

STA261 Summer 2018

Quiz 10

August 13th, 2018

First Name: SOLUTIONS.

Last Name: _____

Student Number: _____

This quiz is out of 10 marks. Do ALL of your work on the back of the quiz, where the questions are. You can use the front for rough work, but nothing on the front will be marked, or even seen by the TAs.

If $X_i \stackrel{iid}{\sim} \text{Unif}(0, \theta)$ then $W_n = X_{(n)}/\theta$ has CDF $F(w) = w^n$.

BELOW SPACE IS FOR ROUGH WORK. NOTHING WRITTEN HERE WILL BE READ OR MARKED.

1. (a) (8 marks) Let $X_i \stackrel{iid}{\sim} \text{Unif}(0, \theta)$. Find a $1 - \alpha$ confidence interval for θ of the form $(X_{(n)}, qX_{(n)})$, where $X_{(n)} = \max(X_i)$ is the sample maximum, and $q > 1$ is a constant that you have to find.

② Want $P(X_{(n)} \leq \theta \leq qX_{(n)}) = 1 - \alpha$

$$= P\left(\frac{1}{qX_{(n)}} \leq \frac{1}{\theta} \leq \frac{1}{X_{(n)}}\right)$$

$$= P\left(\frac{1}{q} \leq \frac{X_{(n)}}{\theta} \leq 1\right)$$

② $= F_W(1) - F_W(1/q)$

$$= 1 - (1/q)^n = 1 - \alpha$$

$$\frac{1}{q}^n = \alpha$$

② $q = \alpha^{-1/n}$

② $\Rightarrow (X_{(n)}, \alpha^{-1/n} X_{(n)})$ is a $1 - \alpha$ CI for θ .

- (b) (2 marks) Suppose I suggest the interval $\left(2\bar{x} - \frac{1}{\sqrt{n}}, 2\bar{x} + \frac{1}{\sqrt{n}}\right)$. Give a clear and very brief explanation as to why your interval from part (a) is preferable.

② The interval $(2\bar{x} - 1/\sqrt{n}, 2\bar{x} + 1/\sqrt{n})$ contains (possibly) values of θ that are less than $X_{(n)}$. We know $\theta \geq X_{(n)}$, so we should make $X_{(n)}$ the lower value of any confidence interval we build for $X_i \stackrel{iid}{\sim} \text{Unif}(0, \theta)$