STA261 Summer 2018

Quiz 7

July 30th, 2018

First Name: SOLUTIONS.

Last Name:_____

Student Number:_____

This quiz is out of 10 marks. Do ALL of your work on the back of the quiz, where the questions are. You can use the front for rough work, but nothing on the front will be marked, or even seen by the TAs.

If $X_i \overset{IID}{\sim} Gamma(\alpha, \beta)$ then the density is $f_X(x) = \frac{1}{\Gamma(\alpha)\beta^\alpha} x^{\alpha-1} \exp\left(-\frac{x}{\beta}\right)$

$\Gamma(x)$ is the "gamma function". $\partial \log \Gamma(x)/\partial x \equiv \psi(x)$ is the "digamma function". $\partial^2 \log \Gamma(x)/\partial x^2 \equiv \psi'(x)$ is the "trigamma function". You can leave terms involving these functions as-is in your equations. None of these functions have inverses.

BELOW SPACE IS FOR ROUGH WORK. NOTHING WRITTEN HERE WILL BE READ OR MARKED.

1. Let $X_i \overset{IID}{\sim} Gamma(\alpha, 1)$, and let α_0 be the true value of α.

(a) (4 marks) Find the log-likelihood and the score statistic for α.

$f(x_i) = \frac{1}{\Gamma(\alpha)} x_i^{\alpha-1} e^{-x_i}$; $L(\alpha) = \Gamma(\alpha)^{-n} \left(\prod_{i=1}^n x_i\right)^{\alpha-1} e^{-\sum x_i}$

(2) $\ell(\alpha) = -n \log \Gamma(\alpha) + (\alpha-1) \sum_{i=1}^n \log x_i - \sum x_i$

(2) $S(\alpha) = -n\psi(\alpha) + \sum \log x_i$

(b) (2 marks) Find the Fisher Information for α.

(1) $J(\alpha) = -\partial S / \partial \alpha = n\psi'(\alpha)$

(1) $I(\alpha) = EJ(\alpha) = n\psi'(\alpha)$

(c) (4 marks) State the asymptotic distribution of $\hat{\alpha}$, the MLE for α. Give the family, the mean, and the variance.

$\hat{\alpha} \overset{\cdot}{\sim} N\left(\alpha_0, \frac{1}{n\psi'(\alpha_0)}\right)$ OR $\frac{\alpha - \alpha_0}{1/\sqrt{n\psi'(\alpha_0)}} \xrightarrow{d} N(0,1)$

-1 if they forget the $_0$ in α_0 (and just write α), EACH TIME.
Because n appears in the variance, $\xrightarrow{d}$ is wrong and $\overset{\cdot}{\sim}$ is right (-1 for this)