## Section Goals:

- Combine functions with elementary operations and interpret the result
- Find an equation for and evaluate the composition of two functions
- Analyze an applied context to determine what elementary combination or composition of functions is appropriate
- Find the domain of a composite function

---

---

Def For two functions, $f(t)$ and $g(t)$, for any $t$ in the domain of both $f$ and $g$, we write

$$\left[\begin{array}{c} (f+g)(t) = f(t) + g(t) \\ (f-g)(t) = f(t) - g(t) \\ (f \cdot g)(t) = f(t) \cdot g(t) \\ \left(\dfrac{f}{g}\right)(t) = \dfrac{f(t)}{g(t)} \text{ (as long as) } g(t) \neq 0 \end{array}\right]$$

If it seems like there is not a great deal of purpose to those definitions, that's because there isn't. It is, however, a necessary formality, for example, to define "the function whose name is $f$ minus $g$" to be the function that takes an input, evaluates it in $f$, then $g$, then subtracts those values.

**Ex 1** Simplify each function expression, given $f(t) = 3t^2 + t$ and $g(t) = \dfrac{8t}{3t+1}$.

a) $(f+g)(1)$ $= f(1) + g(1) = 4 + 2 = 6$

b) $(f \cdot g)(t)$

$$= f(t) \cdot g(t) = (3t^2 + t) \cdot \frac{8t}{3t+1}$$
$$= t(3t+1)\frac{8t}{3t+1}$$
$$= 8t^2$$

**Ex 2** Let $f(t) = 3t - 2$ and $g(x) = x^2 + 1$. Compute $f(g(-1))$ and $g(f(-1))$.

$f(g(-1)) = f(2) = 4$

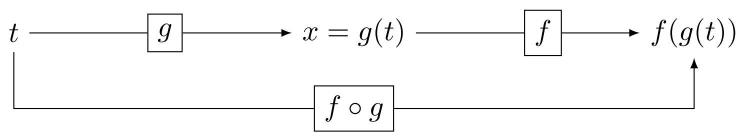
$g(f(-1)) = g(-5) = 26$.

Notice, the order matters greatly!

$\boxed{\text{Def}}$ The composition of $f$ with $g$ is defined to be $f(g(x))$, sometimes written $(f \circ g)(x)$, and read "f of g of x" or "f composed with g of x". (Notice that the order of composition appears reversed (e.g. $(f \circ g)(x)$ means input $x$ goes into function $g$ first, and then into $f$). As a mnemonic device, consider thinking of the composition $\circ$ symbol like a little mouth and you can read $f \circ g$ as "f is eating g", and thus $g$ would be on the inside.

Try not to confuse the composition symbol $\circ$ with multiplication $\cdot$, they are different operators!

Try thinking of this as one function used as the input for another. The diagram below shows a "typical" input $t$ which is passed through first $g$ and then $f$.




**Ex 3** Pretend that the temperature, $T$ (degrees Fahrenheit), can be fairly accurately predicted based on the chirp rate, $n$ (chirps per minute), of crickets, by the formula

$$T(n) = 60 + \left(\frac{n - 72}{4}\right).$$

We also have the relationship

$$C(F) = \frac{5}{9}(F - 32)$$

for converting a temperature $F$ in Fahrenheit to degrees Celsius.

a) Use composition notation to write function that takes in the chirp rate, $n$, and outputs the temperature in Celsius.

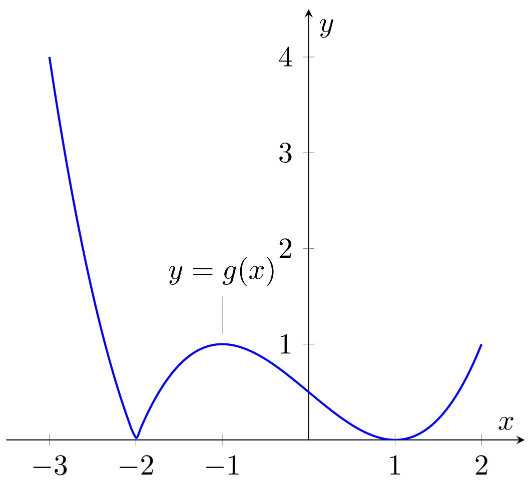
The function should be $(C \circ T)(n)$:

$$\begin{aligned}(C \circ T)(n) &= \frac{5}{9}\left(T(n) - 32\right)\\ &= \frac{5}{9}\left(60 + \left(\frac{n-72}{4}\right) - 32\right)\\ &= \frac{5}{9}\left(28 + \left(\frac{n-72}{4}\right)\right)\end{aligned}$$

b) Use the formula above to find the temperature in Celsius when the chirp rate is 42 chirps per minute.

$$(C \circ T)(42) = \frac{5}{9}\left(28 + \frac{-30}{4}\right) = \frac{5}{9}(20.5) = 11.4°\text{C}$$

**Ex 4** Consider the function given by $y = g(x)$ shown in the graph below, and $y = h(t)$ defined completely by the table. Compute each indicated value or state it to be undefined.




| $t$ | $h(t)$ |
|---|---|
| $-1$ | 3 |
| 0 | $-1$ |
| 1 | 4 |
| 2 | 3 |

a) $(g+h)(1) = 0 + 4 = 4$

b) $(h \circ g)(-1)$ $h(1) = 4$

c) $\left(\dfrac{g}{h}\right)(2) = \dfrac{1}{3}$

d) $(g \circ h)(-1) = g(3)$ is unknown

Def The domain of the composite function $f \circ g$ is the set of all elements in the domain of $g$ such that the image of each element is also in the domain of $f$.

In symbols,

$$\text{Dom}(f \circ g) = \{x \mid x \in \text{dom}(g) \text{ and } g(x) \in \text{Dom}(f)\}$$

In other words, we would check each number, $a$, in the domain of $g$ to see if $g(a)$ is in the domain of $f$. If it is, then $a$ is part of the domain of the composite function.

**Ex 5** Find the domain of the function $f \circ f$, where $f(x) = \frac{1}{x}$.

What is $f \circ f$?

$$(f \circ f)(x) = \frac{1}{f(x)} = \frac{1}{1/x} = x$$

as a formula. But what is the domain of this function? Well, the domain of the first part of the composition is all nonzero $x$, or $(-\infty, 0) \cup (0, \infty)$. The next part requires us to be able to divide by $f(x)$; luckily, $f(x) \neq 0$ ever, so the domain is just $(-\infty, 0) \cup (0, \infty)$.

This example illustrates the main features of domains of compositions: while the final result may be simplifiable and have a big domain, the actual pieces of the composition contribute to the domain.

**Ex 6** Find the domain of $f \circ g$ where $f(x) = \sqrt{x-2}$ and $g(t) = \dfrac{3}{t}$.

Apply the same process. At each step, ask yourself: does this make sense?

$$(f \circ g)(t) = f\left(\frac{3}{t}\right) = \sqrt{\frac{3}{t} - 2}.$$

At the first step, we need $3/t$ to make sense, so we kick out $t = 0$. In the second step, we need the result $\frac{3}{t} - 2 \geq 0$, which places the restriction

$$\frac{3}{t} \geq 2.$$

Before solving for $t$, notice that this requirement is quite subtle. It actually already implies that $t$ must be positive, for a negative $t$ would make it less than 2. So, since $t$ is positive, we can multiply both sides by $t$ and we get

$$\frac{3}{2} \geq t > 0$$

since we can't have $t = 0$. So, the domain is

$$\left(0, \frac{3}{2}\right].$$