Written Assignment 6

Due Monday, November 14th

Disclaimer: Some of the later problems in this assignment are subject to change after Wednesday's lecture, depending on how much material is covered in class.

For extra practice, do (but don't turn in) the odd problems in the book nearby the assigned problems.

1. (1.9 #14, 16) Suppose the volume in the lungs is V, the volume breathed in and out is W, and the ambient concentration is γ mmol/L. For the given parameter values and initial condition, find the following:

   (i) The amount of chemical in the lungs before breathing

   (ii) The amount of chemical breathed out

   (iii) The amount of chemical in the lungs after breathing out

   (iv) The amount of chemical breathed in

   (v) The amount of chemical in the lungs after breathing in

   (vi) The concentration of chemical in the lungs after breathing in.

   Also, answer the following: does your answer for (vi) agree with what you get by using the general lungs discrete dynamical system for c_1?

   (a) $V = 1.0$ L, $W = 0.1$ L, $\gamma = 8.0$ mmol/L, $c_0 = 4.0$ mmol/L.

   (b) $V = 10.0$ L, $W = 0.2$ L, $\gamma = 1.0$ mmol/L, $c_0 = 9.0$ mmol/L.

2. (1.9, #18, 20)

   (a) With the numbers from problem 1a, write down the discrete dynamical system for the concentration c_t, and make a cobweb diagram with at least three steps.

   (b) Do the same with problem 1b.

3. (1.9, #22, 24)

   (a) With the setup of problem 1a, find the equilibrium concentration in the lungs, and verify it agrees with γ.

   (b) Do the same for problem 1b.

4. (1.10, # 16, 18) Identify stable and unstable equilibria on the graphs of the given updating functions (see the book for the graphs).

   (a) The graph in problem 16.

   (b) The graph in problem 18.

5. (1.10, # 20, 22) Find and graph the updating functions for the following cases of the selection model (equation 1.10.7 in the book). Cobweb starting from $p_0 = 0.1$ and $p_0 = 0.9$. Which equilibria are stable?

   (a) $s = 1.8$, $r = 0.8$

   (b) $s = 1.8$, $r = 1.8$.

6. (1.11, #1 - 4) In the following circumstances, compute $\hat{V}_t$ and V_{t+1} and state whether the heart will beat.

   (a) $V_c = 20.0$ mV, $u = 10.0$ mV, $c = 0.5$, $V_t = 30.0$ mV.

   (b) $V_c = 20.0$ mV, $u = 10.0$ mV, $c = 0.6$, $V_t = 30.0$ mV.

   (c) $V_c = 20.0$ mV, $u = 10.0$ mV, $c = 0.7$, $V_t = 30.0$ mV.

   (d) $V_c = 20.0$ mV, $u = 10.0$ mV, $c = 0.8$, $V_t = 30.0$ mV.

7. (Really fun extra credit!) In this bonus problem, you will again consider a zombie outbreak. This time, however, we model the dynamics between the zombie population and the alive human population. Let z_t be the zombie population at time t, and let h_t be the human population at time t.

   Assume that the human population, due to being constantly under a zombie threat, has stopped reproducing. Furthermore, assume that each zombie can both (1) turn two people into zombies each day and (2) slay one human a day (leading to a total of three casualties for the humans per day for each zombie). We then get

$$\begin{aligned} h_{t+1} &= h_t - 3z_t \\ z_{t+1} &= 2z_t \end{aligned}$$

   This is known as a *coupled* discrete dynamical system, since the population variables appear in some of the other equations.

   (a) Follow the derivation given in class to find the DDS for the fraction p_t of humans in the total population of humans and zombies.

   (b) Find the equilibria for the fraction of the human population. Do all of your answers make sense in context?

   (c) Based on reasoning about what's going on with this zombie outbreak, when should the model not make sense anymore? (This is called a breakdown point of the model.)

   (d) Explain how the breakdown of the model appears in the cobweb diagram.