Bidding Heuristics for Thermostatic Load Aggregation

Alexander Yee

***Abstract*—I explore the various control heuristics that a central controller, such as a load aggregator, can employ in order to optimally manage its thermal load portfolio while respecting temperature constraints. I simulate 10,000 thermostatically controlled loads (TCL) with varying building characteristics of which a load controller has complete control of its HVAC system. I use a Model Predictive Controller (MPC) as well a modified Least Laxity First (LLF) heuristic that generate real-time control action using forecasted weather data, forecasted pricing data and current building temperatures. I show that the MPC heuristic is superior to the LLF heuristic, although both are much superior to the no demand response option from a cost perspective. However, both have significant gaps in both costs and temperature violations with the theoretical optimal load scheduling.**

I. Introduction

We are in the midst of a technological upheaval in the power sector. The rise of renewable integration, the widespread use of Information and Communication Technology (ICT) and a spotlight on the new "big" data that it provides have brought many obstacles but they have also opened new doors. Demand response has been made possible through a combination of these factors and it continues to grow and evolve since it offers much value to the grid.

For small thermal loads, such as residences, it is often the case where transaction costs (monitoring price signals or otherwise) are too high to warrant response to grid conditions. However, residential space cooling and heating represent a significant portion of electricity demand due to the shear number of them. We have seen the entrance of a middleman, a load aggregator, who offers to control multiple loads to offer the grid the value added service of demand response at a magnitude that is sufficient to warrant attention in the market. In addition, coordinated control of aggregated resources offers additional value since it offers the opportunity for significant load matching to variable generation.

Previous literature on TCLs have looked at similar problems. Mathieu et al takes a control system approach from both a decentralized perspective as well as a centralized control perspective in [1]. They demonstrate through the use of a proportional controller with the goal of minimizing ON/OFF cycling while using state (temperature) estimation instead of real-time measurement.

Subramanian et al have written on scheduling heuristics in the context of EVs in [2]. They investigate the use of the Earliest Deadline First and Least Laxity First (LLF) decision algorithms which are both originally Processor Time Algorithms. In addition, they consider the use of a Receding Horizon Control(RCF) which is a form of Model Predictive Control. They also prove that *Causal Optimal* scheduling policies do not exist; in essence, this means that one cannot generate optimal decisions with only predictive data for the future. As a result, heuristic must be use to generate decisions which hope to be as close to the optimal solution as possible.

II. Model Formulation

In this paper, I incorporate the control plant model by Mathieu et al found in [1] with some of the heuristics employed by Subramanian et al in [2]. Specifically, I simulate TCLs through an RC-model, but the control employed uses either a modified LLF or MPC heuristic.

A. General Parameters: Time Scale, Exogenous Data and Forecasts

I ran multiple simulations each spanning 24 hours with 1 hour time steps. The model also used two sets of exogenous data: the ambient temperature $T_{amb}(h)$ and electricity prices $P(h)$. The temperature data was procured from the NOAA at a station in Durham, New Hampshire (North). The hourly electricity price data was calculated as the average price of the LMPs across the nodes under the jurisdiction of the New England ISO (ISO-NE). The day used was September 9, 2015 when temperatures reached 38 degrees Celsius and electricity prices peaked at $240 per MWh.

In order to generate forecasted temperatures $\hat{T}^{\mu}_{amb}(h)$, I synthetically added Gaussian noise. Forecasted temperatures are generated by the following formula:

$$\hat{T}^{\mu}_{amb}(h) = T_{amb}(h) + \sum_{n=\mu}^{h} \epsilon_{n,T}, \mu = 1, 2..., 24$$

