4.3: Integration by Parts

- Recall: Product Rule

$$\frac{d}{dx}(f(x)g(x)) = f'(x)g(x) + f(x)g'(x).$$

Example:

$$\frac{d}{dx}(\sin(x)\cos(x)) = \cos(x)\cos(x) - \sin(x)\sin(x).$$

We undo this by integrating both sides!

$$\begin{aligned} \frac{d}{dx}(fg) &= \frac{df}{dx}g + f\frac{dg}{dx} \\ \int \frac{d}{dx}(fg)\,dx &= \int \frac{df}{dx}g\,dx + \int f\frac{dg}{dx}\,dx \\ fg &= \int \frac{df}{dx}g\,dx + \int f\frac{dg}{dx}. \end{aligned}$$

Rewrite:

$$\int \frac{df}{dx}g\,dx = fg - \int g\frac{df}{dx}$$

- Idea: "move a derivative" to the other function.
- Hope: the resulting integral on the right side is easier to compute.
- Ex: $\int xe^x\,dx$.
- $\int \ln(x)\,dx = x\ln(x) - x + C$
- Sometimes you need a slick trick.

  Ex: $\int e^x \sin(x)\,dx = \frac{1}{2}e^x(\sin(x) - \cos(x)) + C$

- Sometimes group differently:

$$\int x^3 \cos(x^2)\,dx = \frac{1}{2}x^2 \sin(x^2) + \frac{1}{2}\cos(x^2) + C$$

(You need to set $dv = x\cos(x^2)\,dx$ and $u = x^2$.)