

02. Power rule for positive integers

$$\frac{d}{dx}(x^n) = nx^{n-1}$$

$$f(x) = x^3$$

$$\frac{d}{dx}(f) = \frac{d}{dx}(x^3) = 3x^{3-1} = 3x^2$$

$$f(x) = x^5$$

$$\frac{d}{dx}(f) = \frac{d}{dx}(x^5) = 5x^{5-1} = 5x^4$$

03. Constant Multiple Rule

$$\frac{df}{dv} = \frac{d}{dv}(cx^n) = c \frac{d}{dv}(nx^{n-1}) = c \cdot x^{n-1}$$

$$f(v) = 3x^4$$

$$\frac{df}{dv} = \frac{d}{dv}(3x^4) = 3 \frac{d}{dv}(4x^{4-1}) = 3(4x^3) = 12x^3$$

04. Sum rule

$$\frac{d}{dx}(u + v) = \frac{du}{dx} + \frac{dv}{dx}$$

$$f(x) = x^3 + 3x$$

$$\frac{d}{dx}(f) = \frac{d}{dx}(x^3 + 3x)$$

$$= \frac{d}{dx}(x^3) + \frac{d}{dx}(3x) = 3x^{3-1} + 3\frac{d}{dx}(x)$$

$$= 3x^2 + 3(1) = 3x^2 + 3$$

$$f(x) = x^4 + 12x$$

$$\frac{d}{dx}(f) = \frac{d}{dx}(x^4 + 12x)$$

$$= \frac{d}{dx}(x^4) + \frac{d}{dx}(12x) = 4x^{4-1} + 12\frac{d}{dx}(x)$$

$$= 4x^3 + 12(1) = 4x^3 + 12$$

$$\text{Why } \frac{d}{dx}(x) = 1?$$

$$x = x^1$$

$$\frac{d}{dx}(x) = x^{1-1} = x^0 = 1$$

$$f(x) = x^3 + \frac{4}{3}x^2 - 5x$$

$$\begin{aligned}\frac{dy}{dx} &= \frac{d}{dx} \left(x^3 + \frac{4}{3}x^2 - 5x \right) \\ &= \frac{d}{dx}(x^3) + \frac{d}{dx}(4x^2) - \frac{d}{dx}(5x) \\ &= 3x^{3-1} + 4\frac{d}{dx}(x^2) - \frac{d}{dx}(x) \\ &= 3x^2 + 4(2x^{2-1}) - 5(1) \\ &= 3x^2 + 4(2x) - 5 \\ &= 3x^2 + 8x - 5\end{aligned}$$

05. Product rule

$$\frac{d}{dx}(uv) = u \frac{dv}{dx} + v \frac{du}{dx}$$

$$f(x) = (x + 1) \cdot (x - 1)$$

$$\begin{aligned}\frac{df}{dx} &= \frac{d}{dx} (x + 1) \cdot (x - 1) \\ &= (x + 1) \frac{d}{dx} (x - 1) + (x - 1) \frac{d}{dx} (x + 1) \\ &= (x + 1) \left[\frac{d}{dx} (x) - \frac{d}{dx} (1) \right] + (x - 1) \left[\frac{d}{dx} (x) + \frac{d}{dx} (1) \right] \\ &= (x + 1)[1 - 0] + (x - 1)[1 + 0] \\ &= (x + 1)(1) + (x - 1)(1) \\ &= (x + 1)(x - 1) = 2x \\ f(x) &= (x^2 + 1)(x^3 + 3)\end{aligned}$$

 d d d

6. Quotient rule

$$\frac{d}{dx} \left(\frac{u}{v} \right) = \frac{v \frac{d}{dx} (u) - u \frac{d}{dx} (v)}{v^2}$$

$$f(x) = \frac{x^2 - 1}{x^2 + 1}$$

$$f(x) = \frac{d}{dx} \left[\frac{x^2 - 1}{x^2 + 1} \right]$$

$$= \frac{(x^2 + 1) \frac{d}{dx} (x^2 - 1) - (x^2 - 1) \frac{d}{dx} (x^2 + 1)}{(x^2 + 1)^2}$$

$$= \frac{(x^2 + 1) \left[\frac{d}{dx} (x^2) - \frac{d}{dx} (1) \right] - (x^2 - 1) \left[\frac{d}{dx} (x^2) + \frac{d}{dx} (1) \right]}{(x^2 + 1)^2}$$

$$= \frac{(x^2 + 1)[2x^{2-1} - 0] - (x^2 - 1)[2x^{2-1} + 0]}{(x^2 + 1)^2}$$

$$= \frac{(x^2 + 1)[2x] - (x^2 - 1)[2x]}{(x^2 + 1)^2}$$

$$= \frac{4x}{(x^2 + 1)^2}$$

$$f = \frac{2x}{4x + 3}$$

$$\frac{d}{dx} \left(\frac{2x}{4x + 3} \right)$$

$$= \frac{(4x + 3) \frac{d}{dx} (2x) - (2x) \frac{d}{dx} (4x + 3)}{(4x + 3)^2}$$

$$= \frac{(4x + 3) \cdot 2 \frac{d}{dx} (x) - (2x) \left[\frac{d}{dx} (4x) + \frac{d}{dx} (3) \right]}{(4x + 3)^2}$$

$$= \frac{2(4x + 3) - 2x[4(1) + (0)]}{(4x + 3)^2}$$

$$= \frac{8x + 6 - 8x}{(4x + 3)^2} = \frac{6}{(4x + 3)^2}$$