$$\epsilon_{n,T} = \mathcal{N}(0, \sigma^2_{n,T})$$

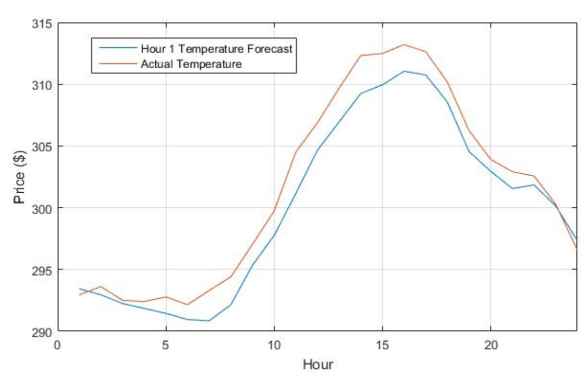

Fig. 1. Sample Temperature Forecast Data

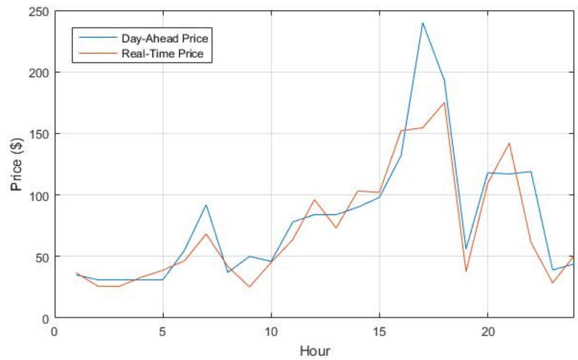

Fig. 2. Price Forecast Data

where σ_n^2 is the historical standard deviation of the difference between forecasted value and the actual value. h is the hour that is being forecasted and μ is the hour that the forecast was made. This creation of synthetic noise was used in [2]. Figure 1 shows the difference between the actual temperature and the temperature forecast in hour 1.

Price forecasts $\hat{P}(h)$ are also generated using synthetic Gaussian noise, but there is no time-dependent error and the error is proportional to the actual price. The price forecasts are a stand-in for the day-ahead hourly prices which are set once and have little time interdependencies.

$$\hat{P}(h) = P(h) \times (1 + \epsilon_P(h)),$$

$$\epsilon_P(h) = \mathcal{N}(0, \sigma_{n,P}^2)$$

As the prices are exogenously set, this assumes that the loads have no effect on the market and are purely price-takers. Figure 2 shows the actual price and the day-ahead price.

B. Thermostatically Controlled Load

TCLs are modeled as a thermal mass which losses (or gains) heat naturally to its environment which is dependent on the temperature difference between the interior temperature and the ambient (external) temperature. I am applying the analogy of an electrical circuit with resistance and capacitance where energy transfer is analogous to current, temperature difference is akin to voltage potential and insulation is similar to resistance. Heat can be removed (or added) against the natural flow through HVAC systems that use electricity. Fundamentally, I am applying the law of conservation of energy to a thermal mass:

$$\Delta Energy = E_{loss/gain} + E_{HVAC}$$

In each hour, TCLs are characterized with one state variable (temperature) and one decision variable (apply cooling if needed). Each TCL state is independent of every other TCL state. Within TCL i during hour h, the state variable of interior temperature $T_{int}^i(h)$ is calculated as a discrete state time model. The decision variable $x^i(h)$ is under the control of the load aggregator for the entire day. The equation is adapted from [3] which uses a similar state time model except where the decision variable is a binary decision (on or off). I justify the use of a continuous variable since I am using a timestep of an hour as opposed to a much smaller timestep: continuity over such a large timestep is achieved by the ability of a load to cycle its equipment over the hour (e.g. a value at 50% of its capacity would merely mean that the equipment is on for half of the hour).

Parameter	Units	Distribution
c, Thermal Capacitance	$[Wh/°Cm^2]$	$U(15,65)$
r, Thermal Resistance	$[kW/°Cm^2]$	$U(333,1000)$
F, COP*	[]	$U(1.5,3.5)$
A, Floor Area	$[m^2]$	$U(150,350)$
x_{max}^i, Max Power	$[kW]$	$U(10,18)$
$T_{max}^{i,b1}(h)$, Max Temp	$[°C]$	$U(20,25)$
$T_{min}^{i,b1}(h)$, Min Temp	$[°C]$	$U(16,19)$
$T_{max}^{i,b2}(h)$, Max Temp	$[°C]$	$U(22,30)$
$T_{min}^{i,b2}(h)$, Min Temp	$[°C]$	$U(12,16)$

$$T_{int}^i(h+1) = T_{int}^i(h) + a^i(T_{amb}^h - T_{int}^i(h)) - a^i R^i F^i x^i(h)$$

$$a^i = e^{-\frac{\tau}{R^i C^i}}$$

The interior temperature of the building in the next hour $h+1$ is a function of the current state and the decision to turn on the cooling. a^i is a dimensionless coefficient which effectively represents the quality of the building's insulation; a low coefficient represents very good insulation whereas a coefficient approaching 1 represents very little insulation capability. a^i is calculated as an exponential function of the timestep τ (1 hour), the thermal resistance R^i and the thermal capacitance C^i of the building i. F^i represents the coefficient of performance of the HVAC system.

