Markov Decision Process

- only present matters
- stationary model (rules of game don't change)

Problem:

- states : s
- model : $T(s, a, s') \sim Pr(s'|s, a)$
- actions : a
- reward : $R, R(s)$

Policy: a "solution" to a MDP problem

- $\pi(s) \rightarrow a$ (policy maps states to actions)
- π^* maximized the total cummulative reward

RL vs Supervised Learning

- in SL the input is $\langle s, a \rangle$ tuples (a is correct action in s)
- in RL the input is $\langle s, a, r \rangle$ tuples (r is reward for a in s)
- MDPs have delayed rewards

Rewards

- temporal credit assingment problem: how to rate past choices in problems with delayed rewards
- small negative reward in each state other than game ending states (goal/lava) trains the ideal policy π^*
    - large positive reward makes a policy that avoids goal (since endless traversal is better)
    - large negative reward makes a policy that can choose the lava (positive of goal not enough to outweigh negative of long journey)
- in MDPs minor changes matter

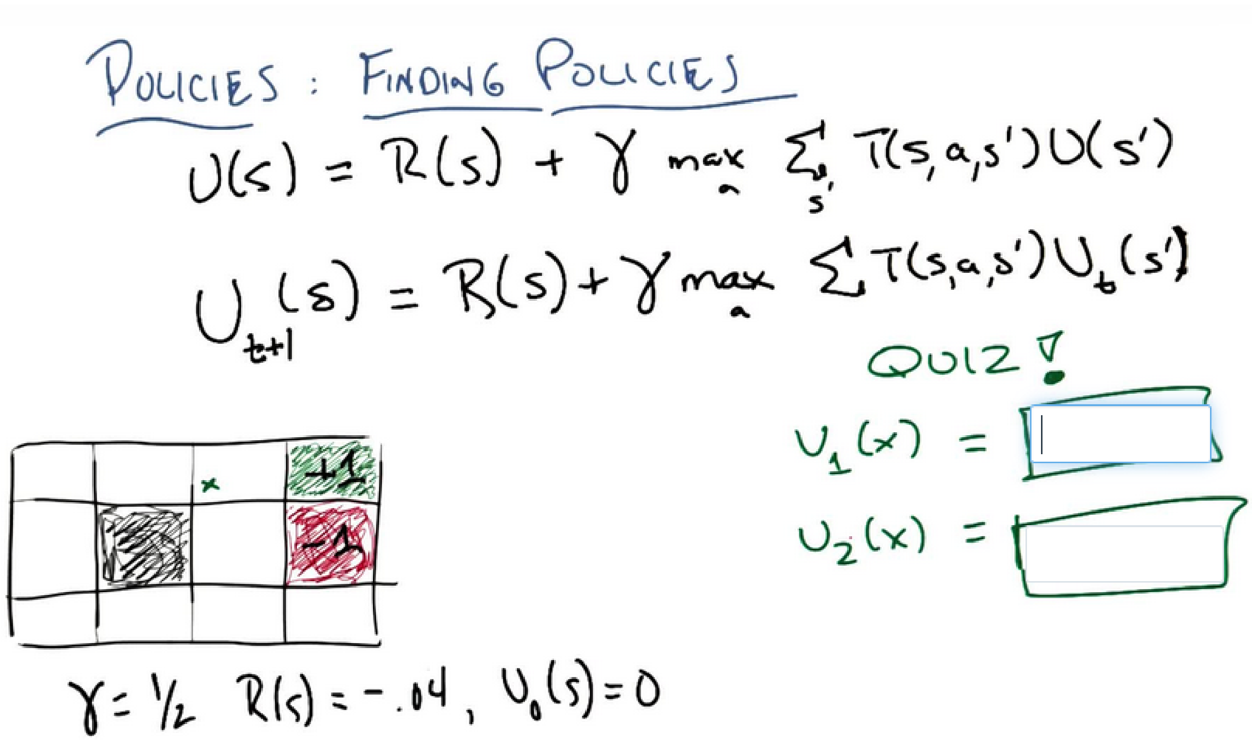
Sequences of Rewards

- infinite vs finite horizons (this course is infinite only)
- utility of sequences: if $U(s0, s1, ..) > U(s0, s1', ..)$, then $U(s1, s2, ..) > (s1', s2', ..)$
    - note this is equivalent to the stationary model requirement of MDP
    - what is a good utility function?
        - then $U(s0, s1, ..) = \sum_t (R(s_t))$, but in this case any countably infinite sum is equivalent for unending games (if you live forever, it doesn't matter what you do)

