1.10: Selection Model

1 Modeling Procedure

1. Define variables. Spend some time to understand them.
2. Derive the Model
3. Analyze the model: Cobweb, equilibria, stability of equilibria.
4. Draw conclusions.

2 Selection Model

- Consider two populations of growing species.
- Example: Slow Zombies, b_t, and fast zombies, m_t. (Notation matches the book).
- Example: Regular cells b_t, mutated cancer cells, m_t.
- Let's derive a DDS for the fraction of mutants, call it p_t.
- $p_t = \dfrac{m_t}{m_t + b_t}$.
- Without DDS: If there are 300 slow zombies and 500 fast zombies, what is the value of p? $p = \frac{500}{300+500} = \frac{500}{800} = 0.625$.
- Ex: If p_t is the fraction of mutants, then what does $1 - p_t$ represent?
- $1 - p_t = \frac{b_t}{m_t+b_t}$ is the fraction of non-mutants.

3 Derive the Model

- Suppose for simplicity both populations grow by a fixed factor each time. (Imagine the doubling example.)
- $b_{t+1} = rb_t$, $m_{t+1} = sm_t$.
- Plug in:

$$\begin{aligned} p_{t+1} &= \frac{m_{t+1}}{m_{t+1} + b_{t+1}} \\ &= \frac{sp_t}{sp_t + r(1-p_t)}. \end{aligned}$$

(The algebra is shown in the book and in class.)

4 Analyzing the Model

- Look at examples.
- Ex: Fast zombies grow more quickly than slow zombies. Say $s = 2$ and $r = 1.5$.
- $p_{t+1} = \dfrac{2p_t}{2p_t + 1.5(1 - p_t)}$.
- Cobweb: note that p_t tends to 1.
- Conclusion: if mutants grow faster, their population "takes over." This does NOT mean that slow zombies DIE. It just means that in the long run, there are waaaaay more fast zombies.
- Ex: Suppose fast zombies grow more slowly than the slow zombies. Say $s = 1$ and $r = 3$.
- $p_{t+1} = \frac{p_t}{p_t + 3(1 - p_t)}$.
- Cobweb: note p_t tends to 0.
- Conclusion: if fast zombies grow more slowly, their fraction tends to 0. This does NOT mean they all die.
- Ex: Find equilibria and analyze stability for $s = 2$, $r = 1.5$:
- solve
$$\frac{2p^*}{2p^* + 1.5(1 - p^*)} = p^*.$$
Get: $p^* = 0$ and $p^* = 1$ from algebra (done in class and in book).
- Use stability theorem to analyze the equilibria:
$$f(x) = \frac{2x}{2x + 1.5(1 - x)}$$
$$f'(x) = \frac{2(2x + 1.5(1 - x)) - 2x(2 - 1.5)}{((2x + 1.5(1 - x))^2}$$
- $f'(0) = \frac{2(1.5) - 0}{(1.5)^2} = \frac{2}{1.5} > 1$ so $p^* = 0$ is stable.
- $f'(1) = \frac{2(2) - 2(0.5)}{(2)^2} = \frac{1.5}{2} < 1$ so $p^* = 1$ is unstable (as expected!)