Exam 1

Name: _____   Student Number: _____

Don't leave anything blank. If you don't know the entire answer, showing a formula or writing something illustrating that you understand any concept involved in the problem will allow me to give partial credit. I have to give you a 0 if you write nothing down.

Check your answers. Take the time before you turn in your test to make sure you have read the directions correctly and in their entirety, that all your work shown is correct, and that you have clearly stated your answer (by boxing or circling it where appropriate).

Pace yourself. If you're stuck on a problem, move on and come back to it later. Don't risk forcing yourself to give partial answers if you run out of time near the end of the test. Do the easy ones first. There are 50 points on this exam. That means you should budget about 1.3 minutes for each point a problem is worth in order to complete the exam in time.

Most importantly:

Read carefully!

Short-Answer Problems Give a short answer response to each problem. Use no more than 3 sentences.

1. [3 pts] State the degree of the following polynomial, and list its leading term and constant term.

$$f(x) = 2x^4 - x^5 + x^2 - 1 + x$$

The degree is 5, and its leading term is $-x^5$. The constant term is -1.

2. [4 pts] Find the domain of the function

$$f(x) = \frac{\sqrt{x}}{x^2 - 4}.$$

Write your answer in both set notation and interval notation.

Need $x \neq \pm 2$ and $x \geq 0$. So,

$$\text{Dom}(f) = \{x \mid x \geq 0 \text{ and } x \neq 2\} = [0, 2) \cup (2, \infty).$$

3. [3 pts] Does the following table define y as a function of x? Why or why not?

x	0	4	7	9	3	4	12
y	0	2	3	1	10	0	14

No. The input $x = 4$ has two outputs, $y = 2$ and $y = 0$.

Free-Response Problems Write your answers clearly and concisely, including all work. If asked to explain something, use complete sentences. Any numerical answers may be written in approximate form as long an exact solving method is used.

4. Geralt of Rivia hunts monsters for a living. He sets out on his travels on March 1st and slays about 35 monsters every day. On March 20th, Geralt fights an ancient noonwraith and suffers a serious injury. After this encounter he becomes more cautious, so he only slays 25 monsters each day afterwards.

   (a) [5 pts] Write a function, $M(t)$, describing the number of monsters Geralt has slain as a function of time, t, in days. Hint: what kind of function must it be if you have more than one formula describing it?)

   $$M(t) = \begin{cases} 35t & t \le 20 \\ 700 + 25(t - 20) & t > 20 \end{cases}$$

   (b) [2 pts] How many monsters has Geralt slain on March 28th?   M(28) = 700 + 25(28-20) = 900

   (c) [3 pts] After how many days has he slain two thousand monsters? (You do not need to convert to month-and-day.)   Solve $M(t) = 2000$: $2000 = 700 + 25(t - 20)$, gives $t = 72$ days.

   (d) [5 pts] Draw a graph of M. Label your axes and relevant points on the graph.
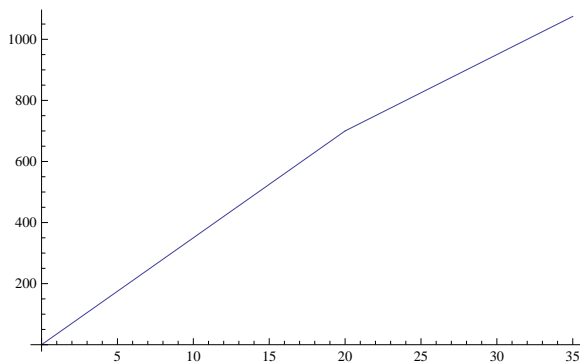
5. The height of a diver above the water is given by $h(t) = h_0 + v_0 t - 4.9t^2$ meters, t seconds after jumping from the platform. h_0 is the initial height and v_0 is the initial upward velocity.

   (a) [5 pts] If the diver begins from a 12-meter-high board with initial velocity upward velocity of 5 meters per second, when will she reach the water?

   The quadratic becomes $h(t) = 12 + 5t - 4.9t^2$. "She reaches the water" means "solve $h(t) = 0$." This is the equation $12 + 5t - 4.9t^2 = 0$, so use the quadratic formula:

   $$t = \frac{-5 \pm \sqrt{5^2 - 4(-4.9)(12)}}{2(-4.9)}$$

   $$t \approx -1.135 \text{ and } 2.156 \text{ seconds.}$$

   So, she reaches the water after 2.156 seconds.

   (b) [5 pts] [**Note: this part has nothing to do with part (a).**] If instead the diver begins from a 8.5-meter-high board and reaches the water 2.2 seconds after leaving the board, after how long will she reach her maximum height?

   The quadratic becomes $h(t) = 8.5 + v_0 t - 4.9t^2$. We know $h(2.2) = 0$, so

   $$8.5 + v_0(2.2) - 4.9(2.2)^2 = 0$$

   $$v_0 = \frac{4.9(2.2)^2 - 8.5}{2.2} \approx 6.9 \text{ m/sec}$$

   Now, the time she reaches the max height is $t_{\text{vert}} = -\frac{b}{2a}$:

   $$t_{\text{vert}} = -\frac{6.9}{2(-4.9)} \approx 0.7 \text{ seconds.}$$

6. Suppose that you take a *rectangular* piece of cardboard with one side length 25 inches and the other side with 10 inches. Suppose also that you cut out a square from each corner of length x-inches and fold the remaining shape into a box.

   (a) [5 pts] Show that the volume of the box, V, is given by $V(x) = x(25 - 2x)(10 - 2x)$. (Hint: you did this on your homework. No algebra is needed. Make a picture.)

   Draw a rectangle, cut out the corners of length x; the resulting sides have length $25 - 2x$ and $10 - 2x$, and when you fold it up the height is x. Volume is base times height times depth, so $V(x) = x(25 - 2x)(10 - 2x)$.

   (b) [5 pts] Compute the average rate of change in V on the interval $[2, 4]$. **Include units.** Interpret your answer in one sentence.

   $$\text{ARC} = \frac{4(17)(2) - 2(21)(6)}{4 - 2} = -58 \text{ in}^3/\text{in}$$

   For each inch we increase x by, the volume decreases by 41 cubic inches.

   (c) [5 pts] Compute the percentage change in V on the interval $[2, 4]$.

   $$\text{PC} = \frac{136 - 252}{252} \cdot 100\% = -46\%$$

   which means that the volume decreased by 46%.

   (d) [5 pts Extra Credit] What is the practical domain of V? The smallest side is $10 - 2x$, and this must be positive. This is zero when $x = 5$, so the practical domain is $[0, 5]$ (parenthesis or brackets are really okay here).

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