2.8 and 2.10: Some more functions

1 Trigonometric Functions

- sin and cos are rather straightforward from the graphs.
- $\frac{d}{dt}\sin(t) = \cos(t)$
- $\frac{d}{dt}\cos(t) = -\sin(t)$.

2 Exponential Functions

- Write down definition of the derivative for $f(t) = b^t$. Get:

$$f'(t) = \left(\lim_{h\to 0}\frac{a^h - 1}{h}\right)a^t.$$

  That is to say, the derivative is a number times itself!

- Turns out: the number $e = 2.71828....$ is the base such that this number is 1:

$$\lim_{h\to 0}\frac{e^h - 1}{h} = 1.$$

- Consequence:

$$\boxed{\frac{d}{dt}e^t = e^t.}$$

  It is its own derivative.

- Definition of ln: at least a new one!

$$\ln(a) = \lim_{h\to 0}\frac{a^h - 1}{h}.$$

  This *will* be justified to you later by me.

- Consequence:

$$\boxed{\frac{d}{dt}a^t = \ln(a)\cdot a^t}.$$

  Ex:

$$\frac{d}{dt}2^t = \ln(2)\cdot 2^t = (0.693)\cdot 2^t.$$

  Note: you cannot combine the 0.693 and the 2; different exponents.

3 Examples

- Ex: find $\frac{d}{dt}\tan(t)$. (Use quotient rule!)
- Principle: turn your gnarly trig functions into combinations of sin and cos and use a bunch of other shortcuts.
- Ex: find $\frac{d}{dt}(\sec(t))$.
- Ex: find $\frac{d}{dt}t^2\cos(t)$.
- Ex: differentiate these functions.

$$\frac{3^t + t^3}{\sin(t)}$$
$$e^{-x}$$
$$te^t$$
$$(0.5)^t \cdot 6^t$$
$$e^t \sin(t)$$
$$\frac{te^t}{\cos(t)}$$