

---

# Development, validation, and sensitivity analysis of circumglobal CO<sub>2</sub> air-sea gas transfer velocity and its parameterisations with a spectral wave model

Andrew W. Smith<sup>1</sup>, Adrian H. Callaghan<sup>1</sup> and Jean-Raymond Bidlot<sup>2</sup>

<sup>1</sup>Imperial College London    <sup>2</sup>European Centre for Medium-Range Weather Forecasts

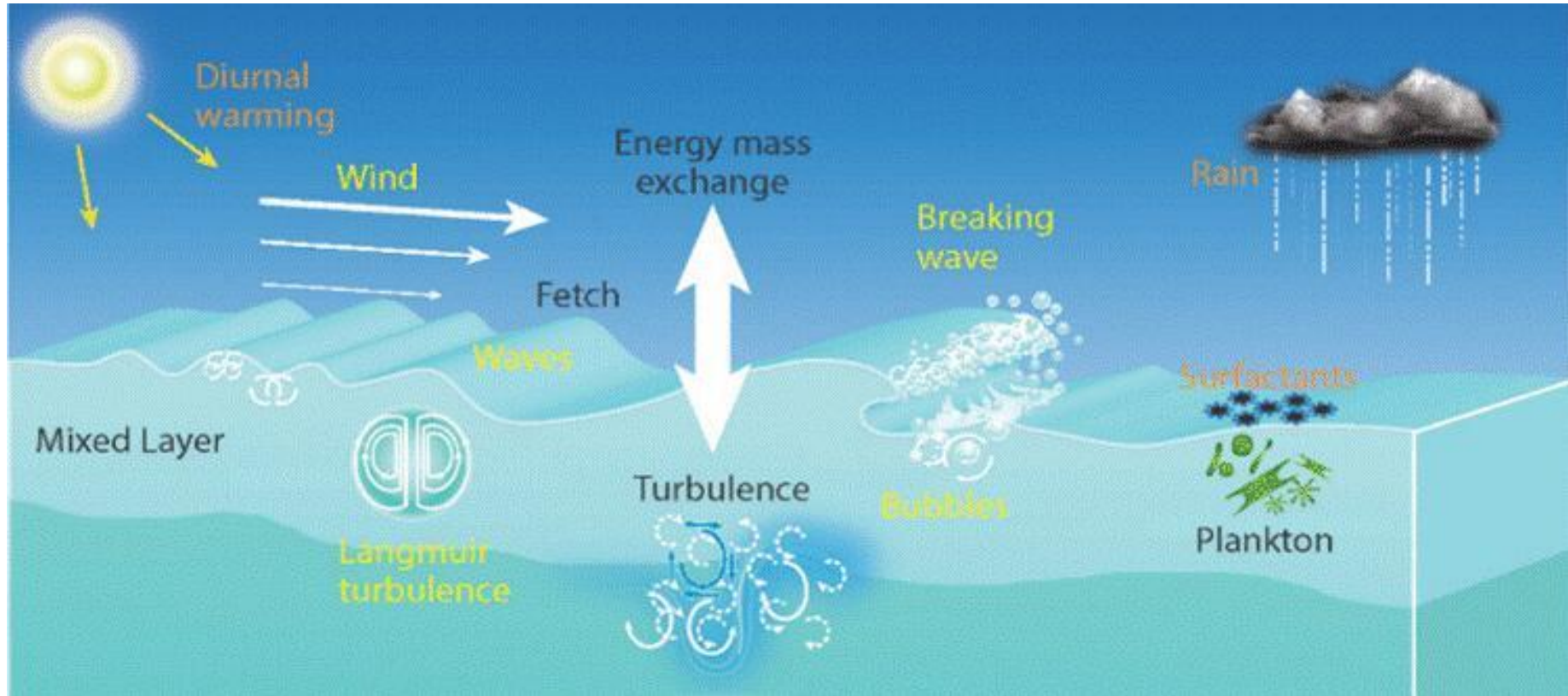


Postdoctoral Fellow Interview, April 19, 2024 | University of Rhode Island, Narragansett, RI

THE  
UNIVERSITY  
OF RHODE ISLAND



## Air-Sea Fluxes



Momentum, heat, material and gas exchanged between atmosphere, waves, and ocean through dynamic and thermodynamic processes

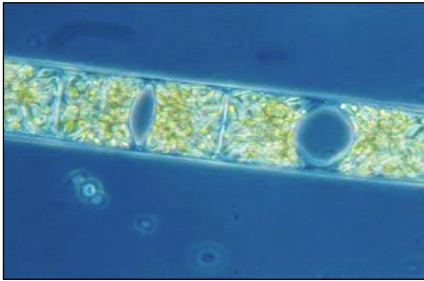
Diffusion • Wave growth and breaking → gas flux

Bubbles, sea spray, spume facilitate exchange and impact up-scale budgets

Air-sea gas fluxes → global climate

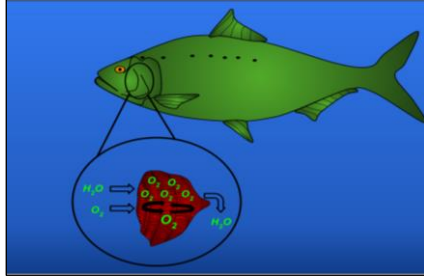
## Notable Gases and their Importance

$N_2$  (62.6 %)



- 1 Fixation by marine bacteria  
 $NH_4^+$   
 $NO_2^{2-}$   
 $NO_3^- \rightarrow$
- 2 Photosynthesis (byproduct)

$O_2$  (34.3 %)



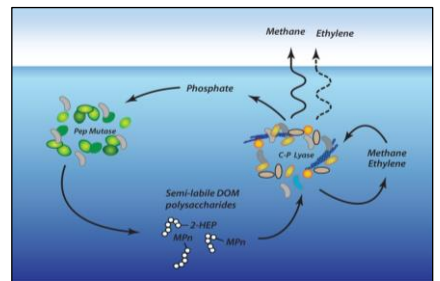
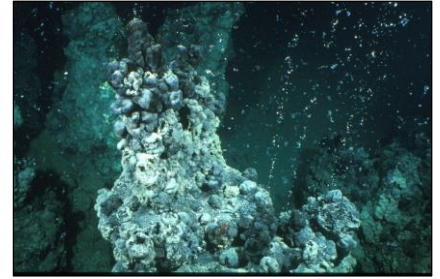
- 1 Aerobic respiration
- 2 Photosynthesis (byproduct)

$CO_2$  (1.4 %)

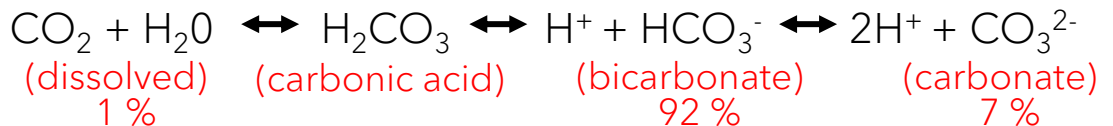


- 1 Aerobic respiration
- 2 Photosynthesis (reactant)
- 3 Carbon cycle/loading

$CH_4$  (0.0004 %)



- 1 Benthic bacterial decomposition
- 2 Hydrothermal release
- 3 Methanogenesis



## Scientific Objectives

1

Develop a hybrid gas transfer velocity parameterization with bubble-mediated gas exchange linked explicitly to breaking wave energy dissipation

Diffusive gas flux  
Bubble-mediated gas flux

2

Evaluate our and other extant parameterisations using outputs from a wind-forced spectral wave model, and compare with field measurements

9 global cruises (2007-2019)

3

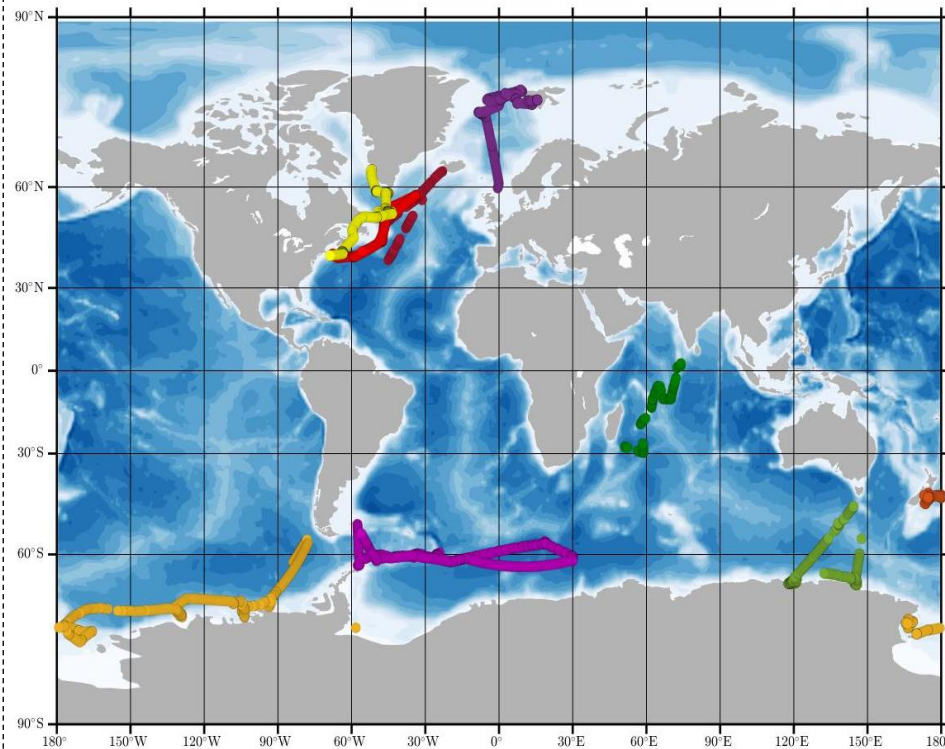
Identify sources of parameterisation success, uncertainty and error considering physical and chemical processes

