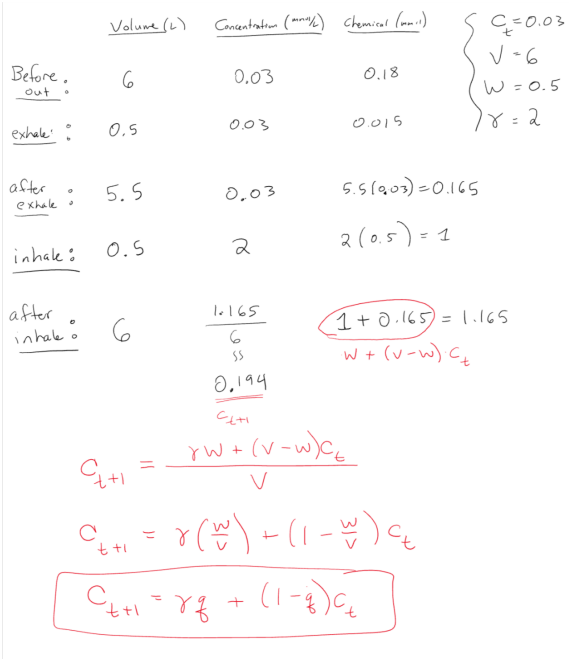
1.9: Lung Concentration Model

1 Walking Through the Model

- Quantities: c_t = concentration of a kind of gas in the lungs. Could be O_2, could be nitrogen, or laughing gas!
- Let V be the total volume of the lungs (average is about 6 Liters)
- W - amount of volume exhaled on normal breath
- γ - outside concentration of the chemical.
- Let's get familiar with these values before modeling.
- Ex: if a person exhales 0.5 L of air, and the concentration of O_2 is 2mmol/L, then how much O_2 did they exhale?
- Ans: 0.5L$*$2mmol/L = 1 mmol of O_2. That's a number of O_2 molecules.
- The quantity we're modeling, c_t, is like a conversion factor. It converts volume (Liters) into a number of moluecules (mmol).

2 One Breath

- Make table:



- Let $q = \frac{W}{V}$. It turns out that neither W or V are that important; what is more important is how big they are relative to each other, meaning their ratio q.

3 Analyze the Model

- What are the equilibria?
- Get: $c^* = \gamma$.
- Is it stable or unstable?
- Note: $f(x) = (1 - q)x + q\gamma$.
- $f'(x) = 1 - q$. Since q is a fraction of air breathed out, q is smaller than 1, so $1 - q$ is also smaller than 1.
- Stability theorem tells us this equilibrium is always stable.
- More analysis to come in your homework.

4 Conclusions to Draw

- No matter the initial starting amount, we know that the concentration will come into equilibrium with the outside equilibrium, as we expected. What is cool is that the math predicts what we expected in reality; nothing told the math to do that, rather it just did that.