Nonlinear Altitude Control of a Quadcopter Drone Using Interval Type-2 Fuzzy Logic

Ayad Al-Mahturi
School of Engineering and Information Technology
The University of New South Wales
Canberra, Australia
A.Al-mahturi@student.adfa.edu.au

Fendy Santoso
School of Engineering and Information Technology
The University of New South Wales
Canberra, Australia
F.Santoso@adfa.edu.au

Matthew A. Garratt
School of Engineering and Information Technology
The University of New South Wales
Canberra, Australia
M.Garratt@adfa.edu.au

Sreenatha G. Anavatti
School of Engineering and Information Technology
The University of New South Wales
Canberra, Australia
S.Anavatti@adfa.edu.au

***Abstract*— The development of Unmanned Aerial Vehicles (UAVs) has become one of the most fruitful research areas in the field of autonomous flight control. Quadcopters are chosen due to their simple mechanical structure which is able to hover in a stationary manner, vertical take-off and landing. Nevertheless, these types of aircraft are highly nonlinear and under-actuated systems. Intelligent control such as fuzzy logic is a suitable choice for controlling nonlinear systems. However, type-1 fuzzy control cannot handle uncertainties of nonlinear systems. As a solution, interval type-2 fuzzy control has the advantage of being able to deal with uncertainties. This research proposes the use of interval type-2 fuzzy for controlling the altitude of a nonlinear quadcopter UAV using Gaussian membership function and utilizing the Enhance Iterative Algorithm with Stop Condition (EIASC) algorithm for type-reduction. A comparison between the proposed interval type-2 fuzzy controller and proportional-Derivative (PD) controller is illustrated. Simulation results demonstrated that the tracking performance of the proposed controller outperformed the PD controller.**

***Index Terms*—Intelligent control, quadcopter UAVs, interval type-2 fuzzy logic, uncertain systems**

I. Introduction

Fuzzy logic was first introduced by Lotfi Zadeh in 1965 [1]–[3] and has been applied in many engineering areas over the past few decades. The basic concept of fuzzy logic is that the belonging of objects can be described with a degree of membership which manipulate results between the values of "0" and "1"; unlike the case of classical Boolean logic where objects belonging are described by a crisp value "True/False or 0/1" [4], [5]. Fuzzy logic imitates human reasoning which derives a conclusion based on a set of expert fuzzy *If-Then* rules in a range between '0' and '1'.

Intelligent systems such as fuzzy logic are found out to be very successful in handling complex, non-linear and time-varying systems. Intelligent control techniques such as fuzzy inference system (FIS), artificial neural networks (NNs) and Genetic Algorithms (GAs) have shown the ability to deal with nonlinear, time-varying systems in the presence of uncertainties. Moreover, due to the fact that quadrotors experience various forms of disturbances and noise, fuzzy logic is chosen due to its capability of dealing with cluttered environments [6]. Fuzzy logic has been successfully applied in many real-time applications. Nevertheless, most of these studies have focused on type-1 fuzzy logic. Type-2 fuzzy logic systems appear to be more capable for handling problems with uncertainties and time-varying systems compared to their type-1 counterpart. However, one main problem which could badly influence the performance of a system in control applications is data uncertainties. In aerial robotics, for instance, parameters always vary at different working conditions, therefore, type-1 fuzzy sets may not be an appropriate method to be used. As a solution, type-2 fuzzy logic systems appear to be a more promising method to deal with uncertainties associated with real-time applications such as in Unmanned Aerial Vehicles (UAVs).

Several control methods have been used to control UAVs including linear controllers such as Proportional Integral Derivative (PID) controller [7], LQR controller [8], H_∞ controller [9] and other nonlinear controllers such as sliding mode controller (SMC) [10], type-1 fuzzy controller [11], [12] and model-predictive controller (MPC) [13]. Nevertheless, a major limitation of the linear controllers can be that they cannot handle uncertainties in nonlinear systems. Also, the effectiveness of conventional controllers degrades for complex, nonlinear and time-variant systems [14]. However, type-2 fuzzy has shown better results and been applied successfully to control Unmanned Aerial Vehicles (UAVs) where uncertainties are taken into account such as in [15] for a UAV lateral control and for trajectory tracking in [16]. Nevertheless, most type-2 fuzzy controllers utilize Karnik-Mendel (KM) algorithm for reducing type-2 to type-1 to obtain a crisp output. The KM algorithm has a high computational cost which hinders type-2 fuzzy from being used in various real-time applications [17].

