Zoning out: an analysis of zoning and property values in Washington, D.C.

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Sampling Frame

Download the data

```
library(tidyverse)
library(knitr)
```

```
# file path to csv with addresses
aru_file_path <-
  "https://opendata.arcgis.com/datasets/c3c0ae91dca54c5d9ce56962fa0dd645_68.csv"

ap_file_path <-
  "https://opendata.arcgis.com/datasets/aa514416aaf74fdc94748f1e56e7cc8a_0.csv"

# create a directory for downloading the data
if (!dir.exists("data/")) {
  dir.create("data")
}

# if the data doesn't already exist, download the data
if (!file.exists("data/aru.csv")) {
  download.file(aru_file_path, "data/aru.csv")
}

if (!file.exists("data/ap.csv")) {
  download.file(ap_file_path, "data/ap.csv")
}
```

Address Residential Units

The first dataset is Address Residential Units

The dataset does not contain a variable for quadrant, so we extract quadrant from the full address.

```
aru <- read_csv("data/aru.csv") %>%
  rename_all(tolower) %>%
  select(unit_id, address_id, fulladdress, status, unitnum, unittype)

# extract quadrant
aru <- aru %>%
  mutate(quadrant = str_sub(fulladdress, start = -2, end = -1))
```

Address Residential Units contains residential units with status set to "RETIRED". We drop these cases as well.

```
count(aru, status) %>%
  kable()
```

status	n
ACTIVE	244046
ASSIGNED	47
RETIRE	7087

```
aru <- aru %>%
  filter(status != "RETIRE")
```

Address Points

```
# load the data and convert the variable names to lower case
ap <- read_csv("data/ap.csv", guess_max = 10000) %>%
  rename_all(tolower) %>%
  select(address_id, status, type_, entrancetype, quadrant, fulladdress,
         objectid_1, assessment_nbhd, cfsa_name, census_tract, vote_prcnct,
         ward, zipcode, anc, census_block, census_blockgroup, latitude,
         longitude, active_res_unit_count, res_type, active_res_occupancy_count)
```

Address Points contains residential units, non-residential units, and mixed-use units. Residential units and mixed-use units contain residences that belong to our sampling frame. We drop non-residential units.

```
count(ap, res_type) %>%
  kable()
```

res_type	n
MIXED USE	473
NON RESIDENTIAL	15807
RESIDENTIAL	131370

```
ap <- ap %>%
  filter(res_type != "NON RESIDENTIAL")
```

Address points contains residential units with status set to "RETIRED". We drop these cases as well.

```r
count(ap, status) %>%
  kable()
```

status	n
ACTIVE	128490
ASSIGNED	668
RETIRE	2675
TEMPORARY	10

```r
ap <- ap %>%
  filter(status != "RETIRE")
```

After the above filtering, there are 98 observations from Address Points and 3,706 observations in Address Residential Units that have missing addresses. We investigated joining the two datasets on `address_id` to fill in the address but all records missing an address in one dataset were missing an address in the other dataset.

We dropped the missing values which represented about 1.5 percent of observations in Address Residential Units and 0.07 percent of observations in Address Points.

```r
ap <- ap %>%
  filter(!is.na(fulladdress))

aru <- aru %>%
  filter(!is.na(fulladdress))
```

Merge variables

Address Points has interesting variables not present in Address Residential Units. So we merge the Address Points dataset with the Address Residential Units dataset. The join works for all but 572 cases, most of which are in a new building at the Wharf.

```r
aru_expanded <- aru %>%
  select(-status) %>%
  left_join(ap, by = c("fulladdress", "address_id")) %>%
  select(quadrant = quadrant.x, everything(), -quadrant.y)

anti_join(aru, ap, by = c("fulladdress", "address_id"))
```

```
## # A tibble: 572 x 7
##    unit_id address_id fulladdress         status unitnum unittype quadrant
##      <dbl>      <dbl> <chr>               <chr>  <chr>   <chr>    <chr>
##  1  223379     276680 600 WATER STREET SW ACTIVE 6-12    RENTAL   SW
##  2  223380     276680 600 WATER STREET SW ACTIVE 6-13    RENTAL   SW
##  3  223381     276680 600 WATER STREET SW ACTIVE 6-14    RENTAL   SW
##  4  223384     276680 600 WATER STREET SW ACTIVE 1-1     RENTAL   SW
```

```
##  5  223389     276680 600 WATER STREET SW ACTIVE 1-6     RENTAL   SW
##  6  223392     276680 600 WATER STREET SW ACTIVE 1-9     RENTAL   SW
##  7  223494     276680 600 WATER STREET SW ACTIVE 8-16    RENTAL   SW
##  8  223497     276680 600 WATER STREET SW ACTIVE 9-3     RENTAL   SW
##  9  223503     276680 600 WATER STREET SW ACTIVE 9-9     RENTAL   SW
## 10  223508     276680 600 WATER STREET SW ACTIVE 9-14    RENTAL   SW
## # ... with 562 more rows
```

```r
rm(aru)
```

Combination

Next, we need to drop addresses in the Address Points dataset that exist in the Address Residential Units dataset so we don't over count addresses in multi-dwelling units.

```r
ap <- ap %>%
  filter(!address_id %in% unique(aru_expanded$address_id))
```

Finally, we can combine the two datasets to create a sampling frame that contains approximately every residential address in Washington D.C.

```r
sampling_frame <- bind_rows(ap, aru_expanded)

rm(ap, aru_expanded)

#summarize_all(addresses, list(~sum(is.na(.))))

write_csv(sampling_frame, "sampling_frame.csv")
```

