Worksheet 11

Math 251, Summer 2017

Name: Key

I have given you the answers on the bottom of the last page. You must figure out how to solve the problem to get the correct answer.

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1. Find all of the critical points and inflection points of $f(x)$, and classify the critical points as local maxima, local minima, or neither.

   (a) $f(x) = x^4 - x^3 - 2x^2$

Crit Pts: $f'(x) = 4x^3 - 3x^2 - 4x = 0$

$x(4x^2 - 3x - 4) = 0$

$x = 0$ or $x = \dfrac{3 \pm \sqrt{9 - 4(4)(-4)}}{8}$

$x = \dfrac{3 \pm \sqrt{73}}{8}$

$x = -0.693$ or $x = 1.443$

$f''(x) = 12x^2 - 6x - 4$

Inflection points: $x = \dfrac{6 \pm \sqrt{228}}{24}$

$x = 0.879$, $x = -0.379$

★Concavity definitely changes b/c of distinct roots

Classify: 1st Der. Test:

$- - - -$ -0.69 $+ + + +$ 0 $- - - - -$ 1.4 $+ + + +$

$\Rightarrow$ -0.69 local min

0 local max

1.4 local min.

   (b) $f(x) = x^5$

$f'(x) = 5x^4$

$f''(x) = 20x^3$.

Crit Pts: $x = 0$

Possible Inflection Pts: $x = 0$.

$f''(0) = 0$, so 2nd deriv. test fails.

1st deri test:

0

So 0 is neither max nor min. "Saddle"

Note: Concavity changes at 0, so 0 is an inflection point.

   (c) $f(x) = \frac{x^2 - x}{e^x}$

$$f'(x) = \frac{(2x-1)e^x - (x^2-x)e^x}{(e^x)^2} = \frac{\cancel{e^x}(2x - 1 - x^2 + x)}{(e^x)^{\cancel{2}}} = \frac{-x^2 + 3x - 1}{e^x}.$$

$$f''(x) = \frac{(-2x+3)e^x - (-x^2+3x-1)e^x}{(e^x)^2} = \frac{\cancel{e^x}(x^2 - 5x + 4)}{(e^x)^{\cancel{2}}} = \frac{x^2 - 5x + 4}{e^x}.$$

Crit Pts: $-x^2 + 3x - 1 = 0$

$x = \dfrac{-3 \pm \sqrt{9 - 4(-1)(-1)}}{-2}$

$x = \dfrac{3 \pm \sqrt{5}}{2} = 2.61, 0.38$

local max (2.61), local min (0.38)

f': $.38$ 2.61

Possible Inflection points:

$x^2 - 5x + 4 = 0$

$x = \dfrac{5 \pm \sqrt{9}}{2} = 4, 1.$

f'': $\cup$ 1 $\cap$ 4 $\cup$

So 1, 4 are definitely inflection pts

(d) $f(x) = e^{-x^2}$

$f'(x) = -2xe^{-x^2}$

$f''(x) = 4x^2e^{-x^2} - 2e^{-x^2} = e^{-x^2}(4x^2-2)$.

f'': $-\frac{1}{\sqrt{2}}$ $\quad$ $1/\sqrt{2}$

Since concavity changes, both are inflection points

Crit pts $x=0$

local Max

f': 0

Inflection Pts: $4x^2-2=0$

$x^2 = \frac{1}{2}$

$x = \pm 1/\sqrt{2}$

2. Find the intervals where $f(x) = 2x^4 - x^2$ is concave down.

$f'(x) = 8x^3 - 2x$

$f''(x) = 24x^2 - 2 = 0$

$x^2 = \frac{1}{12}$

$x = \pm\sqrt{\frac{1}{12}} = \pm\frac{1}{\sqrt{12}}$

$= 0.288$

-0.288 $\quad$ 0.288

Concave down on

$[-2.88, 2.88]$.

3. Optimize the function $f(x) = \ln(x)x^2$ on the interval $[0.2, \infty)$.

$f'(x) = \frac{1}{x}x^2 + \ln(x)\cdot 2x$

$= x + 2x\ln(x) = 0$

$= x(1 + 2\ln(x)) = 0$

$\cancel{x=0}$ or $1 + 2\ln(x) = 0$

not in our interval!

$\ln(x) = -\frac{1}{2}$

$x = e^{-1/2}$

$f(0.2) = -0.064$

$f(e^{-1/2}) = \ln(e^{-1/2})(e^{-1/2})^2$

$= -\frac{1}{2}e^{-1} = -0.184$

$\lim_{x\to\infty}(\ln(x) \cdot x^2) = \infty$ (both function go to ∞, so their product does too.

So, global min is at $(e^{-1/2})$, no global max.

4. Optimize the function $g(x) = \frac{x^2-1}{x+4}$ on the interval $[-1, 2]$.

$g'(x) = \frac{x^2+8x+1}{(x+4)^2} = 0 \rightsquigarrow x = -4-\sqrt{15}$ or $-4+\sqrt{15}$

$\cancel{-7.87}$ or -0.127

outside our interval.

$g(-1) = 0$

$g(2) = \frac{3}{8}$ ← Global Max

$g(-0.127) = -0.25$ ← Global Min

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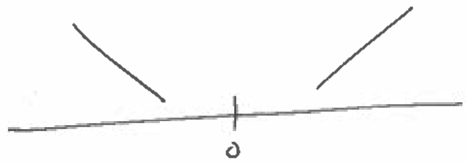
5. The following functions all have $f'(0) = 0$ and $f''(0) = 0$, which means the second derivative test will fail for $x = 0$. Decide if 0 is a local max, local min, or neither.

(a) $f(x) = x^4$

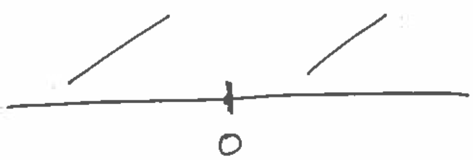
$f'(x) = 4x^3$



→ 0 a local min.

(b) $f(x) = x^3$

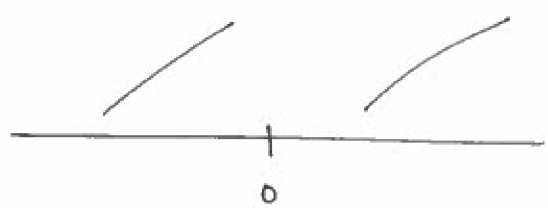
$f'(x) = 3x^2$



→ 0 is a saddle point (neither max nor min)

(c) $f(x) = \tan(x) - x$

$f'(x) = \sec^2(x) - 1 = 0$



→ 0 is also a saddle point here.