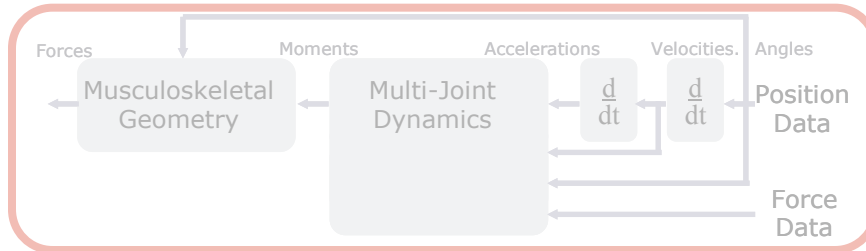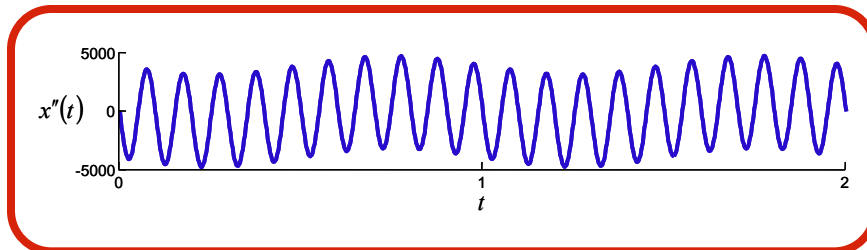**Inverse Dynamics**

Plan for Session

The inverse problem
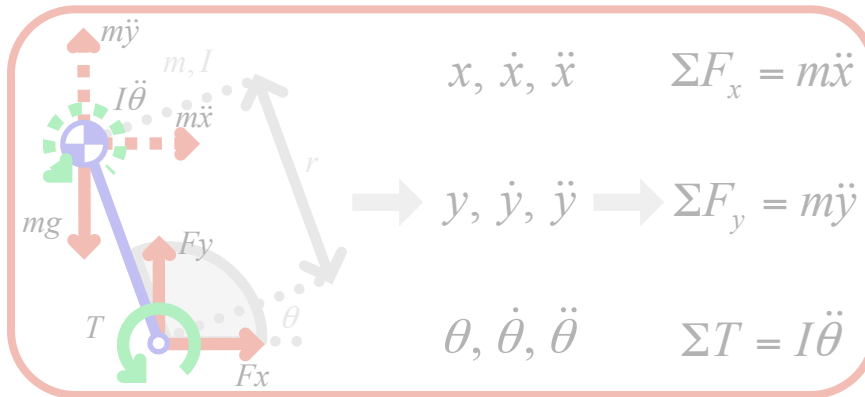

Going from subject motion to joint kinematics

Dealing with noisy data

Plan for Session (cont.)

$$x, \dot{x}, \ddot{x} \qquad \Sigma F_x = m\ddot{x}$$

$$y, \dot{y}, \ddot{y} \implies \Sigma F_y = m\ddot{y}$$

$$\theta, \dot{\theta}, \ddot{\theta} \qquad \Sigma T = I\ddot{\theta}$$

$m\ddot{y}$

m, I

$I\ddot{\theta}$

$m\ddot{x}$

mg

Fy

T

θ

Fx

r

Going from joint kinematics to joint moments

Major flexors

Major extensors

Gastrocnemius

Tibialis anterior

Soleus

Extensor digitorum

Tibialis posterior

Net ankle moment

M_a
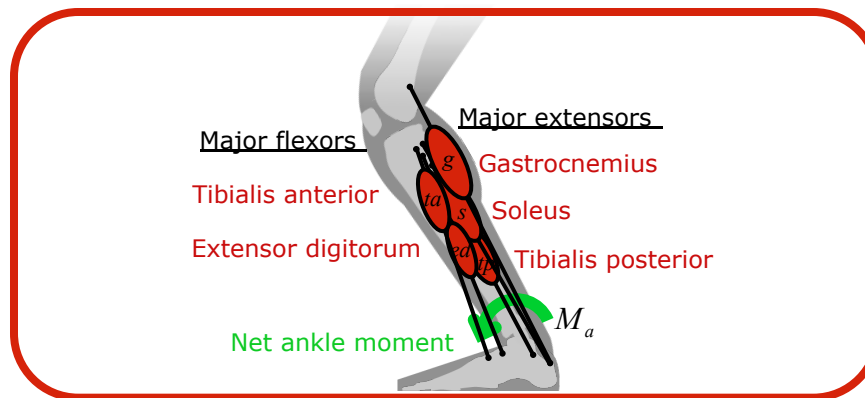
Solving the muscle force distribution problem

The Inverse Problem

Forces ← Musculoskeletal Geometry ← Moments ← Multibody Dynamics ← Accelerations ← $\dfrac{d}{dt}$ ← Velocities. ← $\dfrac{d}{dt}$ ← Angles ← Position Data

Force Data

The Inverse Problem

Forces   Moments   Accelerations   Velocities.   Angles

Musculoskeletal Geometry ← Multibody Dynamics ← $\dfrac{d}{dt}$ ← $\dfrac{d}{dt}$ ← Position Data

