National University of Singapore
School of Computing
CS3243 Introduction to AI

Tutorial 1: Agents, Problems, and Uninformed Search

Issued: Week 2 Discussion in: Week 3

Important Instructions:

- **Tutorial Assignment 1** consists of **Question 4** from this tutorial.
- Your solution(s) must be TYPE-WRITTEN, though diagrams may be hand-drawn.
- You must submit your solution(s) via **Canvas** > **Assignments**, satisfying the deadlines:
    - Pre-tutorial 1 submission by **Week 2 Sunday, 2359 hrs**.
    - Post-tutorial 1 submission by **Week 3 Friday, 2359 hrs**.
- You must make both submissions for your assignment score to be counted.

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1. You are given an n-piece unassembled jigsaw puzzle set (you may assume that each jigsaw piece can be properly connected to either 2, 3, or 4 pieces), which assembles into an $m \times k$ rectangle (i.e., $n = m \times k$. There may be multiple valid final configurations of the puzzle. Figure 1 illustrates an example.

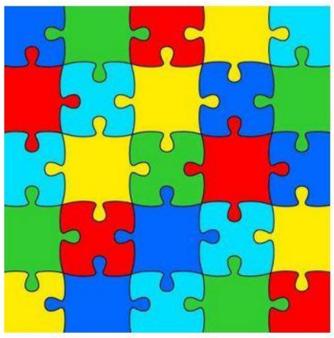


Figure 1: A sample configuration of the jigsaw puzzle.

Formulate the above as a search problem. More specifically, define the following:
- State representation
- Initial state
- Actions
- Transition model
- Action cost
- Goal test

If necessary, you may identify the assumptions you have made. However, assumptions that are contradictory to any instruction in the question, or that are unreasonable, will be invalid.

2. Prove that the **Uniform-Cost Search** algorithm is optimal as long as each action cost exceeds some small positive constant ε. You may also assume the same constraints that make Breadth-First Search complete.

3. (a) Describe the difference between **Tree Search** and **Graph Search** implementations.

   (b) Assuming that ties (when pushing to the frontier) are broken based on ascending alphabetical order (e.g., *A* before *B*), specify the order of the nodes checked (i.e., via the goal test) by the following algorithms. Assume that *S* is the initial state, while *G* is the goal state.

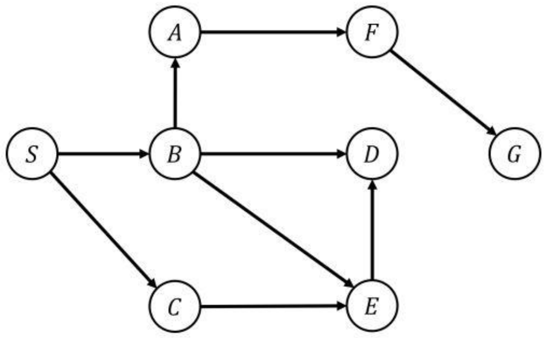


Figure 2: Graph representing the search space for Question **2b**.

You should express your answer in the form *S–B–A–F–G* (i.e., no spaces, all uppercase letters, delimited by the dash (–) character), which, for example, corresponds to the order, *S*, *B*, *A*, *F*, *G*.

i. **Depth-First Search** using a **tree-search** implementation.
ii. **Depth-First Search** using a **graph-search** implementation.
iii. **Breadth-First Search** using a **tree-search** implementation.
iv. **Breadth-First Search** using a **graph-search** implementation.

4. You have just moved to a strange new city, depicted via the graph in Figure 3, and you are trying to learn your way around. More specifically, you wish to learn how to get from your home at *S* to the train station at *G*.

