# Overview of Diffusion Methods for Image Denoising

Axel Largent

**Abstract**—Image denoising takes an important part of the research in image processing. A lot of computer vision applications hold an image denoising module to have a better understanding of the image or make other algorithms more robust and efficient. In this paper we will present the linear and non linear isotropic diffusion. Two methods of image denoising based on a physical principle. After emphasizing the inconvenient of the isotropic linear diffusion, we will show on a office noisy image that isotropic non linear diffusion smooths the homogeneous areas and preserves the discontinuities of an image. What is an important characteristic in image denoising. Also we will discuss about of the Perona Malik function diffusivity and the influence of this parameter on the diffusion.

**Index Terms**—Image Denoising, Adaptative filtering, Isotropic non linear diffusion, Isotropic linear diffusion, Edge detection, Gaussian smoothing.

## 1 Introduction

IMAGE denoising is an important field in image processing. On one hand that permits to have a better interpretation of an image on the other hand that makes more efficient and robust the other steps of algorithms of pattern recognition, image segmentation, tracking, image registration, ect [3]. Traditional methods like gaussian smoothing, Canny-Deriche filter, median filter, morphological filtering provide quite good results. However, on some kind of images, as medical images, these methods can appear to be inefficient. That's why the research in this field stays very active so far. In this article we will be interested in the linear and non linear isotropic diffusion. Two denoising methods based on the physical process of diffusion and often used in medical images. These tools have the particularity to employ EDP in their scheme. The isotropic non linear diffusion acts like an adaptative filter, in smoothing the homogeneous areas and preserving the edges according to the value of the gradient. That is an important characteristic in image denoising.
This paper is organised as follow : Section 2 we introduce the principle of isotropic linear diffusion. We comment the results provided by the diffusion on a office noisy image. Then we discuss about the similarity between gaussian filtering and the isotropic linear diffusion. In section 3 we introduce the isotropic non linear diffusion filtering with the Perona Malik diffusivity. We discuss about the particularity of this method and compare the given results on our test image.

## 2 Isotropic linear diffusion

The diffusion is the movement of molecules from a hight concentration place to a low concentration place. The purpose of this physical process is to equilibrate the concentration differences in the system while preserving the total mass. This process is specified by the following equation

$$\partial_t u = div(g\nabla u) \tag{1}$$

where u is the concentration and g is the diffusivity. If $g$ is a constant then the equation 1 called heat equation and the diffusion is isotropic and linear.
This physical principle in computer vision used to perform noise reduction on a image. In this case $u_0$ represents the original image and at each step the gray scale diffuses on the image.

### 2.1 Application of isotropic linear diffusion

We solved heat equation in the discrete case. Indeed, a image being a scene representation below the matricial form we could consider we are in the discrete case and use finite difference to solve this equation. Thereby the equation 1 becomes

$$\frac{u_{t+1} - u_t}{dt} = div(g\nabla u) \tag{2}$$

$$u_{t+1} = u_t + dt(g\Delta u) \tag{3}$$

In this paper we apply this method on the office noisy image with $g = 1$ and $t = 1, 5, 10, 30, 100$ (cf. figure 1). As we solve the diffusion equation in the discrete case we use zero padding method or border approximations of the image to not deal with boundary condition of the PDE (cf. equation 1).
Figure 1 displays the ouput images. We can see that images are less and less noisy. Thus, the diffusion process performs well noise reduction on the input image. When the diffusion time increase the homogeneous areas and the edges are more and more blurred. The discontinuities are less and less well localised.
The effect of the diffusivity parameter $g$ is to control the strength of the denoising. When $g$ increases the output images are more and more smoothed (cf. figure 2).