Force Data

F_z

M_z

F_y M_y M_x F_x
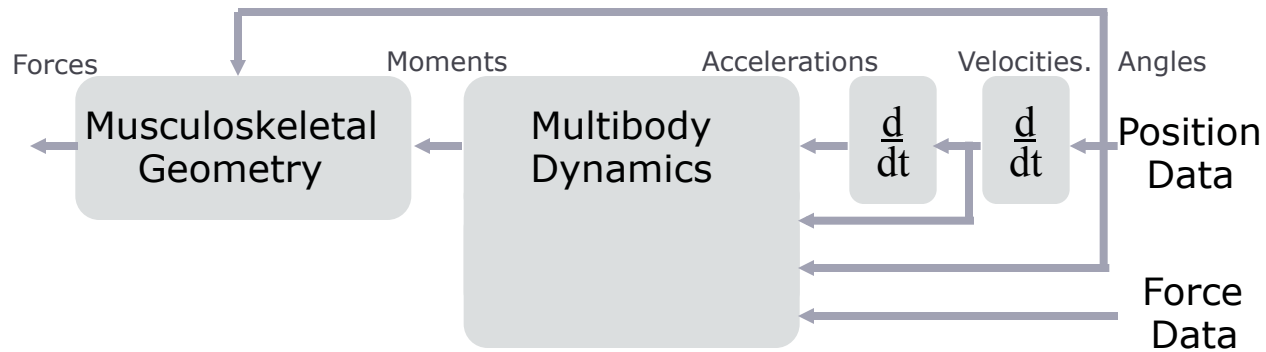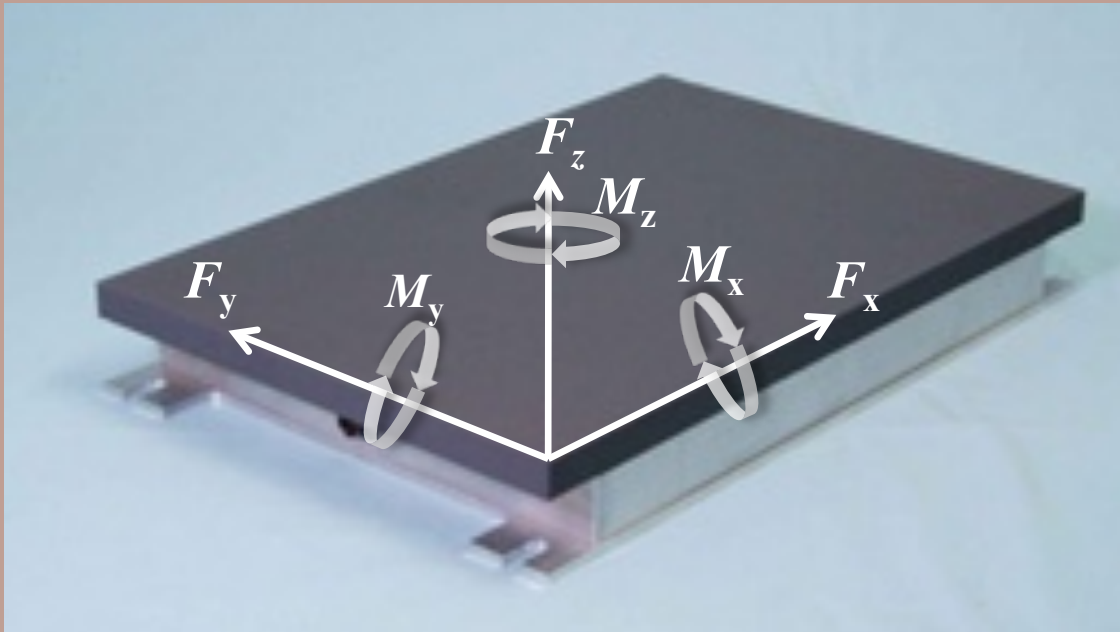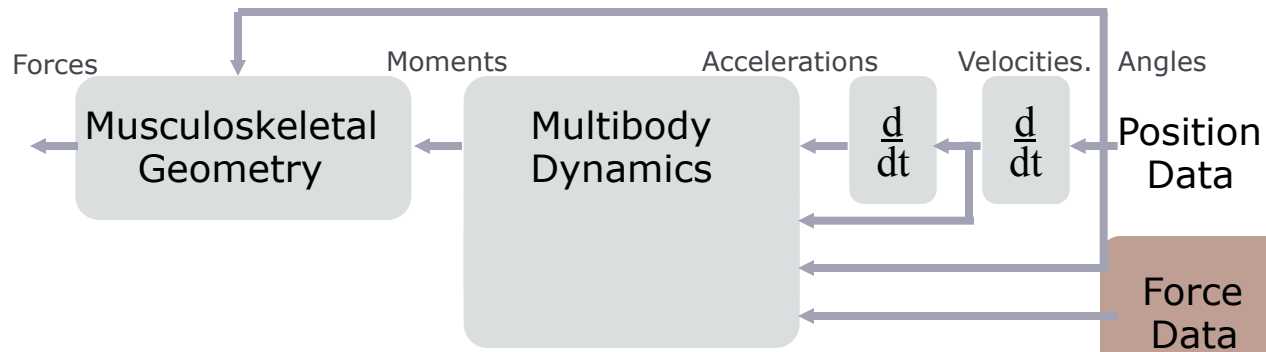
3 Forces
3 Moments

The Inverse Problem

Forces   Moments   Accelerations   Velocities.   Angles

$$\boxed{\text{Musculoskeletal Geometry}} \leftarrow \boxed{\text{Multibody Dynamics}} \leftarrow \boxed{\dfrac{d}{dt}} \leftarrow \boxed{\dfrac{d}{dt}} \leftarrow \boxed{\text{Position Data}}$$