Unmanned Aerial Vehicles are convenient platforms for

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testing algorithms. They are chosen due to their low maintenance, high maneuverability, small size, ease of maintenance, safety for human interaction [18], [11], [19]. UAVs have been widely utilized for various missions such as aerial surveillance, rescue and exploration. Quadcopters are also preferred due to their simple mechanical structure which is able to hover in a stationary manner, vertical take-off and landing (VTOL). The popularity of this platform has stemmed from its simple construction which has smaller diameter rotors as compared with conventional helicopters [20].

While type-1 fuzzy logic controllers are well established, type-2 fuzzy controllers have not gained much attention. In spite the fact that type-1 fuzzy conveys the connotation of uncertainties, it has a limitation of modeling and minimizing the impact of uncertainties. Moreover, most of the designed controllers for quadcopters are based on linear models where the model is linearized around the hovering point which lead to unsatisfactory performance when the system operate at different operating conditions. Furthermore, while some researchers have considered the use of type-2 fuzzy logic control to control quadcopters UAVs using the iterative Karnik-Mendel (KM) algorithm, which have a high computational cost, the implementation of interval type-2 fuzzy logic controller using the enhanced iterative algorithm with stop condition (EIASC) [17] has not been examined in the literature.

Therefore, our contribution in this study can be demonstrated by accommodating a nonlinear model of a quadcopter in line with designing an interval type-2 fuzzy controller for altitude control considering 25% of Footprint of Uncertainties (FOU). We also implemented the enhanced iterative algorithm with stop condition (EIASC) [17] to reduce the computational cost of KM algorithm. In addition, a comparative study is performed on the proposed controller and a PD controller. The simulation results demonstrate that the proposed controller is more efficient than the conventional PD controller.

The rest of the paper is organized as follows: in the following section, an overview of type-2 fuzzy logic is presented, followed by the mathematical model of quadcopters in section III. The fourth section discusses the simulation results. Lastly, the conclusion and future work are demonstrated in section V.

II. Type-2 fuzzy logic controller

The concept of type-2 fuzzy logic was introduced by Zadeh [2] and firstly implemented in [21] to represent uncertainties of data. Type-2 fuzzy logic systems (FLS) are extensions of type-1 FLS. The basic structure of type-2 fuzzy logic is depicted in Fig. 1 and consists of five major elements as follows:

- **Fuzzification**: A process to transform crisp values into fuzzy values.
- **Knowledge Base**: A set of *If-Then* rules and membership functions (MFs) where the *If* part is called *antecedent* and the *Then* part is called the *consequent*. The rule-base is considered as the heart of fuzzy logic systems.
- **Inference System**: A fuzzy reasoning mechanism to get a fuzzy output. The most popular inference systems are the Mamdani and the Takagi-Sugeno inference engines.

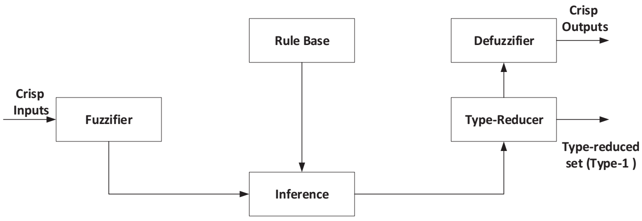

Fig. 1. Type-2 fuzzy logic structure

- **Type-Reduction**: A process to reduce from type-2 to type-1 fuzzy.
- **Defuzzification**: A process to transform fuzzy grade into corresponding crisp values.

