Chapter 3 Image Enhancement in the Frequency Domain

Main Content

- 3.1 Overview
- 3.2 Fourier transform and its properties
- 3.3 Lowpass Filtering
- 3.4 Highpass Filtering
- 3.5 Homomorphic Filtering

3.1 Overview

- Image enhancement approaches are divided into two categories: spatial domain methods and frequency domain methods.

  - **Spatial domain methods:** directly on process the gray value of each pixel

  - **Frequency domain methods:** First transform the image from the spatial domain to the frequency domain by Fourier transform, then process frequency spectrum in the frequency domain, finally, inversely transform it to the spatial domain, thereby obtain the enhanced image.

3.1 Overview

- **Image Transform：** A mathematical calculation to image in order to achieve a particular purpose.
- **Physics Background：** Map the original image to another space, so that certain characteristics of the image are projecting to facilitate the subsequent processing and recognition. Usually for the converted image, most of the energy distribute at the low-frequency spectrum band, which make the image compression, transmission more favorable.
- **Application：** image enhancement, image restoration, image compression, feature extraction.

Image spatial domain $\xrightarrow{\text{Positive transform}}$ Image frequency domain $\xrightarrow{\text{Inverse transform}}$ Image spatial domain

Image processing

- **More effective**
- **More convenient**
- **Faster**

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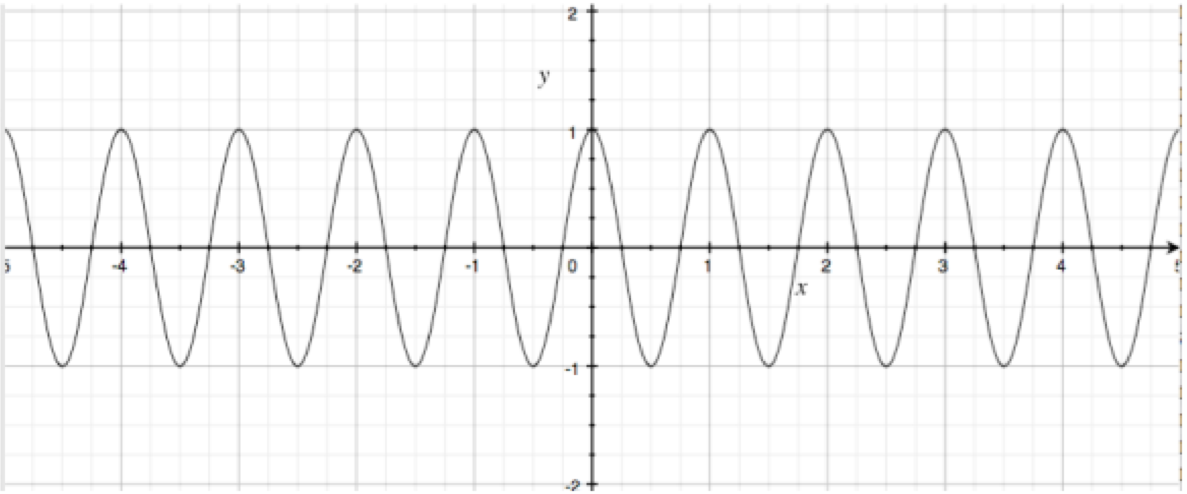
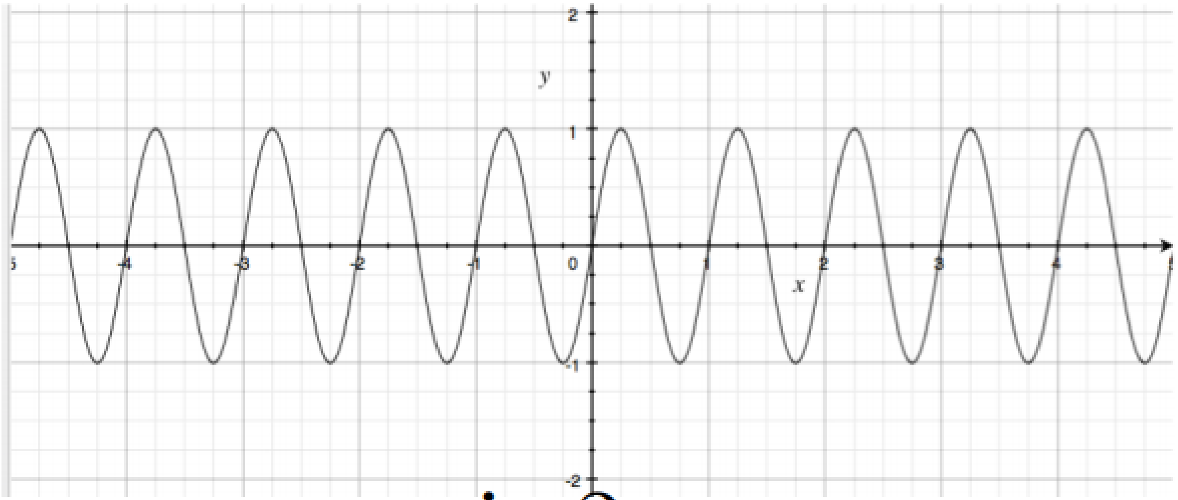
3.2 Fourier transform and its properties

$$f(t) = a_0 + \sum_{n=1}^{\infty} (a_n \cos n\omega_0 t + b_n \sin n\omega_0 t)$$

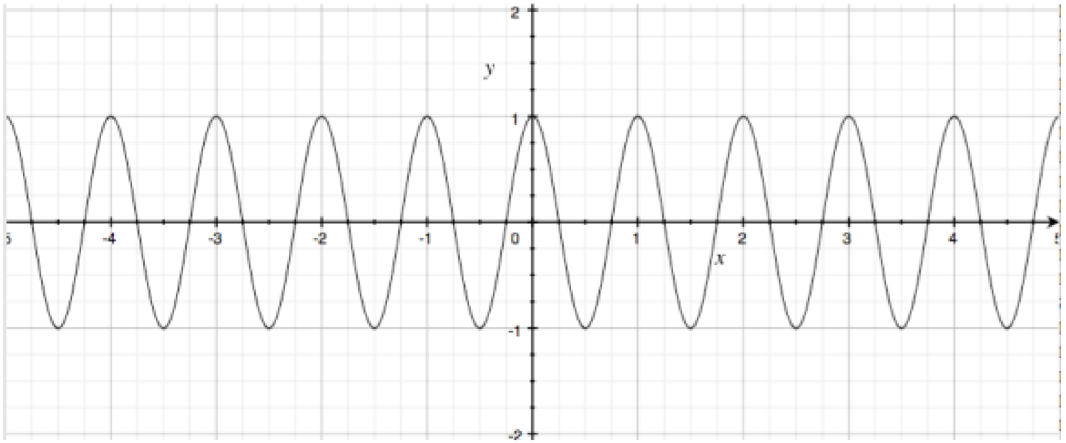
Fourier(1768-1830)



$$\cos 2\pi x$$

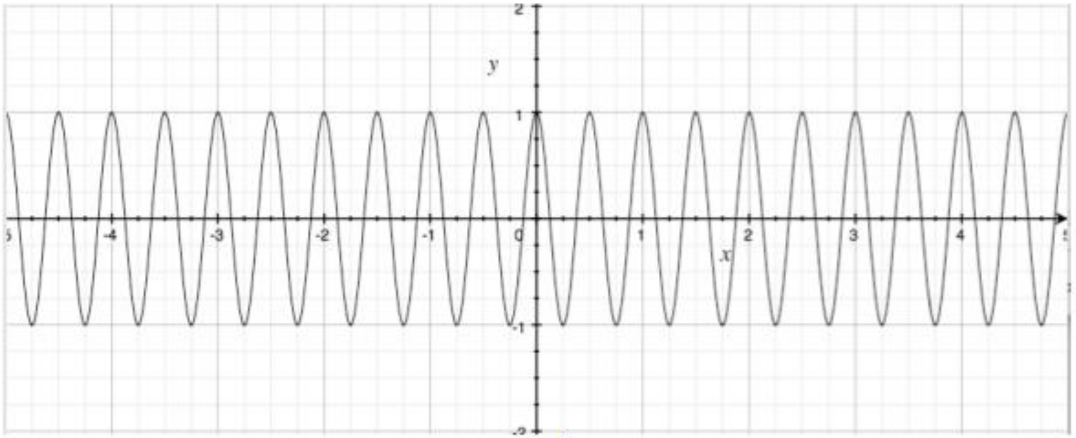


$$\sin 2\pi x$$



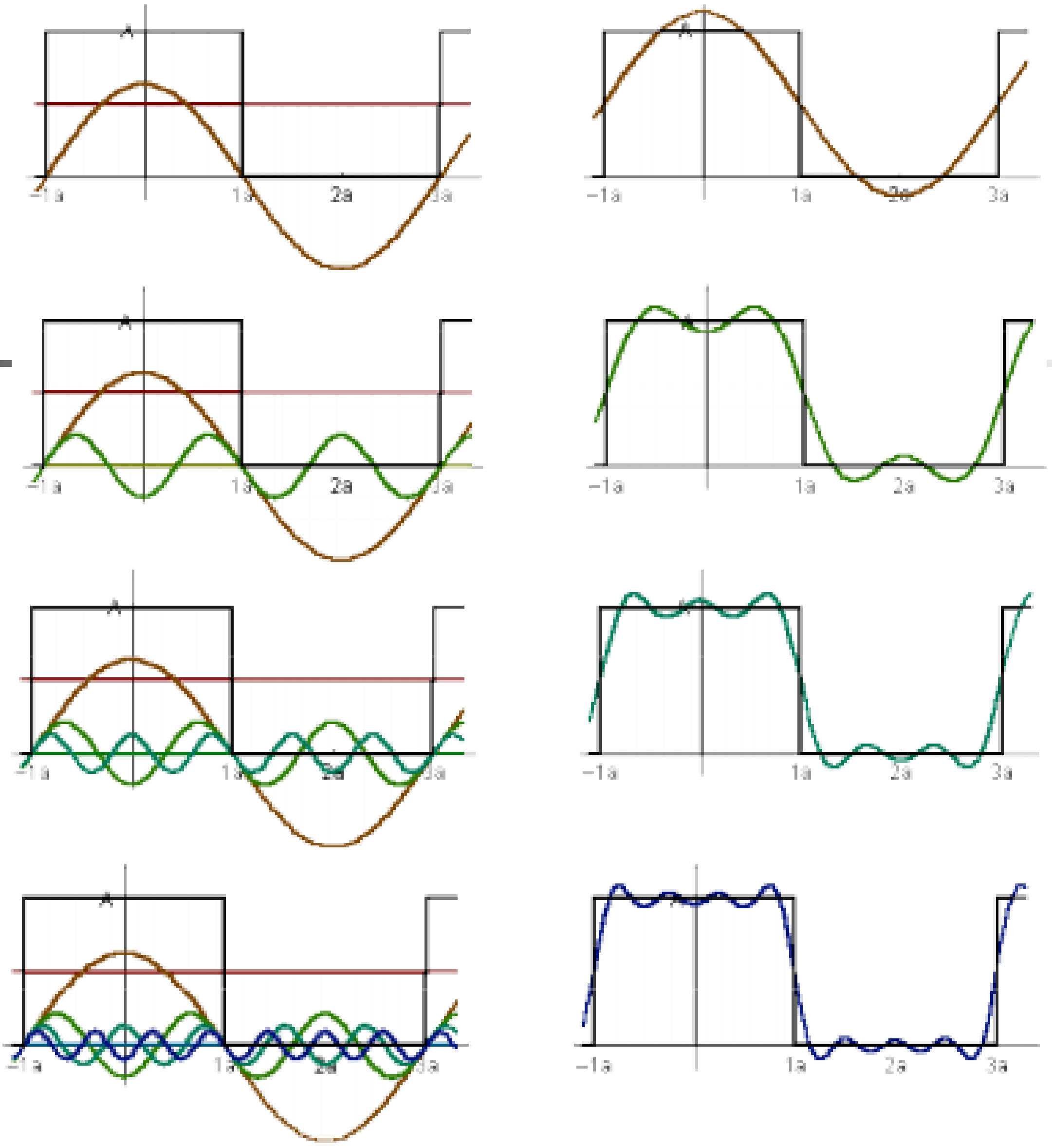
$$f = 1$$

$$\cos 2\pi x$$



$$f = 2$$

$$\cos 4\pi x$$



$$f(x) = \frac{A}{2} + \frac{2A\cos(t\omega)}{\pi} - \frac{2A\cos(3t\omega)}{3\pi} + \frac{2A\cos(5t\omega)}{5\pi} - \frac{2A\cos(7t\omega)}{7\pi} + \cdots$$

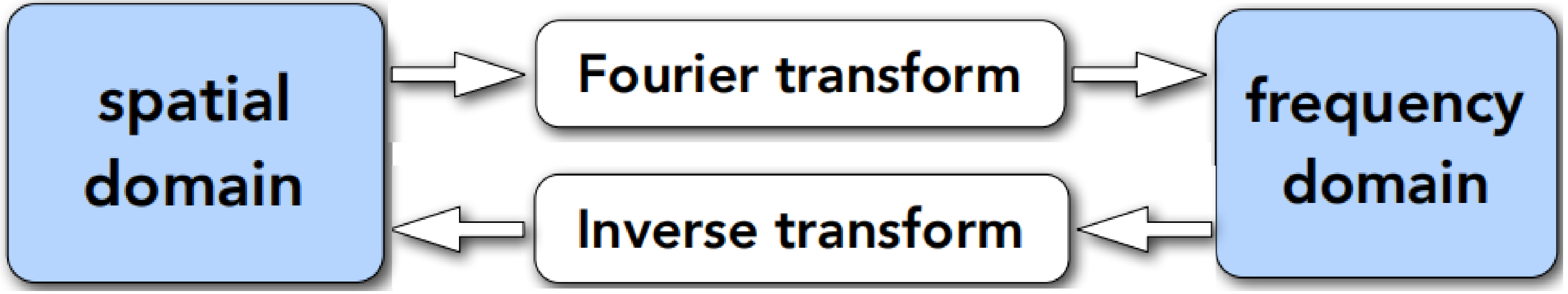
Fourier Transform

$$e^{j\theta} = \cos\theta + j\sin\theta$$

$f(x)$

$$F(u) = \int_{-\infty}^{+\infty} f(x) e^{-j2\pi ux} dx$$

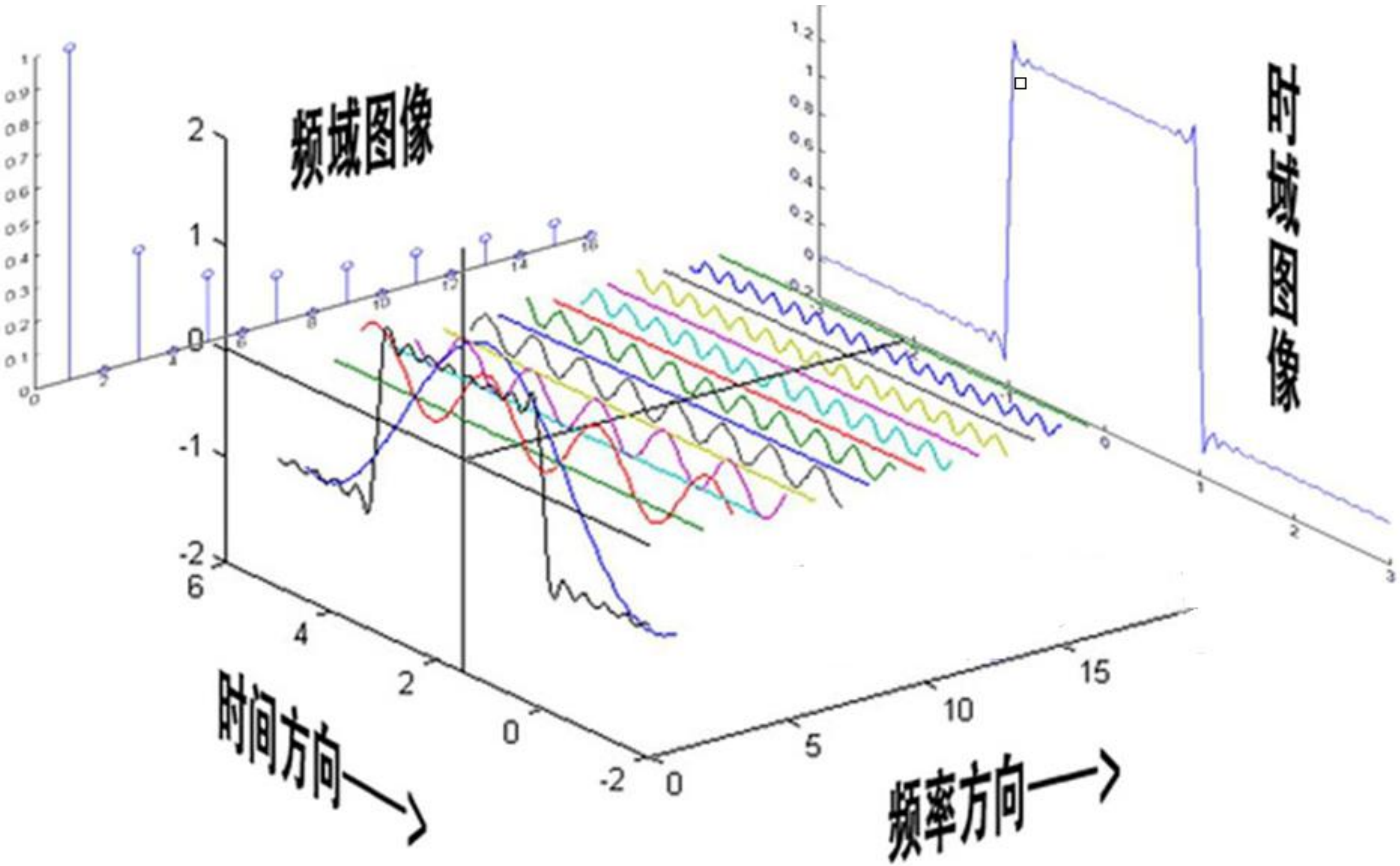
$F(u)$



$$f(x) = \int_{-\infty}^{+\infty} F(u) e^{j2\pi ux} du$$

$$F(u) = R(u) + jI(u) \qquad F(u) = |F(u)| e^{j\theta(u)}$$

$$|F(u)| = \sqrt{R^2(u) + I^2(u)} \qquad \theta(u) = \arctan\left[\frac{I(u)}{R(u)}\right]$$


