Exam 1 Solutions

Name: ____________________________

- Scientific calculators only.
- Show as much work as possible, even if you can't answer the problem entirely. This allows me to give you partial credit.
- Spend time wisely: read through the test first and solve the ones you know how to do.
- There are a total of 40 points on this exam.

True/False

Directions: Indicate True (T) if the statement is always true, or False (F) otherwise. [1 pt each].

1. ______ The antiderivative of $\sin(x)$ is $\cos(x)$. False
2. ______ If $f(-1) < 0$, then the antiderivative F of f is decreasing near $x = -1$. True
3. ______ If the total change in F from 2 to 4 is positive, then $(\int_2^4 F'(t)\,dt)$ is a positive number. True
4. ______ The integral $\displaystyle\int e^{-x^2}\,dx$ is computable with the strategies from our course. False

Multiple Choice

1. [2 pts] Consider the function $f(x) = x^3$ on the interval $[4, 5]$ with 4 steps. Which of the following expressions is the left Riemann sum?

   (a) $\frac{1}{4}\left((4)^3 + (4.25)^3 + (4.5)^3 + (4.75)^3\right)$

   (b) $\frac{1}{4}\left((4.25)^3 + (4.5)^3 + (4.75)^3 + (5)^3\right)$

   (c) $\frac{1}{4}\left((4)^3 + (4.25)^3 + (4.5)^3 + (4.75)^3 + (5)^3\right)$

   (d) $\frac{1}{4}\left((4.25)^3 + (4.5)^3 + (4.75)^3\right)$

   (a) is correct.

2. [2 pts] Suppose $S(t)$ represents the rate of change of Energy expended by a bug t seconds after it starts flying. The bug lands after one minute. Which expression below represents the total change in the bugs' energy for this flight?

   (a) $\displaystyle\int S(t)\,dt$

   (b) $\displaystyle\int_0^{60} S(t)\,dt$

   (c) $\displaystyle\int_0^{1} S(t)\,dt$

   (d) $\displaystyle\int_0^{60} S'(t)\,dt$

   (b) is correct.

Free Response

Directions: answer the following problems. Be sure to show your work.

1. A cell's volume V starts changing at a rate of $(32e^{-0.2t})$ μm^3 per minute at time $t = 0$. The cell stops growing after thirty minutes.

   (a) [2 pts] Express this as a differential equation.

   $$\frac{dV}{dt} = 32e^{-0.2t}$$

   (b) [2 pts] Write down, but do not evaluate, an integral that represents the total change in the cell's volume during its growth period.

   $$\int_0^{30} 32e^{-0.2t}\,dt$$

   (c) [2 pts] Compute the right Riemann sum with $n = 3$ steps that approximates the integral in part (b). (Include units). [Note: you do not need to use summation notation.] $\delta t = \frac{30-0}{3} = 10$, so the right Riemann sum is

   $$32e^{-.2(10)} \cdot 10 + 32e^{-.2(20)} \cdot 10 + 32e^{-.2(30)} \cdot 10 \approx 49.96\ \mu\text{m}^3$$

2. Evaluate the following integrals.

   (a) [2 pts] $\displaystyle\int \left(\frac{1}{x^2} + \frac{1}{x} + 1 + x + x^2\right) dx = -\frac{1}{x} + \ln|x| + x + \frac{1}{2}x^2 + \frac{1}{3}x^3 + C$

(b) [3 pts] $\int 2x\sin(x)\,dx$ $= -2x\cos(x) + 2\sin(x) + C$

(c) [3 pts] $\int \frac{t}{t^2+2}\,dt$ $= \frac{1}{2}\ln(t^2+1) + C$ substitute $u = 1 + x^2$

3. The temperature of an ice cream pop T in degrees Fahrenheit follows Newton's law of cooling, which is described by the differential equation below.

