Worksheet 13

Math 251, Summer 2017

Name: Key

1. Use the "Positivity Test" to find the intervals where the function is positive and negative. (There is absolutely no calculus in this question.)

(a) $G(x) = x\sqrt{x^2+1}$ $=0$

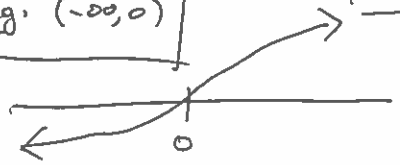
$x=0$ or $\sqrt{x^2+1}=0$

$x^2+1=0$

$x^2=-1$ nope!

Pos: $[0,\infty)$

Neg: $(-\infty,0)$



$G(-1) = (-1)\cdot\sqrt{1+1} < 0$

$G(1) = 1\cdot\sqrt{1+1} > 0$

(b) $h(x) = x^3 - x$ $=0$

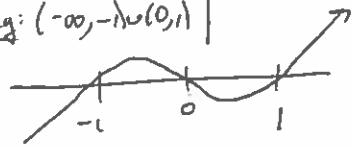
$x(x^2-1)=0$

$x=0$ or $x^2=1$

$x=\pm 1$

Pos: $(-1,0)\cup(1,\infty)$

Neg: $(-\infty,-1)\cup(0,1)$



$h(-2) = -8+2 < 0$

$h(0.5) > 0$

$h(0.5) < 0$

$h(2) > 0$

(c) $R(t) = \arctan(t^2-1)$ $=0$

tan( ) tan( )

$t^2-1 = \tan(0) = 0$

$t^2 = 1$

$t = \pm 1$

$R(-2) = 1.2$

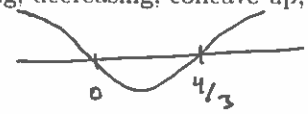
$R(0) = \arctan(-1) = -0.78$

$R(2) = 1.2$

Pos: $(-\infty,-1)\cup(1,\infty)$, Neg: $(-1,1)$

2. Determine the intervals where $f(x) = x^3 - 2x^2 + 1$ is increasing, decreasing, concave up, and concave down.

$f'(x) = 3x^2 - 4x$ ⇝ f':



$3x^2 - 4x = 0$

$x(3x-4) = 0$

$x = 0$ or $x = \frac{4}{3}$

$f'(1) = 3 - 4 = -1$

$f'(-1) = 3 + 4 = 7$

$f'(2) = 12 - 8 = 4$

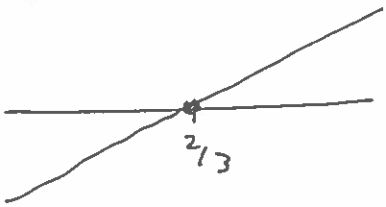
$f''(x) = 6x - 4$

$6x - 4 = 0$

$6x = 4$

$x = \frac{4}{6} = \frac{2}{3}$

f'':



$f''(0) = -4$

$f''(1) = 2$

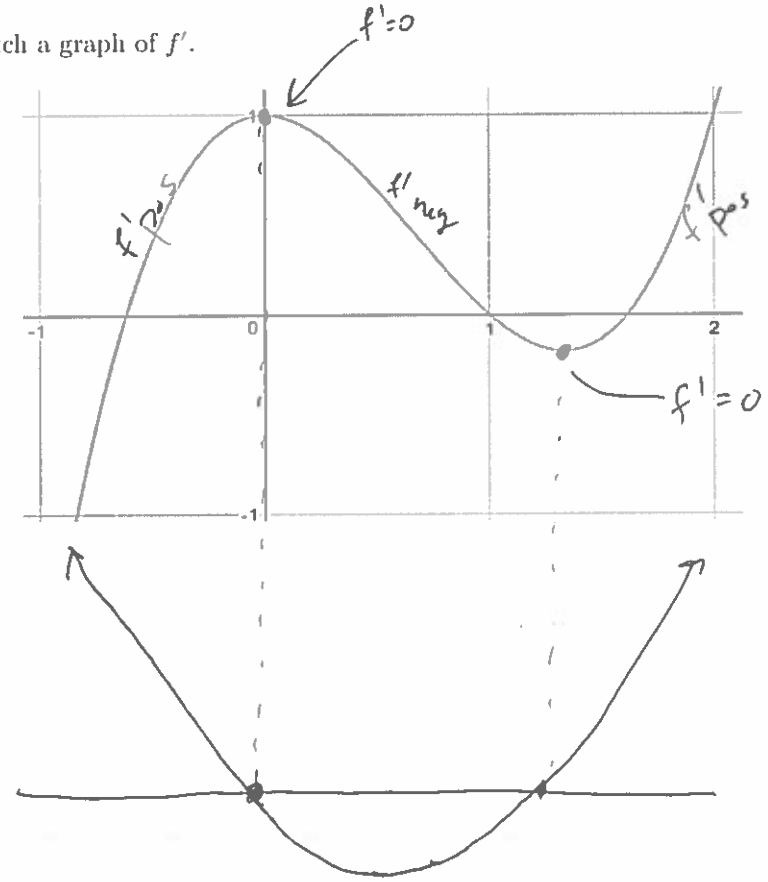
Increasing: $(-\infty,0)\cup(\frac{4}{3},\infty)$