Pilot survey

```r
set.seed(20190714)

pilot_sample <- sampling_frame %>%
  group_by(quadrant) %>%
  sample_n(25)

write_csv(pilot_sample, "data/pilot_sample.csv")

rm(pilot_sample)
```

```r
# load the completed pilot survey and clean the values
pilot_sample <- read_csv("data/pilot_sample_completed.csv") %>%
  mutate(land_value = ifelse(!is.na(rf_land_value),
                             rf_land_value,
                             land_value),
```

```r
        improvement_value = ifelse(!is.na(rf_improvement_value),
                                   rf_improvement_value,
                                   improvement_value)) %>%
  mutate(property_value = land_value + improvement_value) %>%
  mutate(property_value = ifelse(unittype == "RENTAL" &
                                  active_res_occupancy_count > 4 &
                                  property_value > 2000000,
                                property_value / active_res_occupancy_count,
                                property_value
                                ))
```

```r
pilot_sample %>%
  summarize(mean = mean(property_value, na.rm = TRUE),
            s_squared_h = var(property_value, na.rm = TRUE),
            missing_prop = mean(is.na(property_value))) %>%
  kable(caption = "Pilot survey summary statistics")
```

Table 4: Pilot survey summary statistics

mean	s_squared_h	missing_prop
535087.4	297224769021	0.17

```r
pilot_sample %>%
  group_by(quadrant) %>%
  summarize(mean = mean(property_value, na.rm = TRUE),
            s_squared_h = var(property_value, na.rm = TRUE),
            missing_prop = mean(is.na(property_value))) %>%
  kable(caption = "Pilot survey summary statistics by quadrant")
```

Table 5: Pilot survey summary statistics by quadrant

quadrant	mean	s_squared_h	missing_prop
NE	408489.5	55231295979	0.08
NW	928130.1	728182282168	0.12
SE	496448.4	136823871969	0.28
SW	283103.1	25025018879	0.20

Picking stratum sizes

Condition 1: Sample mean

We begin with a derivation of Exact Optimal Sample Allocation for $\bar{y}$.

Decomposition of $V(\bar{y}_h)$:

By Wright (12.4), $V(\bar{y}_{str}) = \sum_{h=1}^{H}(\frac{N_h}{N})^2 V(\bar{y}_h) = \sum_{h=1}^{H}(\frac{N_h}{N})^2 \frac{N_h - n_h}{N_h} \frac{S_h^2}{n_h}$

$V(\bar{y}_h) = (\frac{N_h}{N})^2 \frac{N_h - n_h}{N_h} \frac{S_h^2}{n_h}$

$V(\bar{y}_h) = (\frac{N_h^2}{N^2})(1 - \frac{n_h}{N_h})\frac{S_h^2}{n_h}$

$V(\bar{y}_h) = (\frac{N_h^2 S_h^2}{N^2})(\frac{1}{n_h}) - \frac{N_h^2 n_h S_h^2}{N^2 N_h n_h}$

$V(\bar{y}_h) = (\frac{N_h^2 S_h^2}{N^2})(\frac{1}{n_h}) - \frac{N_h S_h^2}{N^2}$

$V(\bar{y}_h) = (\frac{N_h^2 S_h^2}{N^2})(1 - \frac{1}{1 \cdot 2} - \frac{1}{2 \cdot 3} - ... - \frac{1}{n_h(n_h-1)}) - \frac{N_h S_h^2}{N^2}$

$V(\bar{y}_h) = \frac{N_h(N_h-1)S_h^2}{N^2} - \frac{N_h^2 S_h^2}{N^2 \cdot 1 \cdot 2} - \frac{N_h^2 S_h^2}{N^2 \cdot 2 \cdot 3} - ... - -\frac{N_h^2 S_h^2}{N^2 n_h(n_h-1)}$

Decomposition of $V(\bar{y}_{str})$

$V(\bar{y}_{str}) = \sum_{h=1}^{H} \frac{N_h(N_h-1)S_h^2}{N^2}$

$-\frac{N_1^2 S_1^2}{N^2 \cdot 1 \cdot 2} - \frac{N_1^2 S_1^2}{N^2 \cdot 2 \cdot 3} - ... - -\frac{N_1^2 S_1^2}{N^2 n_1(n_1-1)}$

$\ldots$

$-\frac{N_h^2 S_h^2}{N^2 \cdot 1 \cdot 2} - \frac{N_h^2 S_h^2}{N^2 \cdot 2 \cdot 3} - ... - -\frac{N_h^2 S_h^2}{N^2 n_h(n_h-1)}$

$\ldots$

$-\frac{N_H^2 S_H^2}{N^2 \cdot 1 \cdot 2} - \frac{N_H^2 S_H^2}{N^2 \cdot 2 \cdot 3} - ... - -\frac{N_H^2 S_H^2}{N^2 n_H(n_H-1)}$

For a desired bound V_0 on the sampling variance $V(\bar{y}_{str})$, we may find an optimal allocation using the following algorithm:

1) Assign, for each stratum, 1 unit to be selected for the sample.

2) Fill in the following table and number these values starting from 1, in decreasing order.

$\frac{N_1^2 S_1^2}{N^2 \cdot 1 \cdot 2}$	$\frac{N_1^2 S_1^2}{n^2 \cdot 2 \cdot 3}$	$\frac{N_1^2 S_1^2}{N^2 \cdot 3 \cdot 4}$	$\ldots$
$\frac{N_2^2 S_2^2}{N^2 \cdot 1 \cdot 2}$	$\frac{N_2^2 S_2^2}{N^2 \cdot 2 \cdot 3}$	$\frac{N_2^2 S_2^2}{N^2 \cdot 3 \cdot 4}$	$\ldots$
.	.	.	$\ldots$
.	.	.	$\ldots$
.	.	.	$\ldots$
$\frac{N_H^2 S_H^2}{N^2 \cdot 1 \cdot 2}$	$\frac{N_H^2 S_H^2}{N^2 \cdot 2 \cdot 3}$	$\frac{N_H^2 S_H^2}{N^2 \cdot 3 \cdot 4}$	$\ldots$

3) Since the initial allocation is $(n_{11}, n_{21}, ..., n_{H1}) = (1, 1, ..., 1)$, compute $V(\bar{y}_{str} | n_{11} = 1, n_{21} = 1, ..., n_{H1} = 1) = \sum_{h=1}^{H} \frac{N_h(N_h-1)S_h^2}{N^2}$

4) Pick value (1) from the table and increase the associated stratum's sample

size by 1, o that the updated allocation is $(n_{12}, n_{22}, ..., n_{H2})$, where exactly one of the n_{h2}'s is equal to 2 and the rest are equal to 1. Then, compute $V(\bar{y}_{str}|n_{12}, ..., n_{H2} = V(\bar{y}_{str}|n_{11}, ..., n_{H1}) - \frac{1}{N^2}$ where "(1)" represents the largest value from the table. If $V(\bar{y}_{str}|N_{12}, ..., n_{H2} \leq V_0$, then stop with $n_1 = n_{12}, ..., N_H = N_{H2}$. Otherwise, go to step 5.

