STA261: Lecture 11

Hypothesis Tests & Confidence Intervals II

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Disclaimer

The materials in these slides are intended to be a companion to the course textbook, *Mathematical Statistics and Data Analysis, Third Edition*, by John A Rice. Material in the slides may or may not be taken directly from this source. These slides were organized and typeset by Alex Stringer.

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Two Sample Problems

Consider the following situation: a pharmeceutical company has a new blood pressure medication. They want to know if their medication lowers blood pressure, on average.

They bring in n individuals, and measure their blood pressure before and after taking the drug.

How to tell if the treatment worked?

Paired Samples

This is an example of a **paired samples** problem.

Let X_i represent patient i's blood pressure before treatment, and Y_i represent it after treatment. Clearly X_i and Y_i are not statistically independent, but we may consider pairs of tuples (X_i, Y_i), (X_j, Y_j) to be *mutually independent* of each other (measurements on different patients are independent).

Suppose $X_i \sim N(\mu_x, \sigma^2)$ and $Y_i \sim N(\mu_y, \sigma^2)$.

What hypothesis do we want to test?

Paired Samples

We saw how to do this when we had *independent* samples from two populations using a likelihood ratio test. But our two "populations" here are not independent.

Define $d_i = X_i - Y_i$. Then

$$d_i \overset{IID}{\sim} N(\mu_x - \mu_y, \sigma_d^2)$$

and we may proceed with one-sample inference to solve this problem.

Independent Samples

What about when the observations are not naturally paired- suppose the company brought in two groups of people, one to serve as a treatment group and the other as control.

$X_i \overset{IID}{\sim} N(\mu_x, \sigma^2)$ and $Y_i \overset{IID}{\sim} N(\mu_y, \sigma^2)$.

But X_i and Y_i are not paired in any natural way.

Because of mutual independence, we can still write

$$\bar{X} - \bar{Y} \sim N(\mu_x - \mu_y, 2\sigma^2)$$

...