Wind-wave momentum flux  
Wave-ocean energy dissipation (wave breaking)  
Bubbles and surfactants

---



## Field Data: Global Cruises



### Measurements (#)

1	R/V Knorr (2007)	61
2	R/V Knorr (2011)	215
3	SOAP (2012)	220
4	NBP-1210 (2013)	234
5	HiWinGS (2013)	529
6	NBP-1402 (2014)	68
7	SO-234/235 (2014)	74
8	ANDREXII (2019)	199
9	JR18007 (2019)	293

### Measurements / Methods

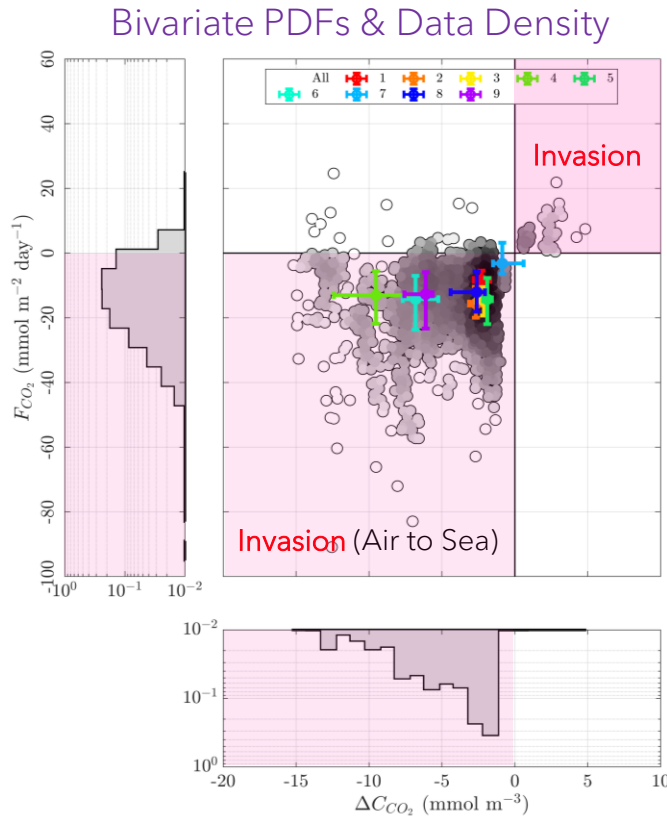
1

4 June 2007 – 29 August 2019: 9 cruises, hourly data in various ocean basins, water depths, time of year, fetch, and wind-wave conditions

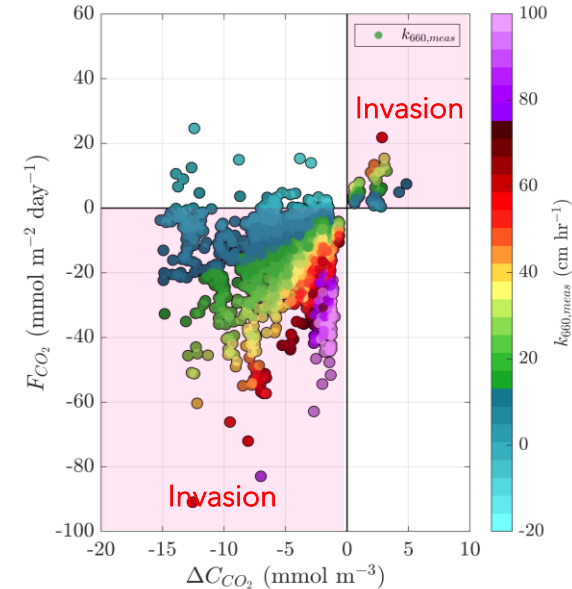
2

High-quality eddy covariance CO<sub>2</sub> fluxes and air-sea CO<sub>2</sub> concentration gradients via closed-path infrared gas analyzers ([LI-COR Li-7200](#), [Li-7500](#)) or cavity-ring down mass spectrometers ([Picarro G1301-f](#), [G2311-f](#))

## Field Data: Global Cruises



### Gas Transfer Velocity $k$ (at $Sc = 660$ )



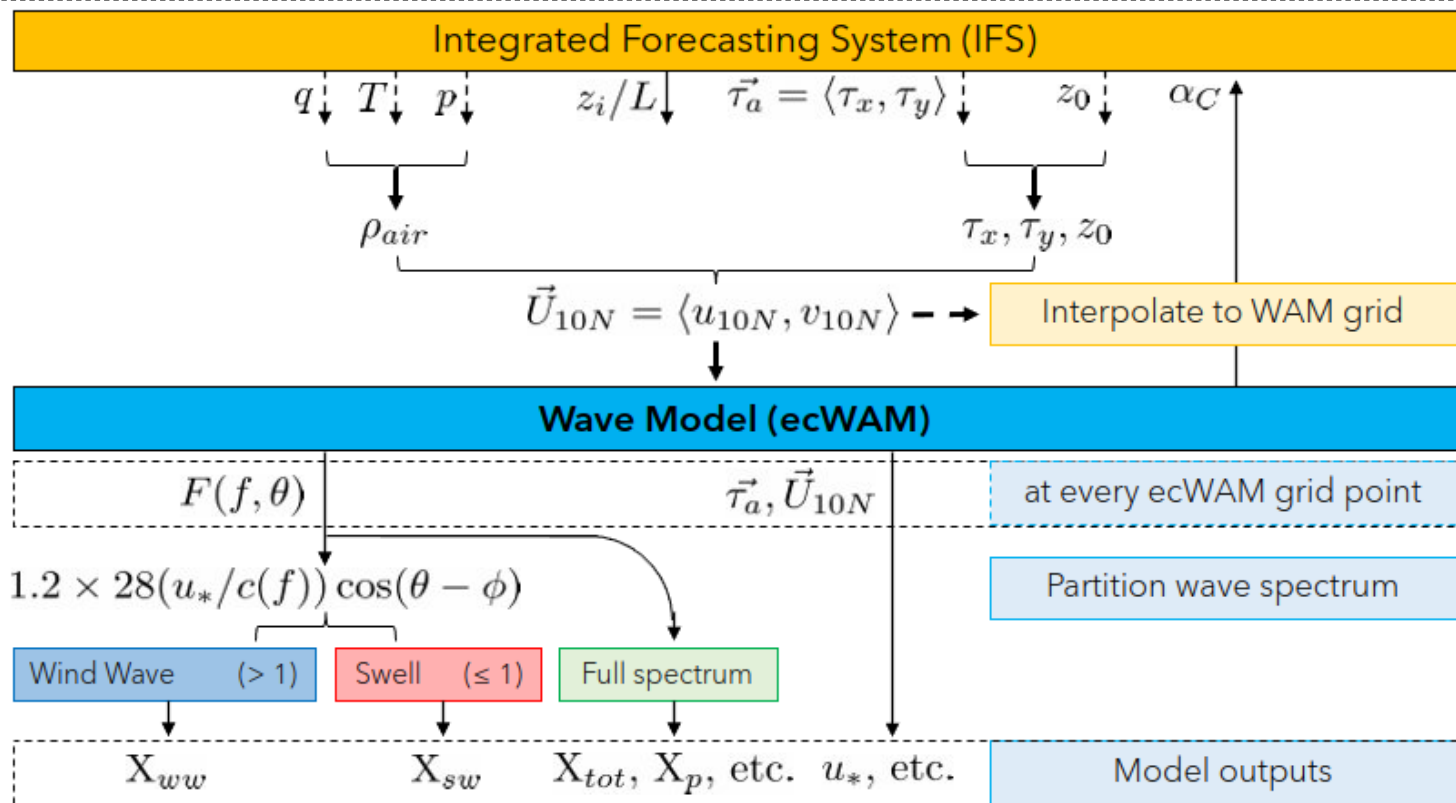
### Measurements / Methods

3

Gas transfer velocity of CO<sub>2</sub> via CO<sub>2</sub> fluxes and concentration gradients; met-ocean state variables also measured in each cruise ( $u_*$ ,  $U_{10N}$ , SST, solubility, Schmidt number)

