Quiz 4

Name: _____

You will have 20 minutes ○ Calculators are allowed ○ Show all work for credit ○ Don't cheat ○ attempts at a problem may count for partial credit. ○ If you get stuck, show as much work as possible.

1. [3 pts] Find the area between the x-axis and the graph of the function $\sin(x)$ between $x = 0$ and $x = 2\pi$.

$$\int_0^{2\pi} \sin(x)\, dx = (-\cos(x))\Big|_0^{2\pi} = (-\cos(2\pi)) - (-\cos(0)) = (-1) - (-1) = 0.$$

2. Suppose the number of fish, f, in a pond grow according to the differential equation

$$\frac{df}{dt} = 7.7e^{0.1t},$$

where t is measured in months. Let's also suppose that there are 10 fish at time $t = 0$.

(a) [3 pts] What is the total change in the number of fish in the first three months?

$$\int_0^8 7.7e^{0.1t}\, dt = 77e^{0.1t}\Big|_0^8 = 26.94 \text{ fish.}$$

(b) [1 pts] How many fish are there at this point in time? There are a total of about 37 fish.

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3. [4 pts] Suppose the amount of energy (in Joules) a cell produces follows the equation

$$E(t) = \frac{2.5}{1+t}$$

over a time length of four milliseconds. Find the average energy produced by the cell during this time.

$$\frac{1}{4}\int_0^4 \frac{2.5}{1+t}\, dt = \frac{2.5}{4}\ln(1+t)\Big|_0^4 = 1.005 \text{ joules}$$

4. [4 pts] The number of beavers along the Willamette river has a density of

$$B(x) = 0.2(x-5)^2 + 1 \qquad \text{hundred beavers/mile,}$$

where x is the number of miles east along the river measured from Eugene. How many beavers live between one and five miles away (to the east) from Eugene along the river?

$$\int_1^5 (0.2(x-5)^2 + 1)\, dx = \left[\frac{0.2}{3}(x-5)^3 + x\right]_1^5 = 5 - \left(\frac{0.2}{3}(-4)^3 + 1\right) = 8.3$$

So there are about 830 beavers in this region.