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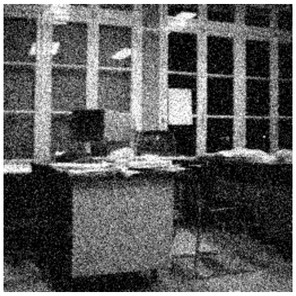
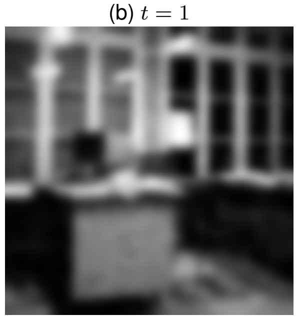
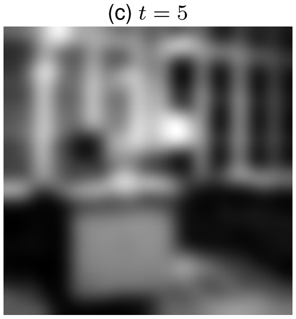

(a) office noisy image


(b) $t = 1$


(c) $t = 5$


(d) $t = 10$


(e) $t = 30$

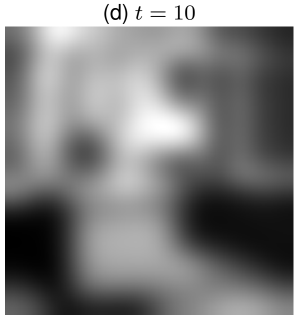
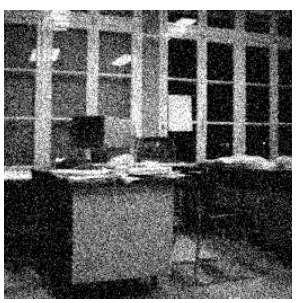
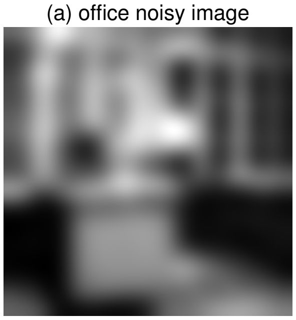

(f) $t = 100$

Fig. 1. Isotropic linear diffusion process with $g = 1$


(a) office noisy image


(b) $g = 1, t = 10$


(c) $g = 5, t = 10$

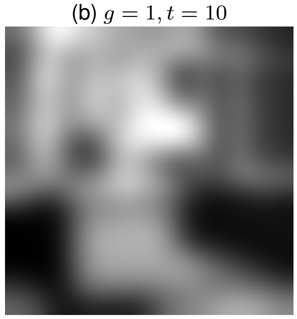

(d) $g = 10, t = 10$

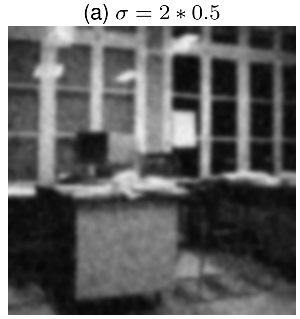
Fig. 2. Isotropic linear diffusion process with variation of $g$

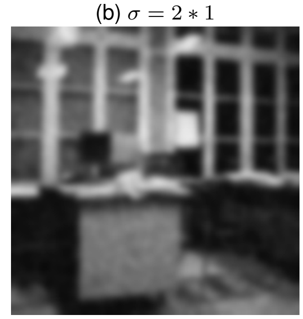
### 2.2 Similarity between linear diffusion process and gaussian filtering

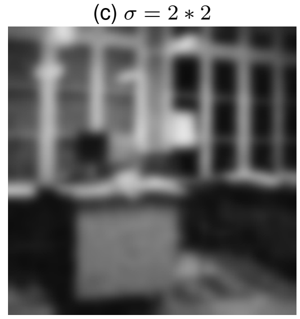
A gaussian smoothing is the application of gaussian kernel on a image due to a convolution [3]. The analytical function


(a) $\sigma = 2 * 0.5$


(b) $\sigma = 2 * 1$


(c) $\sigma = 2 * 2$


(d) $\sigma = 2 * 5$


(e) $\sigma = 2 * 10$

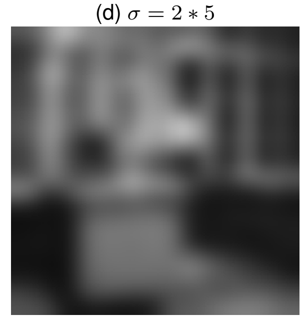

(f) $\sigma = 2 * 50$

Fig. 3. Gaussian smoothing on the office noisy image

used to fill this kernel is the following gaussian function

$$G(x, y) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp(\frac{-(x^2 + y^2)}{2\sigma^2}) \tag{4}$$

This method is often used to perform noise reduction. We applied this gaussian smoothing with $\sigma = 0.5, 1, 2, 5, 10, 30$ on the image office noisy. We find that an increase in the parameter $\sigma$ improve the noise reduction performances of the gaussian smoothing. However the edges and the small details disappear or are blurred (cf. figure 3).