The main difference between type-1 and type-2 fuzzy is in the output processing; type-reduction and defuzzification processes. To develop practical applications using type-2 fuzzy logic, it becomes vital to obtain a crisp value for all the fired fuzzy sets [4], [22]. To do this, a *type-reduction* is implemented where type-2 fuzzy sets are reduced to type-1 fuzzy sets. There are many type-reduction approaches in the literature such as iterative Karnik-Mendel (KM) algorithm [23] and other alternative type-reduction algorithms which mostly have a closed-form representation. The theory and design of interval type-2 fuzzy systems were studied by Liang and Mendel in [24].

A. Type-2 fuzzy sets

A Type-2 fuzzy set (T2FS), denoted by $\tilde{A}$, is made up of $((x,u), \mu_{\tilde{A}}(x,u))$, where for every $x \in X$, there is a primary membership function u where $u \in J_x$, and a secondary membership function $\mu_{\tilde{A}}(x,u))$. Fig. 2 depicts a type-2 Gaussian MF with uncertain mean. The theory of type-2 fuzzy sets can be found in [25]. While type-2 fuzzy has shown a better performance compared to its type-1 counterpart in various applications, the use of type-2 fuzzy in real-world applications is relatively sparse due to the computational cost in the type-reduction process [26]. Therefore, several studies have been conducted to improve the aspects of type-2 structure which led to an interval type-2 fuzzy systems as a simplified version of type-2 fuzzy. Interval type-2 fuzzy was introduced as a method to reduce the computational cost while maintaining the main advantages of type-2 fuzzy sets [27] where the secondary grade values $\mu_{\tilde{A}}(x,u))$ of an interval type-2 fuzzy sets are set to unity as shown in Equation (1).

$$\begin{aligned}\tilde{A} &= ((x,u),1)|\forall x \in X, |\forall u \in J_x \subseteq [0,1] \\ &= \sum_{x\in X}\sum_{u\in J_x}((x,u),1)\end{aligned} \tag{1}$$

B. Fuzzy inference engine

There are several existing fuzzy inference systems. The most widely used models are the: Mamdani inference engine in [28] and the Takagi-Sugeno (TS) inference engine in [29].

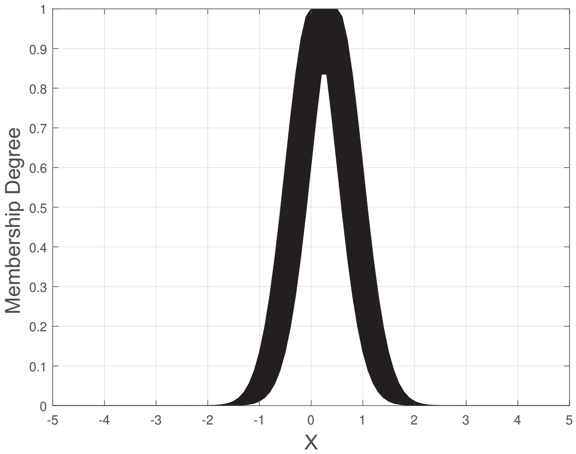

Fig. 2. Interval Type-2 membership function with uncertain mean

1) **The Mamdani inference engine**: In the Mamdani inference engine, fuzzy sets can be performed by combining different algebraic operations. The union operation (∪), which is referred to as (s-norm), and the intersection operation (∩), which is referred to as (t-norm), are the basic logic operations. The fuzzy rules of the Mamdani inference engine can be described as follows:

$$R^i: \ If\ (x_1\ is\ F_1^i)\ and\ (x_2\ is\ F_2^i)... \\ and\ (x_j\ is\ F_j^i),\ Then\ (y^i\ is\ G^i)$$

where R^i denotes the i^{th} fuzzy rule, F^i and G^i label the linguistic terms, $i = 1, ..., M$ where M represents the number of fuzzy rules, $j = 1, ..., N$ where N represents the number of antecedents, x_j and y^i represents the fuzzy system input and output respectively.

