Worksheet 9

Math 251, Summer 2017

Name: Key

I have given you the answers to some of the problems on the bottom of the last page. You need to figure out how to come up with those answers.

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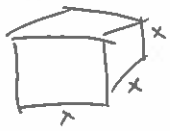
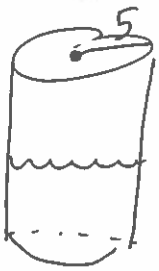
1. If the volume V of a cube with edge length x expands as time passes, find a formula for $\frac{dV}{dt}$ in terms of $\frac{dx}{dt}$.



$V = x^3$

$$\frac{dV}{dt} = 3x^2 \frac{dx}{dt}$$

2. A cylindrical tank with radius 5 m is being filled with water at a rate of $3\text{m}^3/\text{min}$. How fast is the height of the water increasing?



V = volume
h = height of water

$\frac{dV}{dt} = 3$

$V = \pi(5)^2 \cdot h$

$$\frac{dV}{dt} = 25\pi \cdot \frac{dh}{dt}$$

$$\frac{3}{25\pi} = \frac{dh}{dt}$$

$$\frac{dh}{dt} = 0.038 \text{ m/min}$$

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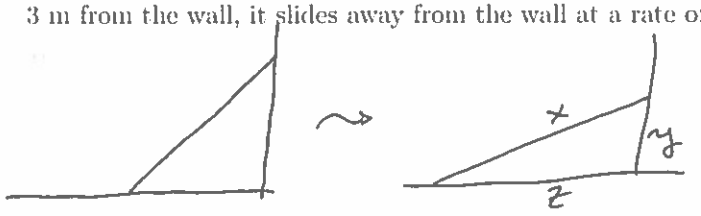
3. The top of a ladder slides down a vertical wall at a rate of 0.15 m/s. At the moment when the bottom of the ladder is 3 m from the wall, it slides away from the wall at a rate of 0.2 m/s. How long is the ladder?



x = length of ladder

y = dist. from top to ground

z = dist. from bottom to wall.

Known values:

* $\frac{dy}{dt} = -0.15$
* $\frac{dz}{dt} = 0.2$ when $z = 3$
* x is constant.

Want: x!

$$x^2 = y^2 + z^2$$

$$0 = 2y\frac{dy}{dt} + 2z\frac{dz}{dt}$$

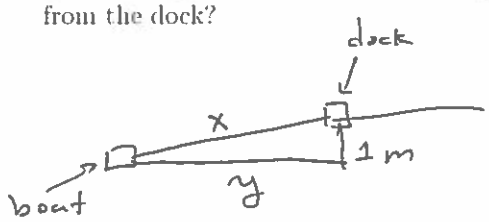
$$0 = y(-.15) + (3)(.2)$$

$$y = \frac{0.6}{0.15} = 4$$

$$\Rightarrow x^2 = 4^2 + 3^2 = 25$$

$$x = 5 \text{ m}$$

4. A boat is pulled to a dock by a rope attached ~~to the bow of~~ the boat, and the rope connects to the dock 1 meter above the bow of the boat. The rope is being pulled in at 1 m/s. How fast is the boat approaching the dock when it is 8 m from the dock?



x = length of rope

y = dist. from boat to dock.

Known values:

* $\frac{dx}{dt} = -1$
* $y = 8$ (at desired time).

Want: $\frac{dy}{dt}$

Relate: $x^2 = 1^2 + y^2$

$$2x\frac{dx}{dt} = 2y\frac{dy}{dt}$$

$$x(-1) = 8 \cdot \frac{dy}{dt}$$

need x:

$$x^2 = 1^2 + 8^2$$

$$x^2 = 1 + 64$$

$$x = \sqrt{65}$$

$$\Rightarrow \frac{dy}{dt} = -\frac{\sqrt{65}}{8} = -1.007$$

Answers: 2) 0.038 m/~~sec~~ min 3) 5 ~~feet~~ m 4) -1.00778 m/sec.