Package 'randtoolbox'

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Type Package

Title Toolbox for Pseudo and Quasi Random Number Generation and Random Generator Tests						
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Author R port by Yohan Chalabi, Christophe Dutang, Petr Savicky and Diethelm Wuertz with some underlying C codes of (i) the SFMT algorithm from M. Matsumoto and M. Saito, (ii) the Knuth-TAOCP RNG from D. Knuth.						
Maintainer Christophe Dutang <christophe.dutang@ensimag.fr></christophe.dutang@ensimag.fr>						
Description Provides (1) pseudo random generators - general linear congruential generators, multiple recursive generators and generalized feedback shift register (SF-Mersenne Twister algorithm and WELL generators); (2) quasi random generators - the Torus algorithm, the Sobol sequence, the Halton sequence (including the Van der Corput sequence) and (3) some generator tests - the gap test, the serial test, the poker test. See e.g. Gentle (2003) <doi:10.1007 b97336="">. The package can be provided without the rngWELL dependency on demand. Take a look at the Distribution task view of types and tests of random number generators. Version in Memoriam of Diethelm and Barbara Wuertz.</doi:10.1007>						
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Description

The randtoolbox-package started in 2007 during an ISFA (France) working group. From then, it grew quickly thanks to the contribution of Diethelm Wuertz and Petr Savicky. It was presented at the Rmetrics workshop in 2009 in Meielisalp, Switzerland.

This package has currently implementations for state-of-the-art pseudo RNGs for simulations as well as the usual quasi RNGs. See pseudoRNG and quasi RNG, respectively. There are also some RNG tests.

We recommend first users to take a look at the vignette 'Quick introduction of randtoolbox', whereas advanced users whould have a look at the vigentte 'A note on random number generation'.

The stable version is available on CRAN https://CRAN.R-project.org/package=randtoolbox, while the development version is hosted on R-forge in the rmetric project.

Details

Package: randtoolbox Type: Package Version: 1.30.0 Date: 2019

License: BSD_3_clause + file LICENSE

Author(s)

Christophe Dutang and Petr Savicky.

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auxiliary

Auxiliary functions for 'randtoolbox' package.

Description

Stirling numbers of the second kind and permutation of positive integers.

Usage

```
stirling(n)
permut(n)
```

Arguments

n

a positive integer.

Details

stirling computes stirling numbers of second kind i.e.

$$Stirl_n^k = k * Stirl_{n-1}^k + Stirl_{n-1}^{k-1}$$

with $Stirl_n^1 = Stirl_n^n = 1$. e.g.

- n = 0, returns 1
- n = 1, returns a vector with 0,1
- n = 2, returns a vector with 0,1,1
- n = 3, returns a vector with 0,1,3,1
- n = 4, returns a vector with 0,1,7,6,1...

Go to wikipedia for more details.

permut compute permutation of 1, ..., n and store it in a matrix. e.g.

• n = 1, returns a matrix with

1

• n=2, returns a matrix with

1 2

• n=3 returns a matrix with

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```
1 3 2
2 3 1
1 2 3
2 1 3
```

Value

a vector with stirling numbers.

Author(s)

Christophe Dutang.

See Also

choose for combination numbers.

Examples

```
# should be 1
stirling(0)

# should be 0,1,7,6,1
stirling(4)
```

coll.test

the Collision test

Description

The Collision test for testing random number generators.

Usage