5) Pick value (2) from the table and increase the associated stratum's sample size by 1, so that the updated allocation is $(n_{13}, ..., n_{H3})$. Then compute $V(\bar{y}_{str}|n_{13}, ..., n_{H3}) = V(\bar{y}_{str}|n_{12}, ..., n_{H2} - \frac{(2)}{N^2}$, where "(2)" represents the second value from the table. If $V(\bar{y}_{str}|n_{13}, ..., N_H = n_{H3}$. Otherwise, continue until step j, where $V(\bar{y}_s tr|n_{1j}, ..., n_{Hj}) \leq V_0$. The final allocation is $n_{1j}, ..., n_{Hj}$ and $n = n_{1j} + \cdots + n_{Hj}$.

```r
# find Nh and s2 for each strata
# (1) and (2)
s_squared_h <- pilot_sample %>%
  group_by(stratum = quadrant) %>%
  summarize(s_squared_h = var(property_value, na.rm = TRUE),
            missing_prop = mean(is.na(property_value)))

Nh <- sampling_frame %>%
  count(stratum = quadrant) %>%
  rename(Nh = n)

strata <- left_join(s_squared_h, Nh, by = "stratum") %>%
  # adjust N because of missingness
  mutate(Nh = Nh * (1 - missing_prop)) %>%
  mutate(N = sum(Nh))

rm(s_squared_h, Nh)

kable(strata)
```

stratum	s_squared_h	missing_prop	Nh	N
NE	55231295979	0.08	68953.08	297153.2
NW	728182282168	0.12	166931.60	297153.2
SE	136823871969	0.28	49423.68	297153.2
SW	25025018879	0.20	11844.80	297153.2

Step 3: $\hat{V}(\bar{y}|1, 1, 1, 1) = \sum_{h=1}^{H}(\frac{N_h}{N})^2 \frac{N_h - n_h}{N_H} \frac{s_h^2}{n_h} = \sum_{h=1}^{H}(\frac{N_h}{N})^2 \frac{N_h - 1}{N_H} \frac{s_h^2}{1}$

(Wright 12.5)

```r
# Let the initial allocation be (n_11, n_21, n_31, n_41) = (1, 1, 1, 1)
# (3) and (6)
starting_variance <- strata %>%
  mutate(strata_variance = Nh * (Nh - 1) * s_squared_h / N^2)
```

```r
kable(starting_variance)
```

stratum	s_squared_h	missing_prop	Nh	N	strata_variance
NE	55231295979	0.08	68953.08	297153.2	2973894581
NW	728182282168	0.12	166931.60	297153.2	229802057101
SE	136823871969	0.28	49423.68	297153.2	3784970868
SW	25025018879	0.20	11844.80	297153.2	39758730

```r
starting_variance <- starting_variance %>%
  summarize(V = sum(strata_variance)) %>%
  pull()

starting_variance
```

```
## [1] 236600681280
```

Step 3:

Priority value $= \frac{N_1^2 \cdot s_1^2}{N_1^2 \cdot n_h(n_h - 1)}$

```r
# create a table of priority values
# (4) and (5)
n_strata <-
  tibble(stratum = rep(strata$stratum, strata$Nh)) %>%
  group_by(stratum) %>%
  mutate(n = row_number()) %>%
  ungroup() %>%
  left_join(strata, by = "stratum")

# step 2
priority_values <- n_strata %>%
  group_by(stratum) %>%
  # rewritten to avoid integer overflow
  # mutate(priority_value = (Nh ^ 2 * s_squared_h) / (n * lag(n) * N ^ 2)) %>%
  mutate(priority_value = (Nh ^ 2 / n) * (s_squared_h / lag(n)) * (1 / N ^ 2)) %>%
  ungroup() %>%
  arrange(desc(priority_value))

kable(head(select(priority_values, -missing_prop), n = 10))
```

stratum	n	s_squared_h	Nh	N	priority_value
NW	2	728182282168	166931.6	297153.2	114901716867
NW	3	728182282168	166931.6	297153.2	38300572289
NW	4	728182282168	166931.6	297153.2	19150286144
NW	5	728182282168	166931.6	297153.2	11490171687
NW	6	728182282168	166931.6	297153.2	7660114458
NW	7	728182282168	166931.6	297153.2	5471510327

stratum	n	s_squared_h	Nh	N	priority_value
NW	8	728182282168	166931.6	297153.2	4103632745
NW	9	728182282168	166931.6	297153.2	3191714357
NW	10	728182282168	166931.6	297153.2	2553371486
NW	11	728182282168	166931.6	297153.2	2089122125

Step 4:

```
# (7)
priority_values <- priority_values %>%
  mutate(agg_priority_value = cumsum(priority_value)) %>%
  mutate(marginal_variance = starting_variance - agg_priority_value) %>%
  mutate(marginal_sd = sqrt(marginal_variance))

kable(head(select(priority_values, -missing_prop, -N), n = 100), digits = 0)
```

