Day 8: Log Differentiation

We add another neat trick.

1 Using Logs to take derivatives

- Trick: take log of a function before differentiating.
- Ex: $y = x^x$. Its derivative is *not* $x(x^{x-1})$.

$$\begin{aligned}
\ln(y) &= \ln(x^x) \\
\ln(y) &= x\ln(x) \\
(\ln(y))' &= \ln(x) + x\frac{1}{x} \\
\frac{1}{y}y' &= \ln(x) + 1 \\
y' &= y \cdot (\ln(x) + 1) \\
y' &= x^x\,(\ln(x) + 1)
\end{aligned}$$

- Apply log diff to a problem we know how to solve: $y = e^x$.

$$\begin{aligned}
\ln(y) &= \ln(e^x) = x \\
\frac{1}{y}y' &= 1 \\
y' &= y \\
y' &= e^x
\end{aligned}$$

which we already knew.

- Ex: $y = (x-1)(x^2+5)\cos(x)$. Can do a bunch of product rule, but easier to take log:

$$\begin{aligned}
\ln(y) &= \ln\big((x-1)(x^2+5)\cos(x)\big)\\
\ln(y) &= \ln(x-1) + \ln(x^2+5) + \ln(\cos(x))\\
\frac{1}{y}y' &= \frac{1}{x-1} + \frac{1}{x^2+5}\,(2x) + \frac{1}{\cos(x)}(-\sin(x))\\
y' &= y\left(\frac{1}{x-1} + \frac{2x}{x^2+5} - \tan(x)\right)\\
y' &= (x-1)(x^2+5)\cos(x)\cdot\left(\frac{1}{x-1} + \frac{2x}{x^2+5} - \tan(x)\right)
\end{aligned}$$

- Ex: $y = (\sin(x))^{\ln(x)}$