

11

8/8

- (a)
- (i)  $(1.0 \text{ L})(4.0 \text{ mmol/L}) = 4.0 \text{ mmol}$
  - (ii)  $(0.1 \text{ L})(4.0 \text{ mmol/L}) = 0.4 \text{ mmol}$
  - (iii)  $(0.9 \text{ L})(4.0 \text{ mmol/L}) = 3.6 \text{ mmol}$
  - (iv)  $(0.1 \text{ L})(8.0 \text{ mmol/L}) = 0.8 \text{ mmol}$
  - (v)  $(3.6 \text{ mmol}) + (0.8 \text{ mmol}) = 4.4 \text{ mmol}$
  - (vi)  $\frac{4.4 \text{ mmol}}{1.0 \text{ L}} = 4.4 \text{ mmol/L}$

+0.5 each

$$C_1 = C_0 \left(1 - \frac{w}{v}\right) + \gamma \left(\frac{w}{v}\right) = (4.0) \left(1 - \frac{.1}{1.0}\right) + (.8) \left(\frac{.1}{1.0}\right)$$

$$= (4.0)(.9) + .8$$

$$= 4.4 \text{ mmol/L}$$

+1

Yes, they agree!

- (b)
- (i)  $(10.0 \text{ L})(9.0 \text{ mmol/L}) = 90 \text{ mmol}$
  - (ii)  $(0.2 \text{ L})(9.0 \text{ mmol/L}) = 1.8 \text{ mmol}$
  - (iii)  $90 - 1.8 = 88.2 \text{ mmol}$
  - (iv)  $(0.2 \text{ L})(1.0 \text{ mmol/L}) = 0.2 \text{ mmol}$
  - (v)  $88.2 \text{ mmol} + 0.2 \text{ mmol} = 88.4 \text{ mmol}$
  - (vi)  $\frac{88.4 \text{ mmol}}{10.0 \text{ L}} = 8.84 \text{ mmol/L}$

+0.5 each

$$C_1 = (9) \left(1 - \frac{0.2}{10}\right) + (1) \left(\frac{0.2}{10}\right)$$

$$= (9)(0.98) + 0.02$$

$$= 8.84 \text{ mmol/L}$$

+1

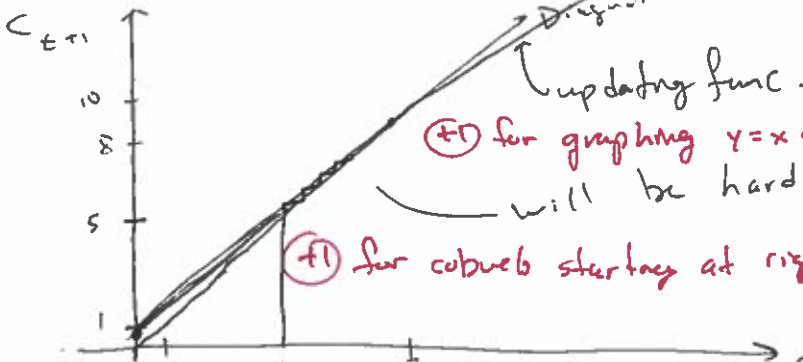
They agree!

21

8/8

(a)  $C_{t+1} = C_t \cdot \left(1 - \frac{.1}{1}\right) + 8(.1) = C_t \cdot (0.9) + 0.8$

+2



(+1) for graphing  $y=x$  and updating func.  
will be hard to see.  
(+1) for cubes starting at right place.

