Day 10: More Related Rates

1 Water leaking in/out

- Coffee is brewing in a conical coffee maker. Water (which is now coffee) is leaking out at 12 in^3/min, and at the same time water is being added to the pot at a constant rate. The coffee maker is 10 inches tall and has a diameter of 6 inches at the top. If the water level is rising at a rate of 2 in/min when the height of the water is 2 in, find the rate at which water is being added into the coffee maker.
- Variables: r=radius of water level at time t
- h= height of water level
- V = volume of water
- E = Entering Rate (volume/min)
- L = leaving rate (volume/min)
- What we know: $\frac{dh}{dt} = 2$ when $h = 2$.
- Also: $L = 12$.
- Want: E.
- We can immediately relate r and h:

$$\frac{r}{3} = \frac{h}{10} \implies r = \frac{3}{10}h$$

So

$$V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi \frac{9}{100}h^3 = \frac{3}{100}\pi h^3.$$

Take the derivative:

$$\frac{dV}{dt} = \frac{9\pi}{100}h^2\frac{dh}{dt} = \frac{9\pi}{100}(4)(2) = \frac{72\pi}{100}$$

Now, $\frac{dV}{dt} = E - L$:

$$\begin{aligned} E - L &= \frac{72\pi}{100} \\ E &= 12 + \frac{72\pi}{100} = 14.26\text{in}^3/\text{min} \end{aligned}$$

2 Man's Shadow

A street light is mounted at the top of a 15 foot pole. A man 6 ft tall walks away from the pole with a speed of 5 ft/s along a straight path. How fast is the tip of his shadow moving when he is 40 ft from the pole?

- Let x = his distance from the pole, and let y = length from pole to end of shadow.
- Know: $\frac{dx}{dt} = 5$. Want: $\frac{dy}{dt}$.
- Use similar triangles to get relationship:

$$\frac{y}{15} = \frac{y-x}{6} \implies 15x = 9y$$

So,

$$15\frac{dx}{dt} = 9\frac{dy}{dt} \implies \frac{dy}{dt} = \frac{15}{9}(5) = 8.33\text{ft/s}.$$

3 People Walking

A man starts walking north at point P at a rate of 4 ft/s. Five minutes later a woman starts walking south at a rate of 5 ft/s from a point 500 ft due east from P. At what rate are the people moving apart 15 min after the woman starts walking?

- Let x be the man's distance, y the woman's distance south. Let z be the distance between them.

- We know: $\frac{dx}{dt} = 4$ and $\frac{dy}{dt} = 5$.

- Want: $\frac{dz}{dt}$ at $t = 20$ (15 mins after the woman starts walking).

- Draw picture.

- Get the relation
$$(x + y)^2 + 500^2 = z^2.$$

- Differentiate:
$$2(x + y)\left(\frac{dx}{dt} + \frac{dy}{dt}\right) + 0 = 2z\frac{dz}{dt}$$
Now, at $t = 20$, the man has been walking 20 minutes, so $x = (20 * 60) * 4 = 4800$. The woman has been walking for 15 mins, so $y = 15 * 60 * 5 = 4500$. Then,
$$z = \sqrt{500^2 + (4500 + 4800)^2} = 9313.4 \text{ ft}.$$
So,
$$9300(4 + 5) = (9313)\frac{dz}{dt}$$
Giving $\frac{dz}{dt} = 8.98$ ft/s.