

STA261: Problems 6

Alex Stringer

July, 2018

This assignment is not for credit. Complete the questions as preparation for the quiz in tutorial 6 on July 25th. The questions on the quiz will be very similar to the questions on the assignment.

1. Let $X_i \stackrel{IID}{\sim} \text{Bern}(p)$ be an IID random sample from a Bernoulli distribution with parameter $p \in (0, 1)$.
 - (a) Find an unbiased estimator for p
 - (b) Figure out whether it is efficient
2. Let $X_i \sim N(\mu, \sigma)$.
 - (a) Suppose σ is known. Find the MLE for μ , and show that it is both unbiased and efficient.
 - (b) Suppose μ is known. Find the MLE for σ^2 , and show that it is both unbiased and efficient.
3. Let $X_i \sim N(\mu, \sigma)$ as in the previous question, but now both parameters are unknown. The Mean Squared Error of an estimator $\hat{\theta}$ of a parameter θ is

$$MSE(\hat{\theta}) = E((\hat{\theta} - \theta)^2)$$

Note this is not the bias squared, that would be $(E(\hat{\theta} - \theta))^2$, which is not the same.

- (a) Show that for any estimator $\hat{\theta}$ for any parameter θ ,

$$MSE(\hat{\theta}) = Var(\hat{\theta}) + bias(\hat{\theta})^2$$

In particular, this means that for unbiased estimators, $MSE(\hat{\theta}) = Var(\hat{\theta})$.

- (b) Show that $\hat{\sigma}^2 = s_n^2$ as defined in lecture has bias

$$bias(s_n^2) = -\frac{\sigma^2}{n}$$

and find its variance $Var(s_n^2)$ and Mean Squared Error $MSE(s_n^2)$.

- (c) Show that s_{n-1}^2 as defined in lecture is unbiased, and find its variance and Mean Squared Error. Compare these to the variance and mean squared error of s_n^2 . Comment on the relative merits of each.