Force Data
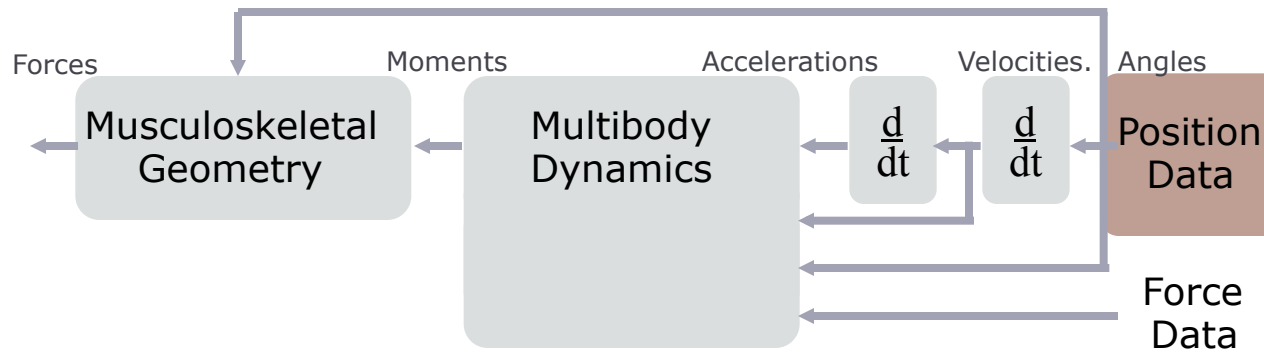
Video Cameras
Reflective Markers

The Inverse Problem

Forces	Moments	Accelerations	Velocities.	Angles

Musculoskeletal Geometry ← **Multibody Dynamics** ← $\dfrac{d}{dt}$ ← $\dfrac{d}{dt}$ ← **Position Data**

Force Data
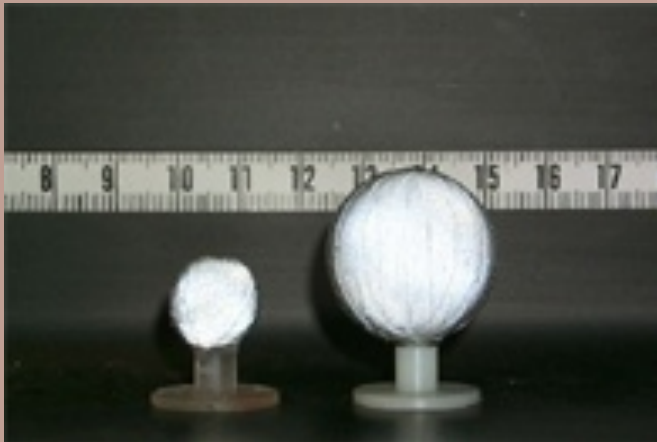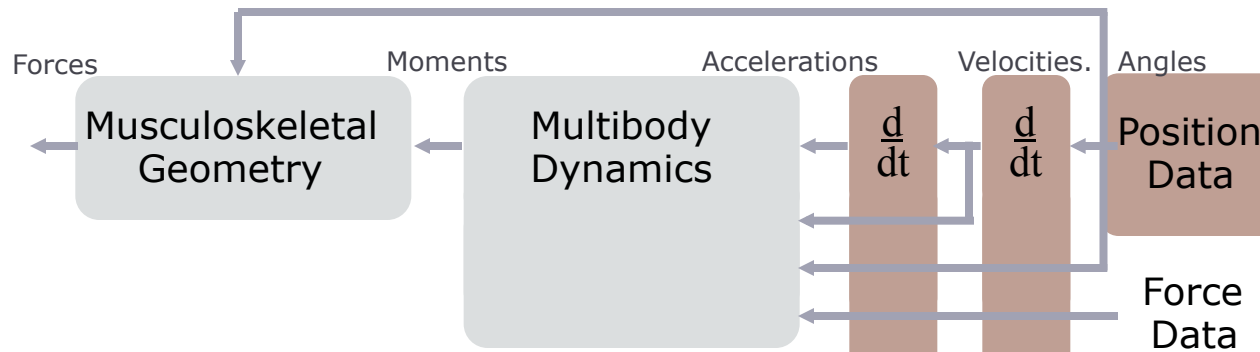
Inverse Kinematics

