Quiz 6 Solutions

Name: _____

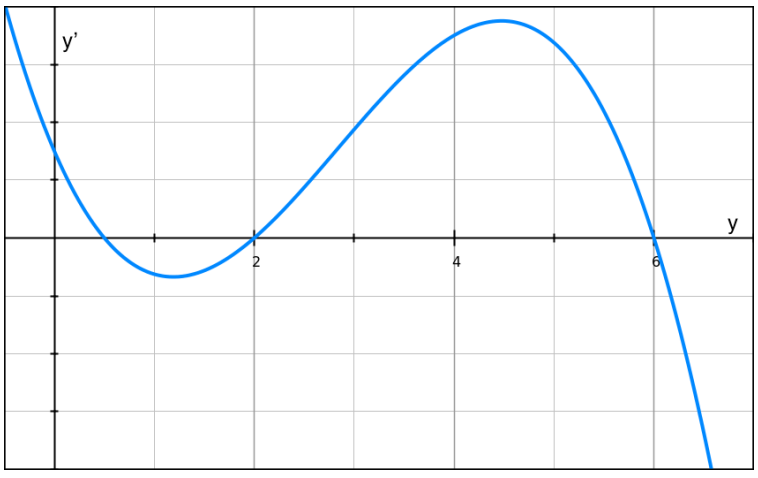
You will have 20 minutes ∘ Calculators are allowed ∘ Show all work for credit ∘ Don't cheat ∘ attempts at a problem may count for partial credit. ∘ If you get stuck, show as much work as possible.

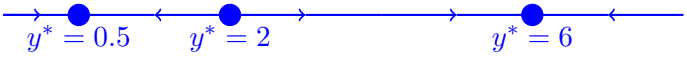
1. Consider the differential equation $\frac{dy}{dt} = 0.5(2y-1)^3$.

   (a) [3 pts] Find the equilibria of this equation.
   Only $y^* = 0.5$.

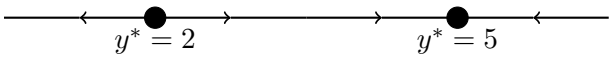
   (b) [2 pts] Is this equilibrium stable, unstable, or neither?
   Plug in other numbers, like $y = 0$ and $y = 1$; you'll get $y' = -0.5$ and $y' = 0.5$ respectively. Hence, the phase-line diagram will tell you it is stable.

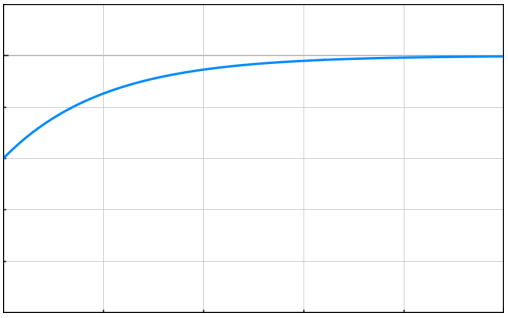
2. [3 pts] From the graph shown below, sketch the phase-line diagram for y. [Note that the axes below are y' and y.]





3. [3 pts] From the phase-line shown below, sketch a solution beginning at $y(0) = 3$. **Label your axes.**





4. [2 pts] Suppose that $\frac{dy}{dt} = f(y)$, and that $y^* = 0$ is an equilibrium. I tell you that $f'(0) = -3$. Is $y^* = 0$ a stable or unstable equilibrium? Why? The stability theorem says that when $f'(y^*) < 0$, y^* is a stable equilibrium, so 0 is a stable equilibrium.

5. [2 pts] Check that $H(t) = 30 - 10e^{-2t}$ is a solution to Newton's law of cooling $\dfrac{dH}{dt} = 2(30 - H)$. [Don't use separation of variables.]

   1. $\dfrac{dH}{dt} = 20e^{-2t}$
   2. $2(30 - H) = 2(30 - (30 - 10e^{-2t}) = 20e^{-2t}$

   Since 1 and 2 match, this is a solution.