STA261 Summer 2018

Quiz 9

August 8th, 2018

First Name: SOLUTIONS

Last Name: _____

Student Number: _____

This quiz is out of 10 marks. Do ALL of your work on the back of the quiz, where the questions are. You can use the front for rough work, but nothing on the front will be marked, or even seen by the TAs.

If $X_i \overset{IID}{\sim} Poisson(\lambda)$ then the mass function is $P(X = x) = \frac{\lambda^x e^{-\lambda}}{x!}$ for $x = 0, 1, \ldots$, and $E(X) = \lambda$.

BELOW SPACE IS FOR ROUGH WORK. NOTHING WRITTEN HERE WILL BE READ OR MARKED.

1. (10 marks) Suppose you are hanging out on the corner counting busses that go by, for some reason. A reasonable model for the number of busses that go by in an hour (say) is $X \sim Poission(\lambda)$. You do this all day (for 12 hours) and observe $X_1, \ldots X_{12}$ with a sample mean of $\bar{x} = 15$ busses per hour. Your friend says "I've been standing on this corner for years and I swear only 12 busses go by per hour on average". Test your friend's claim at the 5% level. Show all your work.

Perform a likelihood ratio test of $\lambda = 12$ for Poisson($\lambda$).

(1) Unrestricted likelihood: $L(\lambda) = \lambda^{\Sigma x_i} e^{-n\lambda} / \Pi x_i!$

(1) $\ell(\lambda) = \Sigma x_i \log\lambda - n\lambda - \Sigma \log x_i!$

(1) $S(\lambda) = \Sigma x_i / \lambda - n = 0 \Rightarrow$ MLE $\hat{\lambda} = \bar{x} = 15$.

(1) $\Lambda = \dfrac{L(12)}{L(15)} = \dfrac{12^{\Sigma x_i} e^{-12} / \Pi x_i!}{15^{\Sigma x_i} e^{-15} / \Pi x_i!}$ Note $n=12$, $\bar{x}=15 \Rightarrow \Sigma x_i = 180$

$= \left(\frac{12}{15}\right)^{180} e^{3}$

(1) $-2\log\Lambda = -2(180\log(12/15) + 3) = -2(-37.17) = 74.3.$

(1) Under $H_0: \lambda = 12$, $-2\log\Lambda \dot{\sim} \chi^2_1$

(4) A value of 74.3 is extremely unlikely under $H_0$. Conclude that observing 15 busses per hour for 12 hours provides substantial evidence that $\lambda \neq 12$.