Quiz 5

Name: ________________________

You will have 20 minutes ○ Calculators are allowed ○ Show all work for credit ○ Don't cheat ○ attempts at a problem may count for partial credit. ○ If you get stuck, show as much work as possible.

1. Suppose a bacteria colony grows with a growth rate of $\frac{1}{1+t}$ bacteria/hour.

   (a) [3 pts] Calculate the total change in the long run for this colony (or show it is divergent).

   (b) [1 pts] Will this colony be able to keep growing this way forever? (Hint: Interpret your answer from part (a).)

2. For the two differential equations below, compute one step of Euler's method with $\Delta t = 0.5$ to estimate $\hat{y}(0.5)$. In both parts, use the initial condition $y(0) = 1$.

   (a) [2 pts] $\frac{dy}{dt} = \cos(t) + 1$

   (b) [2 pts] $\frac{dy}{dt} = 2^y$

3. [2 pts] Translate the following into a differential equation: the number y of yeast bacteria grows at a rate proportional to the <u>square</u> of the number of yeast.

4. [3 pts] Consider the differential equation

$$\frac{dy}{dt} = -y^2.$$

Show that $y(t) = \dfrac{1}{t+5}$ is a solution to the differential equation.

5. [2 pts] Suppose that $y(t)$ is a solution to the equation

$$\frac{dy}{dt} = \frac{y}{1+y}.$$

When $y = 3$, is $y(t)$ increasing, decreasing, or remaining the same?