STA261 Summer 2018

Quiz 4

July 18th, 2018

First Name: SOLUTIONS

Last Name:______________________________

Student Number:______________________________

This quiz is out of 10 marks. Do ALL of your work on the back of the quiz, where the questions are. You can use the front for rough work, but nothing on the front will be marked, or even seen by the TAs.

If $X \sim Bernoulli(\theta)$ then $P(X = x) = \theta^x(1-\theta)^{1-x}$, for $x = 0, 1$.

BELOW SPACE IS FOR ROUGH WORK. NOTHING WRITTEN HERE WILL BE READ OR MARKED.

1. (6 marks) Show that the maximum likelihood estimator (MLE) can depend on the data only through a function of a sufficient statistic. That is, if IID random variables $X_1 \ldots X_n$ have likelihood $L(\theta)$ and maximum likelihood estimator $\hat{\theta}(\mathbf{X})$, then $\theta(\hat{\mathbf{X}})$ is sufficient for θ.

① $L(\theta) = \prod_{i=1}^{n} f(x_i;\theta)$ (by independence and identically dist$^{\underline{n}}$)

Factor $L(\theta)$ into parts depending on θ, and parts depending only on X:

② $L(\theta) = g(\underset{\sim}{x};\theta) \times h(x)$

③ Note $\text{argmax}_\theta f(\underset{\sim}{x};\theta) = \text{argmax}_\theta g(\underset{\sim}{x};\theta)$. Hence MLE depends on $\underset{\sim}{x}$ only through $g(\cdot;\cdot)$. By factorization theorem, MLE is sufficient.

2. (4 marks) Let $X \sim Bernoulli(\theta)$. Find the MLE for θ.

① $P(X=x;\theta) = \theta^{x}(1-\theta)^{1-x} = L(\theta)$ —1 if they use IID sample; not asked.

① $\ell(\theta) = \log L(\theta) = x\log\theta + (1-x)\log(1-\theta)$

① $S(\theta) = d\ell/d\theta = \frac{X}{\theta} - \frac{1-X}{1-\theta}$

① $S(\hat{\theta}) = 0 \Rightarrow \hat{\theta} = X$