Quiz 5 Solutions

Name: ________________________

You will have 20 minutes ∘ Calculators are allowed ∘ Show all work for credit ∘ Don't cheat ∘ attempts at a problem may count for partial credit. ∘ If you get stuck, show as much work as possible.

1. Suppose a bacteria colony grows with a growth rate of $\dfrac{1}{1+t}$ bacteria/hour.

   (a) [3 pts] Calculate the total change in the long run for this colony (or show it is divergent).

$$\int_0^\infty \frac{1}{1+t}\,dt = \ln(1+t)\Big|_0^\infty = \lim_{t\to\infty} \ln(1+t) - \ln(1) = \infty.$$

   (b) [1 pts] Will this colony be able to keep growing this way forever? (Hint: Interpret your answer from part (a).) Since the integral diverges, this means that the bacteria colony will eventually get too big for its environment.

2. For the two differential equations below, compute one step of Euler's method with $\Delta t = 0.5$ to estimate $\hat{y}(0.5)$. In both parts, use the initial condition $y(0) = 1$.

   (a) [2 pts] $\dfrac{dy}{dt} = \cos(t) + 1$

$$\hat{y}(0.5) = 1 + (2)(0.5) = 2.$$

(b) [2 pts] $\dfrac{dy}{dt} = 2^y$

$$\hat{y}(0.5) = 1 + 2^1(0.5) = 1.5.$$

3. [2 pts] Translate the following into a differential equation: the number y of yeast bacteria grows at a rate proportional to the square of the number of yeast.

$$\frac{dy}{dt} = \alpha y^2.$$

4. [3 pts] Consider the differential equation

$$\frac{dy}{dt} = -y^2.$$

Show that $y(t) = \dfrac{1}{t+5}$ is a solution to the differential equation. First,

$$\frac{dy}{dt} = -(t+5)^{-2}.$$

On the other hand,

$$-y^2 = -\left(\frac{1}{t+5}\right)^2 = -(t+5)^2.$$

These are equal, so it's a solution.

5. [2 pts] Suppose that $y(t)$ is a solution to the equation

$$\frac{dy}{dt} = \frac{y}{1+y}.$$

When $y = 3$, is $y(t)$ increasing, decreasing, or remaining the same?

$$\frac{dy}{dt} = \frac{3}{1+3} = \frac{3}{4} > 0,$$

so $y(t)$ is increasing.