Written Assignment 6

Due Tuesday, November 20th

1. Explain, in your own words,

   (a) what a discrete dynamical system is,

   (b) what a solution to a discrete dynamical system is,

   (c) and what you need in order to find a solution.

2. For the following discrete dynamical systems, make a table of the first five values (m_1 through m_5) with the initial seeds shown below. [For this problem show your work and do not use the online calculator to do it for you.]

   (a) $m_{t+1} = m_t + 1$, $m_0 = 0.5$.

   (b) $m_{t+1} = e^{-m_t}$, $\quad m_0 = 2$.

   (c) $m_{t+1} = 0.5m_t(1 - m_t)$, $m_0 = 0.4$.

   (d) $m_{t+1} = 0.5m_t(1 - m_t)$, $m_0 = 1$.

3. For each of the discrete dynamical systems below, do the following:

   (i) Make a cobweb diagram of the DDS with the given initial condition. Be sure to include arrows on your cobweb to indicate direction.

   (ii) Make a plot of the solution (which is a graph of m_t vs. t). Use the first 5 values.

   (iii) Identify any equilibria using the graph of the updating function.

   (a) $m_{t+1} = 0.2(m_t)^4 + 0.5(m_t)^2 - m_t$, $\quad m_0 = 0.6$.

   (b) $m_{t+1} = e^{-m_t}$, $\quad m_0 = 2$.

   (c) $m_{t+1} = \sin(m_t)$, $\quad m_0 = -1.5$.

   (d) $m_{t+1} = \sqrt{m_t}$, $\quad m_0 = 0.5$.

   (e) $m_{t+1} = (m_t)^2 + 1$, $\quad m_0 = 1$.

   (f) $b_{t+1} = \frac{2b_t}{0.5b_t + 2(1-b_t)}$, $b_0 = 0.2$

4. Recall from class that an equilibrium is *stable* if nearby seeds are attracted to the equilibrium, and *unstable* if nearby seeds are repelled by the equilibrium. For each of the following,

   (i) What are the equilibria?

   (ii) Are nearby seeds attracted or repelled from each equilibrium?

   Use cobweb diagrams to illustrate your answers.

   (a) $m_{t+1} = 0.5m_t + 1$.

   (b) $m_{t+1} = 2m_t + 1$.

(c) $m_{t+1} = m_t^2$

(d) $m_{t+1} = \cos(m_t)$ [You won't be able to find this equilibrium algebraically, but you can approximate using a calculator or computer.]

5. Find the equilibria of the discrete dynamical system.

(a) $m_{t+1} = \dfrac{m_t}{m_t + 1}$

(b) $b_{t+1} = 2b_t$

(c) $p_{t+1} = \dfrac{2p_t}{2p_t + 1.5(1 - p_t)}$

6. Using the stability theorem [to be given in class Friday or Monday] determine if the equilibria to the DDS below are stable or unstable.

(a) $m_{t+1} = \dfrac{m_t}{m_t + 1}$

(b) $b_{t+1} = 2b_t$

(c) $p_{t+1} = \dfrac{2p_t}{2p_t + 1.5(1 - p_t)}$

7. Practice with modeling. Write down discrete dynamical systems that model the following scenarios. In each scenario, indicate how much time passes in one iteration of the dynamical system. Include an initial seed if one is given.

(a) A population of bacteria in a petri dish doubles in size every day; then, after it doubles, 100 bacteria are removed from the petri dish.

(b) A population of bacteria is growing, and each day, 100 bacteria are removed, and after the removal their population doubles.

(c) A bank account accrues interest at a rate of 10% each year, and the owner deposits \$50 a month, with an initial down payment of \$500. (Assume the monthly deposits all happen before the 10% interest is applied.)

(d) A population of birds in a jungle begin to adapt by evolving a new gene that helps them hunt insects. The number of individuals with this new gene increases by 15% every generation.