Quiz 23

Name: _____

1. In class we developed the disease model

$$\frac{dI}{dt} = \alpha I(1 - I) - \mu I.$$

(a) Explain what the variables and parameters mean in this equation.

α is the transmission rate, μ is the recovery rate, and I is the fraction of the population that is infected.

(b) In the case where $\alpha > \mu$, show (using the stability test) that the equilibrium $I^* = 1 - \frac{\mu}{\alpha}$ is stable.

$$\begin{aligned} f(I) &= \alpha I - \alpha I^2 - \mu I \\ f'(I) &= \alpha - 2\alpha I - \mu \\ f'(I^*) &= \alpha - 2\alpha\left(1 - \frac{\mu}{\alpha}\right) - \mu \\ &= \alpha - 2\alpha + 2\mu - \mu \\ &= \mu - \alpha. \end{aligned}$$

Since $\alpha > \mu$ this result is negative. Therefore, this equilibrium is stable.