Homework 5
Due Tuesday, February 12th

Instructions: write up solutions to all problems below. Neatness counts: be sure to follow guidelines for homework in the syllabus.

Reading Assignment: chapters 5.1, 5.4.

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1. Chapter 4.7, # 28.
2. Chapter 4.7, # 30.
3. Explain what an improper integral represents in real life models.
4. In this problem we will calculate the value of

$$\int_{-\infty}^{\infty} \frac{1}{1+x^2}\, dx.$$

When looking at a new improper integral, you should first decide if it is convergent before trying to calculate it. In this problem I will guide you through this process.

(a) Sketch a graph of $\dfrac{1}{1+x^2}$ on the interval $-10 \leq x \leq 10$, with a y range of $-1 \leq y \leq 1$. For larger x values (the "tails"), which power of x does this look like? Why should we expect that this integral converges?

(b) Find the antiderivative of $\dfrac{1}{1+x^2}$ by using the substitution $x = \tan(u)$. [Hint: look back at Homework 3, Problem 3 for a similar kind of substitution.]

(c) Make a graph of the antiderivative you found in part (a) (with integration constant $C = 0$). What are the horizontal asymptotes *exactly* (meaning not as a decimal)?

(d) Use these answers to deduce the exact value of the integral (meaning not a decimal).

[Context: This integral shows up in probability, where it relates to the *Lorenz curve*, also named the *Cauchy distribution*. You may see it again later in a biostatistics course.]