ENGG 225
Fundamentals of Electrical Circuits and Machines

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1 Introduction

1.1 Electric Circuits:

The interconnection of circuit elements in a closed path by conductors. The concept of electrical charge is the basics for describing all electrical phenomena. Charge exists in discrete quantities of integer multiples of $1.60 \times 10^{-19} C$. In circuit analysis there are two fundamental electrical quantities voltage and current.

1.2 Electical Current:

Electrical current is defined as the rate of flow of electrical charges.

$$i(t) = \frac{dq(t)}{dt}$$

It is assumed that i is a measure of the equivalent flow of positive charge flow. Given $i(t)$, we can also find $q(t)$

$$q(t) = \int_{t_0}^{t} i(t)\, dt + q(t_0)$$

Normally there is an assigned reference direction for current. Often the direction is unknown and is assumed. The actual direction is determined by the sign of i

1.2.1 Direct and Alternating Current:

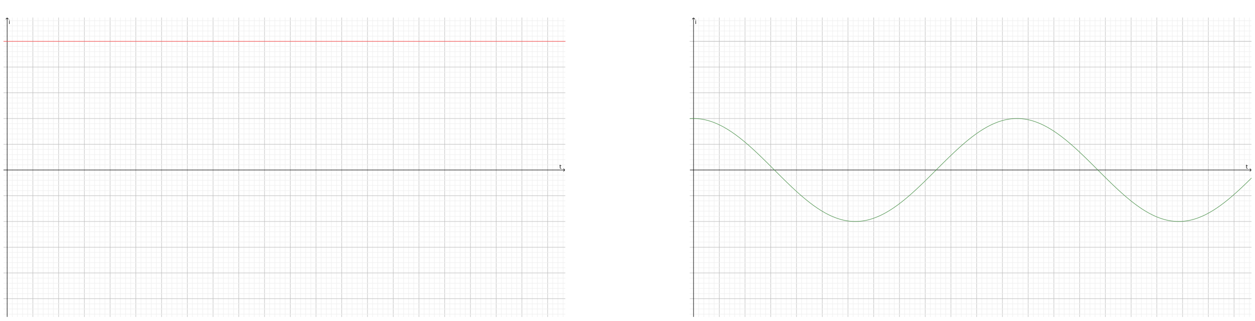

Figure 1: Direct and Alternating Current

1.2.2 Notation for Current

1.3 Voltage

Voltage is the energy transferred to or from a circuit element per unit of charge that flows through it.

$$V(t) = \frac{dw(t)}{dq(t)}$$

Voltage is often thought of as the potential difference across a circuit element. The polarity of the voltage is used to determine which node of a circuit element is at a higher potential then the other. If a circuit element has two nodes a and b with a voltage V across it such that a is the negative terminal and b is the positive terminal then it can be said that b is V volts higher in potential then a. For analysis purposes, reference polarities can be assigned if they are not given. The reference polarities can be chosen arbitrarily and if the actual polarity is the opposite the value of V is negative.

1.4 Ideal Circuit Elements:

All circuit elements are characterized by the voltage current relationship they hold.

1.4.1 Ideal Conductor:

An ideal conductor is a conductor with 0 resistance; this implies that the current through the conductor stays constant, and there is 0 voltage drop across the conductor, or $R = 0\Omega$, $\Delta i = 0A$, and $V = 0V$. The components on either side of the conductor are treated as if they are shorted (connected) together. The absence of a conductor is an open circuit. Since all conductors are ideal, they can be as arbitrarily long or short as necessary as long as the connection remains the same.

1.4.2 Sources:

1. Independent Voltage Sources:
   Tells which terminal is at a higher potential voltage and by what amount. The current is unknown and is dependent on the circuit the voltage source is connected to. A voltage source can be either AC or DC

2. Dependent Voltage Source:
   The voltage provided by a dependent voltage source is based on some other value in the circuit. The voltage current characteristics are the same as a independent voltage source, except the value of the voltage provided depends on a voltage or current elsewhere in the circuit.

3. Independent Current Source:
   Tells the exact value of the current flowing through it. The current provided is unaffected by other elements in the circuit, but the voltage is dependent on the circuit connected to the source.

4. Dependent Current Source:
   A dependent current source has the same properties as an independent current source except the current is dependent on the current or voltage in another element of the circuit.

