Exam 2 Solutions

Name: ____________________________

- Show as much work as possible, even if you can't answer the problem entirely. This allows me to give you partial credit.
- Spend time wisely: read through the test first and solve the ones you know how to do.
- There are a total of 40 points on this exam.

True/False

Directions: Indicate whether the statement is always true or sometimes false. **Explain your reasoning**. [2 pts each]

1. ______ All solutions to an autonomous differential equation must converge to a stable equilibrium.

   False. Solutions may diverge and never approach any equilibrium.

2. ______ The state variable y is the prey population in the predator-prey model $\begin{cases} \dfrac{dx}{dt} = 8x - xy \\ \dfrac{dy}{dt} = xy - y \end{cases}$.

   False. The y-population benefits from interactions because of the $+xy$ term. Hence, y must be the predator.

Short Answer

Directions: Answer each question with a short response (preferably one to two sentences or a quick calculation).

3. [3 pts] Can the function $y(t) = t^2$ be a solution to the differential equation $\dfrac{dy}{dt} = 3y$?

   No way! $\frac{dy}{dt} = 2t$, while $3y = 3t^2$. These are totally different, so it is not a solution.

4. [3 pts] A population of foxes living in the Deschutes National Forest grows at a rate of $f(t)$ thousand foxes per year, where $f(t)$ is a function with the property that $\int_0^\infty f(t)\,dt = 6$. At their current population of five thousand, will their population ever reach ten thousand?

   They will eventually reach 10,000 since their total change in the long run exceeds the needed 5,000.

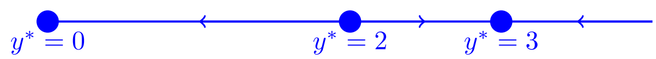
Free Response

Directions: answer the following problems. Be sure to show your work.

5. The number B of yeast bacteria (measured in billions) fermenting in a large jug of Kombucha can be modeled by the differential equation

$$\frac{dB}{dt} = B(2 - B)(B - 3).$$

(a) [4 pts] Draw a phase-line diagram for the differential equation.



(b) [2 pts] Identify the stable and unstable bacteria equilibria for this model.
Both 0 and 3 are stable. 2 is unstable.

(c) [2 pts] How many bacteria need to be alive in order for the bacteria to stay alive?
There needs to be at least 2 billion bacteria, otherwise their population will decline to 0.

6. [4 pts] Suppose the function $M(x) = \dfrac{x}{2 + x^2}$ models the density of buffalo (in thousands per kilometer) x kilometers along the river. The river is 3800 km long in total. How many buffalo live along the first 500 kilometer segment of the river?

$$\int_0^{500} \frac{x}{2 + x^2}\, dx = \frac{1}{2}\ln(2 + 500^2) - \frac{1}{2}\ln(2) \approx 5.868$$

so there are about 5,868 buffalo.

7. [5 pts] The number of penguins in Antarctica follows the differential equation $\frac{dQ}{dt} = (2t+1) \cdot Q$. Use Euler's method with $\Delta t = 0.3$ and initial condition $Q(0) = 2$ to estimate $Q(0.6)$.

t	Q'	$\hat{Q}$
0	2	2.6
0.3	4.16	3.848

So $\hat{Q}(0.6) = 3.848$.

8. [5 pts] Solve the differential equation in problem 7 with the same initial condition $Q(0) = 2$.

$Q(t) = 2e^{t+t^2}$

9. Consider two populations, A and B, of animals. The per capita growth rate of population A is given by $1.12 - 4B$, while the per capita growth rate of poplation B is given by $0.94 - 2A$.

(a) [4 pts] Write down the system of differential equations for these populations.

$$\frac{dA}{dt} = 1.12A - 4AB$$
$$\frac{dB}{dt} = 0.94B - 2AB$$

(b) [2 pts] If there are none of population A's animals, will population B grow or die off? They will grow; $\frac{dB}{dt} = 0.94B$ describes a growing population.

(c) [2 pts] Interpret how these two species interact with each other. Both species grow like usual populations, but both are negatively influenced by the presence of the other. This could be an example of two species competing for the same resources.