STA261 Summer 2018

Quiz 6

July 25th, 2018

First Name: SOLUTIONS

Last Name: ______________________________

Student Number: ______________________________

This quiz is out of 10 marks. Do ALL of your work on the back of the quiz, where the questions are. You can use the front for rough work, but nothing on the front will be marked, or even seen by the TAs.

If $X \sim Bernoulli(p)$ then $P(X = x) = p^x(1-p)^{1-x}$, for $x = 0, 1$.

$Var(\Sigma X_i)$

If $X_i \overset{IID}{\sim} Bernoulli(p)$ then $\sum_{i=1}^{n} X_i \sim Binomial(n, p)$, with $E\left(\sum_{i=1}^{n} X_i\right) = np$, $V\cancel{ar}(X) = np(1-p)$.

BELOW SPACE IS FOR ROUGH WORK. NOTHING WRITTEN HERE WILL BE READ OR MARKED.

1. Let $X_i \overset{IID}{\sim} Bern(p)$ be an IID random sample from a Bernoulli distribution with parameter $p \in (0, 1)$.

(a) (2 marks) Find an unbiased estimator for p

1 $E(\Sigma X_i) = np \Rightarrow E(\frac{1}{n}\Sigma X_i) = p$

1 $\hat{p} = \frac{1}{n}\Sigma X_i$ is ~~th~~ unbiased for p.

(b) (8 marks) Figure out whether it is efficient

Need Fisher info. in sample.

1 $L(p) = p^{\Sigma X_i}(1-p)^{n-\Sigma X_i}$; $\ell(p)$ 1 $= \Sigma X_i \log p + (n-\Sigma X_i)\log(1-p)$

1 $S(p) = \partial \ell / \partial p = \frac{\Sigma X_i}{p} - \frac{n-\Sigma X_i}{1-p}$; $J(p)$ 1 $= \frac{-\partial S}{\partial p} = \frac{\Sigma X_i}{p^2} + \frac{n-\Sigma X_i}{(1-p)^2}$

1 $I(p) = E J(p) = \frac{n}{p} + \frac{n}{(1-p)} = \frac{n}{p(1-p)}$

Hence the cramer-rao lower band is $Var(\hat{p}) \geq \frac{p(1-p)}{n}$ 1 $\forall \hat{p}$ unbiased.

With $\hat{p} = \frac{1}{n}\Sigma X_i$, $Var(\hat{p}) = \frac{np(1-p)}{n^2} = \frac{p(1-p)}{n}$ 1

Hence $\hat{p}$ achieves the CRLB, and is efficient. 1