Final Exam Review Packet

Math 251, Summer 2017

Name: ____

This packet consists of problems from the whole class. Not everything on here will be on the exam, and there may be problems on the exam that do not appear on here. Nonetheless this packet will be very helpful in studying for the final.

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1. Find the following limits.

   (a) $\displaystyle\lim_{x\to\infty} \frac{x^7 + x^2 + 13x^9 - 2\sqrt{x}}{16x^9 + 47x^3 - 10000}$

   (b) $\displaystyle\lim_{x\to 0} x^2 \ln(x)$

   (c) $\displaystyle\lim_{t\to\infty} e^{-t^2}$

   (d) $\displaystyle\lim_{x\to 0} \frac{\sin(2x)}{\tan(3x)}$

2. Find $\frac{dy}{dx}$.

   (a) $y = x^x$

   (b) $y = \arcsin(x^2 - 1)$

   (c) $x^2 + 2xy + y^2 = \dfrac{x}{y}$

   (d) $y = 4x + x^2 + e^3$

3. Find all inflection points of the following functions.

   (a) $g(x) = x^4 + x - 1$

   (b) $T(x) = x^2 e^{-x}$

Find all intervals where f is increasing, decreasing, concave up, and concave down. [Bonus: use this info to construct a graph of f.]

4. (a) $f(x) = xe^x$

   (b) $g(x) = \arctan(x)$

5. A kite 100 ft above the ground moves horizontally at a speed of 8 ft/s. At what rate is the angle between the string and the horizontal decreasing when 200 ft of string has been let out?

6. A baseball diamond is a square with side 90 ft. A batter hits the ball and runs toward first base with a speed of 24 ft/s. At what rate is his distance from second base decreasing when he is halfway to first base? What about third base?

7. Find the linearization of $\sqrt[3]{x}$ near $x = 1$. Use this to approximate the value of $\sqrt[3]{1.1}$.

8. Find the linearization of $\ln(x)$ near $x = 1$, and use it to approximate the value of $\ln(1.1)$.

9. Find the dimensions of the rectangle of largest area that has its base on the x-axis and has the other two corners sitting on the parabola $y = 8 - x^2$.

10. True or false time! Explain your responses.

    (a) A function $f(x)$ can have a vertical tangent line. (Vertical means the slope comes out as $\frac{1}{0}$.)

    (b) If $f(9)$ exists and $\lim_{x\to 9} f(x) = 3$, then $f(9) = 3$.

    (c) If $f'(x) > 0$ for $x > 0$ and $f(0) = 1$, then $f(x) > 0$ for all $x > 0$.

    (d) A horizontal asymptote of $y = 3$ means that either $\lim_{x\to\infty} f(x) = 3$ or $\lim_{x\to-\infty} f(x) = 3$.

    (e) $\dfrac{d}{dx}[f(x) + g(x)] = f'(x) + g'(x)$.

    (f) An equation for the tangent line to $y = x^2$ at $(-2, 4)$ is $y = 2x(x + 2) + 4$.

    (g) If f has an absolute minimum at c, then $f'(c) = 0$.

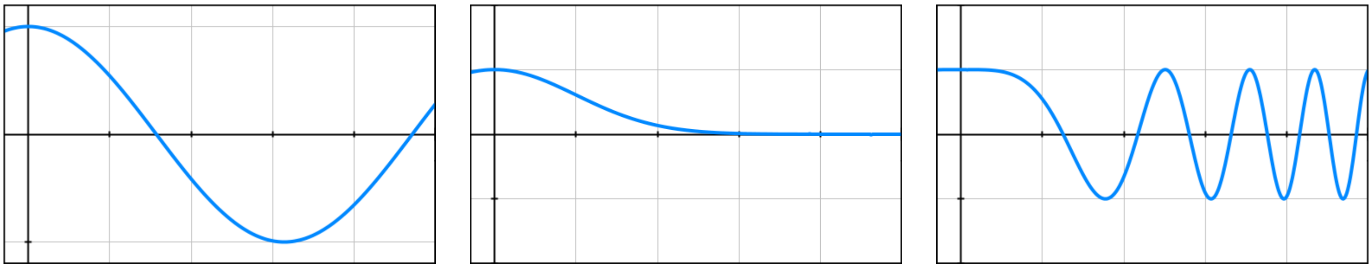
    (h) If $f''(2) = 0$, then $(2, f(2))$ is an inflection point of $f(x)$.

    (i) Two functions $f(x)$ and $g(x)$ with $f'(x) = g'(x)$ must be equal; that is, $f(x) = g(x)$.

