

General Information

The final exam will take place on **Friday, August 12** from 11:00am-1:00pm in our usual room (Deady 306). You may have a graphing calculator. You may bring notes on the front and back of a **half** sheet of paper. This review will not be collected for credit. The exam will cover all material from section 1 to 8. Please keep in mind that this review is not comprehensive. You should also spend some time reviewing notes, reading through the sections, going over old quizzes, exams, written assignments, worksheets, and webwork.

The exam will have a few true/false problems, a few multiple choice problems, and some long answer problems.

Topics

Here are some key words to help you study.

1. Section 1: Introduction to Functions
 - a) Determine if some relationship is a function
 - b) Mathematical and practical domain and image
 - c) Piecewise-Defined Functions
2. Section 2: Linear Functions and Average Rate of Change
 - a) Average rate of change
 - b) Percent change
 - c) Graphing a linear function
3. Section 3: Quadratic Functions and Higher-Order Polynomials
 - a) Optimization
 - b) Second differences
 - c) Forms of a quadratic; finding a formula for a quadratic
 - d) Polynomial functions
 - e) Long term behavior of polynomials

4. Section 4: Rational Functions

- a) Determine if a function is rational
- b) Graphs of basic rational functions
- c) Long-term behavior of rational functions

5. Section 5: Exponential Functions

- a) Forms of an exponential function
- b) Graphs of basic exponential functions
- c) Long term behavior of exponential functions
- d) Domain and image of an exponential function
- e) Compound interest
- f) Continuously compounding interest

6. Section 6: Power and Logarithmic Functions

- a) Converting between logarithmic and exponential forms
- b) Graphs of basic logarithmic functions
- c) Properties of logarithms (sum, difference, and constant multiple rules)
- d) Change of base formula

7. Section 7: Composition and Arithmetic of Functions

- a) Arithmetic operations on functions
- b) Finding domains
- c) Composition of functions
- d) Domain of a composite function

8. Section 8: Function Inverses

- a) One-to-one functions
- b) Computing inverses
- c) Domain and image of an inverse function
- d) Domain restrictions

Practice Problems

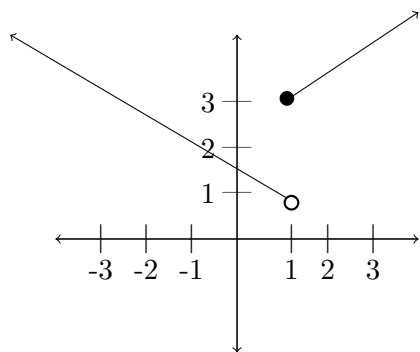
1. Give a precise definition of the following.
 - a) function
 - b) linear function
 - c) piecewise defined function
 - d) quadratic function
 - e) polynomial function
 - f) rational function
 - g) exponential function
 - h) logarithmic function
2. Determine the type of function that could model the following scenarios:
 - a) The price of a cab, given how many miles I've traveled.
 - b) The amount of money in a bank account gaining compound interest.
 - c) A function that can optimize the area of a square, given some constraints on the sides.
 - d) The height of an object after being thrown, given its initial height and velocity.
 - e) The amount of money an employee makes an an hour job, given the number of hours they've worked.
3. Determine if the following statements are true or false. Provide a short explanation for your answer.
 - a) The domain of g is always a subset of the domain of $f \circ g$.
 - b) Let $f(x) = \frac{(x+1)^2}{x+1}$ and $g(x) = x+1$. Then f and g are equal functions.
 - c) If a function is not one-to-one, then it is not invertible.
 - d) Every quadratic function is a polynomial function.
 - e) Every function must pass the horizontal line test.
 - f) The following is a rational function

$$g(x) = x^{-2} + \frac{\sqrt{e}}{x+1}.$$

4. Compute the average rate of change **and** the percent change of the following functions on the specified intervals.

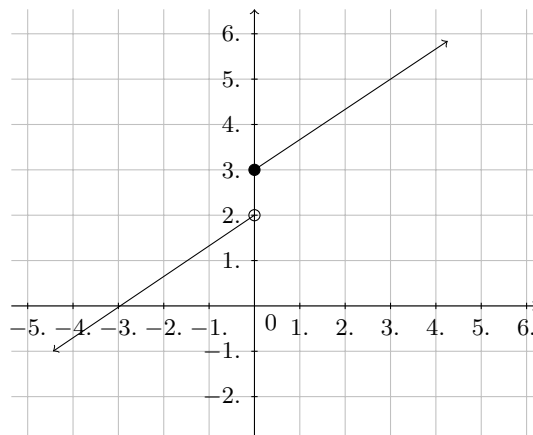
a) $f(x) = x^3 + 4$ on $[-1, 1]$.