$$k = \frac{F_{CO_2}}{\Delta C_{CO_2}} = \frac{F_{CO_2}}{S(\Delta f_{CO_2})}$$

# Spectral Wave Model: ECMWF ERA-5H ecWAM



## Model Details

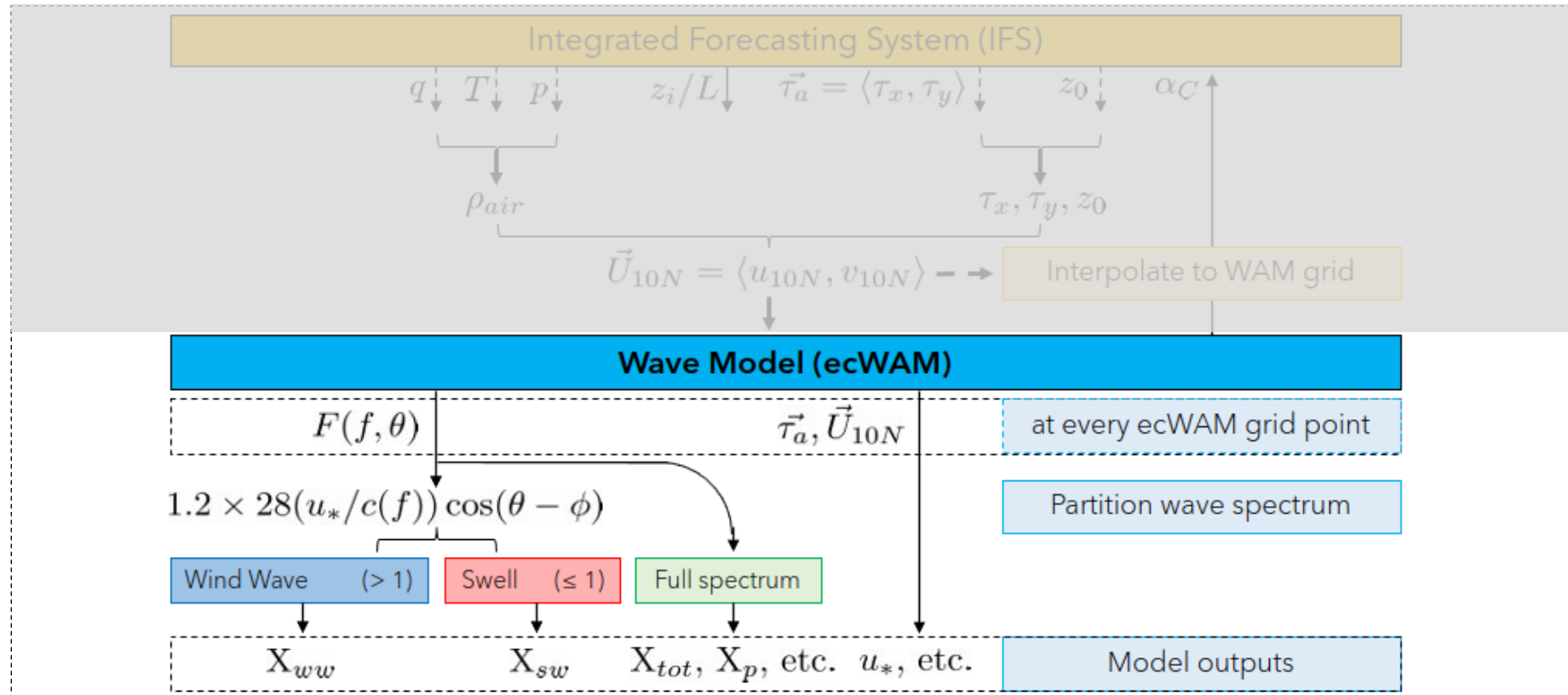
1

Global wave model forced by hourly **ERA-5** wind ( $U_{10N}$ ), surface air density, gustiness, and sea ice cover

2

**14 km × 14 km** ( $0.125^\circ \times 0.125^\circ$ ) spatial resolution  
**36** frequencies ( $f_{\min} = 0.035$  Hz) × **36** directions

# Spectral Wave Model: ECMWF ERA-5H ecWAM



## Model Details

3 4 June 2007 – 29 August 2019: 1-hour outputs

Interpolated in space and time to all cruise coordinates

4

2-D wave spectrum  $F(f, \theta)$  and total atmospheric stress  $\tau_a$  calculated at each model grid-point



# Results: Parameterising Gas Transfer Velocity

Operational (Full) and Hybrid Forms

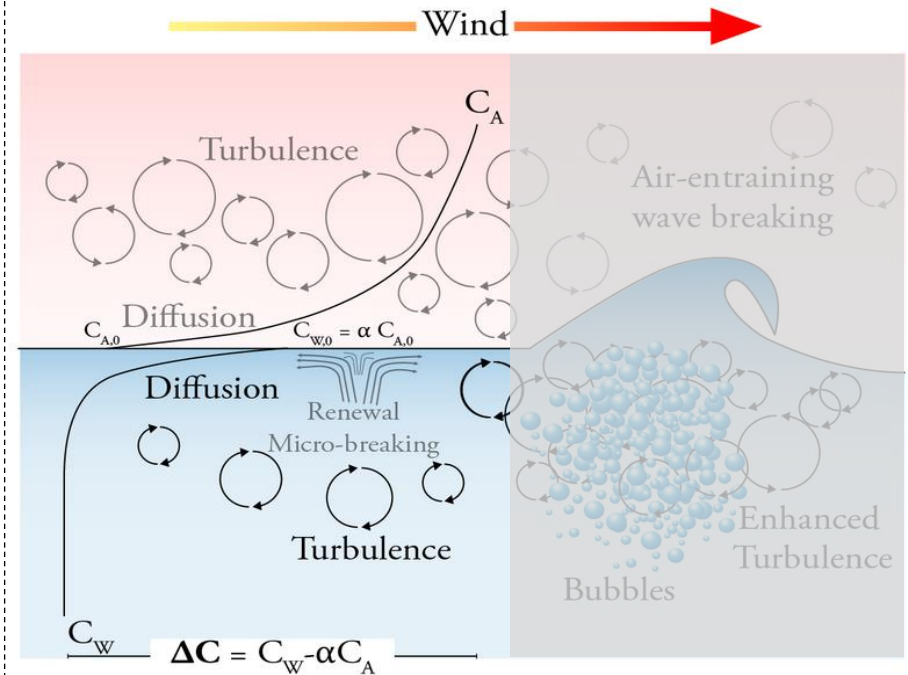
$$k = \frac{F_{CO_2}}{S(\Delta f_{CO_2})} = k_s + k_b$$

Diffusive Gas Transfer Velocity

$$k_s = \frac{D}{\delta_z} \propto [(\varepsilon\nu)^m \cdot (\frac{\nu}{D})^n] \simeq [(\varepsilon\nu)^m \cdot Sc^n]$$

Bubble-Mediated Gas Transfer Velocity

$$k_b = \frac{\int_0^\infty V(r)Q(r)E(r)dr}{\alpha} = \frac{V_b}{\alpha}$$



Source: Sophia E. Brumer

# Results: Parameterising Gas Transfer Velocity

Operational (Full) and Hybrid Forms

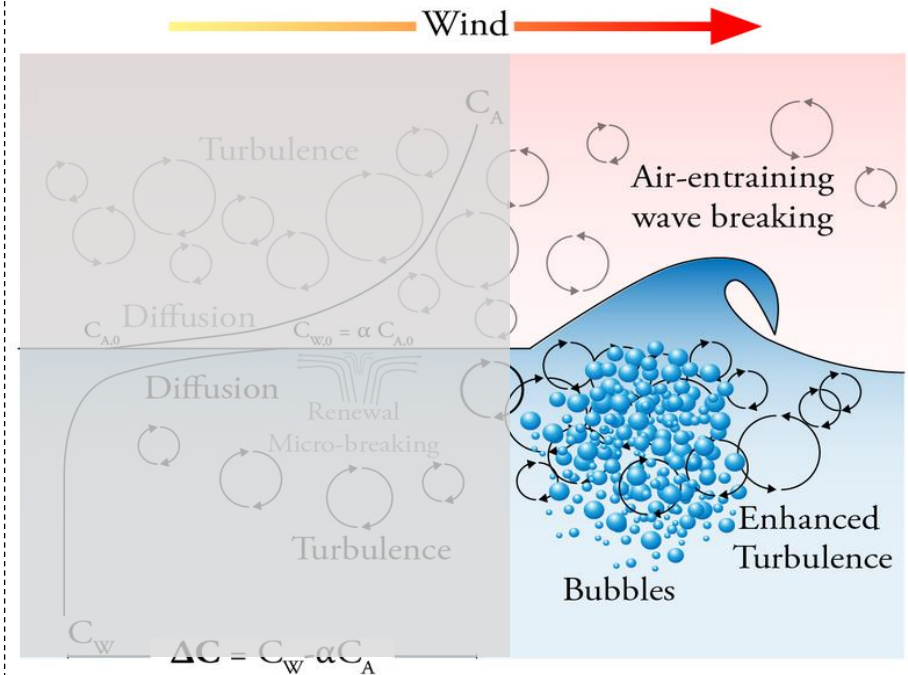
$$k = \frac{F_{CO_2}}{S(\Delta f_{CO_2})} = k_s + k_b$$

Diffusive Gas Transfer Velocity

$$k_s = \frac{D}{\delta z} \propto [(\varepsilon \nu)^m \cdot (\frac{\nu}{D})^n] \simeq [(\varepsilon \nu)^m \cdot Sc^n]$$

Bubble-Mediated Gas Transfer Velocity

$$k_b = \frac{\int_0^\infty V(r)Q(r)E(r)dr}{\alpha} = \frac{V_b}{\alpha}$$



Source: Sophia E. Brumer

## Results: Diffusive Gas Transfer Velocity

$$k_{s,param} = c_1 u_{*\nu}$$

$c_1 = 2.034 \times 10^{-4}$ , optimised constant  $[-]$  •  $u_{*\nu}$ , viscous friction velocity  $[m\ s^{-1}]$

