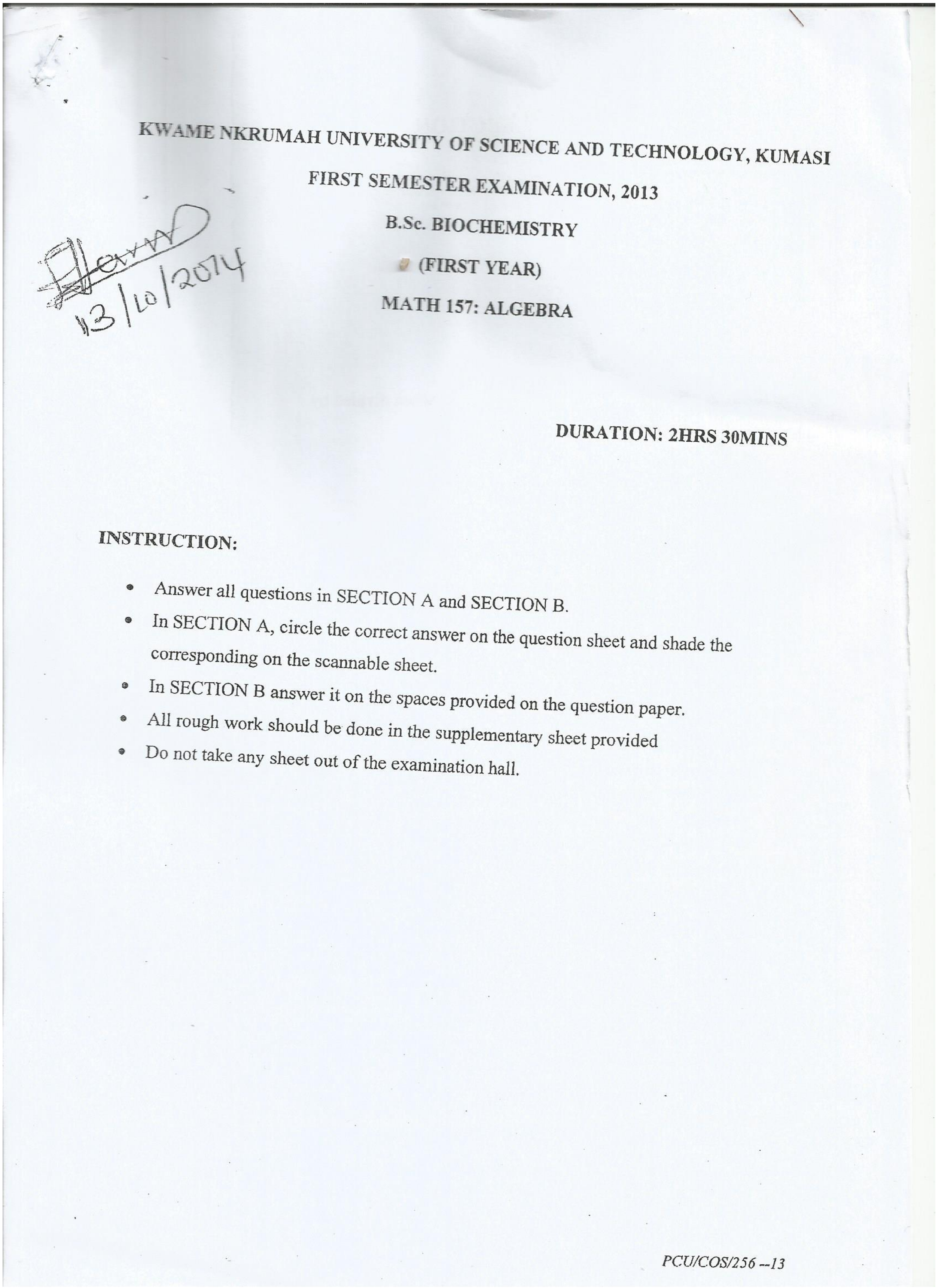
COMPILED BY ADJEI PATRICK

PATO

0542548870

MATHS 157

PAST QUESTIONS



KWAME NKRUMAH UNIVERSITY OF SCIENCE AND TECHNOLOGY, KUMASI
FIRST SEMESTER EXAMINATION, 2013
B.Sc. BIOCHEMISTRY
(FIRST YEAR)
MATH 157: ALGEBRA

13/10/2014

DURATION: 2HRS 30MINS

INSTRUCTION:

- Answer all questions in SECTION A and SECTION B.
- In SECTION A, circle the correct answer on the question sheet and shade the corresponding on the scannable sheet.
- In SECTION B answer it on the spaces provided on the question paper.
- All rough work should be done in the supplementary sheet provided
- Do not take any sheet out of the examination hall.

PCU/COS/256 --13

SECTION A

1. The partial fraction $\frac{2}{(x+1)(x+1)} \equiv \frac{A}{(x+1)} + \frac{B}{x-1}$ corresponds to
   - (a) $A=1, B=1$
   - (b) $A=-1, B=1$
   - (c) $A=0, B=2$
   - (d) $A=x-1, B=x+1$
   - (e) none

2. The function x^3-3x^2+2x-2 has a remainder 2 when divided by
   - (a) $x+1$
   - (b) $x-1$
   - (c) $2x+1$
   - (d) $x+2$
   - (e) none

3. In the expansion of $(a-2b)^3$ the coefficient of b^2 is
   - (a) $2a^2$
   - (b) $-4a$
   - (c) $-8a$
   - (d) $12a$
   - (e) none

4. An order of a matrix is
   - (a) The length of a matrix
   - (b) The number of rows and columns in a matrix
   - (c) The highest power of a matrix
   - (d) The lowest power of a matrix
   - (e) None of the above

5. If $sinB = \frac{2}{5}$ then $\cos(2B) =$
   (a) $\frac{1}{5}$
   (b) $\frac{17}{25}$
   (c) $\frac{11}{25}$
   (d) $-\frac{3}{5}$
   (e) $\frac{2}{5}$

6. The largest solution to the nearest degree of the equation $5sin^2x + 9sinx - 2 = 0$ on the interval $0^o \leq x \leq 360^o$ is which of the following?
   (a) 12^o
   (b) 278^o
   (c) 168^o
   (d) 212^o
   (e) 22^o

7. The value of $sin100^o$ can be expressed equivalently as
   (a) $2sin50^o$
   (b) $sin50^o + cos50^o$
   (c) $2sin50^o cos50^o$
   (d) $cos^2 50^o - sin^2 50^o$
   (e) $sin50^o$

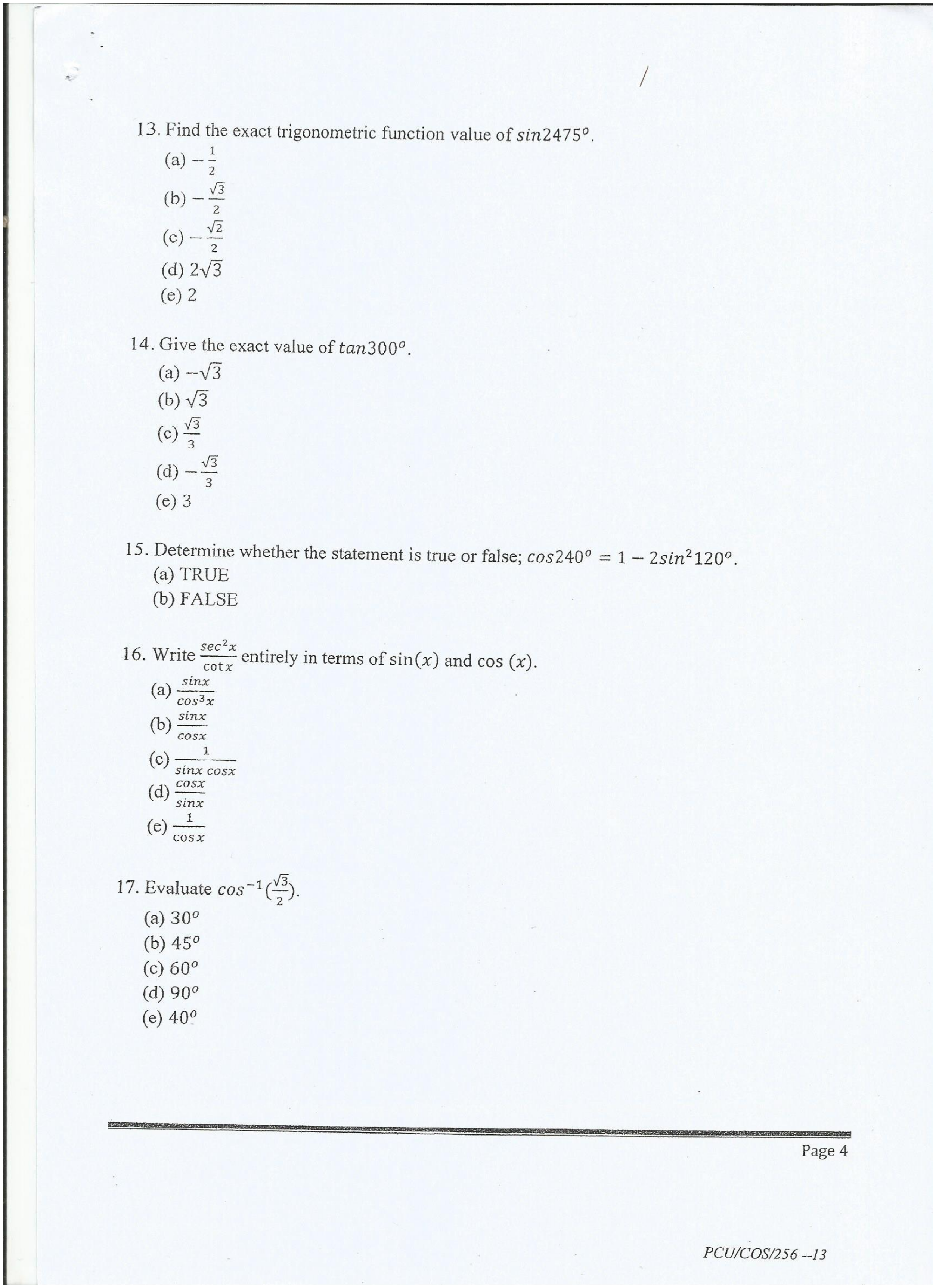
8. For an angle A where $90^o < A < 180^o$, it is given that $sinA = \frac{\sqrt{5}}{3}$. Which of the following is the value of $\cos A$?
   (a) $\frac{4}{9}$
   (b) $-\frac{2}{3}$
   (c) $-\frac{\sqrt{3}}{2}$
   (d) $\frac{\sqrt{3}}{3}$
   (e) $\frac{3}{5}$

9. Which of the following values of x solves $6^{2x-1} = 36^{-x}$?
   - (a) -4
   - (b) $\frac{1}{3}$
   - (c) $\frac{1}{4}$
   - (d) 4
   - (e) $\frac{2}{3}$

10. For a particular real number a and base b, it is known that $log_b^a = 2.75$. Determine the value of $\log_b^{(a^3)}$.
    - (a) 8.25
    - (b) 2.75
    - (c) 3
    - (d) 0.75
    - (e) 2

11. Algebraically determine the intersection point(s) of the two logarithmic functions given below; $y = log_3^{(x-6)}$ and $y = 3 - log_3^x$.
    - (a) $(27, 1)$
    - (b) $(-3, 1)$
    - (c) $(9, 1)$
    - (d) $(1, 9)$
    - (e) (3,1)

12. Solve for x; $(\sqrt{5})^x = 78125$.
    - (a) 31250
    - (b) $log_5^{(2)}$
    - (c) 14
    - (d) 10
    - (e) 25



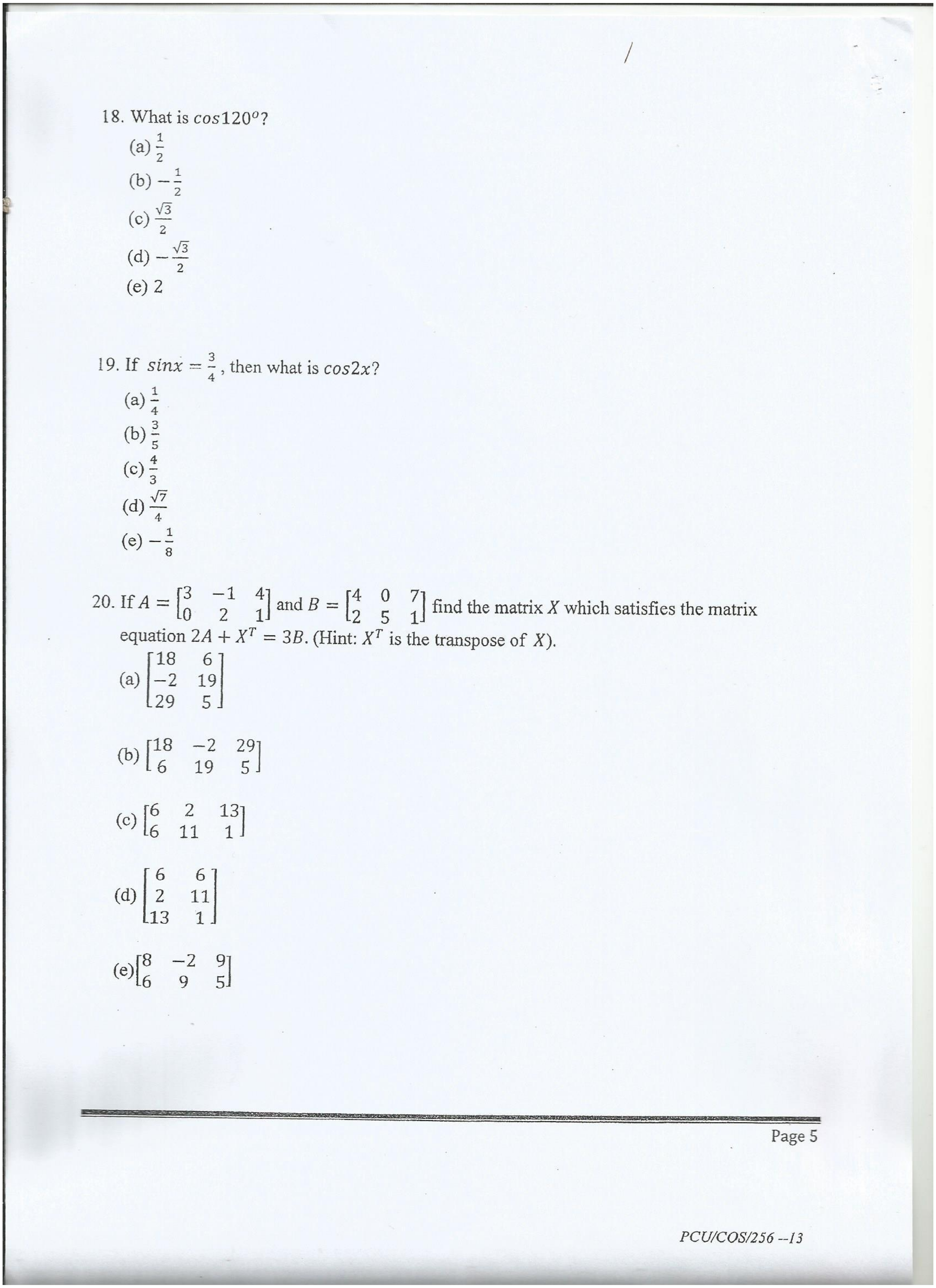
13. Find the exact trigonometric function value of $sin2475^o$.
   (a) $-\frac{1}{2}$
   (b) $-\frac{\sqrt{3}}{2}$
   (c) $-\frac{\sqrt{2}}{2}$
   (d) $2\sqrt{3}$
   (e) 2

