Guide for Section 1 - Functions

Section Goals:

- Model functions in non-mathematical contexts.
- Determine if a relationship between two variables defines a function.
- Determine a function's input or output, given the other.
- Find the domain and image of a function.
- Evaluate and interpret values of a function defined in a table or graph.

Def A **function** is a pairing of some set of inputs and another set of outputs, along with a rule* that determines how pairing happens. Pairs are commonly written (x, y), where x is an input and y is the output paired with x.

*The only requirement of this rule is that each input yields exactly one output.

For a function named f, we write $f(\text{input}) = \text{output}$.

The set of inputs in a function is called the **domain** and the set of outputs the **image**.

The result of evaluating a function for an input not in its domain is that the function is said to be **undefined** at that value.

Ex 1 Three of my friends are Jeff, Joe, and Patrick. I want to describe a function that assigns their hair color. Jeff and Joe have brown hair, while patrick has black hair. Write the input-output pairs that define this function.

Describe the domain and image for the relation in this example. Why does the relation described define a function?

If we reverse the inputs and outputs, do we still have a function?

Ex 2 Below are recorded the new cases of Hansen's disease (leprosy) in the United States during 2010, by month.

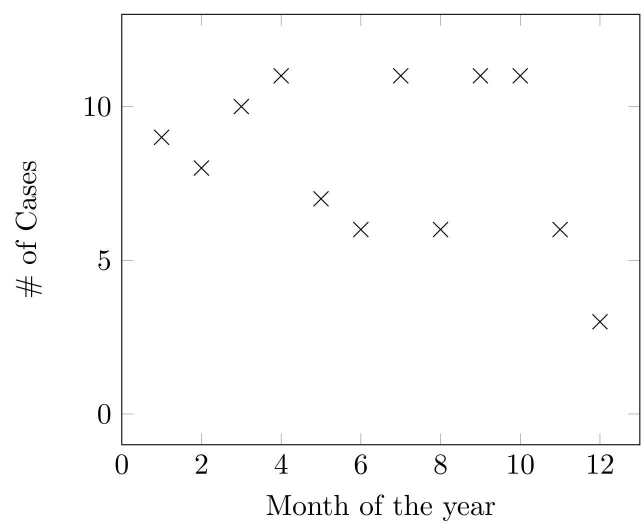
Month	Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec
# of cases	9	8	10	11	7	6	11	6	11	11	6	3

a) Is the number of new cases per month a function of the month? Explain your answer.

b) Is the month a function of the number of new cases per month? Explain.

c) Let $N = f(t)$ be the function giving the number of new cases reported during month t of the year 2010 (January is $t = 1$, February is $t = 2$, etc.). Compute and interpret the value of $f(4)$.

d) The same data is presented below in a graphical format. Use the graph to determine all values of t so that $f(t) \geq 10$.



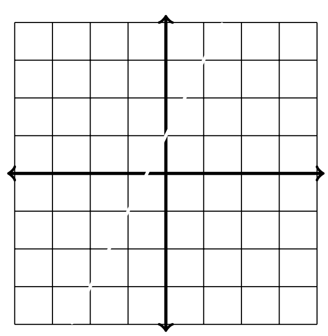
Ex 3 An individual's resting heart rate is 60 beats per minute and at the end of a run 30 minutes later it was 130 beats per minutes.

a) Sketch a plausible graph of the individual's heart rate r as a function of t, in minutes after beginning the run.

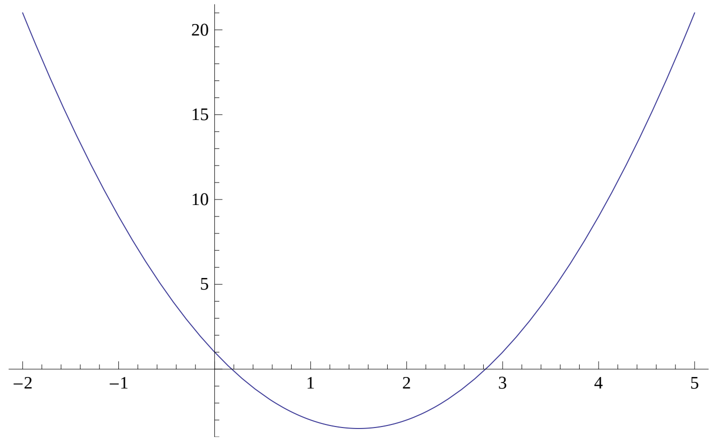
b) What is the value of $r(0)$? What is the only other value of r that we are guaranteed to know?

c) How many solutions are there likely to be to $r(t) = 100$? Explain.

Ex 4 Consider the function $f(x) = 2x + 1$. Graph it on the axes provided below.



Ex 5 (a) With the graph of f shown below, evaluate $f(2)$ and solve the equation $f(x) = 4$.



(b) Assuming the domain of f is just $[-2, 5]$, what is the image (or range) of f? Write your answer in set notation and interval notation.

Ex 6 For the function $G(x) = \sqrt{x + 10}$, find and simplify $G(a + 12)$ along with $G(t^2 - 10)$.

Def When the domain of a function is not specified, use the largest subset of the real numbers for which the function produces a real number.

Ex 7 Find the largest possible domain of each of the following functions. Write your answer in both set and interval notation.

a) $f(x) = \sqrt{x-7}$

b) $g(t) = \dfrac{1}{t}$

c) $h(r) = \dfrac{2^r}{r^2 - 16}$

Note If there is a non-mathematical context applied to the function, the practical domain of the function is the portion of the domain above which also applies to the context.

Unless otherwise stated, the image of a function will correspond to the outputs from the *practical* domain of the function.

Ex 8 What is the practical domain of $B(t) = 20 - 4\sqrt{2t+1}$, where B is approximating the quantity of burgers sold as a function of time, t (in hours)? Assume $t = 0$ is opening time.

$\boxed{\text{Def}}$ A piecewise-defined function is a function defined from several pieces of functions, each of which is defined only on a specific domain.

Ex 9 Let $D(t) = \begin{cases} 2 + 7t & , \text{if } t < 2 \\ t^2 & , \text{if } t \geq 2 \end{cases}$

a) Evaluate $D(1)$ and $D(3)$.

b) Find all real values of t such that $D(t) = 16$.

Def Two variables a and b are proportional if there is a constant k so that $a = k \cdot b$.
Two variables a and b are inversely proportional if there is a constant k so that $a = \dfrac{k}{b}$.
In each case, k is called the constant of proportionality.

Ex 10 Write a formula for G as a function of P assuming that G is proportional to the product of P and $1 - P$, with constant of proportionality 1.5.

Ex 11 A guitar string changes pitch when it becomes tighter. It is known by physicists that the frequency of the guitar string is proportional to the square root of the tension strength in the string.

a) Set up a functional model for how the frequency relates to the tension.

b) After messing around one afternoon, someone figured out that if you play a string with a tension of 30 Newtons (unit of force) produced a nice sounding A (a note of 440 Hz). Determine the unknown constant k.

c) Say you want to make your string play a G, which has a frequency of 784 Hz. How tight must you tune the string?

Ex 12 Let monthly revenue, $R = f(n)$ (in thousands of dollars), be proportional to the number of units sold during that month, n, in thousands. Last month, the company sold $8,500$ units for a revenue of $\$106,250$.

a) Fill out the table of values for R as a function of n.

n	0	8.5	12	16	22
R		106.250			

b) What is the value of the constant of proportionality? What is its significance in the context of revenue?