Project 4

Alex Xu

I. Jacobian Ellipsoid

Each frame of the robot is defined as:

$$T_1^0 = \begin{bmatrix} \cos(\theta_1) & -\sin(\theta_1) & 0 & 0 \\ \sin(\theta_1) & \cos(\theta_1) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_2^1 = \begin{bmatrix} \cos(\theta_2) & -\sin(\theta_2) & 0 & l_1 \\ \sin(\theta_2) & \cos(\theta_2) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_{tool}^2 = \begin{bmatrix} 1 & 0 & 0 & l_2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Using the explicit method, the Jacobian is:

$$J_{\text{exp}} = \begin{bmatrix} -l_2 s_{1+2} - l_1 s_1 & -l_2 s_{1+2} \\ l_2 c_{1+2} + l_1 c_1 & l_2 c_{1+2} \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 1 & 1 \end{bmatrix}$$

For shorter expressions, c_{number} and s_{number} refer to cosine and sine of the angles in the subscript. To make the Jacobian matrix square, we only consider the first 2 rows. Thus:

$$A = JJ^T = \begin{bmatrix} l_2^2 s_{1+2}^2 + (l_2 s_{1+2} + l_1 s_1)^2 & -0.5 s_{2t_1} l_1^2 - s_{2t_1+t_2} l_1 l_2 - s_{2t_1+2t_2} l_2^2 \\ -0.5 s_{2t_1} l_1^2 - s_{2t_1+t_2} l_1 l_2 - s_{2t_1+2t_2} l_2^2 & l_2^2 c_{1+2}^2 + (l_2 c_{1+2} + l_1 c_1)^2 \end{bmatrix}$$

