Quiz 14

Key

1. For the two examples shown below, (a) find the global maxima and minima if they exist and (b) decide if the extreme value theorem applies. If it does not apply, state which hypothesis fails. [All work must be shown for full credit.]

   (a) $f(x) = \dfrac{1}{x^2}$ on the interval $[-1, 1]$

   This function is not continuous, so the extreme value theorem fails. Critical points:

   $$f'(x) = -\frac{2}{x^3}$$

   There are no solutions to the equation $-\frac{2}{x^3} = 0$, so there are no critical points. Endpoints:

   $$f(-1) = 1$$
   $$f(1) = 1$$

   Because $\lim_{x\to 0} f(x) = \infty$, there is no global maximum, but there are global minima at $x = -1$ and $x = 1$.

   (b) $h(x) = \dfrac{x^2}{2x^2+3}$ on the interval $[-2, \infty)$

   The extreme value theorem fails because we aren't on a closed and bounded interval. Find critical points:

   $$h'(x) = \frac{(2x)(2x^2+3) - (4x)(x^2)}{(2x^2+3)^2} = \frac{6x}{(2x^2+3)^2} = 0$$

   This has one solution, $x = 0$. Test y-values at endpoints and critical points:

   $$h(-2) = \frac{4}{11}$$
   $$h(0) = 0$$
   $$\lim_{x\to\infty} h(x) = \lim_{x\to\infty} \frac{x^2}{2x^2+3} = \lim_{x\to\infty} \frac{x^2}{x^2} \cdot \frac{1}{2+\frac{3}{x^2}} = \frac{1}{2}$$

   So there is a global minimum at $x = 0$, but no global maximum ($f(x)$ never reaches $1/2$).

2. Evaluate the following limits:

(a) $\lim_{x\to\infty} \dfrac{\sqrt[3]{x^3-5}}{15x+10}$

$$= \lim_{x\to\infty} \frac{\sqrt[3]{x}}{x} \frac{\sqrt[3]{1-\frac{5}{x^3}}}{15+\frac{10}{x}} = \lim_{x\to\infty} \frac{\sqrt[3]{1-\frac{5}{x^3}}}{15+\frac{10}{x}} = \frac{1}{15}$$

(b) $\lim_{x\to\infty} \dfrac{e^x - e^{-x}}{e^x + e^{-x}}$

$$= \lim_{x\to\infty} \left(\frac{e^x}{e^x}\right) \frac{1-e^{-2x}}{1+e^{-2x}} = 1$$