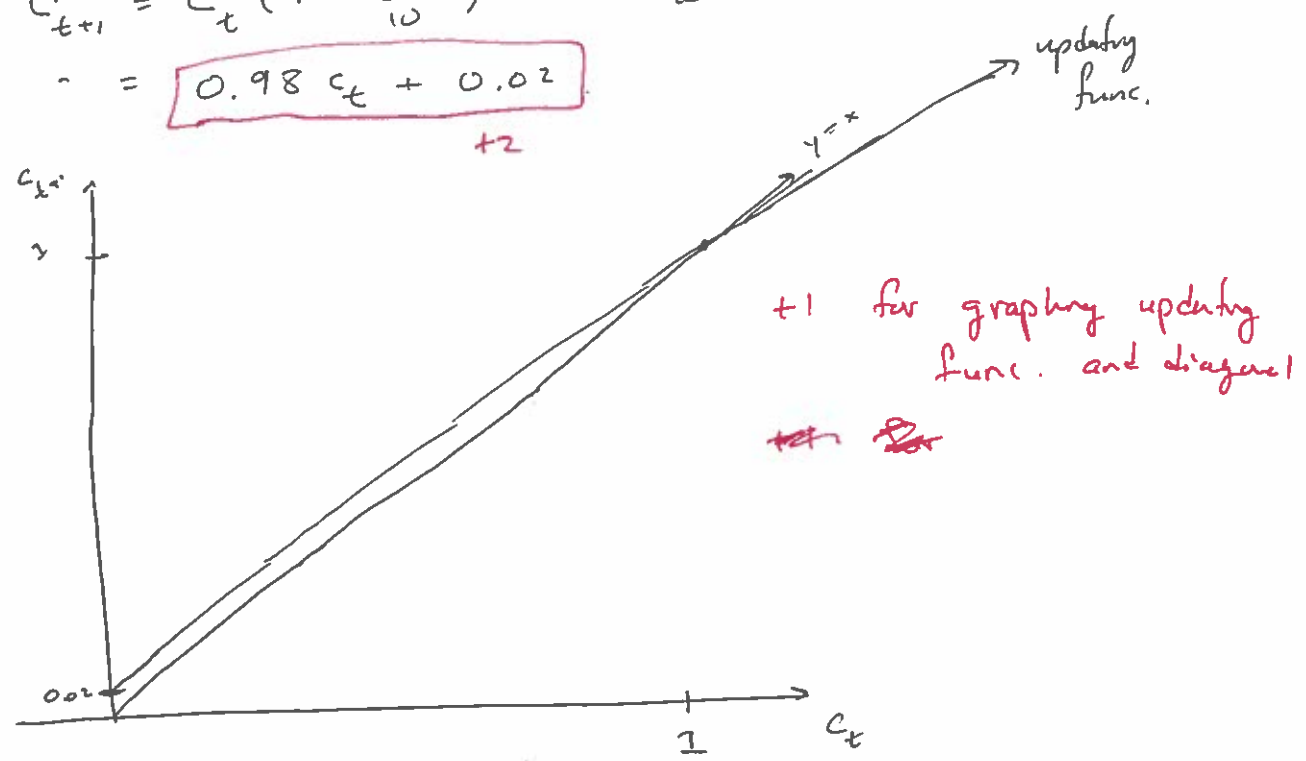
\* The graphs probably won't look great; these updating func's are hard to distinguish from the diagonal.

2b

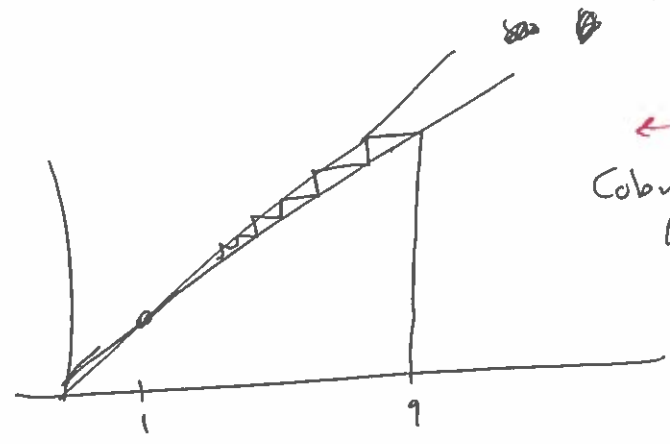
$$C_{t+1} = C_t \left(1 - \frac{0.2}{10}\right) + 1 \cdot \left(\frac{0.2}{10}\right)$$

$$= 0.98 C_t + 0.02$$

+2



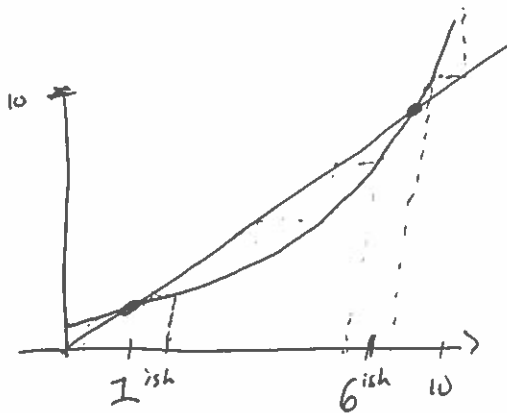
\* very hard to distinguish these lines,



+1 for having a cobweb diagram like this.  
Cobwebbing should look like this.

