STA261: Problems 7

Alex Stringer

July, 2018

This assignment is not for credit. Complete the questions as preparation for the quiz in tutorial 7 on July 30th. The questions on the quiz will be very similar to the questions on the assignment.

1. Let $X_i \overset{IID}{\sim} Unif(a,b)$.
   (a) Write down the likelihood.
   (b) Find the MLE $(\hat{a},\hat{b})$ of (a,b).
   (c) Explain why you can't use the distributional results we proved in Lecture 4 in this example. Consider the "Regularity Conditions" from the lecture slides, and reference explicitly which one(s) is/are violated.
2. Let $X_i \overset{IID}{\sim} Laplace(\theta)$, with density $f_{x_i}(x_i) = \frac{1}{2}\exp\left(-|x_i - \mu|\right)$
   (a) Write down the likelihood.
   (b) Find the MLE $\hat{\mu}$ of μ.
   (c) Can you use the distributional results we derived in Lecture 4? Check each of the "Regularity Conditions" as in question 1.
3. Let $X_i \overset{IID}{\sim} Gamma(\alpha,\beta)$ with density $f_X(x) = \frac{1}{\Gamma\alpha\beta^\alpha}x^{\alpha-1}\exp\left(-\frac{x}{\beta}\right)$.
   (a) Write down the likelihood for (α,β)
   (b) Find the score statistic for α and β
   (c) Find the observed and Fisher informations for α and β
   (d) For fixed α, state the asymptotic distribution of $\hat{\beta}$, the MLE for β
   (e) For fixed β, state the asymptotic distribution of $\hat{\alpha}$, the MLE for α
4. Prove that if $X_i \overset{IID}{\sim} N(\mu,1)$, the CLT for the MLE derived in lecture is exact, not approximate. There are a couple indirect ways of showing this, but I want you to do it this way:
   (a) Write down the log-likelihood and the score statistic
   (b) Take a first-order Taylor expansion of $S(\mu_0)$ about the point $\hat{\mu}$. What is the remainder term equal to, and why?
   (c) Explain why this completes the proof.