2) **Takagi-Sugeno (T-S) inference engine**: The Takagi-Sugeno (TS) inference engine is similar to the Mamdani inference engine. However, in the Takagi and Sugeno approach, a new structure for the *consequent* part is proposed where the *consequent* part of fuzzy rules is usually represented by *first-order polynomial* equations. Although it is possible to represent the consequent part using higher-order polynomial equations, first-order representation is widely used due to its similarity with linear modeling techniques [4]. The fuzzy rule-based of TS type can be described as:

$$R^i: \ If\ (x_1\ is\ F_1^i)\ and\ ...(x_j\ is\ F_j^i) \\ Then\ (y^i = a_0^i + a_1^i x_1 + ... + a_j^i x_j)$$

Where R^i is the i^{th} fuzzy rules, F^i labels the linguistic terms, a_j^i are the consequent polynomial parameters, $i = 1, ..., M$ where M represents the number of fuzzy rules, $j = 1, ..., N$ where N represents the number of antecedents, x_j is the input to the fuzzy system and y^i denotes fuzzy system output. T-S inference structure is used in this work.

Takagi-Sugeno (T-S) fuzzy logic systems are computationally efficient and have the advantage of offering an accurate way for modeling nonlinear systems [4].

C. Footprint of Uncertainty

The Gaussian membership function of a type-1 fuzzy logic depends on two parameters namely: the variance σ and the mean c . In type-2, another third dimensional parameter in the membership function is called *footprint of uncertainty* (FOU) and is represented by the blurred area in Fig. 2. FOU can be defined as the union of all primary membership functions J_x as follows [25]:

$$FOU(\tilde{A}) = \bigcup_{x \in X} J_x \tag{2}$$

FOU is bounded by an upper membership function (UMF), denoted by $\bar{\mu}_{\tilde{A}}(x)$, and a lower membership function (LMF) which is denoted by $\underline{\mu}_{\tilde{A}}(x)$ as follows:

$$\bar{\mu}_{\tilde{A}}(x) = \overline{FOU}(\tilde{A}) \tag{3}$$

$$\underline{\mu}_{\tilde{A}}(x) = \underline{FOU}(\tilde{A}) \tag{4}$$

D. Type-reduction

As we defined in the previous section that type-reduction is a process to reduce type-2 fuzzy into type-1 fuzzy in order to get a crisp output. In this work, the Enhanced Iterative Algorithm with Stop Condition (EIASC) type-reduction method is used for its computational effectiveness. The EIASC algorithm has a better stopping criteria for initialization and termination conditions so the computing technique is optimized where the details of this algorithm can be found in [17].

III. Quadcopter Nonlinear Aerodynamic Modeling

A quadcopter has six Degrees of Freedom (DOF), where six variables are considered. The motion of a quadcopter can be expressed by the following six state variables, namely, $x = [p_x\ \dot{p}_x\ p_y\ \dot{p}_y\ p_z\ \dot{p}_z\ \phi\ \dot{\phi}\ \theta\ \dot{\theta}\ \psi\ \dot{\psi}]^T$, where $[p_x,\ p_y,\ p_z]^T$ indicates the coordinate linear positions of the drone (in meters), $[\dot{p}_x,\ \dot{p}_y,\ \dot{p}_z]^T$ indicates the linear velocities (m/s) across the (x, y, z) axis, whereas the angular position (in radians) and also known as Euler's angles (i.e., roll, pitch, and yaw) of quadcotpers are described as $[\phi,\ \theta,\ \psi]^T$ respectively while $[\dot{\phi}\ \dot{\theta}\ \dot{\psi}]^T$ indicates the angular velocities (rad/s) of a quadcopter [11].

As quadcopters have four independent controllable actuators, the system is under-actuated where there are only four degrees of freedom to be controlled in order to obtain a stable system. The schematic diagram of a quadcopter is shown in Fig. 3 where rotors 1 & 3 rotate clockwise, rotors 2 & 4 rotates anticlockwise so the total torque around the z-axis is cancelled out. To increase the altitude of a quadcopter, the speed of the four rotors must be increased at the same rate and vice versa for decreasing the altitude.

