Competetive Equilibrium for Dynamic Production Economy

Given $\bar{a}_1$, $\{\tau_t\}_t$, a competitive equilibrium is:

- a set of allocations (10, skip d): $\{c_t, l_t, n_t, k_t\}_t$, $\{a_t\}_t$, $\{d_t\}_t$
- a set of prices (4): $\{w_t, r_t\}_t$
- a government policy (2): $\{b_t\}_t$

such that:

1. Household Optimal (4): Given prices, taxes, and transfers, $\{c_t, l_t\}_t, a_t$ solve the Household Problem
   - FOC for labor at each time
   - Euler equation
   - Lifetime budget constraint
2. Firm Optimal (4, skip CRS condition): Given prices, $\{n_t, k_t\}$ solve the Firm problem
   - FOC for labor and capital at each time
   - $\forall t, d_t = 0$ by CRS
3. Labor Market Clears (4): $\forall t, n_t = 1 - l_t$ and $\forall t, w_t = sf_n(n_t, k_t)$
4. Capital Market Clears (4): $\forall t, a_t = k_t$ and $\forall t, r_t + \delta = z_t f_k(n_t, k_t)$
5. Goods Market Clears (2, but skip due to Walras):
   - $c_1 + i_1 = z_1 f(n_1, k_1)$
   - $c_2 = z_2 f(n_2, k_2) + (1-\delta)k_2$

   where investment $s_1 = i_1 = k_2 - (1-\delta)k_1$ is what is saved/invested between the time periods. As noted before $i_1 - \delta k_1 = k_2 - k_1$ and $s_1 = a_2 - \bar{a}_1$, and we have $s_1 = i_1 - \delta k_1$ from national accounts.
6. Government balances budget (2): $\forall t, b_t = \tau_t w_t n - t$

Planner Problem for Dynamic Production Economy

$\max_{c_1, l_1, c_2, l_2, k_2} u(c_1, l_1) + \beta u(c_2, l_2)$ such that

- First period feasible: $c_1 + k_2 - (1-\delta)k_1 = z_1 f(1 - l_1, k_1)$
- Second period feasible: $c_2 = z_2 f(1 - l_2, k - 2) + (1-\delta)k_2$

$$\mathcal{L} = u(c_1, l_1) + \beta u(c_2, l_2) + \lambda_1(z_1 f(1-l_1, k_1) - c_1 - k_2 + (1-\delta)k_1) + \lambda_2(z_2 f(1-l_2, k-2) + (1-\delta)k_2 - c_2)$$

- FOC c_1: $u_c(c_1, l_1) = \lambda_1$
- FOC c_2: $\beta u_c(c_2, l_2) = \lambda_2$
- FOC l_1: $u_l(c_1, l_1) = \lambda_1 z_1 f_n(1 - l_1, k_1)$
- FOC l_2: $\beta u_l(c_2, l_2) = \lambda_2 z_2 f_n(1 - l_2, k_2)$
- FOC k_2: $\lambda_1 = \lambda_2(z_2 f_k(1 - l_2, k - 2) + (1 - \delta))$

This gives us $\frac{u_l(c_t, l-t)}{u_c(c_t, l_t)} = z_t f_n(1 - l_t, k_t)$, which is not the competitive equilibrium becaues it would have been $(1 - \tau_t)w_t$. There is a wedge resulting in workers performing less labor.

The Euler equation comes out to $\frac{u_c(c_1, l_1)}{u_c(c_2, l_2)} = \beta(1 + z_2 f_n(1 - l_2, k_2) - \delta)$, which is equal to $\beta(1 + r_2)$ at competitve equilibrum by the firm MPC.

So the wedge is in the intratemporal first order condition due to the income tax. By contrast, a tax on capital income would show up in the Euler equation (which would be $\beta(1 + (1 - \tau_k)r_2)$ at equilibrium).

Indifference Curves

Given $U = u(c_1, l_1) + \beta u(c_2, l_2)$, we have $0 = u_c(c_1, l_1)dc_1 + \beta u_c(c_2, l_2)dc_2$, or $\frac{dc_2}{dc_1}\mid_{U=\bar{U}} = \frac{-u_c(c_1,l_1)}{\beta u_c(c_2,l_2)}$.

Just as the wage is the slope of the budget constraint in the static economy, $1 + r_2$ is the slope of the budget constraint in the two period economy.

To see this, we look at a simpler model, the **Endowment Economy** , also called **Lucas Tree Economy** because you are 'growing your endowment on a tree'. The one actor has y_1, y_2 as endowments, and responds to the storage rate $1 + r_2$: $\max_{c_1, c_2} u(c_1) + \beta u(c_2)$ such that $c_1 + a_2 = y_1$ and $c_2 = y_2 + (1 + r_2)a_2$. The lifetime budget constraint is $c_1 + \frac{c_2}{1+r_2} = y_1 + \frac{y_2}{1+r_2}$. Here it is clear that the slope is $-(1 + r_2)$.

In general, just as the wage is a relative price (of leisure) it also seems sensible that the rental rate would be a relative price, namely $1 + r_2 = \frac{p_1}{p_2}$. Real interest rate normalizes across changes in intertemporal purchasing power. High interest rate means the goods in period 2 are cheaper in nominal prices. The necessary equations to see this are $p_1c_1 + p_1a_2 = p_1y_1$ and $c_2 = y_2 + \frac{p_1}{p_2}a_2$.