5.4: Solving separable Diffy-Q

- Any differential equation that looks like $\frac{dy}{dt} = f(y)g(t)$ can be solved *analytically* by a technique called separation of variables.
- Example: $\frac{dy}{dt} = 3y$.

$$\begin{aligned}
\frac{dy}{dt} &= 3y \\
\frac{dy}{y} &= 3dt \\
\int \frac{dy}{y} &= \int 3dt \\
\ln|y| &= 3t + C \\
|y| &= e^{3t+C} \\
|y| &= e^C e^{3t} \\
y &= (\pm e^C)e^{3t}.
\end{aligned}$$

  Call $K = \pm e^C$. Thus, the solution is $y = Ke^{3t}$.

- Find the solution with initial condition $y(0) = 5$.

$$5 = y(0) = Ke^0 = K,$$

  so $K = 5$ and the solution is $y(t) = 5e^{3t}$.

- Let them try some: worksheet.