WA 5

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(1) 2/2

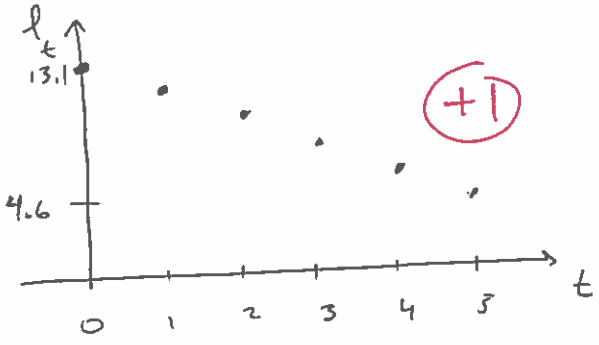
(a) $f(x) = \frac{x}{2}$ ← linear +1

(b) $f(x) = \frac{1}{x+1}$ ← not linear +1

(2) 4/4

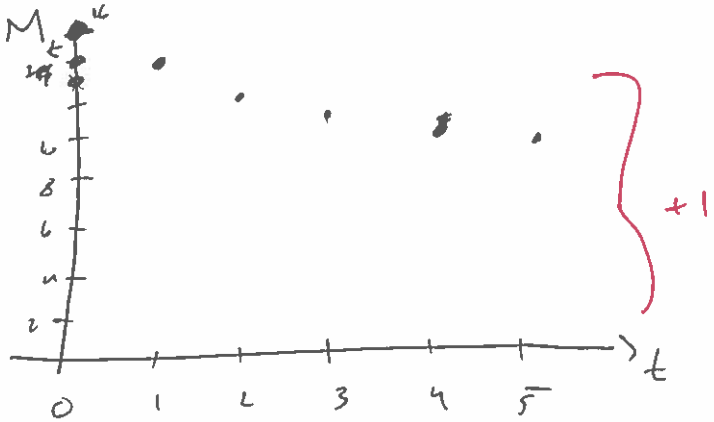
(a) $\ell_{t+1} = \ell_t - 1.7$, $\ell_0 = 13.1\text{ cm}$

$\ell_1 = 11.4$
$\ell_2 = 9.7$
$\ell_3 = 8$
$\ell_4 = 6.3$
$\ell_5 = 4.6$
} +1



(b) $M_{t+1} = .75\,M_t + 2$, $M_0 = 16.0$

$M_1 = 14$
$M_2 = 12.5$
$M_3 = 11.375$
$M_4 = 10.531$
$M_5 = 9.898$
} +1



3 (a) $l_{t+1} = l_t - 1.7$, $l_0 = 13.1$

4/4 Sol: $l_t = 13.1 - 1.7 \cdot t$ (+2)

(b) $M_{t+1} = .75 M_t + 2$

Let $N_t = M_t + c$.

$N_{t+1} = M_{t+1} + c = .75 M_t + 2 + c$

$= .75(N_t - c) + 2 + c$

$= .75 N_t - .75c + 2 + c$

$N_{t+1} = .75 N_t + 2 + .25c.$

so, if $c = -8$,

then $N_{t+1} = .75 N_t$.

$\Rightarrow N_t = 0.75^t \cdot N_0$.

$M_t - 8 = .75^t (M_0 - 8)$

$M_t = .75^t (M_0 - 8) + 8$

$M_t = .75^t (16 - 8) + 8$

$M_t = (.75^t)(8) + 8$. +2

* Shown work is ideal, but it's ok if a guess-and-check method to find this worked.

4 7/7

(a) $f_{t+1} = 1.1 \cdot f_t$ (+2)

(b) The experiment is "wait one generation" (+1)

(c) $f_t = 0.0001 \cdot (1.1)^t$ (+2)

(d)

It does reach 1:

$$1 = f_t = .0001 \cdot 1.1^t$$

$$10{,}000 = 1.1^t$$

$$\ln(10{,}000) = t \cdot \ln(1.1)$$

$$t = \frac{\ln(10{,}000)}{\ln(1.1)} \approx 96.653$$

so f_t goes above 1 on the 97th step.

