Worksheet 5

Math 251, Summer 2017

Name: Key

Directions

- This counts as your "attendance" for the day. You must give this to me today to get credit for it.
- You may leave if you finish the packet.
- You may work in groups, and you can ask me for assistance.
- I grade this worksheet based on completion, not accuracy; however, you should strive for completely correct answers in order to make sure you understand the material.

1. Find the derivatives of the functions shown below. [Suggestion: identify how each function is a *composition* of two simpler functions.]

(a) $h(x) = (2+3x)^{35}$

$h'(x) = 35(2+3x)^{34}$

(b) $F(t) = e^{14t}$

$F'(t) = 14e^{14t}$

(c) $r(t) = \sin(\cos(t))$

$r'(t) = \cos(\cos(t)) \cdot (-\sin(t))$

(d) $S(x) = \dfrac{1}{\sqrt{1-x^2}} = (1-x^2)^{-1/2}$

$S'(x) = -\frac{1}{2}(1-x^2)^{-3/2} \cdot (-2x)$

(e) $T(t) = e^{t^2}$ (Careful: this is not the same as e^{2t}.)

$T'(t) = 2t\, e^{t^2}$

(f) $F(x) = \sqrt{\cos(e^t)} = (\cos(e^t))^{1/2}$

$F'(x) = \frac{1}{2}(\cos(e^t))^{-1/2} \cdot (-\sin(e^t)) \cdot e$

2. A population P of jellyfish can be modeled by the equation $P(t) = 12e^{0.12t}$, where P is measured in thousands of jellyfish and t is in years. Find the instantaneous rate of growth of the population initially and after 3 years.

$$P'(t) = 12\, e^{0.12t} \cdot 0.12 = 1.44\, e^{0.12t}$$

$$P'(0) = 1.44\, e^{0} = 1.44 \text{ thousand Jellyfish/yr}$$

$$P'(3) = 1.44\, e^{0.36} = 2.06 \text{ thousand Jellyfish/yr}$$

3. Optimize (find max's and min's) of the functions below on the given intervals using the method outlined in class.

(a) $f(x) = \frac{1}{1+x^2}$, on $[-1, 1]$.

$$f'(x) = \frac{(1)'(1+x^2) - (1+x^2)'\cdot(1)}{(1+x^2)^2} = 0$$

$$\cancel{(1+x^2)^2} \cdot \frac{-2x}{\cancel{(1+x^2)^2}} = 0 \cdot (1+x^2)^2$$

$$-2x = 0$$

$$x = 0$$

$$f(-1) = \frac{1}{1+(-1)^2} = \frac{1}{2}$$

$$f(0) = \frac{1}{1} = 1$$

$$f(1) = \frac{1}{1+1} = \frac{1}{2}$$

Min @ $x = -1$ or $x = 1$, $y = 1/2$

Max @ $x = 0$, $y = 1$.

(b) $g(x) = \sqrt{x^3 - 2x + 2}$ on $[-1, 1]$ ← my bad

$$g'(x) = \frac{1}{2}\left(x^3 - 2x + 2\right)^{-1/2} \cdot (3x^2 - 2)$$

$$\cancel{(x^3-2x+2)^{1/2}} \cdot \left(\frac{1}{2\cancel{(x^3-2x+2)^{1/2}}} \cdot \frac{3x^2-2}{1} = 0\right)$$

$$\Rightarrow \frac{1}{2} \cdot (3x^2 - 2) = 0$$

$$3x^2 - 2 = 0$$

$$x^2 = 2/3$$

$$x = \pm\sqrt{\frac{2}{3}} = \pm 0.816$$

-1, -0.8, $+0.8$, 1 ← So, test all crit pts.

$g(-1) = 1.73$

$g(-0.816) \approx 1.757$ ← Max

$g(+0.816) \approx 0.955$ ← Min.

$g(1) = 1$

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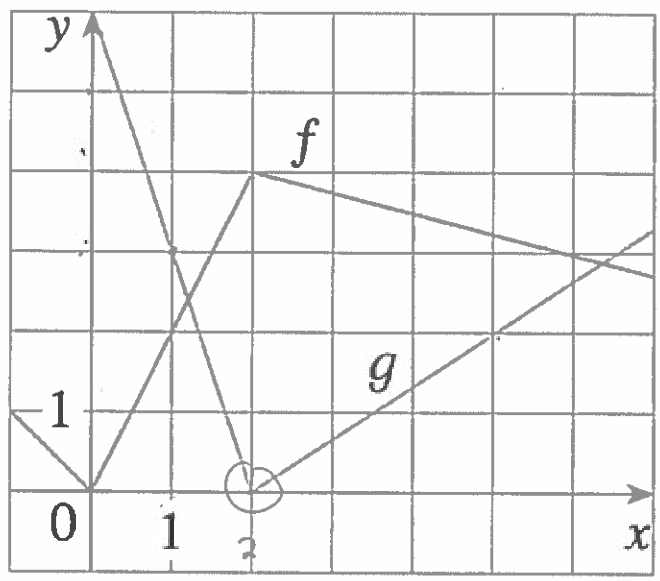
4. Suppose that $u(x) = f(g(x))$ and $v(x) = g(f(x))$, and $w(x) = g(g(x))$, where $f(x)$ and $g(x)$ are the functions graphed below.

   (a) Find $u'(1)$.

   (b) Find $v'(1)$.

   (c) Find $w'(1)$.



$$u'(1) = f'(g(1)) \cdot g'(1)$$
$$= f'(3) \cdot \left(\tfrac{1}{3}\right)$$
$$= \left(-\tfrac{1}{4}\right) \cdot \tfrac{1}{3} = \boxed{-\tfrac{1}{12}}$$

$$v'(x) = g'(f(x)) \cdot f'(x)$$
$$v'(1) = g'(f(1)) \cdot f'(1)$$
$$= g'(2) \cdot 2$$
$$= \text{DNE}$$

($g(x)$ has no well-defined slope at $x = 2$ because of the sharp corner.)

$$w'(x) = g'(g(x)) \cdot g'(x)$$
$$w'(1) = g'(g(1)) \cdot g'(1)$$
$$= g'(3) \cdot g'(1)$$
$$= \tfrac{2}{3} \cdot (-3) = \boxed{-2}$$