- Identify research question for the inverse problem
- Determine what should be measured and modeled
- Compute joint kinematics
- Filter and differentiate joint kinematics data
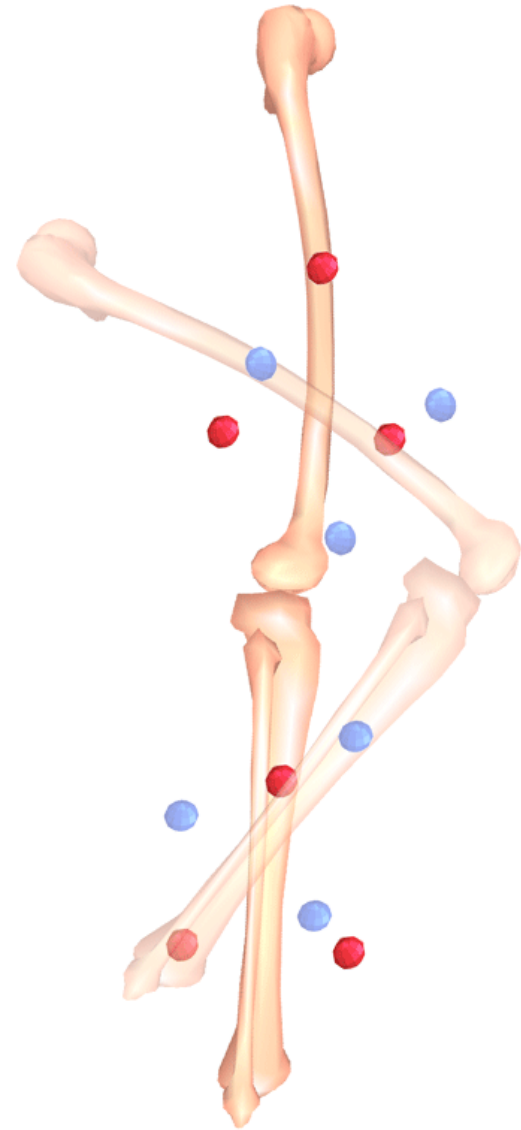
Example Research Questions

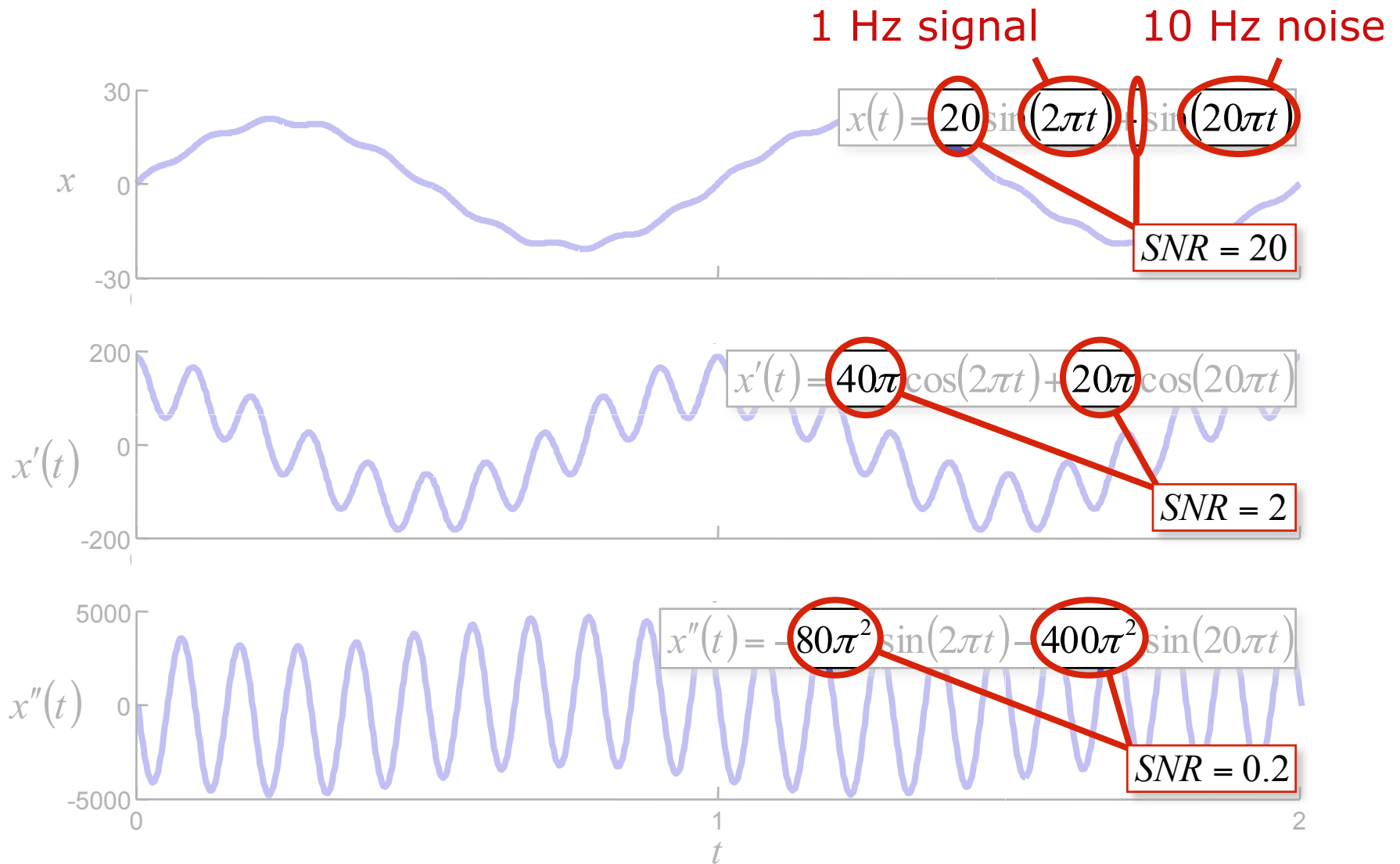Courtesy of Gillette Children's Specialty Healthcare