stratum	n	s_squared_h	Nh	priority_value	agg_priority_value	marginal_variance	marginal_sd
NW	2	728182282168	166932	114901716867	114901716867	121698964413	348854
NW	3	728182282168	166932	38300572289	153202289155	83398392124	288788
NW	4	728182282168	166932	19150286144	172352575300	64248105980	253472
NW	5	728182282168	166932	11490171687	183842746987	52757934293	229691
NW	6	728182282168	166932	7660114458	191502861444	45097819835	212362
NW	7	728182282168	166932	5471510327	196974371771	39626309508	199064
NW	8	728182282168	166932	4103632745	201078004517	35522676763	188475
NW	9	728182282168	166932	3191714357	204269718874	32330962406	179808
NW	10	728182282168	166932	2553371486	206823090360	29777590920	172562
NW	11	728182282168	166932	2089122125	208912212485	27688468795	166399
SE	2	136823871969	49424	1892523726	210804736211	25795945069	160611
NW	12	728182282168	166932	1740935104	212545671315	24055009965	155097
NE	2	55231295979	68953	1486968855	214032640170	22568041110	150227
NW	13	728182282168	166932	1473098934	215505739104	21094942176	145241
NW	14	728182282168	166932	1262656229	216768395333	19832285946	140827
NW	15	728182282168	166932	1094302065	217862697399	18737983881	136887
NW	16	728182282168	166932	957514307	218820211706	17780469574	133343
NW	17	728182282168	166932	844865565	219665077271	16935604008	130137
NW	18	728182282168	166932	750991614	220416068885	16184612395	127219
NW	19	728182282168	166932	671939865	221088008749	15512672530	124550
SE	3	136823871969	49424	630841242	221718849991	14881831288	121991
NW	20	728182282168	166932	604745878	222323595870	14277085410	119487
NW	21	728182282168	166932	547151033	222870746902	13729934377	117175
NW	22	728182282168	166932	497410030	223368156932	13232524348	115033
NE	3	55231295979	68953	495656285	223863813217	12736868063	112858
NW	23	728182282168	166932	454156984	224317970201	12282711079	110827
NW	24	728182282168	166932	416310568	224734280769	11866400511	108933
NW	25	728182282168	166932	383005723	225117286492	11483394788	107161
NW	26	728182282168	166932	353543744	225470830236	11129851043	105498
NW	27	728182282168	166932	327355319	225798185555	10802495725	103935
SE	4	136823871969	49424	315420621	226113606176	10487075104	102406
NW	28	728182282168	166932	303972796	226417578972	10183102308	100911
NW	29	728182282168	166932	283009155	226700588127	9900093153	99499

stratum	n	s_squared_h	Nh	priority_value	agg_priority_value	marginal_variance	marginal_sd
NW	30	728182282168	166932	264141878	226964730005	9635951275	98163
NE	4	55231295979	68953	247828143	227212558147	9388123133	96892
NW	31	728182282168	166932	247100466	227459658613	9141022666	95609
NW	32	728182282168	166932	231656687	227691315301	8909365979	94389
NW	33	728182282168	166932	217616888	227908932189	8691749091	93230
NW	34	728182282168	166932	204815895	228113748083	8486933196	92125
NW	35	728182282168	166932	193112129	228306860212	8293821067	91070
SE	5	136823871969	49424	189252373	228496112585	8104568695	90025
NW	36	728182282168	166932	182383678	228678496263	7922185017	89007
NW	37	728182282168	166932	172525100	228851021363	7749659917	88032
NW	38	728182282168	166932	163444832	229014466195	7586215085	87099
NW	39	728182282168	166932	155063046	229169529241	7431152039	86204
NE	5	55231295979	68953	148696886	229318226126	7282455153	85337
NW	40	728182282168	166932	147309893	229465536020	7135145260	84470
NW	41	728182282168	166932	140124045	229605660065	6995021215	83636
NW	42	728182282168	166932	133451471	229739111536	6861569744	82835
NW	43	728182282168	166932	127244426	229866355962	6734325317	82063
SE	6	136823871969	49424	126168248	229992524211	6608157069	81291
NW	44	728182282168	166932	121460589	230113984799	6486696480	80540
NW	45	728182282168	166932	116062340	230230047139	6370634140	79816
NW	46	728182282168	166932	111016152	230341063291	6259617989	79118
NW	47	728182282168	166932	106292060	230447355351	6153325929	78443
NW	48	728182282168	166932	101863224	230549218575	6051462704	77791
NE	6	55231295979	68953	99131257	230648349832	5952331447	77151
NW	49	728182282168	166932	97705542	230746055374	5854625906	76516
NW	50	728182282168	166932	93797320	230839852694	5760828586	75900
SE	7	136823871969	49424	90120177	230929972871	5670708409	75304
NW	51	728182282168	166932	90118994	231020091865	5580589415	74703
NW	52	728182282168	166932	86652878	231106744743	5493936536	74121
NW	53	728182282168	166932	83382959	231190127702	5410553578	73556
NW	54	728182282168	166932	80294701	231270422403	5330258877	73009
NW	55	728182282168	166932	77374894	231347797296	5252883984	72477
NW	56	728182282168	166932	74611504	231422408801	5178272479	71960
NW	57	728182282168	166932	71993557	231494402357	5106278922	71458
NE	7	55231295979	68953	70808041	231565210398	5035470881	70961
NW	58	728182282168	166932	69511020	231634721419	4965959861	70470
SE	8	136823871969	49424	67590133	231702311552	4898369728	69988
NW	59	728182282168	166932	67154715	231769466266	4831215013	69507
NW	60	728182282168	166932	64916224	231834382491	4766298789	69038
NW	61	728182282168	166932	62787823	231897170314	4703510966	68582
NW	62	728182282168	166932	60762410	231957932724	4642748556	68138
NW	63	728182282168	166932	58833444	232016766168	4583915111	67705
NW	64	728182282168	166932	56994899	232073761067	4526920212	67282
NW	65	728182282168	166932	55241210	232129002277	4471679002	66871
NW	66	728182282168	166932	53567234	232182569511	4418111768	66469
NE	8	55231295979	68953	53106031	232235675542	4365005738	66068
SE	9	136823871969	49424	52570103	232288245645	4312435634	65669
NW	67	728182282168	166932	51968212	232340213858	4260467422	65272
NW	68	728182282168	166932	50439735	232390653593	4210027687	64885
NW	69	728182282168	166932	48977714	232439631307	4161049973	64506
NW	70	728182282168	166932	47578351	232487209657	4113471622	64136
NW	71	728182282168	166932	46238115	232533447773	4067233507	63775