14. Give the exact value of $tan300^o$.
   (a) $-\sqrt{3}$
   (b) $\sqrt{3}$
   (c) $\frac{\sqrt{3}}{3}$
   (d) $-\frac{\sqrt{3}}{3}$
   (e) 3

15. Determine whether the statement is true or false; $cos240^o = 1 - 2sin^2 120^o$.
   (a) TRUE
   (b) FALSE

16. Write $\frac{sec^2 x}{\cot x}$ entirely in terms of $\sin(x)$ and $\cos(x)$.
   (a) $\frac{sinx}{cos^3 x}$
   (b) $\frac{sinx}{cosx}$
   (c) $\frac{1}{sinx\ cosx}$
   (d) $\frac{cosx}{sinx}$
   (e) $\frac{1}{\cos x}$

17. Evaluate $cos^{-1}(\frac{\sqrt{3}}{2})$.
   (a) 30^o
   (b) 45^o
   (c) 60^o
   (d) 90^o
   (e) 40^o



18. What is $cos120^o$?
   (a) $\frac{1}{2}$
   (b) $-\frac{1}{2}$
   (c) $\frac{\sqrt{3}}{2}$
   (d) $-\frac{\sqrt{3}}{2}$
   (e) 2

19. If $sinx = \frac{3}{4}$, then what is $cos2x$?
   (a) $\frac{1}{4}$
   (b) $\frac{3}{5}$
   (c) $\frac{4}{3}$
   (d) $\frac{\sqrt{7}}{4}$
   (e) $-\frac{1}{8}$

20. If $A = \begin{bmatrix} 3 & -1 & 4 \\ 0 & 2 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 4 & 0 & 7 \\ 2 & 5 & 1 \end{bmatrix}$ find the matrix X which satisfies the matrix equation $2A + X^T = 3B$. (Hint: X^T is the transpose of X).
   (a) $\begin{bmatrix} 18 & 6 \\ -2 & 19 \\ 29 & 5 \end{bmatrix}$
   (b) $\begin{bmatrix} 18 & -2 & 29 \\ 6 & 19 & 5 \end{bmatrix}$
   (c) $\begin{bmatrix} 6 & 2 & 13 \\ 6 & 11 & 1 \end{bmatrix}$
   (d) $\begin{bmatrix} 6 & 6 \\ 2 & 11 \\ 13 & 1 \end{bmatrix}$
   (e) $\begin{bmatrix} 8 & -2 & 9 \\ 6 & 9 & 5 \end{bmatrix}$

21. Find the determinant of the matrix $\begin{bmatrix} 5 & -2 & 3 \\ 4 & -1 & -5 \\ -6 & 7 & 9 \end{bmatrix}$.

(a) 340
(b) 14
(c) 364
(d) 76
(e) 100

22. Find the cofactor, A_{23} of the matrix $A = \begin{bmatrix} 5 & -2 & 7 \\ 6 & 1 & -9 \\ -4 & -3 & 8 \end{bmatrix}$.

(a) -7
(b) 23
(c) -23
(d) 7
(e) 0

23. If $A = \begin{bmatrix} 0 & 1 \\ 2 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & -1 \\ 5 & 2 \end{bmatrix}$, which of the following is false?

(a) $A^2 = \begin{bmatrix} 0 & 1 \\ 4 & 9 \end{bmatrix}$
(b) $AB = \begin{bmatrix} 5 & 2 \\ 17 & 4 \end{bmatrix}$
(c) $A + B = \begin{bmatrix} 1 & 0 \\ 7 & 5 \end{bmatrix}$
(d) $3A - 4B = \begin{bmatrix} -4 & 7 \\ -14 & 1 \end{bmatrix}$
(e) All above

24. Solve for x and y in the matrix

$$\begin{bmatrix} 3 & -4 & y \\ 1 & 2 & -1 \end{bmatrix} + \begin{bmatrix} x-y & 4 & 2 \\ 4 & x & -8 \end{bmatrix} = \begin{bmatrix} 10 & 3 & -1 \\ 2 & 6x & -9 \end{bmatrix} - \begin{bmatrix} 4 & 3 & -1 \\ -3 & 3x & 0 \end{bmatrix}$$

(a) $(x, y) = (-1, -2)$
(b) $(x, y) = (1, -2)$
(c) $(x, y) = (-1, 2)$
(d) $(x, y) = (1, 2)$
(e) $(x, y) = (2, -1)$

25. Which of the following matrices is the inverse of $\begin{bmatrix} -4 & 2 \\ 11 & -6 \end{bmatrix}$?

(a) $\begin{bmatrix} -2 & \frac{11}{2} \\ 1 & -3 \end{bmatrix}$

(b) $\begin{bmatrix} 3 & -\frac{11}{2} \\ -1 & 2 \end{bmatrix}$

(c) $\begin{bmatrix} -3 & -1 \\ -\frac{11}{2} & -2 \end{bmatrix}$

(d) The inverse does not exist

(e) $\begin{bmatrix} -2 & -\frac{11}{2} \\ -1 & -3 \end{bmatrix}$

26. The coefficient of xy in the expansion of (x-3y)(2x+y) is

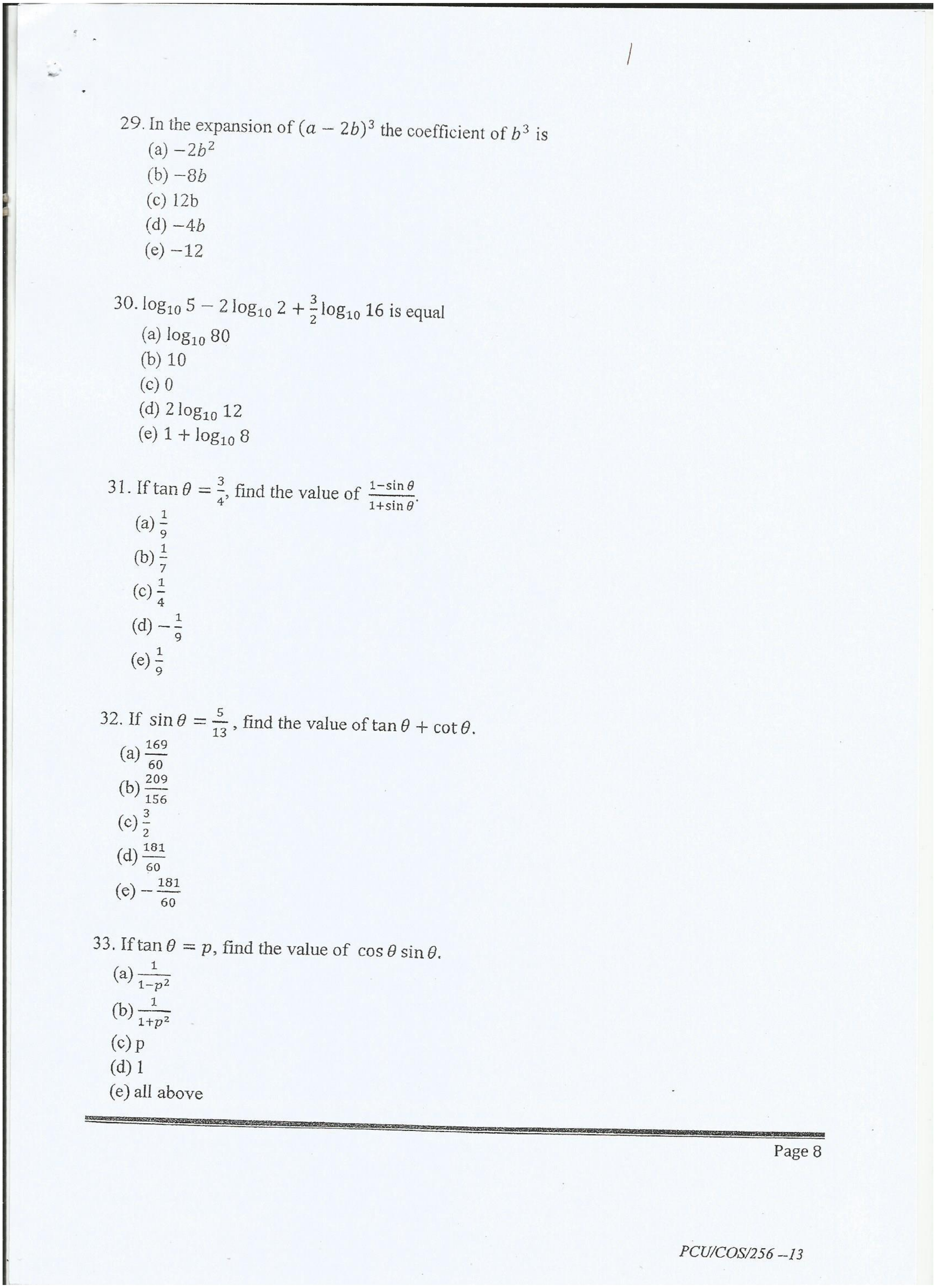
(a) 1
(b) 6
(c) 5
(d) 0
(e) -5

27. The value of $\log_5 0.04$ is

(a) 4
(b) 5
(c) 0.5
(d) -2
(e) 0.25

28. $\frac{p^{-\frac{1}{2}} \times p^{\frac{3}{4}}}{p^{-\frac{1}{4}}}$ simplifies to

(a) 1
(b) $p^{\frac{3}{4}}$
(c) $p^{\frac{1}{2}}$
(d) $p^{-\frac{1}{2}}$
(e) p



29. In the expansion of $(a-2b)^3$ the coefficient of b^3 is
   - (a) $-2b^2$
   - (b) $-8b$
   - (c) 12b
   - (d) $-4b$
   - (e) -12

30. $\log_{10}5-2\log_{10}2+\frac{3}{2}\log_{10}16$ is equal
   - (a) $\log_{10}80$
   - (b) 10
   - (c) 0
   - (d) $2\log_{10}12$
   - (e) $1+\log_{10}8$

31. If $\tan\theta=\frac{3}{4}$, find the value of $\frac{1-\sin\theta}{1+\sin\theta}$.
   - (a) $\frac{1}{9}$
   - (b) $\frac{1}{7}$
   - (c) $\frac{1}{4}$
   - (d) $-\frac{1}{9}$
   - (e) $\frac{1}{9}$

32. If $\sin\theta=\frac{5}{13}$, find the value of $\tan\theta+\cot\theta$.
   - (a) $\frac{169}{60}$
   - (b) $\frac{209}{156}$
   - (c) $\frac{3}{2}$
   - (d) $\frac{181}{60}$
   - (e) $-\frac{181}{60}$

33. If $\tan\theta=p$, find the value of $\cos\theta\sin\theta$.
   - (a) $\frac{1}{1-p^2}$
   - (b) $\frac{1}{1+p^2}$
   - (c) p
   - (d) 1
   - (e) all above

34. If $\sin\theta = \frac{2}{\sqrt{5}}$, find $\tan\theta$.

   (a) 2
   (b) $\frac{1}{2}$
   (c) $\frac{2}{5}$
   (d) $\frac{1}{\sqrt{5}}$
   (e) $-\frac{2}{5}$

35. If $\operatorname{cosec}\theta = \frac{\sqrt{13}}{3}$, find $\cot\theta$.

   (a) $\frac{2}{3}$
   (b) $\frac{3}{2}$
   (c) $\frac{\sqrt{13}}{3}$
   (d) $\frac{2}{\sqrt{13}}$
   (e) $-\frac{\sqrt{13}}{3}$

36. Find the possible value of x for which $\log_5(x-2) + \log_5(x-6) = 1$.

   (a) 1 and 7
   (b) $\frac{9}{2}$
   (c) 4 and 2
   (d) -1.3 and 9.4
   (e) -4.5 and 2

37. Find the possible value of x for which $\log_{\sqrt{3}} 9 = x$.

   (a) 3
   (b) 2
   (c) 1
   (d) 4
   (e) 5

If $\sin C = \frac{1}{\sqrt{5}}$ and C is acute, answer questions 38-39.

38. Find $\cos 2C$
   - (a) $\frac{3}{5}$
   - (b) $\frac{4}{5}$
   - (c) $\frac{4}{3}$
   - (d) $\frac{5}{3}$
   - (e) 2

39. Find $\tan 2C$.
   - (a) $\frac{3}{5}$
   - (b) $\frac{4}{5}$
   - (c) $\frac{4}{3}$
   - (d) $\frac{5}{3}$
   - (e) 1

40. Find the term involving x^8 in the expansion of $(4 + x)^{12}$.
   - (a) 127620
   - (b) 126720
   - (c) 32440320
   - (d) 32032440
   - (e) 78047

The partial fraction $\frac{4x+3}{(x-1)^2}$ can be expressed in the form $\frac{A}{X-1} + \frac{B}{(X-1)^2}$. Use this information to answer questions 41 and 43.

