Modelization

The “minimum dominating groups” problem can be formalized as below.

Minimize:

$$\sum_{k\in[1..m]} y_k$$

Subject to:

$$\forall i \in [1..n], \sum_{j\in A_i} x_j \geq 1$$

$$\forall i \in [1..n], x_i = \prod_{k\in G_i} y_k$$

Given that:

- n is the number of vertices.
- m is the number of groups.
- $x_i \in \{0, 1\}$ indicates if vertex i is part of the dominating set.
- $y_k \in \{0, 1\}$ indicates if group k is part of the dominating groups.
- A_i is the closed neighborhood of i: the set of vertices directly connected to i, including i.
- G_i is the set of groups that include i.

For implementation in a linear programming framework we rewrite the equality above as two linear inequalities:

$$\forall i, 1 - x_i \leq \sum_{k\in G_i} (1 - y_k)$$

$$\forall i, \forall k \in G_i, x_i \leq y_k$$

Note that if we define each group to consist of a single vertex, the minimum dominating group problem simplifies into the well known minimum dominating set problem.