Section Goals:

- Combine functions with elementary operations and interpret the result
- Find an equation for and evaluate the composition of two functions
- Analyze an applied context to determine what elementary combination or composition of functions is appropriate
- Find the domain of a composite function

Def For two functions, $f(t)$ and $g(t)$, for any t in the domain of both f and g, we write

$$\left[\begin{array}{c} (f+g)(t) = f(t) + g(t) \\ (f-g)(t) = f(t) - g(t) \\ (f \cdot g)(t) = f(t) \cdot g(t) \\ \left(\dfrac{f}{g}\right)(t) = \dfrac{f(t)}{g(t)} \text{ (as long as) } g(t) \neq 0 \end{array}\right]$$

If it seems like there is not a great deal of purpose to those definitions, that's because there isn't. It is, however, a necessary formality, for example, to define "the function whose name is f minus g" to be the function that takes an input, evaluates it in f, then g, then subtracts those values.

Ex 1 Simplify each function expression, given $f(t) = 3t^2 + t$ and $g(t) = \dfrac{8t}{3t+1}$.

a) $(f+g)(1)$

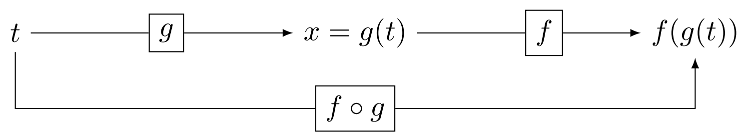
b) $(f \cdot g)(t)$

Ex 2 Let $f(t) = 3t - 2$ and $g(x) = x^2 + 1$. Compute $f(g(-1))$ and $g(f(-1))$.

$\boxed{\text{Def}}$ The composition of f with g is defined to be $f(g(x))$, sometimes written $(f \circ g)(x)$, and read "f of g of x" or "f composed with g of x". (Notice that the order of composition appears reversed (e.g. $(f \circ g)(x)$ means input x goes into function g first, and then into f). As a mnemonic device, consider thinking of the composition $\circ$ symbol like a little mouth and you can read $f \circ g$ as "f is eating g", and thus g would be on the inside.

Try not to confuse the composition symbol $\circ$ with multiplication $\cdot$, they are different operators!

Try thinking of this as one function used as the input for another. The diagram below shows a "typical" input t which is passed through first g and then f.



Ex 3 Pretend that the temperature, T (degrees Fahrenheit), can be fairly accurately predicted based on the chirp rate, n (chirps per minute), of crickets, by the formula

$$T(n) = 60 + \left(\frac{n - 72}{4}\right).$$

We also have the relationship

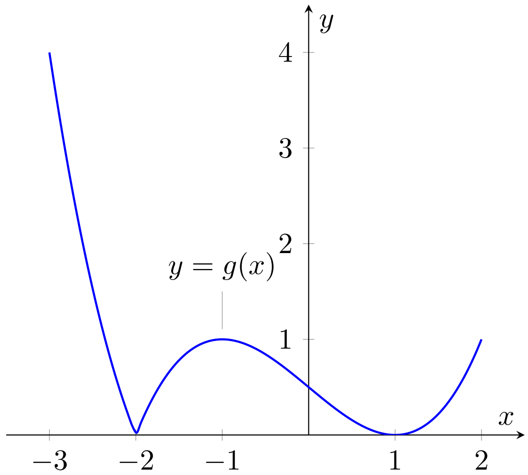
$$C(F) = \frac{5}{9}(F - 32)$$

for converting a temperature F in Fahrenheit to degrees Celsius.

a) Use composition notation to write function that takes in the chirp rate, n, and outputs the temperature in Celsius.

b) Use the formula above to find the temperature in Celsius when the chirp rate is 42 chirps per minute.

Ex 4 Consider the function given by $y = g(x)$ shown in the graph below, and $y = h(t)$ defined completely by the table. Compute each indicated value or state it to be undefined.



t	$h(t)$
-1	3
0	-1
1	4
2	3

a) $(g+h)(1)$

b) $(h \circ g)(-1)$

c) $\left(\frac{g}{h}\right)(2)$

d) $(g \circ h)(-1)$

Def The domain of the composite function $f \circ g$ is the set of all elements in the domain of g such that the image of each element is also in the domain of f.

In symbols,

$$\text{Dom}(f \circ g) = \{x \mid x \in \text{dom}(g) \text{ and } g(x) \in \text{Dom}(f)\}$$

In other words, we would check each number, a, in the domain of g to see if $g(a)$ is in the domain of f. If it is, then a is part of the domain of the composite function.

Ex 5 Find the domain of the function $f \circ f$, where $f(x) = \frac{1}{x}$.

Ex 6 Find the domain of $f \circ g$ where $f(x) = \sqrt{x-2}$ and $g(t) = \dfrac{3}{t}$.