02. Power rule for positive integers

$$\frac{d}{dx}(x^n) = nx^{n-1}$$

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f$(x) = x^3$

$$\frac{d}{dx}(f) = \frac{d}{dx}(x^3) = 3x^{3-1} = 3x^2$$

f$(x) = x^5$

$$\frac{d}{dx}(f) = \frac{d}{dx}(x^5) = 5x^{5-1} = 5x^4$$

03. Constant Multiple Rule

$$\frac{df}{dv} = \frac{d}{dv}(cx^n) = c\frac{d}{dv}(nx^{n-1}) = c.x^{n-1}$$

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f(v) = $3x^4$

$$\frac{df}{dv} = \frac{d}{dv}(3x^4) = 3\frac{d}{dv}(4x^{4-1}) = 3(4x^3) = 12x^3$$

04. Sum rule

$$\frac{d}{dx}(u+v) = \frac{du}{dx} + \frac{dv}{dx}$$

$$f(x) = x^3 + 3x$$

$$\frac{d}{dx}(f) = \frac{d}{dx}(x^3 + 3x)$$

$$= \frac{d}{dx}(x^3) + \frac{d}{dx}(3x) = 3x^{3-1} + 3\frac{d}{dx}(x)$$

$$= 3x^2 + 3(1) = 3x^2 + 3$$

$$f(x) = x^4 + 12x$$

$$\frac{d}{dx}(f) = \frac{d}{dx}(x^4 + 12x)$$

$$= \frac{d}{dx}(x^4) + \frac{d}{dx}(12x) = 4x^{4-1} + 12\frac{d}{dx}(x)$$

$$= 4x^3 + 12(1) = 4x^3 + 12$$

Why $\frac{d}{dx}(x) = 1$?

$x = x^1$

$$\frac{d}{dx}(x) = x^{1-1} = x^0 = 1$$

$$f(x) = \ x^3 + \frac{4}{3}x^2 - 5x$$

$$\frac{dy}{dx} = \frac{d}{dx}\left(x^3 + \frac{4}{3}x^2 - 5x\right)$$

$$= \frac{d}{dx}(x^3) + \frac{d}{dx}(4x^2) - \frac{d}{dx}(5x)$$

$$= 3x^{3-1} + 4\frac{d}{dx}(x^2) - \frac{d}{dx}(x)$$

$$= 3x^2 + 4(2x^{2-1}) - 5(1)$$

$$= 3x^2 + 4(2x) - 5$$

$$= 3x^2 + 8x - 5$$

05. Product rule

$$\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$$

$$f(x) = (x+1).(x-1)$$

$$\frac{df}{dx} = \frac{d}{dx}(x+1).(x-1)$$

$$= (x+1)\frac{d}{dx}(x-1) + (x-1)\frac{d}{dx}(x+1)$$

$$= (x+1)\left[\frac{d}{dx}(x) - \frac{d}{dx}(1)\right] + (x-1)\left[\frac{d}{dx}(x) + \frac{d}{dx}(1)\right]$$

$$= (x+1)[1-0] + (x-1)[1+0]$$

$$= (x+1)(1) + (x-1)(1)$$

$$= (x+1)(x-1) = 2x$$

$$f(x) = (x^2+1)(x^3+3)$$

$$\frac{d}{dx}(x^2+1)(x^3+3) = (x^2+1)\frac{d}{dx}(x^3+3) + (x^3+3)\frac{d}{dx}(x^2+1)$$

$$= (x^2+1)[\frac{d}{dx}(x^3) + \frac{d}{dx}(3)] + (x^3+3)[\frac{d}{dx}(x^2) + \frac{d}{dx}(1)$$

$$= (x^2+1)[3x^{3-1} + (0)] + (x^3+3)[2x^{2-1} + (0)]$$

$$= (x^2+1)[3x^2] + (x^3+3)[2x]$$

$$= 3x^4 + 3x^2 + 2x^4 + 6x$$

$$= 5x^4 + 3x^2 + 6x$$

6. Quotient rule

$$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{d}{dx}(u) - u\frac{d}{dx}(v)}{v2}$$

$$f(x) = \frac{x^2 - 1}{x^2 + 1}$$

$$f(x) = \frac{d}{dx}\left[\frac{x^2 - 1}{x^2 + 1}\right] = \frac{(x^2 + 1)\frac{d}{dx}(x^2 - 1) - (x^2 - 1)\frac{d}{dx}(x^2 + 1)}{(x^2 + 1)^2}$$

$$= (x^2 + 1)\left[\frac{d}{dx}(x^2) - \frac{d}{dx}(1)\right] - (x^2 - 1)\left[\frac{d}{dx}(x^2) + \frac{d}{dx}(1)\right]$$

$$= \frac{(x^2 + 1)\left[\frac{d}{dx}(x^2) - \frac{d}{dx}(1)\right] - (x^2 - 1)\left[\frac{d}{dx}(x^2) + \frac{d}{dx}(1)\right]}{(x^2 + 1)^2}$$

$$= \frac{(x^2 + 1)[2x^{2-1} - 0] - (x^2 - 1)[2x^{2-1} + 0]}{(x^2 + 1)^2}$$