

Homework 2

Due Tuesday, January 22nd

Instructions: write up solutions to all problems below. Neatness counts: be sure to follow guidelines for homework in the syllabus.

Reading Assignment: 4.2, 4.3

1. Perform two steps of Euler's method with $\Delta t = 0.3$ for the following two initial value problems. You can use the online Euler's method to check your work, but you must show the intermediate steps (the slopes and rises).

(a) $\frac{dy}{dt} = t + \sqrt{y}, \quad y(0) = 1$

(b) $\frac{dy}{dt} = \sqrt{t} + 1, \quad y(0) = 2.$

(c) $\frac{dy}{dt} = y - 1, \quad y(0) = 1.$

2. Diffusion across a cell. Recall that in class we came up with the diffusion equation for the concentration of potassium flowing into a cell:

$$\frac{dP}{dt} = k(P_0 - P),$$

where P_0 is the ambient concentration of potassium in the cell's environment. Suppose that the constant of proportionality k is 0.5 per second, and that the ambient concentration P_0 is $3 \mu\text{g/mL}$. Here, t is measured in seconds.

- (a) Using the Euler's method solver online (found on my website), construct a plot of the solution to this equation with the initial condition $P(0) = 0.2$ using a very small step size $\Delta t = 0.01$ seconds. Make the graph show the solution for 60 seconds. You should print out a copy of the graph. Don't print out the long list of data points!
 - (b) Print out another graph, this time with the initial condition $P(0) = 5 \mu\text{g/mL}$. Use the same parameters and time length as the previous problem.
 - (c) Analyze the diffusion model: what happens to the concentration of potassium in the cell in the long run? Why is that expected?
3. Chapter 4.2, # 24.
4. Chapter 4.2, # 28.