National University of Singapore
School of Computing
CS3243 Introduction to AI

Tutorial 2: Informed Search

Issued: Week 3 Discussion in: Week 4

Important Instructions:

- **Tutorial Assignment 2** consists of **Question 5** from this tutorial.
- Your solution(s) must be TYPE-WRITTEN, though diagrams may be hand-drawn.
- You must submit your solution(s) via **Canvas** > **Assignments**, satisfying the deadlines:
  - Pre-tutorial 2 submission by **Week 3 Sunday, 2359 hrs**.
  - Post-tutorial 2 submission by **Week 4 Friday, 2359 hrs**.
- You must make both submissions for your assignment score to be counted.

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1. (a) Provide a counterexample to show that the **tree-search** implementation of the **Greedy Best-First Search** algorithm is **incomplete**.

   (b) Briefly explain why the **graph-search** implementation of the **Greedy Best-First Search** algorithm is **complete**.

   (c) Provide a counterexample to show that neither the **tree-search** nor the **graph-search** implementations of the **Greedy Best-First Search** algorithm are **optimal**.

2. (a) Prove that the **tree-search** implementation of the **A* Search** algorithm is optimal when an **admissible heuristic** is utilised.

   (b) Prove that the **graph-search** implementation of the **A* Search** algorithm is optimal when a **consistent heuristic** is utilised. Assume graph search **Version 3**.

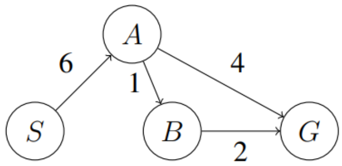
3. (a) Given a **heuristic** h, such that h(t) = 0, where t is any goal state, prove that if h is **consistent**, then it must be **admissible**.

   (b) Give an example of an **admissible heuristic** that is **not consistent**.

4. We have seen various search strategies in class and analysed their worst-case running time. Prove that **any deterministic search algorithm** will, in the worst case, **search the entire state space**. More formally, prove the following theorem.

   Theorem 1. *Let $\mathcal{A}$ be some complete, deterministic search algorithm. Then for any search problem defined by a finite connected graph* $\boldsymbol{G} = \langle \boldsymbol{V}, \boldsymbol{E} \rangle$ *(where* $\boldsymbol{V}$ *is the set of possible states and* $\boldsymbol{E}$ *are the transition edges between them), there exists a choice of start node* s_0 *and goal node* g *so that* $\mathcal{A}$ *searches through the entire graph* $\boldsymbol{G}$.

5. (a) In the search problem below, we have listed 5 heuristics. Indicate whether each **heuristic** is **admissible** and/or **consistent** in the table below.



	S	*A*	*B*	*G*	**Admissible**	**Consistent**
h_1	0	0	0	0		
h_2	8	1	1	0		
h_3	9	3	2	0		
h_4	6	3	1	0		
h_5	8	4	2	0		

(b) Write out the order of the nodes that are explored by the **A* Search** algorithm. Assume the use of a **graph search version 3** implementation that utilises heuristic h_4. You must express your answer in the form *A–B–C* (i.e., no spaces, all uppercase letters, delimited by the dash (–) character), which, for example, corresponds to the order *A*, then *B*, and then *C*.

(c) Which heuristic would you use? Explain why.

(d) Prove or disprove the following statement:

The heuristic $h(n) = \boldsymbol{max}\{h_3(n), h_5(n)\}$ is admissible.