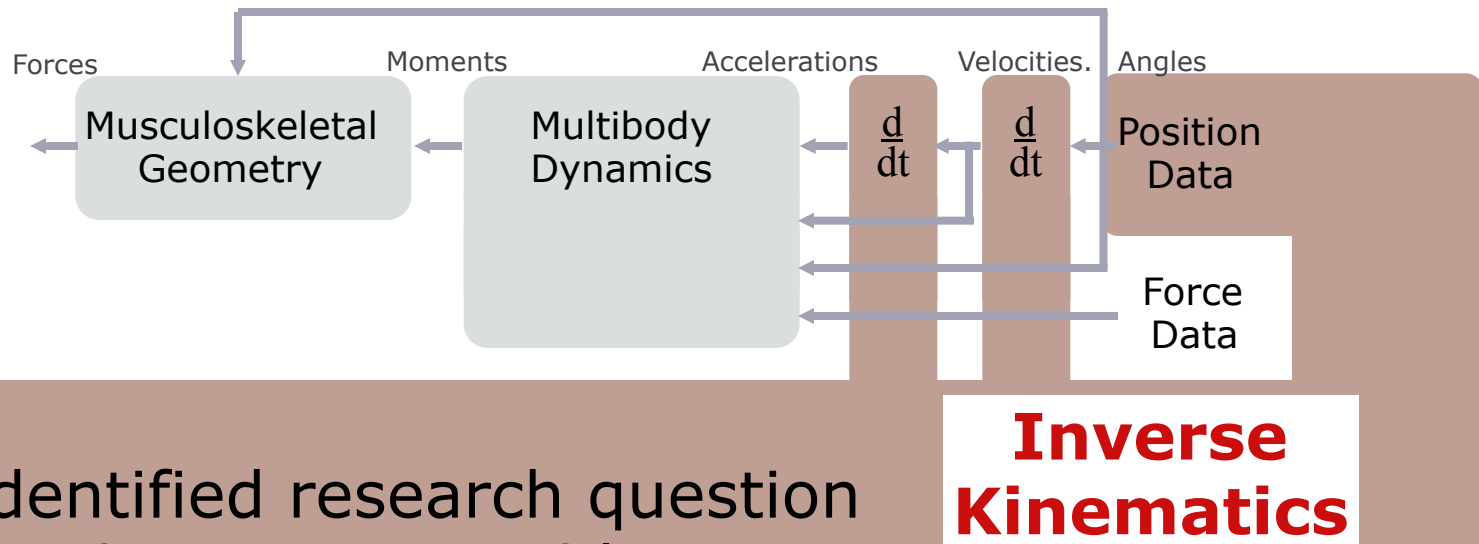Courtesy of John Hutchinson

Computing Joint Kinematics

Marker Error

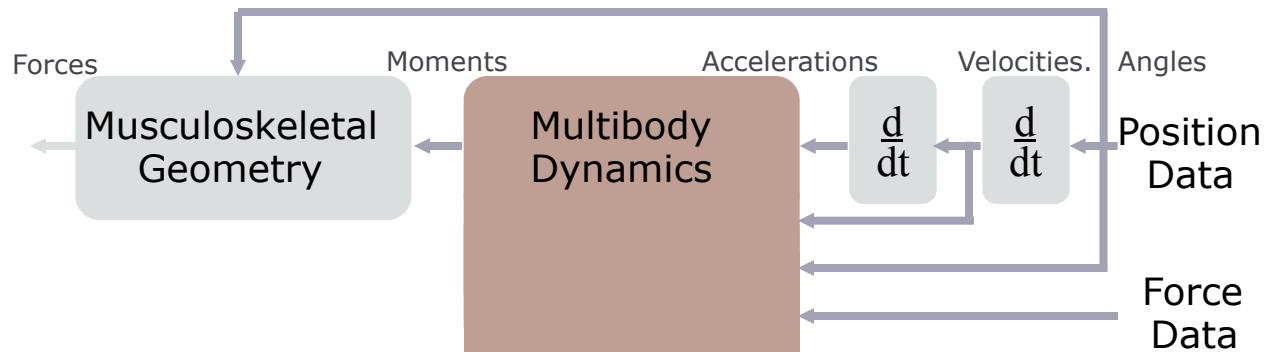# Differentiation Amplifies High-Frequency Noise

1 Hz signal   10 Hz noise

$$x(t) = 20\sin(2\pi t) + \sin(20\pi t)$$

$SNR = 20$

$$x'(t) = 40\pi\cos(2\pi t) + 20\pi\cos(20\pi t)$$

$SNR = 2$

$$x''(t) = -80\pi^2\sin(2\pi t) - 400\pi^2\sin(20\pi t)$$

$SNR = 0.2$

The Inverse Problem

Forces	Moments	Accelerations	Velocities.	Angles

Musculoskeletal Geometry ← Multibody Dynamics ← $\frac{d}{dt}$ ← $\frac{d}{dt}$ ← Position Data
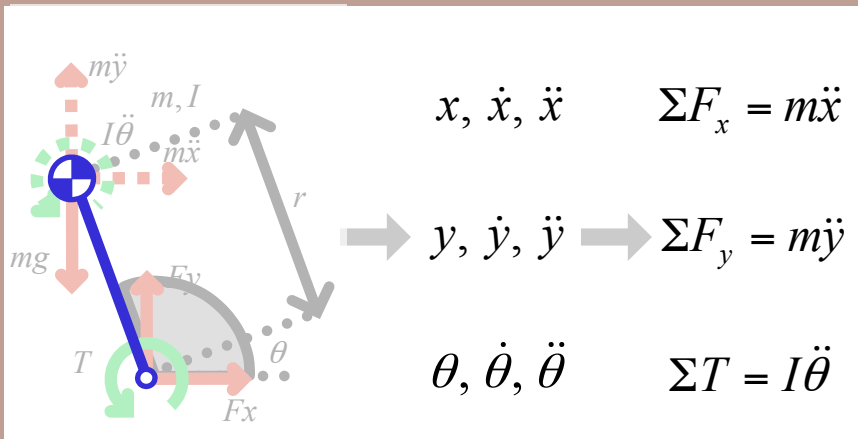
Force Data

