5.8 Part 3: Fitzhugh-Nagumo Equations

The Fitzhugh-Nagumo Equations

- We couple the sodium channel to the potassium channel mechanism as follows:

$$\frac{dv}{dt} = \overbrace{-v(v-a)(v-1)}^{\text{Na-Channel}} \qquad \overbrace{-w}^{\text{K-channel affect on voltage}}$$
$$\frac{dw}{dt} = \underbrace{\epsilon(v-\gamma w)}_{\text{K-channel}}$$

- These are called the Fitzhugh-Nagumo equations.
- Warning: these are a simplified version of a more complicated set of DE's modeling a neuron, called the *Hodgkin-Huxley* equations (which are derived by treating the neuron like a circuit).

Analysis

- Nullclines:
- w-nullcline:
  - $\epsilon(v-\gamma w) = 0$
  - $w = \frac{1}{\gamma}v$
  - This is a straight line in the phase-plane of slope γ.
- v-nullcline:
  - $-v(v-a)(v-1) - w = 0$
  - or, $w = -v(v-a)(v-1)$.

- Neat feature: we can graph both of these.
- Do set of parameters with $\epsilon = 1$, $a = 0.3$, $\gamma = 2.5$.
- Do one with a bunch of equilibria ($\epsilon = 1$, $a = 0.2$, $\gamma = 10$).
- Do one with small γ, with $a = 0.4$ and $\gamma = 1$.
- Sketch the $v(t)$ and $w(t)$ curves for each. (Then check with a computer).

With constant applied voltage

- If the neuron has a bunch of applied voltage, will it make use of it in an interesting way?
- Add in a number I_a to the voltage equation.

$$\frac{dv}{dt} = -v(v-a)(v-1) - w + I_a$$
$$\frac{dw}{dt} = \epsilon(v - \gamma w)$$

- This has the effect of shifting the v-nullcline up.