STA261 Summer 2018

Quiz 1

July 9th, 2018

First Name: SOLUTIONS.

Last Name:

Student Number:

This quiz is out of 10 marks. Do ALL of your work on the back of the quiz, where the questions are. You can use the front for rough work, but nothing on the front will be marked, or even seen by the TAs.

You might find it helpful to recall Chebyshev's inequality: if X is a random variable with $E(X) < \infty$ and $Var(X) < \infty$ then for any $\epsilon > 0$,

$$P\left(\left|X - E(X)\right| > \epsilon\right) \leq \frac{Var(X)}{\epsilon^2}$$

BELOW SPACE IS FOR ROUGH WORK. NOTHING WRITTEN HERE WILL BE READ OR MARKED.

1. (a) (2 marks) Consider a sequence of independent random variables X_n, $n = 1, 2, \ldots$ with common finite mean $E(X_n) = \mu < \infty$ and (not necessarily common) finite variance $Var(X_n) = \sigma_n^2 < \infty$. Fix $k \in \mathbb{R}$. State what it means for X_n to converge in probability to k, $X_n \xrightarrow{p} k$.

2

$X_n \xrightarrow{?} K \Rightarrow \lim_{n\to\infty} P(|X_n - K| > \varepsilon) = 0$

OR $\lim_{n\to\infty} P(|X_n - K| < \varepsilon) = 1 \quad \forall \varepsilon > 0$

They may also give a verbal description; this is fine.

(b) (4 marks) Prove that if $\lim_{n\to\infty} Var(X_n) = 0$, then $X_n \xrightarrow{p} \mu$.

By Chebyshev, $P(|X_n - \mu| > \varepsilon) \leq \dfrac{Var(X_n)}{\varepsilon^2} \quad \forall \varepsilon > 0$

so $0 \leq P(|X_n - \mu| > \varepsilon) \leq \dfrac{Var(X_n)}{\varepsilon^2} \xrightarrow{n\to\infty} 0 \quad \forall \varepsilon > 0$

Hence $X_n \xrightarrow{p} \mu$

(c) (4 marks) State and prove the Law of Large Numbers as in lecture.

1 Let $\bar{X}_n = \frac{1}{n}\sum_{i=1}^{n} X_i$, $E(X_i) = \mu < \infty$, $Var(X_i) = \sigma^2 < \infty \quad \forall i = 1..n \in \mathbb{N}$

1 LLN: $\bar{X}_n \xrightarrow{p} \mu$ as $n \to \infty$

Proof: $E(\bar{X}_n) = \mu$, $Var(\bar{X}_n) = \sigma^2/n \quad \forall n \in \mathbb{N}$

2 $\Rightarrow Var(\bar{X}_n) \to 0$ as $n \to \infty$

So by part b), $\bar{X}_n \xrightarrow{p} \mu$.