2.2 Limits

1 What is a limit?

- Think like you're annoying your sibling: "I'm not touching you!"
- Get as close as possible *without* actually touching
- Given a function $f(t)$, we want to compute a number that "guesses" the value of y when t approaches t_0.
- Ex:

  $$\lim_{t \to 0} t^2$$

  is asking you a question: "What does the value of $y = t^2$ become when t gets close to 0?"

  Answer: well, if you plug in numbers for t close to 0, you'll see small numbers, so

  $$\lim_{t \to 0} t^2 = 0.$$

- Why not actually "touch"?
- Ex:

  $$\lim_{t \to 1} \frac{t^2 - 1}{t - 1}$$

  In above ex, could "plug in the limit." Here, we can't actually see what it is. When you try to plug in $t = 1$, you'll get $\frac{0}{0}$, an absolutely meaningless symbol.

- When you get a meaningless symbol, like 0/0, 1/0, etc.. you should become a scientist: run an experiment!
- Meaning: plug in $t = 1.1, t = 1.01, t = 1.001$, and try to guess the limiting value.
- For $f(t) = \frac{t^2-1}{t-1}$, I find:

  $$f(1.1) = 2.1, \quad f(1.01) = 2.01, \quad f(1.001) = 2.001,$$

  so the natural guess would be 2, and the proper way to phrase your answer is

  $$\lim_{t \to 1} \frac{t^2 - 1}{t - 1} = 2.$$

- Another example:

  $$\lim_{t \to 0} \frac{|t|}{t}.$$

  Plugging in $t = 0.1, 0.01, ...$ you'll constantly get the value 1. But: *must check both sides.* From $t = -0.1, -0.01, ...$, you'll constantly get -1.

- In this case, we say the limit *does not exist.* Proper notation:

$$\lim_{t\to 0} \frac{|t|}{t} \text{ DNE}$$

- Method 3: Sometimes you can do algebra.
- Ex:

$$\lim_{t\to 2} \frac{t^2-4}{t-2}$$

  If you run $t = 2$, you get $\frac{0}{0}$, very bad.

- We can factor:

$$\frac{t^2-4}{t-2} = \frac{(t-2)(t+2)}{(t-2)} = (t+2) \quad \text{when } t \neq 2.$$

  Since *limits don't care about exactly* $t = 2$, we can conclude

$$\lim_{t\to 2} \frac{t^2-4}{t-2} = \lim_{t\to 2}(t+2) = 4.$$

1.1 Summary

When faced with a limit such as

$$\lim_{t\to t_0} f(t)$$

here's what you should try:

1. First see if plugging in $f(t_0)$ works.
2. If step 1 fails (e.g. 0/0, 1/0), attempt to simplify using algebra.
3. If algebra is not feasible, plug in values for t *near* t_0 and make an educated guess for what these values are doing.

ALWAYS do algebra if possible before resorting to numerics. CALCULATORS CAN LIE.

- Ex: let $g(t) = \frac{\sqrt{t^2+4}-2}{t^2}$.
- If you try cheating the system by plugging in $t = 0.00000001$, your calculator will probably say "0" is the right answer.
- However, if you plug in a series of values like $t = 0.1, 0.01, 0.001$, you'll see the outputs of g are homing in on 0.24999. So,

$$\lim_{t\to 0} \frac{\sqrt{t^2+4}-2}{t^2} = \frac{1}{4},$$

  not 0!