Risk Aversion

Consider a bet z with $\mu = 0$. Would pay Π to avoid the bet where $u(c - \Pi) = \text{Ex}[u(c+z)]$.

Consider the Taylor expansion around $\Pi^* = 0$:

$$\text{Ex}[u(c+z)] \approx \text{Ex}\left[u(c+z^*) + u'(u+z^*)(z-z^*) + \frac{1}{2}u''(c+z^*)(z-z^*)^2\right]|_{z=z^*=0}$$

$$= u(c) + 0 + \frac{1}{2}u''(c)\,\text{Ex}\left[z^2\right] - \text{Ex}[z]^2$$

$$u(c) - u'(c)\Pi = u(c) + \frac{1}{2}u''(c)\sigma^2$$

$$\Pi = \frac{-\sigma^2 u''(c)}{2u'(c)}$$

where $\rho^{\text{ABS}}(c) = \frac{-u''(c)}{u'(c)}$ is the coefficient of absolute risk aversion, desire to smooth utility across outcomes.

With log utility, $u(c) = \log c$ then $u'(c) = \frac{1}{c}$ and $u''(c) = \frac{-1}{c^2}$. Then $\rho^{\text{ABS}}(c) = \frac{1}{c}$.

With C-S utility, $u(c) = \frac{c^{1-\gamma}}{1-\gamma}$ then $u'(c) = \frac{1}{c^\gamma}$ and $u''(c) = \frac{-\gamma}{c^{\gamma+1}}$ has $\rho^{\text{ABS}}(c) = \frac{\gamma}{c}$.

These two examples are Decreasing Absolute Risk Aversion ($\frac{d\rho^{\text{ABS}}}{dc} < 0$).

With $u(c) = \frac{-1}{e^{gammac}}$ then $u'(c) = \frac{\gamma}{e^{\gamma c}}$ and $u''(c) = \frac{-\gamma^2}{e^{\gamma c}}$ so $\rho^{\text{ABS}}(c) = \gamma$ (Constant Absolute Risk Aversion).

With $u(c) = \alpha c - \frac{\beta}{2}c^2$ then $\rho^{\text{ABS}}(c) = \frac{\beta}{\alpha - \beta c}$ (Increasing Absolute Risk Aversion).

Note $\rho^{\text{ABS}}(c) \geq 0$ but $\frac{d\rho^{\text{ABS}}}{dc}$ can have any sign. But DARA fits data best.

Precautionary Savings

$\max_{c_0, c_1} u(c_0) + \beta\,\text{Ex}[u(c_1)]$ such that $c_0 + a_1 = y_0$ and $c_1 = Ra_1 + y_1$ where $y_1 = \bar{y} + \varepsilon_1$ and assume $\beta R = 1$.

When $\text{Var}[\varepsilon_1] = 0$ then $y_1 = \bar{y}$ we have $\text{Ex}[u'(Ra_1 + \bar{y})] = u'(Ra_1 + \bar{y})$.

When $\text{Var}[\varepsilon_1] > 0$ we have $\text{Ex}[u'(Ra_1 + \bar{y} + \varepsilon_1)] > u'(\text{Ex}[Ra_1 + \bar{y} + e_1]) = u'(Ra_1 + \bar{y})$.

The above by convexity of marginal utility $u''' > 0$ ("prudence") and Jensen's inequality.

Thus when the utility function displays prudence the equilibrium of the maximization problem at a higher variance is at a higher a_1 and so the household responds to risk by saving.