Quiz 13

Key

1. [5 pts] Find the global maxima and minima for $f(x) = x\ln(x)$ on the interval $0.1 \leq x \leq 2$.

   $f'(x) = \ln(x) + \frac{1}{x}x = \ln(x) + 1$. Setting $f'(x) = 0$ gives the equation $\ln(x) = -1$, so $x = e^{-1} \approx 0.368$ is the only critical point. We have

   $$f(0.1) = -0.23, \quad f(0.368) = -0.368, \quad f(2) = 1.386,$$

   so the global minimum is at the point $(0.368, -0.368)$, and the global maximum is at the point $(2, 1.386)$.

2. [5 pts] Find and classify all critical points of $g(t) = (t-1)\sqrt{t}$ as local maxima, local minima, or neither.

   $$\begin{aligned} g'(t) = \sqrt{t} + \frac{t-1}{2\sqrt{t}} &= 0 \\ \sqrt{t}\left(\sqrt{t} + \frac{t-1}{2\sqrt{t}}\right) &= 0 \cdot \sqrt{t} \\ t + \frac{t-1}{2} &= 0 \\ t &= \frac{1}{3}. \end{aligned}$$

   Note that $g'(0.1) \approx -1.1$ and $g'(1) = 1$, so $t = 1/3$ is a local minimum for g.

Bonus. [+2 EC] What theorem guarantees that the function f in problem 1 *must* have had a global maximum and minimum? Why does the theorem apply?

The extreme value theorem tells us f attains its global maximum and minimum on $[0.1, 2]$. This theorem applies because f is continuous and the given interval is closed.