2.2 Limits, part 2

We'll discuss more stuff about limits.

1 One-sided limits

- Limits from the right: $\lim_{t \to 0^+}$
- Limits from the left: $\lim_{t \to 0^-}$
- These only mean approach from one side.
- Ex: We saw yesterday that

  $$\lim_{t \to 0} \frac{|t|}{t} \text{ DNE.}$$

  What is

  $$\lim_{t \to 0^+} \frac{|t|}{t}?$$

  Since this means from right of 0, $|t|/t = t/t = 1$, so

  $$\lim_{t \to 0^+} \frac{|t|}{t} = 1.$$

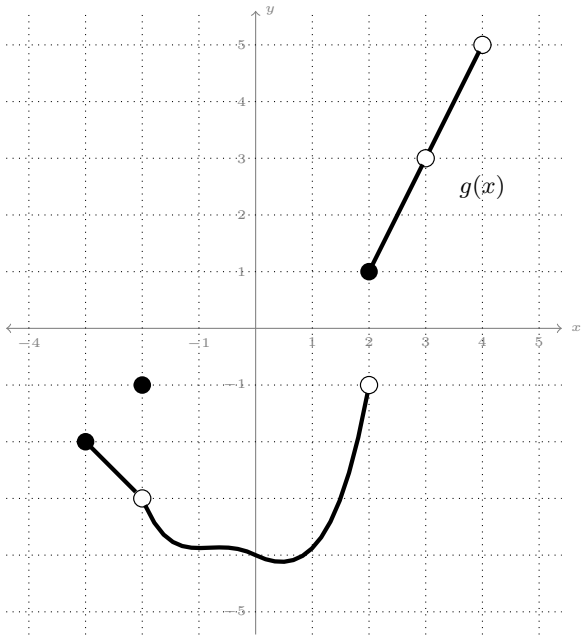
  Similarly,

  $$\lim_{t \to 0^-} \frac{|t|}{t} = -1.$$

- Important fact: A limit will exist exactly when the left and right-sided limits agree.

- Ex: Using the graph, evaluate the following limits.

1. $\lim_{x \to 4^-} g(x)$
2. $\lim_{x \to 2^+} g(x)$
3. $\lim_{x \to 2^-} g(x)$
4. $\lim_{x \to 2} g(x)$
5. $\lim_{x \to 2.5} g(x)$
6. $\lim_{x \to -2} g(x)$
7. $\lim_{x \to -3^-} g(x)$



- Ex: Calculators will not always work for you.

$$\lim_{x \to 0} \frac{1 - \cos(1,000,000x)}{x}.$$

Try plugging in $x = 0.1, 0.01, 0.001$ to numerically estimate. Moral: calculators are LIMITED! Your brain is not.

- Ex:

$$\lim_{x \to 0} \frac{\sqrt{x^2 + 4} - 2}{x^2}$$

Try cheating the system by putting in $x = 0.000001$. (six 0's)