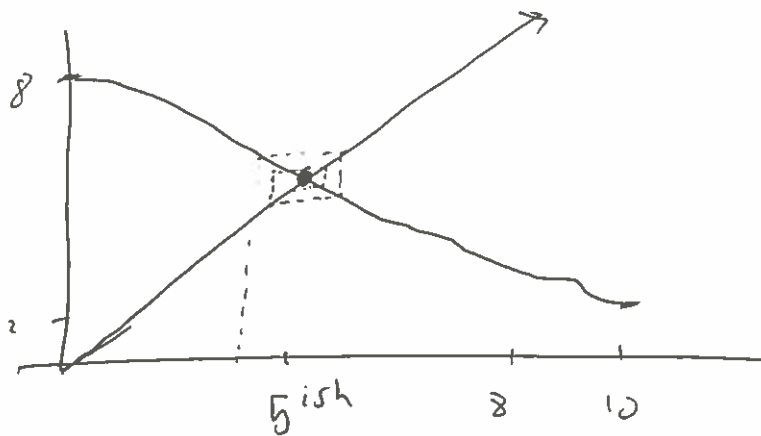
4) (a)

6/6



Equilibria are  $x=1$  and  $x=6$ .  
 $x=1$  is stable (+1) and  $x=6$  is unstable (+1).  
 check by cobwebbing.

(b)



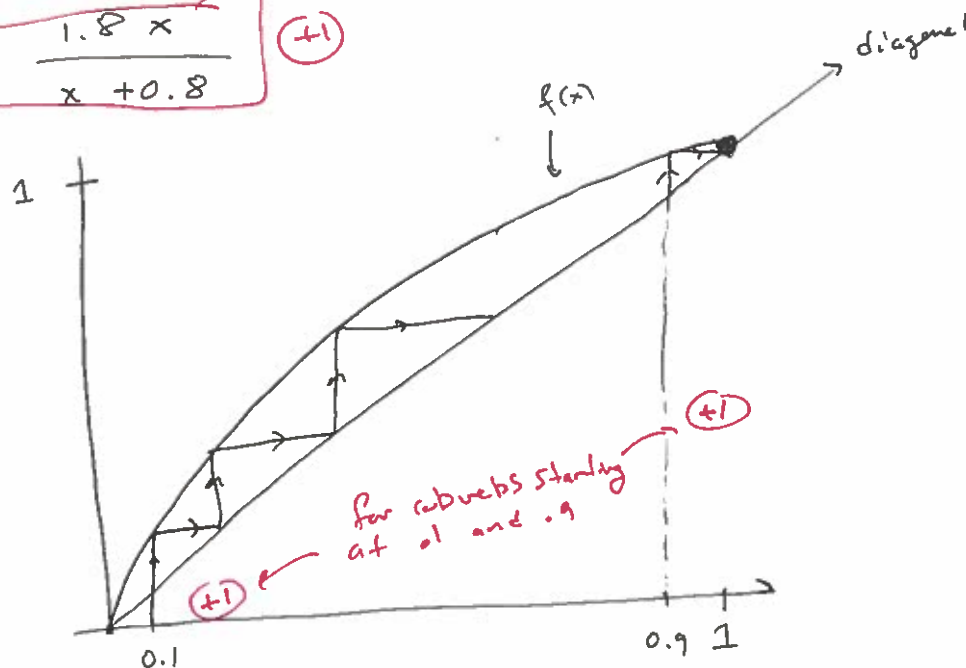
Equilibrium is  $x=5$ .

This is stable (+1), because solutions near  $x=5$  approach the equilibrium.

