Homework 6 sols

Tuesday, February 19, 2019 4:47 PM

$\boxed{1}$ Let P be the concentration of some chemical diffusing across a cell membrane.

ambient concentration.

$$\frac{dP}{dt} = k\left(P_0 - P\right)$$

$\boxed{2}$ P - state variable

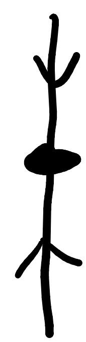
t - independent variable

k - parameter

P_0 - parameter

$\boxed{3}$ $k(P_0 - P) = 0$

$\Rightarrow P^* = P_0$



* for P bigger than P_0, $P_0 - P$ is negative, so $\frac{dp}{dt}$ is negative (down arrow).
* for P smaller than P_0, $P_0 - P$ is positive, so $\frac{dp}{dt}$ is too. (up arrow).

$\boxed{4}$

$$\frac{dP}{dt} = k(P_0 - P)$$

$$\frac{1}{P_0 - P}\,dP = k\,dt$$

$$\int \frac{1}{P_0 - P}\,dP = \int k\,dt$$

$u = P_0 - P$

$du = -dP$

$$\int \frac{1}{u} \cdot (-1)\,du = kt + C$$

$$-\ln|u| = kt + C$$

$$-\ln|P_0 - P| = kt + C$$

$$\ln|P_0 - P| = -kt + C$$

$$|P_0 - P| = e^{-kt}e^{C}$$

$$P_0 - P = \left(\pm e^{c}\, e^{-kt}\right)$$

$$P = P_0 \pm e^{c} e^{-kt}$$

$$\text{OR} \quad P = P_0 \pm C e^{-kt}.$$

$\boxed{5}$

$$\lim_{t\to\infty} P(t) = \lim_{t\to\infty}\left(P_0 \pm C e^{-kt}\right)$$

$$= P_0 \pm C\cdot\left(\lim_{t\to\infty} e^{-kt}\right)$$

(because decaying exponentials tend to 0)

$$= P_0 + 0$$

We have shown that solutions <u>always</u> tend to equilibrium! This is predicted by the phase line <u>and</u> our numerical solutions back in Homework 2.