STA261 Summer 2018

Quiz 3

July 16th, 2018

First Name: SOLUTIONS.

Last Name:

Student Number:

This quiz is out of 10 marks. Do ALL of your work on the back of the quiz, where the questions are. You can use the front for rough work, but nothing on the front will be marked, or even seen by the TAs.

If $X \sim N(\mu, \sigma^2)$ then $f_X(x) = \frac{1}{\sqrt{2\pi\sigma^2}} \times \exp\left(-\frac{1}{2\sigma^2}(x-\mu)^2\right)$, $x \in \mathbb{R}$, $\mu \in \mathbb{R}$, $\sigma \in \mathbb{R}^+$.

BELOW SPACE IS FOR ROUGH WORK. NOTHING WRITTEN HERE WILL BE READ OR MARKED.

1. (5 marks) Suppose random variable X has density $f_X(x;\theta)$ depending on parameter θ, we have a random sample of independent and identically distributed $X_i \stackrel{d}{=} X$, and $T_1(\mathbf{X})$ is a sufficient statistic for θ. Let $r(\cdot)$ be an invertible function, and $T_2 = r(T_1)$. Prove that T_2 is sufficient for θ.

(1) Factorization theorem: $f(x;\theta) = g(T(x);\theta)h(x) \iff T(X)$ suff. for θ.

(1) Write $f(x;\theta) = g(T_1(x);\theta)h(x)$, possible because T_1 suff. for θ.

(2) But $T_1(x) = r^{-1}(T_2(x))$, so $f(x;\theta) = g(r^{-1}(T_2(x));\theta)h(x)$.

(1) By Factorization Theorem, T_2 suff. for θ.

2. (5 marks) Let $X \sim N(\mu, 1)$ and find a sufficient statistic for μ.

$$f(x;\mu) = \frac{1}{\sqrt{2\pi}} \exp\left(\frac{-1}{2}(x-\mu)^2\right)$$

$$= \frac{1}{\sqrt{2\pi}} \exp\left(\frac{-1}{2}x^2 + x\mu - \frac{1}{2}\mu^2\right)$$

(5)
$$= \exp\left(x\mu - \frac{1}{2}\mu^2\right) \cdot \frac{1}{\sqrt{2\pi}} \exp\left(\frac{-1}{2}x^2\right)$$

$$\equiv g(x;\mu) \cdot h(x)$$

By fact. theorem, X is suff. for μ

-1 if they do it for random sample $X_1 \cdots X_n$, that's not what question asks

-3 if they aren't clear about how $f(x;\mu)$ factorizes.