        - "discounted rewards" - instead use $U(s0, s1, ..) = \sum_t (\gamma^t R(s_t))$ where $|\gamma| < 1$ -- this is a geo series that converges
        - so $U(s0, s1, ..) \leq \sum_t (\gamma^t R_{max}) = R_{max}/(1-\gamma)$

Policies

- $\pi^* = \text{argmax}_\pi E[\sum_t \gamma^t R(s_t)|\pi] =$ the policy that maximizes expected value of reward
- $U^\pi(s) = E[\sum_t \gamma^t R(s_t)|\pi, s_0 = s] \neq R(s)$
- Reward is immediate, utility is long term value of policy
- Usually we use $U(s) = U^{\pi^*}(s)$ -- "true utility of a state"
- $\pi^*(s) = \text{argmax}_a \sum_{s'} T(s,a,s')U(s')$
    - knowing true utility means you can find optimal policy
- **Bellman Equation:** $U(s) = R(s) + \gamma \max_a \sum_{s'} T(s,a,s')U(s')$
    - how to solve? n equations (one for each utilities) and n unknowns (utilities), but not linear equations (due to max)
    - algo to solve (**value iteration**): (proof of convergence on slide 25 here)
        - start with arbitrary utilities
        - update utilities based on neighbors
        - repeat update step until convergence
    - Ex.



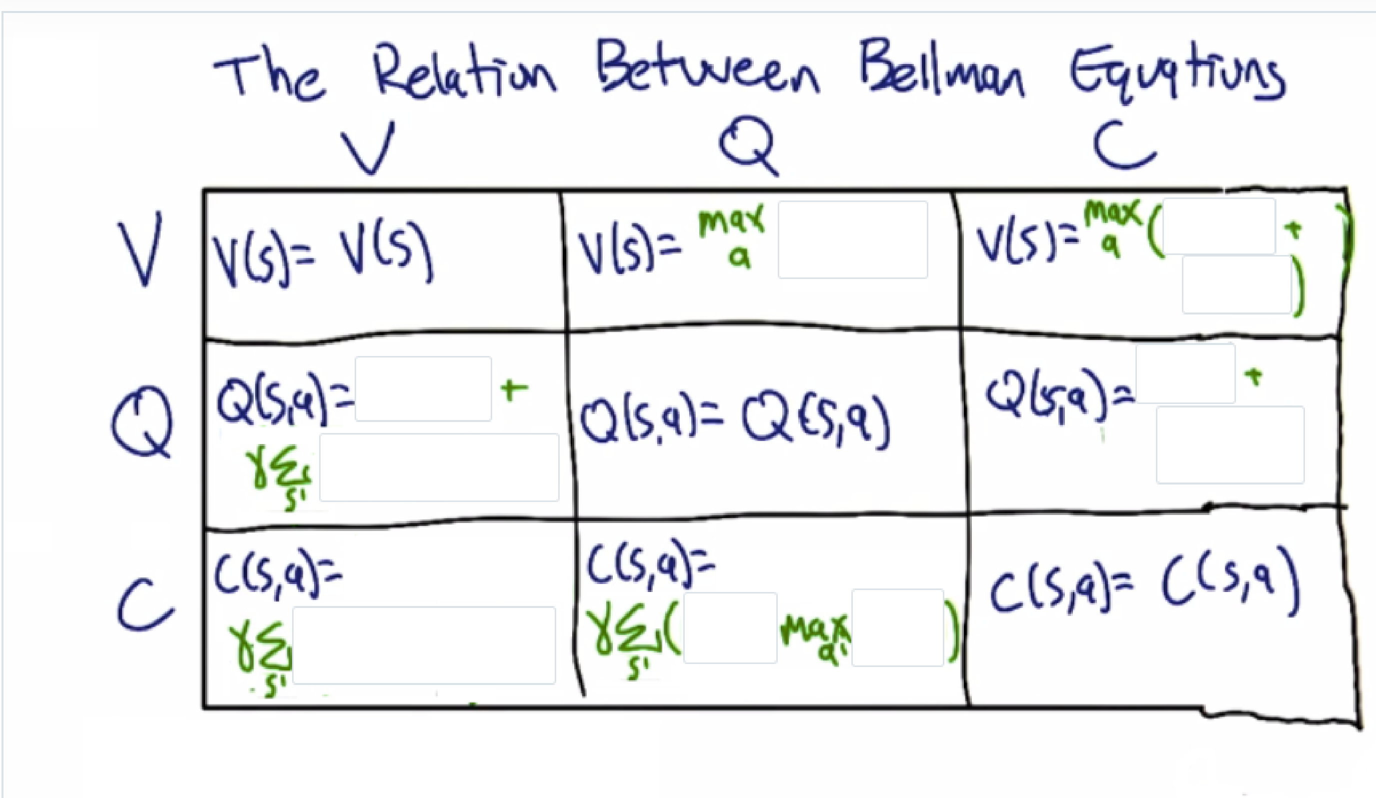
notes: Don't forget the transition probabilities: 0.8 of going in the desired direction, and 0.1 of going in each of the directions at 90-degrees, $U_0(green) = 1$, ~ $U_0(red) = -1$

