Quiz 3 Solutions

Name: ______________________

You will have 20 minutes ∘ Calculators are allowed ∘ Show all work for credit ∘ Don't cheat ∘ attempts at a problem may count for partial credit. ∘ If you get stuck, show as much work as possible.

1. Compute these indefinite integrals. Show your work. [3 pts each]

(a) $\displaystyle\int \frac{1}{5}\sqrt{t^3}\,dt = \tfrac{1}{5}\int t^{3/2}\,dt = \tfrac{2}{25}t^{5/2} + C$

(b) $\displaystyle\int x^{2/3}(1-x)\,dx = \int (x^{2/3} - x^{5/3})\,dx = \tfrac{3}{5}x^{5/3} - \tfrac{3}{8}x^{8/3} + C$

(c) $\displaystyle\int \left(\sin(4x) - \frac{17}{x^7}\right)\,dx = \int \sin(4x)\,dx - 17\int \tfrac{1}{x^7}\,dx = -\tfrac{1}{4}\cos(4x) + \tfrac{17}{6}x^{-6} + C$

(d) $\int \cos(t^4) t^3 \, dt$ Let $u = t^4$, $du = 4t^3 dt$. Then regular substitution gives you $\int \cos(t^4) t^3 \, dt = \frac{1}{4} \sin(t^4) + C$.

2. [3 pts] Solve the following differential equation.

$$\frac{df}{dt} = -\frac{1}{t^2} e^{-1/t}, \quad f(1) = 1.$$

[Hint: try substituting $u = 1/t$.] The integral that solves this equation is

$$\int -\frac{1}{t^2} e^{-1/t} \, dt.$$

Sub $u = \frac{1}{t}$, $du = -\frac{1}{t^2} \, dt$, so the integral equals

$$\int e^{-u} \, du = -e^{-u} + C = -e^{-1/t} + C.$$

So, $f(t) = -e^{-1/t} + C$. Putting $f(1) = 1$, we'll get $C = 1 + e^{-1}$. So the final answer is

$$f(t) = 1 + e^{-1} - e^{-1/t}.$$