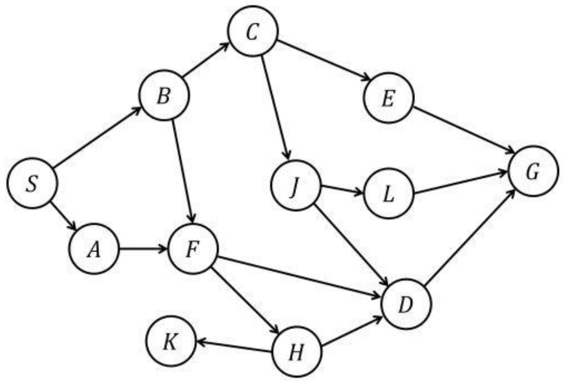


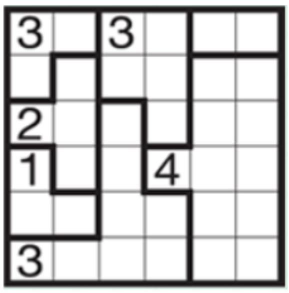
Figure 3: A graphical representation of the strange new city.

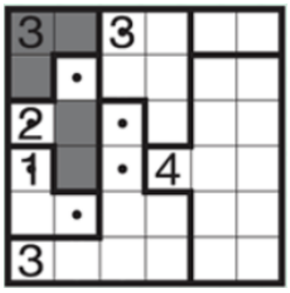
Apply the **Depth-First Search** algorithm with a **tree-search** implementation. Use ascending alphabetical order to break ties when deciding the priority for pushing nodes into the frontier (e.g., *A* before *B*).

Determine the final path found from the start *S* to the goal *G*.

Note that you ***MUST*** express your answer in the form *S–B–C–J–L–G* (i.e., no spaces, all uppercase letters, delimited by the dash (–) character), which, for example, corresponds to the exploration order of *S*, *B*, *C*, *J*, *L*, then *G*.

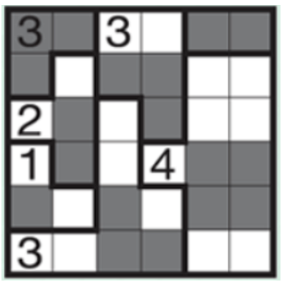
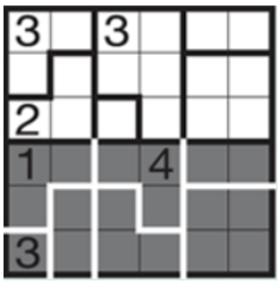
5. **[AY22/23 S3 Midterm Q1]** A certain puzzle game is played on a square *n* by *n* grid (i.e., a grid with *n* rows and *n* columns). The grid is separated into *k* contiguous regions. Each region, $\boldsymbol{R}_i$ (where $i \in [1, k]$), has an associated positive integer value, v_i. An example of this puzzle where $n = 6$ and $k = 7$ is depicted on the right. Notice that each of the *k* regions is bounded by a thick line, with the value for the region stated in the region. However, notice that some regions (e.g., the region in the top-right of the grid) have no associated value – such regions have variable values ranging from 0 to the size of the region (i.e., the number of cells in the region).





To solve this puzzle, one must fill contiguous cells in the region such that for region $\boldsymbol{R}_i$, the number of contiguous cells filled is v_i. When filling cells in this manner, there is a further constraint: filled cells cannot be contiguous across the regions. Continuing from the example puzzle above, suppose that we fill the first two regions in the top-left of the grid. The figure on the left shows a possible partial solution. The shaded cells denote filled cells. Notice that these filled cells only exist within the specified regions and are contiguous. The cells containing dots denote the cells that cannot be filled, since filling them would cause the solution to become inconsistent with the constraint that requires filled cells to not be contiguous across different regions.

Next, we must first fill all the regions (though you may choose to leave regions with no value empty). The puzzle is solved if the following constraint is satisfied. When all filled cells "fall" to the bottom of the grid, retaining their shape and not coexisting in cells with other filled cells, the bottom half of the grid must be completely filled. A solution to the example puzzle shown above is thus as follows.