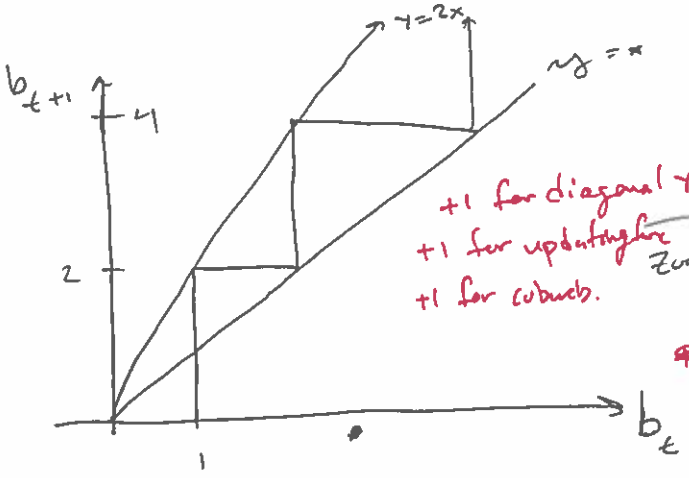
+1 for one of these answers; either is acceptable.

OR: explain that f_t is a growing exponential, so it must eventually reach 1.

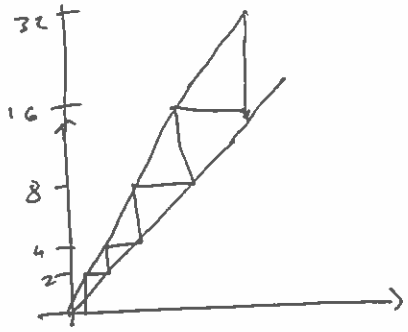
✱ Model only makes sense while $f_t \leq 1$, because f_t is a fraction of the total population with a certain gene; this cannot exceed 1. (+1)

5 a

8/8



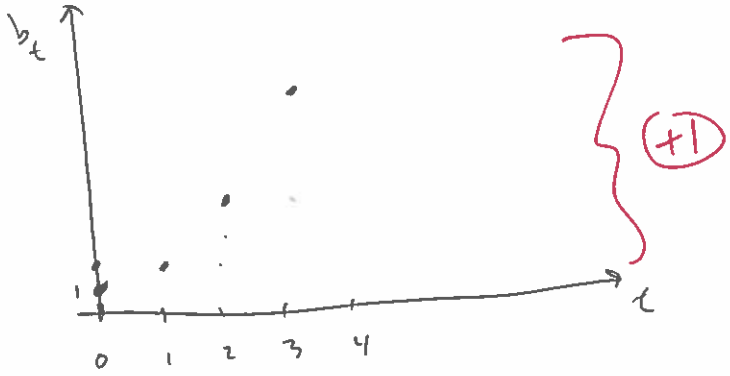
+1 for diagonal $y=x$
+1 for updating fns
+1 for cobweb.
Zoomed out



Things to watch out for:

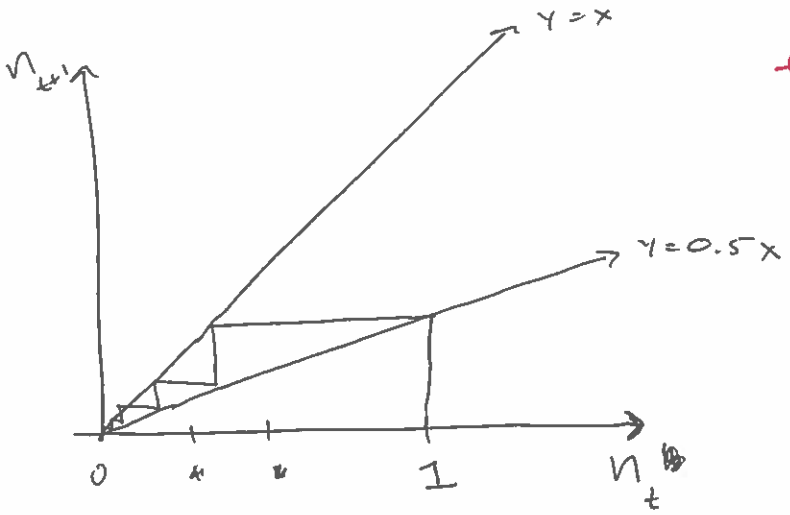
1. Using solution (exponential) as the updating function
2. going left/right to the updating function. (cobwebbing the wrong way)