频域图像
时域图像
时间方向
频率方向

离散傅里叶变换（Discrete Fourier Transform，DFT）

- 在数学中进行傅里叶变换的$f(x)$为连续信号， 而计算机处理的是数字信号；
- 数学上采用无穷大概念，而计算机只能进行有限次计算。

通常， 将受这种限制的傅里叶变换称为离散傅里叶变换（Discrete Fourier Transform，DFT）

One-Dimensional Discrete Fourier transform

$$F(u) = \frac{1}{N}\sum_{x=0}^{N-1} f(x) e^{-j2\pi ux/N} \quad u = 0,1,\cdots,N-1$$

$$f(x) = \sum_{u=0}^{N-1} F(u) e^{j2\pi ux/N} \qquad x = 0,1,\cdots,N-1$$

- Ex. : Calculate Fourier transform of 1D sequence of {1,1,2,2}.

$$F(u) = \frac{1}{N}\sum_{x=0}^{N-1} f(x) e^{-j2\pi ux/N} \qquad u = 0,1,\cdots,N-1$$

Extended to the two-dimensional transform ★

$$F(u,v)=\frac{1}{MN}\sum_{x=0}^{M-1}\sum_{y=0}^{N-1} f(x,y)e^{-j(2\pi/M)ux}e^{-j(2\pi/N)vy}$$

$$u=0,1,\cdots,M-1$$

$$v=0,1,\cdots,N-1$$

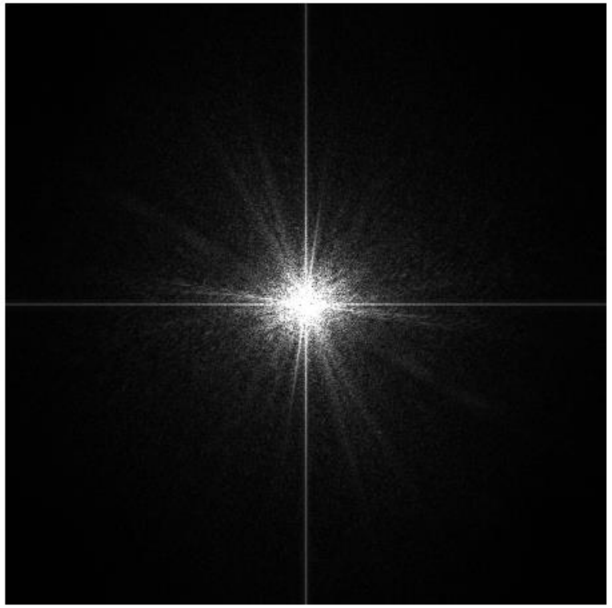
$$F(u,v)=\frac{1}{N^2}\sum_{x=0}^{N-1}\sum_{y=0}^{N-1} f(x,y)e^{-j2\pi(ux+vy)/N}$$

$$f(x,y)=\sum_{u=0}^{M-1}\sum_{v=0}^{N-1} F(u,v)e^{j(2\pi/M)ux}e^{j(2\pi/N)vy}$$

$$x=0,1,\cdots,M-1$$

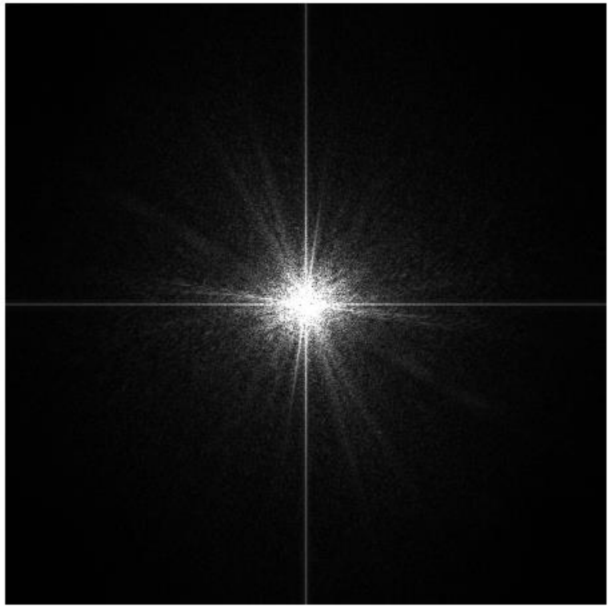
$$y=0,1,\cdots,N-1$$

图像傅里叶变换(Image Fourier transform)

图像处理中，利用傅里叶变换，可将原来在空间域研究的灰度问题，变换为在频率域研究图像的幅度、相位角等，有利于图像分析。

图像傅里叶变换(Image Fourier transform)



$$F(u,v)=\frac{1}{MN}\sum_{x=0}^{M-1}\sum_{y=0}^{N-1}f(x,y)e^{-j(2\pi/M)ux}e^{-j(2\pi/N)vy}$$

$$u=0,1,\cdots,M-1$$
$$v=0,1,\cdots,N-1$$

$$F(u,v)=\left|F(u,v)\right|e^{j\theta(u,v)}$$

$$\left|F(u,v)\right|=\sqrt{R^2(u,v)+I^2(u,v)}$$

$$\theta(u,v)=\arctan\left[\frac{I(u,v)}{R(u,v)}\right]$$

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- Calculate the Fourier transform of 2D function f(x,y), where f(0,0)=1, f(0,1)=1, f(1,0)=1, f(1,1)=1.

$$F(u,v)=\frac{1}{N^2}\sum_{x=0}^{N-1}\sum_{y=0}^{N-1} f(x,y)e^{-j2\pi(ux+vy)/N}$$

The properties of the 2-D discrete Fourier transform

Transform pair:

$$f(x,y) \Leftrightarrow F(u,v), \qquad M = N$$

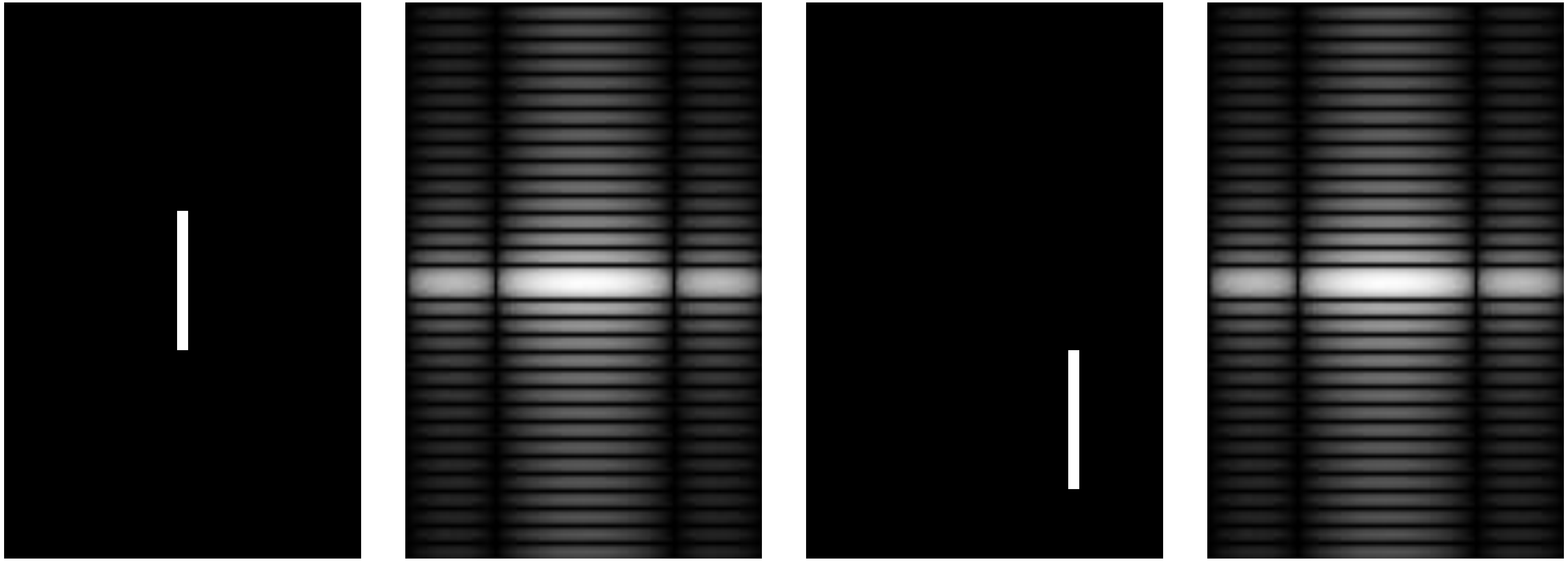
$$F(u,v) = \frac{1}{N^2}\sum_{x=0}^{N-1}\sum_{y=0}^{N-1} f(x,y)e^{-j2\pi(ux+vy)/N}$$

The properties of the 2-D discrete Fourier transform

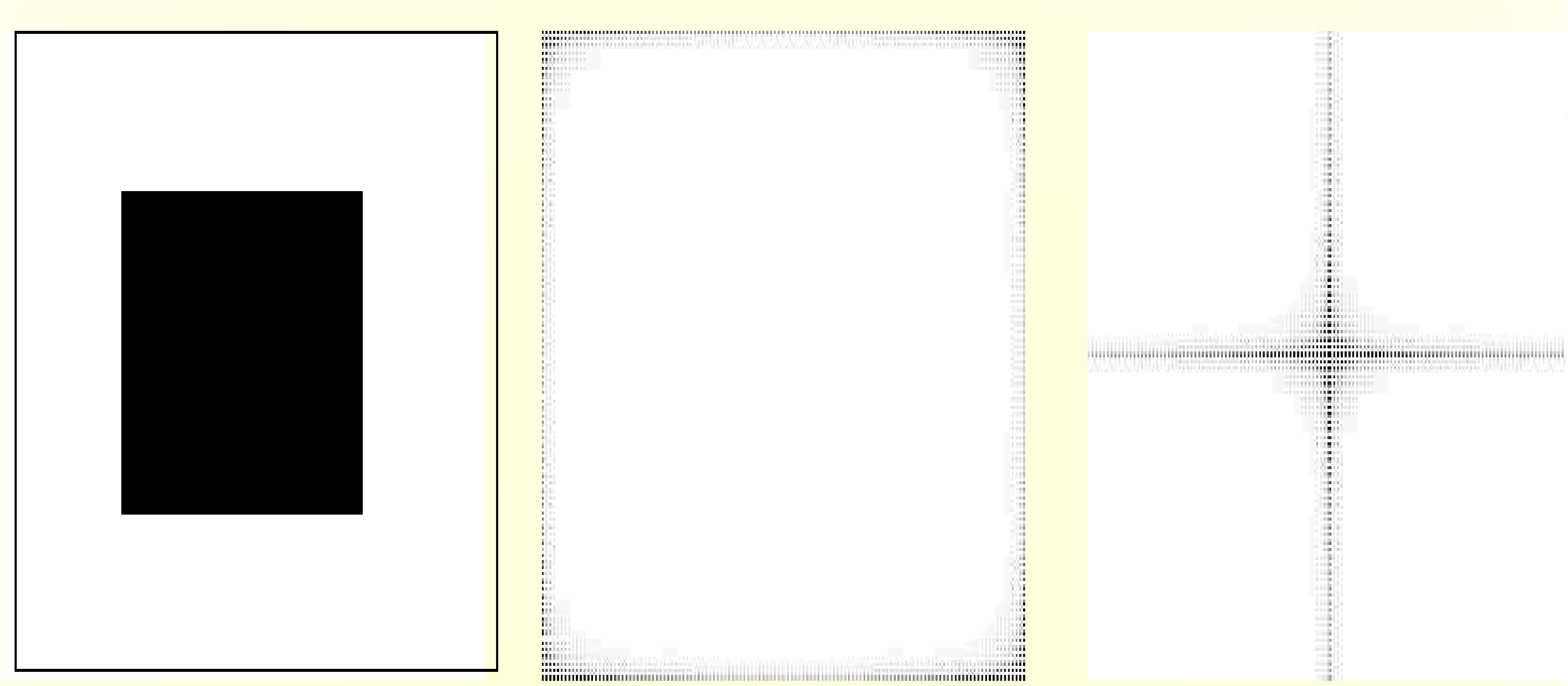
1. Translational property

$$f(x-c, y-d) \Leftrightarrow F(u,v)\mathrm{e}^{-\mathrm{j}2\pi(cu+dv)/N}$$

$$f(x, y)\mathrm{e}^{\mathrm{j}2\pi(cx+dy)/N} \Leftrightarrow F(u-c, v-d)$$



Translational invariance property of Fourier transform



(a) 原图像　(b)无平移的傅里叶频谱　(c)平移后的傅里叶频谱

思考题：如何将图像频谱的原点从初始点（0，0）移到图像的中心点（N/2，N/2）？

$$f(x-c,y-d) \Leftrightarrow F(u,v)\mathrm{e}^{-\mathrm{j}2\pi(cu+dv)/N}$$

$$f(x,y)\mathrm{e}^{\mathrm{j}2\pi(cx+dy)/N} \Leftrightarrow F(u-c,v-d)$$

证明：$f(x,y)\mathrm{e}^{\mathrm{j}2\pi(cx+dy)/N} \Leftrightarrow F(u-c,v-d)$

$$\Im[f(x,y)]=F(u,v)=\frac{1}{N^2}\sum_{x=0}^{N-1}\sum_{y=0}^{N-1} f(x,y)e^{-j2\pi(ux+vy)/N}$$

$$\Im[f(x,y)e^{j2\pi(cx+dy)/N}]$$

$$=\frac{1}{N^2}\sum_{x=0}^{N-1}\sum_{y=0}^{N-1} f(x,y)e^{j2\pi(cx+dy)/N}e^{-j2\pi(ux+vy)/N}$$

$$=\frac{1}{N^2}\sum_{x=0}^{N-1}\sum_{y=0}^{N-1} f(x,y)e^{\{-j2\pi[(u-c)x+(v-d)y)/N\}}$$