```
coll.test(rand, lenSample = 2^14, segments = 2^10, tdim = 2,
nbSample = 1000, echo = TRUE, ...)
```

Arguments

rand	a function generating random numbers. its first argument must be the 'number of observation' argument as in runif.
lenSample	numeric for the length of generated samples.
segments	numeric for the number of segments to which the interval [0,1] is split.
tdim	numeric for the length of the disjoint t-tuples.
nbSample	numeric for the overall sample number.
echo	logical to plot detailed results, default TRUE
	further arguments to pass to function rand

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Details

We consider outputs of multiple calls to a random number generator rand. Let us denote by n the length of samples (i.e. lenSample argument), k the number of cells (i.e. nbCell argument) and m the number of samples (i.e. nbSample argument).

A collision is defined as when a random number falls in a cell where there are already random numbers. Let us note C the number of collisions

The distribution of collision number C is given by

$$P(C=c) = \prod_{i=0}^{n-c-1} \frac{k-i}{k} \frac{1}{k^c} {}_{2}S_{n}^{n-c},$$

where ${}_2S_n^k$ denotes the Stirling number of the second kind and $c=0,\ldots,n-1$.

But we cannot use this formula for large n since the Stirling number need $O(n \log(n))$ time to be computed. We use a Gaussian approximation if $\frac{n}{k} > \frac{1}{32}$ and $n \geq 2^8$, a Poisson approximation if $\frac{n}{k} < \frac{1}{32}$ and the exact formula otherwise.

Finally we compute m samples of random numbers, on which we calculate the number of collisions. Then we are able to compute a chi-squared statistic.

Value

a list with the following components:

statistic the value of the chi-squared statistic.

p. value the p-value of the test.

observed the observed counts.

expected the expected counts under the null hypothesis.

residuals the Pearson residuals, (observed - expected) / sqrt(expected).

Author(s)

Christophe Dutang.

References

Planchet F., Jacquemin J. (2003), *L'utilisation de methodes de simulation en assurance*. Bulletin Français d'Actuariat, vol. 6, 11, 3-69. (available online)

L'Ecuyer P. (2001), Software for uniform random number generation distinguishing the good and the bad. Proceedings of the 2001 Winter Simulation Conference. (available online)

L'Ecuyer P. (2007), *Test U01: a C library for empirical testing of random number generators*. ACM Trans. on Mathematical Software 33(4), 22.

See Also

other tests of this package coll.test.sparse, freq.test, serial.test, poker.test, order.test and gap.test

ks.test for the Kolmogorov Smirnov test and acf for the autocorrelation function.

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Examples

```
# (1) poisson approximation
#
coll.test(runif, 2^7, 2^10, 1, 100)
# (2) exact distribution
#
coll.test(SFMT, 2^7, 2^10, 1, 100)
```

coll.test.sparse

the Collision test

Description

The Collision test for testing random number generators.

Usage

```
coll.test.sparse(rand, lenSample = 2^14, segments = 2^10, tdim = 2, nbSample = 10, ...)
```

Arguments

rand a function generating random numbers. its first argument must be the 'number

of observation' argument as in runif.

lenSample numeric for the length of generated samples.

segments numeric for the number of segments to which the interval [0,1] is split.

tdim numeric for the length of the disjoint t-tuples.

nbSample numeric for the number of repetitions of the test.

... further arguments to pass to function rand

Details

We consider outputs of multiple calls to a random number generator rand. Let us denote by n the length of samples (i.e. lenSample argument), k the number of cells (i.e. nbCell argument).

A collision is defined as when a random number falls in a cell where there are already random numbers. Let us note C the number of collisions

The distribution of collision number C is given by

$$P(C=c) = \prod_{i=0}^{n-c-1} \frac{k-i}{k} \frac{1}{k^c} {}_2S_n^{n-c},$$

where ${}_2S_n^k$ denotes the Stirling number of the second kind and $c=0,\ldots,n-1$.

This formula cannot be used for large n since the Stirling number need $O(n \log(n))$ time to be computed. We use a Poisson approximation if $\frac{n}{k} < \frac{1}{32}$ and the exact formula otherwise.

The test is repeated nbSample times and the result of each repetition forms a row in the output table.

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Value

A data frame with nbSample rows and the following columns. observed the observed counts.

p.value the p-value of the test.

Author(s)

Christophe Dutang, Petr Savicky.

References

P. L'Ecuyer, R. Simard, S. Wegenkittl, Sparse serial tests of uniformity for random number generators. SIAM Journal on Scientific Computing, 24, 2 (2002), 652-668.

L'Ecuyer P. (2007), Test U01: a C library for empirical testing of random number generators. ACM Trans. on Mathematical Software 33(4), 22.

See Also

```
other tests of this package coll.test, freq.test, serial.test, poker.test, order.test and gap.test
```

ks. test for the Kolmogorov Smirnov test and acf for the autocorrelation function.

```
# (1) poisson approximation
#
coll.test.sparse(runif)

# (2) exact distribution
#
coll.test.sparse(SFMT, lenSample=2^7, segments=2^5, tdim=2, nbSample=10)

## Not run:
#A generator with too uniform distribution (too small number of collisions)
#produces p-values close to 1
set.generator(name="congruRand", mod="2147483647", mult="742938285", incr="0", seed=1)
coll.test.sparse(runif, lenSample=300000, segments=50000, tdim=2)

#Park-Miller generator has too many collisions and produces small p-values
set.generator(name="congruRand", mod="2147483647", mult="16807", incr="0", seed=1)
coll.test.sparse(runif, lenSample=300000, segments=50000, tdim=2)

## End(Not run)
```

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freq.test

the Frequency test

Description

The Frequency test for testing random number generators.

Usage