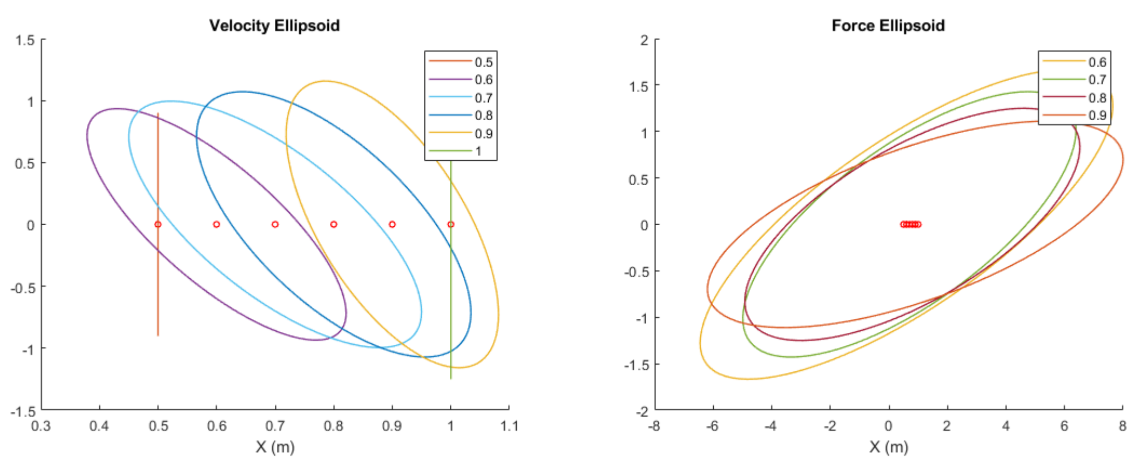

Fig. 1: Velocity and Force Ellipsoids for $l_1 = 0.25m$ and $l_2 = 0.75m$

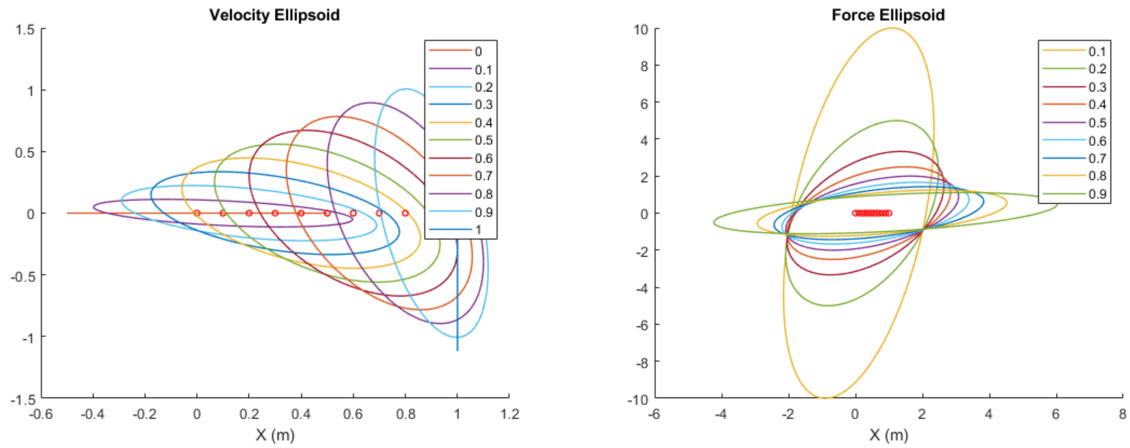

Fig. 2: Velocity and Force Ellipsoids for $l_1 = 0.5m$ and $l_2 = 0.5m$

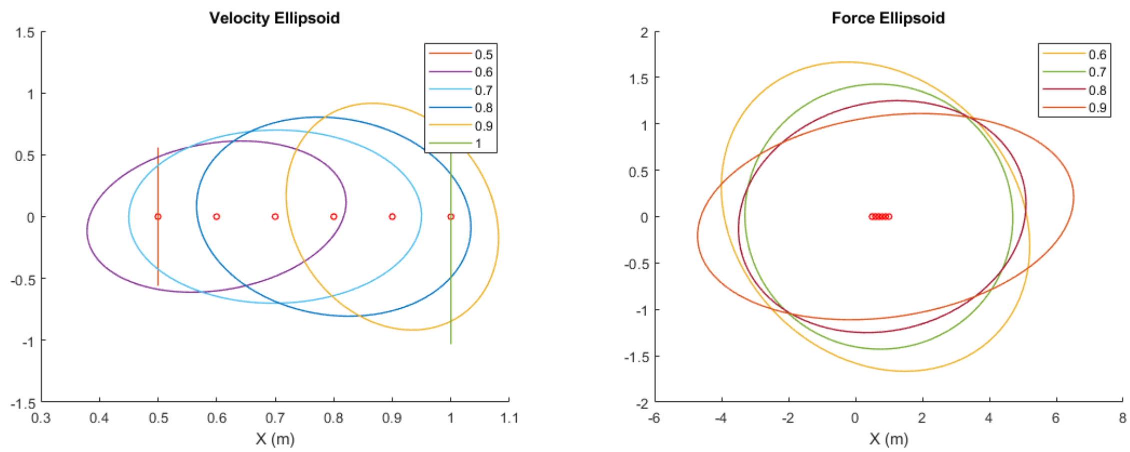

Fig. 3: Velocity and Force Ellipsoids for $l_1 = 0.75m$ and $l_2 = 0.25m$

A. Best Pose

The best pose is where the ellipsoid is large and circular as this means that the robot can move in any direction with great velocity. To quantify this, we can use:

$$manipulability = \frac{\sqrt{\lambda_{max}}}{\sqrt{\lambda_{min}}} \geq 1 \tag{3}$$

Where a smaller manipulability value is good, and an infinite manipulability is a singularity.

Based on Eq. (3), the best position for Fig. 1 is 0.7 m with $manipulability = 5.85$. For Fig. 2 it is 0.4 m with $manipulability = 1.62$. For Fig. 3 it is 0.7 m with $manipulability = 2.80$

B. Relationship Between Velocity and Force Ellipsoid

The velocity and force ellipsoids are opposites, where the major axis of the velocity ellipsoid is parallel to the minor axis of the force ellipsoid. This can be hard to observe sometimes as the scale between the velocity and force ellipsoids can differ by a lot, but in singularity cases, the velocity ellipsoid is usually a vertical line and the force ellipsoid is a horizontal line.

II. Design - Arm Optimization

Fig. 4 shows the values of the goal function for every combination of l_1 and l_2 between 0 and 500 mm.

A. Optimal Combination

The max value of the goal function is $C = 1.53*10^{-6}$ which corresponds to link lengths of $l_1 = 300mm$ and $l_2 = 210mm$.

For the worst combination, we ignore the values where the goal function is equal to 0 as these combinations are where the robot cannot reach all the points in the task space. The smallest non-zero value of the goal function is $C = 2.08 * 10^{-8}$ which corresponds to link lengths of $l_1 = 310mm$ and $l_2 = 500mm$. Fig. 5 shows the value of k for the best and worst configurations plotted over the task space.