b



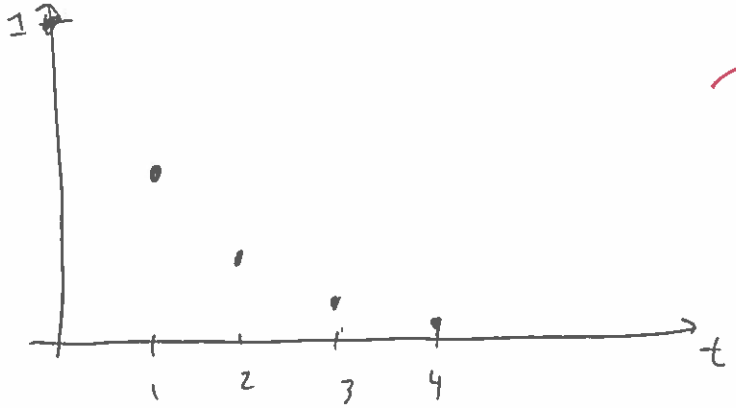
+1

c



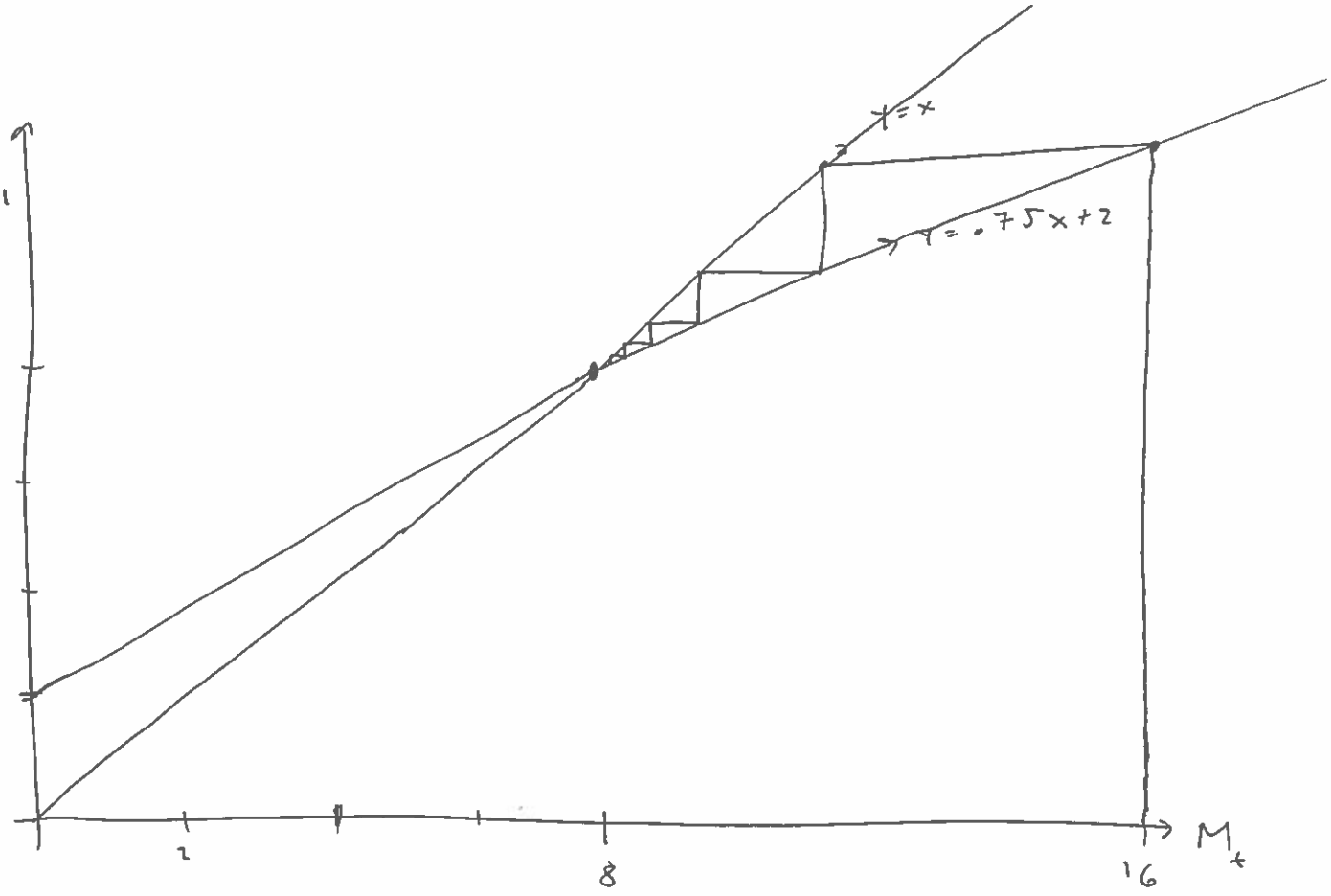
+1 for diag
+1 for $y=x$
+1 for cobweb.

