Quiz 2

1. Is the following function a rational function? If so, identify the polynomials p and q such that f is a ratio of p and q.

$$f(t) = \frac{1}{1-t} - \frac{t^2}{3t+1}$$

Combine fractions:

$$\begin{aligned} f(t) &= \frac{(3t+1)}{(1-t)(3t+1)} - \frac{t^2(1-t)}{(3t+1)(1-t)} \\ &= \frac{t^3 - t^2 + 3t + 1}{-3t^2 + 2t + 1} \end{aligned}$$

so f is a rational function, with $p(t) = t^3 - t^2 + 3t + 1$ and $q(t) = -3t^2 + 2t + 1$.

2. A once thriving company had its monthly profits, in thousands of dollars, modeled by the function

$$P(t) = \frac{3t^2 + 9}{1 + 0.8t^2}$$

where t is the number of months after January 1st, 2010.

(a) How much was their profit on January 1st, 2010?
Set $t = 0$. Then $P(0) = 9/1 = 9$, so their profit was $\$9,000$.

(b) What happens to their profit in the long run?
As $t \to \infty$, $P(t) \sim \frac{3t^2}{0.8t^2} = \frac{3}{0.8} = 3.75$. This means their profit stabilizes at around $\$3,750$.

3. Let t be the time in weeks. At time $t = 0$, waste is dumped into a pond. As time goes on, the oxygen level fluctuates due to the waste. The oxygen level in the pond at time t is given by

$$f(t) = \frac{t^2 - t + 1}{2t^2 + 1}.$$

Assume $f(0) = 1$ is the normal level of oxygen (or 100%).

(a) What is the oxygen level (as a percentage) after four weeks?
$f(4) = \frac{16-4+1}{32+1} = \frac{13}{33} = 0.394,$ or about 39.4%.

(b) What happens to the oxygen level in the lake in the long run?
As $t \to \infty$, $f(t)$ becomes well-approximated by $\frac{t^2}{2t^2} = \frac{1}{2}$, so $f(t) \to \frac{1}{2}$. This means that the oxygen level only returns to half of its initial value.