Social Security

Two systems:

- Fully funded
- Pay-as-you-go

Given endowment economy $y_1 = y$ with work at $t = 1$ and retirement at $t = 2$.

Preferences $U(c_t^y, c_{t+1}^o) = 2\sqrt{c_t^y} + \beta 2\sqrt{c_{t+1}^o}$ with $\beta = 1$

Assume population growth $N_{t+1} = (1+n)N_t$ and output growth $y_{t+1} = (1+g)y_t$.

Under the fully funded system, the budget constraints are $c_t^y + a = y - \tau_y$ and $c_{t+1}^o = (1+r)(a+\tau_y)$. The utility-maximizing $a = \left(\frac{1+r}{2+r} - \tau\right)y$, showing mandated savings τ_y crowding out individual savings at rate of return $1+r$.

Under the pay-as-you-go system, $B_t = \tau y_t N_t$ so $\frac{B_t}{N_{t-1}} = \tau y_t \frac{N_t}{N_{t-1}} = \tau y_t(1+n) = \tau y_{t-1}(1+g)(1+n)$ so the payoff is better when $(1+g)(1+n) > (1+r)$, e.g. approximately when $g + n > r$.

Optimal Taxation

- Government makes choices subject to constraints
  - Benevolent: maximizes welfare of citizens
  - Political economics: simple model: median voter
- Optimal labor income tax
- 2-period income model
- g in the first period with $\{\tau_i\}_{t=1}^2$
- Preferences are quasilinear: $U(c_1, c_2, h_1, h_2) = c_1 - \frac{1}{2}h_1^2 + \beta\big(c_2 - \frac{1}{2}h_2^2\big)$
- Indifferent about consumption when $\beta(1+r) = 1$ and otherwise consume at a corner solution
- $y = Ah \Rightarrow w = A = 1$ we can normalize the wage to 1
- Budget constraint: $c_1 + \frac{c_2}{1+r} = (1-\tau-1)w_1h_1 + \frac{(1-\tau_2)w_2h_2}{1+r}$ tells us $c_1 + \beta c_2 = (1-\tau_1)h_1 + \beta(1-\tau_2)h_2$
- Household problem is $\max\limits_{h_1,h_2}(1-\tau_1)h_1 + \beta(1-\tau_2)h_2 - \frac{1}{2}h_1^2 - \beta\frac{1}{2}h_2^2$
- How should the government set taxes given the FOCs of the above: $h_1^* = (1-\tau_1)$ and $h_2^* = (1-\tau_2)$.
- Government problem $\max_{\tau_1,\tau_2}(1-\tau_1)h_1 + \beta(1-\tau_2)h_2 - \frac{1}{2}h_1^2 - \beta\frac{1}{2}h_2^2$ such that $g = \tau_1h_1 + \beta\tau_2h_2$ and given that the household problem FOCs above are met
- A Ramsey equilibrium is a C.E. with endogenous taxes, i.e. tax levels and household choices such that:
  1. Given taxes, households and firms optimize
  2. Markets clear
  3. Intertemporal budget budget constraint is balanced
  4. Government optimizes the government problem subject to the household reaction function
- $\mathcal{L} = \frac{1}{2}(1-\tau_1)^2 + \beta\frac{1}{2}(1-\tau_2)^2 + \lambda(\tau_1(1-\tau_1) + \beta\tau_2(1-\tau_2) - g)$ then $\tau_1^* = \tau_2^* = \frac{\lambda-1}{2\lambda-1}$.
- This result is called tax smoothing: taxes should be the same in both periods.