Worksheet 1

Math 251, Summer 2017

Name: _____

Directions

- This counts as your "attendance" for the day. You must give this to me today to get credit for it.
- You may leave if you finish the packet.
- You may work in groups, and you can ask me for assistance.
- I grade this worksheet based on completion, not accuracy; however, you should strive for completely correct answers in order to make sure you understand the material.

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1. Evaluate these limits using the "evaluation" strategy.

   (a) $\lim_{x\to 1} \dfrac{x^2+2}{7-x}$

   (b) $\lim_{x\to \pi} \cos(x)$

2. Evaluate these limits using the "estimation" strategy.

   (a) $\lim_{x\to 0} \dfrac{\sin(x)}{x}$ (be sure to be in radians)

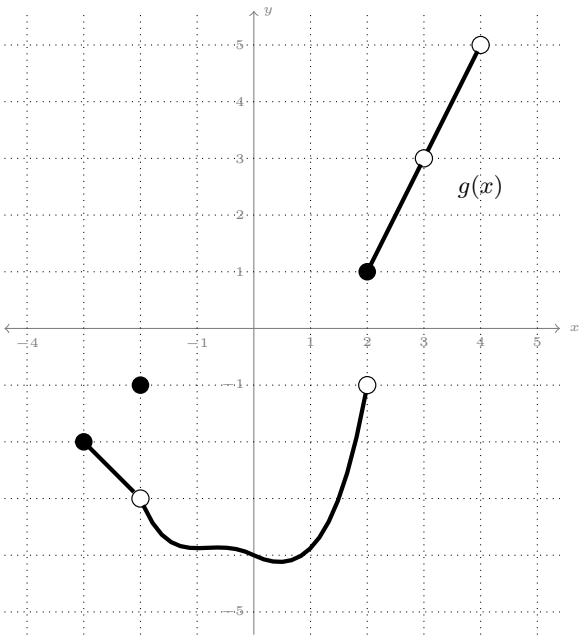
   (b) $\lim_{x\to 0^-} \dfrac{|x|}{x}$

3. Find the limits below by simplifying the expression first. Double check your answer by estimating the limit numerically.

   (a) $\lim_{x\to 2} \dfrac{x^2-4}{x-2}$

   (b) $\lim_{h\to 0} \dfrac{(1+h)^2-1}{h}$

4. Using the graph, evaluate the following limits.



(a) $\lim_{x \to 4^-} g(x)$

(b) $\lim_{x \to 2^+} g(x)$

(c) $\lim_{x \to 2^-} g(x)$

(d) $\lim_{x \to 2} g(x)$

(e) $\lim_{x \to 2.5} g(x)$

(f) $\lim_{x \to -2} g(x)$

(g) $\lim_{x \to -3^-} g(x)$

5. Calculators are not always reliable. Consider the function

$$g(x) = \frac{\sqrt{x^2+4}-2}{x^2}.$$

(a) Evaluate $g(x)$ at $x = 0.1$, 0.01 0.001, and 0.00000001.

x	$g(x)$
0.1	
0.01	
0.0001	
0.00000001	

(b) Based on your first three calculations, what do you expect that $\lim_{x \to 0} g(x)$ is equal to?

The issue with $x = 0.00000001$ is the calculator does not have enough decimal accuracy to handle such small numbers. As such, be careful when using a calculator to evaluate limits.

6. Let $g(x) = \begin{cases} \sin(\pi x) & x < 0 \\ x + 1 & x \geq 0 \end{cases}$

(a) Make a graph of $g(x)$ on the interval $[-1, 1]$

(b) Find $\lim_{x \to 0^-} g(x)$

(c) Find $\lim_{x \to 0^+} g(x)$.

(d) Find $\lim_{x \to 0} g(x)$.

7. Limits as $x \to \infty$ or $x \to -\infty$ sometimes need some care. This problem illustrates a method for evaluating such limits.

(a) Consider the function $g(x) = \dfrac{x^2 - 10x}{13 - x^2}$. Our goal with this problem is to find $\lim\limits_{x\to\infty} g(x) = \lim\limits_{x\to\infty} \dfrac{x^2 - 10x}{13 - x^2}$.

i. Factor out the largest power of x from the top and bottom of the fraction. (You may need to recall that $x = x^2 \cdot \left(\frac{1}{x}\right)$.

ii. Cancel as many factors of x as possible:

iii. Evaluate the limit $\lim_{x\to\infty} g(x)$ by remembering some of the "important limits."

(b) Apply the same strategy to find the following limit.

$$\lim_{x\to\infty} \frac{x^3}{1 + x + x^2}$$