STA261 S19: Test 2

Please write your information clearly and legibly.

First Name:________________________________

Last Name:________________________________

Student Number:________________________________

U of T Email:________________________________

- No aids permitted except a non-programmable calculator,
- 60 minutes.
- Write all answers directly beneath where the question is asked.
- Use the backs of the pages and the final page for rough work. Write all your final answers directly below the question. **Only what is written in the space beneath the question will be marked. Use your space wisely.**
- Test is out of 10 marks. 4 marks are designated "basic" and test base knowledge. 4 marks are designated "adept" and test application of base knowledge to new problems. 2 marks are dedicated "advanced" and require in-depth understanding and problem solving skills. Use your time wisely.

1. *Basic, 4 marks* Let $X_1, \ldots, X_n$ be an IID sample from a parametric family of distributions $\{F_\theta : \theta \in \Theta\}$ with corresponding densities f_θ.

a) (2) Define the Likelihood function in words and very briefly explain how it is used for inference on θ.

b) (2) Give a mathematical definition of the following in terms of f_θ (.5 each):

Likelihood function:

Score function:

Observed Information:

Expected Information:

2. *Adept, 4 marks.* Suppose $X_1, \ldots, X_n$ is an IID sample from a Poisson(λ) distribution, having $P(X = x) = \lambda^x e^{-\lambda}/x!$ for $x = 0, 1, 2, \ldots$

a) (1) Find the Likelihood function for λ.

b) (1) Show that $T = \sum_{i=1}^{n} X_i$ is a sufficient statistic for λ. Is the sample mean $\bar{X} = T/n$ is also sufficient? Briefly explain why or why not.

c) (1) Find the maximum likelihood estimator for λ.

d) (1) Find a random interval (L_n, U_n) such that $\mathbb{P}(\lambda \in (L_n, U_n)) \approx 1 - \alpha$ for some $\alpha \in (0, 1)$.

3. *Advanced, 2 marks.* Suppose I flip a coin 10 times and get 7 heads. I claim that the coin is fair. Find an expression for the probability of seeing data at least this extreme if my claim is true. For full marks, show all work and justify all steps using theorems/results from lecture. A Binomial(n, p) random variable has $P(X = x) = \binom{n}{x} p^x (1-p)^{n-x}$. *This question is intentially vague. You are being tested on your statistical reasoning and ability to make decisions.*

THIS PAGE IS FOR ROUGH WORK. NOTHING ON THIS PAGE WILL BE MARKED.