4.1 The idea of a Differential Equation

Intro

- Recall: discrete dynamical systems model quantities that vary in time (with discrete time chunks)
- Example: concentration c_t of chemical ($t =$# of breaths)

$$c_{t+1} = (1 - c_t)q + c_t\gamma$$

- Some quantities vary continuously in time. E.g.
  - concentration c for *all* times, rather than just after a breath?
  - population at *all* times, rather than after the end of a year?
- Consider $c_{t+1} = 2c_t + 3$ for an example.

$$\begin{aligned} c_{t+1} &= 2c_t + 3 \\ c_{t+1} - c_t &= c_t + 3 \\ \frac{c_{t+1} - c_t}{1} &= c_t + 3. \end{aligned}$$

This resembles:

$$\lim_{h \to 0} \frac{c(t+h) - c(t)}{h} = c(t) + 3$$

which is

$$\frac{dc}{dt} = c + 3$$

This is a *differential equation*, an equation (viewed as a *requirement* for c).

- General: for any function $f(t)$, a differential equation for f is any *equation* involving the function $f(t)$ and its derivatives.

- Idea: a differential equation *determines* a function $f(t)$.
- Compare: an equation $3x + 5 = 7$ determines a solution (a number).
- Compare: a discrete dynamical system is an equation

$$m_{t+1} = (\text{stuff with } m_t),$$

  which determined a solution (which was a bunch of numbers).

- Ideas we will explore:
  1. How to solve differential equations
  2. How to set up differential equations
  3. How differential equations appear in the real world
- 1) will happen first, but here's an example of 3.

  A population of raccoons in the city of Pawnee grows at a rate that is proportional to the number of raccoons. (More racoons = faster growth rate, since there are more of them!) Assume the constant of proportionality is 3.

  Get:

$$\frac{dP}{dt} = 3P$$

- The variable P is known as the *state variable.*
- Ex: Suppose that the population of racoons grows at a constant rate of 20 racoons per year.
  1. Express this as a differential equation.

$$\frac{dP}{dt} = 20$$

2. Suppose that $P(0) = 300$. Find a solution to your differential equation.

$$P(t) = 20t + 300$$

- We've seen two types of Diffy-Q: Pure-time differential equation: $\frac{dP}{dt} = 20$, and autonomous differential equation: $\frac{dP}{dt} = 3P$.