$$=F(u-c,v-d)$$

2. Separability

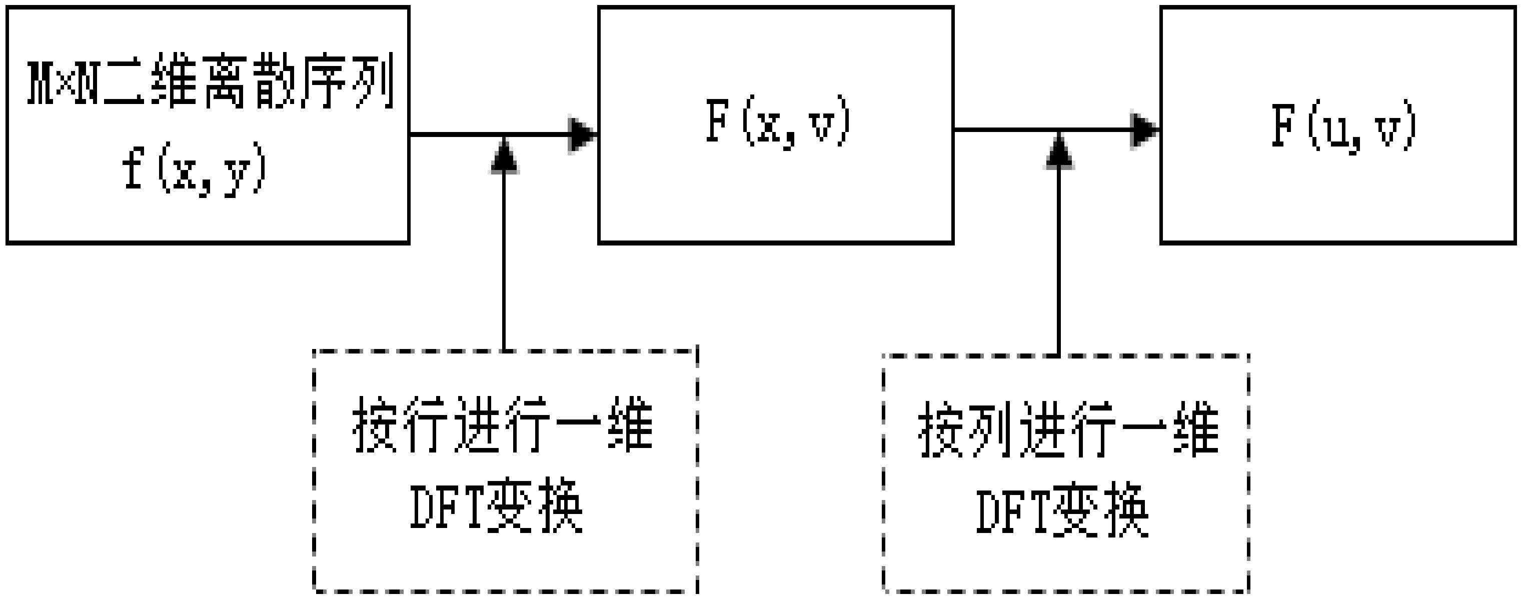
$$F(u,v)=\frac{1}{N^2}\sum_{x=0}^{N-1}\sum_{y=0}^{N-1}f(x,y)\mathrm{e}^{-\mathrm{j}2\pi\frac{ux+vy}{N}}$$

$$=\frac{1}{N}\sum_{x=0}^{N-1}\mathrm{e}^{-\mathrm{j}2\pi ux/N}\,\frac{1}{N}\sum_{y=0}^{N-1}f(x,y)\mathrm{e}^{-\mathrm{j}2\pi vy/N}$$

$$=\frac{1}{N}\sum_{x=0}^{N-1}F(x,v)\mathrm{e}^{-\mathrm{j}2\pi ux/N}$$

$$F(x,v)=\frac{1}{N}\sum_{y=0}^{N-1}f(x,y)\mathrm{e}^{-\mathrm{j}2\pi vy/N}$$

2-D Fourier transform can be decomposed into 1-D transform in two directions.



If 2-D sequence F (u, v) is known, the 2-D Separability is equally adaptable to the inverse Fourier transform.

$$f(x,y)=\sum_{u=0}^{N-1}\sum_{v=0}^{N-1}F(u,v)e^{j2\pi(\frac{ux+vy}{N})}$$

$$=\sum_{u=0}^{N-1}\{[\sum_{v=0}^{N-1}F(u,v)e^{j2\pi\frac{vy}{N}}]e^{j2\pi\frac{ux}{N}}\}$$

$$=\sum_{u=0}^{N-1}[f(u,y)e^{j2\pi\frac{ux}{N}}]$$

$$x,y=0,1,2,\cdots,N-1$$

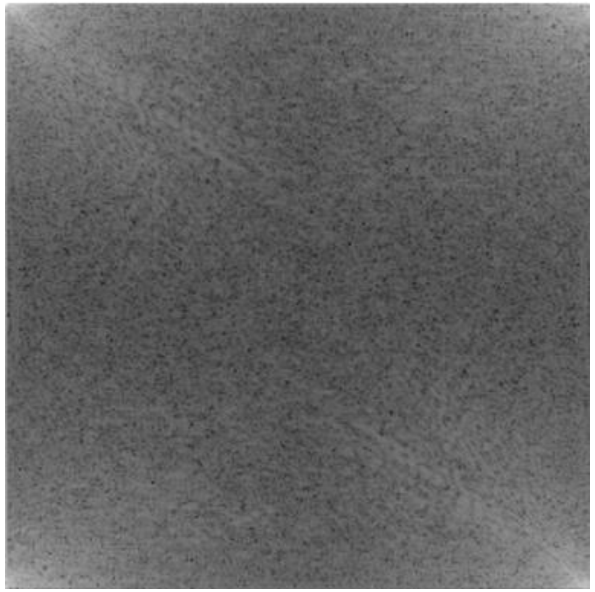
3. Periodic property and Conjugate symmetric property

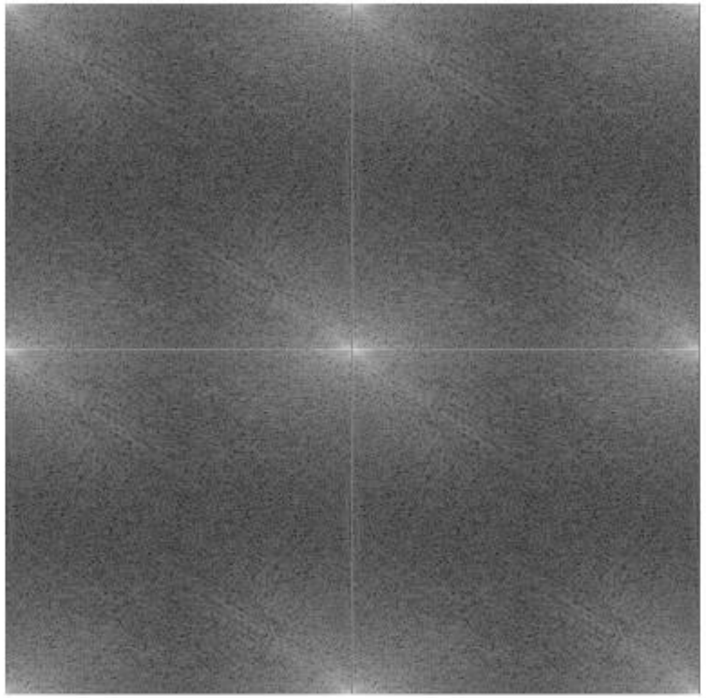
Both the positive and inverse Fourier transform are periodic functions with the cycle N.

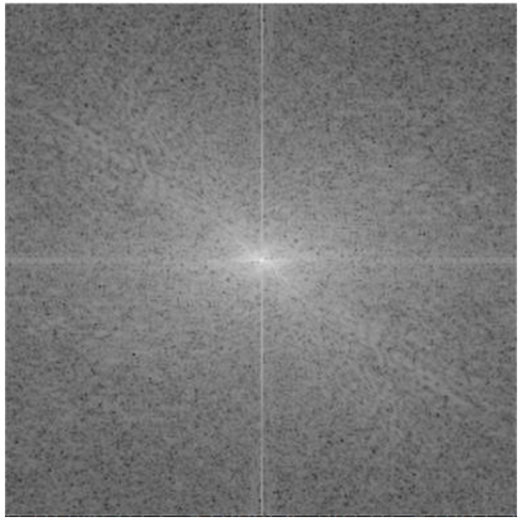
$$F(u,v) = F(u+mN, v+nN)$$

$$f(x,y) = f(x+mN, y+nN)$$

$$m,n = 0, \pm 1, \pm 2, \cdots$$





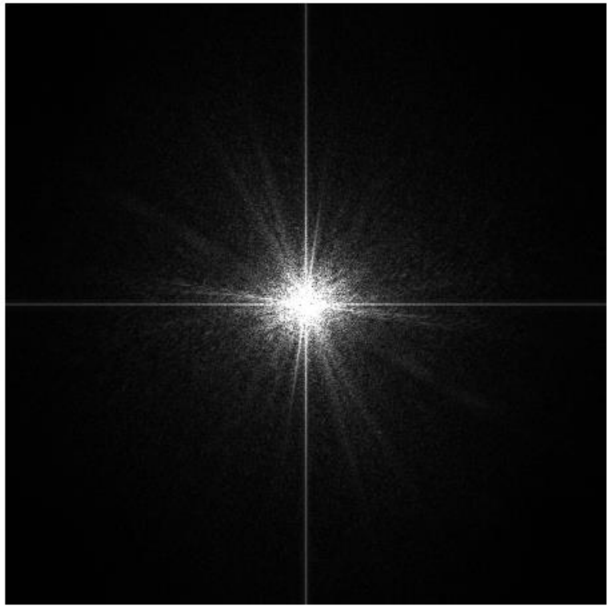


3. Periodic property and Conjugate symmetric property

If f (x, y) is a real function, its Fourier transform is conjugate symmetric, i.e.

$$F^*(u,v) = F(-u,-v)$$

$$|F(u,v)| = |F(-u,-v)|$$



4. Rotating property

Let $x = r\cos\theta, y = r\sin\theta$

$u = w\cos\phi, v = w\sin\phi$

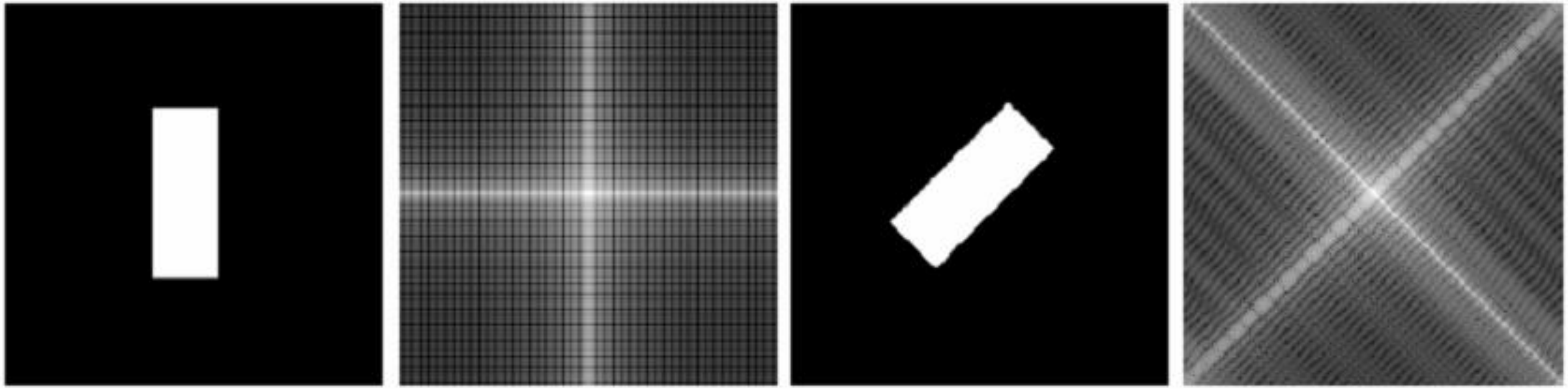
Convert $f(x,y)$ **into** $f(r,\theta)$

$F(u,v)$ **into** $F(w,\phi)$

The transform pair in the polar coordinate system

$$\boxed{f(r,\theta+\theta_0) \Leftrightarrow F(w,\phi+\theta_0)}$$

Rotating property sample of Fourier transform



(a) (b) (c) (d)

傅里叶变换旋转性质示例

5. 尺度定理(Scales theorem)

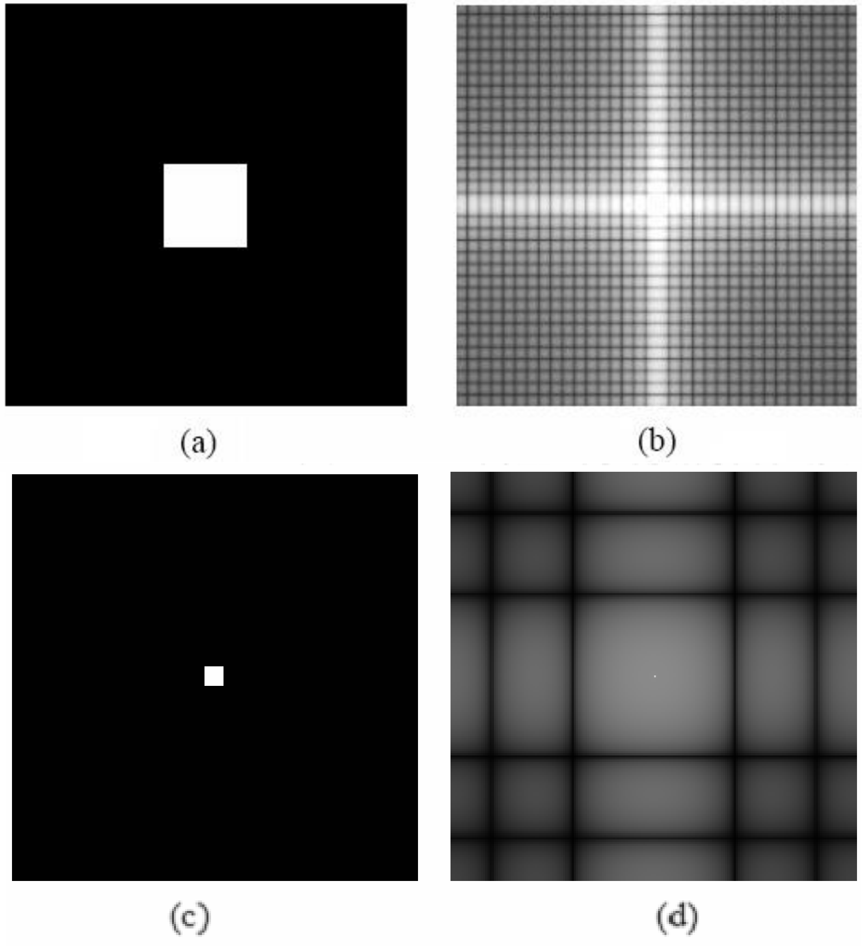
$$f(ax,by) \Leftrightarrow \frac{1}{|ab|}F(\frac{u}{a},\frac{v}{b})$$

对*f*(*x*, *y*)的收缩(对应*a*>1, *b*>1)不仅导致*F*(*u*, *v*)的膨胀，还使*F*(*u*, *v*)的幅度减小。

对*f*(*x*, *y*)的膨胀(对应*a*<1, *b*<1)不仅导致*F*(*u*, *v*)的收缩，还使*F*(*u*, *v*)的幅度变大。

Scale change property sample of Fourier transform

正方形收缩导致其傅里叶频谱网格在频谱空间的增大



6. Linearity

$$F\left\{af_1(x,y)+b\,f_2(x,y)\right\}=aF\left\{f_1(x,y)\right\}+bF\left\{f_2(x,y)\right\}$$

$$F\left\{f_1(x,y)\bullet f_2(x,y)\right\} \neq F\left\{f_1(x,y)\right\}\bullet F\left\{f_2(x,y)\right\}$$

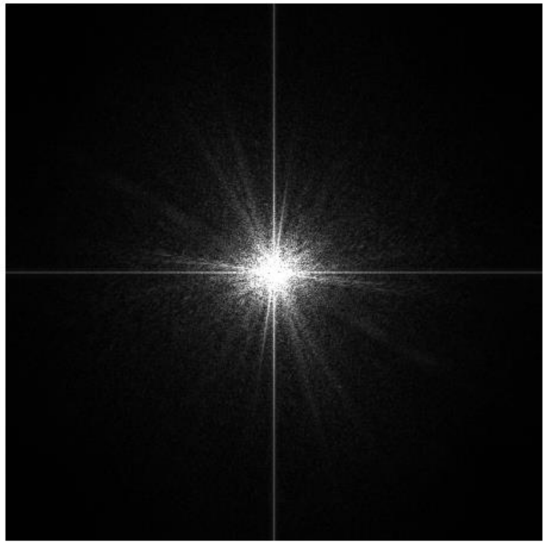
$$F(u,v)=\frac{1}{N^2}\sum_{x=0}^{N-1}\sum_{y=0}^{N-1} f(x,y)e^{-j2\pi(ux+vy)/N}$$