### Atmospheric Stress Decomposition

$$\tau = \tau_\nu + \tau_f$$

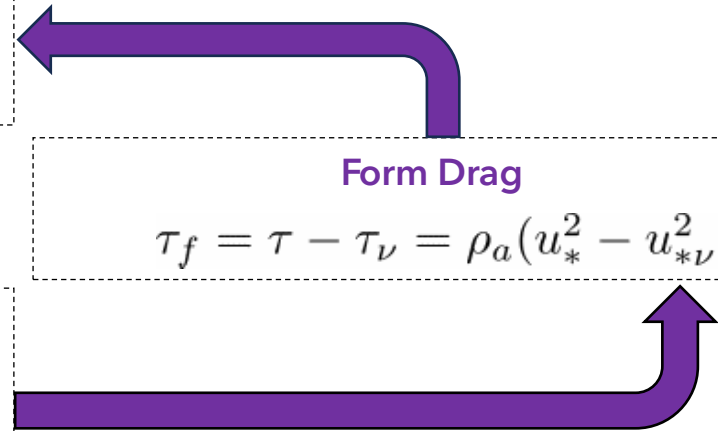
### Form Drag

$$\tau_f = \tau - \tau_\nu = \rho_a(u_*^2 - u_{*\nu}^2)$$

### Viscous Friction Velocity via Mueller & Veron (2009)

$$u_{*\nu} = c_{d\nu}^{1/2} U_{10N} = \frac{\kappa}{\log(10/z_{o\nu})} U_{10N}$$

$$z_{o\nu} = \frac{0.11\nu_a}{u_{*\nu}}$$

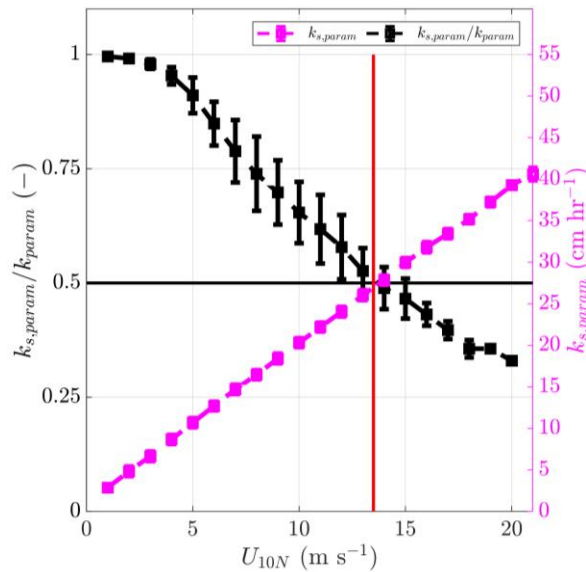


# Results: Diffusive Gas Transfer Velocity

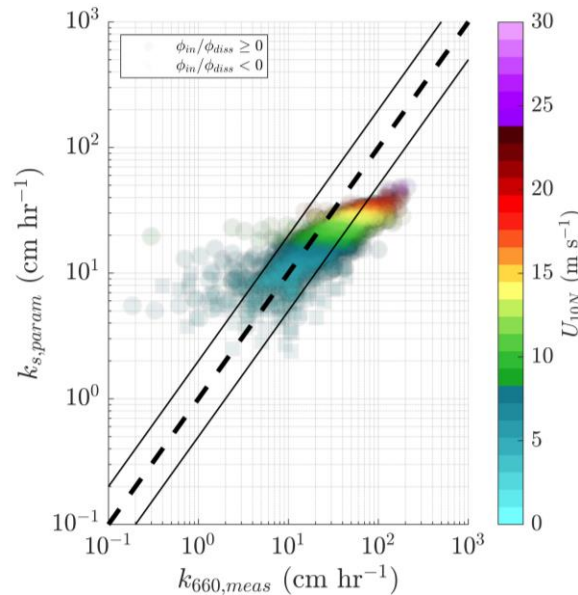
$$k_{s,param} = c_1 u_{*v}$$

$c_1 = 2.034 \times 10^{-4}$ , optimised constant  $[-]$  •  $u_{*v}$ , viscous friction velocity  $[m s^{-1}]$

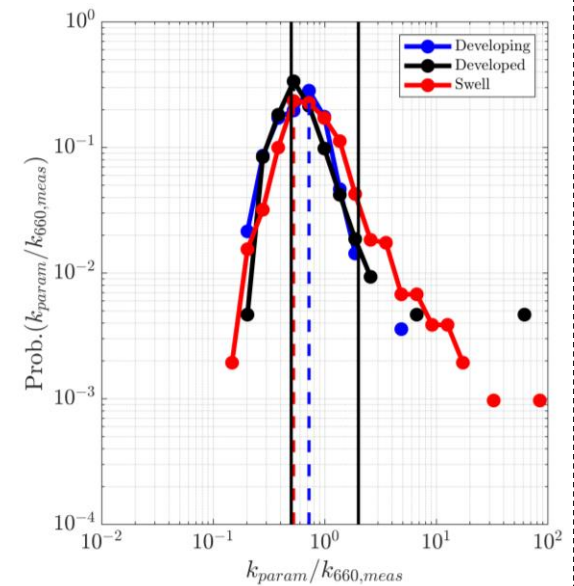
## Overview



## Log-Log vs. Measurements



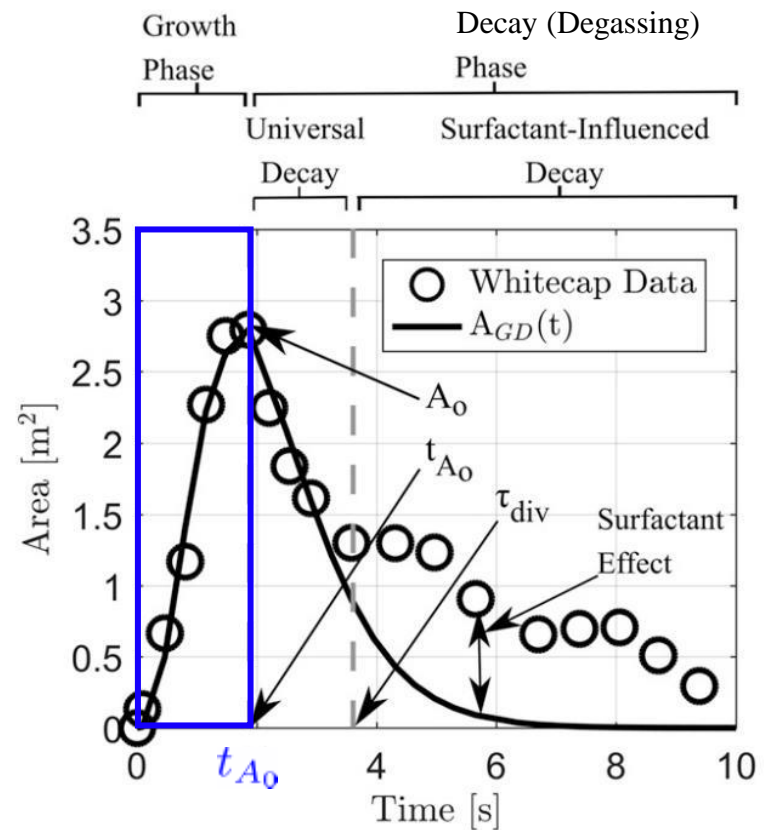
## PDFs by Wave Age



- Diffusive flux dominates gas transfer at wind speeds  $U_{10N} < 13.5 \text{ ms}^{-1}$
- $k_{s,param}$  approximately linear with  $U_{10N}$  via use of  $u_{*v}$  (Fairall, et al. 2011)
- Diffusive flux alone insufficient at wind speeds  $U_{10N} > 8 \text{ ms}^{-1}$
- Across all wave-ages, PDFs show under-estimation of  $k_{660}$  by about 41 percent.

$c_2 = 5.902$ , optimised constant  $[-]$  •  $a_{eff}$ , effective void fraction  $[-]$  •  $\overline{w}_{ent}$ , average entrainment velocity  $[m\ s^{-1}]$   
 $W_{growth}$ , growth-phase whitecap fraction  $[-]$  •  $\xi$ , volume-weighted efficiency factor  $[-]$

$$a_{eff} = \frac{\int_0^{t_{A0}} \alpha(t) V(t) dt}{\int_0^{t_{A0}} V(t) dt}$$



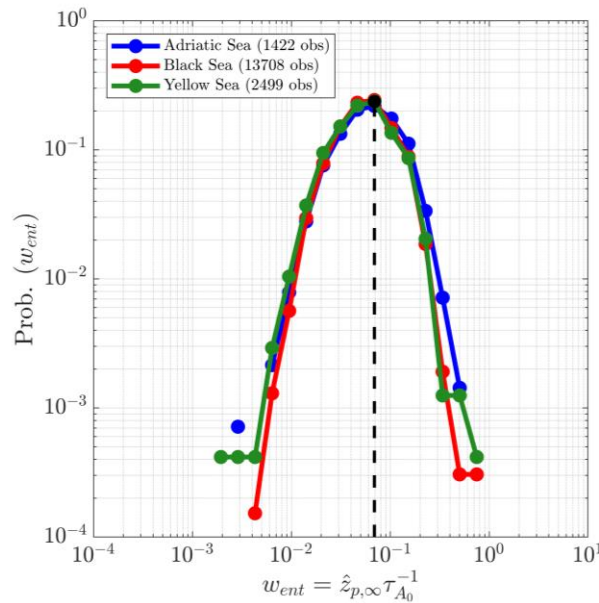


## Results: Bubble-Mediated Gas Transfer Velocity

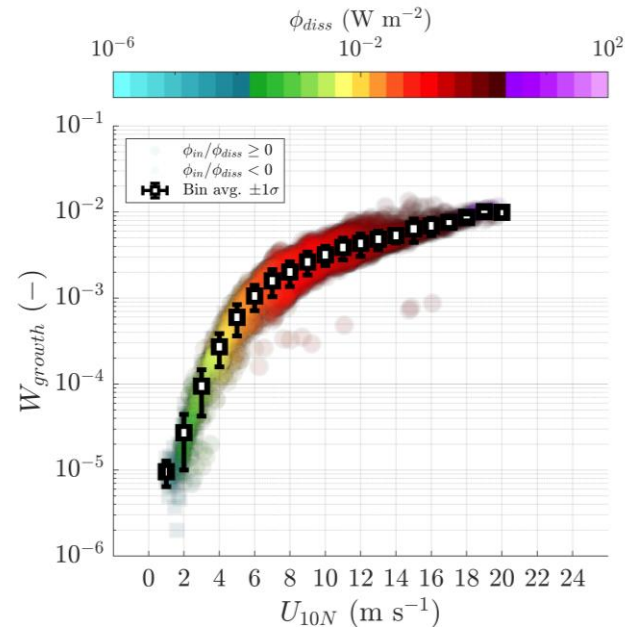
$$k_{b,param} = c_2 \frac{1}{\alpha} a_{eff} \bar{w}_{ent} W_{growth} \xi$$

