STA261 S19: Test 3

Please write your information clearly and legibly.

First Name:____

Last Name:____

Student Number:______________________________________

U of T Email:_______________________________________

- No aids. You do not need a calculator.
- 60 minutes.
- Write all answers directly beneath where the question is asked.
- Use the backs of the pages for rough work.
- Test is out of 10 marks. 4 marks are designated "basic" and test base knowledge. 4 marks are designated "adept" and test application of base knowledge to new problems. 2 marks are dedicated "advanced" and require in-depth understanding and problem solving skills. Use your time wisely.

1. *Basic, 4 marks* Let $X_1, \ldots, X_n$ be an IID sample from a parametric family of distributions $\{F_\theta : \theta \in \Theta\}$ with corresponding densities f_θ.

a) (1) Give a mathematical definition of the likelihood ratio for assesing whether $\theta = \theta_0$ against the alternative that $\theta \neq \theta_0$.

b) (1) Circle True or False: you and I observe two random samples from F_θ, and their sufficient statistics happen to be equal. You and I will make the same decision about whether $\theta = \theta_0$ if we base our decision on the likelihood ratio at the same level.

c) (1) Suppose I go Bayesian on you and put a prior $\pi(\theta)$ on θ. Give an expression for the posterior distribution of $\theta|X$.

d) (1) Circle True or False: consider the situation in part **b)**. You and I will make the same inferences if we base those inferences off of the posterior distribution of $\theta|X$.

2. *Adept, 4 marks.* The gamma function is defined as:

$$\Gamma(n+1) = \int_0^\infty x^n e^{-x} dx \tag{0.1}$$

For $n \in \mathbb{N}$, $\Gamma(n+1) = n! = n \times (n-1) \times \cdots \times 2 \times 1$. Use Laplace approximations to prove Stirling's approximation:

$$n! \approx n^n e^{-n} \sqrt{2\pi n} \tag{0.2}$$

3. *Advanced, 2 marks.* Let $X_1, \ldots, X_n$ be an IID random sample from a $\text{Unif}(0, \theta)$ distribution.

(a) Find $\widehat{\theta}$, the maximum likelihood estimator for θ.

(b) Is the asymptotic distribution of $\widehat{\theta}$ Normal? If yes, state why and give the mean and variance. If no, find the cumulative distribution function of $\widehat{\theta}$, and state why it is *not* normal (what regularity condition is not satisfied?)

THIS PAGE IS FOR ROUGH WORK. NOTHING ON THIS PAGE WILL BE MARKED.

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