Homework 3
Due Tuesday, April 23rd

Instructions: write up solutions to all problems below. Neatness counts: be sure to follow guidelines for homework in the syllabus.

Reading Assignment: Chapters 2.5, 2.7, 3.1.

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1. (Chapter 2.5, # 15 a) Suppose a population, y, is modeled by the logistic equation, with rate constant 0.025 per year. If $y_0 = N/3$ initially, then how long does it take this population to double? Assume time is measured in years here.

2. (Chapter 2.5, # 19, a and b) Suppose a population of fish follows the logistic growth model. A fishery, with a lot of effort, harvests fish at a rate proportional to the number of fish. The new equation for the number of fish would be
$$\frac{dy}{dt} = k\left(1 - \frac{y}{N}\right)y - Ey,$$
where E is the harvest rate, with units of 1/time. (Notice this equation has the format "rate in minus rate out.")

   (a) Show that if $E < k$, then there are two realistic equilibrium points.

   (b) Using a phase line, decide on the stability of each equilibrium.

3. (Chapter 2.5, # 20, modified slightly) In this problem, we assume a similar population model as before, but this time the fishery is less careful and fish are harvested at flat, constant rate, denoted by h. This has the equation
$$\frac{dy}{dt} = k\left(1 - \frac{y}{N}\right)y - h$$
(Note that the term with h now does not have y in it.)

   (a) Find the equilibrium values for this fishery in terms of k, N, and h.

   (b) Choose numbers for k, N, and h to ensure that $h < kN/4$. Then analyze the stability of your equilibria (be sure to write down the actual numbers that your equilibria take!)

   (c) Choose numbers for k, N, and h to ensure that $h = kN/4$. Then analyze the stability of your equilibria.

   (d) Choose numbers for k, N and h to ensure that $h > kN/4$. Then analyze the stability of your equilibria.

4. In this problem you will make a comparison of Euler's method with different Δx's. Please use online software here to do Euler's method for you and to make the graphs.

   Consider the initial value problem

   $$\frac{dy}{dx} = \sin(x)y + 0.5, \quad y(0) = 1.$$

   (a) With $\Delta x = 1, 0.5, 0.1$, and 0.01, use the program to evaluate $y(10)$. I suggest organizing your answer as a table with Δx's and the corresponding values of $y(10)$.

   (b) Print out and attach two graphs, the ones with $\Delta x = 1$ and $\Delta x = 0.01$. Remember, these two graphs are trying to approximate the *same* solution to the DE above. Do you notice anything strikingly different about these graphs?

5. In this problem you will see a shortcoming of numerical techniques, and why it's still important to think critically about what a computer gives you.

   Consider this initial value problem.

   $$\frac{dy}{dt} = \frac{3t^2}{y^2 - 4}, \qquad y(0) = 1.$$

   (a) For which values of y does the derivative $\frac{dy}{dt}$ become undefined?

   (b) Using the same Euler's Method software as the previous problem, make a plot of the solution using a step size of 0.01.

   (c) Connect your observation in part (a) to the graph you make in part (b). What's going on?