5 a)  $P_{t+1} = \frac{1.8 P_t}{1.8 P_t + 0.8(1-P_t)} = \frac{1.8 P_t}{P_t + 0.8}$

6/1

updating func:  
 $f(x) = \frac{1.8x}{x + 0.8}$  (+1)



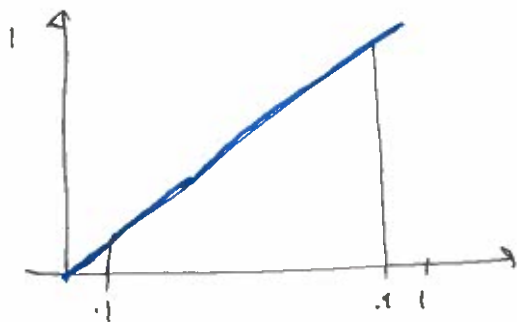
$x=0$  is unstable. (+1)

$x=1$  is stable. (+1)

b)  $P_{t+1} = \frac{1.8 P_t}{1.8 P_t + 1.8(1-P_t)} = \frac{1.8 P_t}{1.8} = P_t$

When both populations ~~are~~ have equal growth rates, the relative fraction is constant.

$f(x) = x$  is the updating function. (+1)



\* cobwebbing just stops at the point you start. \*

(+1) \* check for sensible answers; it's hard to give a right or wrong answer; be generous.

\* every real number between 0 and 1 is an equilibrium, so it doesn't make a lot of sense to ask where ...

6.  $\hat{V}_t = c \cdot V_t.$

12/12

(a)  $\hat{V}_t = 0.5 \cdot 30 \text{ mV} = 15 \text{ mV}$  (+1)

Since  $\hat{V}_t = 15 \text{ mV} \leq V_c$ , the heart will beat. (+1)

and  $V_{t+1} = \hat{V}_t + u = 15 + 10 \text{ mV} = 25 \text{ mV}.$  (+1)

(b)  $\hat{V}_t = 0.6 \cdot 30 \text{ mV} = 18 \text{ mV}.$  (+1)

Since  $\underset{\hat{V}_t}{18 \text{ mV}} \leq \underset{V_c}{20 \text{ mV}}$ , the heart will beat. (+1)

and  $V_{t+1} = \hat{V}_t + u = 18 + 10 \text{ mV} = 28 \text{ mV}.$  (+1)

(c)  $\hat{V}_t = 0.7 \cdot 30 \text{ mV} = 21 \text{ mV}.$  (+1)

since  $21 > 20 \text{ mV}$ , the heart will not beat, (+1)

and  $V_{t+1} = \hat{V}_t = 21 \text{ mV}.$  (+1)

(d)  $\hat{V}_t = 0.8 \cdot 30 = 24 \text{ mV}.$  (+1)

Since  $24 \text{ mV} > 20 \text{ mV}$ , the heart will not beat. (+1)

and  $V_{t+1} = \hat{V}_t = 24 \text{ mV}.$  (+1)

EC

(a)

6/6

$$h_{t+1} = h_t - 3z_t$$

$$z_{t+1} = 2z_t$$

$$P_t \stackrel{\text{def}}{=} \frac{h_t}{h_t + z_t}$$

$$\begin{aligned} P_{t+1} &= \frac{h_{t+1}}{h_{t+1} + z_{t+1}} = \frac{h_t - 3z_t}{h_t - 3z_t + 2z_t} \\ &= \frac{h_t - 3z_t}{h_t - z_t} \\ &= \frac{\frac{h_t}{h_t + z_t} - 3 \frac{z_t}{h_t + z_t}}{\frac{h_t}{h_t + z_t} - \frac{z_t}{h_t + z_t}} \\ &= \frac{P_t - 3(1 - P_t)}{P_t - (1 - P_t)} \end{aligned}$$

$$P_{t+1} = \frac{4P_t - 3}{2P_t - 1} \quad (+2)$$

(b)  $f(x) = \frac{4x - 3}{2x - 1}$

$f(x) = x$  :

$$\frac{4x - 3}{2x - 1} = x$$

$$4x - 3 = (2x - 1)x$$

$$4x - 3 = 2x^2 - x$$

$$0 = 2x^2 - 5x + 3$$

$x = 1$  (+1) and

Makes sense;  
says there are no zombies

$x = 1.5$  (+1)

Doesn't make sense;  
can't have 150% of pop. being human.

(c) The model won't make sense once 50% or more of the total population become zombies, because at the next step there are no humans left. (+1)

(d) You can see in the cobweb diagram that solutions that tend to a value of  $p_t < 0.5$  will jump to a number bigger than 1, which means the model breaks down.

(+1)