Review Handout

1. For the systems shown below, find and graph the nullclines. Find all equilibria.

   (a) $\begin{cases} \frac{dx}{dt} = y^2 - xy + y \\ \frac{dy}{dt} = x^3 - xy^2 \end{cases}$

   (b) $\begin{cases} \frac{du}{dt} & = u \\ \frac{dv}{dt} & = v \end{cases}$

   (c) $\begin{cases} \frac{dz}{dt} & = 13 - z + y \\ \frac{dy}{dt} & = z - y \end{cases}$

2. Find the general solutions to the following differential equations.

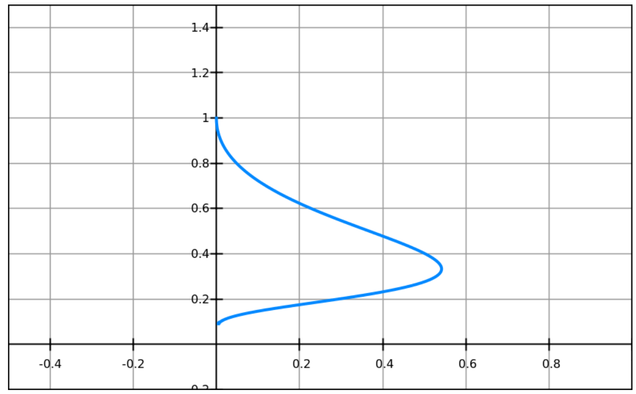
(a) $\dfrac{dx}{dt} = \dfrac{t^2}{x}$

(b) $\dfrac{dx}{dt} = \dfrac{x}{t^2}$

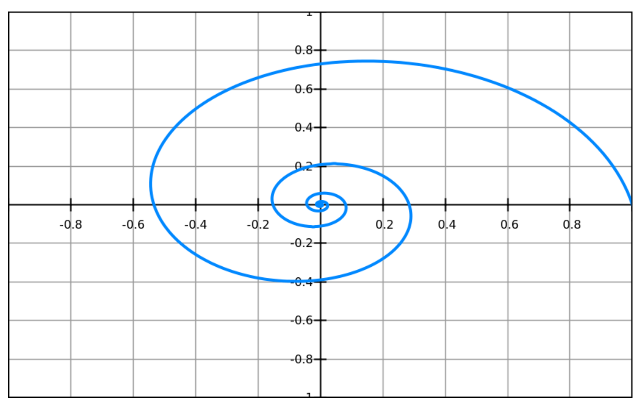
(c) $\dfrac{dx}{dt} = \dfrac{t}{x^2}$

3. The following graphs represents a trajectory in the phase-plane for a system of differential equations for $x(t)$ and $y(t)$. Sketch possible graphs of $x(t)$ and $y(t)$.

(a)



(b)



4. For the following DE, make a phase-line diagram. Classify the equilibria as stable or unstable.

   (a) $\dfrac{dx}{dt} = x(x-2)(x+7)$

   (b) $\dfrac{ds}{dt} = x^2 - 9x$

5. Evaluate the following integrals.

   (a) $\displaystyle\int \sqrt[5]{x^7}\, dx$

   (b) $\displaystyle\int xe^x\, dx$

   (c) $\displaystyle\int_{-2}^{0} \frac{14}{2x}\, dx$

   (d) $\displaystyle\int_{4}^{16} \frac{x}{1+x^2}\, dx$

6. If a population of wolves is currently at one-hundred thousand, and $\int_0^{20} f(t)\,dt = 3000$, where $f(t)$ is the growth rate in wolves per year, then how many wolves are there after 20 years?

7. For the neuron model with a constant applied current, find the nullclines of the system.

8. For the following differential equation, approximate $x(1)$ given $\Delta t = 0.5$ and $x(0) = 1$.

$$\frac{dx}{dt} = \frac{x}{1+t}$$

9. Below is a system of DE's that models the proportion of infected and susceptible people for a disease in a population.

$$\frac{dI}{dt} = \alpha IS - \mu I$$
$$\frac{dS}{dt} = -\alpha IS + \mu I$$

(a) According to this model, can people ever become immune to this disease? What kinds of disease might this apply to?

(b) Find the nullclines of this system and describe the equilibria.

(c) Describe how we might incorporate birth rates into the model.

(d) Describe how we might incorporate death from disease into the model.

(e) Describe how we might incorporate immunity into the model.

10. Find the area of the following regions.

    (a) Between the x-axis and the graph of the function $x - x^2$.

    (b) Between the graphs of e^x and $\sqrt{x}$ from $x = 0$ to $x = 1$.

11. Calculate a Riemann sum for the function $g(t) = 10 - t^2$ for the interval $[1, 3]$ with 5 subintervals. Then write down the integral this approximates, and find the value of this integral. Compare your answers.

12. Below are four differential equations, along with six functions. Identify which function is a solution to which equation (note that some functions may not be solutions to any of the DE's). [Grab some paper.]

$$\frac{dy}{dt} + y = 2$$
$$\frac{dy}{dt} = y^2$$
$$\frac{dy}{dt} = ty^2$$
$$\frac{dy}{dt} = y + t$$

$$y(t) = e^t - t - 1$$
$$y(t) = e^{-t^2}$$
$$y(t) = \frac{e^t}{1+t}$$
$$y(t) = \frac{1}{1-t}$$
$$y(t) = -\frac{2}{1+t^2}$$
$$y(t) = e^{-t} + 2$$

13. The mass density (in kg/meter) of a poorly-made construction beam seems to follow the function

$$D(x) = 12.2xe^{-4x^2},$$

where x is a distance in meters along the beam. How much does the bar weigh?

14. Use the stability theorem to classify the equilibria of the differential equation

$$\frac{dA}{dt} = (A-2)(13-A^2).$$