7. Average property

The average of 2D discrete function is defined as

$$\tilde{f}(x,y)=\frac{1}{N^2}\sum_{x=0}^{N-1}\sum_{y=0}^{N-1} f(x,y)$$

When *u=v=0* ,its Fourier transform



$$F(0,0)=\frac{1}{N^2}\sum_{x=0}^{N-1}\sum_{y=0}^{N-1} f(x,y)$$

$$\tilde{f}(x,y)=F(0,0)$$

8. Convolution theorem

Let f(x,y), h(x,y) be discrete function of M×N,

$$f(x,y) \Leftrightarrow F(u,v) \qquad h(x,y) \Leftrightarrow H(u,v)$$

then

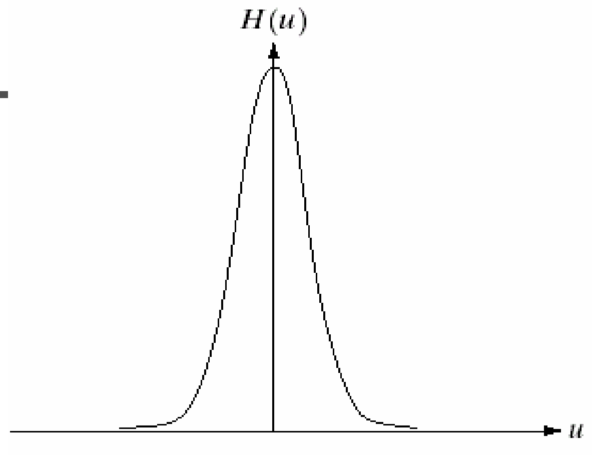
$$f(x,y) * h(x,y) \Leftrightarrow F(u,v)H(u,v)$$

$$f(x,y)h(x,y) \Leftrightarrow F(u,v) * H(u,v)$$

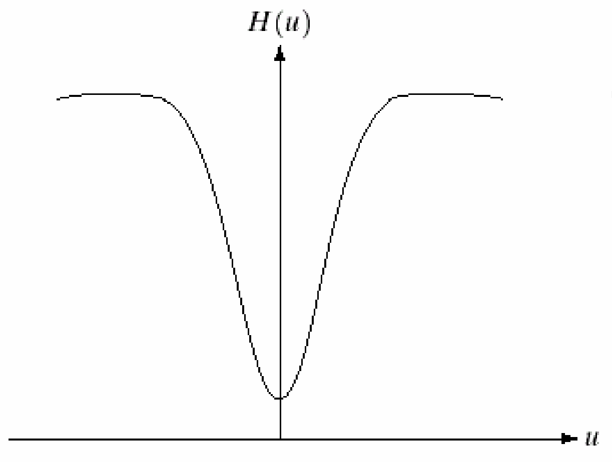
$$f(x,y) * \mathrm{h}(x,y) = \sum_{m=0}^{\mathrm{M}-1} \sum_{n=0}^{N-1} \mathrm{h}(m,n)\mathrm{f}(x-m, y-n)$$

$$g(x,y) = \sum_{s=-a}^{a} \sum_{t=-b}^{b} w(s,t) f(x+s, y+t)$$

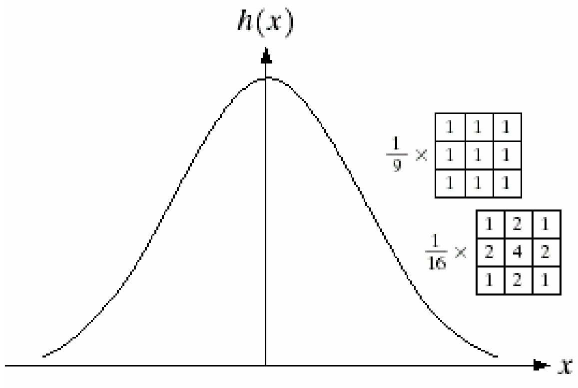
频域高斯低通滤波器



频域高斯高通滤波器



空域高斯低通滤波器及模板



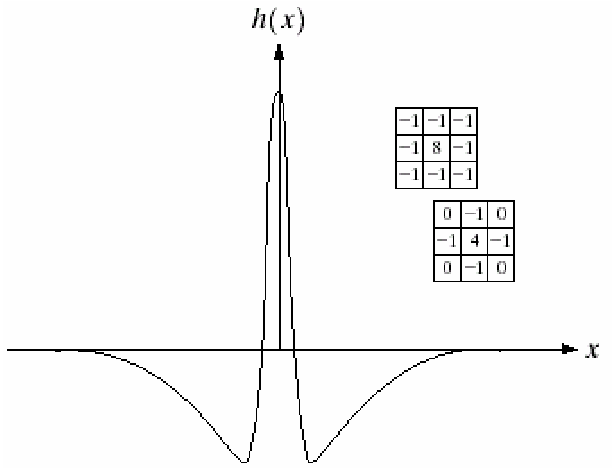
$\frac{1}{9}\times$

1	1	1
1	1	1
1	1	1

$\frac{1}{16}\times$

1	2	1
2	4	2
1	2	1

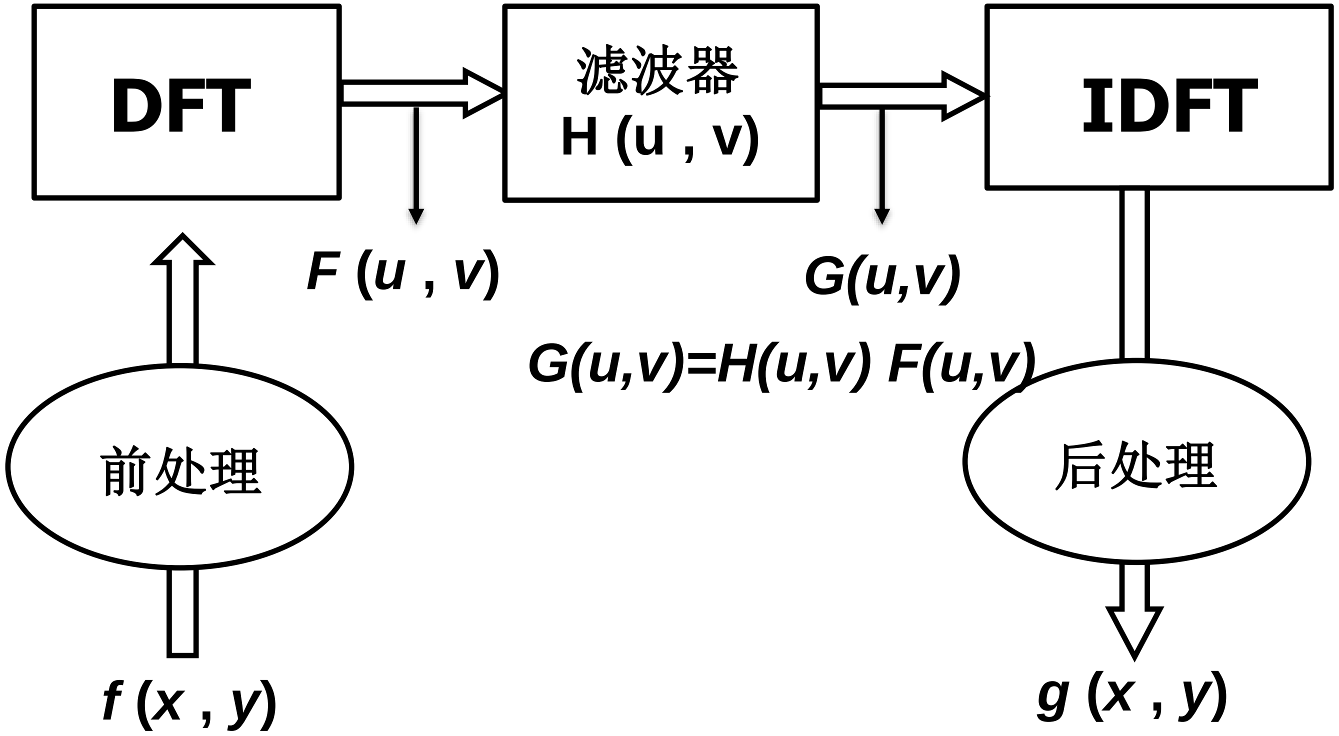
空域高斯高通滤波器及模板



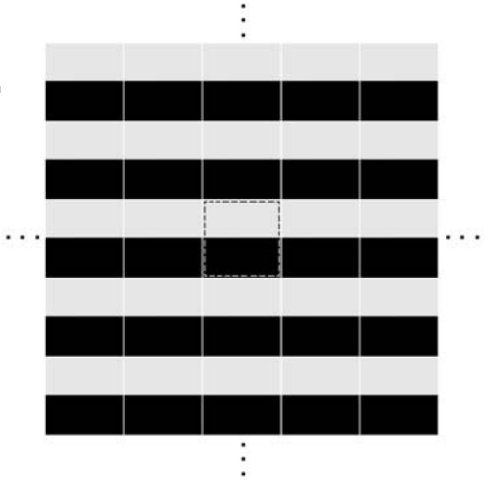
−1	−1	−1
−1	8	−1
−1	−1	−1

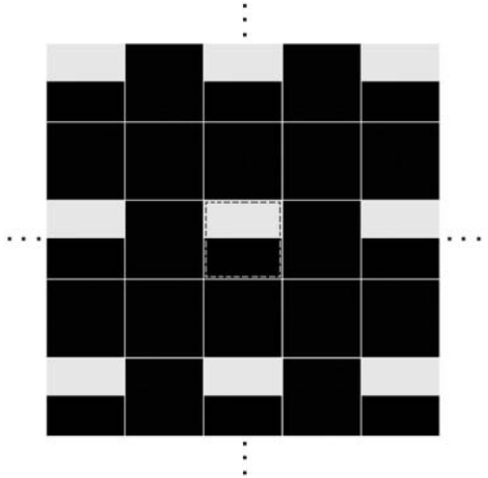
0	−1	0
−1	4	−1
0	−1	0

3.2.6 频率域滤波基础



错误的填充图像会导致错误的结果





a b c

FIGURE 4.5 (a) A simple image of size 256 × 256. (b) Image lowpass-filtered in the frequency domain without padding. (c) Image lowpass-filtered in the frequency domain with padding. Compare the light portion of the vertical edges in (b) and (c).

频率域的滤波步骤：

1、对要滤波的图像 $f_{M\times N}(x,y)$ 进行填充得到 $f_{P\times Q}(x,y)$，典型地：P=2M,Q=2N

2、用 $(-1)^{x+y}$ 乘以输入图像

$$f_{P\times Q}(x,y)\cdot(-1)^{x+y} \Leftrightarrow F(u-\frac{P}{2},v-\frac{Q}{2})$$

3、变换到频域 $F(u,v)=\Im\left[f_{P\times Q}(x,y)(-1)^{x+y}\right]$

4、实的、中心对称的滤波器 $H_{P\times Q}(u,v)$，中心在 $(\frac{P}{2},\frac{Q}{2})$

频域滤波: $G(u,v)=H(u,v)F(u,v)$

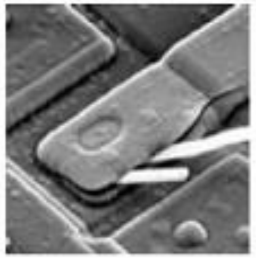
5、变换到空间域: $g_{P\times Q}(x,y)=\Im^{-1}\left[H(u,v)F(u,v)\right]$

取实部: $real\left\{g_{P\times Q}(x,y)\right\}$

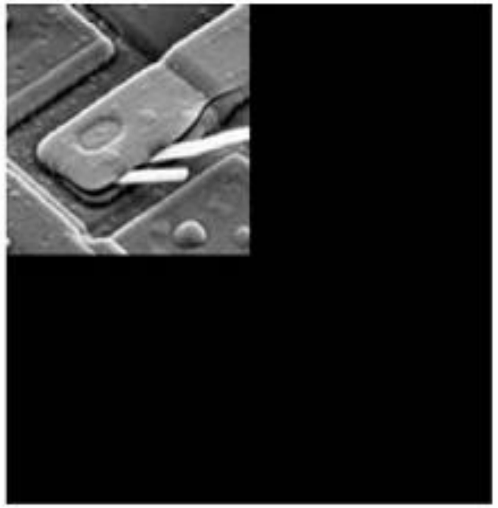
6、取消输入图像的乘数: $g_p(x,y)=real\left\{g_{P\times Q}(x,y)\right\}\cdot(-1)^{x+y}$

7、提取$M\times N$区域: $g_{M\times N}(x,y)=\left\{g_{P\times Q}(x,y)\right\}$的对应部分

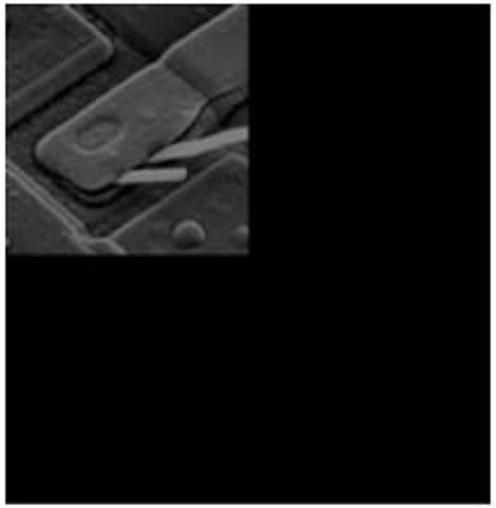
$f_{M\times N}(x,y)$



$f_{P\times Q}(x,y)$

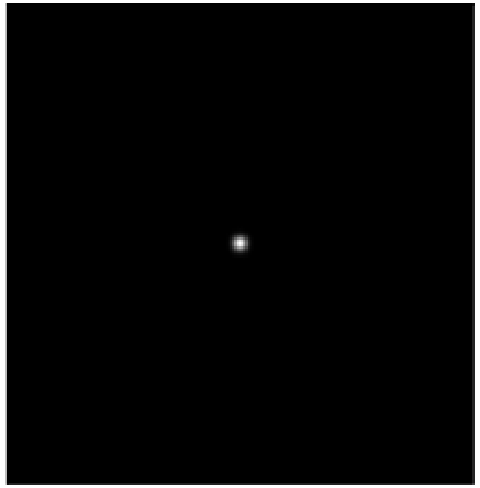


$(-1)^{x+y} f_{P\times Q}(x,y)$

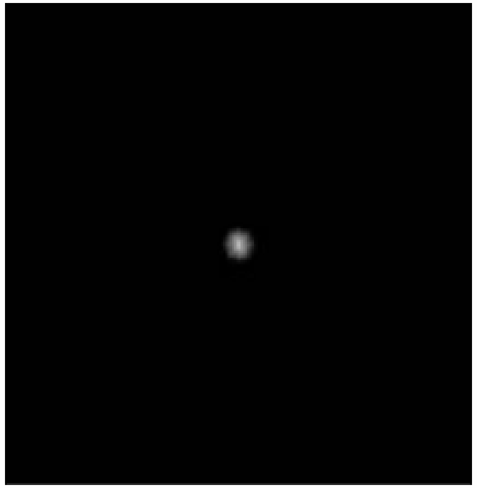


$\left|F_{P\times Q}(u,v)\right|$

$H_{LP}(u,v) : P\times Q$



$H_{LP}(u,v)\cdot F_{P\times Q}(u,v)$



$(-1)^{x+y}\cdot real\left\{g_{P\times Q}(x,y)\right\}$

$g_{M\times N}(x,y)$

- **g (x, y) can enhance a particular aspect of f (x, y), such as the use of the transfer function H (u, v) highlights the high-frequency components, which enhances the edges of the image, i.e., high-pass filtering; if the low frequency component is enhanced, the image will appear smooth, i.e., low-pass filtering.**