- b) $g(x)$, defined by the following graph, on $[0, 1]$.



5. Find the domain of the function $f(x) = \frac{\sqrt{1-x}}{\sqrt{x+7}}$.

6. Find a formula for the function below.



7. Is the function defined by the table below one-to-one? Explain.

x	-1	0	1	2
$f(x)$	4	3	4	7

8. Determine if the following are polynomials, rational functions, or neither. Circle your choice from the options on the right.

a) $f(x) = x^3 + 3$ polynomial function rational function neither

b) $g(t) = \frac{x^{1/2} - 3x^2 + 2}{2x + 1}$ polynomial function rational function neither

c) $h(x) = \frac{\pi x^2 + 2e^2}{2x + x^{-1}} - \frac{1}{3}$ polynomial function rational function neither

9. Find the long term behavior of each of the following functions.

a) $f(t) = (9 - t^2 + t)(2 - t)$

b) $g(t) = 2(0.45)^{-2t}$

c) $h(t) = \frac{t^2 - 4t^5}{t^2 + 6t + 7t^8 - 2}$

10. Write a sentence that describes the inverse of the function defined by: add one to the input and then double the result.

11. Find the inverse of g , defined by

$$g(t) = \log_3(5t + 1) - 4.$$

12. Let $f(x) = \frac{1}{x^2}$ and $g(x) = \sqrt{x+1}$. Find the domain of $(f \circ g)(x)$. Show all work!

13. Consider the functions defined by the table below.

x	3	5	-2	-1
g(x)	5	0	-25	12
h(x)	12	2	25	0

a) Evaluate each function at the specified value.

i. $(g + h)(-1)$

ii. $(h \circ g \circ k)(1)$, where $k(x) = x^2 + x + 1$.

iii. $g^{-1}(5)$

iv. $\left(\frac{g}{h}\right)(-1)$

v. $h^{-1}(2)$

b) Find the domain of $g \circ h$ and $h \circ g$.

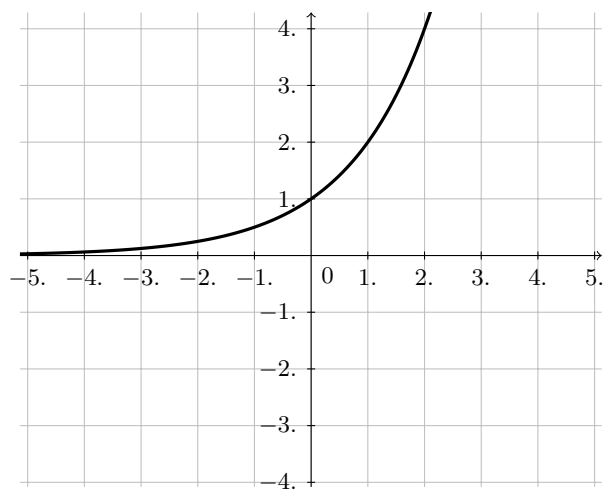
14. The half-life of ibuprofen (used in pain-killers) is around two hours. A typical dose consists of 400 mg. If you take one dose, how long will it take for there to be 20% of the original amount of ibuprofen left?

15. Say I invest \$100 in a bank that accrues 12% interest, compounded continuously.

a) Find a formula for the amount of money in your bank account at time t in years.

b) Find the inverse of the function you found in part (a).

16. Sketch the graph of the inverse to the function below.



17. Recall the formula for projectile motion,

$$h(t) = \frac{-9.8}{2}t^2 + v_0t + h_0,$$

where h is the height of an object, in meters, after t seconds. If a diver reaches their maximum height 2 seconds after leaving a diving board, what must have been their initial velocity? Show all work, and be sure to include units.

18. The area of a circular wave expands across a still pond such that its radius increases by 2cm each second. Write a formula for the circumference of the circle as a function of time t since the wave begins.

19. [6 pts.] Say I have \$5,000 that I'd like to put into savings. When I go to my bank, they tell me that I have two options for savings accounts that will earn me money through compound interest. My first option is to open an account where interest is compounded monthly at a rate of 5%, and my second option is to open an account where interest is compounded continuously at a rate of 4%.

a) Which option will gain the most interest after 10 years?

b) What will be the difference in time between my two options to reach a \$10,000 savings goal?