To solve the heat equation it was demonstrate a unique solution exist. The expression of the solution is

$$u(x, y, t) = \begin{cases} I(x, y) & t = 0 \\ (G_{\sqrt{2t}} * I)(x, y) & t > 0 \end{cases} \tag{5}$$

where $G(x, y)$ is a gaussian kernel (cf. [2]). This solution proves that performing isotropic linear diffusion for $t$ with d = 1 is exactly equivalent to performing gaussian smoothing with $\sigma = 2t$. In figure 4 output images after a linear diffusion process with $g = 1, t = 10$ and a gaussian smoothing with $\sigma = 2 * t$ are identical. The norm between this two output images is equal to $4.97 * 10^{-5}$ and the PSNR distance is equal to $86.0226$ db.

(a) $g = 1, t = 10$ (b) $\sigma = 2 * 10$

Fig. 4. Linear diffusion process vs gaussian smoothing

## 3 Isotropic non linear diffusion

The introduction of non linear diffusion equation dates back to the article of Perona Malik in 1990 [1]. The weakness of the heat equation is that the diffusion is identical in each points of the image. Indeed, as shown in the section 2, the image is as soon as smoothed inside the homogeneous areas than along the edges. The main idea of Perona Malik was to use an other diffusivity function which allows to smooth the image inside the homogeneous area while staying intact the edges. The heat equation with this function becomes

$$\partial_t u = div(g(\mid \nabla u \mid) * \nabla u) \tag{6}$$

with $g$ is the Perona Malik diffusivity.

### 3.1 Properties of Perona Malik diffusivity

The mathematical expression of Perona Malik diffusivity is $g(\mid \nabla u \mid) = 1/(1 + (\|\nabla u\|/\lambda)^2)$ where $g$ is a decreasing function, which has for value $1$ in $0$, and tends towards $0$ in $\infty$. This metric implies a weak diffusivity for the strong values of the gradient norm. That's why a non linear diffusion with this expression is so called edge-preserving diffusion.
As we can see in the figure 5, when the value of $\lambda$ increase the homogeneous areas of the diffusivity image tends toward one. The transitions between the various homogeneous areas are also more significant. So the $\lambda$ parameter allows to handle the diffusivity process in function of the gradient. Hence, most the $\lambda$ is strong and most the edges with small gradient will be blurred. Sometimes the noise can create variations which are not strong enough to be considered by $\lambda$. That's why the diffusion process with $\lambda = 0.5, 1, 2, 5$ on the office image noisy cannot perform the noise reduction expected (cf. figure 7).

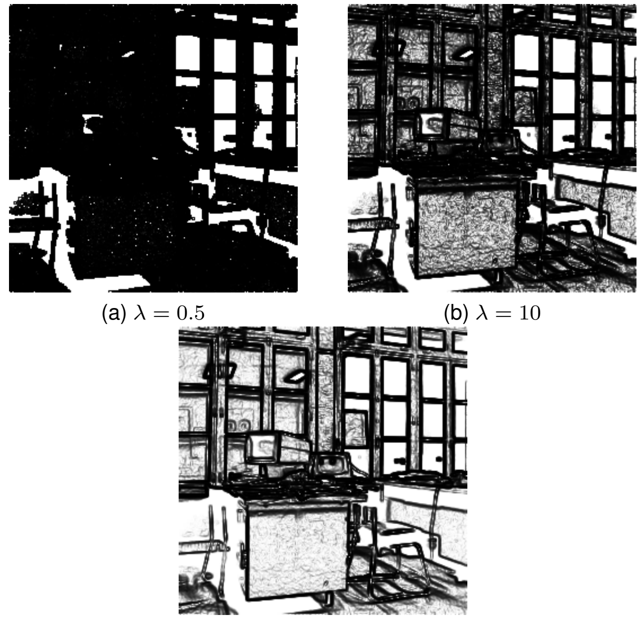
### 3.2 Application of isotropic non linear diffusion

