

Homework 7
Due Tuesday, May 28th

Instructions: write up solutions to all problems below. Neatness counts: be sure to follow guidelines for homework in the syllabus.

1. Practice with input variable substitutions in derivatives. Given the variable substitutions $x = f(t)$ below, convert the derivatives $\frac{dy}{dt}$ and $\frac{d^2y}{dt^2}$ into derivatives involving x . Answers are provided; you must show how to get these answers from scratch.

(a) $x = \cos(t)$ Answer: $\frac{dy}{dt} = \sqrt{1-x^2} \frac{dy}{dx}$ $\frac{d^2y}{dt^2} = (1-x^2) \frac{d^2y}{dx^2} + x \frac{dy}{dx}$

(b) $x = t^2$ Answer: $\frac{dy}{dt} = 2\sqrt{x} \frac{dy}{dx}$ $\frac{d^2y}{dt^2} = 4x \frac{d^2y}{dx^2} + 2 \frac{dy}{dx}$

(c) $x = e^t$ Answer: $\frac{dy}{dt} = x \frac{dy}{dx}$ $\frac{d^2y}{dt^2} = x^2 \frac{d^2y}{dx^2} + x \frac{dy}{dx}$

2. Same as the problem before: given x as a function of t , convert $\frac{dy}{dt}$ and $\frac{d^2y}{dt^2}$ into derivatives with respect to x . Now, the answers are not given!

(a) $x = \sqrt{t}$

(b) $x = \sin(t)$

(c) $x = 4t$

3. Find the general solutions to the following systems of equations. Express your answers in two ways: first, as $\vec{x}(t)$, and second as two functions $x(t)$ and $y(t)$.

(a) $\vec{x}' = \begin{pmatrix} 2 & 3 \\ 0 & 4 \end{pmatrix} \vec{x}$

(b) $\vec{x}' = \begin{pmatrix} 2 & 8 \\ 1 & 0 \end{pmatrix} \vec{x}$

(c) $\vec{x}' = \begin{pmatrix} 6 & 1 \\ 2 & 5 \end{pmatrix} \vec{x}$

4. Convert the following differential equations into a system of equations.

(a) $y'' + 10y' - 9y = 0$

(b) $10y'' - 2y' + y = 0$