Inverse Kinematics

- ✓ Identified research question for the inverse problem

- ✓ Determined what should be measured and modeled

- ✓ Computed joint kinematics

- ✓ Filtered and differentiated joint kinematics data

The Inverse Problem

| Forces | Moments | Accelerations | Velocities. | Angles |

Musculoskeletal Geometry ← Multibody Dynamics ← $\dfrac{d}{dt}$ ← $\dfrac{d}{dt}$ ← Position Data

Force Data

Inverse Dynamics

Inverse Kinematics

$$x, \dot{x}, \ddot{x} \qquad \Sigma F_x = m\ddot{x}$$

$$y, \dot{y}, \ddot{y} \Rightarrow \Sigma F_y = m\ddot{y}$$

$$\theta, \dot{\theta}, \ddot{\theta} \qquad \Sigma T = I\ddot{\theta}$$

- Derive equations of motion defining the model
- Solve equations of motion for joint moments

A Possible Inverse Dynamics Question

Experimental set-up

What are the sagittal plane moments about the ankle, knee, and hip during a maximum height jump?
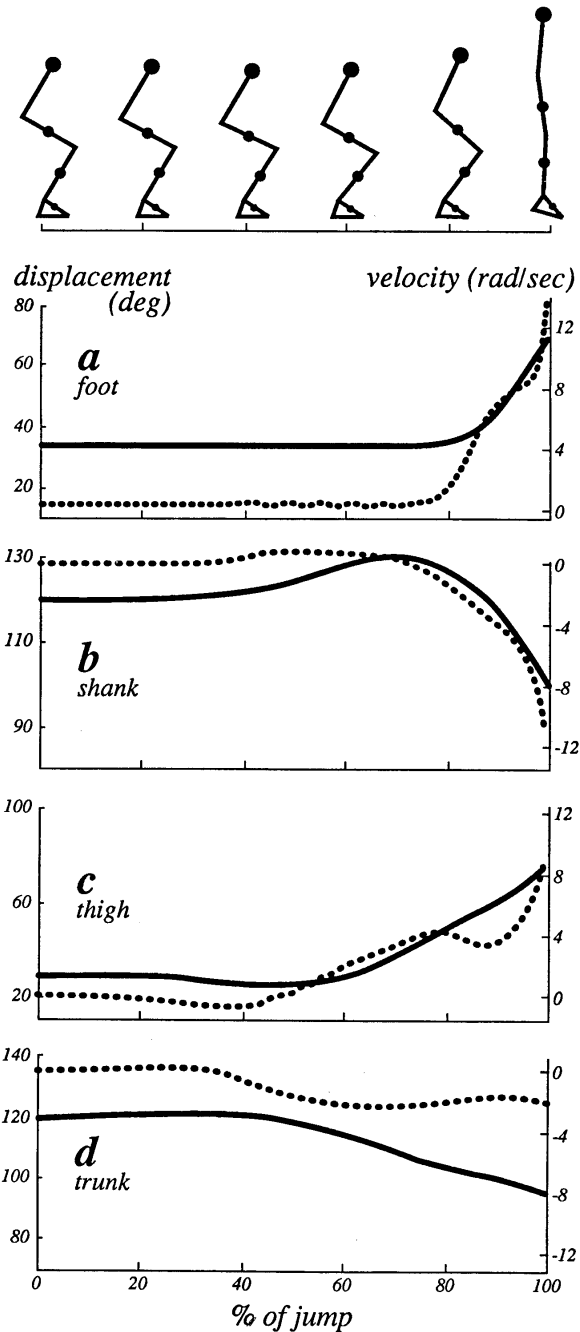
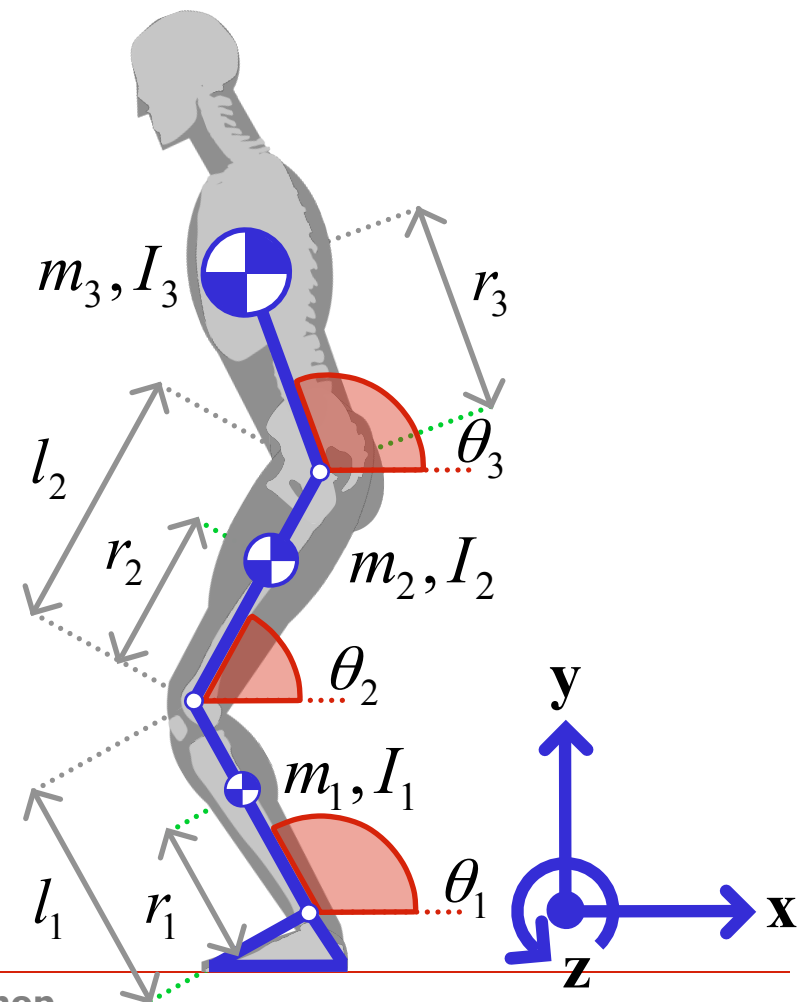# Inverse Dynamics Input: The Experimental Results