```
freq.test(u, seq = 0:15, echo = TRUE)
```

Arguments

u sample of random numbers in]0,1[.
echo logical to plot detailed results, default TRUE
seq a vector of contiguous integers, default 0:15.

Details

We consider a vector u, realisation of i.i.d. uniform random variables U_1, \ldots, U_n .

The frequency test works on a serie seq of ordered contiguous integers (s_1, \ldots, s_d) , where $s_j \in \mathbb{Z}$. From the sample u, we compute observed integers as

$$d_i = |u_i * (s_d + 1) + s_1|,$$

(i.e. d_i are uniformly distributed in $\{s_1, \ldots, s_d\}$). The expected number of integers equals to j is $m = \frac{1}{s_d - s_1 + 1} \times n$. Finally, the chi-squared statistic is

$$S = \sum_{j=1}^{d} \frac{(card(d_i = s_j) - m)^2}{m}.$$

Value

a list with the following components:

statistic the value of the chi-squared statistic.

p.value the p-value of the test.

observed the observed counts.

expected the expected counts under the null hypothesis.

residuals the Pearson residuals, (observed - expected) / sqrt(expected).

Author(s)

Christophe Dutang.

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References

Planchet F., Jacquemin J. (2003), *L'utilisation de methodes de simulation en assurance*. Bulletin Français d'Actuariat, vol. 6, 11, 3-69. (available online)

L'Ecuyer P. (2001), Software for uniform random number generation distinguishing the good and the bad. Proceedings of the 2001 Winter Simulation Conference. (available online)

L'Ecuyer P. (2007), *Test U01: a C library for empirical testing of random number generators*. ACM Trans. on Mathematical Software 33(4), 22.

See Also

```
other tests of this package gap.test, serial.test, poker.test, order.test and coll.test ks.test for the Kolmogorov Smirnov test and acf for the autocorrelation function.
```

Examples

```
# (1)
#
freq.test(runif(1000))
print( freq.test( runif(1000000), echo=FALSE) )
# (2)
#
freq.test(runif(1000), 1:4)
freq.test(runif(1000), 10:40)
```

gap.test

the Gap test

Description

The Gap test for testing random number generators.

Usage

```
gap.test(u, lower = 0, upper = 1/2, echo = TRUE)
```

Arguments

u	sample of random numbers in]0,1[.
lower	numeric for the lower bound, default \emptyset .
upper	numeric for the upper bound, default 1/2.
echo	logical to plot detailed results, default TRUE

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Details

We consider a vector u, realisation of i.i.d. uniform random variables U_1, \ldots, U_n .

The gap test works on the 'gap' variables defined as

$$G_i = \left\{ \begin{array}{ll} 1 & \text{if } lower \leq U_i \leq upper \\ 0 & \text{otherwise} \end{array} \right.$$

Let p the probability that G_i equals to one. Then we compute the length of zero gaps and denote by n_j the number of zero gaps of length j. The chi-squared statistic is given by

$$S = \sum_{j=1}^{m} \frac{(n_j - np_j)^2}{np_j},$$

where p_j stands for the probability the length of zero gaps equals to j $((1-p)^2p^j)$ and m the max number of lengths (at least $\left\lfloor \frac{\log(10^{-1})-2\log(1-p)-\log(n)}{\log(p)} \right\rfloor$).

Value

a list with the following components:

statistic the value of the chi-squared statistic.

p. value the p-value of the test.

observed the observed counts.

expected the expected counts under the null hypothesis.

residuals the Pearson residuals, (observed - expected) / sqrt(expected).

Author(s)

Christophe Dutang.

References

Planchet F., Jacquemin J. (2003), *L'utilisation de methodes de simulation en assurance*. Bulletin Français d'Actuariat, vol. 6, 11, 3-69. (available online)

L'Ecuyer P. (2001), Software for uniform random number generation distinguishing the good and the bad. Proceedings of the 2001 Winter Simulation Conference. (available online)

L'Ecuyer P. (2007), *Test U01: a C library for empirical testing of random number generators*. ACM Trans. on Mathematical Software 33(4), 22.

See Also

other tests of this package freq. test, serial.test, poker.test, order.test and coll.test ks.test for the Kolmogorov Smirnov test and acf for the autocorrelation function.

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Examples

```
# (1)
#
gap.test(runif(1000))
print( gap.test( runif(1000000), echo=FALSE ) )
# (2)
#
gap.test(runif(1000), 1/3, 2/3)
```

get.primes

Get primes for quasi random number generation

Description

Provides a vector of a specified number of smallest primes from the internal table of the package.

Usage

