Worksheet 3

Math 251, Summer 2017

Name: Key

Directions

- This counts as your "attendance" for the day. You must give this to me today to get credit for it.
- You may leave if you finish the packet.
- You may work in groups, and you can ask me for assistance.
- I grade this worksheet based on completion, not accuracy; however, you should strive for completely correct answers in order to make sure you understand the material.

1. Find the derivatives of the functions shown below.

(a) $f(x) = 186.5$

$f'(x) = 0$

(Derivative of constants is always 0!)

(e) $f(x) = e^x + x$

$f'(x) = e^x + 1$

(b) $g(t) = 2 - \frac{2}{3}t$

$g'(t) = -\frac{2}{3}$

(f) $g(t) = \sqrt[4]{t} - 13e^t - \cos(t)$

$\sqrt[4]{t} = t^{1/4}$

$(\cos(t))' = -\sin(t)$

$g'(t) = \frac{1}{4}t^{(\frac{1}{4}-1)} - 13e^t + \sin(t)$

$g'(t) = \frac{1}{4}t^{-3/4} - 13e^t + \sin(t)$

(c) $r(x) = \frac{1}{x} + \sin(x)$

$\frac{1}{x} = x^{-1}$ ← power rule

$r'(x) = (-1)x^{-2} + \cos(x)$

(g) $h(t) = -16t^2 + 20t + 100$

$h'(t) = -32t + 20$

(d) $\ell(s) = 2s^3 + s$

$s = s^1$, power rule

$\ell'(s) = 2(3)s^2 + 1 \cdot s^0$

$= 6s^2 + 1.$

(h) $f(t) = \frac{1}{2}t^6 - 3t^4 + t$

$f'(t) = 3t^5 - 12t^3 + 1$

2. Find the equation of the tangent line to $\sqrt{x}$ at $x = 9$.

$x^{1/2}$

$f'(x) = \frac{1}{2} x^{-1/2}$

$f'(9) = \frac{1}{2} \frac{1}{\sqrt{9}} = \frac{1}{6}$

$\left(x^{-1/2} = \frac{1}{x^{1/2}} = \frac{1}{\sqrt{x}}\right)$

$f(9) = \sqrt{9} = 3$

Tangent Line:

$\underline{y = mx + b}$

$m = \frac{1}{6}.$

$3 = \frac{1}{6} \cdot 9 + b$

$3 = \frac{3}{2} + b$

$3 = 1.5 + b$

$b = 3 - 1.5 = 1.5$

$\boxed{y = \frac{1}{6}x + 1.5}$

3. Find the $\boxed{\text{slope of the tangent line}}$ to $f(x) = \frac{1}{\sqrt{x}}$ at $x = 2$. Make a sketch of $f(x)$ and its tangent line.

$f'(2) = ?$

$f(x) = x^{-1/2}$

$f'(x) = (-\frac{1}{2}) x^{(-\frac{1}{2}-1)}$

$= -\frac{1}{2} x^{-3/2}$

$f'(2) = -\frac{1}{2}(2)^{-3/2}$

$= \boxed{-1.414}$

4. Derivatives don't always work the way you want, especially with products and quotients. Consider $f(x) = (x-2)(2x+3)$.

(a) Find $f'(x)$ by first distributing.

$f(x) = 2x^2 + 3x - 4x - 6$

$f(x) = 2x^2 - x - 6$

$\boxed{f'(x) = 4x - 1}$

(b) Is this the same as doing $\underline{(x-2)' \times (2x+3)'}$?

$= 1 \times 2 \ldots = 3\ldots$ nope, $3 \neq 4x - 1 \ldots$

(c) Based on this, what do you think of the validity of the "formula" $\frac{d}{dx}(f(x)g(x)) = f'(x)g'(x)$?

Seems very wrong!

5. The derivative of a function is *also* a function, so there's no reason why we couldn't take the derivative *again*. We call this (very creatively) the *second derivative*. That is, given $f(x)$, we calculate $(f')'$, the derivative of the derivative. We usually write $f''(x)$ for this instead. Calculate $f''(x)$ for the following functions.

(a) $f(x) = x^2$.

$f'(x) = 2x$

$\boxed{f''(x) = 2}$

(b) $f(x) = \frac{1}{x}$. $= x^{-1}$

$f'(x) = (-1)x^{-2}$

$f''(x) = (-1)(-2)x^{-3}$

$\boxed{f''(x) = \frac{2}{x^3}}$

6. Optimize (find max's and min's) of the functions below on the given intervals using the method outlined in class.

(a) $f(x) = -4x^2 - 2x + 3$, on $[-2, 0]$.

① Crit pts: $f'(x) = -8x - 2 = 0$

$-8x = 2$

$x = -\frac{2}{8}$

$x = -\frac{1}{4}$

Endpts: $-2, 0$.

$f(0) = 3$

$f(-2) = -4(-2)^2 - 2(-2) + 3 = -9$

$-16 \quad +4 \quad +3$

$f(-\frac{1}{4}) = \frac{13}{4} = 3.25$

Max @ $x = -\frac{1}{4}$ of $y = 3.25$

Min @ $x = -2$, of $y = -9$

(b) $g(x) = e^x - 2x$ on $[-1, 1]$

① Crit pts:

$g'(x) = e^x - 2 = 0$

$e^x = 2$

$x = \ln(2) = 0.693$.

② $g(-1) = e^{-1} - 2(-1) = 2.37$

$g(1) = e^1 - 2 = 0.71828$

$g(\ln(2)) = e^{\ln(2)} - 2\ln(2)$

$= 2 - 2\ln(2) = 0.614$

Max @ $x = -1$, $y = 2.37$.

Min @ $x = \ln(2)$, $y = 0.614$.