$c_2 = 5.902$ , optimised constant  $[-]$  •  $a_{eff}$ , effective void fraction  $[-]$  •  $\bar{w}_{ent}$ , average entrainment velocity  $[m\ s^{-1}]$   
 $W_{growth}$ , growth-phase whitecap fraction  $[-]$  •  $\xi$ , volume-weighted efficiency factor  $[-]$

Observed  $w_{ent}$  PDFs



Modeled  $W_{growth}$



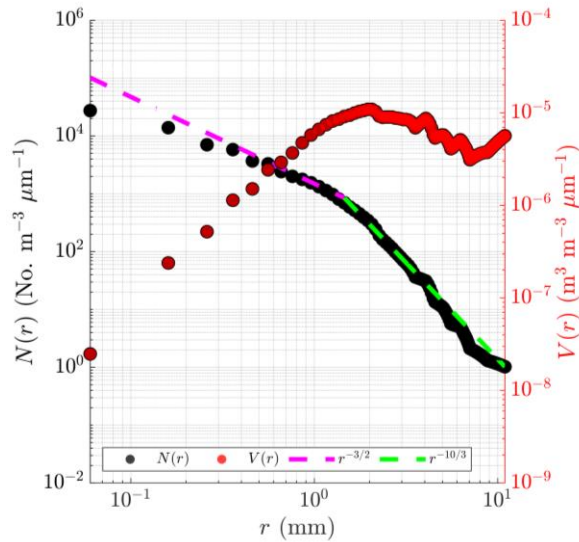
- $a_{eff} = 0.2$  choice within reported values from literature (0.012 - 0.37)
- $w_{ent} = 0.069$  from PDFs, 0.065 from MVCO, we use  $0.066\ m\ s^{-1}$
- $W_{growth}$  follows shape of observed  $W(U_{10N})$  seen in literature
- Larger wave-ocean energy flux at higher winds and slope transition near  $U_{10N} = 7\ m\ s^{-1}$

# Results: Bubble-Mediated Gas Transfer Velocity

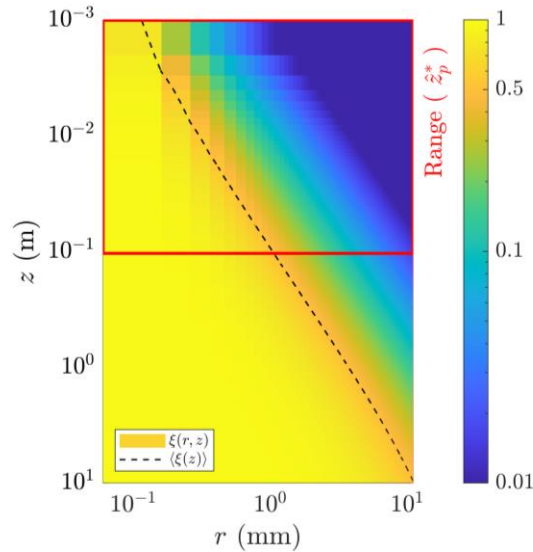
$$k_{b,param} = c_2 \frac{1}{\alpha} a_{eff} \bar{w}_{ent} W_{growth} \xi \rightarrow \xi = \frac{\int_0^\infty V(r) E(r) dr}{\int_0^\infty V(r) dr}$$

$c_2 = 5.902$ , optimised constant  $[-]$  •  $a_{eff}$ , effective void fraction  $[-]$  •  $\bar{w}_{ent}$ , average entrainment velocity  $[m\ s^{-1}]$   
 $W_{growth}$ , growth-phase whitecap fraction  $[-]$  •  $\xi$ , volume-weighted efficiency factor  $[-]$

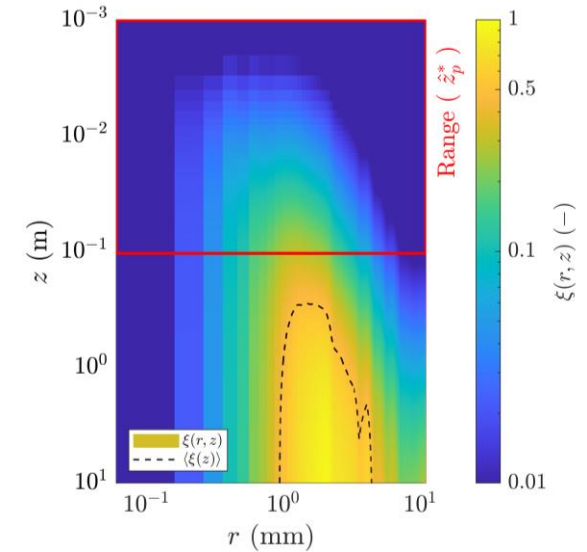
## Bubble size and volume distributions



## Bubble efficiency



## Volume-weighted Bubble efficiency

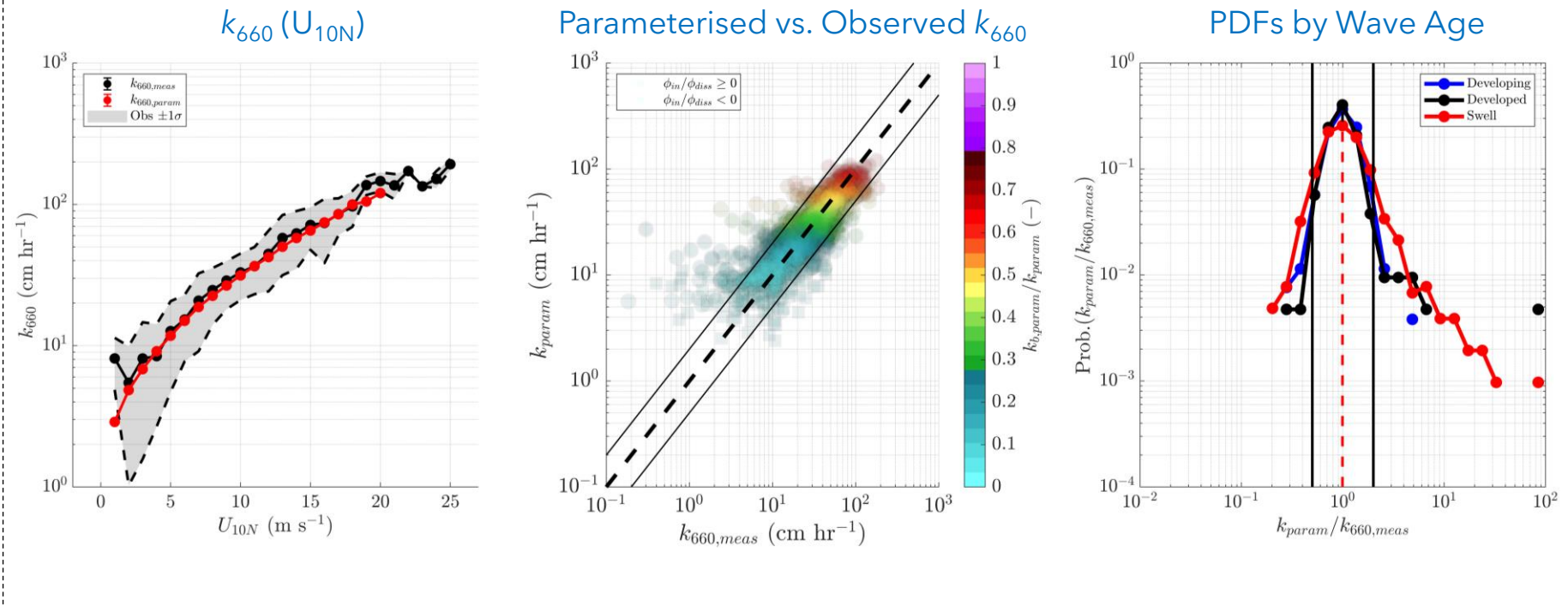


- $N(r)$ ,  $V(r)$  from Deane & Stokes (2002): large bubbles near Hinze scale contribute most
- Bubble efficiency most sensitive to  $r$  at small  $z$ ; bubbles at depth fully dissolve, efficient!
- Large bubbles fewer in number, but contribute more gas volume
- Most efficient bubbles  $r = 1\text{--}4.5$  mm, at  $z \geq 0.25$  m; but occupy just 27.1% of  $r$ - $z$  space

# Results: Total Gas Transfer Velocity

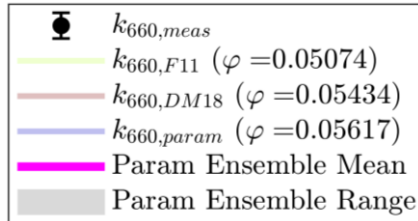
$$k_{param} = \boxed{c_1} u_{*v} + \boxed{c_2} \frac{1}{\alpha} a_{eff} \overline{w}_{ent} \left( \frac{\tau_f c_{eff} \times (c_p U_{10N}^{-1})}{\rho_w \Omega \hat{z}_p^*} \right) \xi$$

\*optimized via  $\epsilon = |k_{660, meas} - (c_1 k_{s, param} + c_2 k_{b, param})|$  such that  $c_1, c_2 = \text{fminsearch}(\epsilon)$