将傅里叶变换和图像中的亮度变化联系起来

- 高频部分对应图像边缘和灰度级突变的部分;
- 低频部分对应图像缓慢变化的分量;
- 直流分量F(0,0)对应一幅图像的平均灰度。

Main Content

- 3.1 Overview
- 3.2 Fourier transform and its properties
- 3.3 Lowpass Filtering
- 3.4 Highpass Filtering
- 3.5 Homomorphic Filtering

3.3 Lowpass Filtering

- **Like smoothing filter, Lowpass filters can eliminate random noise in the image, weaken the edge effect, thereby smooth the image.**
- **There are three common lowpass filters H (u, v): ideal lowpass filters, Butterworth low-pass filters, Gaussian lowpass filters.**

3.3 Lowpass Filtering

1. Ideal Lowpass Filters

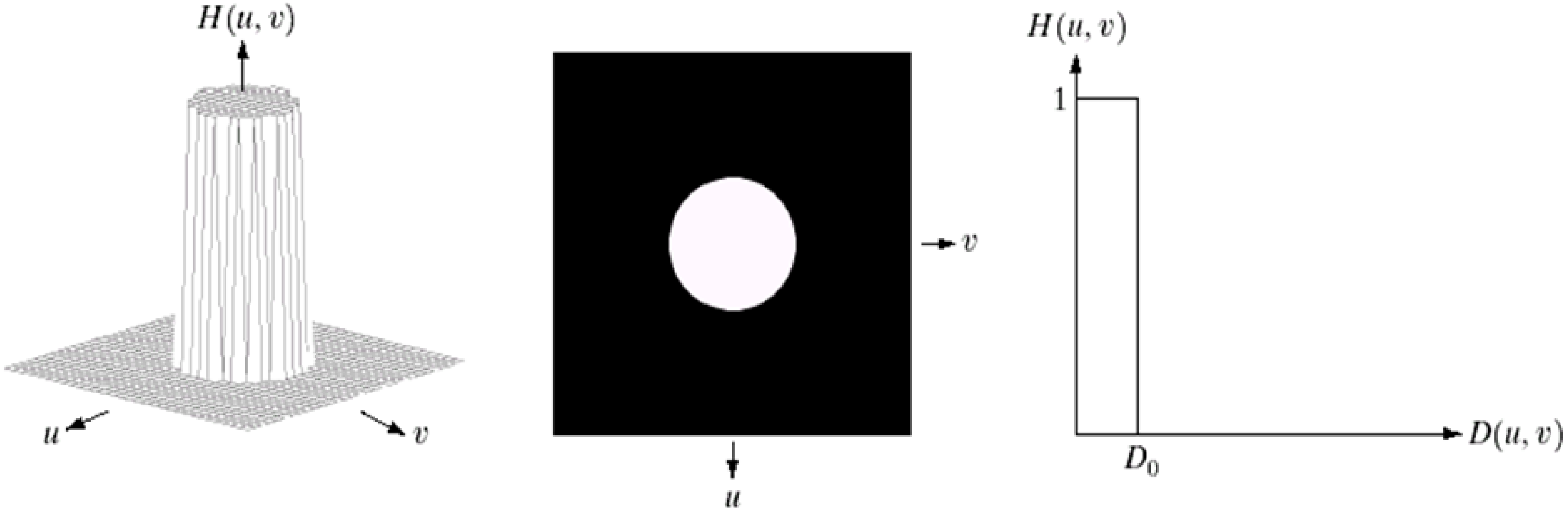
A two-dimensional ideal lowpass filter has the transfer function:

$$H(u,v)=\begin{cases}1 & if \quad D(u,v)\le D_0\\ 0 & if \quad D(u,v)>D_0\end{cases}$$

where D_0 is a specified nonnegative quantity, and D(u,v) is the distance from point (u,v) to the center of the frequency rectangle

$$D(u,v)=\left[\left(u-\frac{P}{2}\right)^2+\left(v-\frac{Q}{2}\right)^2\right]^{\frac{1}{2}}$$

3.3 Lowpass Filtering

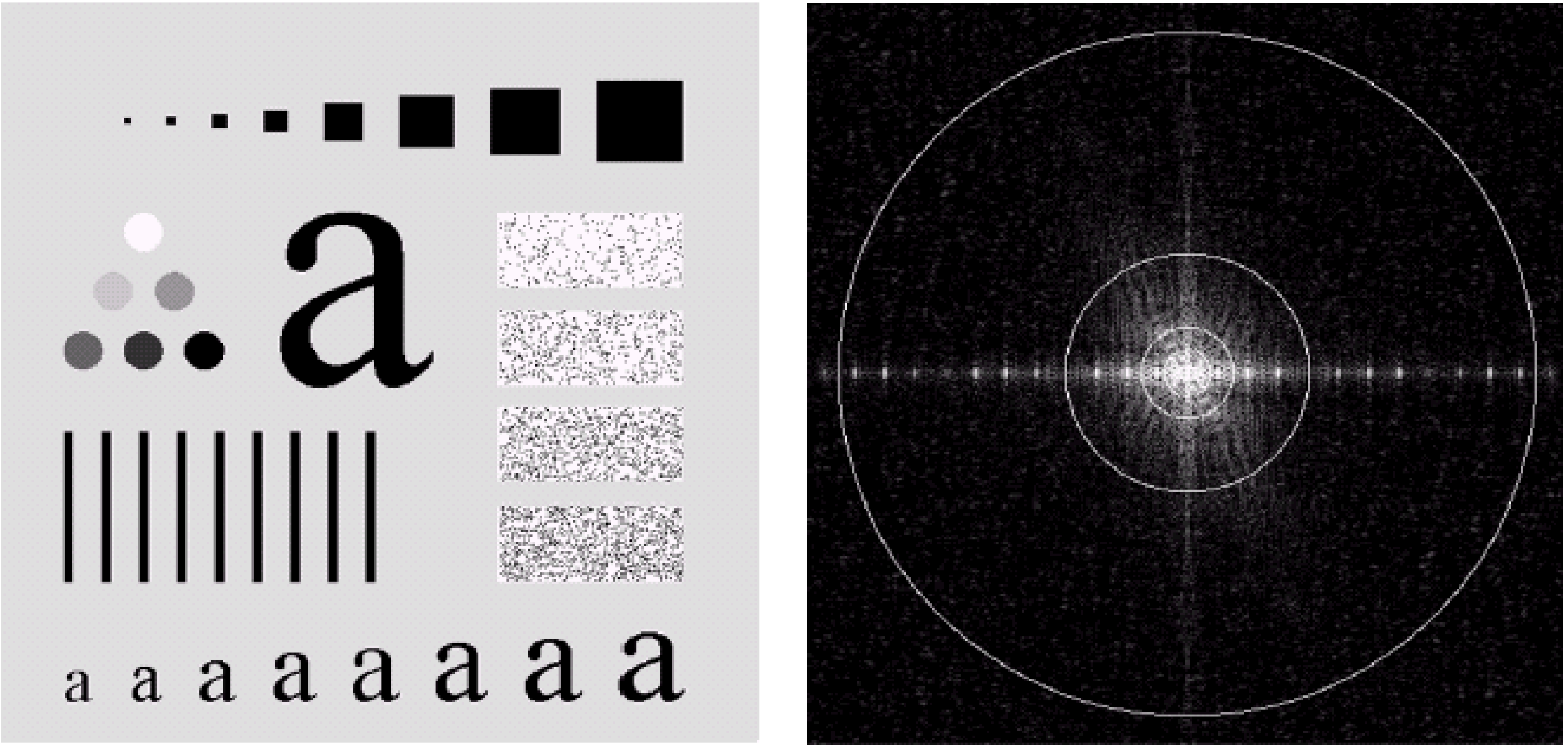


a b c

(a) Perspective plot of an ideal lowpass filter transfer function. (b) Filter displayed as an image. (c) Filter radial cross section.

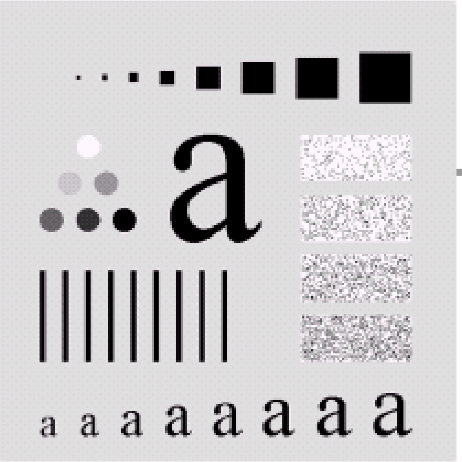
The name *ideal filter* indicates that all frequencies inside a circle of radius D_0 are passed with no attenuation, whereas all frequencies outside this circle are completely attenuated.

3.3 Lowpass Filtering

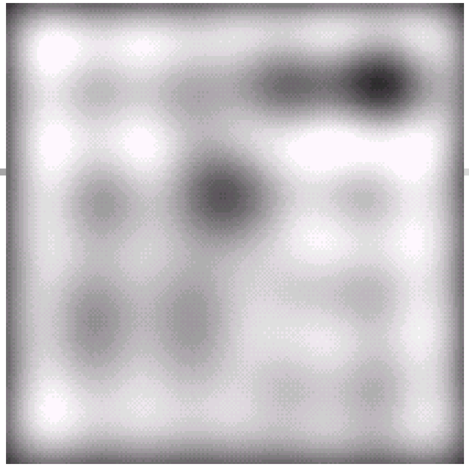


(a)Test pattern of size 688*688 pixels, and (b) its Fourier spectrum. The superimposed circles have radii equal to 10,30,60,160,and 460 with respect to the full-size spectrum image. These radii enclose 87.0, 93.1, 95.7, 97.8,and 99.2% of the padded image power, respectively.

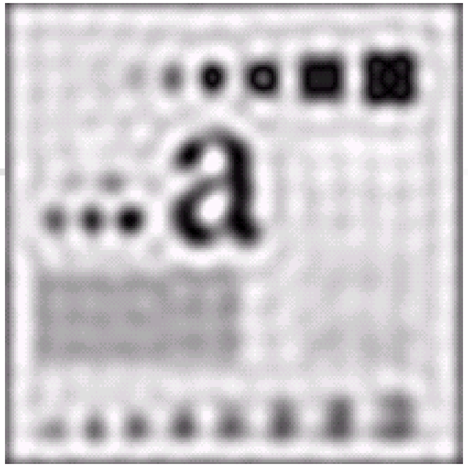
3.3 Lowpass Filtering



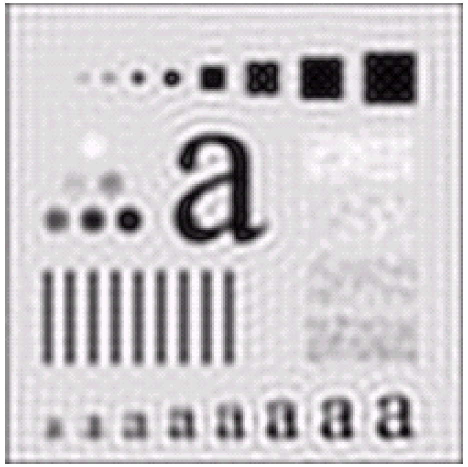
原始图



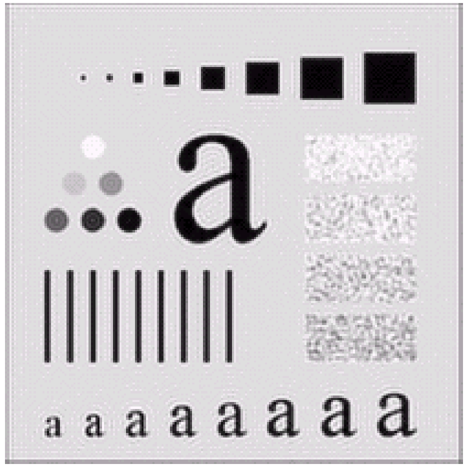
D_0=10的ILPF滤波
损失能量为13%



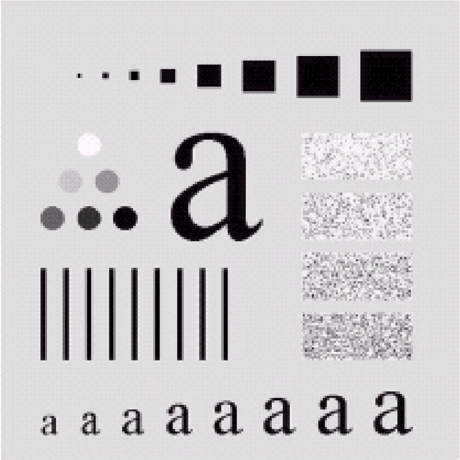
D_0=30的ILPF滤波
损失能量为6.9%



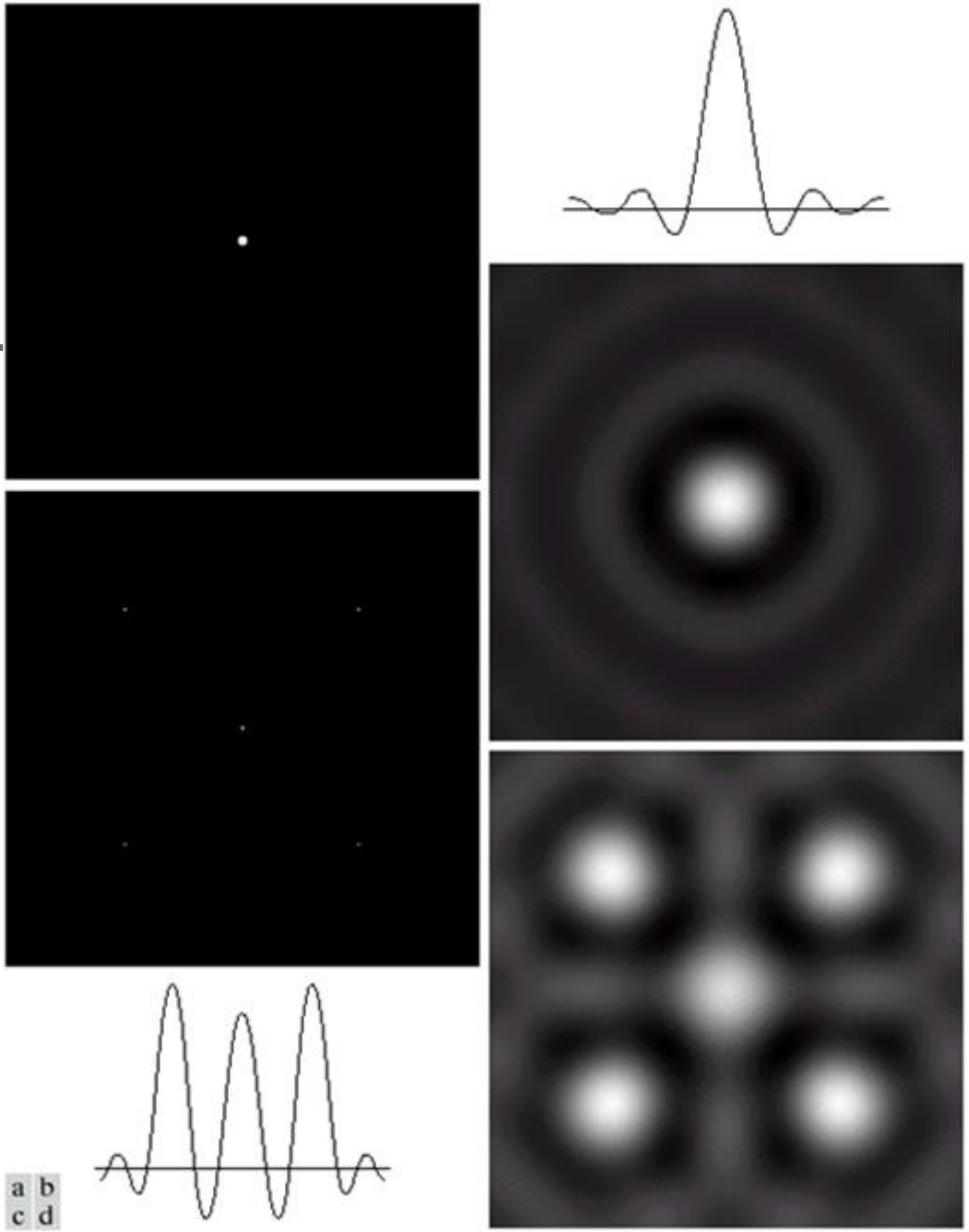
D_0=60的ILPF滤波
损失能量为4.3%



D_0=160的ILPF滤波
损失能量为2.2%



D_0=460的ILPF滤波
损失能量为0.8%



(a) A frequency-domain ILPF of radius 10 (b) Corresponding spatial filter (note the ringing). (c) Five impulses in the spatial domain, simulating the values of five pixels. (d) Convolution of (b) and (c) in the spatial domain.

