Exam 1

Math 251, Summer 2017

Name: Key

- You have the full class time to work on the exam. but it is designed to be 50 minutes.
- There are $\boxed{\cancel{30}}$ 32 points on this exam.
- **Show all of your work and justification** for each answer.
- In each problem, **draw a box around your final answer**.

1. Decide if each of the following statements is true or false and briefly explain why.

(a) [2 pts] The slope of the tangent line is always given by $\dfrac{f(x)-f(a)}{x-a}$.

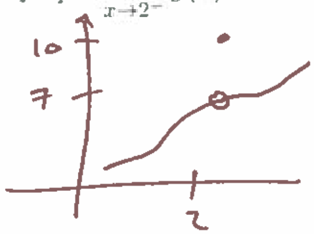
False: for this you need $\lim\limits_{h\to 0} \dfrac{f(x+h)-f(x)}{h}$,

or $\lim\limits_{x\to a} \dfrac{f(x)-f(a)}{x-a}$.

The symbol "$\lim\limits_{x\to a}$" is essential; without it, you only have a secant line.

+1 pt ans
+1 pt explain

(b) [2 pts] If $\lim\limits_{x\to 2^-} g(x) = 7$ and $\lim\limits_{x\to 2^+} g(x) = 7$ but $g(2) = 10$, then $\lim\limits_{x\to 2} g(x) = \text{DNE}$.



False. The limit is definitely 7.

$\boxed{\lim\limits_{x\to 2} g(x) = 7}$.

*Limits don't care about the value at that point.

+1 pt ans
+1 pt explain

2. Find the following limits. Make sure to show work that justifies your answers.

(a) [3 pts] $\lim\limits_{t\to\infty} \dfrac{t^5-4t^3-1}{6t^5+8t^4+3t^3+16} = \lim\limits_{t\to\infty} \dfrac{\cancel{t^5}}{\cancel{t^5}} \cdot \dfrac{\left(1-\frac{4}{t^2}-\frac{1}{t^5}\right)}{\left(6+\frac{8}{t}+\frac{3}{t^2}+\frac{16}{t^5}\right)}$

$= \boxed{\dfrac{1}{6}}$ (all other terms go to 0 as $t\to\infty$.)

+1 pt for factoring t^5
+1 pt for $\frac{1}{t^{n}} \to 0$
+1 pt for ans.

(b) [3 pts] $\lim\limits_{x\to 2} \dfrac{2x^2-4x}{3x^3-6x^2}$

$\dfrac{2(2^2)-4(2)}{3(2^3)-6(2^2)} = \dfrac{8-8}{24-24} = \dfrac{0}{0}$!

$\Rightarrow$ gotta cancel:

$\lim\limits_{x\to 2} \dfrac{2x\,\cancel{(x-2)}}{3x^2\,\cancel{(x-2)}} = \lim\limits_{x\to 2} \dfrac{2x}{3x^2} = \dfrac{4}{12} = \boxed{\dfrac{1}{3}}$

+1 for canceling
+1 for plug in
+1 for ans

3. [3 pts] Find $\frac{d}{dx}\left(\frac{e^x}{1+e^x}\right)$ at $x=2$.

$$= \frac{e^x(1+e^x)-e^{2x}}{(1+e^x)^2}.$$

+1 for quotient
+1 for plugging $x=2$
+1 for ans.

at 2: $$\frac{e^2(1+e^2)-e^4}{(1+e^2)^2} \approx \boxed{0.105}$$

4. [4 pts] Find the slope of the tangent line to the function $y = \ln(x)\cos(3x)$ at $x = 4$.

$$y' = \frac{1}{x}\cos(3x) + \ln(x)\cdot(-\sin(3x)\cdot 3)$$

$$y' = \frac{1}{4}\cos(12) - 3\ln(3)\sin(12) = \boxed{2.443}.$$

(or, if you're incorrectly in degrees, you would have gotten -0.441)

+1 product rule
+1 der. cos
+1 der. ln
+0.5 Chain rule. ← was a minor chain rule...
+0.5 ans.

5. [3 pts] Let $q(x) = \frac{1}{x}$. Use the *definition of the derivative* to find the value of $q'(2)$.

$$q'(2) = \lim_{h\to 0}\frac{1}{h}\left[\frac{1}{2+h} - \frac{1}{2}\right] = \lim_{h\to 0}\frac{1}{h}\left[\frac{2-(2+h)}{2(2+h)}\right]$$

$$= \lim_{h\to 0}\frac{1}{\cancel{h}}\left[\frac{-\cancel{h}}{2(2+h)}\right]$$

$$= \lim_{h\to 0}\frac{-1}{2(2+h)}$$

$$= \boxed{-\frac{1}{4}}$$

+1 def. correctly written
+1 algebraic manipulations
+1 answer.

*Side Note: you can double check with shortcuts:
$q' = -x^{-2}$,
$q'(2) = -(2)^{-2} = -\frac{1}{2^2} = -\frac{1}{4}$ ✓

6. A toy rocket is launched. After t seconds, the height of the rocket is $h(t) = 96t - 16t^2$ feet.

(a) [3 pts] The rocket returns to the ground after $t = 6$ seconds. Find its velocity at this moment. Include units in your answer.

$$v(t) = h'(t) = 96 - 32t.$$

$$v(6) = 96 - 32(6) = \boxed{-96 \text{ ft/sec}}$$

+1 derivative
+1 ans
+1 units

(b) [2 pts] Find $h''(t)$ and write a short sentence explaining what it means. You must include units and must not use the word "derivative".

