Worksheet 4

Math 251, Summer 2017

Name: Key

Directions

- This counts as your "attendance" for the day. You must give this to me today to get credit for it.
- You may leave if you finish the packet.
- You may work in groups, and you can ask me for assistance.
- I grade this worksheet based on completion, not accuracy; however, you should strive for completely correct answers in order to make sure you understand the material.

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1. Find the derivatives of the functions shown below.

(a) $h(x) = (2 + x^3)(2x - 1)$

$$h'(x) = (2+x^3)'(2x-1) + (2+x^3)(2x-1)'$$
$$= 3x^2(2x-1) + 2(2+x^3)$$

(c) $S(x) = \frac{x - \sqrt{x}}{x^{1/3}} = \frac{x}{x^{1/3}} - \frac{x^{1/2}}{x^{1/3}} = x^{2/3} - x^{1/6}$

$$S'(x) = \frac{2}{3}x^{-1/3} - \frac{1}{6}x^{-5/6}.$$

OR:

$$S'(x) = \frac{(x-\sqrt{x})' x^{1/3} - (x^{1/3})'(x-\sqrt{x})}{(x^{1/3})^2}$$
$$= \frac{(1 - \frac{1}{2}x^{-1/2})x^{1/3} - \frac{1}{3}x^{-2/3}(x-\sqrt{x})}{x^{2/3}}$$

this simplifies to that.

(b) $r(t) = \tan(t)$ (hint: write down the definition of tan).

$$r(t) = \frac{\sin(t)}{\cos(t)}$$

$$r'(t) = \frac{(\sin t)' \cos t - (\cos t)' \sin(t)}{\cos^2(t)}$$

$$= \frac{\cos^2 t + \sin^2(t)}{\cos^2(t)}$$

$$= \frac{1}{\cos^2(t)} \quad \left(\text{Recall: } \cos^2 t + \sin^2 t = 1\right)$$

$$\boxed{r'(t) = \sec^2(t)}$$

(d) $T(t) = (t + e^t)(3 - \sqrt{t})$

$$T'(t) = (t+e^t)'(3-\sqrt{t}) + (t+e^t)(3-\sqrt{t})'$$
$$= (1+e^t)(3-\sqrt{t}) + (t+e^t)\left(-\frac{1}{2}t^{-1/2}\right)$$

2. Find the equation of the tangent line to the function $\frac{1+\sqrt{t}}{\sqrt{t}}$ at $t = 1$.

$$f'(t) = \frac{(1+\sqrt{t})'\sqrt{t} - (\sqrt{t})'(1+\sqrt{t})}{(\sqrt{t})^2}$$

$$= \frac{\frac{1}{2}t^{-1/2}\cdot\sqrt{t} - \frac{1}{2}t^{-1/2}\cdot(1+\sqrt{t})}{t}$$

$$f'(1) = \frac{\frac{1}{2}(1)(1) - \frac{1}{2}(1)(1+1)}{1} = \frac{\frac{1}{2} - \frac{1}{2}(2)}{1} = -\frac{1}{2}$$

$$y = mt + b$$
$$y = -\frac{1}{2}t + b$$
$$2 = -\frac{1}{2}(1) + b$$
$$2.5 = b$$

$$y = -\frac{1}{2}t + 2.5$$

3. Suppose $h(x) = x^2 f(x)$, where f is a function with the property that $f(1) = 4$ and $f'(1) = 2$. Find the value of $h'(1)$.

$$h'(x) = (x^2)' f(x) + x^2 f'(x)$$
$$= 2x\, f(x) + x^2 f'(x)$$

$$h'(1) = 2\cdot 1\cdot f(1) + 1^2\cdot f'(1)$$
$$= 2\cdot 4 + 1\cdot 2 = 10$$

4. Suppose that $f(2) = 3$, $f'(2) = 4$, $g(2) = 1$, and $g'(2) = -2$. Find $h'(2)$, where $h(x) = \frac{f(x)}{1+g(x)}$.

$$h'(x) = \frac{f'(x)(1+g(x)) - (1+g(x))' f(x)}{(1+g(x))^2}$$

$$= \frac{f'(x)(1+g(x)) - g'(x)f(x)}{(1+g(x))^2}$$

$$h'(2) = \frac{f'(2)\cdot(1+g(2)) - g'(2)f(2)}{(1+g(2))^2}$$

$$= \frac{3(1+1) - (-2)(3)}{(1+1)^2} = \frac{6+6}{4} = \frac{12}{4} = \boxed{3}$$

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5. Optimize (find max's and min's) of the functions below on the given intervals using the method outlined in class.

(a) $f(x) = xe^x$, on $[-2, 0]$.

$$f'(x) = xe^x + e^x = 0$$
$$e^x(1+x) = 0$$

never! $=0$.

$$\Rightarrow \quad x = -1$$

$$f(-2) = -2e^{-2} \approx -0.27$$
$$f(-1) = -1e^{-1} \approx -0.37$$
$$f(0) = 0$$

Min @ $x=-1$, $y=-0.37$

Max @ $x=0$, $y=0$

(b) $g(x) = \frac{x}{e^x}$ on $[0, 3]$

$$g'(x) = \frac{(x)'e^x - x\cdot(e^x)'}{(e^x)^2} = 0$$
$$1e^x - xe^x = 0\cdot(e^x)^2$$
$$e^x(1-x) = 0$$

not 0, so divide by it.

$$x = 1$$

$$g(0) = 0$$
$$g(1) = \frac{1}{e} = 0.369$$
$$g(3) = \frac{3}{e^3} = 0.149$$

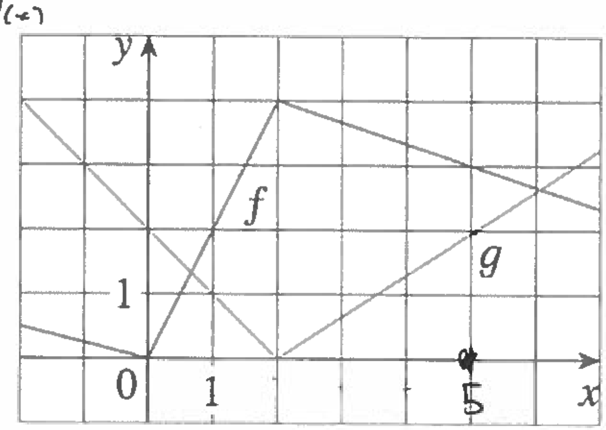
Min @ $x=0$, $y=0$ | Max @ $x=1$, $y=.369$

6. Suppose that $u(x) = f(x)g(x)$ and $v(x) = f(x)/g(x)$, where $f(x)$ and $g(x)$ are the functions graphed below.

(a) Find $u'(1)$.

(b) Find $v'(5)$.

$$u'(x) = f'(x)g(x) + f(x)g'(x)$$
$$u'(1) = f'(1)g(1) + f(1)g'(1)$$
$$= (2)(1) + (2)(-1)$$
$$\boxed{= 0}$$



$$v'(x) = \frac{f'(x)g(x) - f(x)g'(x)}{(g(x))^2}$$

$$v'(5) = \frac{f'(5)g(5) - f(5)g'(5)}{(g(5))^2} = \frac{\left(-\frac{1}{3}\right)(2) - 3\left(\frac{2}{3}\right)}{2^2}$$

$$= \frac{-\frac{2}{3} - \frac{6}{3}}{4} = -\frac{8}{3}\cdot\frac{1}{4}$$

$$= \boxed{-2/3}$$