stratum	n	s_squared_h	Nh	priority_value	agg_priority_value	marginal_variance	marginal_sd
NW	72	728182282168	166932	44953723	232578401496	4022279783	63421
NW	73	728182282168	166932	43722114	232622123611	3978557669	63076
NW	74	728182282168	166932	42540436	232664664046	3936017233	62738
SE	10	136823871969	49424	42056083	232706720129	3893961150	62402
NW	75	728182282168	166932	41406024	232748126153	3852555126	62069
NE	9	55231295979	68953	41304690	232789430844	3811250436	61735
NW	76	728182282168	166932	40316392	232829747236	3770934044	61408
NW	77	728182282168	166932	39269213	232869016448	3731664831	61087
NW	78	728182282168	166932	38262310	232907278758	3693402521	60773
NW	79	728182282168	166932	37293644	232944572402	3656108877	60466
NW	80	728182282168	166932	36361303	232980933705	3619747574	60164
NW	81	728182282168	166932	35463493	233016397198	3584284082	59869
NW	82	728182282168	166932	34598530	233050995728	3549685552	59579
SE	11	136823871969	49424	34409522	233085405250	3515276030	59290
NW	83	728182282168	166932	33764830	233119170080	3481511200	59004

```r
rm(n_strata)
```

```r
condition1 <- priority_values %>%
  mutate(stratum = factor(stratum)) %>%
  filter(marginal_variance >= ((0.1 * (mean(pilot_sample$property_value, na.rm = TRUE))) ^ 2))

condition1 <- condition1 %>%
  count(stratum, .drop = FALSE)
```

Condition 2: Sample means within strata

We are interested in comparing $\bar{y}_h$ from the four different quadrants.

$n = \frac{N\sigma^2}{(N-1)\frac{e^2}{z^2_{\frac{\alpha}{2}}}+\sigma^2}$

We can use s^2 from our pilot survey as an unbiased estimate for σ^2.

$n = \frac{Ns^2}{(N-1)\frac{e^2}{z^2_{\frac{\alpha}{2}}}+s^2}$

We want $50,000 precision at a 90% confidence level for the mean of property value in each strata.

```r
condition2 <- strata %>%
  mutate(n = (N * s_squared_h) / ((N - 1) * (50000 ^ 2 / qnorm(0.95) ^ 2) + s_squared_h))

condition2 %>%
  kable()
```

stratum	s_squared_h	missing_prop	Nh	N	n
NE	55231295979	0.08	68953.08	297153.2	59.76045

stratum	s_squared_h	missing_prop	Nh	N	n
NW	728182282168	0.12	166931.60	297153.2	785.96977
SE	136823871969	0.28	49423.68	297153.2	147.99992
SW	25025018879	0.20	11844.80	297153.2	27.08013

Condition 3: Sample proportion

We begin with a derivation of Exact Optimal Sample Allocation for $\hat{p}$.

Decomposition of $V(\hat{p}_{str})$

By Wright (12.14), $V(\hat{p}_{str}) = \sum_{h=1}^{H}(\frac{N_h}{N})^2 V(p_h) = \sum_{h=1}^{H}(\frac{N_h}{N})^2 \frac{N_h - n_h}{N_h - 1}\frac{p(1-p)}{n_h}$

$V(\hat{p}_h) = (\frac{N_h}{N})^2 \frac{N_h - n_h}{N_h - 1}\frac{p(1-p)}{n_h}$

$V(\hat{p}_h) = \frac{N_h^2}{N^2}\frac{N_h}{N_h-1}\frac{p(1-p)}{n_h} - \frac{N_h^2}{N^2}\frac{n_h}{N_h-1}\frac{p(1-p)}{n_h}$

$V(\hat{p}_h) = \frac{N_h^2}{N^2}\frac{N_h}{N_h-1}\frac{p(1-p)}{n_h} - \frac{N_h^2}{N^2}\frac{n_h}{N_h-1}\frac{p(1-p)}{n_h}$

$V(\hat{p}_h) = \frac{N_h^3 p(1-p)}{N^2(N_h-1)}\frac{1}{n_h} - \frac{N_h^2 p(1-p)}{N^2(N_h-1)}$

$V(\hat{p}_h) = \frac{N_h^3 p(1-p)}{N^2(N_h-1)}(1 - \frac{1}{1\cdot 2} - \frac{1}{2\cdot 3} - \cdots - \frac{1}{n_h(n_h-1)}) - \frac{N_h^2 p(1-p)}{N^2(N_h-1)}$

$V(\hat{p}_h) = \frac{N_h^3 p(1-p)}{N^2(N_h-1)} - \frac{N_h^3 p(1-p)}{N^2(N_h-1)\cdot 1\cdot 2} - \frac{N_h^3 p(1-p)}{N^2(N_h-1)\cdot 2\cdot 3} - \cdots - \frac{N_h^3 p(1-p)}{N^2(N_h-1)\cdot n_h(n_h-1)} - \frac{N_h^2 p(1-p)}{N^2(N_h-1)}$

$V(\hat{p}_h) = \frac{(N_h^3 - N_h^2)p(1-p)}{N^2(N_h-1)} - \frac{N_h^3 p(1-p)}{N^2(N_h-1)\cdot 1\cdot 2} - \frac{N_h^3 p(1-p)}{N^2(N_h-1)\cdot 2\cdot 3} - \cdots - \frac{N_h^3 p(1-p)}{N^2(N_h-1)\cdot n_h(n_h-1)}$

$V(\hat{p}_h) = \frac{N_h^2(N_h-1)p(1-p)}{N^2(N_h-1)} - \frac{N_h^3 p(1-p)}{N^2(N_h-1)\cdot 1\cdot 2} - \frac{N_h^3 p(1-p)}{N^2(N_h-1)\cdot 2\cdot 3} - \cdots - \frac{N_h^3 p(1-p)}{N^2(N_h-1)\cdot n_h(n_h-1)}$