3.3 Lowpass Filtering

characteristics of Ideal low-pass filter：

- **The ideal lowpass filter eliminates high frequency components, so that the image is blurred;**
- **Due to a steep waveform transform, the inverse transform of H (u, v) has strong ringing characteristics;**
- **This ideal low-pass filter can not be used in practice.**

3.3 Lowpass Filtering

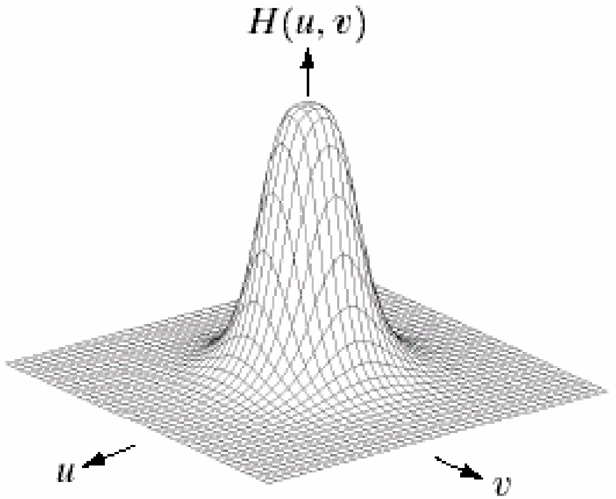
2. Butterworth Lowpass Filters

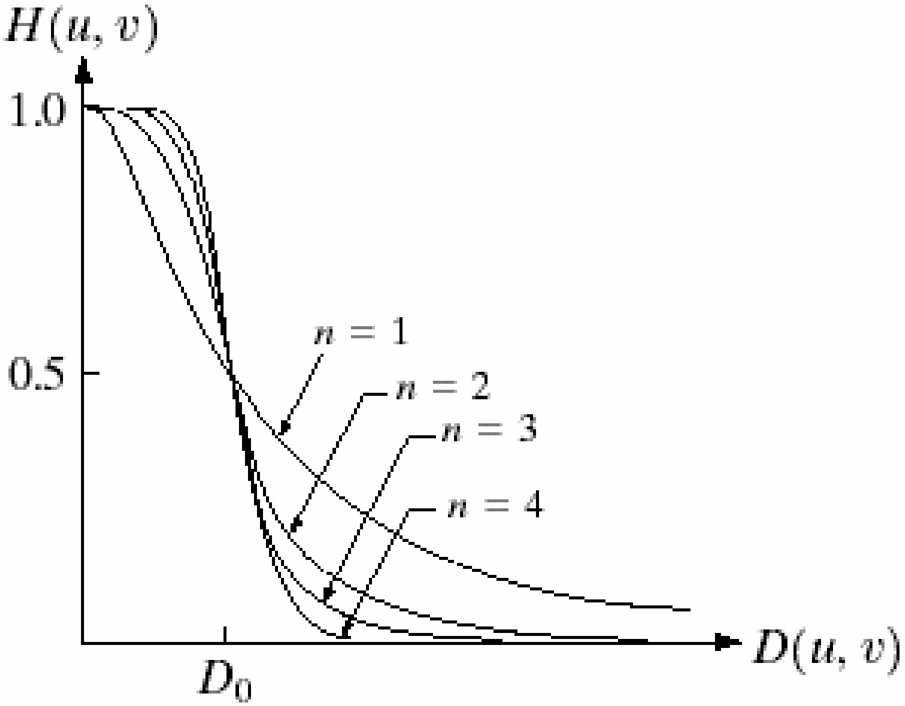
The transfer function of a Butterworth lowpass filter of order n, and with cutoff frequency at a distance D_0 from the origin, is defined as：

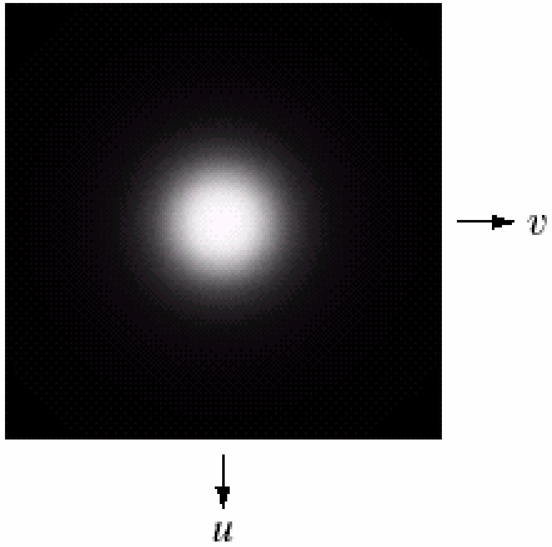
$$H(u,v) = \frac{1}{1 + [D(u,v)/D_0]^{2n}}$$

When $D(u,v)=D_0$, $H(u,v)=0.5$(down 50% from its maximum value of 1).

3.3 Lowpass Filtering







3.3 Lowpass Filtering

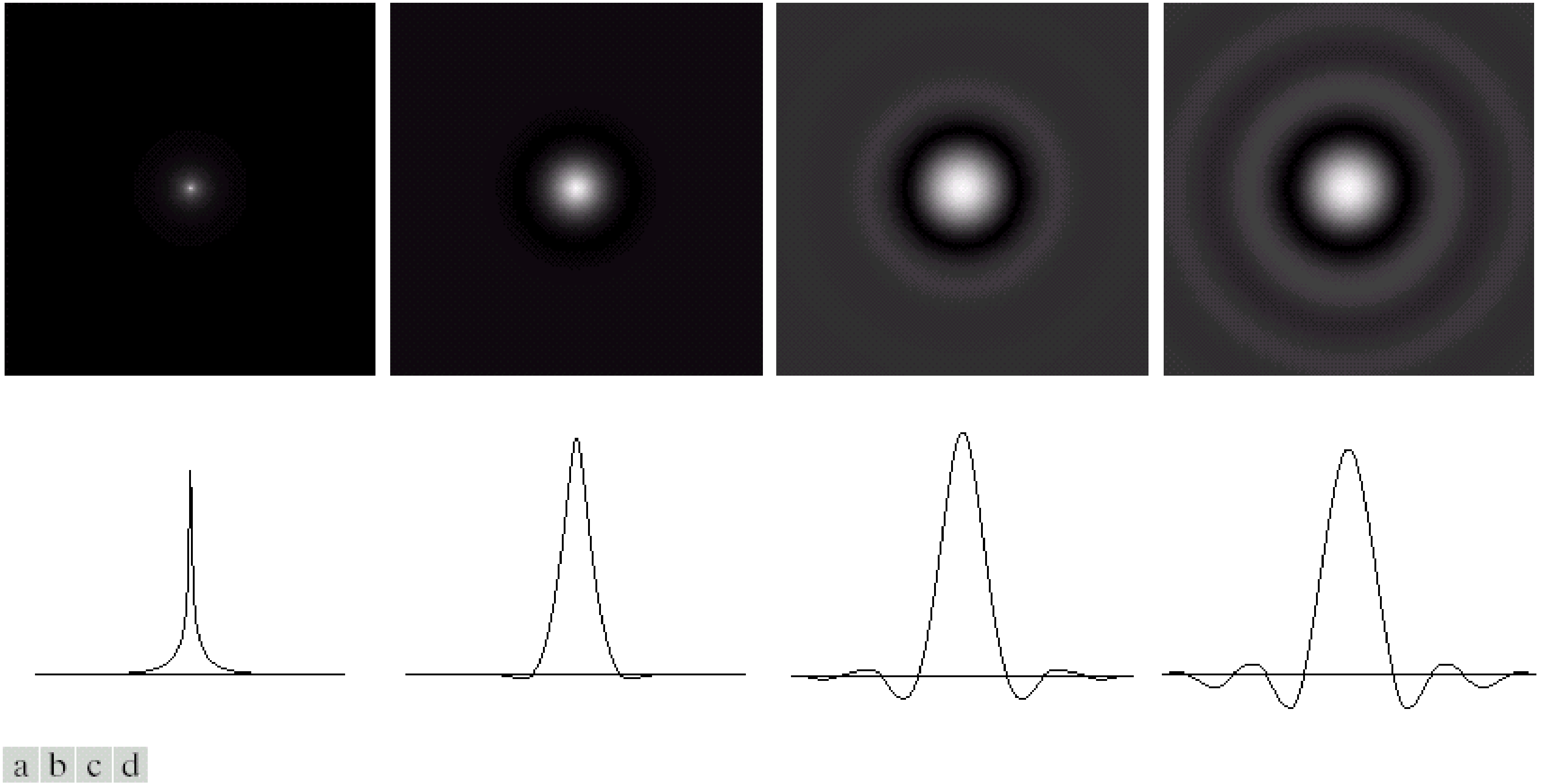
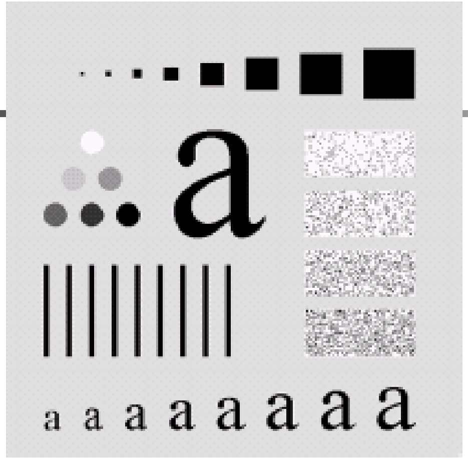
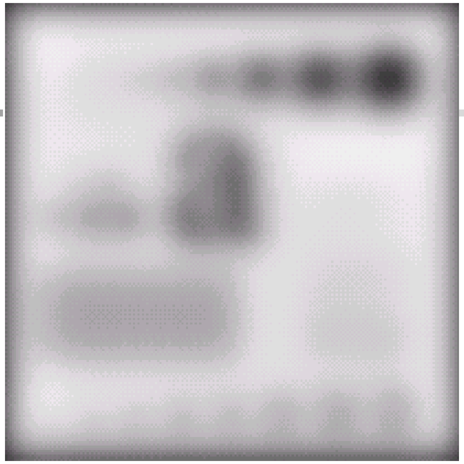


FIGURE 4.16 (a)–(d) Spatial representation of BLPFs of order 1, 2, 5, and 20, and corresponding gray-level profiles through the center of the filters (all filters have a cutoff frequency of 5). Note that ringing increases as a function of filter order.

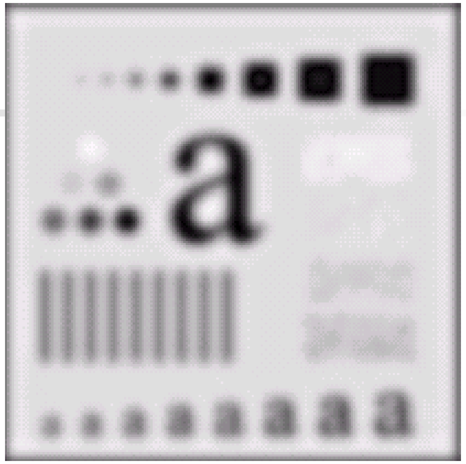
3.3 Lowpass Filtering



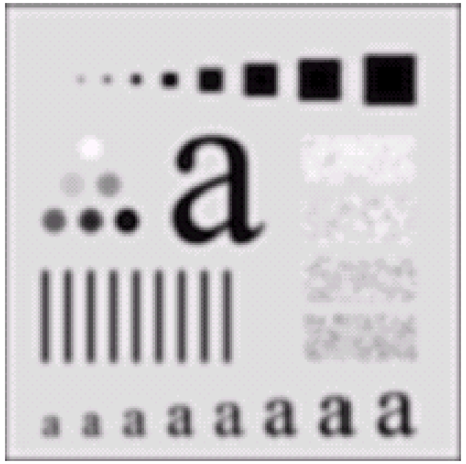
原始图



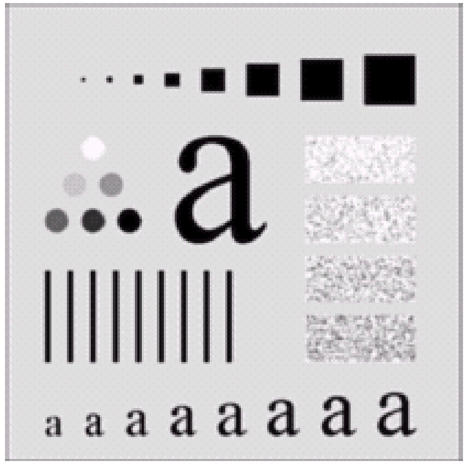
D_0=10的BLPF滤波



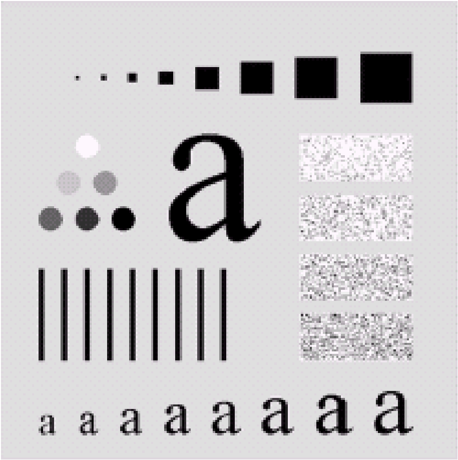
D_0=30的BLPF滤波



D_0=60的BLPF滤波



D_0=160的BLPF滤波



D_0=460的BLPF滤波

3.3 Lowpass Filtering

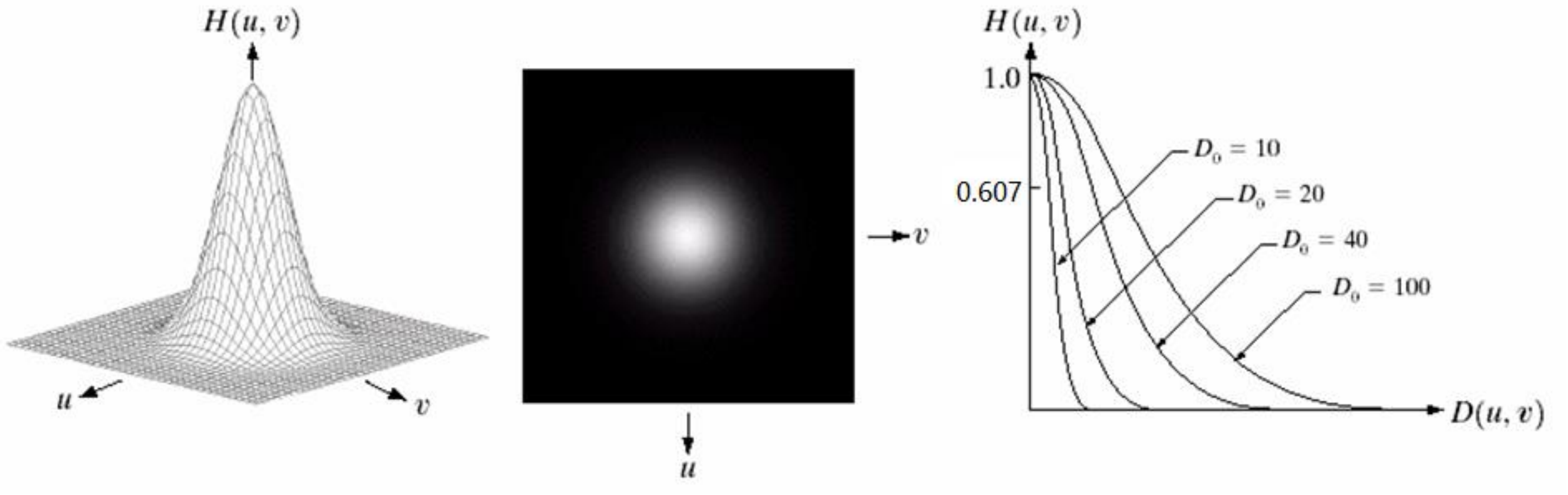
3. Gaussian Lowpass Filters

The transfer function of an Gaussian lowpass filter is:

$$H(u,v) = e^{-D^2(u,v)/2D_0^2}$$

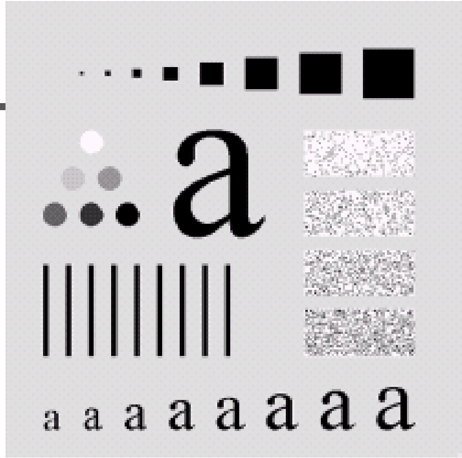
When $D(u,v)=D_0$, $H(u,v)=0.607$(down 60.7% from its maximum value of 1).