*COP: Coefficient of Performance

1) TCL Parameters: Each building in reality will have its own unique combination of parameters; as such I simply use a range of empirically accepted values and randomize each TCL's parameters using a uniform distribution over that range. I also vary the size of the TCL floor areas; thermal resistance R and thermal capacitance are functions of random variables r and c, respectively, as well as floor area A. The maximum power that the HVAC unit can draw is also randomized through a uniform distribution. The table above contains the information on the ranges of values these parameters can take.

$$R^i = Ar^i$$

$$C^i = Ac^i$$

In addition, there are temperature profiles that must be set by each user that describe the allowable temperatures in each hour $T_{max}^i(h)$ and $T_{min}^i(h)$. I created two blocks of hours (6pm-9am - b1 and 10am-5pm - b2) that had uniform maximums and minimums, but the actual values are generated on a uniform distribution like all other parameters.

C. Bidding Process and Demand Response

I require the load aggregator to 'declare' or bid into the market the amount of demand that it wishes to schedule in each hour. While the prices are not affected by these quantity, the aggregator is constrained by its declaration or the Demand Response constraint imposed by the grid operator (the lower of the two). I assume that the aggregator knows about the

Demand Response action at the beginning of the day and can plan accordingly. The aggregator is allowed to change its demand bids or 'declarations' up to 2 hours prior to consumption (representing the closure of the bidding window). As a result, a load aggregator's maximum consumption is determined 2 hours prior to consumption although real-time exogenous factors are not yet known. These form the crux of my problem: 1) how much should a load aggregator declare in the two hour ahead time frame and 2) how should a load aggregator dispatch its loads in real-time to meet these declarations.

III. Algorithms and Heuristics

A. Optimal Load Dispatch

I formulate the optimal dispatch if the load aggregator had perfect forecasting for both temperature and weather. This is used as a reference and baseline for the heuristics that will be employed later. The linear programming formulation of the optimal dispatch is below:

$$min \sum_{h=1}^{24} \sum_{i \in I} (P(h)x^i(h) + G_{over}T^i_{over}(h) + G_{under}T^i_{under}(h))$$

subject to

$$\begin{array}{lll}
(1) & T^i_{int}(0) = T^i_0 & \forall i \\
(2) & T^i_{int}(h) - T^i_{over}(h) \leq T^i_{max}(h) & \forall i, h \\
(3) & T^i_{int}(h) + T^i_{under}(h) \geq T^i_{min}(h) & \forall i, h \\
(4) & T^i_{int}(h+1) = T^i_{int}(h) + a^i(T_{amb}(h) & \forall i, h \\
 & \quad -T^i_{int}(h)) - a^i R^i F^i x^i(h) & \\
(5) & x^i(h) \leq x^i_{max} & \forall i, h \\
(6) & \sum_{i \in I} x^i(h) \leq X(h) & \forall h
\end{array}$$

In the objective function, I impose a arbitrarily large penalty G for every degree centigrade of temperature that is outside of the threshold $[T^i_{max}(h), T^i_{max}(h)]$. Constraint (1) sets the initial state of the system (the interior temperatureT^i_{int}) and is defined as the threshold maximum temperature of that time. Constraints (2) and (3) determines the total temperature violation that might exist; if G is sufficiently large, then a temperature violation will only occur if there is no feasible solution where all $T^i_{int}(h)$ stay within the bounds of $T^i_{max}(h)$ and $T^i_{max}(h)$. This results in a non-zero $T^i_{under}(h)$ or $T^i_{over}(h)$ which applies the penalty in the objective function.

Constraint (4) is the state equation presented in the previous section which ensures the intertemporal temperature relationships are held. Constraint (5) restricts the total power drawn from any building i by its capacity. Constraint (6) represents the total allowed electricity demand by the aggregator $X(h)$ and represents the demand response cap when it is needed. $X(h)$ will generally be the total electricity capacity of all buildings $\sum_{i \in I} x^i_{max}$ except in the hours that demand response is required by the grid.

B. Model Predictive Control

The MPC is a linear programming control that is formulated in a very similar way as the optimal load dispatch except the exogenous values are the forecasts; the MPC is run 24 times (1 for each hour) where each run is one hour forward from the previous run. The MPC model runs are linked through constraint (7) which sets the boundary conditions as the temperature set by the previous MPC run.

