# Final Review

This review sheet is meant to give you practice with some of the later material in the class; also, this sheet may not contain everything we did in the course, but it is a starting point.

1. Summarize the three types of hearts we saw in class via their cobweb diagrams. What distinguishes their behavior?

2. With the usual parameter values of $V_c = u = 1\,\text{mV}$ and $T = 1\,\text{s}$ and with a decay rate of $\alpha = \ln(2)$, make a cobweb diagram and classify the health of the heart. Does it show any signs of second-degree block?

3. With the same parameter values but $\alpha = \ln(1.2)$, make a cobweb diagram. Analyze this heart's health. With these parameter values, if $V_t = 4\,\text{mV}$, will the heart beat on the next SA signal?

4. Suppose a bacteria strain grows by 40% per hour, and then a mutation occurs, producing a new strain that triples every hour. (a) Write down the model for the fraction of mutated bacteria. (b) If originally 20% of the population mutated, then what is their fraction after two hours? (c) What happens to both populations in the long run? Explain using the known equilibria and stability.

5. Find the inflection points of the following functions. Be sure to justify that any points you find are actually inflection points.

   (a) $f(x) = e^{-x^2}$

   (b) $g(x) = x \ln(x)$

6. Suppose a person breathes in 15% of their lung capacity, and that the ambient air contains a concentration of 1.7 mmol/L of nitrogen. If their lungs contain a concentration of 1.3 mmol/L, then what is the concentration in their lungs after three breaths?

7. Find and classify the critical points of these functions as either local minima, maxima, or neither.

   (a) $f(x) = e^x + 3e^{-2x}$

   (b) $g(t) = t^2 \ln(13t)$

   (c) $h(t) = t - \sqrt{t}$.

8. Using only the limit definition of the derivative, calculate the following derivatives. Do not use l'Hopital's rule.

   (a) $f(x) = x^2 - x$

   (b) $g(x) = \frac{1}{x}$

   (c) $r(t) = \frac{1}{t^2}$

   (d) $s(t) = \sqrt{t}$

9. Evaluate the following limits.

(a) $\lim_{x \to \infty} \dfrac{e^x + x}{x^2}$

(b) $\lim_{x \to 2} \dfrac{x^3 - 7x^2 + 10x}{x^2 + x - 6}$

(c) $\lim_{w \to -4} \dfrac{\sin(\pi w)}{w^2 - 16}$

(d) $\lim_{t\to\infty} \dfrac{\ln(3t)}{t^2}$

(e) $\lim_{t\to 0} \dfrac{\sin(2z) + 7z^2 - 2z}{z^2(z+1)^2}$