2.4 and 2.5: Derivatives from the Definition

1 Derivatives from the Definition

- We now understand enough background on functions to give derivatives a second look.
- Summary so far:
- We know what limits mean.
- We know that we need continuous functions to talk about tangent lines.
- We're ready to study the derivative.
- Suppose $f(x)$ is a function and $x = a$ is a basepoint. We write

  $$f'(a) = \lim_{\Delta t \to 0} \frac{f(a + \Delta t) - f(a)}{\Delta t}$$

  for its *derivative at* $t = a$ (or instantaneous rate of change at a).
- Sometimes we use different notation:

  $$\frac{df}{dt} \quad \text{or} \quad \frac{dy}{dt}.$$

- Let's put the definition to use.
- Ex: with $f(t) = 2t + 7$, calculate $f'(4)$, $f'(2)$, and $f'(1)$. Does it matter what basepoint you use? Why?
- With $f(t) = t^2 + t$, calculate $f'(3)$.
- You can also instead leave the basepoint arbitrary, and this defines *the derivative function*, written $f'(t)$.
- Ex: With $f(t) = t^2$, calculate the derivative function $f'(t)$.
- Ex: With $f(t) = \sqrt{t}$, (a) calculate the derivative function, and (b) use the result to find the equation of the tangent line at $t = 1$. A: $y = \frac{1}{2}t + \frac{1}{2}$
- Ex: With $f(t) = \frac{1}{t}$, calculate the derivative function and find the equation of the tangent line at $t = -1$.

2 More on the Derivative Function

- We saw that with $f(t) = t^2$, $f'(t) = 2t$.
- Graph both of these.
- Note distinction between what y-values represent on each graph.
- y-values on $f'(t)$ represent *slopes*.
- Read $f'(t)$ graphs as the rates.
- Ex: Of these graphs, which is the derivative function and which is the "parent" function? (Note to self: Draw a cubic and a quadratic.)

2.1 Applications

- We said that when $f(t) =$ height of an object at time t, then $f'(t)$ represents the velocity of the object (velocity = speed with direction).
- Ex: Physics tells us that my wallet, if thrown upward with initial speed $v_0 = 0.5$ m/sec from my head level of 2 m will follow the height function

$$h(t) = -4.9t^2 + 0.5t + 2.$$

  (a) Find the velocity function. (b) When does the object hit the ground? (c) When does the object have a speed of 0? (d) Describe the acceleration of the wallet. A: (a) $v(t) = h'(t) = -9.8t + 0.5$. (b) after about 0.691 seconds. (c) after about 0.05 seconds. (d) constant acceleration of -9.8 m/sec^2.