

Day 13: Graph Sketching

1 Review Key Points

- f is increasing $\iff f'$ is positive.
- f is concave up $\iff f'$ is increasing $\iff f''$ is positive.
- Statements about f are translated to statements about positivity/negativity of its derivatives.

2 Determine positive/negative of a function

- $g(x)$ any function. How to find which intervals it is positive or negative?
- I'll call this "Positivity Test."
 1. Solve $g(x) = 0$ for x .
 2. Put those values on a number line.
 3. Test numbers in between in $g(x)$.
 4. Positive means positive, negative means negative!

Notice, no calculus is involved in this step!

- Example: determine the intervals where $f(x) = x^2 - 5x + 4$ is positive or negative.

3 Application of this idea

- Ex: Find all intervals where $f(x) = \frac{1}{12}x^4 - \frac{5}{6}x^3 + 3x^2 + x$ is concave down.

- $f''(x) = x^2 - 5x + 6$.
- Apply the “Positivity Test:”
- $f''(x) = 0$, solve for $x = 2, 3$. Make number line. Then try $x = 0, x = 2.5, x = 4$.
- Again, what makes this work is that $f''(x)$ is just a function, and we apply the “Positivity Test” to it. There is no calculus in the positivity test; just facts about continuous functions.

4 Sketching f' from f

- Given a graph of f , we can sketch f' .
- Do example.
- Strategy: mark off special locations, such as critical points.
- Important points: f increasing $\iff f'$ positive, and similar for decreasing.

5 Sketching f from f'

- Given a graph of f' , we can sketch a graph of f .
- Note: many functions have same f' . They are all related by vertical shifts; a $+C$ if you will.
- As such, any problem giving f' asking to graph f will come with extra info: “Assume f goes through the point...”
- Good strategy: start from the specified point, and use f' to decide how to move your pencil.
- Ex: graph f from f' given the graph of f' below. Assume that f goes through the origin.