Worksheet 10

Math 251, Summer 2017

Name: Key

I have given you the answers on the bottom of the last page. You must figure out how to solve the problem to get the correct answer.

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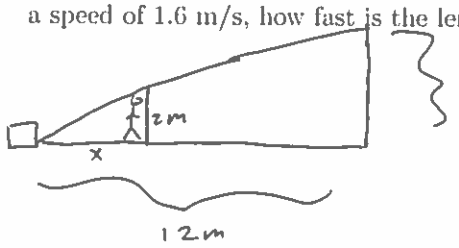
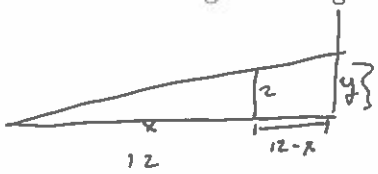
1. A spotlight on the ground shines on a wall 12m away. If a man 2m tall walks from the spotlight toward the building at a speed of 1.6 m/s, how fast is the length of his shadow on the building decreasing when he is 4m from the building?





x = dist. of man from spotlight.

y = height of shadow.

$$\frac{y}{12} = \frac{2}{x}$$

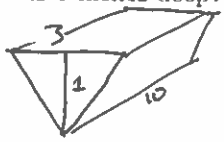
$$y = \frac{24}{x}$$

$$\frac{dy}{dt} = -\frac{24}{x^2} \cdot \frac{dx}{dt}$$

Know: $x = 8$, $\frac{dx}{dt} = 1.6$

$$\frac{dy}{dt} = -\frac{24}{(8)^2} \cdot (1.6) = -0.6 \text{ m/s}$$

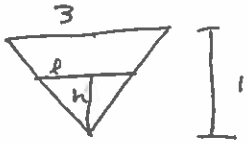
2. A trough is 10 ft long and its ends have the shape of isosceles triangles that are 3 ft across at the top and have a height of 1 ft. If the trough is being filled with water at a rate of 12 ft^3/min, how fast is the water level rising when the water is 6 inches deep?



h = height of water

l = width of water at top.

V = volume of water.



Know: $\frac{dV}{dt} = 12$, $h = 0.5$

Want: $\frac{dh}{dt}$ when $h = 0.5$.

$V = \frac{1}{2} l \cdot h \cdot 10$ and $\frac{h}{1} = \frac{l}{3}$

$l = 3h$

$$V = \frac{1}{2} 3 \cdot h^2 \cdot 10$$

$$V = \frac{3}{2} h^2 \cdot 10$$

$$12 = \frac{dV}{dt} = 3h \frac{dh}{dt} = 3 \cdot (.5) \frac{dh}{dt} \cdot 10$$

$$\frac{12}{(10)3(.5)} = \frac{dh}{dt} = 0.8 \text{ ft/min}$$

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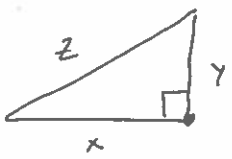
3. Two cars start moving from the same point. One travels north at speed 60 mi/hr, and the other travels west at 25 mi/hr. At what rate is the distance between the cars changing at two hours later?



x = west car's distance
y = North car's distance.

Know: $\frac{dx}{dt} = 25$

$\frac{dy}{dt} = 60$

Want: $\frac{dz}{dt}$ when $x = 50$, $y = 120$.

$z^2 = x^2 + y^2$

$2z\frac{dz}{dt} = 2x\frac{dx}{dt} + 2y\frac{dy}{dt}$

$\Rightarrow z^2 = (50)^2 + (120)^2$

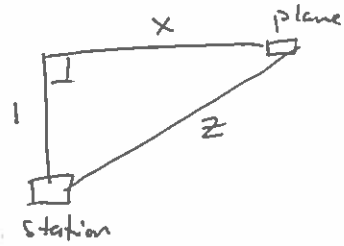
$z^2 = 16900$

$z = 130$

$(130)\frac{dz}{dt} = (50)(25) + (120)(60)$

$\frac{dz}{dt} = \frac{1250 + 7200}{130} = 65$ mi/hr.

4. A plane flying horizontally at an altitude of 1 mi and a speed of 500 mi/hr passes directly over a radar station. Find the rate at which the distance from the plane to the station is increasing when it is 2 mi away from the station.



x = dist. from plane to above the station
z = dist. from plane to station.

Note: You can interpret the last sentence as saying either $x = 2$ or $z = 2$.

Let's assume $z = 2$.

$1 + x^2 = z^2$

Know: $\frac{dx}{dt} = 500$ mi/hr

$2x\frac{dx}{dt} = 2z\frac{dz}{dt}$

Want: $\frac{dz}{dt}$ at $z = 2$.

Need x: $1 + x^2 = 2^2$

$x^2 = 3$

$x = \sqrt{3}$

$\sqrt{3}\frac{dx}{dt} = 2\frac{dz}{dt}$

$\sqrt{3} \cdot 500 = 2\frac{dz}{dt}$

$\frac{dz}{dt} = \frac{\sqrt{3}}{2} \cdot 500 \approx 433.0$ mi/hr.

Answers: 1) 0.6 m/s. 2) 0.8ft/~~sec~~ min. 3) ~~[illegible]~~ 65 mi/hr. 4) ~~[illegible]~~ 433.0 mi/hr.