Experiments provide

- joint angles
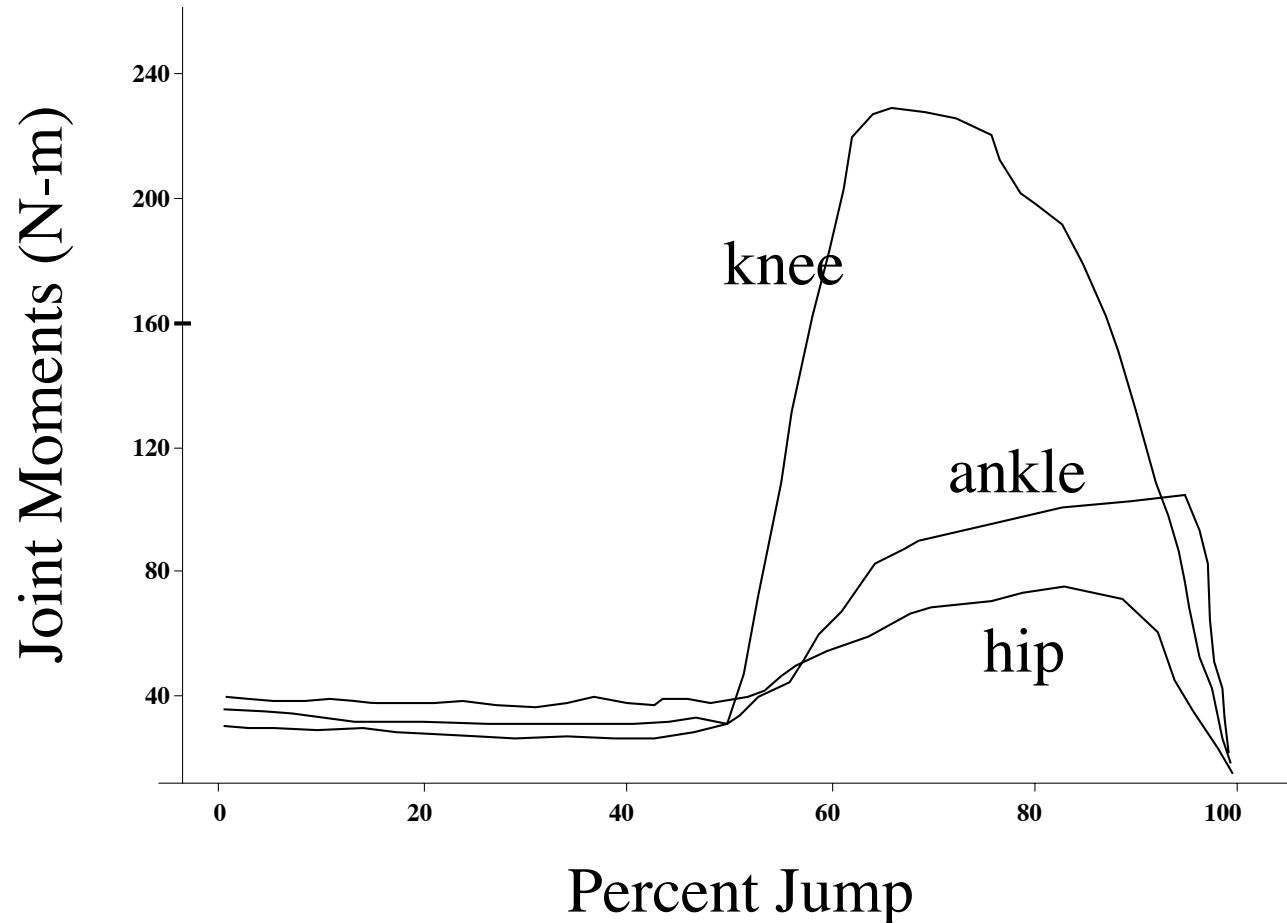- angular velocities
- ground reaction forces
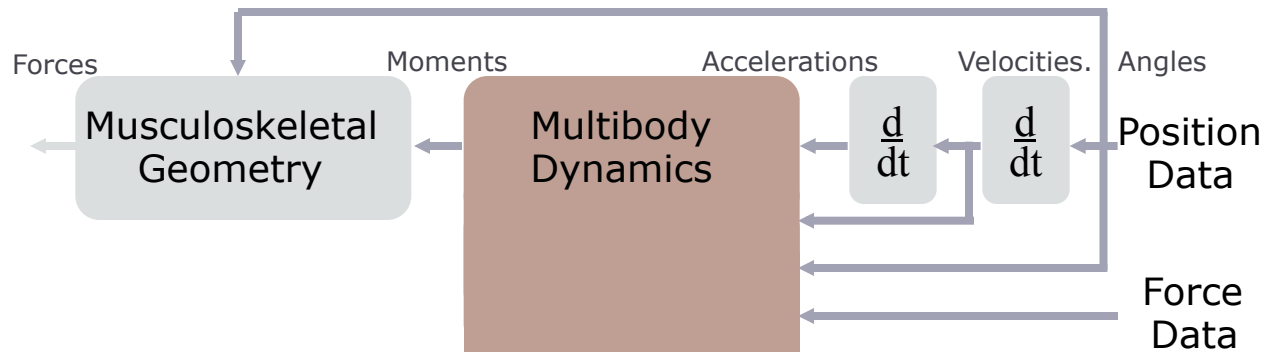
Inverse Dynamics Equations: Multibody Dynamics