- Agreement between  $k_{660, meas}$  and  $k_{param}$  good, median error 8.25%,  $R^2 = 0.818$
- Largest errors:  $U_{10N} < 3 \text{ m s}^{-1}$  and  $U_{10N} \geq 19 \text{ m s}^{-1}$
- Outliers comprise 13.49% of the data (outside 0.5x-2x solid black lines)
- PDFs narrower for wind sea (•, •) vs swell (•), although peak for all wave ages is 1

# Results: Comparing Total Gas Transfer Velocity from Literature

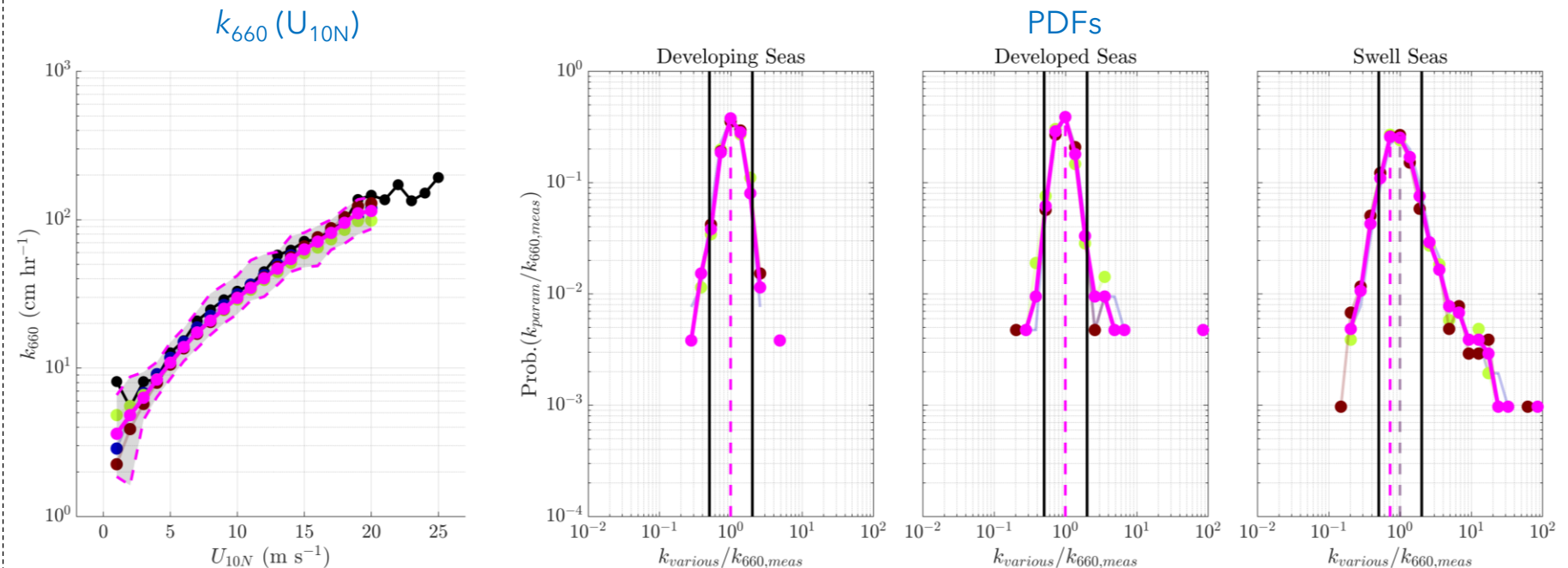


- 1  $k_{SC23} = c_1 u_{*v} + c_2 \frac{1}{\alpha} a_{eff} \bar{w}_{ent} \left( \frac{\tau_f c_{eff} \times (c_p U_{10N}^{-1})}{\rho_w \Omega \bar{z}_p^*} \right) \xi$
- 2  $k_{F11} = 37.5 A \psi u_{*v} + \frac{BV_o(T)}{\alpha(20)} f_{wh}(u_{*f})$
- 3  $k_{DM18} = A_{NB} u_* + A_B \frac{1}{\alpha} \left[ u_*^{5/3} \sqrt{g H_s}^{4/3} \right]$

Smith, et al. (2023; in prep.)

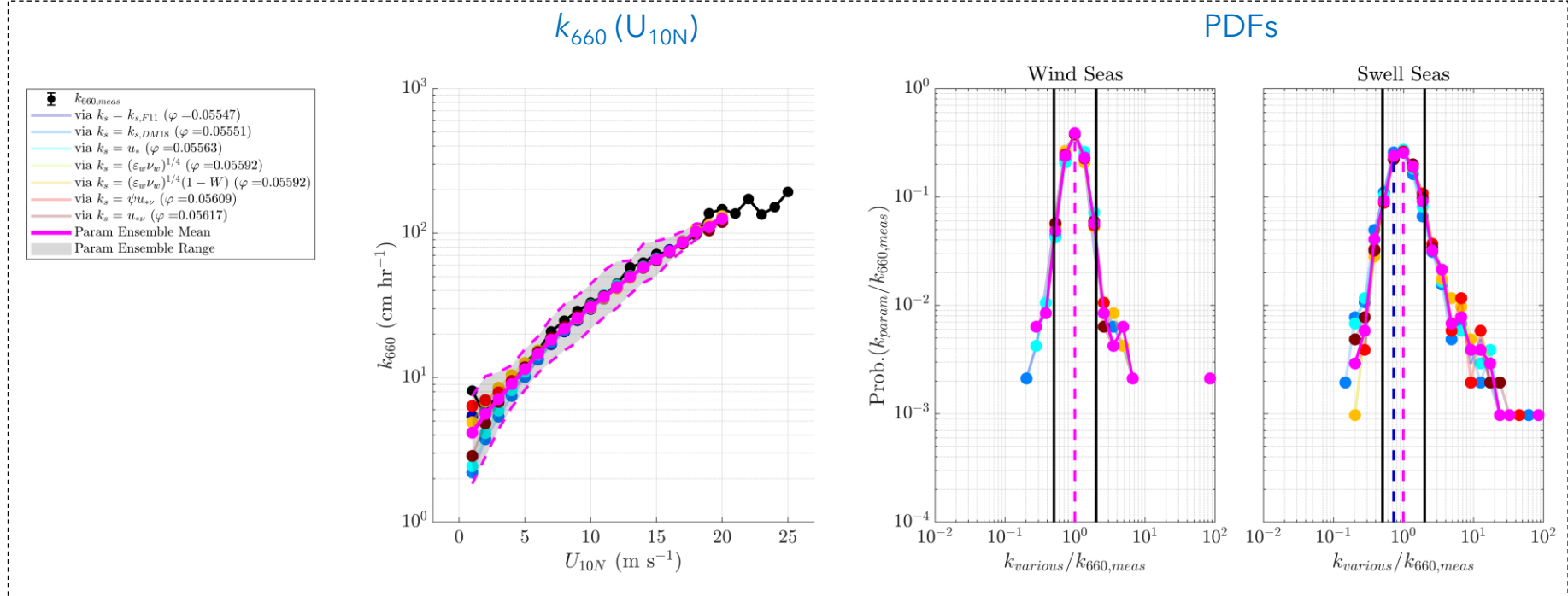
Fairall, et al. (2011)

Deike and Melville (2018)



- $\varphi = R^2/RMSE$ : SC23 outperforms F11 (by 9.7%) and DM18 (by 3.3%)
- Ensemble range larger at  $U_{10N} \leq 4$  ms<sup>-1</sup>, swell PDF shows low-wind over-estimation
- Additional spread emerges at  $U_{10N} > 12$  ms<sup>-1</sup> ... swell underestimation by F11
- All parameterisations perform well where wave spectrum peak is wind-sea

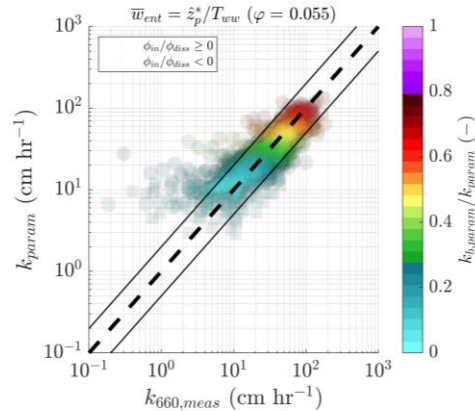
# Results: Sensitivity to Diffusive Gas Flux Parameterisation



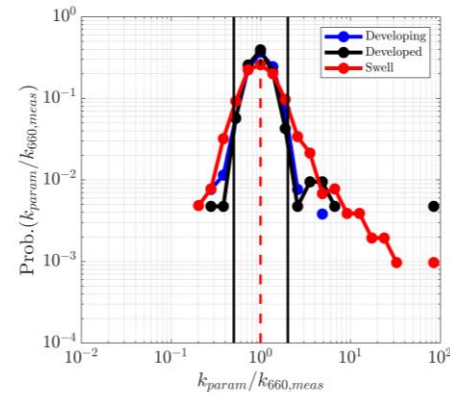
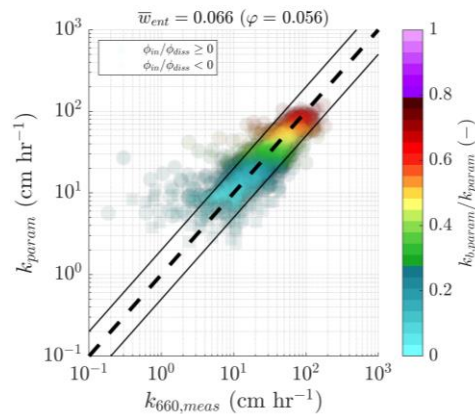
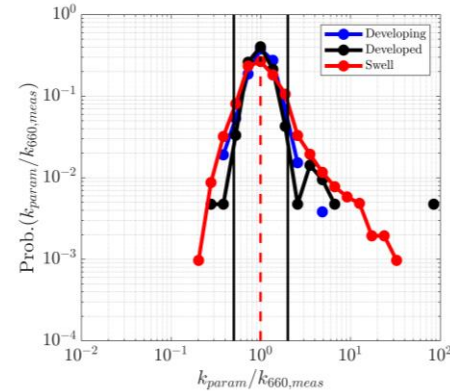
- Only 1.3% difference between worst-performing, best-performing via  $\varphi$
- Greatest improvement: using  $(\epsilon_w \nu_w)^{1/4}$  vs.  $u_*$ , but dependent on which  $k_b$  you use
- Largest spread in ensemble at very low winds
- Ensemble convergence around  $U_{10N} = 12 \text{ ms}^{-1}$