41. Find the value of A.
   - (a) 4
   - (b) 7
   - (c) 6
   - (d) 5
   - (e) -4

42. Find the value of 2A+B.
   (a) 15
   (b) 7
   (c) 4
   (d) 6
   (e) −6

43. Find the value of $\frac{1}{A} - \frac{2}{B}$.
   (a) $\frac{5}{14}$
   (b) $\frac{1}{28}$
   (c) $-\frac{1}{28}$
   (d) $-\frac{5}{14}$
   (e) $\frac{5}{14}$

44. Find the value of x for $5^{2x+1} + 4 = 21 \times 5^x$.
   (a) 3
   (b) 9
   (c) 0.86
   (d) 6
   (e) 4

45. Find the value of x for $\log_3 x + \log_x 9 = 3$.
   (a) 3 and 9
   (b) 0.86
   (c) 6
   (d) 3 and 1

46. If $\log_{10} 3 = a \text{ } and \text{ } \log_{10} 5 = b$, write down in terms of a and b, the value of $\log_{10} 75$.
   (a) 2ab
   (b) ab^2
   (c) a+2b
   (d) $a+b^2$
   (e) ab

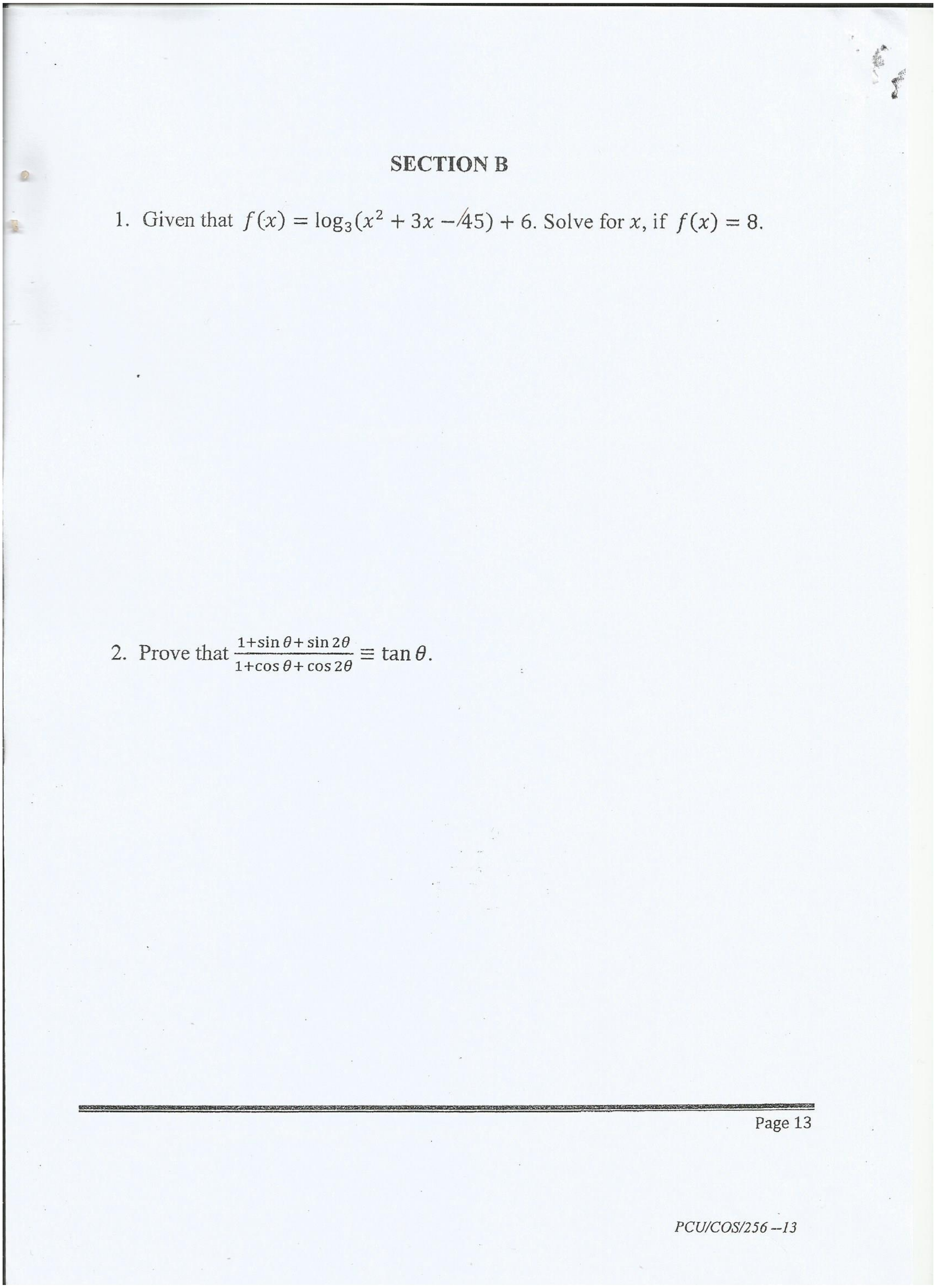
In a group of 120 musicians, 41 play highlife music, 47 play jazz and 42 play reggae music. 14 of them play both highlife and jazz, 15 play both highlife and reggae while 19 play both jazz and reggae. 8 of the musicians play all the three types of music. Use this preamble to answer questions 47 – 49.

47. Find the number of musicians who played at least one type of music
    (a) 66
    (b) 82
    (c) 71
    (d) 90
    (e) 180

48. Find the number of musicians who played none of the three types of music
    (a) 120
    (b) 90
    (c) 30
    (d) 60
    (e) 40

49. Find the number of musicians who played exactly one type of music
    (a) 58
    (b) 42
    (c) 38
    (d) 36
    (e) 16

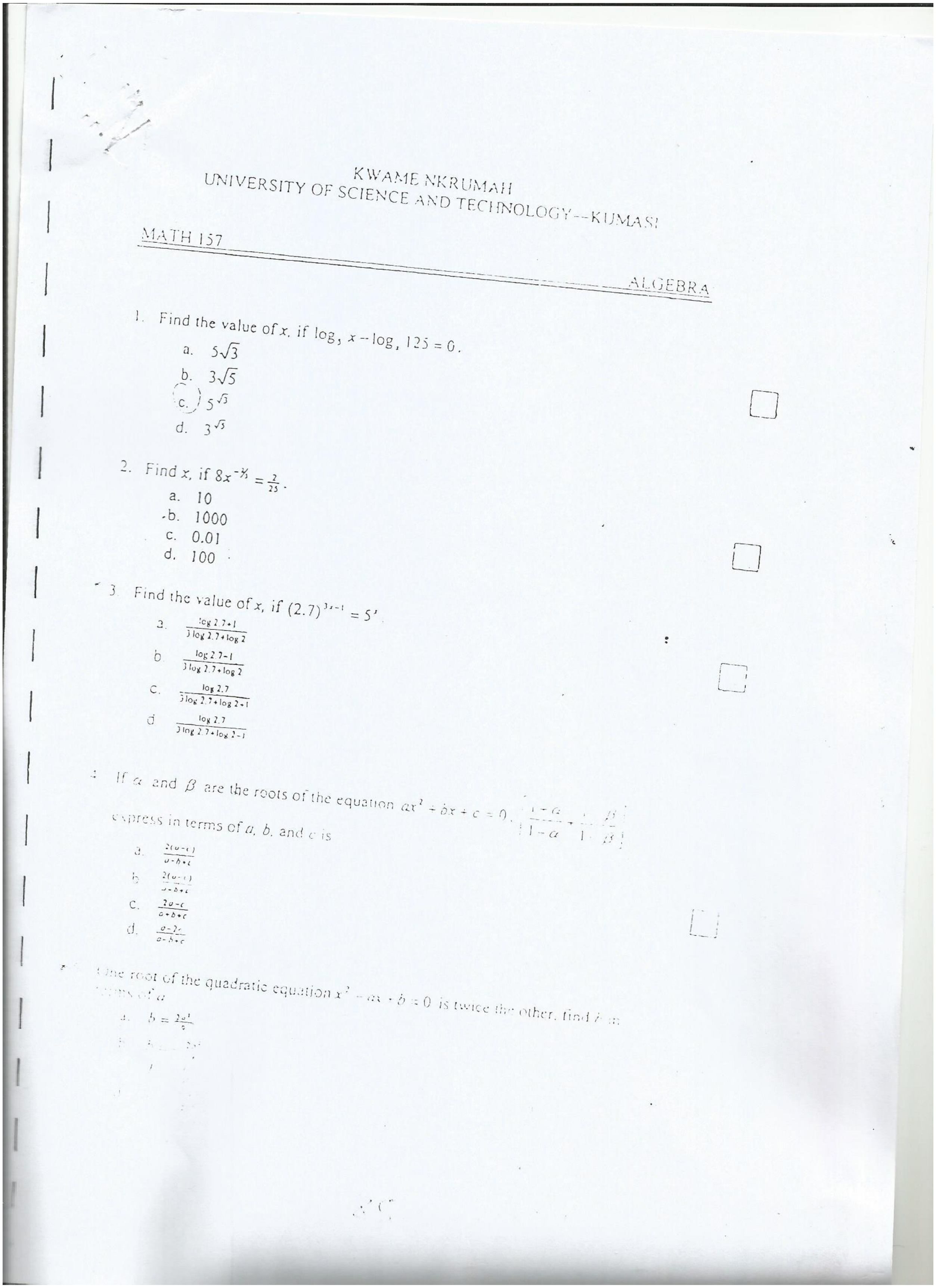
50. Find the value of x for which $9^{2x+1} = \frac{81^{x-2}}{3^x}$.
    (a) 2.5
    (b) 10
    (c) 7
    (d) -10
    (e) 1



SECTION B

1. Given that $f(x) = \log_3(x^2 + 3x - 45) + 6$. Solve for x, if $f(x) = 8$.

2. Prove that $\frac{1+\sin\theta+\sin 2\theta}{1+\cos\theta+\cos 2\theta} \equiv \tan\theta$.



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MATH 157

ALGEBRA

1. Find the value of x, if $\log_5 x - \log_x 125 = 0$.
   a. $5\sqrt{3}$
   b. $3\sqrt{5}$
   c. $5^{\sqrt{3}}$
   d. $3^{\sqrt{5}}$

☐

2. Find x, if $8x^{-\frac{2}{3}} = \frac{2}{25}$.
   a. 10
   b. 1000
   c. 0.01
   d. 100

☐

3. Find the value of x, if $(2.7)^{3x-1} = 5^x$.
   a. $\frac{\log 2.7+1}{3\log 2.7+\log 2}$
   b. $\frac{\log 2.7-1}{3\log 2.7+\log 2}$
   c. $\frac{\log 2.7}{3\log 2.7+\log 2+1}$
   d. $\frac{\log 2.7}{3\log 2.7+\log 2-1}$

☐

4. If α and β are the roots of the equation $ax^2+bx+c=0$, $\left[\frac{1+\alpha}{1-\alpha}+\frac{1+\beta}{1-\beta}\right]$ express in terms of a, b, and c is
   a. $\frac{2(a-c)}{a-b+c}$
   b. $\frac{2(a-c)}{a+b+c}$
   c. $\frac{2a-c}{a+b+c}$
   d. $\frac{a-2c}{a-b+c}$

☐

5. One root of the quadratic equation $x^2-ax+b=0$ is twice the other, find b in terms of a
   a. $b = \frac{2a^2}{9}$
   b. b [illegible] $2a^2$

6 The range of values of x for which $(2x - 1)(x - 3) \le 7$ is

a. $x \le -4$ and $x \ge \frac{1}{2}$

b. $-4 \le x \le \frac{1}{2}$

c. $x \le -\frac{1}{2}$ and $x \ge 4$

d. $-\frac{1}{2} \le x \le 4$

Use the following to Answer Questions 21—30.

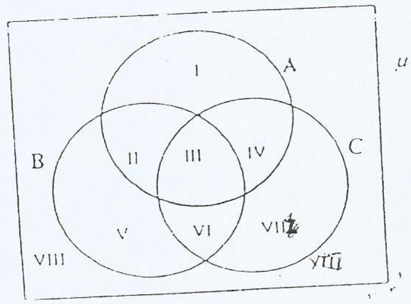
A person's blood group depends on the antigens in his blood. The three most important antigens are labelled A, B, and Rh. The Venn diagram below represents three overlapping sets:

A = {people whose blood contains antigen A}, B = {people whose blood contains antigen B}, C = {people whose blood contains antigen Rh}.

The blood groups are determined as follows:

Both A and B present .... Group AB
Only A present .... Group A
Only B present .... Group B
Neither A nor B present .... Group O

Also, the blood is +ve if Rh is present; otherwise −ve.