We considered we are in the discrete case and we used finite difference to solve the equation 6. Thereby the expression of isotropic non linear diffusion equation becomes

$$\partial_t u = g(\mid \nabla u \mid).\Delta u + \nabla g.\nabla u \tag{7}$$

then

$$u_{t+1} = u_t + dt[c_N.\nabla_N u + c_S.\nabla_S u + c_E.\nabla_E u + c_W.\nabla_W u] \tag{8}$$



(a) $\lambda = 0.5$ (b) $\lambda = 10$

(c) $\lambda = 30$

Fig. 5. Perona-Malik diffusivity image with different values of $\lambda$

where $0 \leq dt \leq 1/4$ for the stability of numerical scheme with

$$\begin{aligned}
\nabla_N u_{i,j} &= u_{i-1,j} - u_{i,j} \\
\nabla_S u_{i,j} &= u_{i+1,j} - u_{i,j} \\
\nabla_E u_{i,j} &= u_{i,j+1} - u_{i,j} \\
\nabla_W u_{i,j} &= u_{i,j-1} - u_{i,j}
\end{aligned} \tag{9}$$

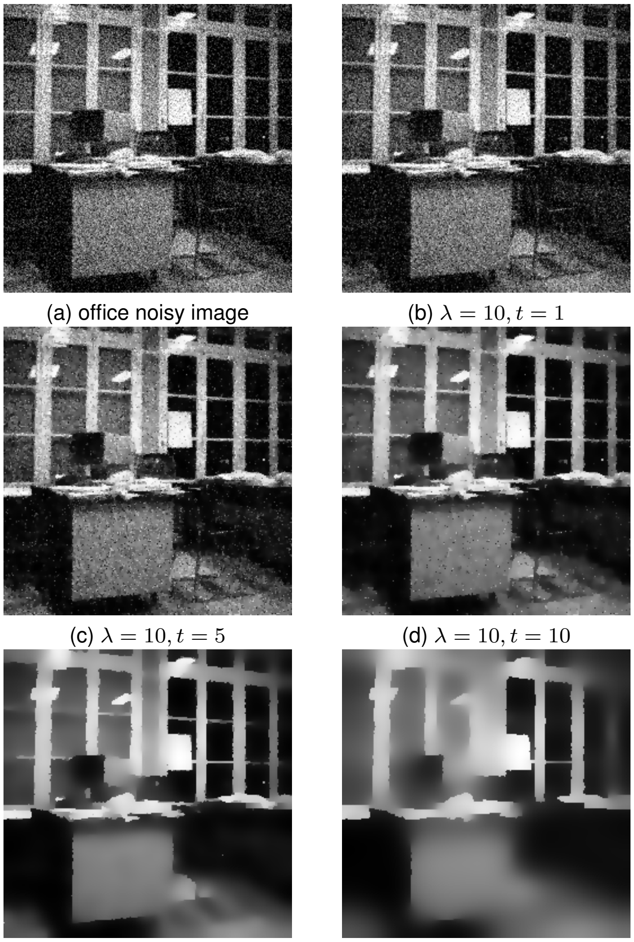
and the conduction coefficients equal to

$$\begin{aligned}
c^t_{N_{i,j}} &= g(\mid \nabla_N u^t_{i,j} \mid) \\
c^t_{S_{i,j}} &= g(\mid \nabla_S u^t_{i,j} \mid) \\
c^t_{E_{i,j}} &= g(\mid \nabla_E u^t_{i,j} \mid) \\
c^t_{W_{i,j}} &= g(\mid \nabla_W u^t_{i,j} \mid)
\end{aligned} \tag{10}$$

As we can see in the figure 6 the diffusion process perform noise reduction on the input image. The output images are less noisy than the input image.
The diffusion time has for effect to handle this noise reduction. When it increases the main discontinuities stay well localised but the small details and homogeneous areas on the image are most and most blurred.
The edges in the output images obtained from non linear diffusion are better preserved than the output images obtained from linear diffusion.

