Homework 1
Due Tuesday, January 15th

Instructions: write up solutions to all problems below. Neatness counts: be sure to follow guidelines for homework in the syllabus.

Reading Assignment: Sections 2.1.4, Section 4.1 (all).

---

1. Verify that the following functions are solutions to the given differential equation.

   (a) $\dfrac{dy}{dx} + y = 0,$

   $\quad y = e^{-x}$

   (c) $\dfrac{dy}{dt} = 2t,$

   $\quad y = t^2$

   (b) $f'(x) = -\left(f(x)\right)^2,$

   $\quad f(x) = \frac{1}{x}.$

   (d) $2xy' = y,$

   $\quad y = \sqrt{x}$

2. A differential equation can tell you conceptual information about a function. Suppose that the function $y = f(x)$ is a solution to the equation

$$\frac{dy}{dx} = y^4$$

   with the initial condition $f(0) = 2$. Use the differential equation to find the slope of the tangent line at 0, and then find the equation of the tangent line.

3. Explain in your own words what a differential equation is. Also, explain what it means to have a solution to a differential equation.

4. Practicing Modeling. Take the following sentences and model the situation with a differential equation. Define any variables you use.

   (a) The voltage in a capacitor increases at a constant rate of $1.5\,\mathrm{mV}$.

   (b) The rate of temperature increase of a cup of coffee is proportional to the difference between the coffee's temperature and room temperature.

(c) The growth rate of feral cats in Eugene is proportional to the number of feral cats.

5. Find a solution to these differential equations by using an educated guess. Verify your guess is correct.

(a) $\dfrac{dy}{dt} = 2t$

(b) $f'(x) = \frac{1}{x}$

(c) $y' = e^x$

(d) $z' = z$