Homework 6
Due Tuesday, May 14th

Instructions: write up solutions to all problems below. Neatness counts: be sure to follow guidelines for homework in the syllabus.

---

1. Something with gravity and springs Suppose we model a spring that hangs *vertically* from the ceiling. Assume, as before, that there is a mass m at the end of a spring. The spring has a spring constant k.

   (a) Assume first that the spring has no friction. Draw a picture of the spring and the forces on the mass m. Use this to derive a differential equation for the spring. Is your equation homogeneous or non-homogeneous?

   (b) Find the general solution to the differential equation.

   (c) Now assume that there is a drag force on this spring. (Maybe the mass is in the shape of a frisbee, so it would feel a lot of air resistance as the spring moves.) Write down a new differential equation that takes the drag force into account.

   (d) How do your equations differ from those of a horizontal spring?

2. (Chapter 4.2, # 8, 15, 16) Find the general solution to these differential equations. Use Wolfram Alpha or an equivalent website to get the roots of the characteristic polynomials.

   (a) $y''' - y'' - y' + y = 0$

   (b) $y^{(8)} + 8y^{(4)} + 16y = 0$

   (c) $y^{(4)} + 2y'' + y = 0$

3. (Chapter 4.3 # 1, 3) For these problems, find the general solution of the non-homogeneous equations.

   (a) $y''' - y'' - y' + y = 2e^{-t} + 3$

   (b) $y^{(6)} + y''' = t$

4. For the given differential equations, perform the requested substitutions.

   (a) $y' - (4x - y + 1)^2 = 0$; use the substitution $u = 4x - y + 1$. [Cool side note: this substitution turns a non-separable DE into a separable one!]

   (b) $y'(t) = 2y(t)$, make the substitution $x = 2t$.

   (c) $T'(t) = k(A - T)$, where A and k are constants. Make the substitutions $u = \dfrac{T}{A}$ and $x = kt$.

   (d) $x''(t) + \beta x'(t) + \omega^2 x(t) = 0$; make the substitution $\tau = \omega t$. [Make your τ's look distinct from your t's.]