3.3 Lowpass Filtering

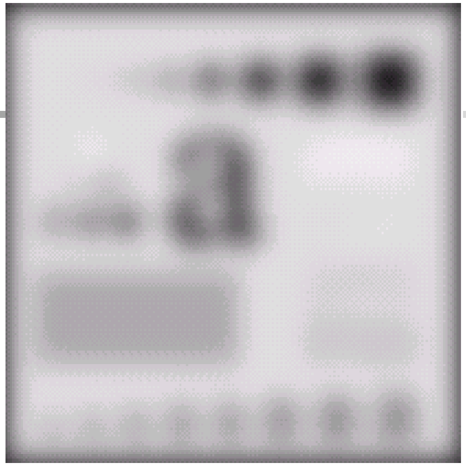


- The inverse Fourier transform of the GLPF is Gaussian also.
- A spatial Gaussian filter have no ringing.

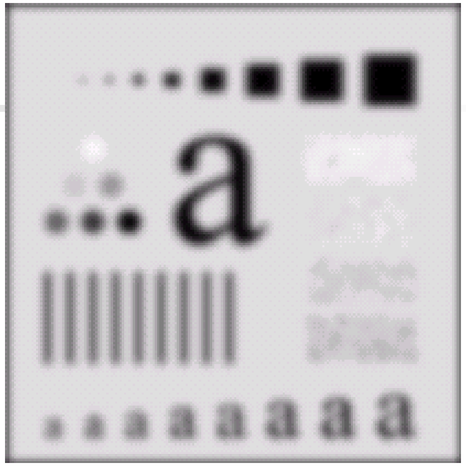
3.3 Lowpass Filtering



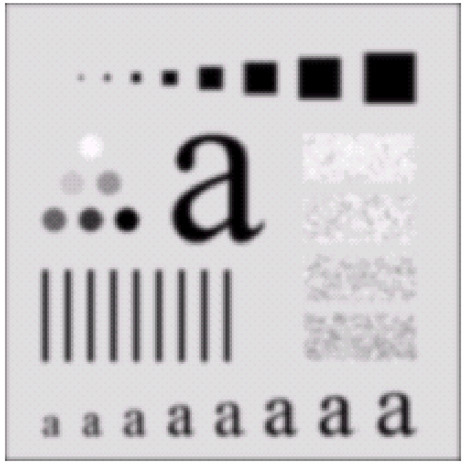
原始图



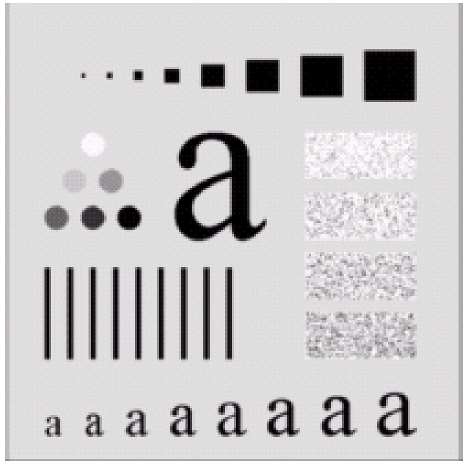
D_0=10的GLPF滤波



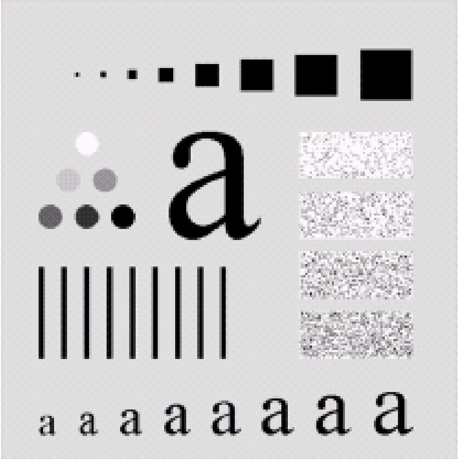
D_0=30的GLPF滤波



D_0=60的GLPF滤波

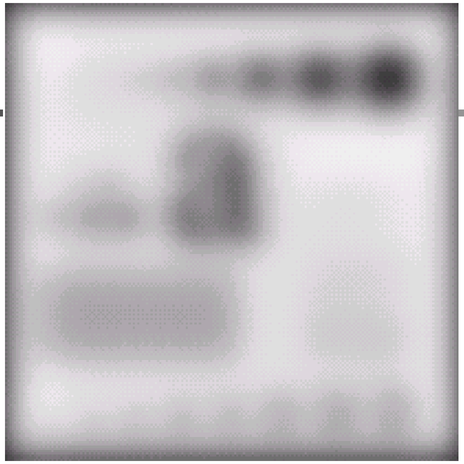


D_0=160的GLPF滤波

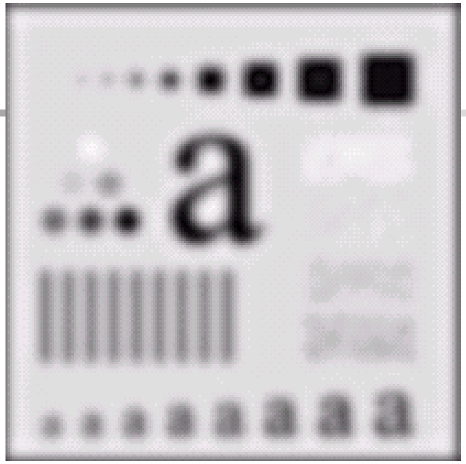


D_0=460的GLPF滤波

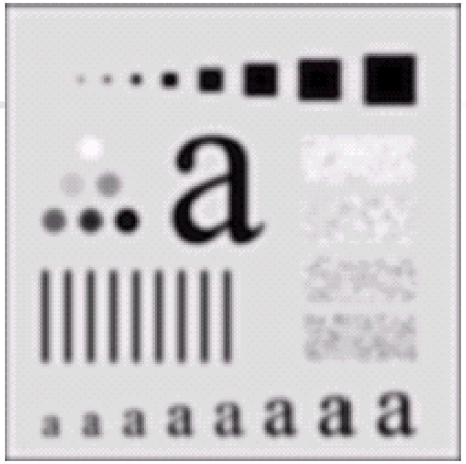
3.3 Lowpass Filtering



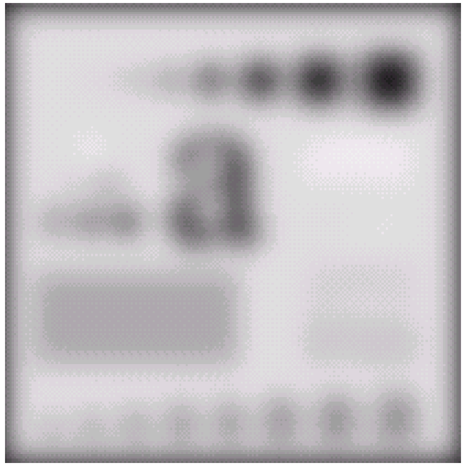
D_0=10的BLPF滤波



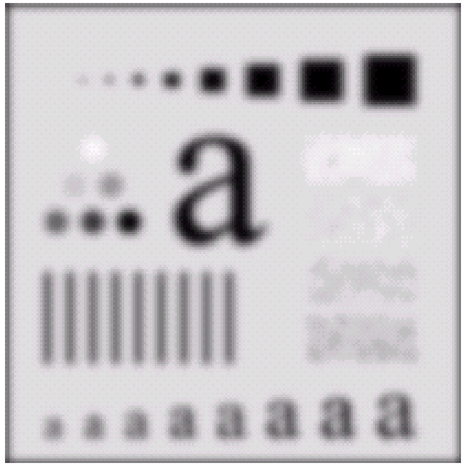
D_0=30的BLPF滤波



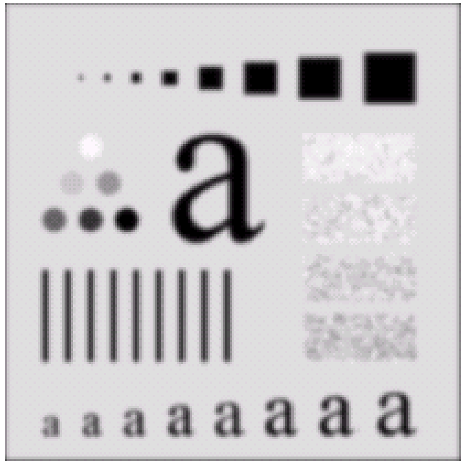
D_0=60的BLPF滤波



D_0=10的GLPF滤波



D_0=30的GLPF滤波

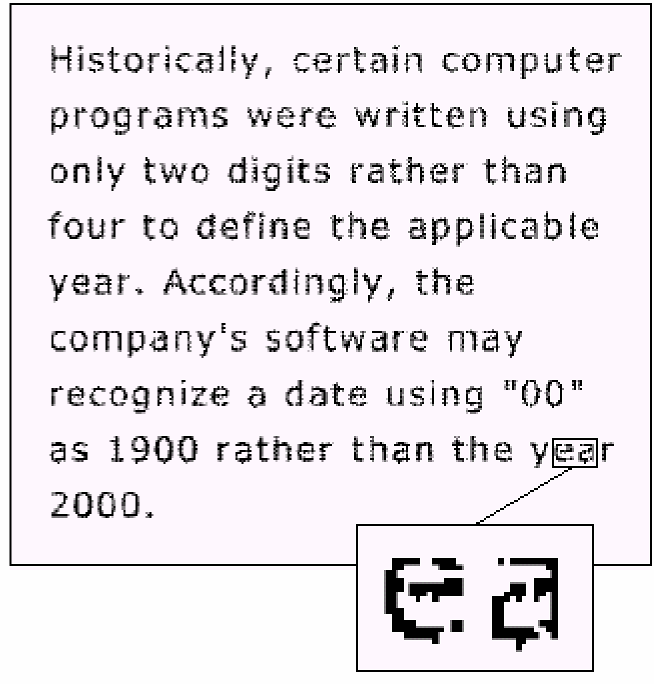


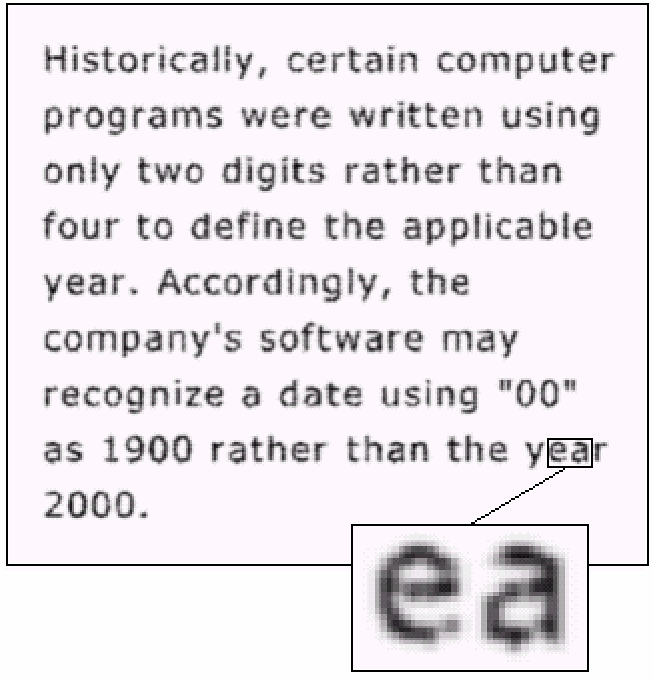
D_0=60的GLPF滤波

3.3 Lowpass Filtering

实例1：来自机器感知领域——字符识别应用

通过GLPF(D_0=80)滤波模糊输入图像，桥接这些裂缝





3.3 Lowpass Filtering

实例2：来自印刷和出版业——”美容”处理（减少皮肤细纹的锐化程度和小斑点）

D_0=100的GLPF滤波

D_0=80的GLPF滤波

3.3 Lowpass Filtering

实例3：卫星和航空图像，墨西哥湾（暗的）和佛罗里达（亮的）的一部分，图像存在“扫描线”

D_0=50的GLPF滤波　D_0=20的GLPF滤波

Main Content

- 3.1 Overview
- 3.2 Fourier transform and its properties
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- 3.4 Highpass Filtering
- 3.5 Homomorphic Filtering

3.4 Highpass Filtering

- **High-pass filtering suppresses the low-frequency components, enhances high-frequency components, thereby the image edge or line become clear.**
- **Commonly used high-pass filters include the ideal highpass filters, Butterworth highpass filters, Gaussian highpass filters.**

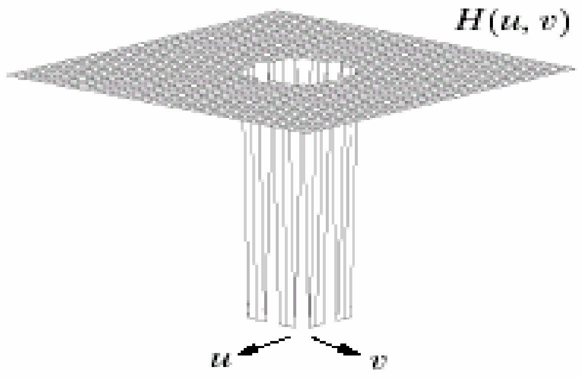
$$H_{hp}(u,v) = 1 - H_{lp}(u,v)$$

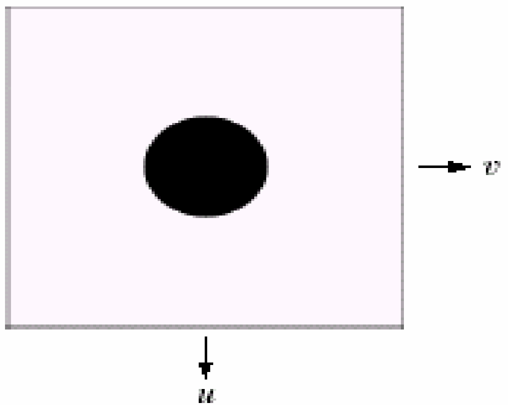
3.4 Highpass Filtering

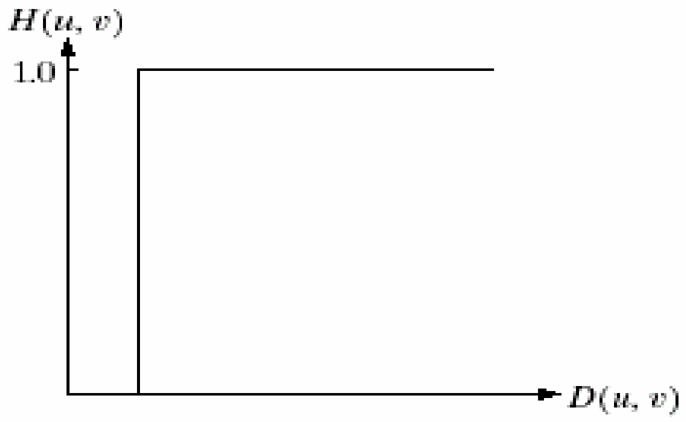
1. Ideal Highpass Filters

Transfer function: $$H(u,v)=\begin{cases}0 & D(u,v)\le D_0\\ 1 & D(u,v)>D_0\end{cases}$$

Cutaway view and perspective plot is







3.4 Highpass Filtering

a b c

FIGURE 4.54 Results of highpass filtering the image in Fig. 4.41(a) using an IHPF with $D_0 = 30, 60$, and 160.

