Day 1: Limits

1 Motivation

- Point of this class: general functions = hard. Lines = easy. How to approximate hard functions by easy lines? Tangent lines.
- Draw picture.
- Definition: slope of the tangent line is called the *derivative* of $f(x)$. Notation: $\frac{df}{dx}$ or $f'(x)$. At $x = a$,

$$f'(a) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}.$$

- Take secant lines (lines connecting two points on $f(x)$) and limit as points close in ($x \to a$ or their difference, $h \to 0$).
- Right now: understand what this "Limit" symbol means in general.

2 Limits

- The limit of a function $g(x)$, as x approaches a, is a number L such that $g(x)$ is close to L whenever x is arbitrarily close to a (but not $x = a$). We notate this as $\lim_{x \to a} g(x) = L$.
- To evaluate limits, there are a couple of different strategies (in order of quickness):
  1. Plug in $x = a$.
  2. Look at graph.
  3. Plug in values of x *close* to the limit.

4. Try simplifying algebraically and then repeat all of 1 – 3.

- The limit $\lim_{x\to a} g(x)$ does not care about $g(a)$ always. Imagine like there's a little cloud around $x = a$ on the graph of $g(x)$. The limit can only see what's around the cloud (the values of $g(x)$ for x near a) and not underneath the cloud (at $x = a$).

- Warning: calculators may not always work. Example in worksheet. Sometimes the numbers get too hard for the calculator to handle and it'll blow up.

- Limits don't always exist. Graphical example with a "jump" discontinuity.

3 Left and Right Limits

- Left limit: $\lim_{x\to a^-} g(x)$

- Right limit: $\lim_{x\to a^+} g(x)$.

- Evaluated same as above, but only plugging in numbers coming from left or right.

- Important Fact: $\lim_{x\to a} g(x)$ exists when

$$\lim_{x\to a^-} g(x) = \lim_{x\to a^+} g(x).$$

Limits at infinity

- Same definition works to understand $\lim\limits_{x\to\infty} g(x)$ or $\lim_{x\to-\infty} g(x)$.

- Intuition: it detects asymptotes.

- Strategies to evaluate:

  1. Plug in large values of x (positive or negative)
  2. look at graph and find asymptotes
  3. Useful limits:
     - $\lim_{x \to \infty} \frac{1}{x} = 0$
     - $\lim_{x \to \infty} \frac{1}{x^2} = 0$
     - $\lim_{x \to \infty} \frac{1}{x^n} = 0$, n a positive whole number
     - $\lim_{x \to \infty} e^{-x} = 0.$

4 Limit Laws

- You can move limits all over the place, when everything is "nice."