STA261: Problems 9

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This assignment is not for credit. Complete the questions as preparation for the quiz in tutorial 9 on August 8th. The questions on the quiz will be very similar to the questions on the assignment.

1. For IID random samples from the following distributions, derive a likelihood ratio test for testing $H_0 : \theta = \theta_0$ against $H_1 : \theta \neq \theta_0$.
   (a) Exponential, $f(x|\theta) = \frac{1}{\theta}e^{-x/\theta}$, $X > 0, \theta > 0$
   (b) Poisson, $P(X = x|\theta) = \frac{\theta^x e^{-\theta}}{x!}$, $X > 0, \theta > 0$
   (c) Bernoulli, $P(X = x) = \theta^x(1-\theta)^{1-x}$
   (d) Binomial, $P(X = x) = \binom{m}{x}\theta^x(1-\theta)^{m-x}$
2. Recall the categorical data analysis example discussed in lecture. Suppose instead of testing whether the row-probabilities are all equal, we want to test the hypothesis that the row and column factors are statistically independent.
   (a) Under H_0, p_{ij}, the probability of an observation being in the i, j^{th} cell of the table, is equal to $p_{ij} = p_i p_j$, the product of the probabilities of being in the i^{th} row and the j^{th} column. Find the Likelihood under this hypothesis.
   (b) Show that the MLEs are $\hat{p}_i = r_i/N$, $\hat{p}_j = c_j/N$, where $r_i = \sum_{j=1}^{J} y_{ij}$ is the sum of the cells in the i^{th} row, and similarly for c_j.
   (c) State the appropriate alternative hypothesis H_1 and find the likelihood under this hypothesis
   (d) Show that the likelihood ratio test statistic is

$$-2\log\Lambda = 2\sum_{i=1}^{R}\sum_{j=1}^{C} y_{ij}\log\left(\frac{Ny_{ij}}{r_i c_j}\right)$$

   (e) For the following 2×2 table, test the hypothesis of independence at the 5% significance level. Report the test statistic, p-value, and your conclusion. I got a p-value of about 0.042.

65	11
98	35