7 Which region of the Venn diagram represents AB^{-} ☐

a. I

b. II

[✓ c. III]

d. IV

8 Which region of the Venn diagram represents O^{+} ☐

[✓ a. VIII]

b. VII

c. VI

d. IV

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9. Which region of the Venn diagram represents Ø+
   - a. V
   - b. VI
   - [✓ c. VII]
   - d. VIII

10. Which region of the Venn diagram represents A
   - a. III
   - [✓ b. IV]
   - c. V
   - d. VI

☐

11. Which region of the Venn diagram represents B
   - a. III
   - b. II
   - [✓ c. V]
   - d. VI

☐

12. The region VI can be described in terms of the sets A, B, and C as:
   - a. $B \cap C$
   - [✓ b. $A' \cap B \cap C$]
   - c. $A' \cap (B' \cap C')$
   - d. $A' \cap (B' \cup C')'$

☐

13. The region I is described in terms of the sets A, B and C is
   - a. $A \cap B' \cap C$
   - b. $A \cap (B \cup C)$
   - [✓ c. $A \cap (B \cup C)'$]
   - d. $A' \cup B' \cup C'$

☐

14. The region II is described in terms of the sets A, B and C is
   - a. $A \cap B' \cap C'$
   - [✓ b. $A' \cap B \cap C'$]
   - c. $A' \cap B' \cap C$
   - d. $A' \cap B' \cap C'$

☐

15. The region III is described in terms of the sets A, B and C is
   - a. $(A' \cap B' \cap C')$
   - b. $A \cap B' \cap C$
   - c. $A' \cap B \cap C$
   - [✓ d. $A \cap B \cap C$]

16. The region VIII is described in terms of the sets A, B and C is
   - a. $A' \cap B' \cap C$
   - b. $A \cap B \cap C$
   - c. $(A \cap B$ [illegible]$)'$

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d $A' \cup B, \cup C'$

17. There are 200 boys in a mixed school, 30 of them being in the sixth form. Altogether, there are 50 students in the sixth form. How many students in the school are either boys or sixth formers.

a. 250
b. [illegible]
c. 220
d. 280

18. How many subsets has the empty set.

a. 0
b. 1
c. 2
d. None of the above

☐

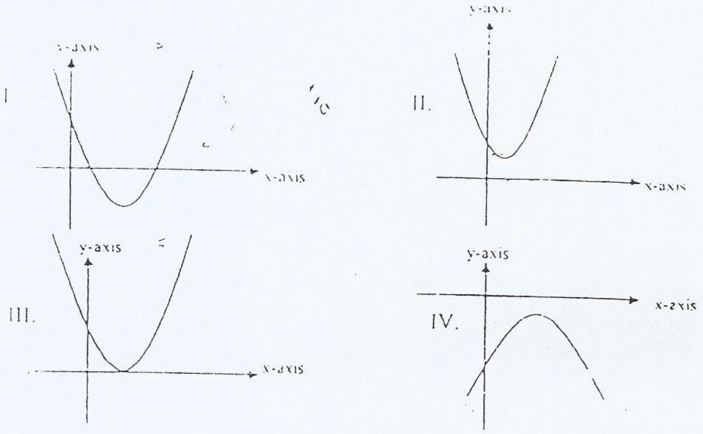
19. Find the number of subsets of the set $A=\{1, 2, 3\}$?

a. 8
b. 9
c. 3
d. None of the above

☐

Use the following to Answer Questions 34—36.

Below are graphs of the quadratic function $y = dx^2 + ex + f$ and $d \neq 0$, and $D = e^2 - 4df$;



20. Which graph(s) have $D < 0$.

a. I and II
b. II and IV
c. II and III
d. III and IV

☐

21. Which graph(s) have $d > 0$
   a. II and IV
   b. I and II
   c. IV only
   d. II only

☐

22. Which graph(s) have equal roots?
   a. I
   b. II
   c. III
   d. IV

☐

Use the table below to answer Questions 24—25.

X_{ij}		j					
		1	2	3	4	5	6
i	0	-4	3	2	0	5	4
	1	1	3	-1	-2	1	3
	2	-5	4	1	2	-2	2
	3	1	1	-1	2	3	4
	4	3	-3	-2	-1	-4	0
	5	1	2	3	4	5	6

23. Find the sum $\sum_{i=0}^{5}\sum_{j=1}^{i+1} x_{ij}$.
   a. 10
   b. 11
   c. 12
   d. 13

☐

24. Find the sum $\sum_{i=0}^{3}\sum_{j=i+1}^{i+3} x_{ij}$.
   a. 10
   b. 12
   c. 13
   d. 14

☐

25. Find the number of terms (addends) in the given sum $\sum_{i=2}^{5}\sum_{j=1}^{i-1} x_{ij}$
   a. 10
   b. 11
   c. 12
   d. 13

Find the number of terms (addends) in the given sum $\sum_{i=0}^{2}\sum^{3i}$
   a. 10

27. Write the following using the sigma notation

(a) $10 + 5 + \frac{5}{2} + \cdots + \frac{10}{1-\frac{1}{2}}$

(b) $-\frac{1}{4}, -\frac{1}{5}, -\frac{4}{25}, \cdots$

$\sum_{n=1}$

$\frac{10}{n}$

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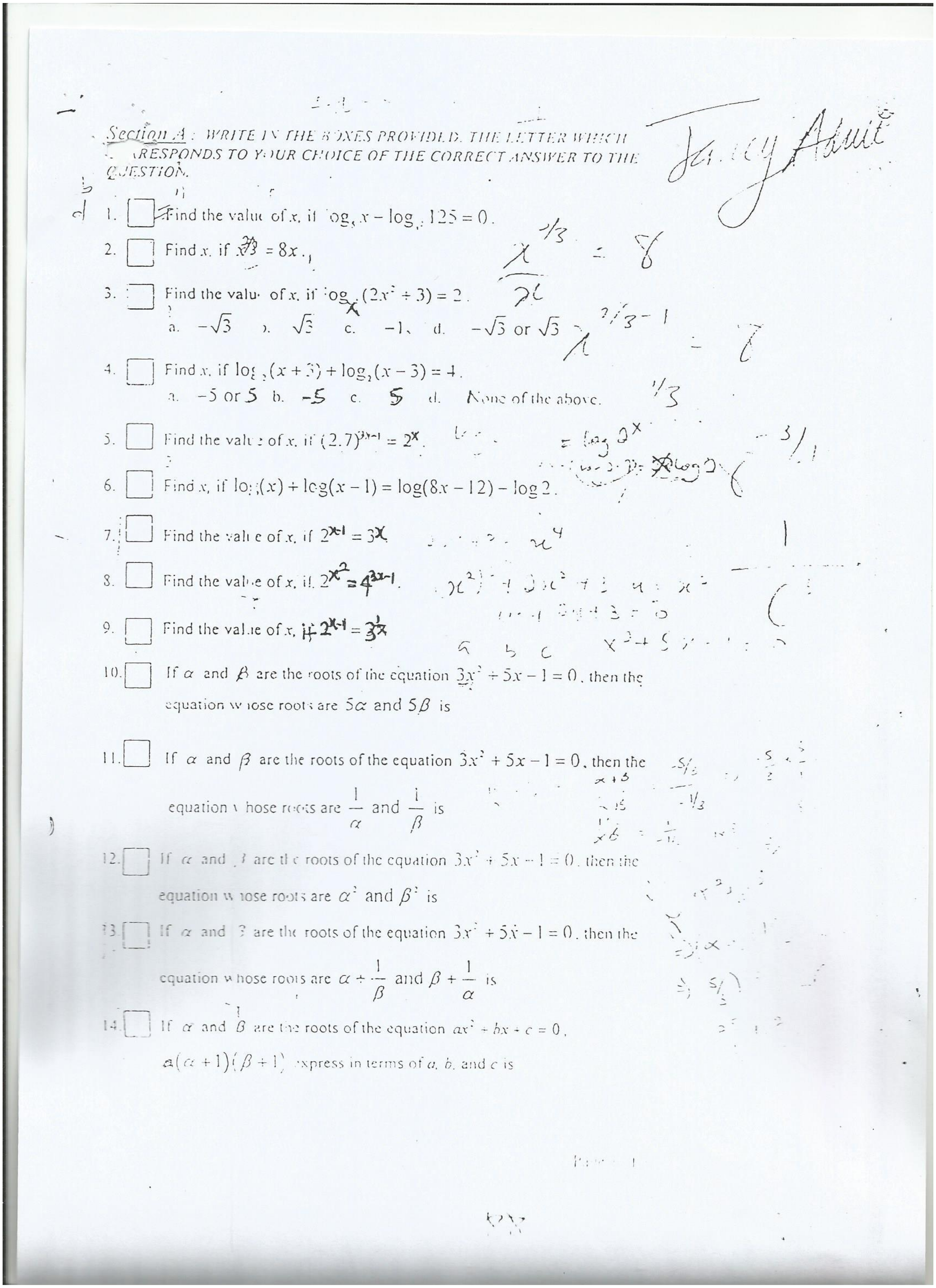


Section A : WRITE IN THE BOXES PROVIDED, THE LETTER WHICH CORRESPONDS TO YOUR CHOICE OF THE CORRECT ANSWER TO THE QUESTION.

1. ☐ Find the value of x, if $\log_5 x - \log_x 125 = 0$.
2. ☐ Find x, if $x^{\frac{2}{3}} = 8x$.
3. ☐ Find the value of x, if $\log_x(2x^2 + 3) = 2$.
   a. $-\sqrt{3}$ b. $\sqrt{3}$ c. -1 d. $-\sqrt{3}$ or $\sqrt{3}$
4. ☐ Find x, if $\log_2(x + 3) + \log_2(x - 3) = 4$.
   a. -5 or 5 b. -5 c. 5 d. None of the above.
5. ☐ Find the value of x, if $(2.7)^{3x-1} = 2^x$.
6. ☐ Find x, if $\log(x) + \log(x - 1) = \log(8x - 12) - \log 2$.
7. ☐ Find the value of x, if $2^{x-1} = 3^x$
8. ☐ Find the value of x, if $2^{x^2} = 4^{3x-1}$.
9. ☐ Find the value of x, if $2^{x-1} = 3^{\frac{1}{x}}$
10. ☐ If α and β are the roots of the equation $3x^2 + 5x - 1 = 0$, then the equation whose roots are 5α and 5β is
11. ☐ If α and β are the roots of the equation $3x^2 + 5x - 1 = 0$, then the equation whose roots are $\frac{1}{\alpha}$ and $\frac{1}{\beta}$ is
12. ☐ If α and β are the roots of the equation $3x^2 + 5x - 1 = 0$, then the equation whose roots are α^2 and β^2 is
13. ☐ If α and β are the roots of the equation $3x^2 + 5x - 1 = 0$, then the equation whose roots are $\alpha + \frac{1}{\beta}$ and $\beta + \frac{1}{\alpha}$ is
14. ☐ If α and β are the roots of the equation $ax^2 + bx + c = 0$, $a(\alpha + 1)(\beta + 1)$ express in terms of a, b, and c is

$\frac{x^{2/3}}{x} = 8$

$x^{2/3-1} = 8$

$\frac{1}{3}$

$= \log 2^x$

$-\frac{5}{3}$

$-\frac{1}{3}$

[illegible] equation which has [illegible] roots equal [illegible] and the difference of the squares of its roots equal to 5

16 ☐ One root of the quadratic equation $2x^2 - x + c = 0$ is twice the other. find the value of c

17 ☐ The range of values of x for which $(2x + 1)(x + 3) \leq 7$ is

18 ☐ The number subsets of the empty set is

αβ = c/2

Use the table below to answer Questions 19—20.

x_{ij}	1	2	3	4	5	6
0	-4	3	2	0	5	4
1	1	3	-1	-2	1	3
2	-5	4	1	2	2	2
3	1	1	-1	2	3	4
4	3	-3	-2	-1	4	0
5	1	2	3	4	5	6

19 ☐ Find the sum $\sum_{i=1}^{4}\sum_{j=1}^{i} x_{i-1,j}$.

20 ☐ Find the sum $\sum_{i=0}^{2}\sum_{j=i+1}^{i+2} x_{ij}$

21 ☐ Find the number of terms (addends) in the given sum $\sum_{i=2}^{5}\sum_{j=1}^{i+1} x_{ij}$

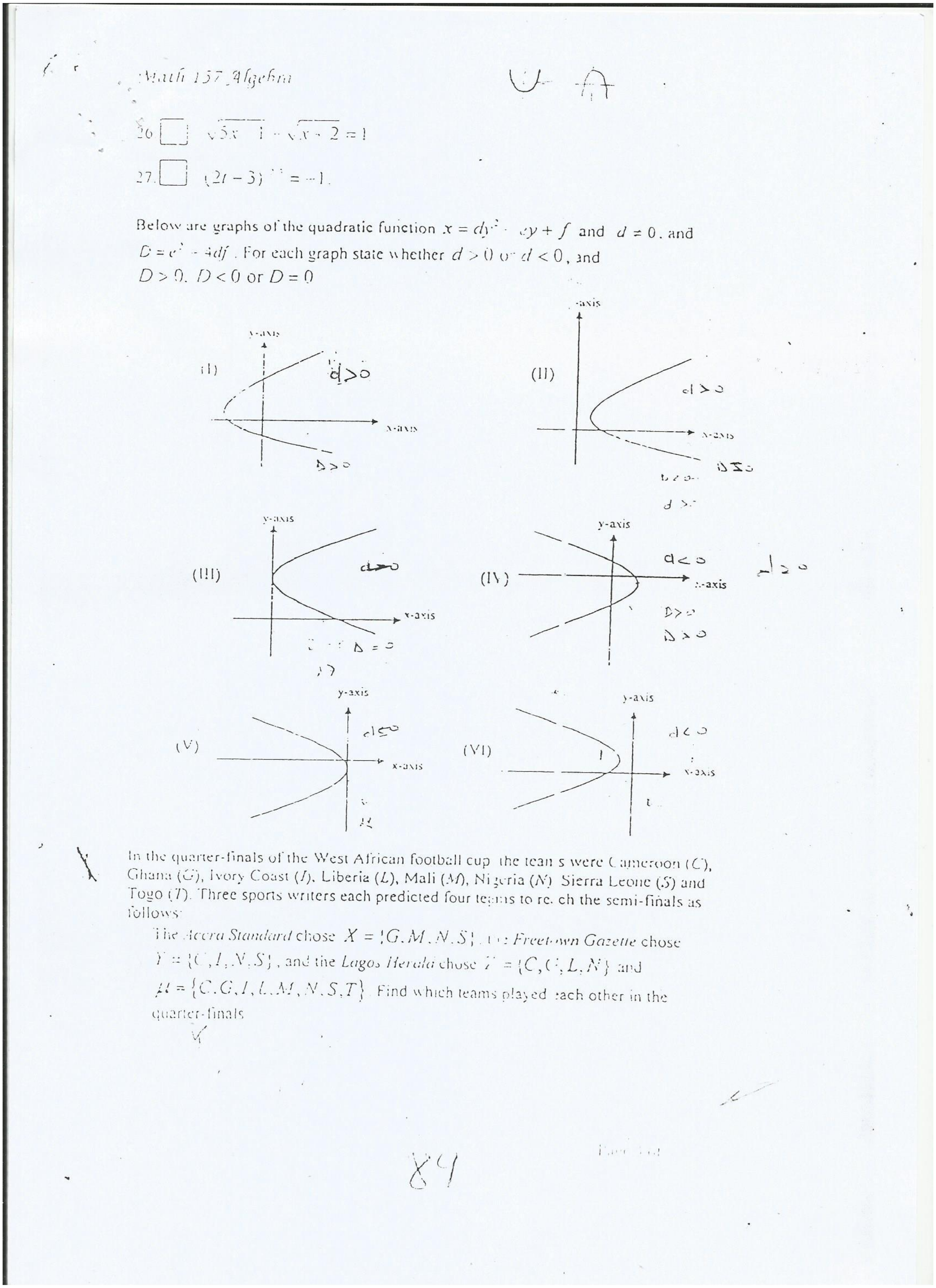
22 ☐ Find the number of terms (addends) in the given sum $\sum\sum_{j=i+1} x_{ij}$

23 ☐ The expanded form of $\sum\sum^{i+1} x_{ij}$ is

24 ☐ The expanded form of $\sum_{i=0}\sum x_{ij}$ is

Solve the equations in Questions 25 – 27:-

25 ☐ $(w - 1)^{2/3} = 4$



Math 157 Algebra

26. ☐ $\sqrt{5x-1} - \sqrt{x-2} = 1$

27. ☐ $(2t-3)^{[illegible]} = -1$.