The dynamics of quadcopter using Newton's motion equations and Euler's rotational motion equations are as follows [30]:

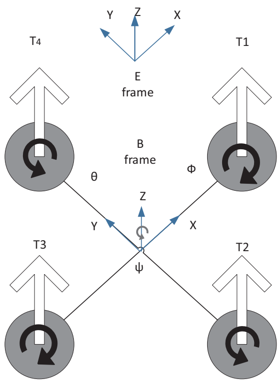

Fig. 3. Quadcotper model

$$
\begin{aligned}
\ddot{X} &= (\ \sin\ \psi\ \sin\ \phi + \cos\ \psi\ \sin\ \theta\ \cos\ \phi)\frac{U_1}{m} + \frac{D_x}{m} \\
\ddot{Y} &= (-\cos\ \psi\ \sin\ \phi + \sin\ \psi\ \sin\ \theta\ \cos\ \phi)\frac{U_1}{m} + \frac{D_y}{m} \\
\ddot{Z} &= -g + (\cos\theta\cos\phi)\frac{U_1}{m} + \frac{D_z}{m} \\
\ddot{\phi} &= \frac{(I_{yy} - I_{zz})\dot{\theta}\dot{\psi} - J_r\dot{\theta}\omega_r + U_2}{I_{xx}} \\
\ddot{\theta} &= \frac{(I_{zz} - I_{xx})\dot{\phi}\dot{\psi} - J_r\dot{\phi}\omega_r + U_3}{I_{yy}} \\
\ddot{\psi} &= \frac{(I_{xx} - I_{yy})\dot{\phi}\dot{\theta} + U_4}{I_{zz}}
\end{aligned} \tag{5}
$$

The control input is achieved by changing the speed of each individual rotor using electronics speed controllers coupled to the DC motors powering each rotor. The control inputs to the quadcopter are represented by $U1$, which determines the total thrust in the z-axis, $U2$, $U2$ and $U4$ for roll, pitch and yaw control respectively as defined in the Equations (6) to (9). These control inputs are given as [30]:

$$U1 = b(\Omega_1^2 + \Omega_2^2 + \Omega_3^2 + \Omega_4^2) \tag{6}$$

$$U2 = b(-\Omega_2^2 + \Omega_4^2) \tag{7}$$

$$U3 = b(\Omega_1^2 - \Omega_3^2) \tag{8}$$

$$U4 = d(-\Omega_1^2 + \Omega_2^2 - \Omega_3^2 + \Omega_4^2) \tag{9}$$

where d is the drag coefficient, b is the thrust/lift coefficient. The description of the parameters is given in Table I.

IV. Results and Discussion

The nonlinear model of the quadcopter was obtained from [30]. The controller design was done in MATLAB/SIMULINK with a stop time of 30 (sec) and a time step of 0.01 (sec) as shown in Fig. 4. The interval type-2 fuzzy controller was designed using five Gaussian membership functions for both the error and the derivative of error. The foot print of uncertainty (FOU) is chosen to be between 1 and 0.75 for both of the error and the derivative of error. The Gaussian functions have an uncertain mean and a fixed variance as shown in Fig. 5 and Fig. 6.

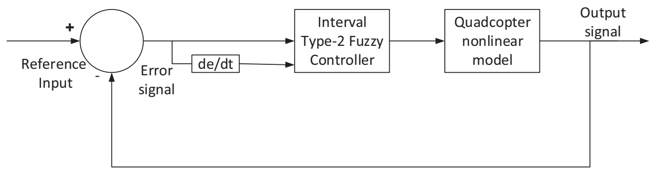

Fig. 4. Block diagram of interval type-2 fuzzy control for a nonlinear plant of a quadcopter

