Written Assignment 7

Due Monday, November 29th

For extra practice, do (but don't turn in) the odd problems in the book nearby the assigned problems.

1. (3.1 # parts of 5-12) Graph the updating functions on the indicated intervals for each of the following discrete dynamical systems, and check the stability of the equilibria using the Stability Theorem. Check that your answers match what you get from cobwebbing. [Be sure to show all work for any derivatives you take.]

   (a) $c_{t+1} = 0.5c_t + 8$ for $0 \leq c_t \leq 30$

   (b) $m_{t+1} = (m_t)^2$ for $0 \leq m_t \leq 2$

   (c) $b_{t+1} = (b_t)^2 - 1$ for $0 \leq b_t \leq 2$

   (d) $x_{t+1} = \frac{x_t}{1+x_t}$, $-1 \leq x_t \leq 1$

2. (3.1 # 13, 14, 15) The Stability Theorem for (un)stability of an equilibrium does not tell us anything if we get a slope of 1, meaning the updating function is tangent to the diagonal. In this problem you will explore different outcomes when the Slope Criterion fails.

   (a) Graph an updating function that lies above the diagonal both to the left and to the right of an equilibrium. Is this equilibrium stable or unstable or neither?

   (b) Graph an updating function that is tangent to the diagonal at an equilibrium but crosses from below to above. Use cobwebbing to decide if the equilibrium is stable or unstable. Also, would the second derivative of the updating function be positive or negative?

   (c) Graph an updating function that is tangent to the diagonal at an equilibrium but crosses from above to below. Use cobwebbing to decide if the equilibrium is stable or unstable. Also, would the second derivative of the updating function be positive or negative?

3. (My brain) Recall our mutation model has the updating function

$$f(x) = \frac{rx}{rx + s(1-x)}.$$

   Let's suppose that r represents the growth factor for a mutated subpopulation, while s is the original subpopulation. Suppose that $r > s$, meaning that the mutated population grows more quickly. The equilibria will happen at $x = 0$ and $x = 1$. Use the Slope Criterion to evaluate the stability of these equlibria.

4. (3.2 # 3 and 4 with altered instructions) For each discrete dynamical systems, make two cobweb diagrams: (i) one with the actual updating function, and (ii) one where the updating function is the tangent line at the specified equilibrium. Use the stability theorem to decide if the indicated equilibrium is stable or unstable.

   (a) The logistic model $x_{t+1} = 1.5x_t(1 - x_t)$ at the equilibrium $x^* = 0$.

   (b) The logistic model $x_{t+1} = 1.5x_t(1 - x_t)$ at the equilibrium $x^* = 1/3$.

5. (3.3, # 8, 10, and my brain) Find the global minima and maxima of the following functions on the given intervals.

   (a) $f(x) = 1 + 2x - x^2$ for $0 \leq x \leq 2$.

   (b) $g(t) = \frac{t}{1+t^2}$ for $0 \leq t \leq 2$.

   (c) $h(r) = r^3 e^{-r}$ on the interval $[0, \infty)$.

   (d) $s(t) = \sin(t)$ on the interval $[0, 4\pi]$.

6. (My brain) Find all local minima and maxima of the following functions.

   (a) $f(x) = e^{-x^2}$

   (b) $g(t) = 10 + \dfrac{7t}{100 + t^2}$

7. (Extra Credit, my brain) Our mutation model did not incorporate a phenomenon known as **frequency dependence**, meaning that the per capita production of the different types does not depend on the fraction of types in the population. [Meaning in example: for our slow and fast zombie model, we don't take into account that the number of slow zombies might stop growing when there are more fast zombies taking over.] In this problem you will explore a more complex model that includes this phenomenon.

   Let b_t denote the original population (say, slow zombies), and let m_t denote the mutated population (say, fast zombies). Let p_t denote the fraction of the mutated population. Assume we come up with the equations

$$
\begin{aligned}
b_{t+1} &= 2(1 - p_t)b_t \\
m_{t+1} &= (1 + p_t)m_t
\end{aligned}
$$

   (a) Identify the term in the equations that represents the per capita production for each population above.

   (b) Describe in words what happens to each subpopulation when the fraction of fast zombies becomes large (i.e. when p_t is close to 1) and when it becomes small (i.e. p_t is close to 0).

   (c) Find the discrete dynamical system for the fraction p_t.

   (d) Find the equilibria for this system.

   (e) Evaluate the stability of the equilibria using the stability theorem.