Final Exam Overview (Chapters 1, 2, and 3)

The exam covers 1.5, 1.6, 1.9 - 1.11, 2.1 - 2.10, and 3.1 - 3.6. The final exam is cumulative.

The exam may contain True/False, Multiple Choice, and Fill in the Blank in addition to Free Response questions. You should be able to finish the exam in the 120 minutes allotted. It is approximately the length of one and a half midterm exams.

You may bring a 5x8 note card (or a half sheet of paper), including any formulas (e.g. slope criterion for stability) that you might want to have with you during the exam. That means you are responsible for the formulas you have with you.

Skills List

The following is a list of skills that you will need to be able to apply on the exam. This list is intended to be comprehensive, whereas the practice problems and those done in class are only a sampling. By its nature it is somewhat more vague than a sampling of practice problems.

With the aid of a scientific calculator and relevant formulas, you should be able to...

- Compute the average rate of change for a given function over a specified interval.
- Determine intervals of continuity of a function at a point from a graph of the function.
- Determine continuity of a function at a point using the limit definition of continuity.
- Set up and algebraically compute the limit definition of the derivative for simple functions, such as linear, quadratic, or reciprocal functions.
- Compute the derivative using shortcut rules for power, root, trigonometric, exponential, and logarithmic functions, as well as sums, products, quotients, and compositions of those functions.
- For any applied problem, interpret the result of any function or rate of change computation in context, including units.
- Given any function for which the first and second derivatives are calculable, find the critical points, inflection points, and intervals on which the function is increasing, decreasing, concave up, and concave down.
- From the graph of the derivative of a function, estimate relative steepness and concavity of the function, as well as find critical points or inflection points.
- Set up formulas for one or more discrete-time dynamical systems given a description in English.
- Sketch the graph of an updating function, including correctly labeling axes.
- Determine values of a discrete-time dynamical system given an initial value.
- Find equilibria of a discrete-time dynamical system using a graph and with algebra.
- Identify the stability of an equilibrium of a discrete-time dynamical system using cobwebbing and/or employing the slope criterion for stability.
- Given a template for discrete-time dynamical systems of the concentration of a chemical in the lungs or voltage in the AV node of the heart, identify the significance of the parameters involved.
- Find maximum and minimum values of functions using the first and second derivatives.

- Use stable equilibria in discrete-time dynamical systems along with optimization in order to determine maximum harvest in a system.
- Explain results of optimization, equilibria and stability in terms of the context of the model.
- Know when the Extreme Value Theorem applies and when it doesn't.
- Identify the behavior of a function at infinity using leading behavior or l'Hôpital's rule.
- Compute a limit of an indeterminate of the form $\frac{0}{0}$ or $\frac{\infty}{\infty}$ using l'Hôpital's rule.

Practice Problems for Recent Material

In general, the practice sheet is at least as hard as the exam, which is at least as hard as the quizzes. A question appearing on this practice virtually guarantees that the exact same question will not appear on the exam itself. I would like to test your understanding, not your memorization. While quizzes generally test whether or not you can do a problem from homework without assistance, exams may ask more involved questions that synergize material from various sections.

Make sure that your studying eventually includes **you being able to do problems like these without assistance from notes, textbook, or other people**. If you cannot do the problems without assistance before the exam, there's no reason to believe that when you get into the test you will do any better.

The homework and quizzes are also good places to review (there will be routine questions on the exam as well as conceptual and more involved ones). The questions here and in the chapter review will help you practice how to tackle a question without knowing the section it is from, a skill necessary for success on the exam.

Textbook Problems

It would be a very good idea to do additional problems from Chapter 3 in the textbook. In particular, here are some good practice problems from the text:

3.3: # 23, 24, 45a
3.5: # 1-8
3.6: #7-16, 17, 21
Ch. 3 Supplementary (pg. 341): #1, 2, 7, 8, 15, 16, 20

Additional Problems

Here are several more problems that you can practice with. These come from the entire term, so you may have to think about content from Chapters 1 and 2 to answer them.

True/False (Clearly indicate whether each of the following statements is True (T) or False (F).)

1. $\lim_{x \to 1^+} \sqrt{1-x}$ does not exist.
2. If V is a function of y and y is a function of x, then $V'(x) = V'(y) \cdot y'(x)$.
3. The function $C(x) = \sin(\pi x)$ has a period of π.
4. $N_{t+1} = a(1 - N_t) + b(N_t + 1)$, for constants a and b, is a linear discrete-time dynamical system.
5. An equilibrium of the system defined by $D_{t+1} = pD_t + q$ is stable exactly when $p < 1$.

Multiple Choice (Choose the best answer from among the choices given)

6. l'Hôpital's rule can be used to determine the value of the limit as long as it is of the form...
   (a) $\frac{0}{0}$
   (b) $\frac{\infty}{\infty}$
   (c) $\frac{\infty}{0}$
   (d) Only (a) and (b).
   (e) All of the above.

7. The number of predators P of a species of ape is a function of number of male apes a in the group. The number of viable mates M for the apes is also a function of a. Using these functions, the most reasonable expression that the apes would want to maximize is...
   (a) $P(a) \cdot M(a)$
   (b) $P(a) - M(a)$
   (c) $M(a) + P(a)$
   (d) $\dfrac{P(a)}{M(a)}$
   (e) $\dfrac{M(a)}{P(a)}$

8. The units on M are dollars while the units of radiation r are in milli-Curie's (mCu). Then the units on $M'(r)$ would be...
   (a) \$
   (b) mCu
   (c) \$ / mCu
   (d) mCu / \$

Free Response (Write your answers clearly and concisely, including all work. If asked to explain something, use complete sentences. Any numerical answers may be written in approximate form as long an exact solving method is used.)

9. In a Ricker model for population growth, the discrete-time dynamical system is $p_{t+1} = 4p_t e^{-p_t}$. Find any equilibria of this system and classify them as stable or unstable using the slope criterion.

10. Let $f(x) = 0.1e^x \sin\left(\frac{\pi}{4}x\right)$ on the interval $[0, 5]$.

    (a) Does the extreme value theorem apply to this situation? What do we learn from it? Find the global maxima and minima if they exist.

    (b) What if we took our interval to instead be $[1, \infty)$? That is, does the extreme value theorem still apply? Does f have a global maximum on this interval?