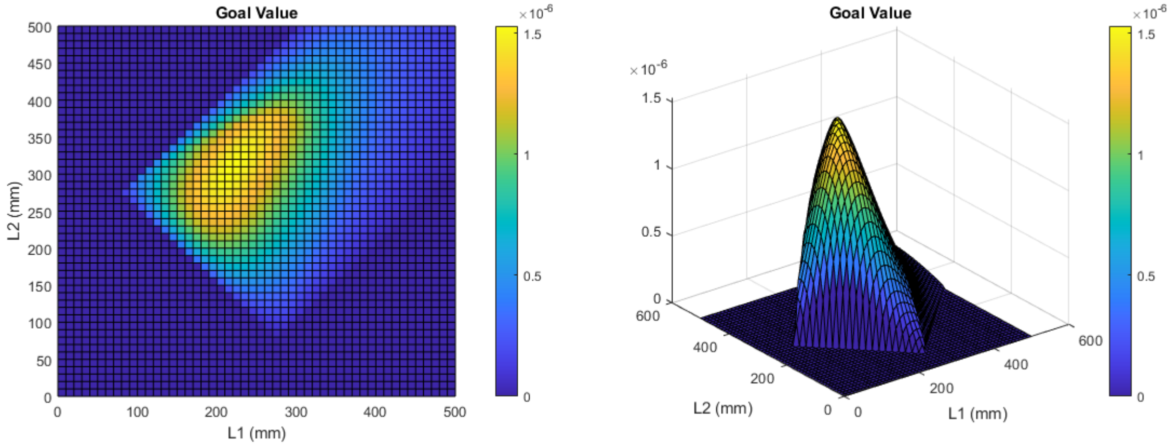

Fig. 4: Goal Function for $l_1 = [0, 500mm]$ and $l_2 = [0, 500mm]$

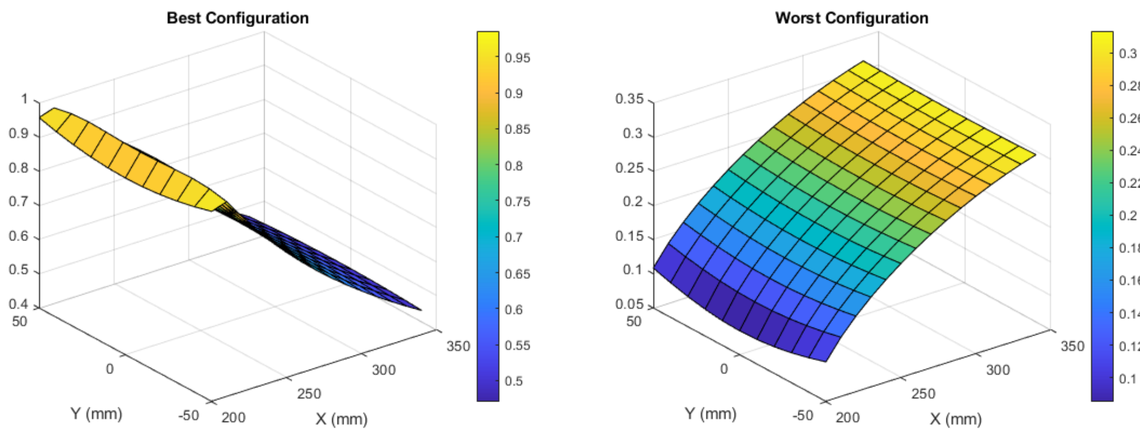

Fig. 5: Best and Worst Combinations' k Values over Task Space

The position with the best manipulability is when k is highest, and the position with the worst manipulability is when k is lowest. For the best configuration, the points with the highest manipulability are when x = 210 mm and the points with the worst manipulability is when x = 350 mm. The values of k do not differ significantly along the y-axis.

For the worst configuration, the situation is reversed, which shows that the two configurations have symmetrical manipulabilities along the X-Z plane.

B. *Singularity*

The points closest to a singularity have a small eigenvalue. For the best configuration, the point closest to a singularity is $(340, \pm 50)$ with an eigenvalue of $2.94 * 10^4$. For the worst configuration, the point closest to a singularity is $(200, 0)$ with an eigenvalue of $2.086 * 10^3$.

C. *Arm Selection*

The optimal lengths were $l_1 = 300mm$ and $l_2 = 210mm$. Based on the listing from Mitsubishi, the closest configurations are the RH-3FRH5515 and RH-3FRH5512C models with link lengths of $l_1 = 325mm$ and $l_2 = 225mm$. The RH-6FRH5520/C/M and RH-6FRH5534/C/M also have the same link lengths.