Worksheet 1

Name: ______________________________

Directions

- This counts as your "attendance" for the day. You must give this to me today to get credit for it.
- You may leave if you finish the packet.
- You may work in groups, and you can ask me for assistance.
- I grade this worksheet based on completion, not accuracy; however, you should strive for completely correct answers in order to make sure you understand the material.

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1. Evaluate these limits using the "evaluation" strategy.

(a) $\lim_{x \to 1} \frac{x^2 + 2}{7 - x}$

$= \frac{1+2}{7-1} = \frac{3}{6} = \frac{1}{2}$

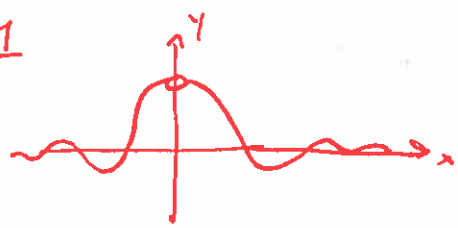
(b) $\lim_{x \to \pi} \cos(x)$ $= \cos(\pi) = -1$

2. Evaluate these limits using the "estimation" strategy.

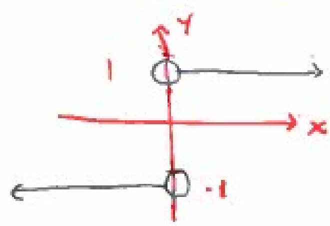
(a) $\lim_{x \to 0} \frac{\sin(x)}{x}$ (be sure to be in radians)

$= 1$

Graph:



(b) $\lim_{x \to 0^-} \frac{|x|}{x}$ $= -1$



3. Find the limits below by simplifying the expression first. Double check your answer by estimating the limit numerically.

(a) $\lim_{x \to 2} \frac{x^2 - 4}{x - 2} = \lim_{x \to 2} \frac{\cancel{(x-2)}(x+2)}{\cancel{(x-2)}}$

$= \lim_{x \to 2} (x+2)$

$= 2 + 2$

$= \boxed{4}$

(b) $\lim_{h \to 0} \frac{(1 + h)^2 - 1}{h} = \lim_{h \to 0} \frac{\cancel{1} + 2h + h^2 - \cancel{1}}{h}$

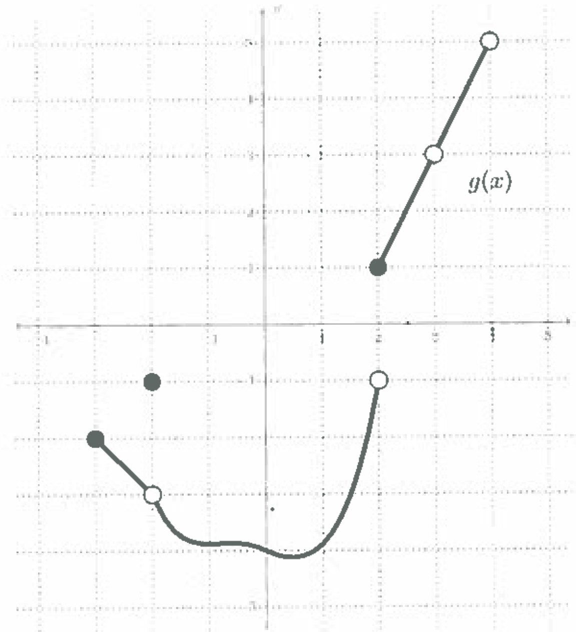
$= \lim_{h \to 0} \frac{2h + h^2}{h}$

$= \lim_{h \to 0} \frac{\cancel{h}(2+h)}{\cancel{h}}$

$= \lim_{h \to 0} (2+h)$

$= 2 + 0 = \boxed{2}$

4. Using the graph, evaluate the following limits.



(a) $\lim_{x \to 4^-} g(x)$ = 5

(b) $\lim_{x \to 2^+} g(x)$ = 1

(c) $\lim_{x \to 2^-} g(x)$ = −1

(d) $\lim_{x \to 2} g(x)$ DNE

(e) $\lim_{x \to 2.5} g(x)$ = 2

(f) $\lim_{x \to -2} g(x)$ = −3

(g) $\lim_{x \to -3^-} g(x)$ DNE (No graph to the left of −3)

5. Calculators are not always reliable. Consider the function

$$g(x) = \frac{\sqrt{x^2+4}-2}{x^2}.$$

(a) Evaluate $g(x)$ at $x = 0.1$, 0.01 0.001, and 0.00000001.

x	$g(x)$
0.1	0.2498
0.01	0.2499
0.0001	0.249999...
0.00000001	0 ⟵ ?? weird!

