STA261 Summer 2018

Quiz 2

July 11th, 2018

First Name: SOLUTIONS.

Last Name:

Student Number:

This quiz is out of 10 marks. Do ALL of your work on the back of the quiz, where the questions are. You can use the front for rough work, but nothing on the front will be marked, or even seen by the TAs.

If $X \sim N(\mu, \sigma^2)$ then $E(X) = \mu$ and $Var(X) = \sigma^2$.

If $X \sim Exp(\theta)$, the density is $f_X(x) = \frac{1}{\theta}e^{-x/\theta}$ for $\theta > 0, x > 0$.

BELOW SPACE IS FOR ROUGH WORK. NOTHING WRITTEN HERE WILL BE READ OR MARKED.

1. (4 marks) Let $X \sim N(\mu, \sigma^2)$. Show that $s^2 = \frac{1}{n-1}\sum_{i=1}^{n}\left(X_i - \bar{X}\right)^2$, with $\bar{X} = \frac{1}{n}\sum_{i=1}^{n} X_i$, is consistent for σ^2.

2 By LLN, $\bar{X} \xrightarrow{P} \mu$ and $\overline{X^2} = \frac{1}{n}\sum X_i^2 \xrightarrow{P} E(X^2) = \sigma^2 + \mu^2$

$$s^2 = \frac{n}{n-1} \times \frac{1}{n}\left(\sum X_i^2 - n\bar{x}^2\right)$$

2 $$= \frac{n}{n-1} \times \left(\overline{X^2} - \bar{X}^2\right)$$

$$\xrightarrow{P} 1 \times \left(\sigma^2 + \mu^2 - \mu^2\right)$$

$$= \sigma^2$$

2. (6 marks) Let $X \sim Exp(\theta)$. Find a Method of Moments estimator for θ.

$$f(x) = \frac{1}{\theta}e^{-x/\theta}, \quad \theta > 0, \quad x > 0$$

$$E(X) = \frac{1}{\theta}\int_0^\infty x e^{-x/\theta}$$

Let $u = x$, $du = dx$, $dv = \frac{1}{\theta}e^{-x/\theta}dx$, $V = -e^{-x/\theta}$

3 $$= \left(-xe^{-x/\theta}\right)\Big|_0^\infty - \int_0^\infty -e^{-x/\theta}dx$$

$$= \left(0 \times e^{-0/\theta} - \lim_{x\to\infty} xe^{-x/\theta}\right) - \theta\left(e^{-x/\theta}\Big|_0^\infty\right)$$

$$= \theta.$$

3 Set $E(X) = \bar{X}$

$$\Rightarrow \hat{\theta} = \bar{x}$$