Homework 5
Due Tuesday, May 7th

Instructions: write up solutions to all problems below. Neatness counts: be sure to follow guidelines for homework in the syllabus.

---

1. Suppose that a second order, linear, homogeneous differential equation has constant coefficients $ay'' + by' + cy = 0$, and that the corresponding characteristic equation has two distinct, real solutions. We know that one set of fundamental solutions are $y_1 = e^{r_1 x}$ and $y_2 = e^{r_2 x}$. Show that, if $r_1 \neq r_2$, then the Wronskian $W[y_1, y_2]$ is never zero, thus proving that these solutions are independent.

2. Find the general solution to the equation
$$y'' - 6y' + 9y = 0.$$

3. (Chapter 3.4, # 16). This is a cool way to see that you always have to get te^{rt} for a system with repeated roots. Suppose your differential equation $ay'' + by' + cy = 0$ has two distinct roots, r_1 and r_2. Now imagine the numbers a, b, c changing in a way so that r_1 becomes closer and closer to r_2.

   (a) Show, that when you have distinct roots $r_1 \neq r_2$, that
   $$\frac{e^{r_2 t} - e^{r_1 t}}{r_2 - r_1}$$
   is also a solution to the differential equation. [Hint: what are c_1 and c_2?]

   (b) Now, think of r_1 as being fixed, and take the limit
   $$\lim_{r_2 \to r_1} \frac{e^{r_2 t} - e^{r_1 t}}{r_2 - r_1}.$$
   [Hint: Use l'Hopital's rule.] The answer you get for this limit will be the other fundamental solution that goes with $e^{r_1 t}$ for the repeated roots scenario!

4. (Chapter 3.5, # 3, 5) Solve these non-homogeneous equations:

   (a) $y'' + y' - 6y = 12e^{3t} + 12e^{-2t}$.

   (b) $y'' + 2y' = 3 + 4\sin(2t)$

5. (Chapter 3.5, # 8,9) Solve these non-homogeneous equations:

   (a) $x'' + \omega_0^2 x = \cos(\omega t)$, $\omega \neq \omega_0$

   (b) $x'' + \omega_0^2 x = \cos(\omega_0 t)$.