Exam 2 Practice Problems

These problems highlight what I will be testing you on. You may have a small notecard (no exact size restrictions, but just be reasonable). **These problems are probably more difficult than what will appear on the exam.** Here are some ideas I think are very important from class:

- Knowing how to check that a given function is or is not a solution to a differential equation.
- How to make and read a phase-line diagram.
- What equilibria are and mean, and how to determine their stability.
- How to read and interpret terms in a (system of) diffy-Q, such as interaction terms, what they mean, and how they positively or negatively affect populations.
- That improper integrals represent a long-run net change.

Some things will be different on this exam. I will ask you fewer True/False questions, but I will ask you to *justify* your answer for them. Also, instead of multiple choice, I will ask you two short-answer questions. Do not just study this handout; you should review old homework, quizzes, and handouts as well.

True/False

Practice justifying your answers to the following problems.

1. An autonomous diffy-Q must have a stable equilibrium.
2. A system of differential equations must have a predator and a prey population.
3. A numerical solution to a differential equation can always be computed, no matter what kind of differential equation is given.
4. An analytic solution to a differential equation can always be found, no matter what kind of differential equation is given.

Free Response Problems

1. Use Euler's method with $\Delta t = 0.5$ on the diffy-Q
$$\frac{dS}{dt} = \frac{e^S}{2+t}, \qquad S(0) = 0$$
to estimate $S(1.5)$. Then solve the differential equation exactly.

2. A population of dingos in Australia grows according to the differential equation
$$\frac{dD}{dt} = 30e^{-0.4t},$$
where D is measured in thousands. Assuming their population is currently five thousand, will their numbers ever reach one hundred thousand?

3. Suppose that two bacteria are growing in the same environment. Colony A has a per-capita growth rate $-\alpha + \beta B$, and colony B has a per-capita growth rate $-\gamma + \eta A$.

(a) Write down a system of equations that will model the dynamics of these two bacteria populations.

(b) Describe the interactions between the bacteria. Is one hunting the other; are they working together; or they competing for resources? Explain.