Decreasing: $(0,4/3)$

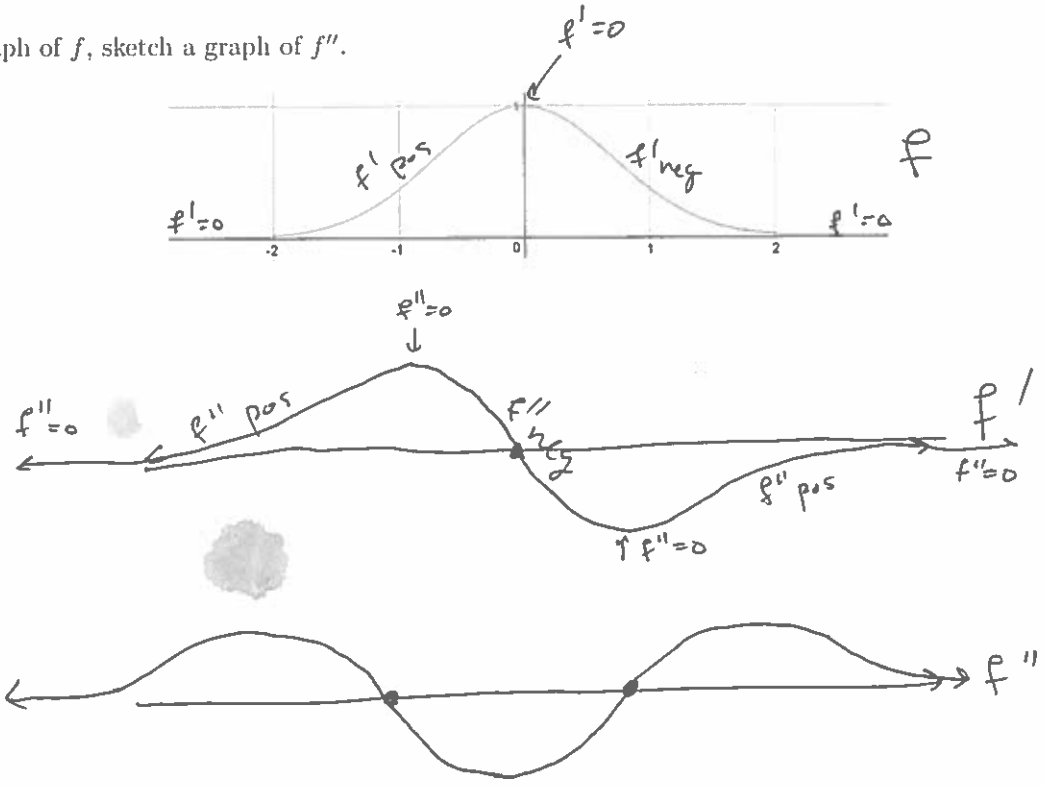
Concave up: $(\frac{2}{3},\infty)$

Concave down: $(-\infty,2/3)$

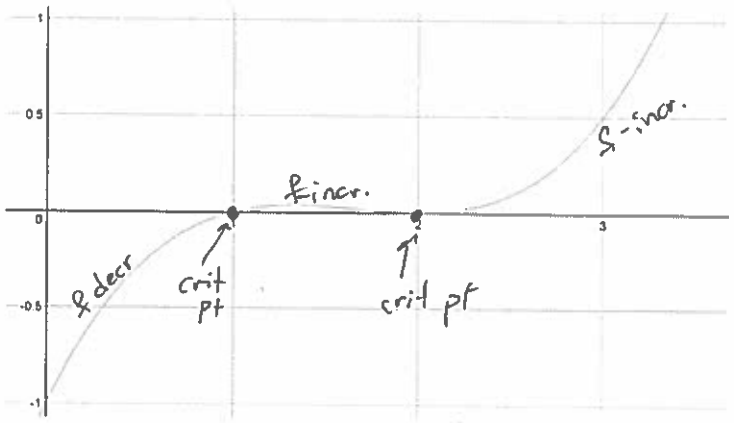
3. Given the graph of f, sketch a graph of f'.

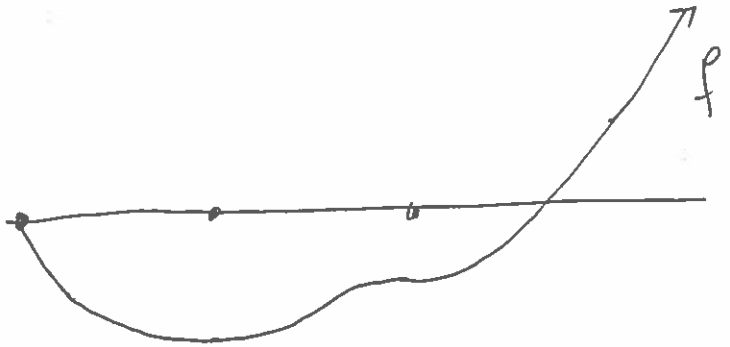


4. Given the graph of f, sketch a graph of f''.



5. Given the graph of f' below, sketch the graph of f assuming that f goes through the origin.





6. Given the graph of f' below, sketch the graph of f assuming that f goes through the point $(0, -1)$.