Worksheet 7

Math 251, Summer 2017

Name: Key

1. Find the derivatives of the following functions.

(a) 4^x

$$\left[\ln(4)\right] \cdot 4^x$$

outside of the $(\ )^x$.

(b) $\log_{10}(x) = \dfrac{\ln(x)}{\ln(10)}$

$$\frac{d}{dx}\log_{10}(x) = \frac{1}{\ln(10)}\frac{d}{dx}(\ln(x))$$

$$= \boxed{\frac{1}{x \cdot \ln(10)}}$$

(c) $\arcsin(e^x)$

outside: $\arcsin(\ )$

inside: e^x.

$$\underbrace{\frac{1}{\sqrt{1-(e^x)^2}}}_{f'(g(x))} \cdot \underbrace{e^x}_{g'(x)}$$

(d) $x\arctan(x) - x$

$$(x)' \cdot \arctan(x) + x \cdot (\arctan(x))' - 1$$

$$= \arctan(x) + x \cdot \frac{1}{1+x^2} - 1$$

$$= \boxed{\arctan(x) + \frac{x}{1+x^2} - 1}$$

(e) $\frac{x}{\ln(x)}$

$$\Rightarrow \frac{(x)' \cdot \ln(x) - x \cdot (\ln(x))'}{(\ln(x))^2}$$

$$= \frac{\ln(x) - x \cdot \left(\frac{1}{x}\right)}{(\ln(x))^2}$$

$$= \frac{\ln(x) - 1}{(\ln(x))^2}.$$

(f) $\ln(x^2e^x)$

Two ways:

① Out: $\ln(\ )$

In: x^2e^x

$$\Rightarrow f'(x) = \frac{1}{(x^2e^x)} \cdot (x^2e^x)'$$

$$= \frac{1}{x^2e^x}\left((x^2)'e^x + x^2(e^x)'\right)$$

$$= \frac{1}{x^2e^x}\left(2xe^x + x^2e^x\right).$$

② Alternative: log rules.

$$f(x) = \ln(x^2e^x) = \ln(x^2) + \ln(e^x)$$

$$= 2\ln(x) + x$$

$$\boxed{f'(x) = 2 \cdot \frac{1}{x} + 1}$$

these are equal if you do algebra

2. Find the linearization for $f(x) = x^2\ln(3x)$ at $x = 3$.

$$f'(x) = 2x\ln(3x) + x^2 \cdot \frac{1}{\cancel{3}x} \cdot \cancel{3}$$
$$= 2x\ln(3x) + x.$$

$$f'(3) = 2(3)\ln(3\cdot 3) + 3 \approx 16.18$$

Linearization = Tangent Line : $y = mx + b$

$x = 3, \quad y = f(3) = 9\ln(9) = 19.8$

$19.8 = 16.2(3) + b$

$b = -28.8$

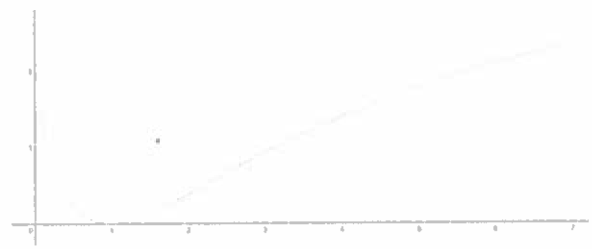
$$\boxed{y = 16.2x - 28.8}$$

3. Find y' from the implicit equation $\ln(x) + 2^y = x$.

$$\frac{1}{x} + \ln(2)\cdot 2^y \cdot y' = 1$$

$$\ln(2)\cdot 2^y\, y' = 1 - \frac{1}{x}$$

$$\boxed{y' = \frac{1 - \frac{1}{x}}{\ln(2)\cdot 2^y}}$$



4. Optimize the function $g(x) = x^2\ln(x)$ on the interval $[0.1, 1]$.

$$g'(x) = x^2\cdot\frac{1}{x} + 2x\ln(x) = 0$$
$$x + 2x\ln(x) = 0$$
$$x(1 + 2\ln(x)) = 0$$

$x = 0$ or $1 + 2\ln(x) = 0$

not in the interval!

$\ln(x) = -\frac{1}{2}$

$x = e^{-1/2} \approx 0.607$

$g(0.1) = (.1)^2\ln(.1) = -0.023$

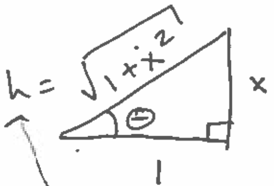
$g(0.607) = -0.184 \leftarrow$ Min

$g(1) = 1^2\ln(1) = 0 \leftarrow$ Max

5. In this problem you will prove the formula $\frac{d}{dx}\arctan(x)$. $= \frac{1}{1+x^2}$

   (a) By letting $\theta = \arctan(x)$, draw a right triangle in order to simplify $\sec^2(\arctan(x))$.

$\tan\theta = x = \frac{x}{1} = \frac{opp}{adj}$



hyp = h = ?

$h^2 = x^2 + 1^2$

$h = \sqrt{x^2+1}$

$\sec(\theta) = \frac{hyp}{adj} = \frac{\sqrt{1+x^2}}{1}$

$\sec^2\theta = 1 + x^2$ ✓

$\boxed{\sec^2(\arctan(x)) = 1 + x^2}$

   (b) Apply the formula for $(f^{-1})'$ to get the formula for $\frac{d}{dx}\arctan(x)$.

$$(f^{-1})'(x) = \frac{1}{f'(f^{-1}(x))} = \frac{1}{\sec^2(\arctan(x))} = \boxed{\frac{1}{1+x^2}}$$

$f(x) = \tan(x)$

$f'(x) = \sec^2(x)$

6. When electricity flows through a wire, you can measure the amount of charge, Q, at a spot along the wire (the shaded section of the figure below). Charges in the form of electrons move through the wire, so the amount of charge Q at the slice is really a function of time. In applications, one measures the *current*, defined as the rate of change of charge. That is, current is the *derivative* of charge.

$$I = \frac{dQ}{dt}.$$

Suppose that the amount of charge flowing in the wire is given by the function

$$Q(t) = 10.5\sin(2\pi 60t)$$

(which, say, represents the amount of charge coming through the sockets in your wall). Find the current $I(t)$ as a function of time.

$$Q'(t) = 10.5 \cdot \cos(2\pi \cdot 60t) \cdot (2\pi \cdot 60)$$

$$\boxed{I = (3958.4)\cos(2\pi \cdot 60t)}$$