# Results: Sensitivity to Entrainment Velocity Choice

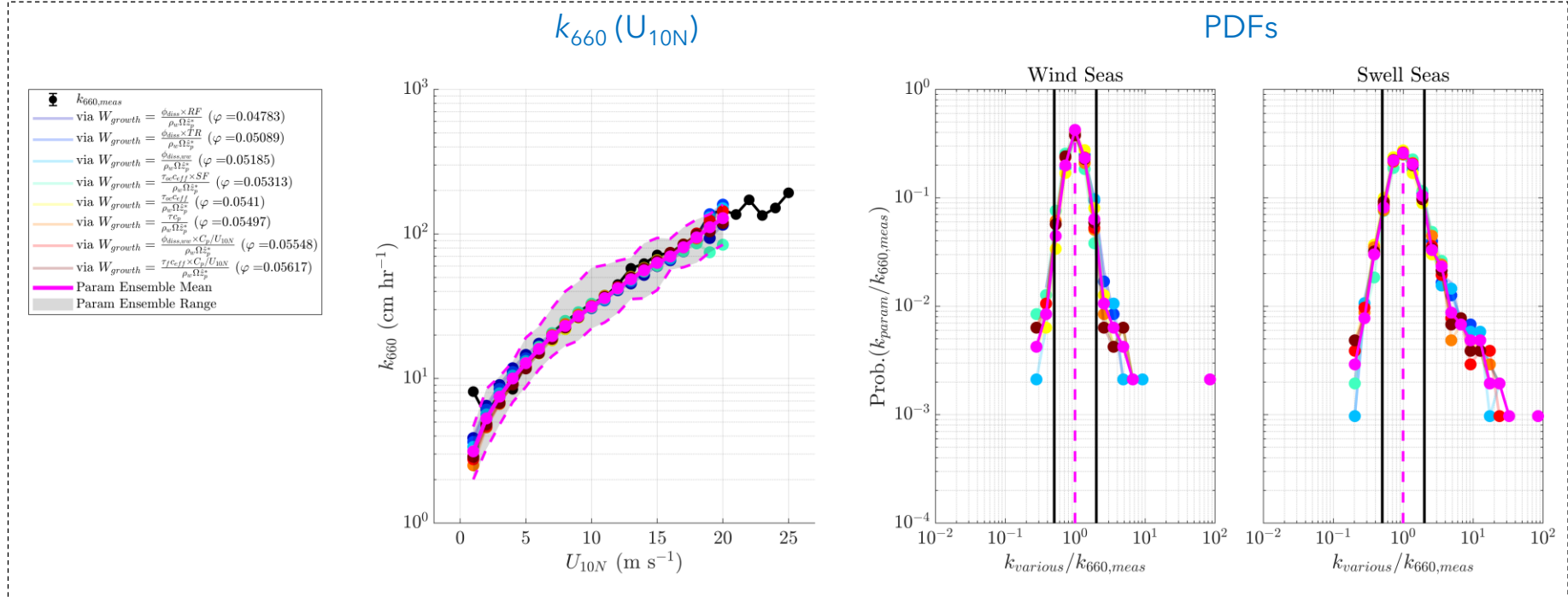
Parameterised vs. Observed  $k_{660}$ 

PDFs



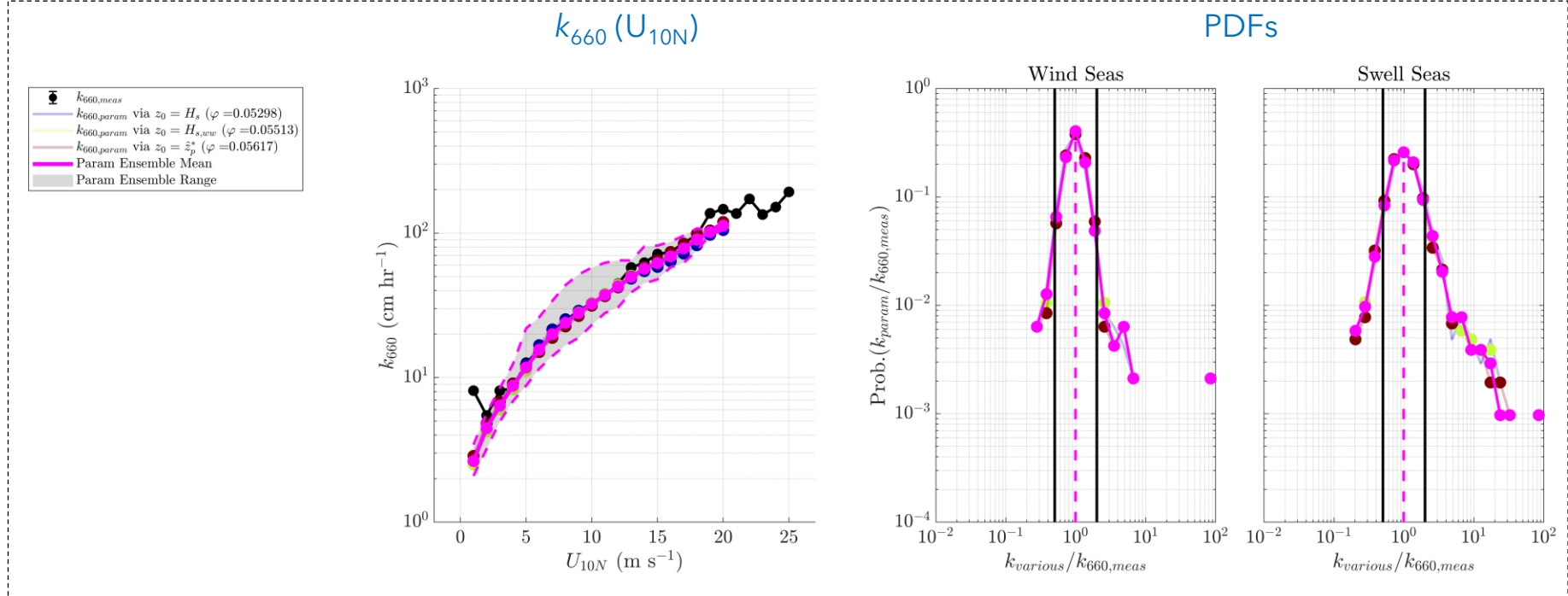
- Entrainment velocity either way almost visually identical and according to  $\varphi$
- Using constant value reduces low-wind swell over-estimation somewhat
- Don't have to worry about the variability of  $z_p$  or  $T_{ww}$  by using constant

# Results: Sensitivity to Growth-Phase Whitecap Fraction Parameterisation



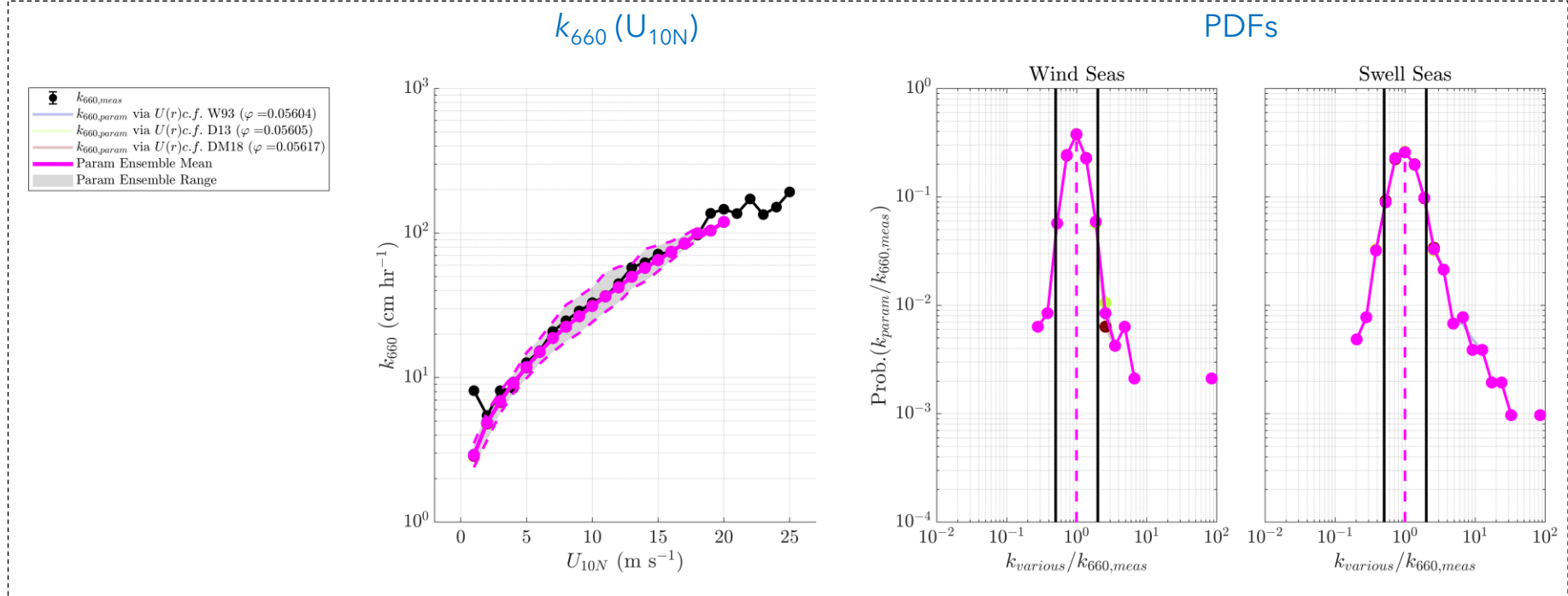
- Choice of  $W_{growth}$ : 17.4% difference between worst-performing, best-performing via  $\phi$
- Over-estimation of  $k$  at low winds using full  $\phi_{diss}$  or omitting wave-age scaling in  $W_{growth}$
- All similar for  $U_{10N} = 7-12$  ms<sup>-1</sup>; sample-size issue - very large spread at  $U_{10N} \geq 18$  ms<sup>-1</sup>
- Most impactful choice

