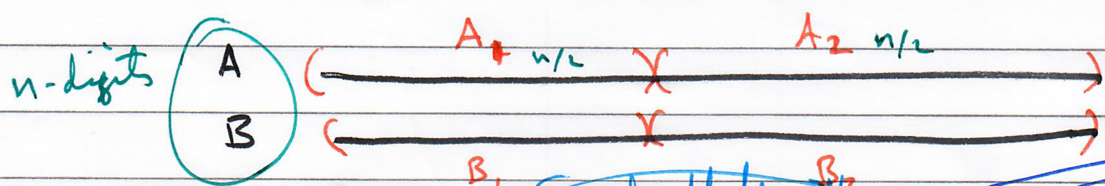
$$\Rightarrow t_i = \Theta(i\,2^i)$$

$i = \lg n$

$$\boxed{T(n) = \Theta(n \lg n)}$$

Multiply Integers





3 half length products $\Rightarrow \Theta(n^{1.58\ldots})$

$$\boxed{AB} = (A_2 + 10^{n/2} A_1)(B_2 + 10^{n/2} B_1)$$

$$A_2 B_2 + 10^{n/2}(A_1 B_2 + A_2 B_1) + 10^n A_1 B_1$$

$\Theta(n)$

$$\boxed{n = 2^i} \qquad t_i = T(2^i)$$

$$T(n) = \underbrace{\cancel{4}\,3\,T(n/2)}_{\text{DIVIDE}} + \underbrace{\Theta(n)}_{\text{CONQUER}}$$

$$t_i = \underbrace{\cancel{4}\,3\,t_{i-1}}_{E-\cancel{4}\,3} + \underbrace{\Theta(2^i)}_{E-2}$$

$$(E-\cancel{4}\,3)(E-2) \Rightarrow t_i = u3^i + v2^i$$

$$= \Theta(3^i)$$

$$3 = 2^{\lg 3} \qquad \lg 3 \approx 1.58$$

$$\Rightarrow 3^i = (2^{1.58\ldots})^i = (2^{i \cdot 1.58\ldots}) = (\underbrace{2^i}_{n})^{1.58\ldots}$$

$$AB = 10^n \boxed{A_1 B_1} + 10^{n/2} \boxed{(A_1 B_2 + B_1 A_2)} + \boxed{A_2 B_2}$$

$$(A_2 + A_1)(B_2 + B_1) = A_2 B_2 + \boxed{A_2 B_1 + A_1 B_2} + A_1 B_1$$

$$\underline{A_2 B_1 + A_1 B_2} = (A_2 + A_1)(B_2 + B_1) - A_2 B_2 - A_1 B_1$$

$$T(n) = 3T(n/2) + \underline{\Theta(n)}$$

$$\Rightarrow \Theta(n^{1.58\ldots})$$

$(A_1 \mid A_2 \mid A_3)\;(B_1 \mid B_2 \mid B_3)$

How many 1/3-length mults would result in a better algorithm

{ $\frac{1}{3}$ length mults.

lg n ~~√x~~ pieces

$\Theta(n \lg n \lg\lg n \lg\lg\lg n \; \ldots)$

Closest Pair of Points (Shamos)

preprocessing $\Theta(n \lg n)$

D + C $T(n) = 2T(n/2) + \Theta(n)$

$n = 2^i$ $t_i = 2t_{i-1} + \Theta(2^i)$

$(E-2)^2$ $t_i = \Theta(i 2^i)$

$\Rightarrow T(n) = \Theta(n \lg n)$

$\Theta(n \lg n)$