Below are graphs of the quadratic function $x = dy^2 + ey + f$ and $d \neq 0$, and $D = e^2 - 4df$. For each graph state whether $d > 0$ or $d < 0$, and $D > 0$, $D < 0$ or $D = 0$

In the quarter-finals of the West African football cup the teams were Cameroon (C), Ghana (G), Ivory Coast (I), Liberia (L), Mali (M), Nigeria (N) Sierra Leone (S) and Togo (T). Three sports writers each predicted four teams to reach the semi-finals as follows:

The *Accra Standard* chose $X = \{G, M, N, S\}$, the *Freetown Gazette* chose $Y = \{C, I, N, S\}$, and the *Lagos Herald* chose $Z = \{C, G, L, N\}$ and $\mu = \{C, G, I, L, M, N, S, T\}$. Find which teams played each other in the quarter-finals

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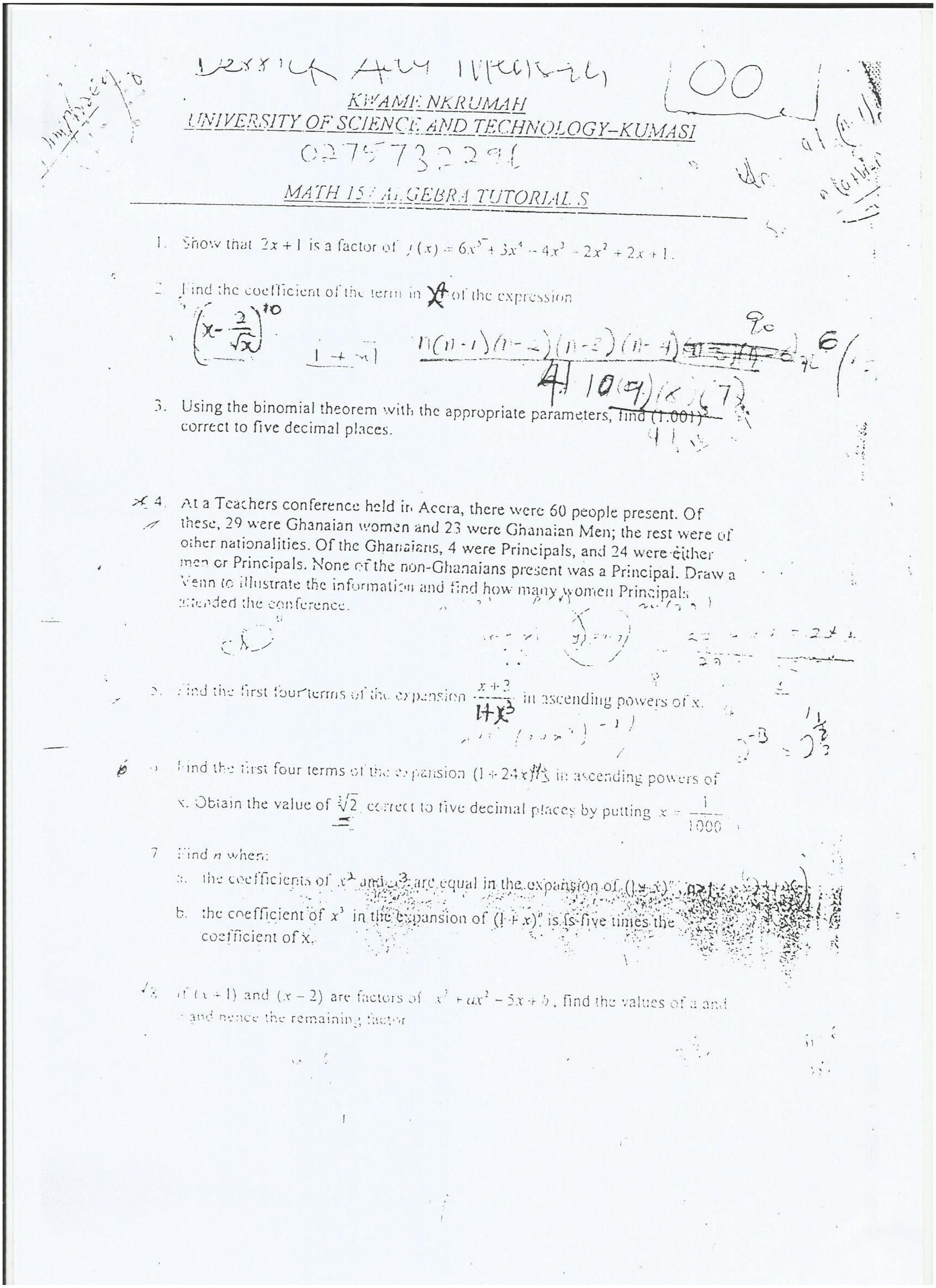

Derrick Atta Mensah

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KWAME NKRUMAH
UNIVERSITY OF SCIENCE AND TECHNOLOGY–KUMASI

0275732291

MATH 15? ALGEBRA TUTORIALS

1. Show that $2x+1$ is a factor of $f(x)=6x^5+3x^4-4x^3-2x^2+2x+1$.

2. Find the coefficient of the term in x^4 of the expression

$$\left(x-\frac{2}{\sqrt{x}}\right)^{10}$$

$$\frac{n(n-1)(n-2)(n-3)(n-4)}{4!}\quad 10(9)(8)(7)$$

3. Using the binomial theorem with the appropriate parameters, ~~find $(1.001)^5$~~ correct to five decimal places.

4. At a Teachers conference held in Accra, there were 60 people present. Of these, 29 were Ghanaian women and 23 were Ghanaian Men; the rest were of other nationalities. Of the Ghanaians, 4 were Principals, and 24 were either men or Principals. None of the non-Ghanaians present was a Principal. Draw a Venn to illustrate the information and find how many women Principals attended the conference.

5. Find the first four terms of the expansion $\frac{x+2}{1+x^3}$ in ascending powers of x.

6. Find the first four terms of the expansion $(1+24x)^{1/3}$ in ascending powers of x. Obtain the value of $\sqrt[3]{2}$ correct to five decimal places by putting $x=\frac{1}{1000}$

7. Find n when:
   a. the coefficients of x^2 and x^3 are equal in the expansion of $(1+x)^n$
   b. the coefficient of x^3 in the expansion of $(1+x)^n$ is five times the coefficient of x.

8. If $(x+1)$ and $(x-2)$ are factors of x^3+ax^2-5x+b, find the values of a and b and hence the remaining factor

3

4

0

9. [illegible] a factor of [illegible] $x^2 + ax + b$. When the expression is divided by [illegible] the remainder is [illegible] the values of a and b

10. For what values of a and b will the polynomial ax [illegible] $bx + 3$ have remainders of 4 and -3 when divide by $(x - 1)$ and $(x - 4)$ respectively

11. When a polynomial $f(x)$ is divided by $(x - a)$ and $(x - b)$ the remainder is $f(a)$ and $f(b)$ respectively. What is the remainder when $f(x)$ is divided by $(x - a)(x - b)$

12. Without using tables or calculators, solve the equation $8x^{-2/3} = \frac{2}{25}$.

13. Without using tables or calculators, evaluate

$$\tfrac{2}{3}\log 27 + 5\log 2 + \tfrac{1}{3}\log 8 + 3\log \tfrac{5}{2} - 2\log 3 - 3\log 10$$

$\left(\frac{27}{10}\right)^{3x-1} = 5^x$

14. Find the value of $\frac{\log x^3}{\log x^2}$.

15. Find x if $(2.7)^{3x-1} = 5^x$

$\frac{1}{8x} = \frac{2}{25}$

16. Find x if $(\log_{10} x)^2 - \log_{10} x = 0$.

$\log_{10} x = 2\log_{10} y$

17. Find y if $\log_3 y = 2\log_3 7$.

18. If $3 + \log_{10} x = 2\log_{10} y$, express x in terms of y.

19. Solve the equation $(0.2)^x = (0.5)^{x+7}$

20. If α and β are the roots of the equation $ax^2 + bx + c = 0$, express $\left[\frac{1+\alpha}{1-\alpha} + \frac{1+\beta}{1-\beta}\right]$ in terms of a, b, and c.

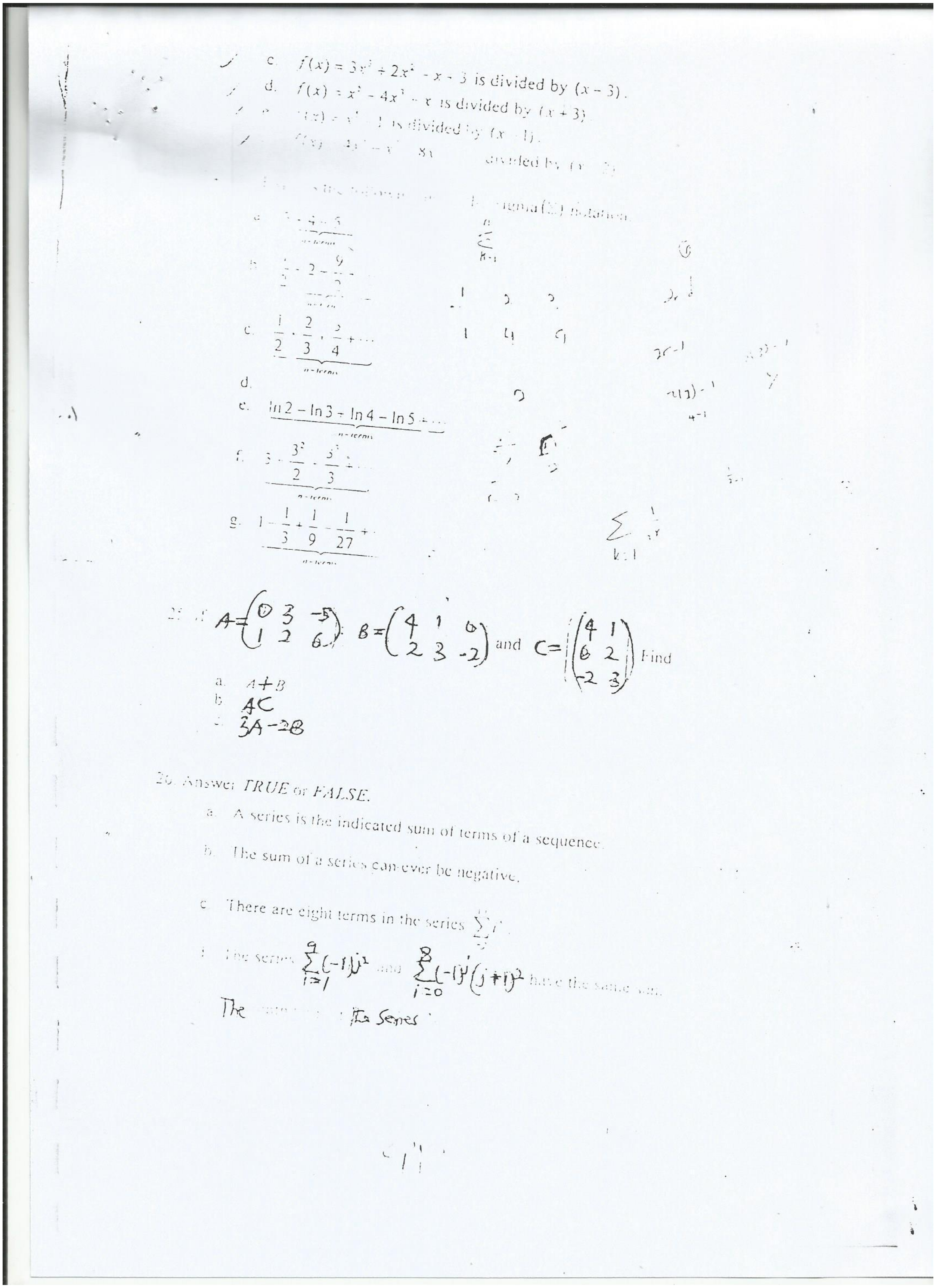
21. If one roots of the equation $16x^2 + ax + 1 = 0$ is the cube of the other, find the possible values of a.

