Worksheet 14

Math 251, Summer 2017

Name: Key

Remember the list of indeterminate forms: $\frac{0}{0}, \frac{\infty}{\infty}, 0^0, \infty - \infty, 1^\infty, 0 \cdot \infty$.

1. Find the limit. Use l'Hopital's rule when appropriate. If there is a more elementary method, consider using it. If l'Hopital's rule doesn't apply, explain why.

(a) $\lim_{x \to 1} \frac{x^2 - 1}{x - 1}$ $\quad \frac{1-1}{1-1} = \frac{0}{0}$!

$= \lim_{x \to 1} \frac{2x}{1} = \boxed{2}$

(d) $\lim_{x \to \infty} \frac{\ln(x)}{x}$ $\quad \frac{\infty}{\infty}$

$= \lim_{x \to \infty} \frac{1/x}{1} = \boxed{0}$

(b) $\lim_{x \to 0} \frac{\sin(4x)}{\tan(5x)}$ $\quad \frac{0}{0}$ ✓

$= \lim_{x \to 0} \frac{\cos(4x) \cdot 4}{\sec^2(5x) \cdot 5} = \boxed{\frac{4}{5}}$

(e) $\lim_{x \to 0} \frac{x}{\arctan(4x)}$ $\quad \frac{0}{0}$ ✓

$= \lim_{x \to 0} \frac{1}{\frac{1}{1+(4x)^2} \cdot 4}$

$= \frac{1}{\frac{1}{1+0} \cdot 4} = \boxed{\frac{1}{4}}$

(c) $\lim_{t \to 0} \frac{e^{3t} - 1}{t}$ $\quad \frac{e^0 - 1}{0} = \frac{0}{0}$ ✓

$= \lim_{t \to 0} \frac{3e^{3t}}{1} = \boxed{3}$

(f) $\lim_{x \to \pi/2} \frac{1 - \sin(\theta)}{\csc(\theta)}$. $= \frac{1 - \sin(\pi/2)}{\csc(\pi/2)}$

$= \frac{1-1}{\frac{1}{\sin(\pi/2)}} = \frac{0}{1} = \boxed{0}$

*We didn't get an indeterminate form, so L'Hopital does not apply.

2. Sometimes algebra is needed to apply l'Hopital's rule. In each of the following, (a) identify the type of indeterminate form, and (b) do algebra to be able to apply l'Hoptial's rule.

(a) $\lim_{x\to 0} \csc(x) - \cot(x)$ $= \lim_{x\to 0} \left[\frac{1}{\sin x} - \frac{\cos(x)}{\sin x} \right]$

$\infty - \infty$

$= \lim_{x\to 0} \frac{1-\cos(x)}{\sin(x)} \to \frac{0}{0}$

$\overset{L}{=} \lim_{x\to 0} \frac{\sin(x)}{\cos(x)} = \boxed{0}$

(b) $\lim_{x\to 0^+} \frac{1}{\sin(x)} - \frac{1}{x}$ $= \lim_{x\to 0} \left[\frac{x - \sin(x)}{x\sin(x)} \right] \to \frac{0}{0}$ ✓

$\infty - \infty$ again.

$\overset{L}{=} \lim_{x\to 0} \left[\frac{1-\cos(x)}{\sin(x) + x\cos(x)} \right]$ $\frac{1-1}{0-0} = \frac{0}{0}$

$\overset{L}{=} \lim_{x\to 0} \frac{+\sin(x)}{\cos(x) + \cos(x) + x(-\sin(x))}$

$= \frac{0}{1+1+0} = \frac{0}{2} = \boxed{0}$

(c) $\lim_{x\to\infty} x\sin\left(\frac{\pi}{x}\right)$

$= \infty \cdot \sin\left(\frac{\pi}{\infty}\right) = \infty \cdot \sin(0) = 0 \cdot \infty$

$=$ ~~∞~~

$= \lim_{x\to\infty} \frac{\sin(\pi/x)}{1/x} \overset{L}{=} \lim_{x\to\infty} \frac{\cos(\pi/x)\cdot\left(\frac{-\pi}{x^2}\right)}{-1/x^2}$

$= \lim_{x\to\infty} \cos(\pi/x)\cdot\pi$ $\left(\left(\frac{-1/x^2}{-1/x^2} \right) \text{can} \right.$

$= \cos(0)\cdot\pi$

$= \boxed{\pi}$

3. (a) Try applying l'Hopital's rule to the following limit:

$$\lim_{x\to\infty} \frac{x}{\sqrt{x^2+1}}. \qquad \frac{\infty}{\infty}$$

(You should find that there's a kind of "cycle"-like behavior.)

$$\overset{L}{\Rightarrow} \lim_{x\to\infty} \frac{1}{\frac{1}{\cancel{2}}(x^2+1)^{-1/2}\cdot \cancel{2}x} = \lim_{x\to\infty} \frac{(x^2+1)^{1/2}}{x} = \lim_{x\to\infty} \frac{\sqrt{x^2+1}}{x}$$

they flipped!

again: $$\overset{L}{=} \lim_{x\to\infty} \frac{\frac{1}{\cancel{2}}(x^2+1)^{-1/2}\cdot \cancel{2}x}{1} = \lim_{x\to\infty} \frac{x}{\sqrt{x^2+1}}$$

← back where you started!

Wow!

(b) Evaluate the limit using a different strategy.

$$\lim_{x\to\infty} \frac{x}{\sqrt{x^2\left(1+\frac{1}{x^2}\right)}} = \lim_{x\to\infty} \frac{x}{\sqrt{x^2}\sqrt{\left(1+\frac{1}{x^2}\right)}}$$

$$= \lim_{x\to\infty} \frac{\cancel{x}}{\cancel{x}\sqrt{1+\frac{1}{x^2}}}$$

$$= \lim_{x\to\infty} \frac{1}{\sqrt{1+\frac{1}{x^2}}} = \frac{1}{\sqrt{1}} = \boxed{1}$$