L0101

STA261 ~~L5101~~: Quiz 3

March 21st, 2018

Last Name: SCHEME
First Name: MARKING
Student Number: 42

You may use a non-programmable calculator. Any other aids are prohibited. Use pen; questions done in pencil will be ineligible for remark requests. Circle your final answer to each question. The quiz is out of 10 points. Write all your answers on the front of the quiz; use the back for rough work. Nothing on the back will be marked.

1. (10 marks) Let $X_i \sim Bern(\theta)$, $\theta \in (0,1)$ with

$$P(X_i = x) = \theta^x(1-\theta)^{1-x}, x = 0, 1$$

We wish to test $H_0 : \theta = \theta_0$ against $H_1 : \theta \neq \theta_0$ using a likelihood ratio test.

(a) (2 marks) Is the null hypothesis simple or composite? Circle the correct answer.

(i) Simple

(ii) Composite

(b) (6 marks) Given that the MLE is $\hat{\theta} = \bar{X}$, find the likelihood ratio test statistic $-2\log\Lambda$.

$$L(\theta_0) = \theta_0^{\sum x_i}(1-\theta_0)^{n-\sum x_i} \quad (1)$$

MLE: $\ell(\theta) = \sum x_i \log\theta + (n - \sum x_i)\log(1-\theta)$

$$S(\theta) = \sum x_i/\theta - (n-\sum x_i)/(1-\theta)$$

$$S(\hat{\theta}) = 0 \Rightarrow \hat{\theta} = \bar{x} = \frac{1}{n}\sum x_i \quad (1)$$

$L(\hat{\theta}) = \bar{x}^{n\bar{x}}(1-\bar{x})^{n(1-\bar{x})}$ or any equivalent statement.

$$\Lambda = \frac{L(\theta_0)}{L(\hat{\theta})} = \left(\frac{\theta_0}{\bar{x}}\right)^{n\bar{x}}\left(\frac{1-\theta_0}{1-\bar{x}}\right)^{n(1-\bar{x})} \quad (2)$$

$-2\log\Lambda = 2\left(n\bar{x}\log\left(\frac{\bar{x}}{\theta_0}\right) + n(1-\bar{x})\log\left(\frac{1-\bar{x}}{1-\theta_0}\right)\right)$ or equivalent. (2)

(c) (2 marks) What is the corresponding distribution under the null hypothesis, including the correct degrees of freedom?

$$-2\log\Lambda \overset{\text{appox}}{\sim} \chi^2_1$$

(1) χ^2

(1) df.