STA261 L5101: Quiz 2

March 7th, 2018

Last Name:___________________________
First Name:___________________________________
Student Number:___________________________________

You may use a non-programmable calculator. Any other aids are prohibited. Use pen; questions done in pencil will be ineligible for remark requests. Circle your final answer to each question. The quiz is out of 10 points. Write all your answers on the front of the quiz; use the back for rough work. Nothing on the back will be marked.

1. (6 marks) Suppose $X_i \sim N(0, \sigma^2), i = 1 \dots n$ is an IID random sample from a normally distributed population with unknown variance σ^2 with true value σ_0^2. You can use the fact that

$$\frac{\sum_{i=1}^{n} X_i^2}{\sigma_0^2} \sim \chi_n^2$$

Use the symbol $\chi^2_{n,\alpha}$ to denote the α quantile of the χ^2_n distribution; that is if $Y \sim \chi^2_n$,

$$P(Y < \chi^2_{n,\alpha}) = \alpha$$
$$P(Y < \chi^2_{n,1-\alpha}) = 1 - \alpha$$

(a) (6 marks) Derive a 95% confidence interval for σ^2. If you need space for rough work, use the back of the page.

(b) (4 marks) Suppose we want to test $H_0 : \sigma^2 = 1$ against $H_1 : \sigma^2 \neq 1$ at the 95% significance level. We compute the above interval and find that it contains 1. Circle the most appropriate conclusion:

(i) Reject H_0 in favour of H_1
(ii) Fail to reject H_0 in favour of H_1
(iii) Accept H_0
(iv) Fail to accept H_0