Math 375
Differential Equations for Engineers and Scientists

Andy Smit

Spring 2019

1 Differential Equations

Any equation that relates a quantity to its rate of changes. A solution for a differential equation is any formula that makes the equation true.

1.1 Classification of Differential Equations

There is two methods by which to classify differential equations. The first method to classify differential equations is to look at the highest order derivative in the equation. The largest n such that the equation has an nth order derivative is an nth order differential equation. The second method to classify differential equations is by the number of variables that are differentiated with respect to. If a differential equation involves derivatives with respect to a single variable, it is a ordinary differential equation. If a differential equation involves derivatives with respect to multiple variable, it is a partial differential equation.

2 First Order Differential Equations

3 Second Order Linear Differential Equations

3.1 Separable Equations:

A separable equation is a differential equation that can be written as

$$\frac{dy}{dx} = f(x)\,g(y) \tag{1}$$

where the x and y are separated into factors. (1) can be rearranged to obtain

$$\frac{1}{g(y)}\frac{dy}{dx} = f(x)$$

Now both sides can be integrated with respect to x to solve

$$\int \frac{1}{g(y)}\frac{dy}{dx}\,dx = \int f(x)\,dx + C$$

3.1.1 An Important Separable Equation

Consider the differential equation with the initial value,

$$\frac{dy}{dx} = ky,\ y(0) = y_0 \tag{2}$$