# Results: Sensitivity to Initial Bubble Injection Depth in Bubble Efficiency



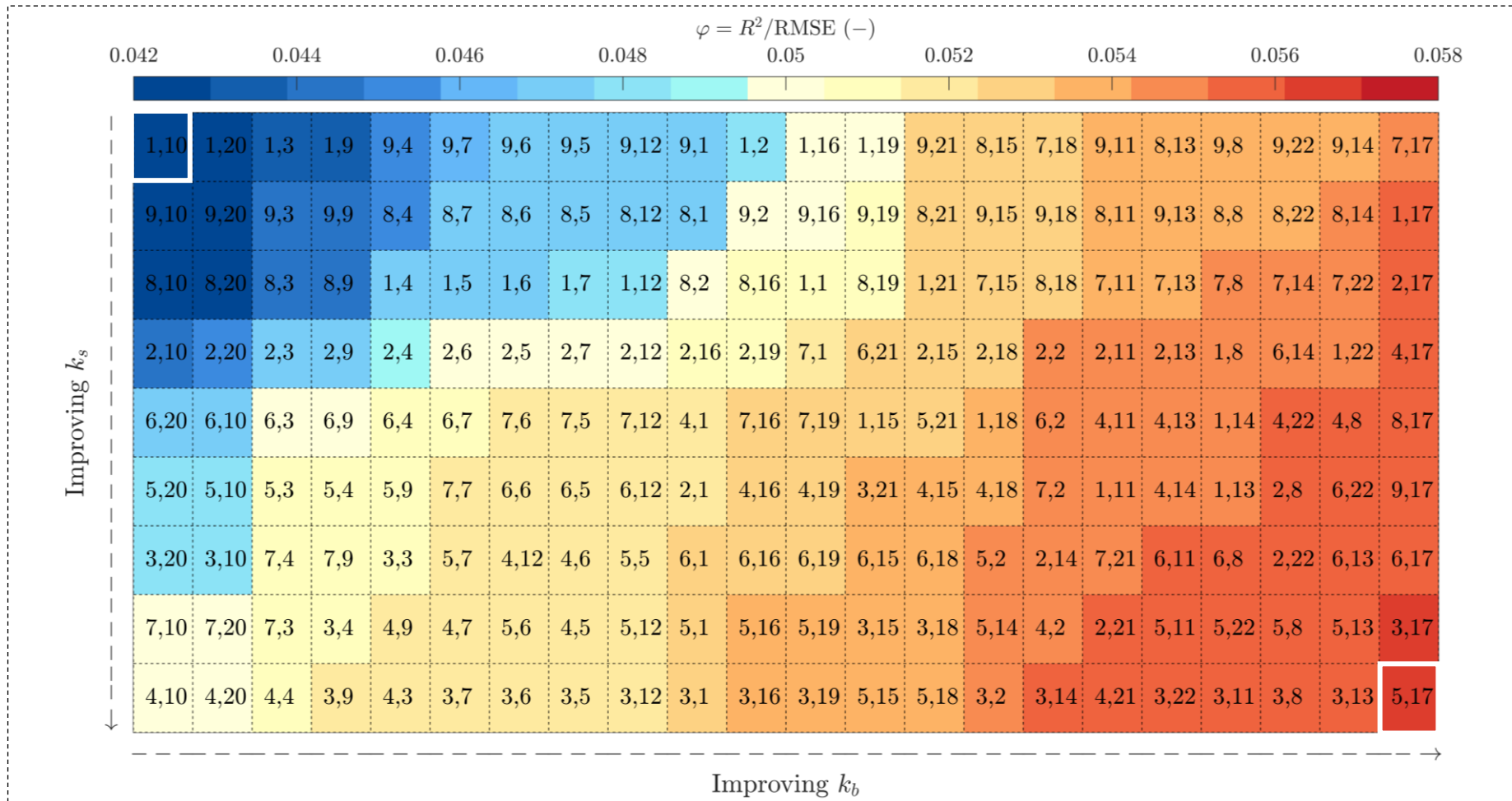
- $z_0$  in E(r) : 6% difference between worst-performing, best-performing via  $\varphi$
- $H_s$  over-estimates at low winds and under-estimates at high winds
- Bubble injection depth critically tied to wind-wave spectrum, higher-frequency waves
- Here, the choice is easy

# Results: Sensitivity to Bubble Rise Velocity Parameterisation



- Only 0.2% difference between worst-performing, best-performing via  $\varphi$
- Bubble of constant radius, terminal velocity, quiescent liquid, clean or dirty (W93)
- Bubble of constant radius, terminal velocity, turbulent liquid, dirty (D13, DM18)
- Least numerically impactful choice, but considering surfactants physically important

# Results: Total $k$ Performance Matrix



- Worst-performing overall:

$$k = 37.5 A \psi u_{*v} + c_2 \frac{1}{\alpha} a_{eff} [\bar{w}_{ent} \neq 0.066] [W_{growth} = \frac{\phi_{diss} \times (\frac{H_{s,ww}}{H_{s,tot}})^2}{\rho_w \Omega_p^*}] \neq$$

- Best-performing overall:

$$k = c_1 u_{*v} + c_2 \frac{1}{\alpha} a_{eff} [\bar{w}_{ent} \neq 0.066] [W_{growth} = \frac{\tau_f c_{eff} \times (c_p U_{10N}^{-1})}{\rho_w \Omega_p^*}] \neq$$



## Conclusions

- 1 Diffusive gas transfer velocity inadequate by itself to parameterise  $k$
- 2 Constant effective void fraction & entrainment velocity are reasonable to implement in bubble-mediated gas transfer velocity parameterisations
- 3  $W_{\text{growth}}$  is the most important physical quantity to get correct
- 4 Bubble rise velocity least impactful choice to parameterisation performance
- 5 Optimised comparison of 198 parameterisations shows partition of stress in hybrid  $k$  parameterisations performs very well

# Additional Links

## Presentations

1

Investigating gas exchange processes using noble gases in very high winds: observations of bubbles and turbulence beneath breaking waves

Andrew W. Smith<sup>1</sup>, Brian K. Haus<sup>1</sup> and Rachel H.R. Stanley<sup>2,3</sup>  
<sup>1</sup>University of Miami / RSMAS <sup>2</sup>Woods Hole Oceanographic Institution <sup>3</sup>Wellesley College



Postdoctoral Fellow Interview, April 19, 2024 | University of Rhode Island, Narragansett, RI

THE  
UNIVERSITY  
OF RHODE ISLAND

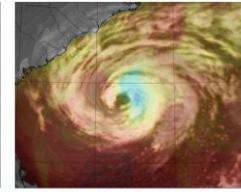
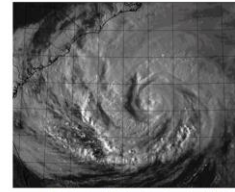


Investigating gas exchange processes using noble gases in very high winds: observations of bubbles and turbulence beneath breaking waves

2

The role of atmosphere-ocean coupling in the structure and intensity evolution of Hurricane Ophelia (2005)

Andrew W. Smith<sup>1</sup>  
<sup>1</sup>University of Miami Rosenstiel School of Marine and Atmospheric Sciences



Postdoctoral Fellow Interview, April 19, 2024 | University of Rhode Island, Narragansett, RI

THE  
UNIVERSITY  
OF RHODE ISLAND



The role of atmosphere-ocean coupling in the structure and intensity evolution of Hurricane Ophelia (2005)

## Associated Publications and Manuscripts

Smith, A. W., Haus, B. K., & Stanley, R. H. R. (2022). Bubble-turbulence dynamics and dissipation beneath laboratory breaking waves. *Journal of Physical Oceanography*, 1, 2159–2181. <https://doi.org/10.1175/JPO-D-21-0209.1>

Stanley, R. H. R., Kinjo, L., Smith, A. W., Aldrett, D., Alt, H., Kopp, E., Krevanko, C., Cahill, K., & Haus, B. K. (2022). Gas fluxes and steady state saturation anomalies at very high wind speeds. *Journal of Geophysical Research: Oceans*, 1–19. <https://doi.org/10.1029/2021jc018387>

Smith, A. W. (2016). The role of air-sea interaction in structure and intensity change in Hurricane Ophelia (2005): Coupled modeling and RAINEX observations. [https://scholarship.miami.edu/esploro/outputs/graduate/The-Role-of-Air-Sea-Interaction-in/991031447708902976?institution=01UOML\\_INST](https://scholarship.miami.edu/esploro/outputs/graduate/The-Role-of-Air-Sea-Interaction-in/991031447708902976?institution=01UOML_INST)