$$\frac{dT}{dt} = 0.5(80 - T)$$

(a) [1 pts] Is this equation pure-time, autonomous, or neither? autonomous

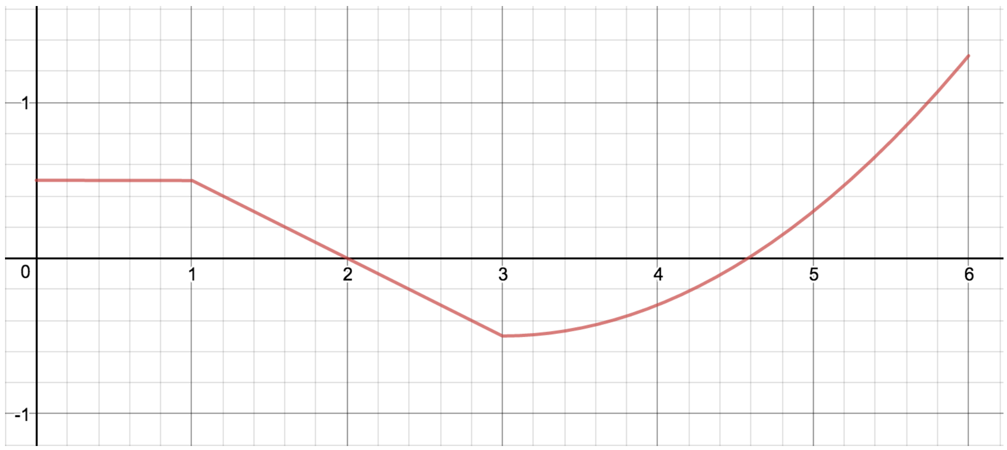
(b) [2 pts] Verify that $T(t) = 80 - 2e^{-0.5t}$ is a solution to this equation. You must take the derivative, and then separately compute $0.5(80 - T)$. This second part looks like

$$0.5(80 - (80 - 2e^{-0.5t})) = 0.5(2e^{-0.5t}) = e^{0.5t},$$

which is equal to $\frac{dT}{dt}$.

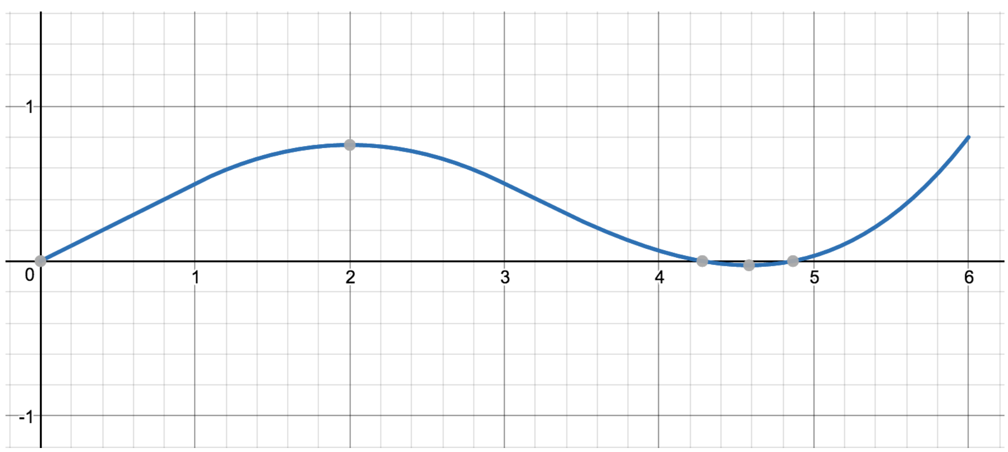
(c) [1 pts] What is the initial condition for the solution given in part (b)? $T(0) = 80 - 2 = 78$.

4. Suppose $f(x)$ is the function graphed below. Let $F(x)$ be the antiderivative of $f(x)$ with $F(0) = 0$.



(a) [3 pts] Is $F(2)$ positive, negative, or 0? Briefly explain. Since $F(0) = 0$ and $F'(x) > 0$ all the way from $x = 0$ to $x = 2$, the value of F only increases from 0, and so $F(2)$ must be positive.

(b) [3 pts] Sketch a graph of $F(x)$ below.



5. Suppose that for a certain chemical reaction to start 50 mL of carbon is needed. The total amount of carbon, C, is given by

$$\frac{dC}{dt} = -\frac{3}{10t+1}$$

where t is measured in nanoseconds.

(a) [1 pts] Write down the integral corresponding to this differential equation. $-\int \frac{3}{10t+1}\,dt$

(b) [3 pts] Find the solution of this differential equation. $C(t) = -0.3\ln(10t+1) + 400$

(c) [2 pts] Find the value of the left Riemann sum for $\frac{dC}{dt}$ on the interval $[0, 4]$ with four subdivisions. (Include units.)

$$\frac{-3}{10(0)+1} + \frac{-3}{10(1)+1} + \frac{-3}{10(2)+1} + \frac{-3}{10(3)+1} \approx -3.51 \text{ mL}$$

(d) [2 pts] Interpret the result of part (c) in the context of the problem (that is, in terms of what is going on with the carbon).

This number approximates the total amount of carbon used up in the reaction. Very roughly, the total amount of Carbon decreases by about 3.5 mL over the first four nanoseconds.