        - $U_1(x) = -0.04 + 0.5[0 + 0 + 0.8 * 1] = 0.36$

- $U_2(x) = -0.04 + 0.5((0.1)(-0.4) + (0.1)(0.36) + 0.8 * 1) = 0.376$
  - the optimal policy can be found even if the true utility is not found (order of actions is all that is needed)
  - find policy (**policy iteration**):
    - start with π_0
    - evaluate: given π_t calculate $U_t = U^{\pi_t}$, where $U_t = R(s) + \gamma \sum_{s'} T(s, \pi_t(s), s') U_t(s') =$ reward + gamma*expected utility
    - improve: $\pi_{t+1} = \text{argmax}_a \sum T(s, a, s') U_t(s')$
  - the algo above is now linear (no max to find U_t)

More on Bellman

- other ways to express Bellman:
  - value of state $= V(s) = \max_a (R(s, a) + \gamma \sum_{s'} T(s, a, s') V(s'))$
  - quality of a state,action $= Q(s, a) = R(s, a) + \gamma \sum_{s'} T(s, a, s') \ \max_{a'} Q(s', a')$ -- useful when you don't know R and T
  - continuation of state,action $= C(s, a) = \gamma \sum_{s'} T(s, a, s') \ \max_{a'} (R(s', a') + C(s', a'))$
- Ex.



View Intro

VIEW ANSWER

SUBMIT ANSWER

For this quiz, each answer will be an arithmetic expression containing the terms V(s), Q(s,a), C(s,a), R(s,a), and/or T(s,a,s'). (There may be primes on some of the arguments, as well. For example: Q(s',a').)

You can use the *character, a space, or concatenation to represent multiplication. (For example, "V(s)* R(s,a)", "R(s,a) V(s)", and "R(s,a)V(s)" are all equal expressions.)

Soln:

The Relation Between Bellman Equations

	V	Q	C
V	$V(s) = V(s)$	$V(s) = \max_a Q(s,a)$	$V(s) = \max_a \left(R(s,a) + C(s,a)\right)$
Q	$Q(s,a) = R(s,a) + \gamma \sum_{s'} T(s,a,s')\, \underline{V}(s')$	$Q(s,a) = Q(s,a)$	$Q(s,a) = R(s,a) + C(s,a)$
C	$C(s,a) = \gamma \sum_{s'} T(s,a,s')\, V(s')$	$C(s,a) = \gamma \sum_{s'} T(s,a,s') \max_{a'} Q(s',a')$	$C(s,a) = C(s,a)$