- Planar 3 degrees of freedom
- Position (orientation) in global coordinate system
- Segment length = l_i
- Distance to mass center = r_i
- Moments of inertia about mass center
- Foot has no mass and remains on ground

m_3, I_3

r_3

θ_3

l_2

r_2

m_2, I_2

θ_2

m_1, I_1

l_1

r_1

θ_1

y

x

z

Inverse Dynamics Output: Net Joint Moments

The Inverse Problem

Forces   Moments   Accelerations   Velocities. Angles

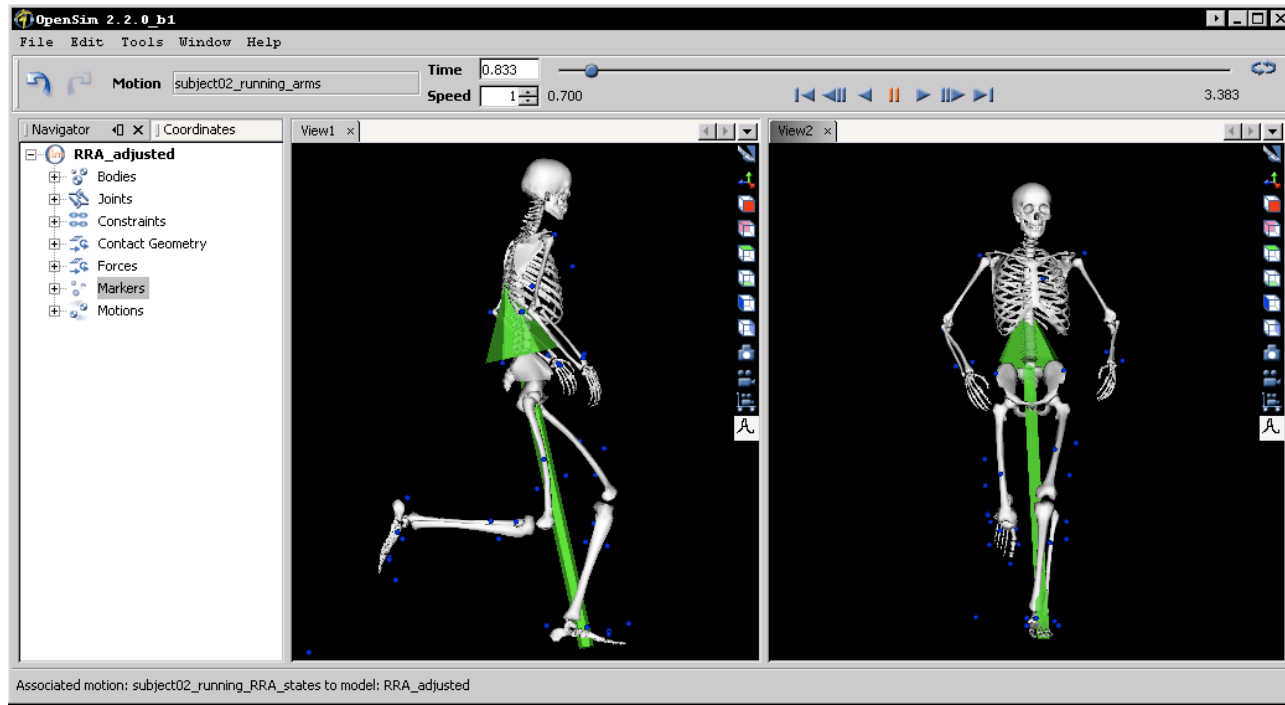| Musculoskeletal Geometry | ← | Multibody Dynamics | ← | $\dfrac{d}{dt}$ | ← | $\dfrac{d}{dt}$ | ← | Position Data |

Force Data

Inverse Dynamics   **Inverse Kinematics**

✓ Derived equations of motion defining the model

✓ Solved equations of motion for joint moments

Inverse Dynamics

TIPS & TRICKS
Filter your raw coordinate data
Check residuals for RRA and to make sure GRFs were applied correctly
Compare to previous literature data (if available)