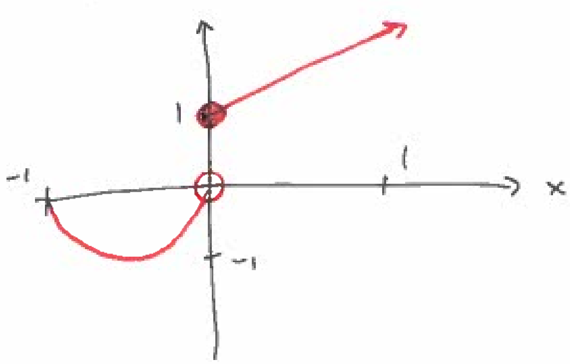
(b) Based on your first three calculations, what do you expect that $\lim_{x\to 0} g(x)$ is equal to?

0.25 !

The issue with $x = 0.00000001$ is the calculator does not have enough decimal accuracy to handle such small numbers. As such, be careful when using a calculator to evaluate limits.

6. Let $g(x) = \begin{cases} \sin(\pi x) & x < 0 \\ x+1 & x \geq 0 \end{cases}$

(a) Make a graph of $g(x)$ on the interval $[-1, 1]$



(b) Find $\lim_{x\to 0^-} g(x)$ = 0

(c) Find $\lim_{x\to 0^+} g(x)$. = 1

(d) Find $\lim_{x\to 0} g(x)$. DNE.

Worksheet 1

7. Limits as $x \to \infty$ or $x \to -\infty$ sometimes need some care. This problem illustrates a method for evaluating such limits.

(a) Consider the function $g(x) = \frac{x^2 - 10x}{13 - x^2}$. Our goal with this problem is to find $\lim_{x\to\infty} g(x) = \lim_{x\to\infty} \frac{x^2 - 10x}{13 - x^2}$.

i. Factor out the largest power of x from the top and bottom of the fraction. (You may need to recall that $x = x^2 \cdot (\frac{1}{x})$.

$$g(x) = \frac{\cancel{x^2}\left(1 - \frac{10}{x}\right)}{\cancel{x^2}\left(\frac{13}{x^2} - 1\right)}$$

ii. Cancel as many factors of x as possible:

$$g(x) = \frac{\left(1 - \frac{10}{x}\right)}{\left(\frac{13}{x^2} - 1\right)}$$

iii. Evaluate the limit $\lim_{x\to\infty} g(x)$ by remembering some of the "important limits."

$$\lim_{x\to\infty} \frac{\left(1 - \frac{10}{x}\right)}{\left(\frac{13}{x^2} - 1\right)} = \frac{1 - \underbrace{\left(\lim_{x\to\infty} \frac{10}{x}\right)}_{=0}}{\underbrace{\left(\lim_{x\to\infty} \frac{13}{x^2}\right)}_{=0} - 1} = \frac{1-0}{0-1} = \frac{1}{-1} = \boxed{-1}$$

(b) Apply the same strategy to find the following limit.

$$\lim_{x\to\infty} \frac{x^3}{1 + x + x^2}$$

$$\frac{x^3}{x^2\left(\frac{1}{x^2} + \frac{1}{x} + 1\right)} = \frac{x}{\left(\frac{1}{x^2} + \frac{1}{x} + 1\right)}$$

So,

$$\lim_{x\to\infty} \frac{x}{\left(\frac{1}{x^2} + \frac{1}{x} + 1\right)} = \frac{\left(\lim_{x\to\infty} x\right)}{\left(\lim_{x\to\infty} \frac{1}{x^2}\right) + \left(\lim_{x\to\infty} \frac{1}{x}\right) + 1}$$

$$= \frac{\infty}{0 + 0 + 1} = \boxed{\infty}$$