Decomposition of $V(\hat{p}_{str})$

$V(\hat{p}_{str}) = \sum_{h=1}^{H} \frac{N_h^2(N_h-1)p(1-p)}{N^2(N_h-1)}$

$-\frac{N_1^3 p(1-p)}{N^2(N_1-1)\cdot 1\cdot 2} - \frac{N_1^3 p(1-p)}{N^2(N_1-1)\cdot 2\cdot 3} - \ldots - \frac{N_1^3 p(1-p)}{N^2(N_1-1)n_h(n_h-1)}$

$\ldots$

$-\frac{N_h^3 p(1-p)}{N^2(N_h-1)\cdot 1\cdot 2} - \frac{N_h^3 p(1-p)}{N^2(N_h-1)\cdot 2\cdot 3} - \ldots - \frac{N_h^3 p(1-p)}{N^2(N_h-1)n_h(n_h-1)}$

$\ldots$

$-\frac{N_H^3 p(1-p)}{N^2(N_H-1)\cdot 1\cdot 2} - \frac{N_H^3 p(1-p)}{N^2(N_H-1)\cdot 2\cdot 3} - \ldots - \frac{N_H^3 p(1-p)}{N^2(N_H-1)n_h(n_h-1)}$

For a desired bound on V_0 on the sampling variance $V(\hat{p}_{str})$, we may find an optimal allocation using the following algorithm:

1) Assign, for each stratum, 1 unit to be selected for the sample.

2) Fill in the following table and number these values starting from 1, in decreasing order. We assume $p_h = 0.5$ because that is where the variance reaches its global maximum.

$\frac{\frac{1}{4}N_1^3}{N^2(N_1-1)\cdot 1\cdot 2}$	$\frac{\frac{1}{4}N_1^3}{N^2(N_1-1)\cdot 2\cdot 3}$	$\frac{\frac{1}{4}N_1^3}{N^2(N_1-1)\cdot 3\cdot 4}$	...
$\frac{\frac{1}{4}N_2^3}{N^2(N_2-1)\cdot 1\cdot 2}$	$\frac{\frac{1}{4}N_2^3}{N^2(N_2-1)\cdot 2\cdot 3}$	$\frac{\frac{1}{4}N_2^3}{N^2(N_2-1)\cdot 3\cdot 4}$	...
.	.	.	...
.	.	.	...
.	.	.	...
$\frac{\frac{1}{4}N_H^3}{N^2(N_H-1)\cdot 1\cdot 2}$	$\frac{\frac{1}{4}N_H^3}{N^2(N_H-1)\cdot 2\cdot 3}$	$\frac{\frac{1}{4}N_H^3}{N^2(N_H-1)\cdot 3\cdot 4}$	...

3) Since the initial allocation is $(n_{11}, n_{21}, ..., n_{H1}) = (1, 1, ..., 1)$, compute $V(\hat{p}_{str}|n_{11} = 1, n_{21} = 1, ..., n_{H1} = 1) = \frac{1}{N^2}\sum_{h=1}^{H}((N_h^2 - N_h)S_h^2)$

4) Pick value (1) from the table and increase the associated stratum's sample size by 1, o that the updated allocation is $(n_{12}, n_{22}, ..., n_{H2})$, where exactly one of the n_{h2}'s is equal to 2 and the rest are equal to 1. Then, compute $V(\hat{p}_{str}|n_{12}, ..., n_{H2} = V(\hat{p}_{str}|n_{11}, ..., n_{H1}) - \frac{1}{N^2}$ where "(1)" represents the largest value from the table. If $V(\hat{p}_{str}|N_{12}, ..., n_{H2} \leq V_0$, then stop with $n_1 = n_{12}, ..., N_H = N_{H2}$. Otherwise, go to step 5.

5) Pick value (2) from the table and increase the associated stratum's sample size by 1, so that the updated allocation is $(n_{13}, ..., n_{H3})$. Then compute $V(\hat{p}_{str}|n_{13}, ..., n_{H3}) = V(\hat{p}_{str}|n_{12}, ..., n_{H2} - \frac{(2)}{N^2}$, where "(2)" represents the second value from the table. If $V(\hat{p}_{str}|n_{13}, ..., N_H = n_{H3}$. Otherwise, continue until step j, where $V(\hat{p}_{str}|n_{1j}, ..., n_{Hj}) \leq V_0$. The final allocation is $n_{1j}, ..., n_{Hj})$ and $n = n_{1j} + \cdots + n_{Hj}$.

```
#
strata <- sampling_frame %>%
  count(stratum = quadrant) %>%
  rename(Nh = n) %>%
  mutate(N =sum(Nh),
         s_squared_h = 0.5 * (1 - 0.5))

kable(strata)
```

stratum	Nh	N	s_squared_h
NE	74949	348094	0.25
NW	189695	348094	0.25
SE	68644	348094	0.25
SW	14806	348094	0.25

```
# Let the initial allocation be (n_11, n_21, n_31, n_41) = (1, 1, 1, 1)
# (3) and (6)
starting_variance <- strata %>%
  mutate(strata_variance = (Nh / N) ^ 2 * ((Nh - 1) / Nh) * (s_squared_h / 1))

kable(starting_variance)
```

stratum	Nh	N	s_squared_h	strata_variance
NE	74949	348094	0.25	0.0115897
NW	189695	348094	0.25	0.0742432
SE	68644	348094	0.25	0.0097218
SW	14806	348094	0.25	0.0004523

```r
starting_variance <- starting_variance %>%
  summarize(V = sum(strata_variance)) %>%
  pull()

starting_variance
```

```
## [1] 0.09600692
```

```r
# create a table of priority values
# (4) and (5)

n_strata <-
  sampling_frame %>%
  count(quadrant)

n_strata <- tibble(stratum = rep(n_strata$quadrant, n_strata$n)) %>%
  group_by(stratum) %>%
  mutate(n = row_number()) %>%
  left_join(strata, by = "stratum")

# step 2
priority_values <- n_strata %>%
  group_by(stratum) %>%
  mutate(priority_value = (0.25 * Nh ^ 3) / (N ^ 2 * (Nh - 1) * n * lag(n))) %>%
  ungroup() %>%
  arrange(desc(priority_value))

kable(head(priority_values, n = 10))
```