For MPC run $j \in 1..24$

$$min \sum_{h=j}^{24} \sum_{i \in I} (\hat{P}^j(h)x^i(h) + G_{over}T^i_{over}(h) + G_{under}T^i_{under}(h))$$

subject to

$$\begin{array}{ll}
(7) T^i_{int}(j-1) = T^i_{j-1} & \forall i \\
(8) T^i_{int}(h) - T^i_{over}(h) \leq T^i_{max}(h) & \forall i, h \in j..24 \\
(9) T^i_{int}(h) + T^i_{under}(h) \geq T^i_{min}(h) & \forall i, h \in j..24 \\
(10) T^i_{int}(h+1) = T^i_{int}(h) + a^i(\hat{T}^j_{amb}(h) & \forall i, h \in j..24 \\
\quad -T^i_{int}(h)) - a^i R^i F^i x^i(h) & \\
(11) x^i(h) \leq x^i_{max} & \forall i, h \in j..24 \\
(12) \sum_{i \in I} x^i(h) \leq \min(X(h), \sum_{i \in I} x^i_{j-2}(h)) & \forall h \in j..24
\end{array}$$

Also note in constraint 12, the total electricity constraint $X_{j-2}(h)$ is the minimum between the exogenously determined Demand Response and as the demand declaration by the load aggregator in timestep $j-2$.

C. Least Laxity First

The LLF heuristic determines the latest time it can completely neglect resources and at which point it must service them at the maximum rate in order to meet its constraint. In other words, it determines the time at which the "do nothing" option is no longer available which I will call the *deferrable deadline*. Typically, LLF is used for independent tasks each with singular deadlines which yields a single *deferrable deadline* for each task. However in the case of loads, we have independent tasks with multiple deadlines. I consider every hour a deadline to meet the temperature requirement of that hour although actions taken to meet the other hourly deadlines will no doubt effect this. In order to adapt LLF, I search for the hour that has the strictest (earliest) *deferrable deadline* where I must service the load.

To determine the *deferrable deadline* ϕ of hour η starting in hour h, I generate a function $F^i_h(s)$ which is the temperature of the load starting at hour h until hour η where no cooling is performed. Next, I generate second function $F^i_\eta(s)$ which looks backward from hour η to hour h and this represents the load as if it had the cooling system on at its maximum capacity. *deferrable deadline* ϕ is the rounded down hour that these two function intersect.

$$\phi^i(h, \eta) = floor[\arg_s(F^i_h(s) = F^i_\eta(s))], \forall i, \eta \in h..24$$

Next, I search amongst the buildings for the earliest deferrable deadline and service them first. I repeat this process until there is no longer any capacity remaining. The capacity is determined by running the MPC model and determining the total capacity that would be optimal in a system. However, unlike the MPC model, the real-time dispatch decisions of loads is not governed by the predicted optimal allocation, but based on the laxity of the loads.

IV. Results and Discussion

Overall, both MPC and LLF heuristics achieve considerable demand response: over 10,000 TCLs each averaging 14 kW in size, I show in Figure 3 that achieve almost 80 MW in demand reduction in hour 18 and around 25 MW in the adjoining hours. This is done by intelligently shifting demand to earlier in the day when it is both cooler and the electricity price is cheaper. The TCLs use their heat storage capability to maintain temperatures within acceptable conditions. I also show that the majority of demand response is caused by demand shifting as a reaction to prices as the demand response cap is tight for only 1 hour (hour)

I find that both the MPC and LLF heuristics are prone to severe error relative to the optimal dispatch due to the difference between in the day-ahead price and real-time price previously shown in Figure 2. Notably, Figure 3 shows that both the MPC and LLF models scheduled a significant amount of load in hours 4 and 10, whereas the optimal schedule had consumption spikes in hours 6, 9 and 11.

In terms of avoiding temperature breaches, neither MPC nor LLF do a perfect job. As shown in Figure 4, the optimal schedule has no temperature breaches (i.e. incurs no temperature penalties due to having insufficient capacity) 0°C. However, both MPC and LLF have temperature breaches due to suboptimal scheduling; there are suboptimal scheduling decisions that are made in prior hours based on predictions that differ from the real-time values. These improper decisions force them into situations where either:

1) they have a tight energy constraint due to demand response caps which was the case in hour 18,
2) they underestimated their energy requirement in $j - 2$ hours and therefore their declared energy consumption was too low to allow them to meet their temperature obligation, or
3) individual TCLs are in a position where their HVAC capacity is insufficient to meet their temperature obligation.

