Exam 1 Practice Problems

These problems highlight some of the main ideas I'll be testing you on for Exam 1. A good way to know that you're ready for the exam is if you can do all of these problems **on your own**, which means without the help of notes, the book, the internet, or other people.
Note: These problems are at least as difficult as what will appear on the exam. That is, you should expect the exam problems to be slightly easier than these.
You will not be allowed to use a graphing calculator on the exam. Please purchase a scientific calculator and bring it to the exam!

True/False

1. The antiderivative of $\ln(x)$ is $\frac{1}{x}$.
2. If $f(2) < 0$, then the antiderivative of $F(2) < 0$.
3. The integral $\displaystyle\int \sqrt{x^3+1}\,dx$ is computable with the strategies from our course.
4. The integral $\displaystyle\int x^2\sqrt{x^3+1}\,dx$ is computable with the strategies from our course.

Multiple Choice

1. Consider a function $f(x)$ on the interval $[2,5]$ with 3 steps. Which of the following expressions is the correct expression for the right Riemann sum?
   (a) $f(2)+f(3)+f(4)$
   (b) $f(3)+f(4)+f(5)$
   (c) $f(2)0.5+f(3)0.5+f(4)0.5$
   (d) $f(3)0.4+f(4)0.4+f(5)0.4$

2. Suppose $R(t)$ represents the rate that air is converted into CO_2 in the lungs t seconds after inhaling. Which expression below represents the total amount of CO_2 converted in a single breath, assuming a breath lasts 1 second?
   (a) $\displaystyle\int R(t)\,dt$
   (b) $\displaystyle\int_0^{60} R(t)\,dt$
   (c) $\displaystyle\int_0^{1} R(t)\,dt$
   (d) $\displaystyle\int_0^{1} tR'(t)\,dt$

Free Response Problems

1. Suppose $\int f(x)\cos(x)\,dx = \frac{1}{4}\sin(x)+9$. What is $f(x)$?
2. Evaluate the following integrals.

(a) $\int \ln(x)\,dx$

(b) $\int 4x\cos(x)\,dx$

(c) $\int \frac{3}{6}e^{-2t}\,dt$

(d) $\int \pi\,dx$

(e) $\int \frac{t^2}{t^3+1}\,dt$

(f) $\int dt$

(g) $\int (x-5)^{4/3}x\,dx$

(h) $\int \frac{1}{6-2x}\,dx$

3. Suppose that in a chemical reaction there is an amount $C(t)$ of water t milliseconds after the beginning of the reaction. The process is modeled with the differential equation

$$\frac{dC}{dt} = -te^{-t^2}$$

Suppose there is initially 200 mL of water.

(a) Describe in words what happens over time with the amount of water.

(b) When is the water being used up the fastest?

(c) Find the solution of this differential equation.

(d) About how much water is used up in this chemical reaction?

4. A mammal's basal metabolic rate, B, is the rate of change in energy consumed. Assume that this rate is given by $B(M) = 980M^{2/3}$ kCal per week at M kilograms. Also, mass M changes over a short period of time according to $M(t) = 100 + 0.5t$ kilograms, t weeks into a period of muscle growth. Write down, but do not evaluate, an integral that represents the total change in energy consumed over the first five weeks. What are the units of this quantity? [Hint: first write B as a function of t.]

5. The growth rate (in kg per year) of an animal follows the differential model $\frac{dm}{dt} = k \cdot (10 - m)$, for some constant k.

(a) Is the differential equation pure-time, autonomous, or neither?

(b) Verify that the general solution is $m(t) = 10 - Ce^{-0.2t}$ as long as $k =$ ________.

(c) Find the solution to the equation given the initial condition that the animal was born weighing 6 kg.