Exam 1 Review

The exam will take place on Friday, June 8th. You will have the entire class period for the exam. No notes will be allowed. Calculators are allowed, but all work must be shown on the exam to receive full credit.

The exam has about three short-answer problems and three long answer problems. Any material from chapters 1-3 is fair game, excluding the portion of chapter 3 on "Long-term behavior."

This review sheet is not for turn-in; it is for you to practice with the kinds of questions I could ask. You should spend time reviewing notes, reading the sections in the book, going over old quizzes and homework/webwork problems.

Disclaimer: If this review packet is *all* you study, you will not do very well on this exam.

Concepts

1. Know the definition of a function and be able to use it to explain why a given rule, table, graph, etc. is or is not a function.
2. Be able to compute the average rate of change and percent change of a function and explain what they mean.
3. Be able to use the quadratic formula or vertex formula to solve problems involving quadratic models.
4. Be able to define a polynomial and identify key features of polynomials, such as the degree and leading term.
5. Know what the domain and range of a function are, and be able to compute them.
6. Be able to construct linear models.
7. Be able to recognize when a piecewise function is necessary for describing real-world scenarios.

Practice Problems

1. Since starting graduate school, your instructor's nightly sleep has been following the model
$$S(t) = -\frac{1}{4}t + 8,$$
where t is the number of days since the quarter began, and S is the number of hours of sleep per night. By what percent did your instructor's nightly sleep decrease between the fourth and the tenth day of the quarter?

$$\text{PC} = \frac{S(10) - S(4)}{S(4)} \cdot 100\% = \frac{5.5 - 7}{7} \cdot 100\% = -21.4\%$$

2. A feather is *dropped* from a 100-foot high building. Recall that the height follows the quadratic model

$$h(t) = -16t^2 + v_0 t + h_0$$

for t in seconds since falling. When does the feather hit the ground?

The feather is dropped, so $v_0 = 0$. The initial height is 100, so $h_0 = 100$. The quadratic is then $h(t) = 0 - 16t^2 + 100$. The ground is at $h = 0$, so set $h(t) = 0$ and solve:

$$\begin{aligned} -16t^2 + 100 &= 0 \\ 16t^2 &= 100 \\ t^2 &= \frac{100}{16} \\ t &= \pm\sqrt{\frac{100}{16}} = \pm\frac{10}{4} = \pm 2.5. \end{aligned}$$

We want positive time values, so $t = 2.5$ seconds.

3. Compute the average rate of change and percent change of the function $P(x) = (x - 20)(2x - 17)(x - 5)$ over the interval $[3, 9]$.

Note $P(9) = (-11)(1)(4) = -44$ and $P(3) = (-17)(-11)(-2) = -374$, so

$$\text{ARC} = \frac{-44 - (-374)}{9 - 3} = \frac{330}{6} = 55.$$

4. Find the largest possible domain of the function

$$g(t) = \frac{\sqrt{20 - 9x}}{x^2 - 4x - 5}.$$

We need to ensure there is no division by 0 or negatives inside the square root. For the square root, we need $20 - 9x \geq 0$, which can be solved for x to get $x \leq 2.22$. This is the first requirement on x.

To make sure we don't divide by 0, we need to know when the denominator is equal to 0. This happens if $x^2 - 4x - 5 = 0$, which can be solved by factoring or with the quadratic formula:

$$x = \frac{4 \pm \sqrt{(-4)^2 - 4(1)(-5)}}{2} = \frac{4 \pm 6}{2} = 5 \text{ and } -1.$$

Therefore, we must exclude $x = 5$ and $x = -1$. Drawing a number line will then help you see that the domain is

$$\text{Dom}(g) = \{x \mid x \leq 2.22 \text{ and } x \neq -1\} = (-\infty, -1) \cup (-1, 2.22].$$

(Note 5 is bigger than 2.22, so it is automatically excluded by the requirement $x \leq 2.22$.)

5. Describe the degree and leading terms of the polynomials $f(t) = (9-t)^2(t+1)$ and $r(n) = n^{29} - 21n^{30} + n^{19}$.

   The leading term of f is t^3, so its degree is 3.

   The leading term of r is $-21n^{30}$, so its degree is 30.

6. A set of keys is thrown up in the air. Assume that the height of the keys follows the model $h(t) = -16t^2 + v_0 t + h_0$. They are thrown at an initial height of 3 feet, and they hit the ground after about 1.5 seconds. What was the maximum height of the keys?

   We know $h_0 = 3$, but we do not know v_0. We know the input-output pair $(1.5, 0)$ (because the keys hit the ground after 1.5 seconds). So,

$$\begin{aligned} 0 &= -16(1.5)^2 + v_0(1.5) + 3 \\ 0 &= -36 + 1.5v_0 + 3 \\ 33 &= 1.5v_0 \\ v_0 &= 22. \end{aligned}$$

   So, the quadratic is

$$h(t) = -16t^2 + 22t + 3.$$

   Finally, we find the vertex. $t_{\text{vert}} = -\frac{b}{2a} = -\frac{22}{2(-16)} = 0.6875$ seconds. Thus, the maximum height is

$$h_{\text{vert}} = h(t_{\text{vert}}) = -16(0.6875)^2 + 22(0.6875) + 3 \approx 10.6 \text{ feet}.$$

7. Graph the piecewise function

$$g(x) = \begin{cases} 2x+1 & x < 10 \\ -x^2 + 12 & x \geq 10 \end{cases}$$