## 5.1 Differential equations

1. A simplistic population model. “The more people there are, the more quickly the population grows.” More rigorously: The rate of population growth is proportional to the total population. Translate this into a differential equation.

2. Newton’s Law of Cooling. Consider the following: The rate at which heat is lost is proportional to the difference between the temperature an ambient temperature of 50 degrees.

   (a) Translate this into a differential equation.

   (b) If the temperature of the object after some time is 60 degrees, does your diffy-Q predict it is cooling or heating up?

3. Diffusion across a membrane. Translate the following into a diffy-Q: The rate of change of the concentration is equal to the difference between the rate at which a chemical enters and which it leaves.

**Tips for Euler's Method**:

- Euler's method is doing the simplest possible thing: add (rate)$\times$(time) to the current value to get the next one.
- Sometimes organizing your work into a table will make the problem easier to handle.

4. Consider the differential equation
$$\frac{df}{dt} = 2t + 5, \quad f(0) = 1$$
   (a) Use Euler's method with $\Delta t = 1$ and two steps to estimate the value of $f(2)$.

   (b) Use Euler's method with $\Delta t = 0.5$ and four steps to estimate the value of $f(2)$.

   (c) Using integration, find the exact value of $f(2)$. How do your answers compare?

5. Consider the diffy-Q
$$\frac{dy}{dx} = y^2, \quad y(0) = 0.5.$$
   Use Euler's method with $\Delta t = 0.4$ to estimate $y(1.2)$.