STA261: Problems 4

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This assignment is not for credit. Complete the questions as preparation for the quiz in tutorial 4 on July 18th. The questions on the quiz will be very similar to the questions on the assignment.

1. Likelihood: Let $X_i \sim Bern(p)$ be a sequence of independent coin flips. Find the likelihood function for $\mathbf{x} = (x_1, \ldots, x_n)$. Compare your answer to the binomial probability mass function, and explain why they are different.

2. Maximum Likelihood: Show that the maximum likelihood estimator can depend on the data only through a function of a sufficient statistic. That is, if IID random variables $X_1 \ldots X_n$ have likelihood $L(\theta)$ and maximum likelihood estimator $\hat{\theta}(\mathbf{X})$, then $\theta\hat{\mathbf{X}}$ is sufficient for θ.

3. Maximum Likelihood: for random variables following the following distributions, find the likelihood function and the maximum likelihood estimator(s) for the given parameter(s), based on a random sample of independent, identically distributed $X_i, i = 1 \ldots n$.

   (a) $X \sim N(\mu, \sigma^2)$: find the MLE for (μ, σ^2).
   (b) $X \sim Exp(\theta)$ with density $f_X(x;\theta) = \frac{1}{\theta} \times \exp\left(-\frac{x}{\theta}\right)$: find the MLE for θ.
   (c) $X \sim Exp(\theta)$ with density $f_X(x;\theta) = \theta \times \exp\left(-x\theta\right)$: find the MLE for θ.
   (d) $X \sim Bernoulli(\theta)$: find the MLE for θ
   (e) $X \sim Binomial(\theta)$: find the MLE for θ, except for this question, only consider a single observation X (rather than an IID sample like in the other questions).

4. Maximum Likelihood:

   (a) Show that any two distributions whose likelihoods are proportional as functions of the parameter θ give the same maximum likelihood estimates. That is if $L_1(\theta;\mathbf{X}) = cL_2(\theta;\mathbf{X})$, and $\hat{\theta_1}$ and $\hat{\theta_2}$ are the corresponding maximum likelihood estimators for θ, then $\hat{\theta}_1 = \hat{\theta}_2$.
   (b) Explain why your answers to the MLE for the Bernoulli and Binomial distributions in the previous question were the same.