

Diffrentiation = अवकलन

07. Quotient rule

$$\frac{d}{dx} \left(\frac{u}{v} \right) = \frac{v \frac{du}{dx} - u \frac{dv}{dx} v}{v^2}$$

$$y = \frac{x^2 - 1}{x^2 + 1}$$

$$\begin{aligned}\frac{d}{dx} \left(\frac{x^2 - 1}{x^2 + 1} \right) &= \frac{(x^2 + 1) \frac{d}{dx}(x^2 - 1) - (x^2 - 1) \frac{d}{dx}(x^2 + 1)}{(x^2 + 1)^2} \\&= \frac{(x^2 + 1) \left[\frac{d}{dx}(x^2) - \frac{d}{dx}(1) \right] - (x^2 - 1) \left[\frac{d}{dx}(x^2) + \frac{d}{dx}(1) \right]}{(x^2 + 1)^2} \\&= \frac{(x^2 + 1)[2x^{2-1} - 0] - (x^2 - 1)[2x^{2-1} + 0]}{(x^2 + 1)^2} \\&= \frac{(x^2 + 1)[2x - 0] - (x^2 - 1)[2x + 0]}{(x^2 + 1)^2} \\&= \frac{(x^2 + 1)[2x] - (x^2 - 1)[2x]}{(x^2 + 1)^2} \\&\frac{4x}{(x^2 + 1)^2} \text{ Ans}\end{aligned}$$

$$y = \frac{2x}{4x + 3}$$

$$\begin{aligned} \frac{d}{dx} \left(\frac{2x}{4x + 3} \right) &= \frac{(4x + 3) \frac{d}{dx}(2x) - (2x) \frac{d}{dx}(4x + 3)}{(4x + 3)^2} \\ &= \frac{(4x + 3)2 \frac{d}{dx}(x) - (2x) \left[\frac{d}{dx}(4x) + \frac{d}{dx}(3) \right]}{(4x + 3)^2} \\ &= \frac{2(4x + 3) - 2x[4(1) + (0)]}{(4x + 3)^2} \\ &= \frac{8x + 6 - 8x}{(4x + 3)^2} = \frac{6}{(4x + 3)^2} \text{ Ans} \end{aligned}$$

$$y = \frac{2x - 3}{2x + 1}$$

$$\begin{aligned} \frac{d}{dx} \left(\frac{2x - 3}{2x + 1} \right) &= \frac{(2x + 1) \frac{d}{dx}(2x - 3) - (2x - 3) \frac{d}{dx}(2x + 1)}{(2x + 1)^2} \\ &= \frac{(2x + 1) \left[\frac{d}{dx}(2x) - \frac{d}{dx}(3) \right] - (2x - 3) \left[\frac{d}{dx}(2x) + \frac{d}{dx}(1) \right]}{(2x + 1)^2} \\ &= \frac{(2x + 1) \left[2 \frac{d}{dx}(x) - \frac{d}{dx}(3) \right] - (2x - 3) \left[2 \frac{d}{dx}(x) + \frac{d}{dx}(1) \right]}{(2x + 1)^2} \\ &= \frac{(2x + 1)[2(1) - (0)] - (2x - 3)[2(1) + (0)]}{(2x + 1)^2} \\ &= \frac{(2x + 1)[2] - (2x - 3)[2]}{(2x + 1)^2} \\ &= \frac{(1)[2] - (-3)[2]}{(2x + 1)^2} = \frac{8}{(2x + 1)^2} \text{ Ans.} \end{aligned}$$