Homework 4
Due Tuesday, April 30th

Instructions: write up solutions to all problems below. Neatness counts: be sure to follow guidelines for homework in the syllabus.

Reading Assignment: 3.1, 3.3, 3.4

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1. (a) What is the value of the complex number $e^{i\pi/4}$? What angle does it rotate by?
   (b) Generalize your observations: what angle does the complex number $e^{i\theta}$ rotate by?

2. (Fun with Euler's Formula.) Recall that Euler's formula is

$$e^{i\theta} = \cos\theta + i\sin\theta.$$

   While these applications are not strictly related to differential equations, you should be aware of how awesome Euler's formula is.

   (a) (Avoiding integration by parts.) Compute the integral $\int e^x \sin(x)\,dx$ by realizing that the integrand is the imaginary part of $e^{(1+i)x}$.
   (b) (Deriving trig identities.) Compute $e^{i(a+b)}$ and $e^{ia}e^{ib}$ and compare the real and imaginary parts. Which trig identities does this produce?
   (c) Show, using Euler's formula, that

$$\cos(x) = \frac{e^{ix} + e^{-ix}}{2}, \qquad \text{and} \qquad \sin(x) = \frac{e^{ix} - e^{-ix}}{2i}.$$

   Do you notice any relation to the hyperbolic trig functions, $\sinh(x)$ and $\cosh(x)$?

3. In class, we solved the equation

$$y'' - y = 0$$

   with two different sets of fundamental solutions: one set was e^t, e^{-t}, and the other set was $\cosh(t), \sinh(t)$.

   (a) Using the description by e^t and e^{-t}, compute the solution to the IVP $y'' - y = 0$, $y(0) = 0, y'(0) = 1$.
   (b) Using the description by $\cosh(t)$ and $\sinh(t)$, compute the solution to the IVP $y'' - y = 0$, $y(0) = 0, y'(0) = 1$. Which fundamental set was easier to use? Do you get the same answer both ways?

4. (Chapter 3.3, # 8) Compute the general solution to the differential equation

$$y'' + 6y' + 13y = 0.$$