stratum	n	Nh	N	s_squared_h	priority_value
NW	2	189695	348094	0.25	0.0371220
NW	3	189695	348094	0.25	0.0123740
NW	4	189695	348094	0.25	0.0061870
NE	2	74949	348094	0.25	0.0057950
SE	2	68644	348094	0.25	0.0048610
NW	5	189695	348094	0.25	0.0037122
NW	6	189695	348094	0.25	0.0024748
NE	3	74949	348094	0.25	0.0019317
NW	7	189695	348094	0.25	0.0017677
SE	3	68644	348094	0.25	0.0016203

```r
# (7)
priority_values <- priority_values %>%
  mutate(agg_priority_value = cumsum(priority_value)) %>%
  mutate(marginal_variance = starting_variance - agg_priority_value) %>%
  mutate(marginal_sd = sqrt(marginal_variance))

kable(head(select(priority_values, -N), n = 100), align = "l")
```

stratum	n	Nh	s_squared_h	priority_value	agg_priority_value	marginal_variance	marginal_sd
NW	2	189695	0.25	0.0371220	0.0371220	0.0588849	0.2426622
NW	3	189695	0.25	0.0123740	0.0494960	0.0465110	0.2156640
NW	4	189695	0.25	0.0061870	0.0556830	0.0403240	0.2008083
NE	2	74949	0.25	0.0057950	0.0614780	0.0345289	0.1858197
SE	2	68644	0.25	0.0048610	0.0663390	0.0296679	0.1722438
NW	5	189695	0.25	0.0037122	0.0700512	0.0259557	0.1611078
NW	6	189695	0.25	0.0024748	0.0725260	0.0234809	0.1532349
NE	3	74949	0.25	0.0019317	0.0744577	0.0215493	0.1467966
NW	7	189695	0.25	0.0017677	0.0762254	0.0197815	0.1406469
SE	3	68644	0.25	0.0016203	0.0778457	0.0181612	0.1347635
NW	8	189695	0.25	0.0013258	0.0791715	0.0168354	0.1297513
NW	9	189695	0.25	0.0010312	0.0802027	0.0158042	0.1257149
NE	4	74949	0.25	0.0009658	0.0811685	0.0148384	0.1218130
NW	10	189695	0.25	0.0008249	0.0819934	0.0140135	0.1183785
SE	4	68644	0.25	0.0008102	0.0828036	0.0132033	0.1149056
NW	11	189695	0.25	0.0006749	0.0834786	0.0125284	0.1119302
NE	5	74949	0.25	0.0005795	0.0840581	0.0119489	0.1093108
NW	12	189695	0.25	0.0005625	0.0846205	0.0113864	0.1067071
SE	5	68644	0.25	0.0004861	0.0851066	0.0109003	0.1044045
NW	13	189695	0.25	0.0004759	0.0855825	0.0104244	0.1020999
NW	14	189695	0.25	0.0004079	0.0859905	0.0100164	0.1000822
NE	6	74949	0.25	0.0003863	0.0863768	0.0096301	0.0981331
NW	15	189695	0.25	0.0003535	0.0867303	0.0092766	0.0963149
SE	6	68644	0.25	0.0003241	0.0870544	0.0089525	0.0946177
NW	16	189695	0.25	0.0003093	0.0873638	0.0086432	0.0929685
NE	7	74949	0.25	0.0002760	0.0876397	0.0083672	0.0914724
NW	17	189695	0.25	0.0002730	0.0879127	0.0080942	0.0899680
NW	18	189695	0.25	0.0002426	0.0881553	0.0078516	0.0886093
SE	7	68644	0.25	0.0002315	0.0883868	0.0076201	0.0872934
SW	2	14806	0.25	0.0002262	0.0886129	0.0073940	0.0859882
NW	19	189695	0.25	0.0002171	0.0888300	0.0071769	0.0847165
NE	8	74949	0.25	0.0002070	0.0890370	0.0069699	0.0834861
NW	20	189695	0.25	0.0001954	0.0892324	0.0067745	0.0823076
NW	21	189695	0.25	0.0001768	0.0894091	0.0065978	0.0812267
SE	8	68644	0.25	0.0001736	0.0895828	0.0064242	0.0801509
NE	9	74949	0.25	0.0001610	0.0897437	0.0062632	0.0791403
NW	22	189695	0.25	0.0001607	0.0899044	0.0061025	0.0781184
NW	23	189695	0.25	0.0001467	0.0900512	0.0059558	0.0771736
SE	9	68644	0.25	0.0001350	0.0901862	0.0058207	0.0762937
NW	24	189695	0.25	0.0001345	0.0903207	0.0056862	0.0754071
NE	10	74949	0.25	0.0001288	0.0904495	0.0055575	0.0745483
NW	25	189695	0.25	0.0001237	0.0905732	0.0054337	0.0737137
NW	26	189695	0.25	0.0001142	0.0906874	0.0053195	0.0729349