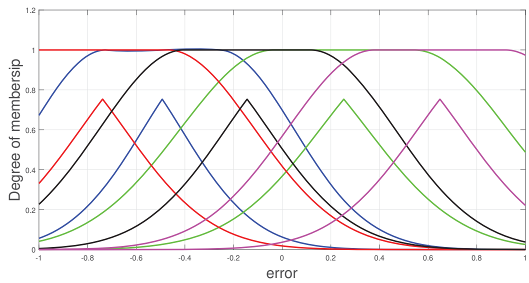

Fig. 5. Type-2 membership function for error with FOU between 1 and 0.75

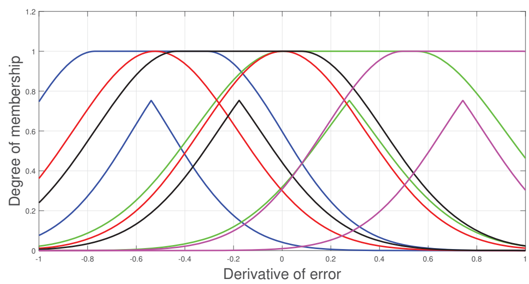

Fig. 6. Type-2 membership function for derivative of error with FOU between 1 and 0.75

The consequent part is based on centroid interval type-2 fuzzy and is chosen arbitrarily and no rule ordering is required for using the EIASC algorithm. The EIASC algorithm [17] was used for the type-reduction. To evaluate the performance of the interval type-2 fuzzy controller, the results are compared with a PD controller. The execution time of the PD controller is relatively shorter than the proposed controller. The reason is that PD controller has only two parameters to execute while the proposed controller has more steps to obtain the control

TABLE I
DESCRIPTION OF THE QUADCOPTER PARAMETERS

Symbol	Description	Value	Units
I_x	Quadrotor moment of inertia around X axis	0.0075	kg.m^2
I_y	Quadrotor moment of inertia around Y axis	0.0075	kg.m^2
I_z	Quadrotor moment of inertia around Z axis	0.013	kg.m^2
J_r	Total rotational moment of inertia around the propeller axis	6.5×10^{-5}	kg.m^2
b	Thrust factor	3.13×10^{-5}	N.s^2
d	Drag factor	7.5×10^{-7}	N.m.s^2
m	Mass of the quadcopter	0.65	kg
g	Gravitational constant	9.81	m/s^2
l	Distance to the center of the quadcopter	0.23	m

signal. However, both algorithms have short execution time under Core i7-6700 CPU @ 3.4GHz with 16.0 GB of RAM. Furthermore, five membership functions were utilized for both inputs with five rules. In this study, several desired trajectories are chosen for tracking the amplitude of the quadcopter as follows: a sinusoidal signal with an amplitude of 1 meter and frequency of 1 Hz, a step input with constant height of 1 meter, a square wave with an amplitude of 1 meter and frequency of 0.025 Hz and a summation of two sinusoidal waves. Firstly, the tracking performance of a step input is witnessed in Fig. 7 where the result shows that the interval type-2 tracking performance is better than the PD controller. The RMSE error is decreased from 0.2069 to 0.1578 using the proposed controller. It is also shown that the PD controller has a steady state error and oscillation. Secondly, the tracking performance is observed for a sinusoidal wave and summation of sinusoidal waves reference and the improvement is noticeable as shown in Fig. 8 and Fig. 9 where there is a tracking error of the PD controller while interval type-2 fuzzy has a smooth tracking. Lastly, a square wave reference is also used to evaluate the proposed type-2 controller, where our controller has shown better performance compared with the PD controller as shown in Fig. 10. Table II summarizes the performance of both the PD controller and the proposed interval type-2 controller using root mean square error (RMSE) for all the reference signals, where lowest RMSE values are achieved from the proposed interval type-2 controller in all the scenarios.