22. If one roots of the equation $x^2 - 4x + b = 0$ is double the other, find b in terms of a.

23. Find the remainder and quotient when:

a. $f(x) = 0.1x^3 + 0.2x$ is divided by $(x + 1.1)$

b. $f(x) = x^3 - x^2 + 2x$ [illegible] is divided by $(x - 2)$



c. $f(x) = 3x^3 + 2x^2 - x + 3$ is divided by $(x - 3)$.

d. $f(x) = x^3 - 4x^2 + x$ is divided by $(x + 3)$

e. $f(x) = x^3 - 1$ is divided by $(x - 1)$.

f. $f(x) = 4x^3 - x^2$ [illegible] $8x$ [illegible] divided by $(x - 2)$

[illegible] the following [illegible] sigma (Σ) notation.

a. $\underbrace{3 + 4 + 5 + \dots}_{n-terms}$ $\sum_{k=1}^{n}$

b. $\underbrace{\frac{1}{2} + 2 + \frac{9}{2} + \dots}_{n-terms}$

c. $\underbrace{\frac{1}{2} + \frac{2}{3} + \frac{3}{4} + \dots}_{n-terms}$

d.

e. $\underbrace{\ln 2 - \ln 3 + \ln 4 - \ln 5 + \dots}_{n-terms}$

f. $\underbrace{3 + \frac{3^2}{2} + \frac{3^3}{3} + \dots}_{n-terms}$

g. $\underbrace{1 - \frac{1}{3} + \frac{1}{9} - \frac{1}{27} + \dots}_{n-terms}$ $\sum_{k=1} \frac{1}{3^k}$

25. If $A = \begin{pmatrix} 0 & 3 & -5 \\ 1 & 2 & 6 \end{pmatrix}$, $B = \begin{pmatrix} 4 & 1 & 0 \\ 2 & 3 & -2 \end{pmatrix}$ and $C = \begin{pmatrix} 4 & 1 \\ 0 & 2 \\ -2 & 3 \end{pmatrix}$ Find

a. $A + B$

b. AC

c. $3A - 2B$

26. Answer *TRUE* or *FALSE*.

a. A series is the indicated sum of terms of a sequence.

b. The sum of a series can ever be negative.

c. There are eight terms in the series $\sum i$.

d. The series $\sum_{j=1}^{9}(-1)^j j^2$ and $\sum_{j=0}^{8}(-1)^j (j+1)^2$ have the same sum.

The [illegible] the Series

f. $\sum (-1)^i 2^i = 2$

g. $\sum 3i = 3\left(\sum i\right)$

h. $\sum_{i=1} 5 = 20$

i. $\sum_{i=1}^{5} 2i + \sum_{i=1}^{5} 7i = \sum_{i=1} 9i$

j.

k. $\sum_{i=1}^{3}(2i+1) = 3\left(\sum_{i=1} 2i\right) + 1$

l. $\frac{1}{2} = \sqrt{2}$

27. Answer *TRUE* or *FALSE*.

a. $\sqrt{2^{-4}} = \frac{1}{4}$

b. $\sqrt[3]{a^{27}} = a^3$

c. $\frac{\sqrt{6}}{\sqrt{3}} = \sqrt{2}$

d. $\frac{\sqrt{10}}{2} = \sqrt{5}$

e. $2^{2/3} = \sqrt[3]{4}$

f. The equation $a^3 = 2$ is equivalent to $\log_a(2) = 3$ T

g. If (a, b) satisfies $y = 8^x$ then (a, b) satisfies $y = \log_8 x$

h. The sequence 2, 6, 24, … is a geometric sequence.

i. For $a_n = 2^n$ there is a constant difference between adjacent terms..

j. The common ratio for the geometric sequence $a_n = 3(0.5)^{n-1}$ is 0.5.

k. If $a_n = 3(2)^{-n+3}$ then $a_n = 12$ F

l. The terms of a geometric series are the terms of a geometric sequence.

m. To evaluate $\sum_{i=1}^{10} 2^i$, we must list all of the terms

n. $-\frac{1}{3}, -\frac{1}{4}, -\frac{3}{16}, \ldots$

o. $\sum_{i=1}^{5} 6\left(\frac{3}{4}\right)^{i-1} = \frac{9\left[1-\left(\frac{3}{4}\right)\right]}{1-\frac{3}{4}}$

p. $10 + 5 + \frac{5}{2} + \cdots = \frac{10}{1-\frac{1}{2}}$

q. $2 + 4 + 8 + 16 + \cdots = \frac{2}{1-2}$

r. Completing the square is used to develop the quadratic formula.

s. If $dx^2 + ex + f = 0$ and $d \neq 0$, then $x = \frac{-e \pm \sqrt{e^2 - 4df}}{2d}$.

t. The quadratic formula will not work on the equation $x^2 - 3 = 0$.

u. If the discriminant of a quadratic equation is zero; then there are no imaginary solutions.

v. Some quadratic equations have one real and one imaginary solution.

28. Suppose you deposit one cedi into your piggy bank on the first day of December and, on each day of December after that, you deposit twice as much as on the previous day. How much will you have in the bank after the last deposit on the last day of December?

29. Write a formula for the nth term of each geometric sequence:

a. $\frac{1}{3}, 1, 3, 9, \ldots$

b. $64, 8, 1, \ldots$

c. $-9, 3, -1, \ldots$

d. $100, 10, 1, \ldots$

e. $-\frac{1}{2}, 2, -8, 32, \ldots$

f. $-\frac{1}{3}, -\frac{1}{4}, -\frac{3}{16}, \ldots$

g. $-\frac{1}{4}, -\frac{1}{5}, -\frac{4}{25}, \ldots$

30. Find the remainder and quotient when:

a. $f(x) = 0.1x^3 - 0.2x$ is divided by $(x + 1.1)$.

b. $f(x) = x^3 - x^2 + 2x + 4$ is divided by $(x - 2)$.

c. $f(x) = 3x^3 + 2x^2 - x + 3$ is divided by $(x - 3)$.

d. $f(x) = x^3 - 4x^2 - x$ is divided by $(x - 3)$.

e. $f(x) = x^4 - 1$ is divided by $(x - 1)$

f. $f(x) = 4x^3$ [illegible] $8x$ [illegible] is divided by [illegible]

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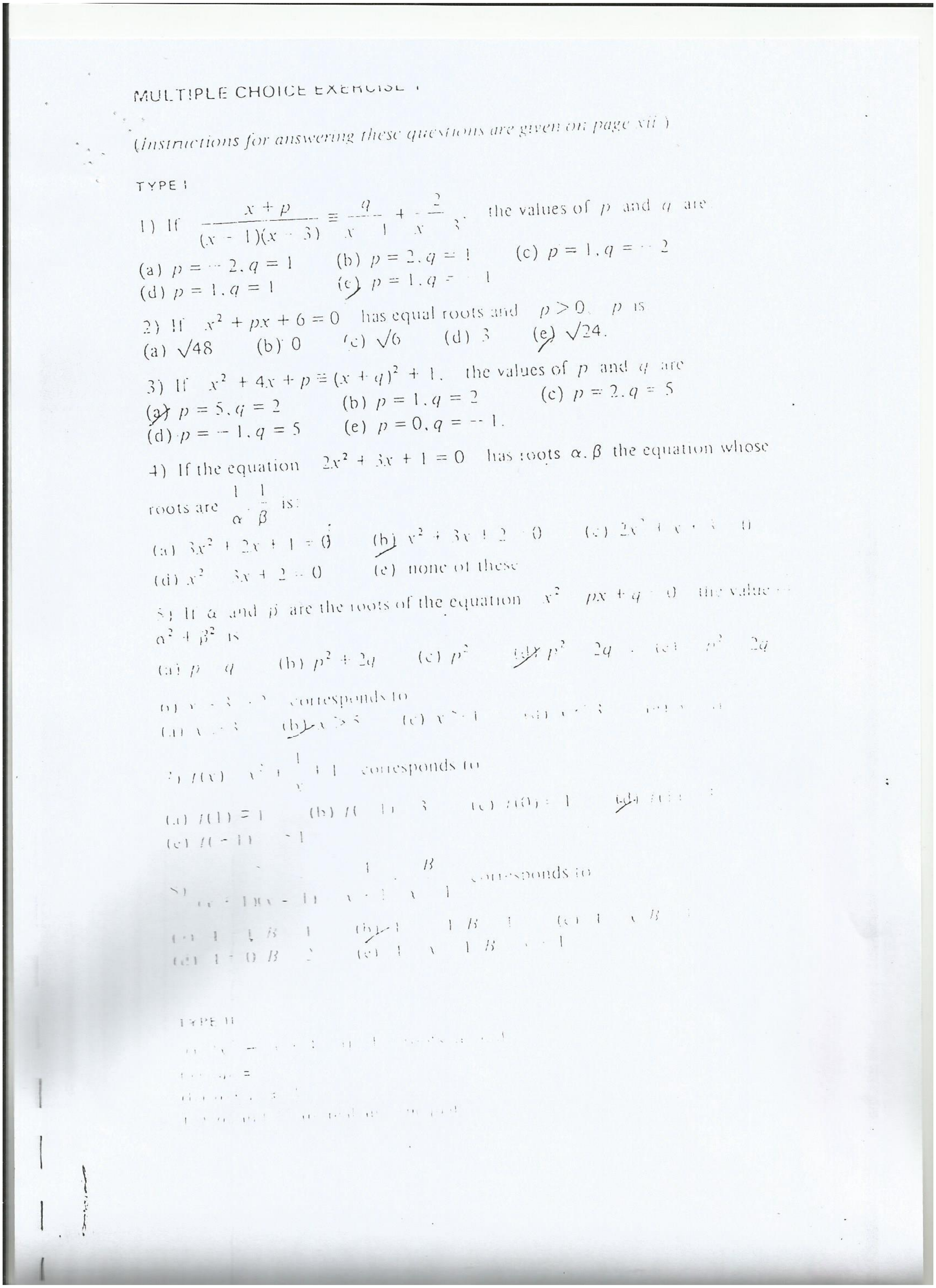


MULTIPLE CHOICE EXERCISE 1

(Instructions for answering these questions are given on page xii)

TYPE I

1) If $\dfrac{x+p}{(x-1)(x-3)} \equiv \dfrac{q}{x-1} + \dfrac{2}{x-3}$, the values of p and q are:

(a) $p = -2, q = 1$ (b) $p = 2, q = 1$ (c) $p = 1, q = -2$
(d) $p = 1, q = 1$ [✓ (e) $p = 1, q = -1$]

2) If $x^2 + px + 6 = 0$ has equal roots and $p > 0$, p is

(a) $\sqrt{48}$ (b) 0 (c) $\sqrt{6}$ (d) 3 [✓ (e) $\sqrt{24}$.]

3) If $x^2 + 4x + p \equiv (x+q)^2 + 1$, the values of p and q are

[✓ (a) $p = 5, q = 2$] (b) $p = 1, q = 2$ (c) $p = 2, q = 5$
(d) $p = -1, q = 5$ (e) $p = 0, q = -1$.

4) If the equation $2x^2 + 3x + 1 = 0$ has roots α, β the equation whose roots are $\dfrac{1}{\alpha}, \dfrac{1}{\beta}$ is:

(a) $3x^2 + 2x + 1 = 0$ [✓ (b) $x^2 + 3x + 2 = 0$] (c) $2x^2 + x + 3 = 0$
(d) $x^2 - 3x + 2 = 0$ (e) none of these

5) If α and β are the roots of the equation $x^2 - px + q = 0$ the value of $\alpha^2 + \beta^2$ is

(a) $p - q$ (b) $p^2 + 2q$ (c) p^2 [✓ (d) $p^2 - 2q$] (e) p^2 [illegible] $2q$

6) $x - 3$ [illegible] corresponds to

(a) x [illegible] 3 [✓ (b) $x > 5$] (c) x [illegible] 1 (d) x [illegible] 3 (e) [illegible]

7) $f(x) = x^2 + \dfrac{1}{x} + 1$ corresponds to

(a) $f(1) = 1$ (b) $f($[illegible]$)$ [illegible] 3 (c) $f(0) = 1$ [✓ (d) $f($[illegible]$)$ [illegible]]
(e) $f(-1)$ [illegible] -1

8) [illegible] $\dfrac{1}{(x+1)(x-1)}$ [illegible] $\dfrac{B}{x-1}$ corresponds to

(a) [illegible] B [illegible] [✓ (b) [illegible] B [illegible]] (c) [illegible] B [illegible]
(d) [illegible] B [illegible] (e) [illegible] B [illegible]

TYPE II

[illegible]

19) Express $f(x)$ in partial fractions

(a) $f(x)$ is a proper fraction

(b) $f(0) = 1$.

(c) $f(x) \equiv \dfrac{A}{(x-1)^2(x+1)}$

TYPE V

20) A quadratic equation always has two real solutions.

21) The relationship $x(x^2+4) = x^2 + 4x$ is an identity.

22) The expression $\dfrac{x-2}{(x-1)(x+1)^2}$ can be expressed as three separate fractions.

23) The inequality $x + 5 > 3$ is satisfied by a finite number of values of x.

24) The equation $ax^2 + bx + c = 0$ has two roots.

25) If $f(x) \equiv \dfrac{x}{(x^2+1)(x-1)}$ then $f(x) \equiv \dfrac{A}{x^2+1} + \dfrac{B}{x+1} + \dfrac{C}{x-1}$

MISCELLANEOUS EXERCISE 1

1) Express $\dfrac{9x}{(2x+1)^2(1-x)}$ as a sum of partial fractions with constant numerators. (U of L)p

2) Resolve the expression $\dfrac{(x-2)}{(x^2+1)(x-1)^2}$ into its simplest partial fractions. (U of L)p

3) Express the function $\dfrac{7x+4}{(x-3)(x+2)^2}$ as the sum of three partial fractions with numerators independent of x. (JMB)p

4) If α, β are the roots of the equation

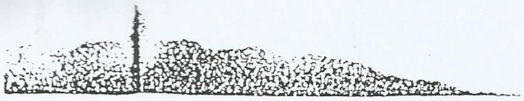
$$ax^2 + bx + c = 0$$

form the equation whose roots are $\alpha + \dfrac{1}{\alpha}, \beta + \dfrac{1}{\beta}$. (U of L)p

5) If the roots of $x^2 + px + q = 0$ are α and β, where α and β are non-zero, form the equation whose roots are $\dfrac{2}{\alpha}, \dfrac{2}{\beta}$. (U of L)p

6) If α and β are the roots of the equation $ax^2 + bx + c = 0$ show that the roots of the equation $acx^2 - (b^2 - 2ac)x + ac = 0$ are $\dfrac{\alpha}{\beta}$ and $\dfrac{\beta}{\alpha}$.

