Homework 8
Due Tuesday, June 4th

Instructions: write up solutions to all problems below. Neatness counts: be sure to follow guidelines for homework in the syllabus.

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1. For each system given below, (a) find the solution to the initial value problem, (b) sketch the solution functions $x(t)$ and $y(t)$, and (c) draw the trajectory associated to this particular solution in the xy-plane.

   (i)
   $$\begin{cases} x' = x - 0.5y \\ y' = y - 0.5x \end{cases}$$
   initial condition $(x, y) = (1, 2)$.

   (ii)
   $$\begin{cases} x' = x + 3y \\ y' = -2y \end{cases}$$
   initial condition $(x, y) = (4, 1)$.

   (iii)
   $$\begin{cases} x' = -2x + 3y \\ y' = -3y \end{cases}$$
   initial condition $(x, y) = (-3, 0)$.

2. For each of the systems in problem 1, sketch the phase plane.

3. In this problem you will connect this new method of solving a system to solving a differential equation you already know how to solve. Consider the follwing differential equation.
   $$y'' - 2y' - 3y = 0.$$
   (a) Solve the equation using the old method, which is by guessing a solution of the form $y = e^{rt}$.
   (b) Convert this differential equation into a system by defining the variable $v = y'(t)$.
   (c) Find the eigenvectors and eigenvalues for the associated matrix.
   (d) Write down the general solution to the equation in vector form.
   (e) By combining your solutions in the previous part, what do you get for the solution $y(t)$? What is $y'(t)$ for this (still general) solution? How different is $y(t)$ from part (a)?
   (f) Make a phase plane for this system.