1.4.3 Resistors

Resistors resist the flow of current by causing a drop in voltage. The relationship between the voltage across a resister and the current through the resistor is given by Ohm's Law. Ohm's Law states that if the current through a resistor is shown in the direction of the voltage drop then the voltage is given by

$$V = i\,R$$

where V is the voltage measured in volts V, i is the current measured in amperes (amps) A and R is the resistance of the resistor measured in ohms Ω. If the current through a resistor is shown in the direction of the voltage rise then ohms law states

$$V = -i\,R$$

Likewise Ohms Law states that the current through a resistive element is

$$i = \frac{1}{R}V$$

The quantity $G = \dfrac{1}{R}$ is referred to as the conductance of a circuit element. Conductance is measured in units of Siemens Ω^{-1}

1.5 Power and Energy

Power is given as the change in energy W over a change in time t

$$P = \frac{dW}{dt}$$

In electric circuits, power can be given as the product of voltage and current

$$P = V\,i$$

Power is measured in watts where $1W = 1J/s$. If current is labelled in the direction of a voltage drop over an element (passive reference direction) which implies the element is absorbing power then use

$$P = V\,i$$

If the current is labelled in the direction of a voltage increase then use

$$P = -V\,i$$

In both cases, if $P > 0$ then the element is absorbing power, and conversely if $P < 0$ the element is supplying power.

1.5.1 Power in Resistors

As stated above power in a circuit element is given by

$$P = V\,i$$

if it is absorbing power. However voltage and current are related by Ohms Law for a resistive circuit element. From Ohms law the following is obtained,

$$V = i\,R$$

$$\therefore \qquad P = i^2R \qquad P = \frac{V^2}{R}$$

As $i^2 \geq 0$, $V^2 \geq 0$ and $R > 0$ for any resistor necessarily $P \geq 0$ for any resistor.

1.5.2 Energy

As stated above

$$P = \frac{dW}{dt}$$

$$\therefore W = \int_{t_1}^{t_2} P(t)dt$$

1.6 Kirchoff's Laws

1.6.1 Kirchoff's Current Law (KCL)

Consider a node in a circuit. The total current flowing into the node is equal to the current out, or the algebraic sum of all currents at a node must be 0.

$$\sum i_n = 0$$

The convention used will be add all currents entering the node and subtract currents leaving the node. In the node voltage method the opposite convention is used.

1.6.2 Kirchoff's Voltage Law (KVL)

Kirchoff's voltage law states the algebraic sum of the voltages equals 0 for any closed path in an electric circuit.

$$\sum V_n = 0$$

When summing voltages around a loop the direction of the loop is arbitrary. If the loop crosses the positive terminal of an element add V. Conversely if the loop crosses the negative terminal of an element subtract V.

1.7 Series and Parallel Circuits

1.7.1 Series Circuits

Two elements are in series if they are connected at one node only and there is no other element connected to that node. Elements in series have the same current and direction of current.

1.8 Parallel Circuits

Two elements are in parallel if their terminals are directly connected to each other. In other words they are connected to the same two nodes. Elements in parallel must have the same actual voltage.

2 Resistive Circuits

2.1 Series and Parallel Resistance

2.1.1 Series Resistance

A series combination of resistors has an equivalent resistance equal to the sum of the original resistances. For n resistors of resistance R_k, the equivalent resistance R_{eq} is given by,

$$\sum_{k=1}^{n} R_k = R_{eq}$$

2.1.2 Parallel Resistance

A parallel combination of resistors has an equivalent resistance equal to the reciprocal of the sum of the reciprocal of the original resistances. For 2 resistors of resistance R_1 and R_2 in parallel the equivalent resistance R_{eq} is given by

$$R_{eq} = \frac{R_1 R_2}{R_1 + R_2}$$

From the above result it is show able that for two resistors of resistance R in parallel have an equivalent resistance

$$R_{eq} = \frac{1}{2} R$$

More generally for n resistors of resistance R_k, the equivalent resistance R_{eq} is given by

$$\frac{1}{R_{eq}} = \sum_{k=1}^{n} \frac{1}{R_k}$$

or

$$R_{eq} = \frac{1}{\sum\limits_{k=1}^{n} \frac{1}{R_k}}$$

2.2 Voltage and Current Division

2.2.1 Voltage Division

In a series connection of resistors, the total voltage across the series branch of resistors divides among the resistors proportional to their size. For n resistors of resistance R_m with a total voltage across them of V the voltage in the k^{th} resistor V_k is given by

$$V_k = \frac{R_k}{\sum\limits^{n} R_m} V$$

2.3 Current Division

In a parallel connection of resistors, the total current available to the parallel resistor divides among them inversely proportionally to their size. For 2 resistors of resistance R_1 amd R_2 and a total current of i then the current i_1 through R_1 is

$$i_1 = \frac{R_2}{R_1 + R_2}$$

2.4 Node Voltage Analysis

The node voltage method allows analysis of any circuit where other methods may fail. The node voltage method follows the following steps

1. Identify nodes and known node voltages
2. Apply KCL at nodes; Develop a system of equations where the unknowns are the node voltages
3. Solve system of equations to determine node voltages.

A node voltage is the potential difference between a node and the reference node. If a voltage source exists the negative terminal of the node should be chosen as the reference (ground) node.

3 Operational Amplifiers

4 Capacitors and Inductors

5 Sinusoidal Currents and Voltages

6 DC Machines