stratum	n	Nh	s_squared_h	priority_value	agg_priority_value	marginal_variance	marginal_sd
SE	10	68644	0.25	0.0001080	0.0907954	0.0052115	0.0721905
NW	27	189695	0.25	0.0001058	0.0909012	0.0051057	0.0714543
NE	11	74949	0.25	0.0001054	0.0910066	0.0050003	0.0707131
NW	28	189695	0.25	0.0000982	0.0911048	0.0049021	0.0700153
NW	29	189695	0.25	0.0000914	0.0911962	0.0048107	0.0693593
SE	11	68644	0.25	0.0000884	0.0912846	0.0047223	0.0687192
NE	12	74949	0.25	0.0000878	0.0913724	0.0046345	0.0680773
NW	30	189695	0.25	0.0000853	0.0914577	0.0045492	0.0674476
NW	31	189695	0.25	0.0000798	0.0915376	0.0044694	0.0668532
SW	3	14806	0.25	0.0000754	0.0916130	0.0043940	0.0662870
NW	32	189695	0.25	0.0000748	0.0916878	0.0043191	0.0657200
NE	13	74949	0.25	0.0000743	0.0917621	0.0042448	0.0651523
SE	12	68644	0.25	0.0000737	0.0918357	0.0041712	0.0645846
NW	33	189695	0.25	0.0000703	0.0919060	0.0041009	0.0640380
NW	34	189695	0.25	0.0000662	0.0919722	0.0040347	0.0635193
NE	14	74949	0.25	0.0000637	0.0920359	0.0039710	0.0630160
NW	35	189695	0.25	0.0000624	0.0920983	0.0039086	0.0625190
SE	13	68644	0.25	0.0000623	0.0921606	0.0038463	0.0620186
NW	36	189695	0.25	0.0000589	0.0922195	0.0037874	0.0615417
NW	37	189695	0.25	0.0000557	0.0922753	0.0037316	0.0610872
NE	15	74949	0.25	0.0000552	0.0923305	0.0036765	0.0606337
SE	14	68644	0.25	0.0000534	0.0923839	0.0036230	0.0601916
NW	38	189695	0.25	0.0000528	0.0924367	0.0035702	0.0597514
NW	39	189695	0.25	0.0000501	0.0924868	0.0035201	0.0593307
NE	16	74949	0.25	0.0000483	0.0925351	0.0034718	0.0589223
NW	40	189695	0.25	0.0000476	0.0925827	0.0034242	0.0585171
SE	15	68644	0.25	0.0000463	0.0926290	0.0033780	0.0581201
NW	41	189695	0.25	0.0000453	0.0926742	0.0033327	0.0577294
NW	42	189695	0.25	0.0000431	0.0927173	0.0032896	0.0573547
NE	17	74949	0.25	0.0000426	0.0927600	0.0032470	0.0569821
NW	43	189695	0.25	0.0000411	0.0928011	0.0032058	0.0566202
SE	16	68644	0.25	0.0000405	0.0928416	0.0031653	0.0562613
NW	44	189695	0.25	0.0000392	0.0928808	0.0031261	0.0559115
NE	18	74949	0.25	0.0000379	0.0929187	0.0030882	0.0555718
SW	4	14806	0.25	0.0000377	0.0929564	0.0030505	0.0552316
NW	45	189695	0.25	0.0000375	0.0929939	0.0030130	0.0548911
NW	46	189695	0.25	0.0000359	0.0930298	0.0029772	0.0545634
SE	17	68644	0.25	0.0000357	0.0930655	0.0029414	0.0542349
NW	47	189695	0.25	0.0000343	0.0930998	0.0029071	0.0539173
NE	19	74949	0.25	0.0000339	0.0931337	0.0028732	0.0536022
NW	48	189695	0.25	0.0000329	0.0931666	0.0028403	0.0532943
SE	18	68644	0.25	0.0000318	0.0931984	0.0028085	0.0529954
NW	49	189695	0.25	0.0000316	0.0932300	0.0027769	0.0526967
NE	20	74949	0.25	0.0000305	0.0932605	0.0027464	0.0524065
NW	50	189695	0.25	0.0000303	0.0932908	0.0027161	0.0521166
NW	51	189695	0.25	0.0000291	0.0933199	0.0026870	0.0518365
SE	19	68644	0.25	0.0000284	0.0933483	0.0026586	0.0515616
NW	52	189695	0.25	0.0000280	0.0933763	0.0026306	0.0512894
NE	21	74949	0.25	0.0000276	0.0934039	0.0026030	0.0510197
NW	53	189695	0.25	0.0000269	0.0934308	0.0025761	0.0507550
NW	54	189695	0.25	0.0000259	0.0934568	0.0025501	0.0504988
SE	20	68644	0.25	0.0000256	0.0934824	0.0025245	0.0502448

stratum	n	Nh	s_squared_h	priority_value	agg_priority_value	marginal_variance	marginal_sd
NE	22	74949	0.25	0.0000251	0.0935075	0.0024995	0.0499946
NW	55	189695	0.25	0.0000250	0.0935325	0.0024745	0.0497439
NW	56	189695	0.25	0.0000241	0.0935566	0.0024504	0.0495010
NW	57	189695	0.25	0.0000233	0.0935798	0.0024271	0.0492655
SE	21	68644	0.25	0.0000231	0.0936030	0.0024039	0.0490300

```
rm(n_strata)
```

```
condition3 <- priority_values %>%
  filter(marginal_variance >= ((0.1 * 0.5) ^ 2))

condition3 <- count(condition3, stratum)
```

Condition 4: Sample proportion within strata

We are interested in comparing $\hat{p}_h$ from the four different quadrants.

$n = \frac{Np(1-p)}{(N-1)\frac{e^2}{z^2_{\frac{\alpha}{2}}}+p(1-p)}$

We can assume that $p = 0.5$.

$n = \frac{\frac{1}{4}N}{(N-1)\frac{e^2}{z^2_{\frac{\alpha}{2}}}+\frac{1}{4}}$

We want 0.1 precision at a 90% confidence level for the mean of proportion with multi-family zoning in each strata.

```
condition4 <- strata %>%
  mutate(n = (N * 0.25) / ((N - 1) * (0.1 ^ 2 / qnorm(0.95) ^ 2) + 0.25))

condition4 %>%
  kable()
```

stratum	Nh	N	s_squared_h	n
NE	74949	348094	0.25	67.62564
NW	189695	348094	0.25	67.62564
SE	68644	348094	0.25	67.62564
SW	14806	348094	0.25	67.62564

Combining the above conditions

We want to sample at a rate that meets the four different requirements from above

1. $V_0 > V(\bar{y}_{str})$ for the sample mean

2. \$50,000 precision at a 90% confidence level for $\bar{y}_h$ in each strata
3. $V_0 > V(\hat{p}_h)$ for the sample proportion
4. 0.1 precision at a 90% confidence level for $\hat{p}$ in each strata

```
tibble(`1.` = condition1$n,
       `2.` = condition2$n,
       `3.` = condition3$n,
       `4.` = condition4$n) %>%
  kable(caption = "Recommended strata sizes across the four conditions")
```

Table 18: Recommended strata sizes across the four conditions

1.	2.	3.	4.
10	59.76045	20	67.62564
100	785.96977	53	67.62564
12	147.99992	19	67.62564
0	27.08013	3	67.62564