Review Handout

1. For the systems shown below, find and graph the nullclines. Find all equilibria.

(a) $\begin{cases} \frac{dx}{dt} = y^2 - xy + y \\ \frac{dy}{dt} = x^3 - xy^2 \end{cases}$

(b) $\begin{cases} \frac{du}{dt} & = u \\ \frac{dv}{dt} & = v \end{cases}$

(c) $\begin{cases} \frac{dz}{dt} & = 13 - z + y \\ \frac{dy}{dt} & = z - y \end{cases}$

2. Find the general solutions to the following differential equations.

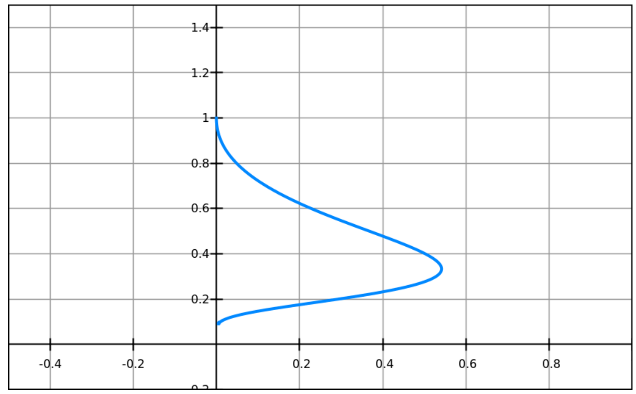
(a) $\dfrac{dx}{dt} = \dfrac{t^2}{x}$

(b) $\dfrac{dx}{dt} = \dfrac{x}{t^2}$

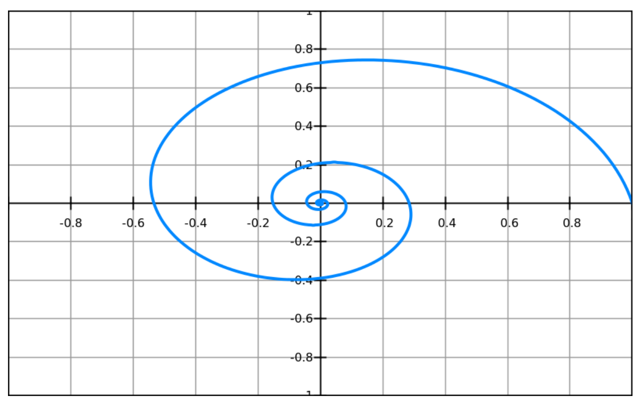
(c) $\dfrac{dx}{dt} = \dfrac{t}{x^2}$

3. The following graphs represents a trajectory in the phase-plane for a system of differential equations for $x(t)$ and $y(t)$. Sketch possible graphs of $x(t)$ and $y(t)$.

(a)



(b)



4. For the following DE, make a phase-line diagram. Classify the equilibria as stable or unstable.

   (a) $\dfrac{dx}{dt} = x(x-2)(x+7)$

   (b) $\dfrac{ds}{dt} = x^2 - 9x$

5. Evaluate the following integrals.

   (a) $\displaystyle\int \sqrt[5]{x^7}\,dx$

   (b) $\displaystyle\int xe^x\,dx$

   (c) $\displaystyle\int_{-2}^{0} \frac{14}{2x}\,dx$

   (d) $\displaystyle\int_{4}^{16} \frac{x}{1+x^2}\,dx$

6. If a population of wolves is currently at one-hundred thousand, and $\int_0^{20} f(t)\,dt = 3000$, where $f(t)$ is the growth rate in wolves per year, then how many wolves are there after 20 years?

7. For the neuron model with a constant applied current, find the nullclines of the system.

8. For the following differential equation, approximate $x(1)$ given $\Delta t = 0.5$ and $x(0) = 1$.

$$\frac{dx}{dt} = \frac{x}{1+t}$$

9. Consider our system of DE for the disease model:

$$\frac{dI}{dt} = \alpha IS - \mu I$$
$$\frac{dS}{dt} = -\alpha IS + \mu I$$

(a) Describe exactly what the variables I and S are, as well as the parameters α and μ.

(b) Find the nullclines of this system and describe the equilibria.

(c) Describe how one might incorporate birth rates into the model.

(d) Describe how one might incorporate death from disease into the model.

10. Find the area of the following regions.

    (a) Between the x-axis and the graph of the function $x - x^2$.

    (b) Between the graphs of e^x and $\sqrt{x}$ from $x = 0$ to $x = 1$.

11. Calculate the left and right Riemann sums for the function $g(t) = 10 - t^2$ on the interval $[0, 2]$ with 5 subintervals. How does your answer compare to the exact value for the integral?

12. Below are four differential equations, along with six functions. Identify which function is a solution to which equation (note that some functions may not be solutions to any of the DE's).

$$\frac{dy}{dt} + y = 2$$
$$\frac{dy}{dt} = y^2$$
$$\frac{dy}{dt} = ty^2$$
$$\frac{dy}{dt} = y + t$$

$$y(t) = e^t - t - 1$$
$$y(t) = e^{-t^2}$$
$$y(t) = \frac{e^t}{1+t}$$
$$y(t) = \frac{1}{1-t}$$
$$y(t) = -\frac{2}{1+t^2}$$
$$y(t) = e^{-t} + 2$$

13. The mass density (in kg/meter) of a poorly-made construction beam seems to follow the function

$$D(x) = 12.2xe^{-4x^2},$$

where x is a distance in meters along the beam. How much does the bar weigh?

14. Use the stability theorem to classify the equilibria of the differential equation

$$\frac{dA}{dt} = (A-2)(13-A^2).$$