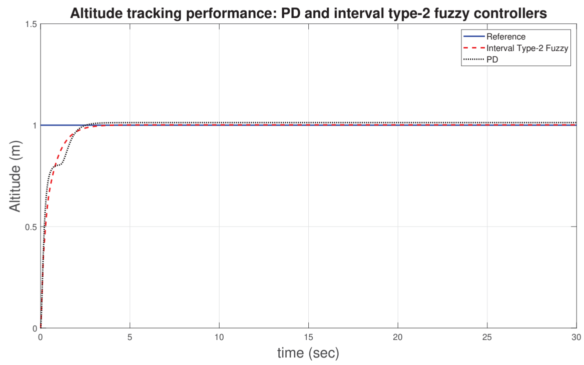

Fig. 7. Altitude tracking performance of a quadcopter using interval type-2 controller (Step reference)

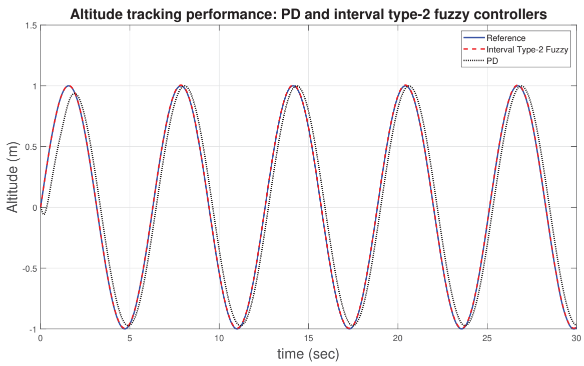

Fig. 8. Altitude tracking performance of a quadcopter using interval type-2 controller (Sinusoidal reference)

TABLE II
INTERVAL TYPE-2 AND PD CONTROLLERS PERFORMANCE (RMSE)

Reference Signal	RMSE	
	PD	Interval Type-2 fuzzy
Step input	0.2069	0.1578
Sinusoidal wave	0.1616	0.0053
Square signal	0.3012	0.2230
Constant altitude	0.9793	0.6767

V. CONCLUSION

Despite the challenges of both understanding and designing interval type-2 fuzzy controllers contrasted with other controllers, the advantages of the former remain a favored research zone for its robustness through nonlinearities and uncertainties. The tracking performance using interval type-2 fuzzy controller of a quadcopter is investigated in this study. The computer simulation results depict that the proposed interval type-2 fuzzy controller performance outperforms the PD controller. Several reference signals were tested to evaluate the tracking performance where the outcomes demonstrate that the proposed controller was able to minimize the error and bring the quadcopter to the desired trajectory and reached to a steady state in short time period. In addition, RMSE error was reduced significantly using the proposed controller. Lastly, the proposed controller can be an encouraging technique for

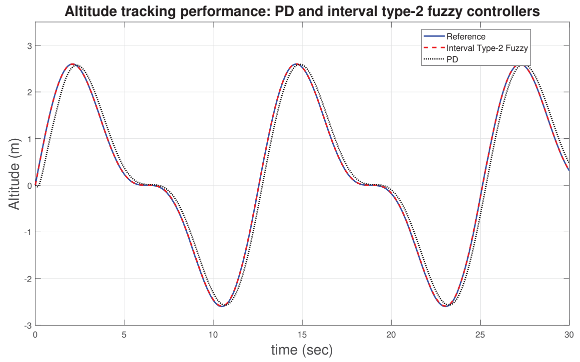

Fig. 9. Altitude tracking performance of a quadcopter using interval type-2 controller (Reference: sum of sinusoidal signals)

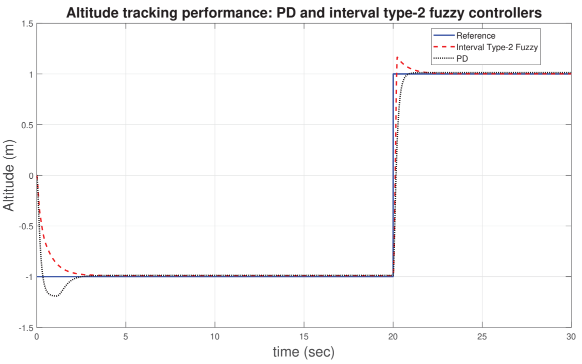

Fig. 10. Altitude tracking performance of a quadcopter using interval type-2 controller (Square wave reference)

real-time control of quadcopters. For future work, an adaptive interval type-2 fuzzy will be designed for more efficient results as well as conducting real-time experiments of a quadcopter using the proposed controller.

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