Worksheet 12

Math 251, Summer 2017

Name: Key

I have given you the answers to these problems on the bottom of the last page. You need to make sure you know how to get these answers.

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1. The sum of two positive numbers is 16. What is the smallest value of the sum of their squares? Prove that your answer is a minimum.

$x+y=16$. Minimize $S:=x^2+y^2$, Want the output

$S = x^2+(16-x)^2$

$S' = 2x + 2(16-x)\cdot(-1) = 0$

$x - (16-x) = 0$

$2x - 16 = 0$

$2x = 16$

$x = 8$

Check if it is a minimum: 1st der. test!

S' — 8

Yes, it's a min.

So, min. value is:

$S = 8^2 + (16-8)^2 = 64+64$

$= 128$

2. Find two numbers whose difference is 100 and whose product is a minimum. Prove that it is indeed a minimum.

$x-y=100$, Minimize $F = x\cdot y$. Want: x and y themselves.

$F = x(x-100)$,

Solve for y:

$x-y=100$

$x = 100+y$

$-100 = y$

$F' = x-100+x = 2x-100=0$

$x = 50$

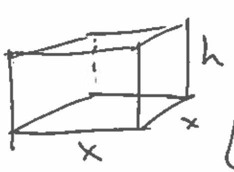
$\Rightarrow y = 50-100 = -50$

Minimum? $F'' = 2 > 0$, so F is concave up.

So $x=50$ must be a minimum by 2nd Derivative Test

3. A box with a square base and open top (meaning no lid) must have a volume of 32,000 cm^3. Find the dimensions of the box that minimize the amount of material used.

Minimize Surface area with fixed volume.



$V = x^2\cdot h$

$32000 = x^2 h.$

$h = \frac{32000}{x^2}$

Min: $S = x^2 + 4hx$

$S = x^2 + 4\left(\frac{32000}{x^2}\right)x$

$S = x^2 + \frac{128000}{x}$

$S' = 2x - \frac{128000}{x^2} = 0$

$2x^3 = 128000$

$\rightarrow x = 40$

Is it a Min?

$S'' = 2 + \frac{128000}{x^3}$

$S'' > 0$, so yes!

$\Rightarrow h = \frac{32000}{(40)^2} = 20$

Dimensions:

$40\times 40\times 20$

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4. A cylindrical can without a top needs to be designed to hold 2000 cm^3 of liquid. Find the dimensions that will minimize the cost to make such a can.

$V = \pi r^2 h$

$2000 = \pi r^2 h$

$h = \dfrac{2000}{\pi r^2}$

Min: $S = \pi r^2 + 2\pi r h$

$S = \pi r^2 + 2\pi r \left(\dfrac{2000}{\pi r^2}\right)$

$S = \pi r^2 + \dfrac{4000}{r}$.

$S' = 2\pi r - \dfrac{4000}{r^2} = 0$ (Mult. by r^2)

$2\pi r^3 - 4000 = 0$

$2\pi r^3 = 4000$

$r^3 = \dfrac{4000}{2\pi}$

$r = \sqrt[3]{\dfrac{4000}{2\pi}} \approx \boxed{8.6\,cm}$

$h = \dfrac{2000}{\pi (8.6)^2} = \boxed{8.6}$!

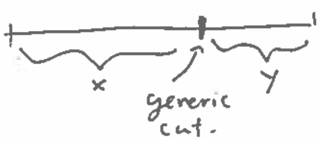
Min? $S'' = 2\pi + \dfrac{8000}{r^3}$

pos pos.

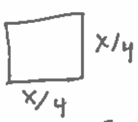
$\Rightarrow S'' > 0 \Rightarrow$ concave up

$\Rightarrow$ it's a min!

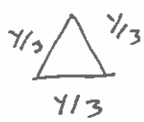
5. A piece of wire 10 m long is cut into two pieces. One piece is bent into a square, and the other is bent into an equilateral triangle. How should the wire be cut so that the total area enclosed by both figures is (a) minimum? (b) maximum?

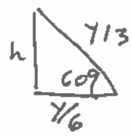


$x + y = 10$



Area = $\dfrac{x^2}{16}$





$\dfrac{h}{y/6} = \tan(60°) = \sqrt{3}$

$h = \dfrac{\sqrt{3}}{6} \cdot y$

$A = \frac{1}{2} b \cdot h$

$= \frac{1}{2}\left(\frac{y}{3}\right)\left(\frac{\sqrt{3}}{6} \cdot y\right)$

Area $= \dfrac{x^2}{16} + \dfrac{1}{2}\cdot\dfrac{1}{3}\cdot\dfrac{\sqrt{3}}{6}\cdot y^2$

$A = \dfrac{1}{16}x^2 + \dfrac{\sqrt{3}}{36}(10-x)^2$.

$A' = \frac{1}{8}x + \frac{\sqrt{3}}{18}(10-x)(-1) = 0$

$\frac{1}{8}x = (10-x)\cdot\frac{\sqrt{3}}{18}$

$\left(\frac{1}{8} + \frac{\sqrt{3}}{18}\right)x = \frac{10\sqrt{3}}{18}$

$x = \left(\frac{10\sqrt{3}}{18}\right)\cdot\dfrac{1}{\left(\frac{1}{8}+\frac{\sqrt{3}}{18}\right)}$

$x \approx 4.35$.

$A'' = \frac{1}{8} + \frac{\sqrt{3}}{18}(-1)(-1) = \frac{1}{8} + \frac{\sqrt{3}}{18} > 0$

So, A concave up at $x = 4.35$. Hence, 4.35 m is a min.

For Max, check endpoints: $x = 0$ or $x = 10$

all triangle · all square.

$A(0) = \frac{\sqrt{3}}{36}(10)^2 = 4.8\,m^2$, $A(10) = \frac{100}{16} = 6.25\,m^2$

So, $x = 10$ m (all square) gives max. area.

Answers: 1) 128. 2) 50 and −50. 3) 40 × 40 × 20. 4) r = 8.6cm, h = 8.6cm. 5a) Cut piece to be 4.35m b) cut one piece to be ~~[illegible]~~ 10 m.