Assumptions:

- Rationality
- Competitive behavior
- Prices as parameters (no adverse selection)

Household problem

$\max_{c,l} U(c,l) + V(g)$ such that $c \leq w(1-l) + d - t$, $c \geq 0$, $l \geq 0$, $l \leq 1$

Note from Inada Conditions the nonnegativity of c, l are not binding conditions, and optimization of $V(g)$ is independent of optimization of $U(c,l)$ (though non-independence could be a good exam question!)

Now reduced to $\max_{c,l} U(c,l)$ such that $c \leq w(1-l) + d - t$, $l \leq 1$

Solve using Lagrangian $\mathscr{L}(c, l, \lambda, \mu) = U(c,l) + \lambda\left[w(1-l) + d - t - c\right] + \mu(1-l)$

Note λ is the marginal utility of an value, μ is the marginal utility of time.

$\frac{\partial \mathscr{L}}{\partial c} = 0 \Rightarrow U_c(c^*, l^*) - \lambda = 0 \Rightarrow U_c(c^*, l^*) = \lambda$

Assuming the latter constraint is slack, e.g. at the solution ($\mu = 0$)

$\frac{\partial \mathscr{L}}{\partial l} = 0 \Rightarrow U_l(c^*, l^*) - \lambda w = 0 \Rightarrow\Rightarrow U_l(c^*, l^*) = \lambda w$

The marginal rate of subsitution is equal to the ratio of the prices (wage being price of time): in real terms, $MRS_{c^*, l^*} = w$ ("Cornerstone of marginal economics")

The solution is where the indifference curve is tangent to the budget constraint.

To solve for c^*, l^*, use $\frac{U_l(c^*, l^*)}{U_c(c^*, l^*)} = w$ and $c^* = w(1-l^*) + d - t$.

The household problem has a unique solution because the indifference curve will only lie tangent to the budget constraint once by convexity of preferences.

If the optimum has $l^* > 1$ then the optimum feasible has $l^* = 1$, $c^* = d - t$ as a corner solution.

Example: $U(c,l) = \log c + A \log l$ (natural log)

- $\mathscr{L}(c, l, \lambda, \mu) = \log c + A \log l + \lambda\left[w(1-l) + d - t - c\right] + \mu(1-l)$
- FOC for c is $\frac{1}{c} = \lambda$; for l is $\frac{A}{l} = \lambda w$. Then $\frac{Ac}{l} = w$.
- Substitute for budget constraint then $A\left[w(1-l) + d - t\right] = wl$ has $l^* = \frac{A(w+d-t)}{(1+A)w}$, $h^* = 1 - l^* = \frac{1 - A\left(\frac{d-t}{w}\right)}{1+A}$
- Solve budget constraint, get $c^* = \frac{w+d-t}{1+A}$
- Agent thinks of the above as functions of the parameters d, t, w
- Effect of an increase in nonwage income (proxy for wealth) $d - t$ resulting in both increased leasure and consumption (sanity-check: both are normal goods). "Negative wealth effect on labor supply."

From $c + wl = w + (d - t)$, you see that high net worth individuals behavior is dominated by the substitution effect, whereas others' behaviors are dominated by the income effect.