	← After all regions are filled but before the filled cells have fallen		← After cells have fallen, notice that the fallen cells are perfectly tessellated

Assume that the puzzle input is a list of 2-tuples, $\boldsymbol{P} = \{(\boldsymbol{R}_1, v_1), (\boldsymbol{R}_2, v_2), \ldots, (\boldsymbol{R}_k, v_k)\}$, where each $\boldsymbol{R}_i = \{(r_{i,1}, c_{i,1}), (r_{i,2}, c_{i,2}), \ldots\}$, and where each $(r_{i,j}, c_{i,j})$ refers to the row and column coordinates respectively for the *j*-th cell in the *i*-th region $\boldsymbol{R}_i$. Note that the sequence of row-column coordinates in each $\boldsymbol{R}_i$ is sorted primarily in terms of descending order of row coordinate value, and secondarily in terms of ascending order of column coordinate value – i.e., cells on the top row will be ordered before cells on lower rows, and within each row, cells are ordered from left to right; however, note that there is no specific order over the regions. Finally, note the following properties for $\boldsymbol{P}$.

- For the bottom row, the leftmost cell is (1, 1), while the rightmost is (1, *n*).
- For the top row, the leftmost cell is (*n*, 1), while the rightmost is (*n*, *n*).
- Each $v_i \leq |\boldsymbol{R}_i|$, $\sum_{i=1}^{k} v_i$, $= n^2/2$, and for cases where $\boldsymbol{R}_i$ has no value (i.e., is variable from 0 to $|\boldsymbol{R}_i|$), then $v_i = null$.

Thus, for the example puzzle above, we have $\boldsymbol{P}$ = {($\boldsymbol{R}_1$, 3), ($\boldsymbol{R}_2$, 3), ($\boldsymbol{R}_3$, null), ($\boldsymbol{R}_4$, 2), ($\boldsymbol{R}_5$, 4), ($\boldsymbol{R}_6$, 3), ($\boldsymbol{R}_7$, 1)}, where: $\boldsymbol{R}_1$ = {(6, 1), (6, 2), (5, 1)}; $\boldsymbol{R}_2$ = {(6, 3), (6, 4), (5, 3), (5, 4), (4, 4)}; $\boldsymbol{R}_3$ = {(6, 5), (6, 6)}; $\boldsymbol{R}_4$ = {(5, 2), (4, 1), (4, 2), (3, 2)}; $\boldsymbol{R}_5$ = {(5, 5), (5, 6), (4, 5), (4, 6), (3, 4), (3, 5), (3, 6), (2, 5), (2, 6), (1, 5), (1, 6)}; $\boldsymbol{R}_6$ = {(4, 3), (3, 3), (2, 3), (2, 4), (1, 1), (1, 2), (1, 3), (1, 4)}; and $\boldsymbol{R}_7$ = {(3, 1), (2, 1), (2, 2)}.

Your task is to model the puzzle as a search problem so that we may apply uninformed, informed, or local search algorithms to solve the problem. In doing this, you are to assume the following partial search problem formulation.

- **State**: each state corresponds to partially assigned solutions, $\boldsymbol{S} = \{\boldsymbol{F}_1, \boldsymbol{F}_2, \ldots, \boldsymbol{F}_k\}$, where each $\boldsymbol{F}_i \subseteq \boldsymbol{R}_i$, $|\boldsymbol{F}_i| \leq v_i$, and such that all coordinates in $\boldsymbol{F}_i$ are contiguous.
- **Initial State**: with the initial state, each $\boldsymbol{F}_i \in \boldsymbol{S}$ is empty, i.e., $\forall \boldsymbol{F}_i \in \boldsymbol{S}$, $|\boldsymbol{F}_i| = \emptyset$.
- **Action Costs**: all action costs correspond to a positive constant – e.g., 1.
- **Goal Check**: this function, *isGoal*($\boldsymbol{S}$), returns *True* if $\forall \boldsymbol{F}_i \in \boldsymbol{S}$, $|\boldsymbol{F}_i| = v_i$, and after all the filled cells have fallen, the bottom half of the grid is completely filled. Note that this function automatically returns *False* if $\exists \boldsymbol{F}_i \in \boldsymbol{S}$ where $|\boldsymbol{F}_i| \neq v_i$.

(a) Complete the search problem formulation by defining the **actions** and **transition model**. Your formulation must be efficient (i.e., significantly better than brute-force search).

(b) Define the runtime complexity of the ***actions*** function you specified in Part **(a)** and define the size of the resultant search tree.