HW 5

Tuesday, February 12, 2019 9:49 AM

$\boxed{1}$ C = concentration of toxin.

$$\frac{dc}{dt} = 50e^{-2t}$$

$$\text{Change in long run} = \int_0^\infty 50e^{-2t}\,dt$$

$$= -25e^{-2t}\Big|_0^\infty$$

$$= \underbrace{\lim_{N\to\infty}\left(-25e^{-2N}\right)}_{=0} - \left(-25e^{0}\right)$$

$$= 25 \ \mu mol/L.$$

Since $(10+25)\,\mu mol/L = 35\,\mu mol/L > 30\,\mu mol/L$, the concentration will exceed $30\,\mu mol/L$ at some point in time.
Hence this cell dies. Also, because this integral converges, this model can be followed indefinitely.

$\boxed{2}$ $\frac{dP}{dt} = \frac{1000}{(2+3t)^{1.5}}$, $P(0) = 1000.$

$$\text{Change in Pop} = \int_0^\infty \frac{1000}{(2+3t)^{1.5}}\,dt$$

$$= 1000 \cdot \frac{1}{3} \cdot \frac{1}{(-.5)} (2+3t)^{-0.5} \Big|_0^\infty$$

$$= \frac{-1000}{1.5}\left[\underbrace{\lim_{N\to\infty} \frac{1}{(2+3t)^{.5}}}_{=0} - \frac{1}{2^{0.5}}\right]$$

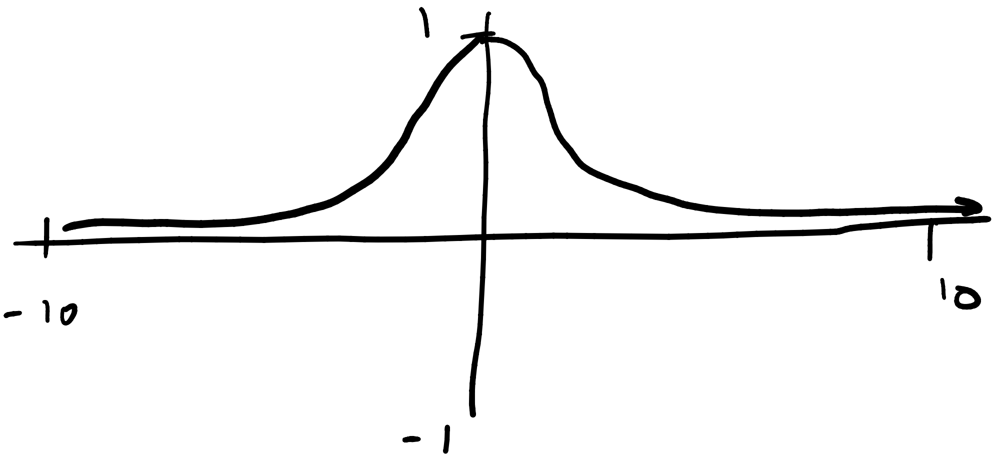
$$= \frac{1000}{1.5\sqrt{2}} \approx 471.4.$$

Since this integral converges, this population's growth can be maintained indefinitely.

The long-run change is 471.4, meaning the population will never exceed $1000 + 471.4 = 1471.4$ individuals.

Thus it cannot reach a size of 2000.

$\boxed{4}$ ⓐ $\frac{1}{1+x^2}$:



For large x, $\frac{1}{1+x^2} \approx \frac{1}{x^2}$ because $+1$ pales in comparison to a large number.

So, on the right tail, this is like $\int_1^\infty \frac{1}{x^2}\,dx$, which converges.

The left tail also converges by symmetry.

(b) $\int \frac{1}{1+x^2}\,dx$, $\quad x = \tan(u).$

$dx = \sec^2 u\,du$

$\rightarrow \int \frac{1}{1+\tan^2 u} \cdot \sec^2 u\,du.$

trig identity: $1+\tan^2 u = \sec^2 u$:

$\int \frac{1}{\cancel{\sec^2 u}} \cdot \cancel{\sec^2 u}\,du$

$= \int 1\,du = u + C$

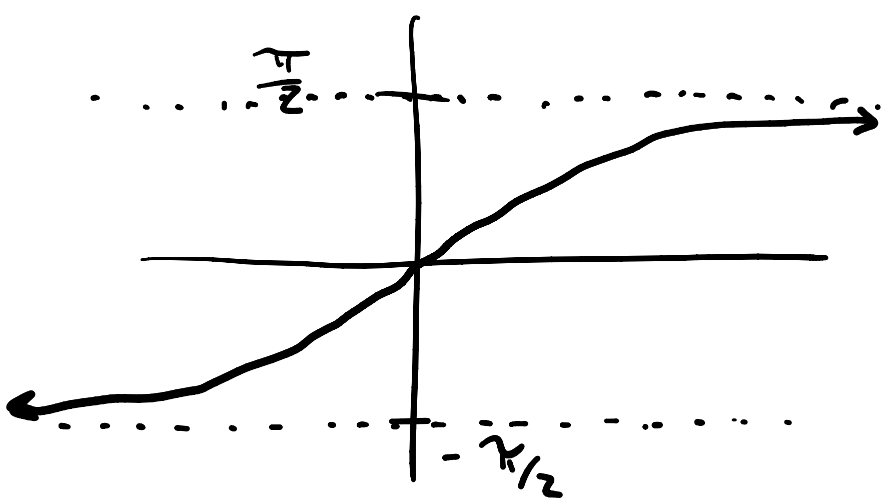
$= \arctan(x) + C.$

$x = \tan u$

$\arctan x = \cancel{\arctan}(\tan u)$

$= u.$

(c) Graph $\arctan(x)$.



like a $\tan(x)$ graph on its side.

$1 - \pi/2$

(d) Right asymptote $= \frac{\pi}{2} = \lim_{N \to \infty} \arctan(N)$

left Asymptote $= -\frac{\pi}{2} = \lim_{N \to -\infty} \arctan(N)$

$$\Rightarrow \int_{-\infty}^{\infty} \frac{1}{1+x^2}\,dx = \lim_{N \to \infty} \arctan(N) - \lim_{N \to -\infty} \arctan(N)$$

$$= \left(\frac{\pi}{2}\right) - \left(-\frac{\pi}{2}\right) = \boxed{\pi}$$

WILD.