Exam 2

Name: _____

- Show as much work as possible, even if you can't answer the problem entirely. This allows me to give you partial credit.
- Spend time wisely: read through the test first and solve the ones you know how to do.
- There are a total of 40 points on this exam.

True/False

Directions: Indicate whether the statement is always true or sometimes false. **Explain your reasoning**. [2 pts each]

1. _____ All solutions to an autonomous differential equation must converge to a stable equilibrium.

2. _____ The state variable y is the prey population in the predator-prey model $\begin{cases} \frac{dx}{dt} = 8x - xy \\ \frac{dy}{dt} = xy - y \end{cases}$.

Short Answer

Directions: Answer each question with a short response (preferably one to two sentences or a quick calculation).

3. [3 pts] Can the function $y(t) = t^2$ be a solution to the differential equation $\frac{dy}{dt} = 3y$?

4. [3 pts] A population of foxes living in the Deschutes National Forest grows at a rate of $f(t)$ thousand foxes per year, where $f(t)$ is a function with the property that $\int_0^\infty f(t)\,dt = 6$. At their current population of five thousand, will their population ever reach ten thousand?

Free Response

Directions: answer the following problems. Be sure to show your work.

5. The number B of yeast bacteria (measured in billions) fermenting in a large jug of Kombucha can be modeled by the differential equation

$$\frac{dB}{dt} = B(2 - B)(B - 3).$$

   (a) [4 pts] Draw a phase-line diagram for the differential equation.

   (b) [2 pts] Identify the stable and unstable bacteria equilibria for this model.

   (c) [2 pts] How many bacteria need to be alive in order for the bacteria to stay alive?

6. [4 pts] Suppose the function $M(x) = \dfrac{x}{2 + x^2}$ models the density of buffalo (in thousands per kilometer) x kilometers along the river. The river is 3800 km long in total. How many buffalo live along the first 500 kilometer segment of the river?

7. [5 pts] The number of penguins in Antarctica follows the differential equation $\frac{dQ}{dt} = (2t + 1) \cdot Q$. Use Euler's method with $\Delta t = 0.3$ and initial condition $Q(0) = 2$ to estimate $Q(0.6)$.

8. [5 pts] Solve the differential equation in problem 7 with the same initial condition $Q(0) = 2$.

9. Consider two populations, A and B, of animals. The per capita growth rate of population A is given by $1.12 - 4B$, while the per capita growth rate of poplation B is given by $0.94 - 2A$.

   (a) [4 pts] Write down the system of differential equations for these populations.

   (b) [2 pts] If there are none of population A's animals, will population B grow or die off?

   (c) [2 pts] Interpret how these two species interact with each other.