I hypothesize that, other than hour 18, that situation 2 was the more likely issue rather than situation 3. I point towards Figure 1 which shows that temperature forecasts were consistently under the actual temperature; as a result loads were consistently taking in more heat than was anticipated and as a result could not cool themselves sufficiently to meet their temperature obligation given their declared commitment. This time lag coupled with a persistent forecast error meant that the system was consistently underestimating its consumption needs which meant that its demand declaration 2 hours prior to consumption was constantly insufficient to meet the heat load.

This points for a need for a more stochastic model that will take into account the chance that the temperature will deviate from its predicted path. Another approach would be to add an arbitrary factor of safety in the demand declarations so that buildings are able to consume more in real-time than what is anticipated. Yet another method to solving this issue of temperature breaches is to cool the buildings more aggressively during low cost hours. This would mean forcing minimum loads or decreasing the maximum temperature thresholds artificially to ensure that the heuristic is more conservative in its reliance on future hours for cooling.

In terms of energy, the least energy solution remains the option with no demand response: demand response in this case causes TCLs to store cooled air for some non-trivial time. There is a transactional energy cost (heat loss) of storing and as a result the act of purchasing energy during lower cost hours requires more energy than meeting cooling demand as it arises. However since there are large price differentials, it makes economic sense to purchase energy during lower cost hours albeit the quantities purchased will be larger. However if the price differentials were not as large, it is likely that there would be less of a demand response due to price shifting. In a hypothetical case where the price differentials are small relative to the energy losses, the demand response exogenous cap would become more important in ensuring sufficient demand response and it would be likely this exogenous cap would be the binding constraint. The energy consumption of all of the demand response options are very similar, but all consume about 50% more energy than the no demand response option.

In terms of total costs, the demand response options all show significant savings compared to the scenario with no demand response. While consumers are not exposed to the price, they are insensitive to wild price spikes like the ones that occurred on this day. However, the costs to the system are real and they are spread across all hours and consumers as a result. I show that any of the demand response heuristics demonstrate significant savings during days with significant price differentials. In this scenario, the MPC and LLF methods save 49% and 45%, respectively, of the total cost of the no demand response scenario. However, both involve the intangible decrease in comfort to building occupants. However, the optimal solutions shows that if the load aggregator was omniscient, it could lower its costs further and with no temperature violations as the MPC and LLF methods were 11% and 20% more expensive than the optimal load dispatch. The optimal dispatch is unlikely to be ever be achieved, so it is likely the load aggregator can either spend more money to reduce temperature breaches or achieve a lower cost through acceptance of more temperature breaches.

A. *MPC vs. LLF*

I show that the MPC method of allocating load dispatches based on the predictive model rather than load laxity is superior through lower costs and lower temperature violations (by two orders of magnitude). The issue with LLF likely lies in its focus on deadlines causes it to overcommit to units that have early deadlines while undercommitting to units with later deadlines. This does not affect the short term, but has cascading effects in the long term. Figure 4 shows that the majority of temperature breaches occurred later in the day as the loads that LLF neglected could no longer fulfill their deadline. The MPC approach considers this issue and allocates load based on its predicted need in the future. It is likely the additional temperature breaches in LLF emerged from situation 3 described above.

