1.5: Discrete Dynamical Systems

What are they?

- A <u>discrete dynamical system</u> is a way of modeling any phenomenon that happens in a non-continuous way, like taking steps.
- discrete: non-continuous chunks
- dynamical: interactions or evolutions
- Here's just an example of what one looks like.
- Model: a single bacterium can split in two. This means the population doubles for each hour. With an initial population of one bacterium, a DDS modeling this is given by

  $$m_{t+1} = 2m_t, \qquad m_0 = 1.$$

  Note: $m_0 = 1$ is the <u>initial data</u> or <u>seed</u>.
- The subscript t is different than the variable t we use in function notation.
- t only takes values $0, 1, 2, 3, \ldots$. So, for example, m_{13} makes sense, but $m_{0.5}$ does not.
- Here's how to use the DDS: compute the first 4 values:

t	m_t
0	1
1	2
2	4
3	8
4	16

- You must keep track of the subscripts. They will be your guide through the DDS.
- Plot m_t versus t. Note it is not a continuous graph!
- Ex: Find the first 4 values for the DDS

  $$m_{t+1} = 2m_t, \qquad m_0 = 0.5.$$

  Plot the solution. Compare with the previous example.

t	m_t
0	0.5
1	1
2	2
3	4
4	8

- Ex: Find the first 4 values of the DDS

$$m_{t+1} = -m_t, \qquad m_0 = 1.$$

t	m_t
0	1
1	-1
2	1
3	-1
4	1

- Ex: Find the first 4 values of m_t with the DDS

$$m_{t+1} = (m_t)^2, \qquad m_0 = 2.$$

t	m_t
0	2
1	4
2	16
3	256
4	65536

- Ex: Find the first 4 values of m_t with the DDS

$$m_{t+1} = (m_t)^2, \qquad m_0 = -1.$$

t	m_t
0	-1
1	1
2	1
3	1
4	1

- Notice: the initial condition *heavily* changes the resulting sequence.

Model

- Concentration of medication in bloodstream: mg/L of a drug.
- Model: suppose each day a person takes a pill, adding 1 mg/L of the drug into their bloodstream.
- Suppose also each day, half of the medication gets removed from the bloodstream.
- Write down a DDS for this person.

$$C_{t+1} = 0.5C_t + 1, \qquad C_0 = 0$$

  First 4 values:

t	C_t
0	0
1	1
2	1.5
3	1.75
4	1.875

Plot: notice it tends towards $C = 2$.

- What if the person starts by taking twice their normal dose of 2 mg/L?

t	C_t
0	2
1	2
2	2
3	2
4	2

It's completely stable! We call this concentration an equilibrium.

- Note that in this model, the concentration will tend towards equilibrium, as a good drug should.