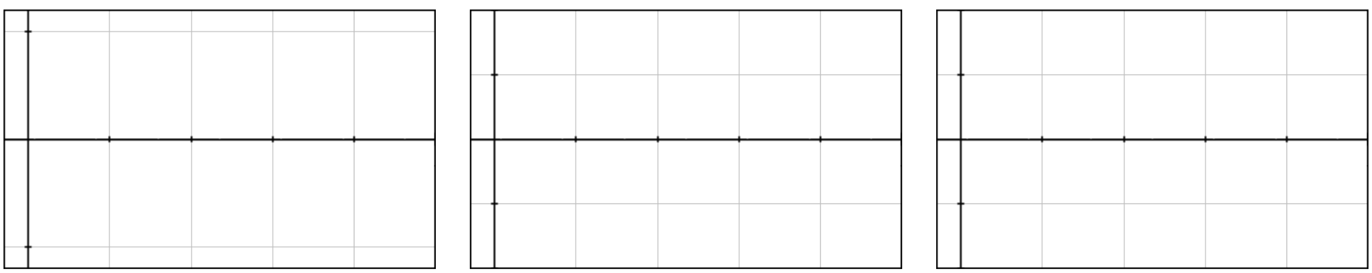
    (j) $\displaystyle\lim_{x\to 0} \frac{x}{e^x} = 1$.

11. Given below are graphs of $f'(x)$ for some function $f(x)$. Sketch both f and f'' (you can choose a point on the graph of f).

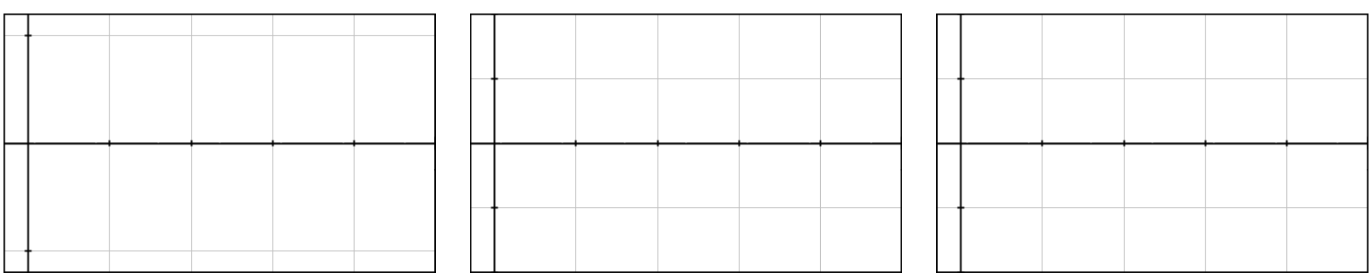
f'



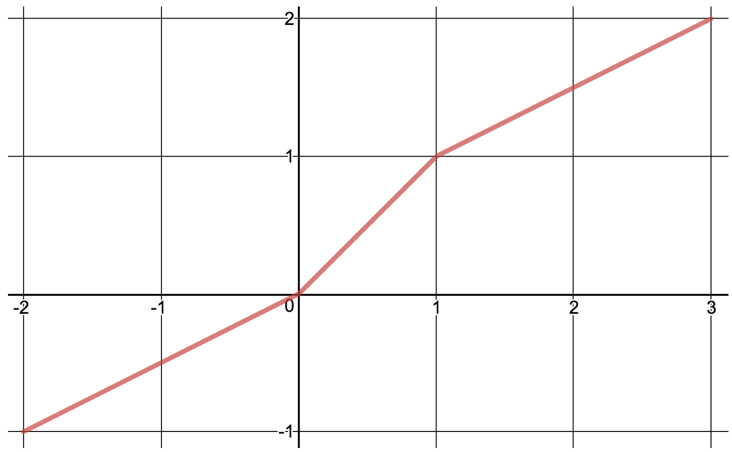
f



f''



12. Below is the graph of the function $f(x)$. Find each of the following values or state why they do not exist.



(a) $f'(1)$

(b) $\dfrac{d}{dx}\left(\dfrac{x^4}{f(x)}\right)$ at $x = 2$.

(c) $(f \circ f)'(2)$

(d) $(f^{-1})'(1.5)$.

(e) $(f^{-1})'(1)$.

(f) $(f^{-1})'(0.5)$

(g) Assuming that $g'(x) = f(x)$ (in which case we might call g the *antiderivative* of f), and that $g(2) = 3$, what is the equation of the tangent line for $g(x)$ at $x = 2$?

13. Evaluate the following limits.

(a) $\lim_{x\to\infty} \dfrac{x^2}{e^x}$

(b) $\lim_{t\to-\infty} \dfrac{x^2}{e^x}$

(c) $\lim_{x\to 0} \dfrac{\cos(x)}{1-\sin(x)}$

(d) $\lim_{x\to 0} \dfrac{x^4}{\sin(x)}$