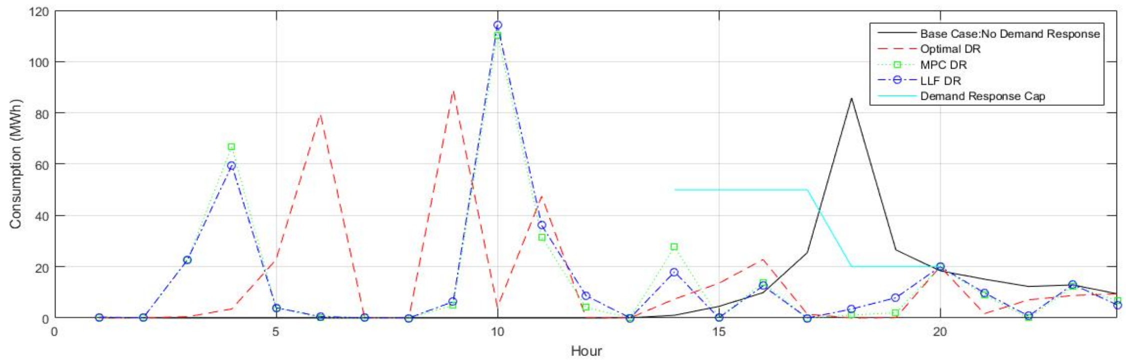

Fig. 3. Total Electricity Consumption in Each Scenario

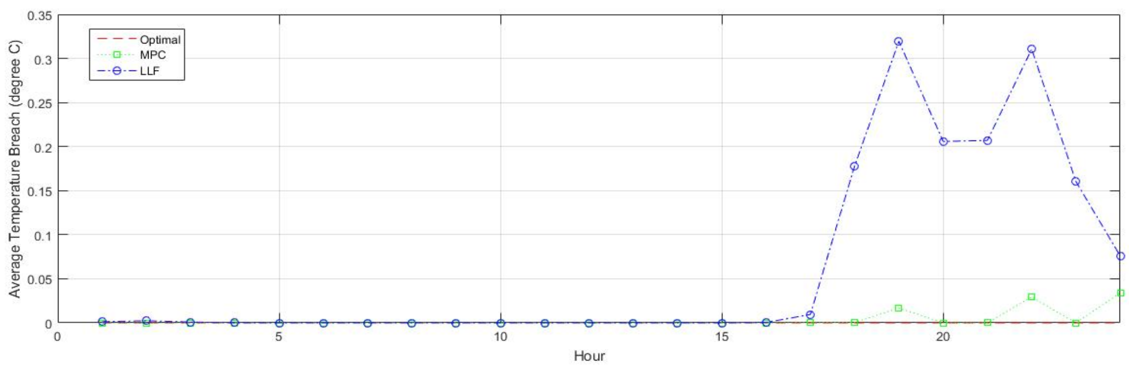

Fig. 4. Average Temperature Breach in Each Scenario

Scenario	Total Cost($)	Energy Consumed (MWh)	Average Temperature Breach over day (° C hr)
No Demand Response	36,709	222	0
Optimal DR	16,858	340	0
MPC DR	18,647	338	0.08
LLF DR	20,225	343	1.47

V. Conclusion

My models have shown that a MPC and LLF approach to demand response show significant savings when compared to the no demand response option. However, they both are deficient in cost and temperature violation when compared to the theoretical optimal demand response schedule. MPC offers significant empirical advantages over LLF in both costs and temperature violations. Further research on a stochastic MPC heuristic should be conducted to determine its gains (if any) in reduced temperature breaches. It should be determined if this solution is strictly better in terms of costs and temperature breaches or if there exists a trade-off. Finally, research should be conducted of the overall performance of both heuristics and the proposed stochastic heuristic across all types of days instead of just exceptional days.

Acknowledgment

I would like to acknowledge the help of the various professors who have imparted the essential knowledge for this paper: Dr. Carlos Batlle, Dr. Javier Garcia-Gonzales, Dr. Andres Ramos and Dr. Pablo Rodilla. The same goes for the teaching assistant, Michael Davidson, who's clarifications were succinct. Special thanks also goes to my classmates who endured my disturbing late entrances to class as well as specifically to Jordan and Turner who kept me company when writing this paper. Finally, a huge thank you to Dr. Claudio Vergara for organizing and teaching this course and without him this course would not have run.

References

[1] J.L. Mathieu, S. Koch, D.S. Callaway, *State Estimation and Control of Electric Loads to Manage Real-Time Energy Imbalance*. IEEE TRANSACTIONS ON POWER SYSTEMS, 2013.

[2] A. Subramanian, M. Garcia, A. Dominguez-Garcia, D. Callawayx, K. Poolla, P. Varaiya, *Real-time Scheduling of Deferrable Electric Loads*. Presented at American Control Conference, 2012.

[3] D.S. Callaway *Tapping the energy storage potential in electric loads to deliver load following and regulation*. Energy Conversion and Management, 2009.

Alexander Yee received his B.A.Sc. in Engineering Science as an Energy Systems Major from the University of Toronto in 2014. Previously, he has worked as a market analyst at the Independent Electricity System Operator of Ontario, Canada. He is currently pursuing an S.M. from the Massachusetts Institute of Technology in the Technology and Policy Program. In addition, he works as a Research Assistant at the MIT Energy Initiative.