Worksheet 2

Name: ____________________

Directions

- This counts as your "attendance" for the day. You must give this to me today to get credit for it.
- You may leave if you finish the packet.
- You may work in groups, and you can ask me for assistance.
- I grade this worksheet based on completion, not accuracy; however, you should strive for completely correct answers in order to make sure you understand the material.

---

1. Find the average rate of change of the following functions on the indicated intervals.

(a) $f(x) = 12x^2 + 2$, interval: $[3, 4]$

$$\frac{f(4)-f(3)}{4-3} = \frac{194-110}{1} = \boxed{84}$$

(b) $f(t) = \frac{2}{t}$, interval: $[1, 6]$.

$$\frac{f(6)-f(1)}{6-1} = \frac{\left(\frac{2}{6} - \frac{2}{1}\right)}{5} = \frac{1}{5}\left(\frac{-10}{6}\right) = \boxed{-\frac{1}{3}}$$

2. Find the instantaneous rate of change of the functions at the indicated x-value.

(a) The function from 1a, at $x = 2$.

$$f'(2) = \lim_{h\to 0} \frac{f(2+h)-f(2)}{h}$$

$$= \lim_{h\to 0} \frac{12(2+h)^2 + \cancel{2} - (12(2^2) + \cancel{2})}{h}$$

$$= \lim_{h\to 0} \frac{12(4+4h+h^2) - 48}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{48} + 48h + 12h^2 - \cancel{48}}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{h}(48+12h)}{\cancel{h}}$$

$$= 48 + 12(0)$$

$$= \boxed{48}$$

(b) The function from 1b, at $x = 3$.

$$f'(3) = \lim_{h\to 0} \frac{\left(\frac{2}{3+h} - \frac{2}{3}\right)}{h}$$

$$= \lim_{h\to 0} \frac{1}{h}\left[\frac{3}{3}\cdot\frac{2}{(3+h)} - \frac{2}{3}\frac{(3+h)}{(3+h)}\right]$$

$$= \lim_{h\to 0} \frac{1}{h}\left[\frac{6}{3(3+h)} - \frac{6+2h}{3(3+h)}\right]$$

$$= \lim_{h\to 0} \frac{1}{h}\left[\frac{\cancel{6}-\cancel{6}-2h}{3(3+h)}\right]$$

$$= \lim_{h\to 0} \frac{1}{\cancel{h}}\left[\frac{-2\cancel{h}}{3(3+h)}\right]$$

$$= \frac{-2}{3(3+0)} = \boxed{\frac{-2}{9}}$$

Worksheet 2

3. For the function $f(x) = x^2 + 2$, find the equation of the tangent line at $x = 3$.

$f'(3) = 6$ (similar to Problem 2a)

Tangent Line: $y = mx + b$

$(m = 6)$ ⟷ Slope = derivative

$y = 6x + b$

Note: $x = 3$, $y = f(3) = 11$.

$11 = 6(3) + b \rightsquigarrow b = 11 - 18 = -7$

$\boxed{y = 6x - 7}$

4. Suppose $f(x) = c$ is a constant function (meaning c is a fixed number not depending on x). Calculate $f'(x)$ from the definition. (If it makes the problem easier, you may pretend $c = 2$; your result, however, will work for any constant.)

$f(1) = c$

$f(2) = c$

$f(\text{any}) = c$

$f($  $) = c$

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h\to 0} \frac{c - c}{h} = 0 \text{ !}$$

⇒ constant functions have 0 slope. (not "no" slope).

5. In all of the preceding problems, we calculated the derivative $f'(x)$ at specific x-values. If instead we leave x unspecified as a variable, then $f'(x)$ defines a new *function*. In the following problems, calculate the derivative function for given $f(x)$.

(a) $f(x) = \dfrac{1}{x}$

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h\to 0} \boxed{\frac{1}{h}} \cdot \left[\frac{1}{x+h}\frac{(x)}{(x)} - \frac{1}{x}\frac{(x+h)}{(x+h)} \right]$$

$$= \lim_{h\to 0} \frac{1}{h} \left[\frac{\cancel{x} - \cancel{x} - h}{x(x+h)} \right]$$

$$= \lim_{h\to 0} \frac{1}{\cancel{h}} \left[\frac{-\cancel{h}}{x(x+h)} \right]$$

$$= \lim_{h\to 0} \frac{-1}{x(x+h)}$$

$$\boxed{f'(x) = \frac{-1}{x^2}}$$

(b) $f(x) = x^2$

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h\to 0} \frac{(x+h)^2 - x^2}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{x^2} + 2hx + h^2 - \cancel{x^2}}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{h}(2x+h)}{\cancel{h}}$$

$$\boxed{f'(x) = 2x}$$

6. Calculate the derivative function of $f(x) = x$.

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$
$$= \lim_{h\to 0} \frac{\cancel{x}+h - \cancel{x}}{h}$$
$$= \lim_{h\to 0} \frac{h}{h} = 1$$

$$\boxed{f'(x) = 1}$$

7. Calculate the derivative function of $f(t) = \sqrt{t}$. [Hint: after setting up the difference quotient, multiply the top and bottom of the fraction by $(\sqrt{x+h} + \sqrt{x})$.]

$$f(t) = \lim_{h\to 0} \frac{(\sqrt{t+h} - \sqrt{t})}{h} \cdot \frac{(\sqrt{t+h} + \sqrt{t})}{(\sqrt{t+h} + \sqrt{t})}$$

$$= \lim_{h\to 0} \frac{(\sqrt{t+h})^2 - (\sqrt{t})^2}{h(\sqrt{t+h} + \sqrt{t})}$$

Difference of squares!

$$= \lim_{h\to 0} \frac{\cancel{t}+h - \cancel{t}}{h(\sqrt{t+h} + \sqrt{t})} = \lim_{h\to 0} \frac{\cancel{h}}{\cancel{h}(\sqrt{t+h} + \sqrt{t})}$$

$$= \lim_{h\to 0} \frac{1}{\sqrt{t+h} + \sqrt{t}} = \frac{1}{\sqrt{t} + \sqrt{t}} = \boxed{\frac{1}{2\sqrt{t}}}$$

8. A really frustrated calculus student decides to drop their copy of the textbook off of the top floor of deady. The height of the book (in feet) as a function of time (in seconds) can be modeled by

$$y(t) = -16t^2 + 70,$$

where y is the height of the book measured from the ground. What is the speed of the book when it hits the ground?

Set $y=0$ to find when it hits ground.

$$0 = -16t^2 + 70$$
$$16t^2 = 70$$
$$t^2 = \frac{70}{16}$$
$$t = \pm\sqrt{\frac{70}{16}} = \oplus 2.09$$

Calculate $y'(2.09)$:

$$y'(2.09) = \lim_{h\to 0} \frac{-16(2.09+h)^2 + 70 - (-16(2.09)^2 + 70)}{h}$$

$$= \lim_{h\to 0} \frac{-16[\cancel{(2.09)^2} + 2(2.09)h + h^2] + \cancel{16(2.09)^2}}{h}$$

$$= -16 \lim_{h\to 0} \frac{2(2.09)\cancel{h} + h^{\cancel{2}}}{\cancel{h}}$$

$$= -16(2)(2.09) \approx \boxed{-66.93 \text{ ft/sec}}$$

* It's a negative velocity because it's traveling down!