Quiz 29

Name: ____________________________

Possibly useful equations:

$$c_{t+1} = (1-q)c_t + q\gamma$$
$$x_{t+1} = rx_t(1-x_t)$$

1. You walk outside the library after studying for your chemistry final and are exposed to an elevated level of oxygen in the air. Suppose that the air in your lungs was at 3 mmol/L of oxygen, and the fresh air has a concentration of 4 mmol/L. You breathe in 15% of your lung capacity during each breath. What is the concentration of oxygen in your lungs after two breaths of fresh air?

   We have $q = 0.15$, $\gamma = 4$, and $c_0 = 3$. The system is $c_{t+1} = 0.85c_t + 0.6$. So,

   $$c_1 = 0.85(3) + 0.6 = 3.15\text{mmol/L},$$
   $$c_2 = 0.85(3.15) + 0.6 = 3.28\text{mmol/L}.$$

   So the concentration of oxygen is now at 3.28 mmol/L.

2. With a growth factor of 1.7 in the logistic model we found in class yesterday that there is an equilibrium value of $x^* = 0.41$.

   (a) What is the meaning of the number 0.41 in context of the population?

   This means the population is currently 41% of the carrying capacity.

   (b) Use the Stability Theorem to decide if this equilibrium is stable or unstable.

   $f(x) = 1.7x(1-x) = 1.7x - 1.7x^2$ is the updating function. So,

   $$f'(x) = 1.7 - 3.4x,$$
   $$f'(0.41) = 1.7 - 3.4(0.41) \approx 0.306.$$

   Since this number is smaller than 1, this is a stable equilibrium.