(AEB)p



) $f(x) \equiv \frac{2x}{(x+2)(x-2)}$

) $f(x)$ is an improper fraction.

) $f(0) = 2$

) $f(x) \equiv \frac{1}{x-2} + \frac{1}{x+2}$.

1) $\frac{2}{(x-1)(x^2+1)} \equiv \frac{A}{x-1} + \frac{Bx+C}{x^2+1}$.

a) $A = 1$

b) $2 \equiv A(x^2+1) + (Bx+C)(x-1)$.

c) $A + B = 0$.

2) $f(x) \equiv x^2 - 2x + 2$.

a) $f(x) \equiv (x-1)^2 + 1$.

b) $f(1) = 0$.

c) $f(x) = 0$ has equal roots.

TYPE III

13) (a) $x(x-2) \equiv x^2 - 2x$.

(b) $x = 2$.

14) (a) $x + 1 > 2$

(b) $x < 0$.

15) (a) $f(3) = -5$

(b) $f(x) = x^2 - 6x + 4$.

16) (a) $x^2 - 4x + 2 = 0$ has roots α and β.

(b) $2x^2 - 4x + 1 = 0$ has roots $\frac{1}{\alpha}$ and $\frac{1}{\beta}$.

TYPE IV

17) Solve the equation $ax^2 + bx + c = 0$.

(a) $a = 1$

(b) One root is twice the other root.

(c) $c = 2$

18) Write down the value of $f(2)$

(a) $f(x)$ is a polynomial of degree 1

(b) $f(0) = 1$

(c) [illegible]

12

7) The roots of the quadratic equation $x^2 - px + q = 0$ are α and β. Form, in terms of p and q, the quadratic equation whose roots are $\alpha^3 - p\alpha^2$ and $\beta^3 - p\beta^2$. (AEB)'75p

8) Form a quadratic equation with roots which exceed by 2 the roots of the quadratic equation $3x^2 - (p-4)x - (2p+1) = 0$.
Find the values of p for which the given equation has equal roots. (U of L)p

9) Given that $g(x) \equiv \dfrac{5-x}{(1+x^2)(1-x)}$, express $g(x)$ in partial fractions. (U of L)p

10) Find the constants A and B in the identity

$$\frac{x+7}{(2x-1)(x+2)} \equiv \frac{A}{(2x-1)} + \frac{B}{(x+2)}$$ (U of L)

11) The equation $ax^2 + bx + c = 0$ has roots α, β.
Express $(\alpha+1)(\beta+1)$ in terms of a, b and c. (U of L)

12) Given that the roots of the equation $ax^2 + bx + c = 0$ are β and $n\beta$, show that $(n+1)^2ac = nb^2$ (U of L)

13) Express $\dfrac{(x-1)}{x(x+1)}$ in partial fractions. (U of L)

Expand the following binomial expressions

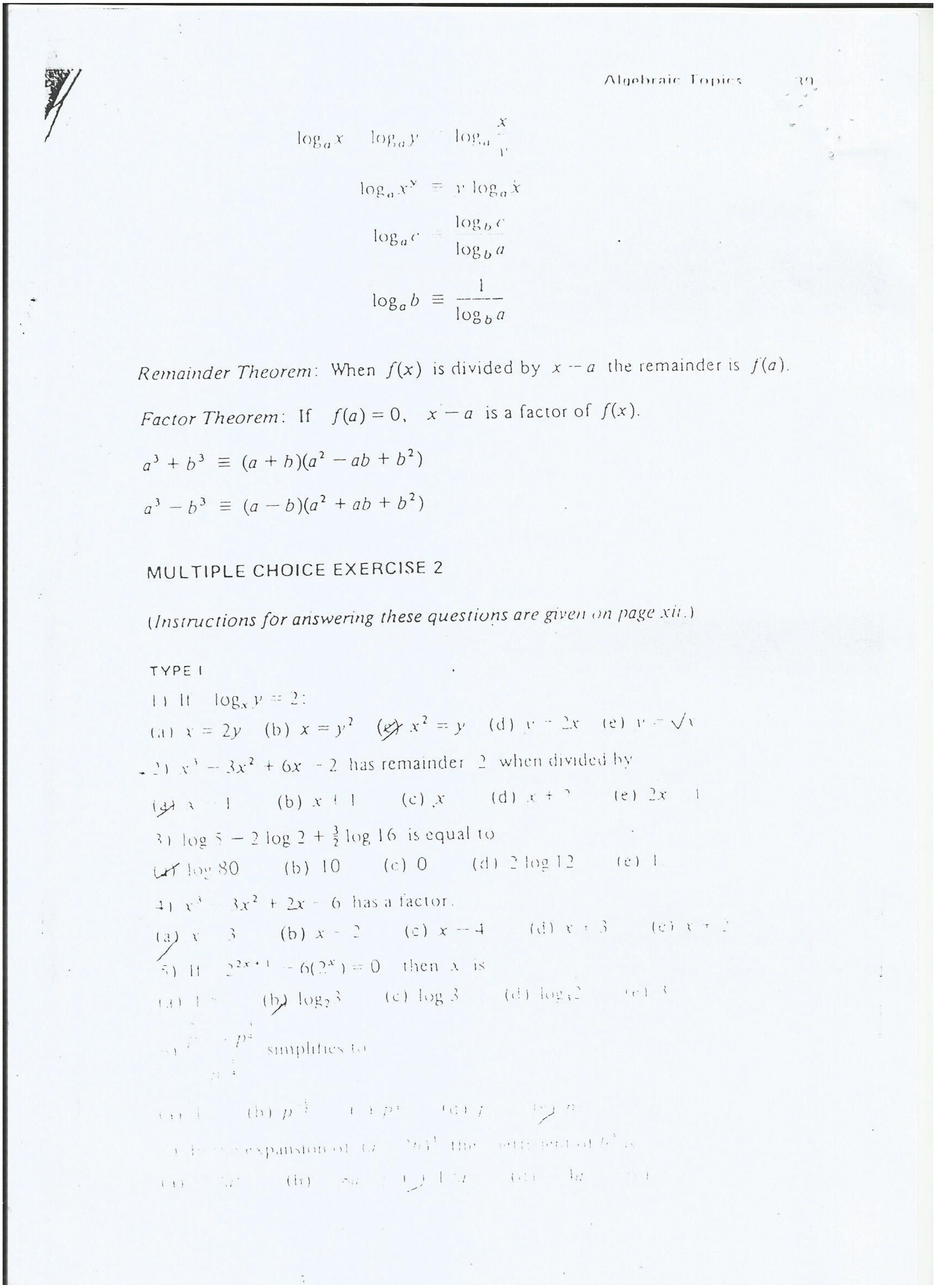
1. $(1+2x)^4$
2. $(1-x)^3$
3. $(x-1)^3$
4. $(1-2y)^4$
5. $(1+x)^7$
6. $(2x-1)^3$
7. $(2x+y)^3$
8. $(x+\frac{1}{x})^4$
9. $(p-2q)^3$
10. $(x^2-y)^5$
11. $(x-\frac{1}{x})^3$
12. $(a-b)^3(a+b)^3$

Use Pascal's Triangle to simplify

13. $(1+\sqrt{2})^3$ 14. $(\sqrt{3}-\sqrt{2})^4$ 15. $(1+\sqrt{3})^3 + (1-\sqrt{3})^3$

16. By expanding $(1+0.01)^3$ evaluate $(1.01)^3$ without using tables

17. Find without using tables, the value of $(2.1)^4$.



$$\log_a x - \log_a y = \log_a \frac{x}{y}$$

$$\log_a x^y = y \log_a x$$

$$\log_a c = \frac{\log_b c}{\log_b a}$$

$$\log_a b \equiv \frac{1}{\log_b a}$$

Remainder Theorem: When $f(x)$ is divided by $x - a$ the remainder is $f(a)$.

Factor Theorem: If $f(a) = 0$, $x - a$ is a factor of $f(x)$.

$a^3 + b^3 \equiv (a + b)(a^2 - ab + b^2)$

$a^3 - b^3 \equiv (a - b)(a^2 + ab + b^2)$

MULTIPLE CHOICE EXERCISE 2

(Instructions for answering these questions are given on page xii.)

TYPE I

1) If $\log_x y = 2$:
(a) $x = 2y$ (b) $x = y^2$ (c) $x^2 = y$ (d) $y = 2x$ (e) $y = \sqrt{x}$

2) $x^3 - 3x^2 + 6x - 2$ has remainder 2 when divided by
(a) $x - 1$ (b) $x + 1$ (c) x (d) $x + 2$ (e) $2x - 1$

3) $\log 5 - 2 \log 2 + \frac{3}{2} \log 16$ is equal to
(a) $\log 80$ (b) 10 (c) 0 (d) $2 \log 12$ (e) 1

4) $x^3 - 3x^2 + 2x - 6$ has a factor.
(a) $x - 3$ (b) $x - 2$ (c) $x - 4$ (d) $x + 3$ (e) $x + 2$

5) If $2^{2x+1} - 6(2^x) = 0$ then x is
(a) [illegible] (b) $\log_2 3$ (c) $\log 3$ (d) $\log_3 2$ (e) 3

6) [illegible] $p^{\frac{1}{2}}$ simplifies to
(a) 1 (b) $p^{-\frac{1}{2}}$ (c) [illegible] (d) [illegible] (e) [illegible]

7) In the expansion of [illegible] the coefficient of [illegible] is
(a) [illegible] (b) [illegible] (c) [illegible] (d) [illegible] (e) [illegible]

8) $f(x) \equiv x^3 - 7x^2 + 3x - 1$.
(a) $f(x)$ has a remainder -15 when divided by $x - 2$,
(b) $f(x)$ has no linear factors with integral coefficients,
(c) $f(x)$ is a polynominal of degree 3.

9) $\frac{1}{2}\log 16 - 1$:
(a) can be expressed as a single logarithm,
(b) has an exact decimal value,
(c) is equal to $\log 7$.

10) $f(x) \equiv 2x^2 + 3x - 2$.
(a) $f(x)$ can be expressed as the sum of two partial fractions,
(b) the equation $f(x) = 0$ has two real distinct roots,
(c) $x + 2$ is a factor of $f(x)$.

11) $\dfrac{2\sqrt{3} - 2}{2\sqrt{3} + 2}$
(a) can be expressed as a fraction with a rational denominator,
(b) is an irrational number,
(c) is equal to -1.

12) $f(x) \equiv (3 - 5x)^4$.
(a) $f(x)$ has a remainder 16 when divided by $x - 1$,
(b) the expansion of $f(x)$ contains four terms,
(c) the equation $f(x) = 0$ is satisfied by only one value of x.

TYPE III

13) (a) $a = \log x$.
(b) $a \log_x 10 = 1$.

14) (a) $f(x) \equiv x^2 + 2x + 1$.
(b) $f(x)$ has a remainder 1 when divided by x.

15) (a) $\log_b a = c$
(b) $a = b^c$.

16) (a) $a = \log c + \log b$.
(b) $a = \log(c + b)$.

TYPE V

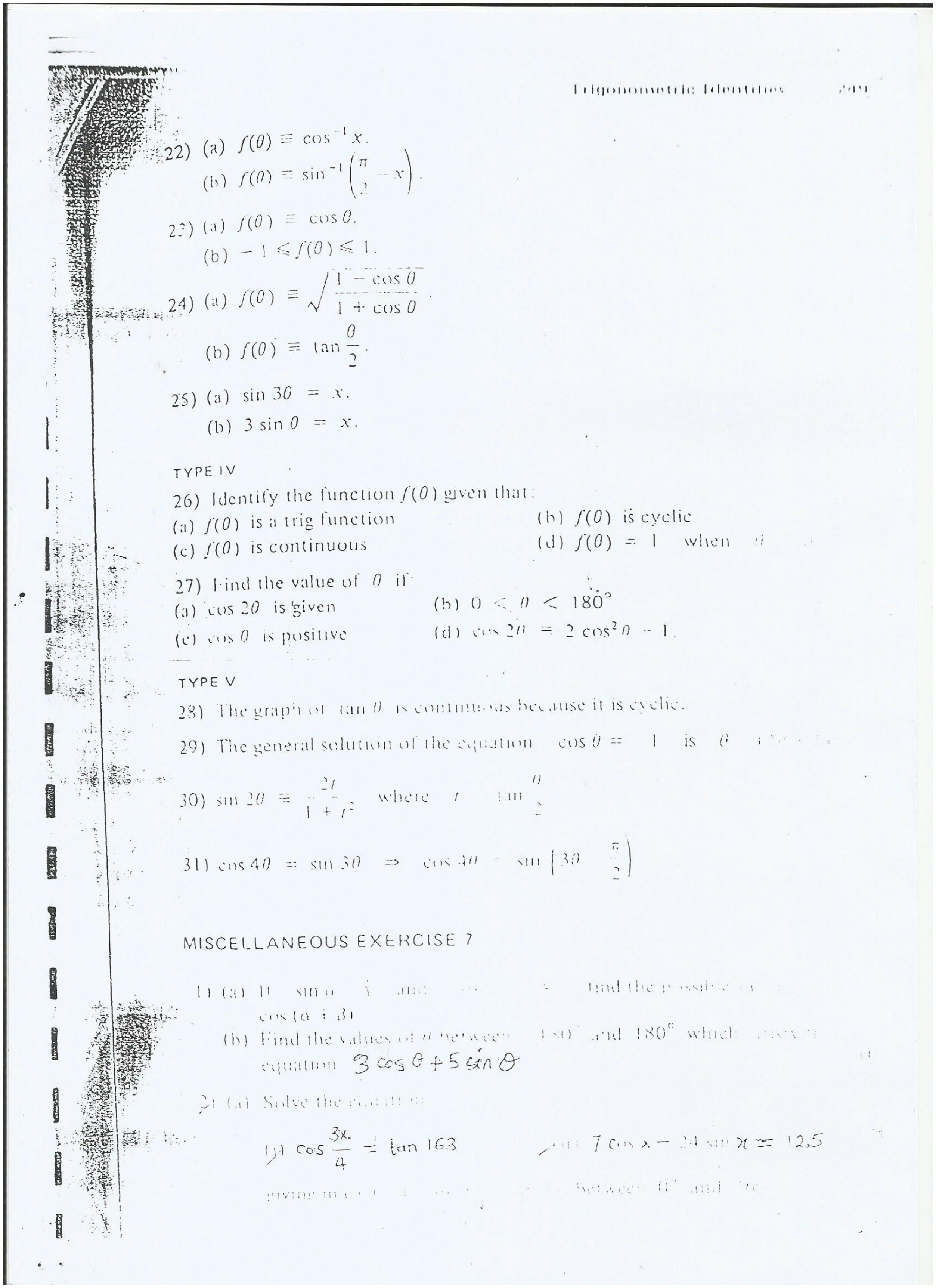
17) If $f(x)$ is divided by $ax - 1$ the remainder is $f\left(\frac{1}{a}\right)$.

