**Exam 1**

Name: _____

- Scientific calculators only.
- Show as much work as possible, even if you can't answer the problem entirely. This allows me to give you partial credit.
- Spend time wisely: read through the test first and solve the ones you know how to do.
- There are a total of 40 points on this exam.

## True/False

Directions: Indicate True (T) if the statement is always true, or False (F) otherwise. [1 pt each].

1. _____ The antiderivative of $\sin(x)$ is $\cos(x)$.
2. _____ If $f(-1) < 0$, then the antiderivative $F$ of $f$ is decreasing near $x = -1$.
3. _____ If the total change in $F$ from 2 to 4 is positive, then $(\int_2^4 F'(t)\,dt)$ is a positive number.
4. _____ The integral $\int e^{-x^2}\,dx$ is computable with the strategies from our course.

## Multiple Choice

1. [2 pts] Consider the function $f(x) = x^3$ on the interval $[4, 5]$ with 4 steps. Which of the following expressions is the left Riemann sum?

   (a) $\frac{1}{4}\Big((4)^3 + (4.25)^3 + (4.5)^3 + (4.75)^3\Big)$

   (b) $\frac{1}{4}\Big((4.25)^3 + (4.5)^3 + (4.75)^3 + (5)^3\Big)$

   (c) $\frac{1}{4}\Big((4)^3 + (4.25)^3 + (4.5)^3 + (4.75)^3 + (5)^3\Big)$

   (d) $\frac{1}{4}\Big((4.25)^3 + (4.5)^3 + (4.75)^3\Big)$

2. [2 pts] Suppose $S(t)$ represents the rate of change of Energy expended by a bug $t$ seconds after it starts flying. The bug lands after one minute. Which expression below represents the total change in the bugs' energy for this flight?

   (a) $\int S(t)\,dt$

   (b) $\int_0^{60} S(t)\,dt$

   (c) $\int_0^{1} S(t)\,dt$

   (d) $\int_0^{60} S'(t)\,dt$

## Free Response

Directions: answer the following problems. Be sure to show your work.

1. A cell's volume $V$ starts changing at a rate of $(32e^{-0.2t})$ $\mu\text{m}^3$ per minute at time $t = 0$. The cell stops growing after thirty minutes.

   (a) [2 pts] Express this as a differential equation.

   (b) [2 pts] Write down, but do not evaluate, an integral that represents the total change in the cell's volume during its growth period.

   (c) [2 pts] Compute the right Riemann sum with $n = 3$ steps that approximates the integral in part (b). (Include units). [Note: you do not need to use summation notation.]

2. Evaluate the following integrals.

   (a) [2 pts] $\displaystyle\int \left(\frac{1}{x^2} + \frac{1}{x} + 1 + x + x^2\right) dx$

   (b) [3 pts] $\displaystyle\int 2x \sin(x)\, dx$

(c) [3 pts] $\displaystyle\int \frac{t}{t^2+2}\,dt$

3. The temperature of an ice cream pop $T$ in degrees Fahrenheit follows Newton's law of cooling, which is described by the differential equation below.

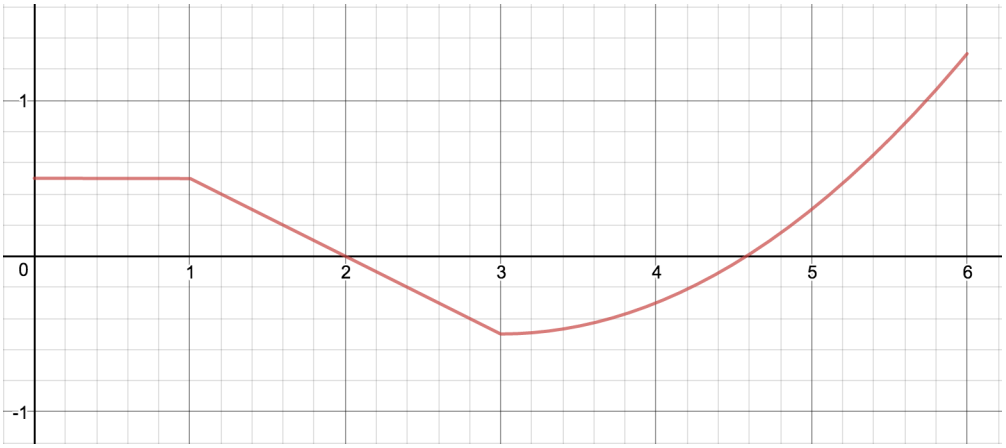
$$\frac{dT}{dt} = 0.5(80 - T)$$

(a) [1 pts] Is this equation pure-time, autonomous, or neither?

(b) [2 pts] Verify that $T(t) = 80 - 2e^{-0.5t}$ is a solution to this equation.

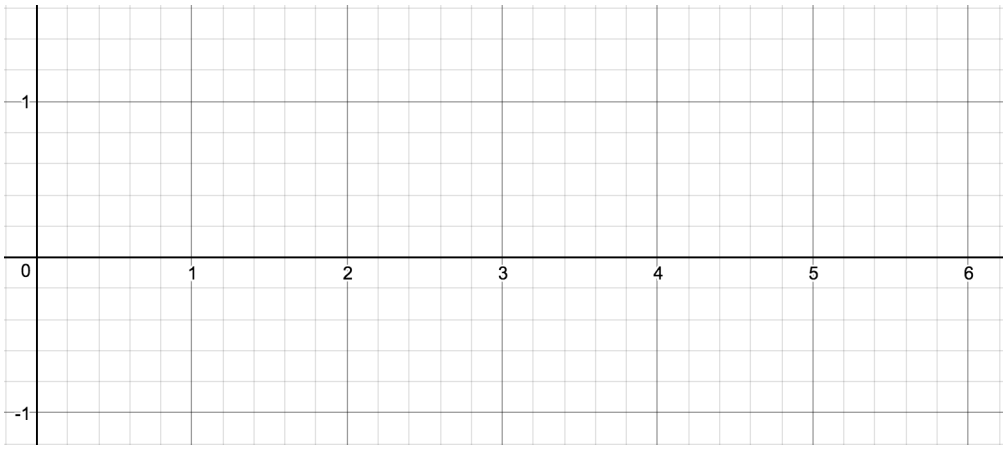
(c) [1 pts] What is the initial condition for the solution given in part (b)?

4. Suppose $f(x)$ is the function graphed below. Let $F(x)$ be the antiderivative of $f(x)$ with $F(0) = 0$.




(a) [3 pts] Is $F(2)$ positive, negative, or 0? Briefly explain.

(b) [3 pts] Sketch a graph of $F(x)$ below.



5. Suppose that for a certain chemical reaction to start 50 mL of carbon is needed. The total amount of carbon, $C$, is given by

$$\frac{dC}{dt} = -\frac{3}{10t+1}$$

where $t$ is measured in nanoseconds.

(a) [1 pts] Write down the integral corresponding to this differential equation.

(b) [3 pts] Find the solution of this differential equation.

(c) [2 pts] Find the value of the left Riemann sum for $\frac{dC}{dt}$ on the interval $[0, 4]$ with four subdivisions. (Include units.)

(d) [2 pts] Interpret the result of part (c) in the context of the problem (that is, in terms of what is going on with the carbon).