HW 1

Friday, January 18, 2019 11:01 AM

1 @ $\frac{dy}{dx} + y = 0$, $y = e^{-x}$

$\frac{dy}{dx} = -e^{-x}$

$\Rightarrow \underbrace{(-e^{-x})}_{\frac{dy}{dx}} + \underbrace{(e^{-x})}_{y} = 0$ ✓

(b) $f'(x) = -(f(x))^2$

$f(x) = \frac{1}{x} = x^{-1}$

$f'(x) = -x^{-2}$ ← equal!

$-(f(x))^2 = -(x^{-1})^2 = -x^{-2}$ ✓

(c) $\frac{dy}{dt} = 2t$, $y = t^2$.

$\frac{dy}{dt} = \frac{d}{dt}(t^2) = 2t$ ✓

(d) $2x y' = y$, $y = \sqrt{x} = x^{1/2}$

$y' = \frac{1}{2} x^{-1/2}$

$2x y' = 2x\left(\frac{1}{2} x^{-1/2}\right) = x^{+1/2} = y$ ✓

(2) $\left.\frac{dy}{dx}\right|_{x=0} = y(0)^4 = 2^4 = 16.$

$y = mx + b$, $m = 16$

$2 = 16(0) + b \Rightarrow b = 2$

Tangent line is $\boxed{y = 16x + 2}$.

(4) (a) V: voltage in capacitor.

$\frac{dV}{dt} = 1.5$.

(b) T: temp of coffee.
T_0: ambient room temp.

$$\frac{dT}{dt} = k(T - T_0)$$

or

$$\frac{dT}{dt} = c(T_0 - T).$$

(k would be equal to $-c$.)

(c) C = # of feral cats.

$$\frac{dC}{dt} = kC$$

(5) (a) $\frac{dy}{dt} = 2t \Rightarrow$ $y = t^2$

(b) $f'(x) = 1/x \Rightarrow$ $f(x) = \ln(x)$

(c) $y' = e^x \Rightarrow$ $y = e^x$

(d) $z' = z \Rightarrow$ $z = e^x$ or e^t.

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