Project 2

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I. DH Parameters

The model of the robot arm and gripper is a SCARA Mitsubishi Arm - Model RH-3FRH5515 and a Yamaha YRG-4220W. The DH parameters used for the robot are described in Table I.

i-1	i	a (mm)	α	d (mm)	θ
0	1	0	0	400	θ_1
1	2	325	0	0	θ_2
2	3	225	0	0	θ_3
3	4	0	0	d	0
4	tool	0	0	30	0

TABLE I: DH parameters

II. Jacobian

Each frame of the robot is defined as:

$$T_1^0 = \begin{bmatrix} \cos(\theta_1) & -\sin(\theta_1) & 0 & 0 \\ \sin(\theta_1) & \cos(\theta_1) & 0 & 0 \\ 0 & 0 & 1 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_2^1 = \begin{bmatrix} \cos(\theta_2) & -\sin(\theta_2) & 0 & a_1 \\ \sin(\theta_2) & \cos(\theta_2) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_3^2 = \begin{bmatrix} \cos(\theta_3) & -\sin(\theta_3) & 0 & a_2 \\ \sin(\theta_3) & \cos(\theta_3) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_4^3 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_4 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_4^0 = \begin{bmatrix} c_3(c_1c_2 - s_1s_2) - s_3(c_1s_2 + c_2s_1) & -c_3(c_1s_2 + c_2s_1) - s_3(c_1c_2 - s_1s_2) & 0 & a_2(c_1c_2 - s_1s_2) + a_1c_1 \\ c_3(c_1s_2 + c_2s_1) + s_3(c_1c_2 - s_1s_2) & c_3(c_1c_2 - s_1s_2) - s_3(c_1s_2 + c_2s_1) & 0 & a_2(c_1s_2 + c_2s_1) + a_1s_1 \\ 0 & 0 & 1 & d_1 + d_4 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

For shorter expressions, c_{number} and s_{number} refer to cosine and sine of the angles in the subscript.
Using the explicit method, the Jacobian is:

$$J_{\text{exp}} = \begin{bmatrix} -a_2s_{1+2} - a_1s_1 & -a_2s_{1+2} & 0 & 0 \\ a_2c_{1+2} + a_1c_1 & a_2c_{1+2} & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 \end{bmatrix}$$

Using the velocity propagation method we get:

$$\begin{bmatrix} v_x \\ v_y \\ v_z \\ \omega_x \\ \omega_y \\ \omega_z \end{bmatrix} = \begin{bmatrix} -a_1\dot{\theta}_1 s_1 - a_2\dot{\theta}_1 s_{1+2} - a_2\dot{\theta}_2 s_{1+2} \\ a_1\dot{\theta}_1 c_1 + a_2\dot{\theta}_1 c_{1+2} + a_2\dot{\theta}_2 c_{1+2} \\ \dot{d}_4 \\ 0 \\ 0 \\ \dot{\theta}_1 + \dot{\theta}_2 + \dot{\theta}_3 \end{bmatrix}$$

Factoring out the velocity terms, we get:

$$\begin{bmatrix} v_x \\ v_y \\ v_z \\ \omega_x \\ \omega_y \\ \omega_z \end{bmatrix} = \begin{bmatrix} -a_2 s_{1+2} - a_1 s_1 & -a_2 s_{1+2} & 0 & 0 \\ a_2 c_{1+2} + a_1 c_1 & a_2 c_{1+2} & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \\ \dot{\theta}_3 \\ \dot{d}_4 \end{bmatrix}$$

This is identical to the Jacobian found through the explicit method. Similarly, solving for the Jacobian using the force/torque method gives the same answer:

$$\begin{bmatrix} f_x \\ f_y \\ f_z \\ \tau_x \\ \tau_y \\ \tau_z \end{bmatrix} = \begin{bmatrix} -a_2 s_{1+2} - a_1 s_1 & -a_2 s_{1+2} & 0 & 0 \\ a_2 c_{1+2} + a_1 c_1 & a_2 c_{1+2} & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} \tau_1 \\ \tau_2 \\ \tau_3 \\ f_4 \end{bmatrix}$$

III. SINGULARITIES

Since the robot arm only has 4 degrees of freedom (DOF), the Jacobian is a 6x4 matrix. In order to make the Jacobian a square matrix, we use a reduced form of the Jacobian described below:

$$J_{\text{red}} = \begin{bmatrix} -a_2 s_{1+2} - a_1 s_1 & -a_2 s_{1+2} & 0 & 0 \\ a_2 c_{1+2} + a_1 c_1 & a_2 c_{1+2} & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix}$$

The matrix J_{red} removes the rows corresponding to the velocities ω_x and ω_y, since there is no actuation in those degrees. The determinant of J_{red} is:

$$det(J_{\text{red}}) = -a_1 a_2 sin(\theta_2) \tag{1}$$

The values of θ_2 that make Eq. (1) singular are 0 and π, which is when the second link of the arm is parallel to the first link (fully extended or retracted).

A. Velocity

If we were to substitute in a value of 0 for θ_2 into the velocity Jacobian and calculate the velocity for the end effector, we would get the following for v_x and v_y:

$$\begin{bmatrix} v_x \\ v_y \end{bmatrix} = \begin{bmatrix} -a_2 s_1 - a_1 s_1 & -a_2 s_1 \\ a_2 c_1 + a_1 c_1 & a_2 c_1 \end{bmatrix} \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix} \tag{2}$$

Eq. (2) shows that the magnitude of v_{xy} is a constant, which means that the end effector can only move tangentially to the arm and not in the radial direction. The singularity in this case is a loss of a degree of freedom.

B. Force

If we were to substitute in a value of 0 for θ_2 and θ_1 into the force Jacobian and calculate the forces on the end effector, we would get the following for f_x and f_y:

$$\begin{bmatrix} f_x \\ f_y \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ a_2 + a_1 & a_2 \end{bmatrix} \begin{bmatrix} \tau_1 \\ \tau_2 \end{bmatrix} \tag{3}$$

Eq. (3) shows that a nonzero horizontal force f$_x ontheendeffectorproducesnotorqueinjoint1, causingtherobotarmtolockup.$

C. Mathematical

Mathematically, if the determinant of a matrix is equal to 0, then it cannot be used to solve linear equations as there are dependent vectors. Finding the reduced-row echelon form of a matrix with dependent vectors will result in a matrix with zeros in a row or column. When using this matrix to solve a linear equation such as joint velocity or joint torques, this will result in zero velocity or torque in some joints, which is a loss of a degree of freedom.

References