d



+1

(6) 3/3



(7) $M_{t+1} = .75\,M_t + 2$

2/2 @ Set $.75x + 2 = x$:

$2 = .25x$

$\frac{2}{.25} = x$

$x = 8$ ✓

+2

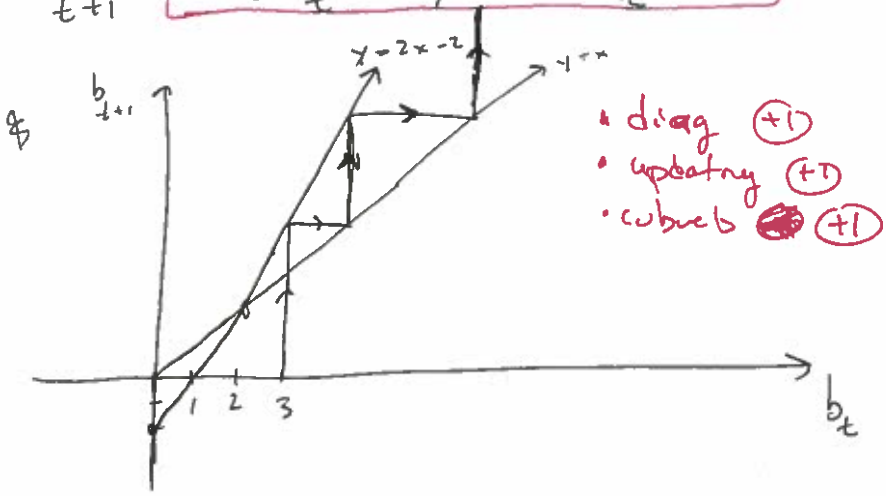
* on cobweb, make sure diagonal (+1), updating function (+1), and cobweb (+1) are graphed correctly (see comments on #5)

8 6/6

(a) $b_{t+1} = 2(b_t - 1) = 2b_t - 2$. +2

(-1 : if $2b_t - 1$)

(b)



- diag +1
- updating +1
- cobweb +1

(c) Set $2(x-1) = x$

$2x - 2 = x$

$x - 2 = 0$

$x = 2$ ✓ +1