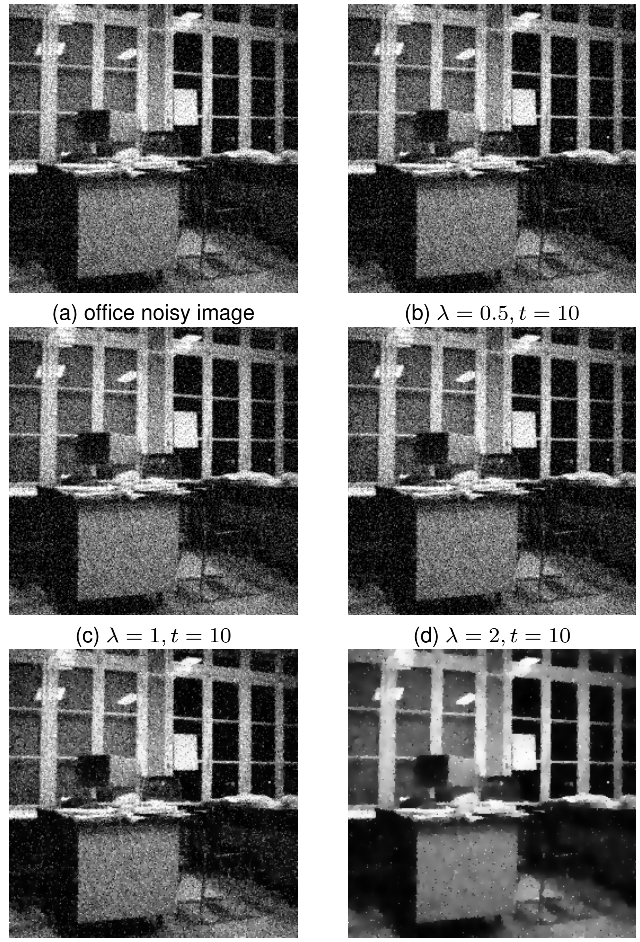
## 4 Conclusion

In this paper, we shown linear diffusion performs well a noise reduction on our test image. We saw this method don't preserve the edges and details. The homogeneous areas and the discontinuities are blurred in the same way on the whole image. We had also noticed there is a connection between the gaussian smoothing and the isotropic linear diffusion. Indeed, for $g = 1$ and a time process t the linear



(a) office noisy image (b) $\lambda = 10, t = 1$ (c) $\lambda = 10, t = 5$ (d) $\lambda = 10, t = 10$ (e) $\lambda = 10, t = 30$ (f) $\lambda = 10, t = 100$

Fig. 6. Office noisy image smoothed with Isotropic non linear diffusion



(a) office noisy image (b) $\lambda = 0.5, t = 10$ (c) $\lambda = 1, t = 10$ (d) $\lambda = 2, t = 10$ (e) $\lambda = 5, t = 10$ (f) $\lambda = 10, t = 10$

Fig. 7. Isotropic non linear diffusion with variation of $\lambda$

diffusion is equivalent to a gaussian smoothing with the parameter $\sigma = 2t$. By contrast with the linear diffusion the isotropic non linear diffusion preserves the edges. It acts as an adaptive filter by having a different behaviour when it meet a homogeneous area or a discontinuity. We shown also non linear diffusion is highly dependent on some crucial parameters, such as diffusivity function parameter and time diffusion. We saw when the diffusion time increases the main discontinuities stay well localised but the small details and homogeneous areas on the image are more and more blurred. In term of perspectives, we think some works to determinate a stopping criterion and the better diffusivity function parameter can carry out to obtain the better smoothing image with well localised edges after an isotropic non linear diffusion.

## Acknowledgments

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## References

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