**5.6: Phase-Plane Diagrams**

# Phase-Planes

- Generalize the idea of a phase-line.
- Make an $xy$-plane, where $x$ and $y$ are state variables.
- At each point $(x, y)$ in this plane, we draw an arrow (or vector) indicating the slopes.
- Ex:
$$\frac{dx}{dt} = -x + 2xy$$
$$\frac{dy}{dt} = y - xy$$
at the point $(x, y) = (1, 1)$,
$$x' = -1 + 2 = -1,$$
$$y' = 1 - 1 = 0$$
while at the point $(x, y) = (2, 3)$,
$$x' = -2 + 2(2)(3) = 10,$$
$$y' = 3 - (2)(3) = -3.$$
- We represent solutions on the phase plane by a trajectory: solutions to a system are two *functions* $(x(t), y(t))$, which, as time moves on, traces out a path in the plane. The phase-plane tells us qualitatively how solutions will evolve over time.

# Nullcines and Equilibria

- Equilibria: BOTH $\frac{dx}{dt} = 0$ and $\frac{dy}{dt} = 0$.

- Nullcline: ONE OF $\frac{dx}{dt}$ or $\frac{dy}{dt}$ is 0.
- Ex: Predator-Prey:

$$\frac{db}{dt} = b - 0.01pb$$
$$\frac{dp}{dt} = -0.5p + 0.0005pb$$

- $b$-nullclines:
$$\frac{db}{dt} = 0 = b - (0.01)pb = b(1 - 0.01p)$$
get: $b = 0$ or $p = 100$.

- $p$-nullclines:
$$\frac{dp}{dt} = 0 = -0.5p + 0.05pb = p(-0.5 + 0.0005b)$$
get: $p = 0$ or $b = 1000$.

- Tip: draw $b$-nullclines in one color, and $p$-nullclines in different color.
- Note: Equilibria are where two nullclines intersect (as long as nullclines come from different state variables).
- Idea: nullclines give you a better idea of how solutions behave, even without making a whole phase-plane.