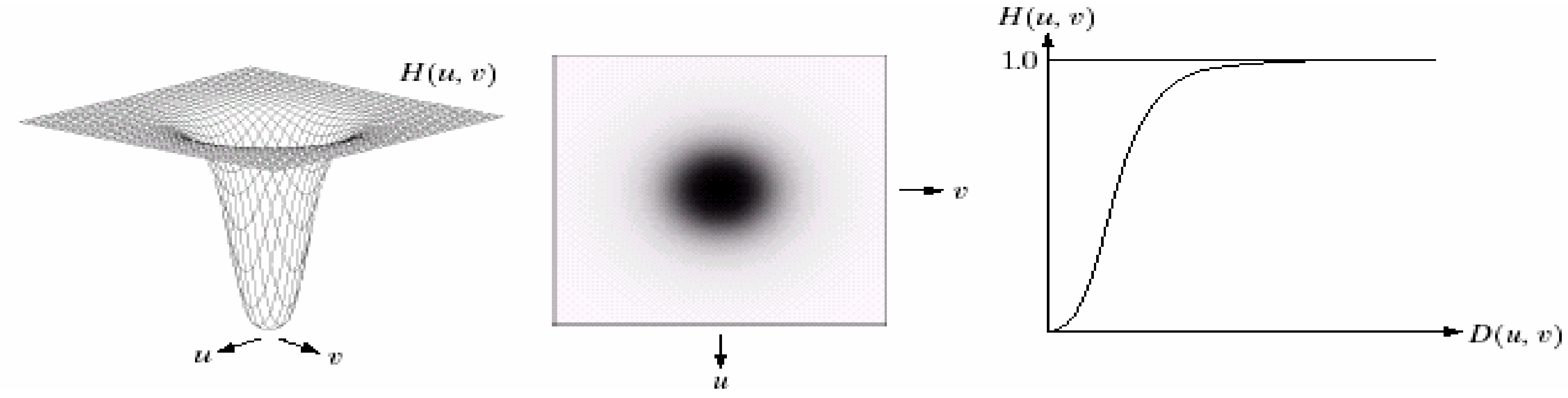
- 可用计算机实现，物理不可实现；
- 存在振铃效应。

3.4 Highpass Filtering

2. Butterworth Highpass Filters

The transfer function of a Butterworth highpass filter of order n, and with cutoff frequency at a distance D_0 from the origin, is defined as：

$$H(u,v) = 1/[1+(D_0/D(u,v))^{2n}]$$



3.4 Highpass Filtering

a b c

FIGURE 4.55 Results of highpass filtering the image in Fig. 4.41(a) using a BHPF of order 2 with $D_0 = 30, 60$, and 160, corresponding to the circles in Fig. 4.41(b). These results are much smoother than those obtained with an IHPF.

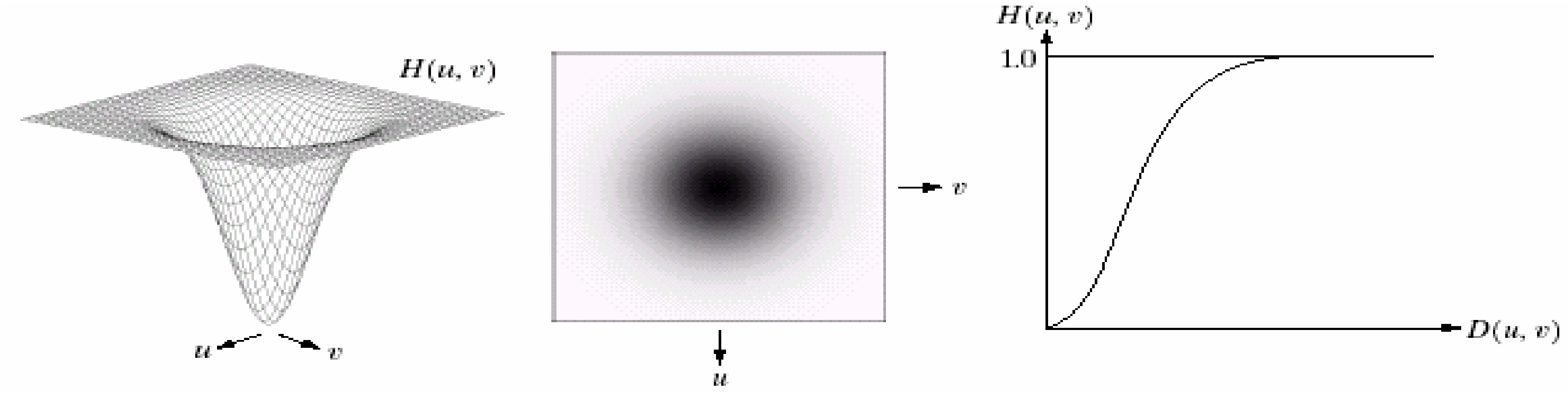
➢ BHPF 比IHPF 平滑，相同设置的BHPF边缘失真比IHPF 小得多。

3.4 Highpass Filtering

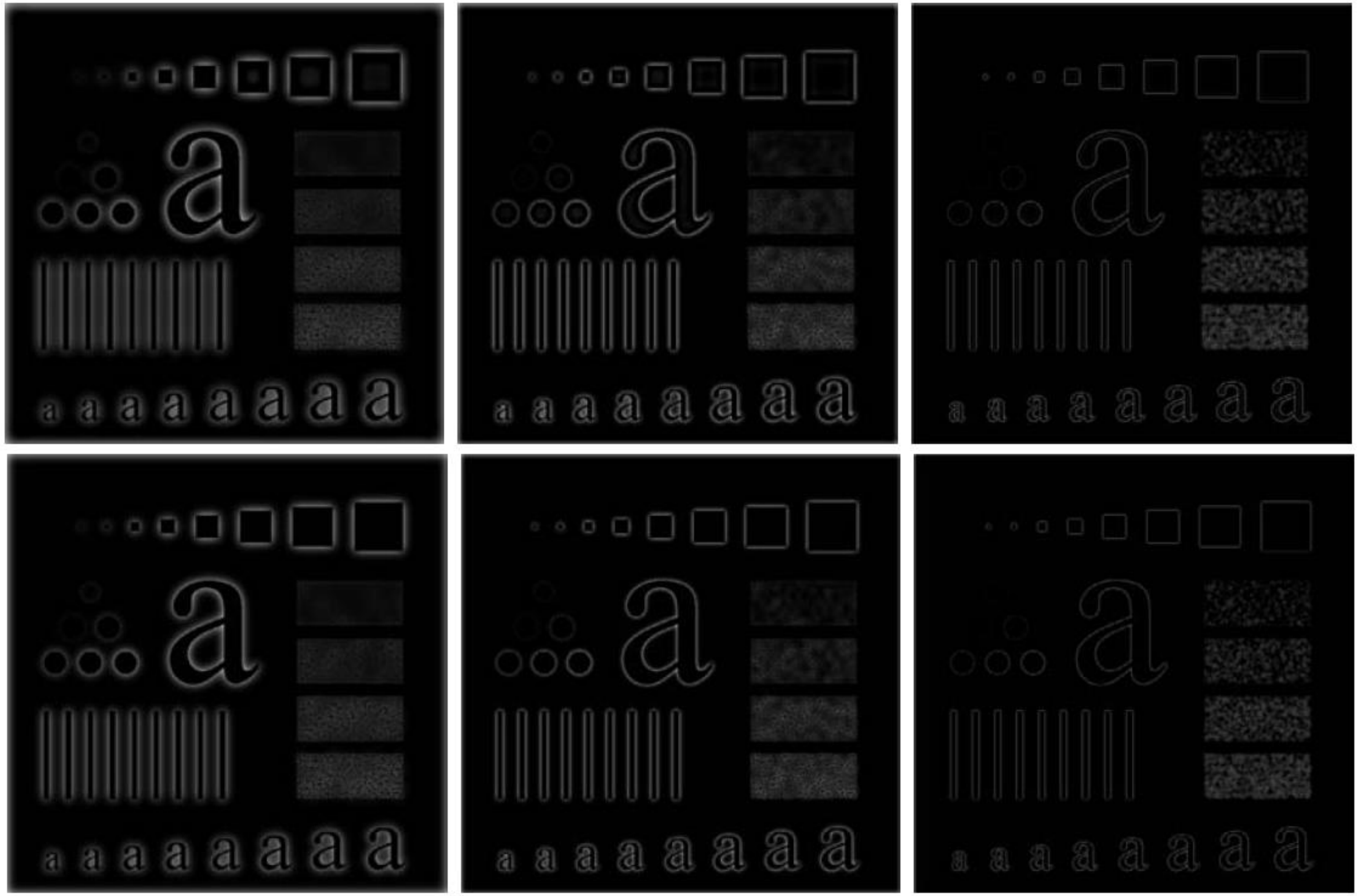
3. Gaussian highpass filters

The transfer function of a Gaussian highpass filter of order n, and with cutoff frequency at a distance D_0 from the origin, is defined as:

$$H(u,v) = 1\text{-}e^{-D^2(u,v)/2D_0^2}$$



3.4 Highpass Filtering



3.4 Highpass Filtering

1026×962 $D_0 \approx 962 \times 5\% \approx 50$ 二值化的结果

a b c

FIGURE 4.57 (a) Thumb print. (b) Result of highpass filtering (a). (c) Result of thresholding (b). (Original image courtesy of the U.S. National Institute of Standards and Technology.)

Main Content

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- 3.5 Homomorphic Filtering

3.5 Homomorphic Filtering

- Homomorphic Filtering is a frequency domain procedure for improving the appearance of an image by simultaneous gray-level range compression and contrast enhancement.

3.5 Homomorphic Filtering

An image $f(x, y)$ can be expressed as the product of illumination $i(x, y)$ and reflectance components $r(x, y)$:

$$f(x, y) = i(x, y)r(x, y)$$

$$\Im[i(x, y)r(x, y)] \neq \Im[i(x, y)] \bullet \Im[r(x, y)]$$

•The equation cannot be used directly to operate separately on the frequency components of illumination and reflectance.

3.5 Homomorphic Filtering

We define:

$$z(x,y) = \ln f(x,y) = \ln i(x,y) + \ln r(x,y)$$

then: $$F\{z(x,y)\} = F\{\ln f(x,y)\} = F\{\ln i(x,y)\} + F\{\ln r(x,y)\}$$

$$Z(u,v) = I(u,v) + R(u,v)$$

If we process Z(u,v) by means of a filter function H(u,v), then

$$S(u,v) = H(u,v)Z(u,v) = H(u,v)I(u,v) + H(u,v)R(u,v)$$

Inverse transform to the spatial domain:

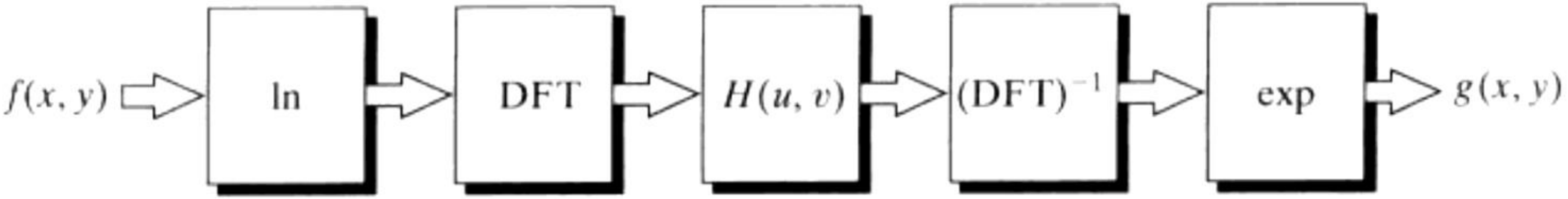
$$h_f(x,y) = h_i(x,y) + h_r(x,y)$$

Take the logarithm to get the final result

$$g(x,y) = \exp\left[h_f(x,y)\right] = \exp\left[h_i(x,y)\right] \bullet \exp\left[h_r(x,y)\right]$$

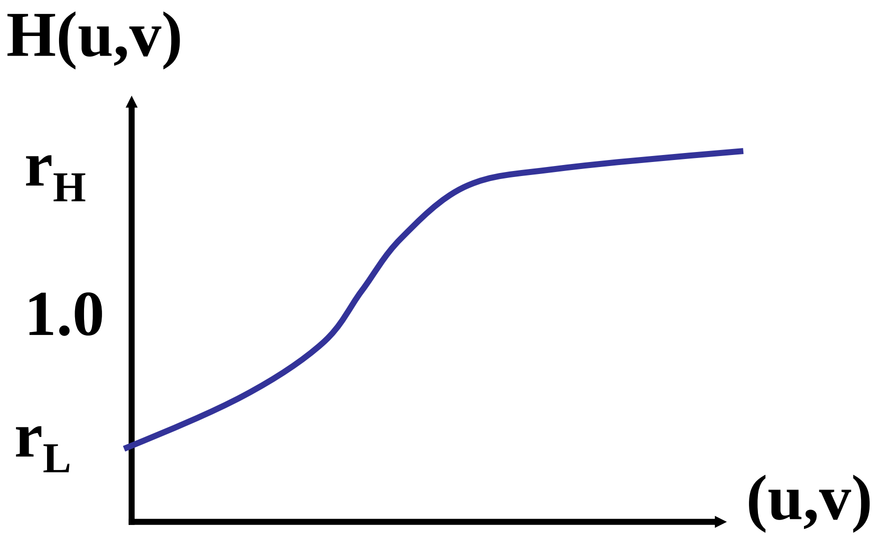
3.5 Homomorphic Filtering

Homomorphic filtering process is as follows:



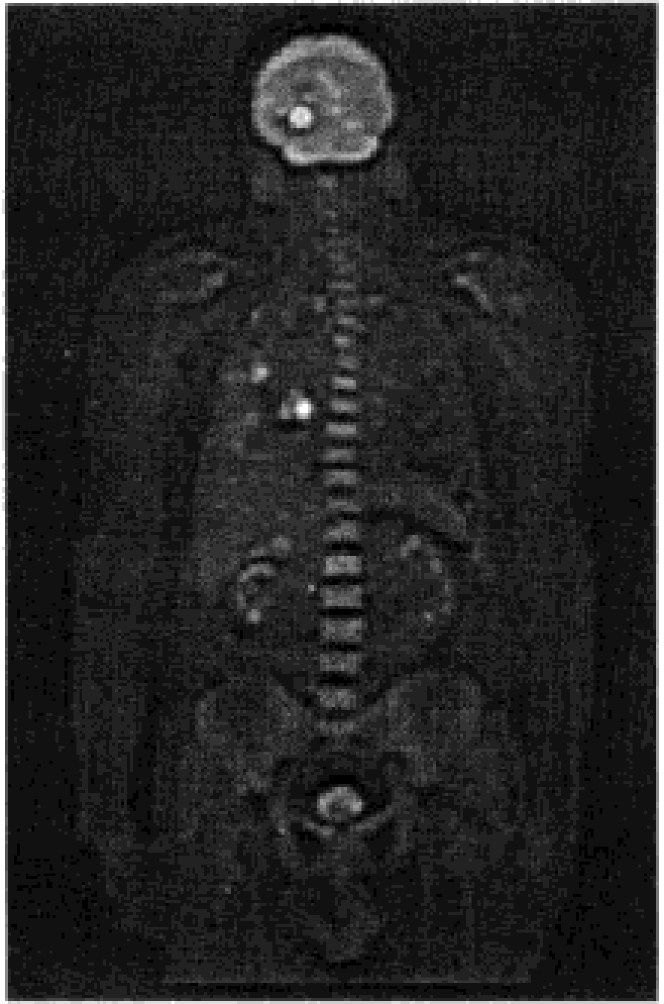
The key to homomorphic filtering is the separation of the illumination and reflectance components, the homomorphic filter function H(u,v) can then operate on these components separately.

3.5 Homomorphic Filtering



$$H(u,v) = (\gamma_H - \gamma_L)[1\text{-}e^{-c[D^2(u,v)/D_0^2]}] + \gamma_L$$

3.5 Homomorphic Filtering



$$H(u,v) = (\gamma_H - \gamma_L)[1 - e^{-c[D^2(u,v)/D_0^2]}] + \gamma_L$$

总结

- 空间域的卷积对应于频率域的乘积
- 低频成分对应图像灰度变化缓慢区域、高频成分对应图像的边缘和细节
- 低通滤波器类似空间域的平滑滤波器
- 高通滤波器类似空间域的微分算子
- 同态滤波器可同时降低灰度动态范围，提高图像对比度

The End!