Economics with Uncertainty

Random variable $s \in \mathcal{S} = \{\bar{s}_i\}_{i=1}^N$ represents the state of nature, the world, the economy.

In the 2-period economy, at $t = 1$ we have no uncertainty, and at $t = 2$ we go to one of the outcomes of s with probability $\Pi(s)$.

W.L.O.G. assume $y_2(\bar{s}_i) \leq y_2(\bar{s}_j) \Leftrightarrow i \leq j$. Then the household problem is:

$$\max \mathrm{Ex}\left[\sum_{t=1}^{2} \beta^{t-1} u(c_t)\right] = \max u(c_1) + \beta\, \mathrm{Ex}_1\left[u(c_2(S))\right] = \max u(c_1) + \beta \sum_{s \in \mathcal{S}} \Pi(s) u(c_2(s))$$

such that the budget constraints $c_1 + a_2 = y_1$ and (w.p. $\Pi(s)$) $c_2(s) = y_2(s) + R(a_2)$. Note $R = 1 + r$ is not dependent on s iff our savings are stored in a 'safe asset,' otherwise they are a 'risky asset.'

$$\mathcal{L} = u(c_1) + \sum_{s \in \mathcal{S}} \Pi(s)\left(\beta u(c_2(s)) + \lambda_s\left(y_1 + \frac{y_2(s)}{R} - c_1 - \frac{c_2(s)}{R}\right)\right)$$

$$\frac{dL}{dc_1} = 0 \Rightarrow u_c(c_1) = \sum_{s \in \mathcal{S}} \Pi(s)\lambda_s$$

$$\frac{dL}{dc_2(s)} = 0 \Rightarrow \Pi(s)\beta u_c(c_2(s)) = \frac{\Pi(s)\lambda_s}{R}$$

$$\beta \sum_{s \in S} \Pi(s) u_c(c_2(s)) = \frac{1}{R} \sum_{s \in S} \Pi(s)\lambda_s$$

$$\beta R\, \mathrm{Ex}_1\left[u_c(c_2(S))\right] = \sum_{s \in S} \Pi(s)\lambda_s = u_c(c_1)$$

So the Euler equation is exactly as expected, but in expectation in period 1.

Example

Let $u(c_t) = \alpha c_t - \frac{1}{2}\gamma c_t^2$ then set $\beta R = 1$ to simplify $u_c(c_t) = \alpha - \gamma c_t$. Then the Euler equation $\alpha - \gamma c_1 = \mathrm{Ex}_1\left[\alpha - \gamma c_2(s)\right]$ simplifies to $\mathrm{Ex}_1\left[c_2(s)\right] = c_1$. Optimal consumption smoothing under uncertainty is make consumption choice so that expected consumption in period 2 is equal to consumption in period 1. This generalizes deterministic consumption smoothing.

Stochastic models

Consider the $AR(1)$ stochastic process $X_{t+1} = \rho X_t + \varepsilon_{t+1}$ for $\rho \leq 1$ and $\forall t, \varepsilon_t \sim \mathcal{N}(0, \sigma)$. This process has mean reversion since $\bar{x}_{t+1} - \mu = \rho \bar{x}_t - \mu$ in expectation when $\rho < 1$. When $\rho = 1$ this is a random walk.

In the general case, consumption is a random walk.