18) If $x - a$ is a factor of $x^2 + px + q$, the equation $x^2 + px + q = 0$ has a root equal to a

19) $3 \log x + 1 = \log 10x^3$ is an equation

20) In the expansion of $(1 + x)^6$ the coefficient of x is 6

100



22) (a) $f(\theta) \equiv \cos^{-1} x$.

(b) $f(\theta) \equiv \sin^{-1}\left(\frac{\pi}{2} - x\right)$.

23) (a) $f(\theta) \equiv \cos\theta$.

(b) $-1 \leqslant f(\theta) \leqslant 1$.

24) (a) $f(\theta) \equiv \sqrt{\frac{1-\cos\theta}{1+\cos\theta}}$.

(b) $f(\theta) \equiv \tan\frac{\theta}{2}$.

25) (a) $\sin 3\theta = x$.

(b) $3\sin\theta = x$.

TYPE IV

26) Identify the function $f(\theta)$ given that:

(a) $f(\theta)$ is a trig function

(b) $f(\theta)$ is cyclic

(c) $f(\theta)$ is continuous

(d) $f(\theta) = 1$ when θ [illegible]

27) Find the value of θ if:

(a) $\cos 2\theta$ is given

(b) $0 < \theta < 180°$

(c) $\cos\theta$ is positive

(d) $\cos 2\theta = 2\cos^2\theta - 1$.

TYPE V

28) The graph of $\tan\theta$ is continuous because it is cyclic.

29) The general solution of the equation $\cos\theta = 1$ is θ [illegible]

30) $\sin 2\theta \equiv \frac{2t}{1+t^2}$ where t [illegible] $\tan\frac{\theta}{2}$

31) $\cos 4\theta = \sin 3\theta \Rightarrow \cos 4\theta$ [illegible] $\sin\left(3\theta \; \frac{\pi}{2}\right)$

MISCELLANEOUS EXERCISE 7

1) (a) If $\sin\alpha$ [illegible] and [illegible] find the possible [illegible] $\cos(\alpha + \beta)$

(b) Find the values of θ between [illegible] and 180° which [illegible] equation 3 cos θ + 5 sin θ

2) (a) Solve the equation

(i) $\cos\frac{3x}{4} = \tan 163$

(ii) $7\cos x - 24\sin x = 12.5$

giving [illegible] between 0° and [illegible]

a factor of $f(x)$.
When k has this value, find another factor of $f(x)$, of the form $x + a$, where a is a constant. (C)p

13) If $f(x) \equiv ax^2 + bx + c$ leaves remainders $1, 25, 1$ on division by $x - 1$, $x + 1$, $x - 2$ respectively, show that $f(x)$ is a perfect square. (U of L)p

14) Without using tables, solve each of the following equations for x, expressing your answers as simply as possible:

(a) $9 \log_x 5 = \log_5 x$,

(b) $\log_8 \dfrac{x}{2} = \dfrac{\log_8 x}{\log_8 2}$. (JMB)

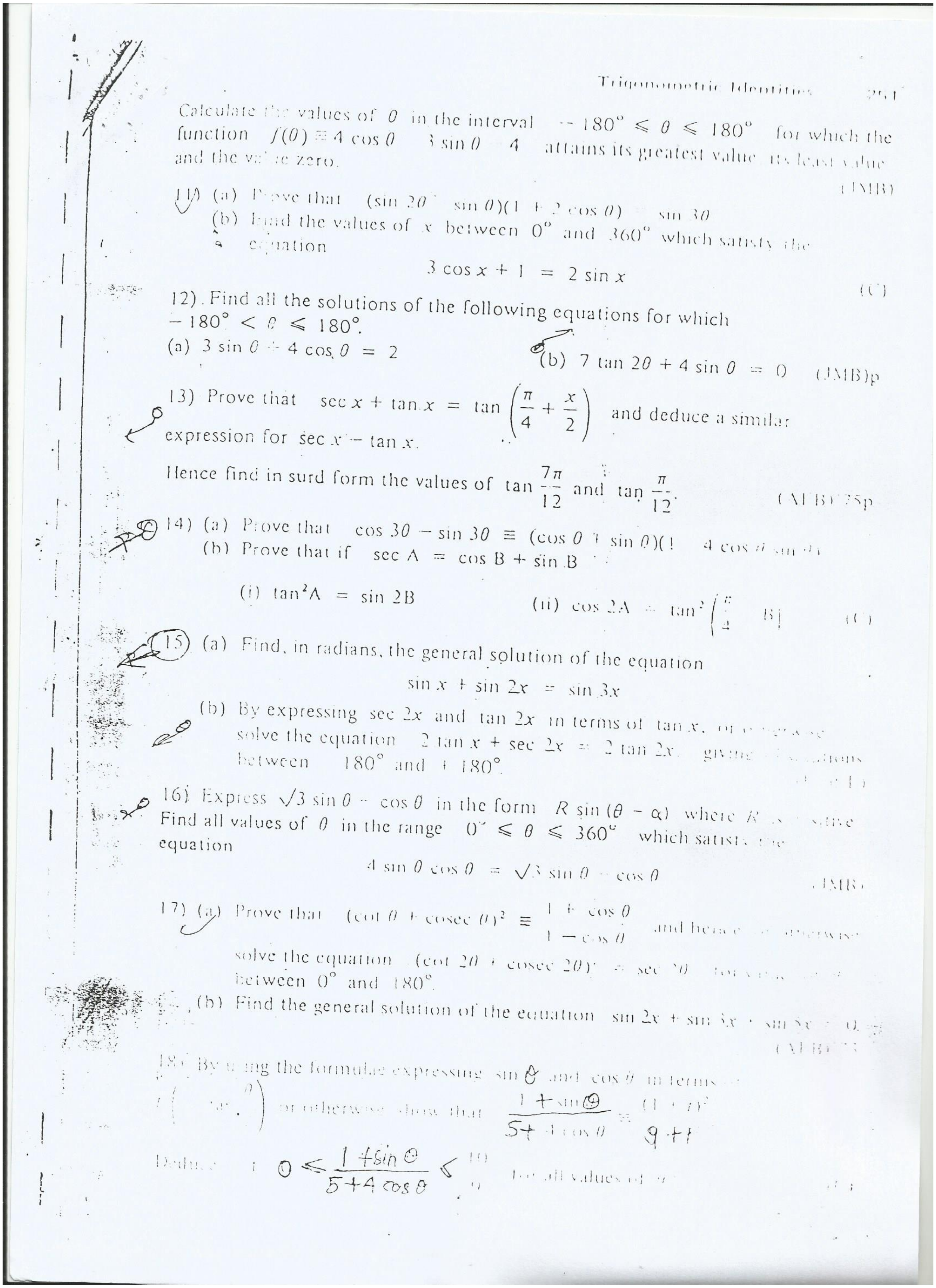
15) Prove that when a polynomial $f(x)$ is divided by $ax + b$, where $a \neq 0$, the remainder is $f(-b/a)$.
Find the polynomial in x of the third degree, which vanishes when $x = -1$ and $x = 2$, has the value 8 when $x = 0$ and leaves the remainder $16/3$ when divided by $3x + 2$. (JMB)

16) Express $\log_9 xy$ in terms of $\log_3 x$ and $\log_3 y$.
Without using tables, solve for x and y the simultaneous equations

$$\log_9 xy = \frac{5}{2}$$

$$\log_3 x \log_3 y = -6$$

expressing your answers as simply as possible. (JMB)



Calculate the values of θ in the interval $-180^\circ \leqslant \theta \leqslant 180^\circ$ for which the function $f(\theta) = 4\cos\theta \;\; 3\sin\theta \;\; 4$ attains its greatest value, its least value and the value zero.

(JMB)

11) (a) Prove that $(\sin 2\theta \;\; \sin\theta)(1 + 2\cos\theta) = \sin 3\theta$

(b) Find the values of x between 0° and 360° which satisfy the equation

$$3\cos x + 1 = 2\sin x$$

(C)

12) Find all the solutions of the following equations for which $-180^\circ < \theta \leqslant 180^\circ$.

(a) $3\sin\theta + 4\cos\theta = 2$

(b) $7\tan 2\theta + 4\sin\theta = 0$ (JMB)p

13) Prove that $\sec x + \tan x = \tan\left(\frac{\pi}{4} + \frac{x}{2}\right)$ and deduce a similar expression for $\sec x - \tan x$.

Hence find in surd form the values of $\tan\frac{7\pi}{12}$ and $\tan\frac{\pi}{12}$. (AEB) 75p

14) (a) Prove that $\cos 3\theta - \sin 3\theta \equiv (\cos\theta + \sin\theta)(1 \;\; 4\cos\theta\sin\theta)$

(b) Prove that if $\sec A = \cos B + \sin B$

(i) $\tan^2 A = \sin 2B$

(ii) $\cos 2A = \tan^2\left(\frac{\pi}{4} \;\; B\right)$ (C)

15) (a) Find, in radians, the general solution of the equation

$$\sin x + \sin 2x = \sin 3x$$

(b) By expressing $\sec 2x$ and $\tan 2x$ in terms of $\tan x$, or otherwise solve the equation $2\tan x + \sec 2x = 2\tan 2x$, giving solutions between 180° and $+180^\circ$.

(C)

16) Express $\sqrt{3}\sin\theta - \cos\theta$ in the form $R\sin(\theta - \alpha)$ where R is positive. Find all values of θ in the range $0^\circ \leqslant \theta \leqslant 360^\circ$ which satisfy the equation

$$4\sin\theta\cos\theta = \sqrt{3}\sin\theta - \cos\theta$$

(JMB)

17) (a) Prove that $(\cot\theta + \operatorname{cosec}\theta)^2 \equiv \frac{1 + \cos\theta}{1 - \cos\theta}$ and hence or otherwise solve the equation $(\cot 2\theta + \operatorname{cosec} 2\theta)^2 = \sec^2\theta$ for values of θ between 0° and 180°.

(b) Find the general solution of the equation $\sin 2x + \sin 3x + \sin 5x = 0$.

(AEB) 73

18) By using the formulae expressing $\sin\theta$ and $\cos\theta$ in terms of $t \left(= \tan\frac{\theta}{2}\right)$, or otherwise show that $\frac{1 + \sin\theta}{5 + 4\cos\theta} = \frac{(1 + t)^2}{9 + t^2}$

Deduce that $0 \leqslant \frac{1 + \sin\theta}{5 + 4\cos\theta} \leqslant \frac{10}{9}$ for all values of θ.

$$\sin^2\frac{\pi}{8} - \cos^4\frac{3\pi}{8}$$

(U of L)

3) (a) If A is the acute angle such that $\sin A = \frac{3}{5}$ and B is the obtuse angle such that $\sin B = \frac{5}{13}$, find without using tables the values of $\cos(A + B)$ and $\tan(A - B)$.

(b) Find the solutions of the equation $\tan\theta + 3\cot\theta = 5\sec\theta$ for which $0 < \theta < 2\pi$.

(U of L)

4) Calculate the values of θ in the range $0 \leqslant \theta \leqslant 180°$ which satisfy the equations

(a) $2\sin\theta + \cos\theta = 1$ (b) $2\sin\theta + \cos 2\theta = 1$. (AEB)'72p

5) (a) Find the values of θ between 0° and 180° for which $\tan^2\theta = 5 - \sec\theta$.

(b) Find the values of θ between 0° and 360° which satisfy the equation

$$6\cos\theta + 7\sin\theta = 4$$

(C)

6) (a) Find the values of x between 0° and 360° which satisfy the equation

$$\sin 2x + 2\cos 2x = 1$$

(b) Find the general solution of the equation $\cos 3x + \cos x = \sin 2x$

(U of L)p

7) (a) If $\sin(\theta - \alpha) = k\sin(\theta + \alpha)$ find $\tan\theta$ in terms of $\tan\alpha$ and k and so determine the possible values of θ between 0 and 360° when $k = \frac{1}{2}$ and $\alpha = 150°$.

(b) Show without the use of tables or calculator, that $x = \frac{\pi}{10}$ satisfies the equation $\cos 3x = \sin 2x$. By expressing this equation in terms of $\sin x$ and $\cos x$ show that $\sin\frac{\pi}{10}$ is a root of the equation

$$\cancel{4s^2} + 2s - 1 = 0$$

(C)

8) (a) Find the values of θ between 0 and 2π for which $\sin 2\theta - \sin\frac{\pi}{6}$

(b) Show that $(2\cos\phi + 3\sin\phi)^2 \leqslant 13$ for all values of [illegible] (U of L)p

9) (a) [illegible] values of θ in the range 0 [illegible]

$$\sin\theta + \sin 3\theta = \cos\theta + \cos 3\theta$$

(b) [illegible] the nearest minute the acute angle [illegible]

$$4\cos\theta + 3\sin\theta = 5\cos(\theta + \alpha)$$