+0.5 ans
+0.5 units
+1 "accel...."

$$h''(t) = \boxed{-32 \text{ ft/sec}^2}$$

This number is the acceleration (due to gravity) of the toy rocket.

7. [4 pts] Find the equation of the tangent line to the hyperbola $x^2 - y^2 + 4xy = 1$ at the point $(1, 0)$.

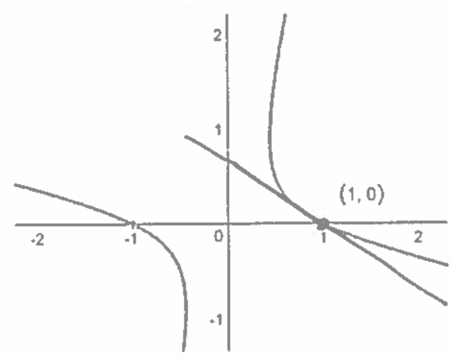
Find y':

$$2x - 2y \cdot y' + 4y + 4xy' = 0$$

$$-2yy' + 4xy' = -2x - 4y$$

$$y'(-2y + 4x) = -2x - 4y$$

$$y' = \frac{-2x - 4y}{-2y + 4x}$$



+2 formula for y'
+1 slope
+1 b

$$\rightarrow y' = \frac{-2(1) - 4(0)}{-2(0) + 4(1)} = \frac{-2}{4} = \boxed{-\frac{1}{2}}$$

$$y = -\frac{1}{2}x + b$$

$$0 = -\frac{1}{2} + b$$

$$b = \frac{1}{2}$$

$$\Rightarrow \boxed{y = -\frac{1}{2}x + \frac{1}{2}}$$

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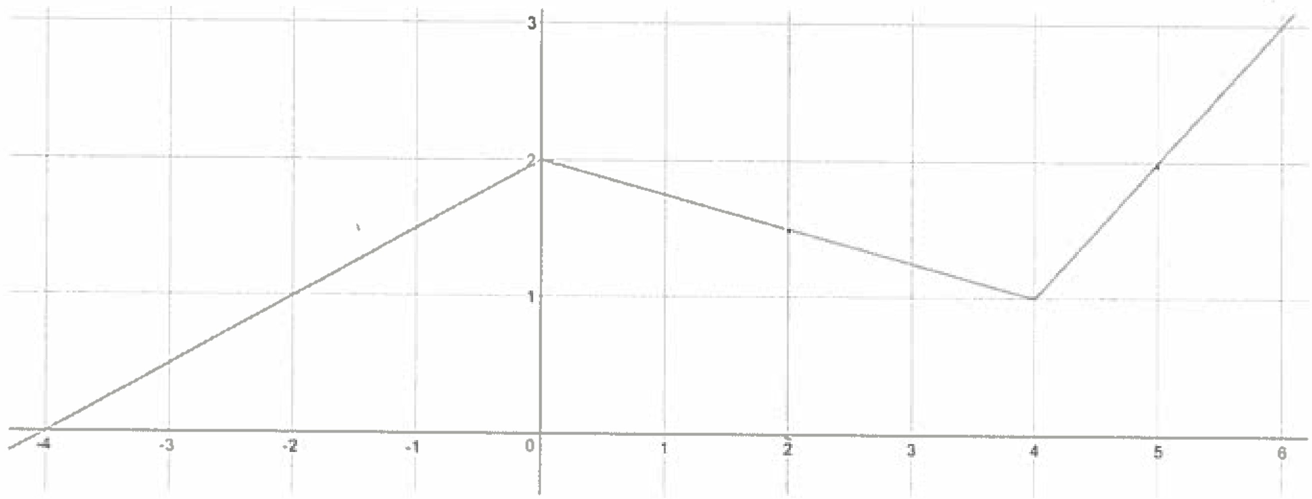
8. [~~[illegible]~~ pts] Let $f(x) = x^2$ and $g(x)$ be the function graphed below. Let $u(x) = f(x)g(x)$, and let $v(x) = g(f(x))$. Find the following values.



(a) $u'(2)$

$= f'(2)g(2) + f(2)g'(2)$

$= 4\cdot(1.5) + 4\left(-\frac{1}{4}\right)$

$= 6 - 1$

$= \boxed{5}$

(b) $v'(-1)$

$= g'\left((-1)^2\right)\cdot 2(-1)$

$= g'(1)\cdot(-2)$

$= -\frac{1}{4}(-2)$

$= \boxed{\frac{1}{2}}$

(c) $u'(5)$

$= 2(5)\cdot g(5) + (5)^2\cdot g'(5$

$= 10\cdot 2 + 25\cdot(1)$

$= \boxed{45}$

+1 pt each

$u'(x) = f'(x)g(x) + f(x)g'(x)$

$= 2x\,g(x) + x^2\cdot g'(x)$

$v'(x) = g'(f(x))\cdot f'(x).$

$= g'(x^2)\cdot 2x$