Pure-Time Equations: A qualitative example

- Example: Suppose a company produces a chemical compound in a way where the *rate* of production of this chemical decreases over time. Let's say the total chemical P follows
$$\frac{dP}{dt} = e^{-t}.$$
- Graph both rate and qualitative solution. (Practice with sketching antiderivatives)

Solving Pure-Time Differential Equations

- Main strategy: Guess and check, along with experience.
- Main format:
$$\frac{dF}{dt} = f(t)$$
(no F's on the right side).
- A solution to this equation (F) is called an *antiderivative* for the function f.
- Notation:
$$F(t) = \int f(t)\,dt$$
We also call this the *indefinite integral* of $f(t)$.
- Same question, different wording:

1. Find a solution to the (pure-time) diffy-Q $\frac{dF}{dt} = f(t)$
2. Find (the) antiderivative of $f(t)$.

Computing Antiderivatives: First steps

- What is $\int x\,dx$? Here, $f(x) = x$, and $F(x) = \frac{1}{2}x^2 + C$.
- The "$+C$" is there because there is not just *one* antiderivative: we can add any constant to an antiderivative and get the same $f(x)$! (Derivatives of constants are 0!)
- Another reason why $+C$: from viewpoint of diffy-Q's, initial conditions influence solutions. The $+C$ is showing us there needs to be an initial condition to get a unique solution.
- $\int x\,dx = \frac{1}{2}x^2 + C$.
- Ex: $\int x^2\,dx$, $\int x^3\,dx$, $\int x^4\,dx$.
- Pattern: power rule for antiderivatives:
$$\int x^n\,dx = \frac{1}{n+1}x^{n+1} + C.$$
- $\int \frac{1}{x^2}\,dx$? $f(x) = \frac{1}{x^2}$...
$$F(x) = -\frac{1}{x} + C$$
- Not random: observe that $\frac{1}{x^2} = x^{-2}$, so
$$\int \frac{1}{x^2}\,dx = \int x^{-2}\,dx = \frac{1}{-1}x^{-1} + C = -\frac{1}{x} + C$$
- $\int \frac{1}{x}\,dx$?
- Recall: $\frac{d}{dx}\ln(x) = \frac{1}{x}$, so $\int \frac{1}{x}\,dx = \ln(x) + C$.