TD and Friends

RL Context

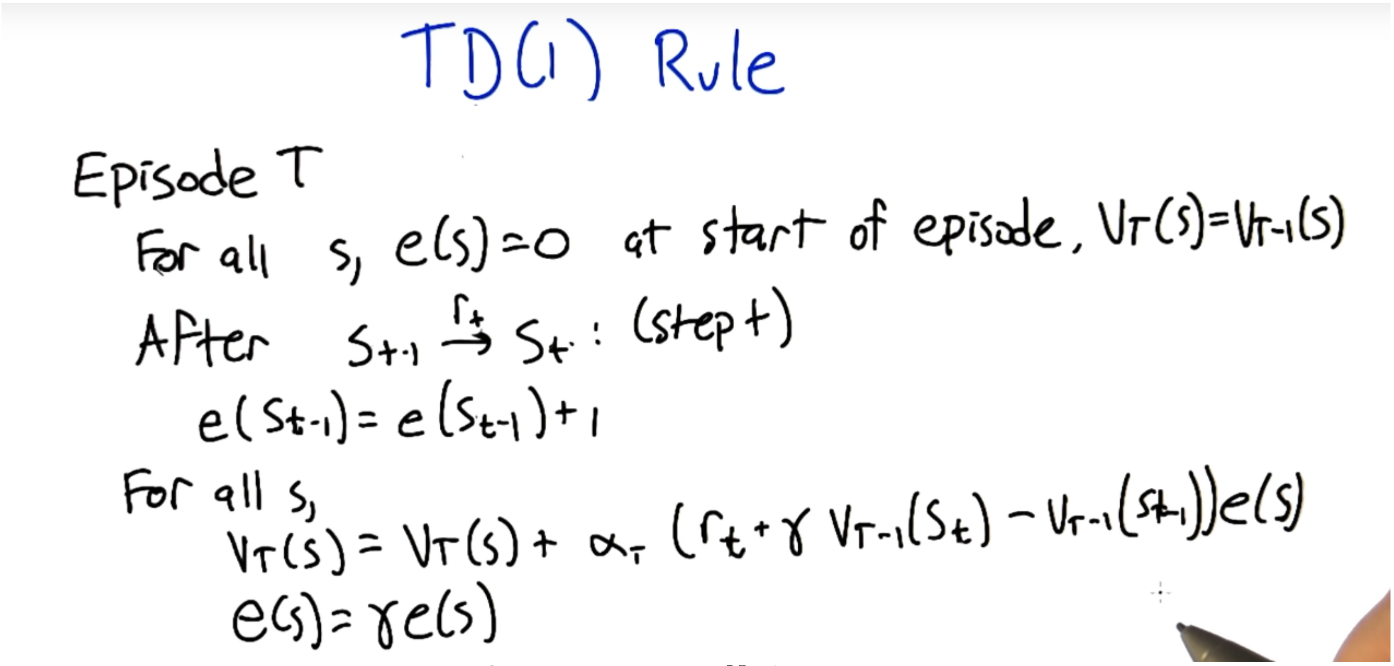
- $\langle s, a, r\rangle^* \rightarrow$ RL algo $\rightarrow \pi$
- model-based (more supervised, less direct learning):
  - $\langle s, a, r\rangle^* \leftrightarrow$ model learner $\rightarrow T, R \rightarrow$ MDP solve $\rightarrow Q^* \rightarrow$ argmax $\rightarrow \pi$
- value-function based (model-free):
  - $\langle s, a, r\rangle^* \leftrightarrow Q \rightarrow$ argmax $\rightarrow \pi$
- policy search (more supervised, less direct learning):
  - $\langle s, a, r\rangle^* \rightarrow$ policy update $\leftrightarrow \pi$

TD(λ) basics

- Computing estimates incrementally
  - $V_T(s) = \frac{(T-1)V_{T-1}(s) + R_T(s)}{T} = \frac{T-1}{T} V_{T-1} + \frac{1}{T} R_T(s) = V_T(s) + \alpha_T(R_T(s) - V_{T-1}(s))$, where $\alpha_T = \frac{1}{T}$
  - interpretation: value is updated by the difference b/w the return at current time and value at previous time
  - Isbell: looks like perceptron update (the diff is error)
- properties of learning rates
  - if $\sum_T \alpha_T = \infty$ and $\sum_T \alpha_T^2 < \infty$, then $\lim_{T\rightarrow\infty} V_T(s) = V(s)$
  - interpretation: sum of alphas must be big enough to move to true value, but not so big they hide noise

TD(1)

- let $e(s)$ be the elligibility of s (kind of like a filter for application of the rule that reflects age of states)
- $V_T(s) = V_T(s) + \alpha_T(r_t + \gamma V_{T-1}(s_t) - V_{T-1}(s_{t-1}))e(s)$
- Big idea: telescoping series on updating values
- behaves like an infinte-step ahead estimator
- update rule:



- $\Delta V_T(s_n) = \alpha(r_n + \gamma r_{n+1} + \gamma^2 r_{n+2} + ... + \gamma^{N-1} V_{T-1}(s_N) - V_{T-1}(s_n)) =$ "what you predicted for val of s vs. what value of s was last episode"
- TD(1) is the same outcome-based updates (if no repeated states)
  - TD(1) allows for extra learning when repeating a state
- maximum likelihood estimate can be better than outcome-based TD(1) since it uses more data (see here for more)

TD(0)

- equivalent to maximum likelihood estimate?
- removes elligibility vector
- behaves like a 1-step ahead estimator

TD(λ)

- updates based on differences b/w temporally successive predictions
- rules comparison:

TD(λ) Rule

Episode T

For all s, $e(s)=0$ at start of episode, $V_T(s)=V_{T-1}(s)$

After $s_{t-1} \xrightarrow{r_t} s_t$: (step t)

$e(s_{t-1}) = e(s_{t-1}) + 1$

For all s,

$V_T(s) = V_T(s) + \alpha_T \left(r_t + \gamma V_{T-1}(s_t) - V_{T-1}(s_{t-1})\right) e(s)$

$e(s) = \cancel{\gamma e(s)}\ \lambda \gamma e(s)$

✓ TD(1)

Episode T

For all s, $e(s)=0$ at start of episode, $V_T(s)=V_{T-1}(s)$

After $s_{t-1} \xrightarrow{r_t} s_t$: (step t)

$e(s_{t-1}) = e(s_{t-1}) + 1$

For all s,

$V_T(s) = V_T(s) + \alpha_T \left(r_t + \gamma V_{T-1}(s_t) - V_{T-1}(s_{t-1})\right) e(s)$

$e(s) = \gamma e(s)$

TD(0) ✓

Episode T

For all s, at start of episode, $V_T(s)=V_{T-1}(s)$

After $s_{t-1} \xrightarrow{r_t} s_t$: (step t)

For all $s = s_{t-1}$

$V_T(s) = V_T(s) + \alpha_T \left(r_t + \gamma V_{T-1}(s_t) - V_{T-1}(s_{t-1})\right)$

- if λ is not 0 or 1, TD(λ) is a weighted combination of all k-step estimators