```
get.primes(n)
```

Arguments

n

The required number of primes. Should be at most 100 000.

Details

The package contains an internal table of the smallest $100\,000$ primes, which may be used in torus algorithm.

Value

Vector of min(n, 100000) smallest primes.

See Also

torus

```
p <- get.primes(20)
torus(5,dim=10,prime=p[11:20])</pre>
```

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getWELLState Get the state of a WELL generator implemented in randtoolbox package.

Description

Get the state of a WELL generator implemented in randtoolbox package to be used in function rngWELLScriptR().

Usage

```
getWELLState()
```

Value

A 0,1-matrix, whose columns represent 32-bit integers.

See Also

rngWELLScriptR

order.test

the Order test

Description

The Order test for testing random number generators.

Usage

```
order.test(u, d = 3, echo = TRUE)
```

Arguments

u sample of random numbers in]	0,1[.
---------------------------------	-------

echo logical to plot detailed results, default TRUE

d a numeric for the dimension, see details. When necessary we assume that d is a

multiple of the length of u.

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Details

We consider a vector u, realisation of i.i.d. uniform random variables U_1, \ldots, U_n .

The Order test works on a sequence of d-uplets (x,y,z) when d=3) of uniform i.i.d. random variables. The triplet is build from the vector u. The number of permutation among the components of a triplet is 3! = 6, i.e. x < y < z, x < z < y, y < x < z, y < z < x, z < x < y and z < y < x. The Marsaglia test computes the empirical of the different permutations as well as the theoretical one n/6 where n is the number of triplets. Finally the chi-squared statistic is

$$S = \sum_{j=0}^{6} \frac{n_j - n/6)^2}{n/6}.$$

Value

a list with the following components:

statistic the value of the chi-squared statistic.

p. value the p-value of the test.

observed the observed counts.

expected the expected counts under the null hypothesis.

residuals the Pearson residuals, (observed - expected) / sqrt(expected).

Author(s)

Christophe Dutang.

References

Planchet F., Jacquemin J. (2003), *L'utilisation de methodes de simulation en assurance*. Bulletin Français d'Actuariat, vol. 6, 11, 3-69. (available online)

L'Ecuyer P. (2001), Software for uniform random number generation distinguishing the good and the bad. Proceedings of the 2001 Winter Simulation Conference. (available online)

L'Ecuyer P. (2007), *Test U01: a C library for empirical testing of random number generators*. ACM Trans. on Mathematical Software 33(4), 22.

See Also

other tests of this package freq.test, serial.test, poker.test, gap.test and coll.test ks.test for the Kolmogorov Smirnov test and acf for the autocorrelation function.

```
# (1) mersenne twister vs torus
#
order.test(runif(6000))
order.test(torus(6000))
# (2)
```

poker.test

```
#
order.test(runif(4000), 4)
order.test(torus(4000), 4)
# (3)
#
order.test(runif(5000), 5)
order.test(torus(5000), 5)
```

poker.test

the Poker test

Description

The Poker test for testing random number generators.

Usage

```
poker.test(u , nbcard = 5, echo = TRUE)
```

Arguments

u sample of random numbers in]0,1[.

echo logical to plot detailed results, default TRUE

nbcard a numeric for the number of cards, we assume that the length of u is a multiple

of nbcard.

Details

We consider a vector u, realisation of i.i.d. uniform random variables U_1, \ldots, U_n .

Let us note k the card number (i.e. nbcard). The poker test computes a serie of 'hands' in $\{0, \ldots, k-1\}$ from the sample $h_i = \lfloor u_i d \rfloor$ (u must have a length dividable by k). Let n_j be the number of 'hands' with (exactly) j different cards. The probability is

$$p_{j} = \frac{k!}{k^{k}(k-j)! * S_{k}^{j}} * (\frac{j}{k})^{(k-j)},$$

where S_k^j denotes the Stirling numbers of the second kind. Finally the chi-squared statistic is

$$S = \sum_{j=0}^{k-1} \frac{n_j - np_j/k)^2}{np_j/k}.$$

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Value

```
a list with the following components:
statistic the value of the chi-squared statistic.
p.value the p-value of the test.
observed the observed counts.
expected the expected counts under the null hypothesis.
residuals the Pearson residuals, (observed - expected) / sqrt(expected).
```

Author(s)

Christophe Dutang.

References

Planchet F., Jacquemin J. (2003), *L'utilisation de methodes de simulation en assurance*. Bulletin Français d'Actuariat, vol. 6, 11, 3-69. (available online)

L'Ecuyer P. (2001), Software for uniform random number generation distinguishing the good and the bad. Proceedings of the 2001 Winter Simulation Conference. (available online)

L'Ecuyer P. (2007), *Test U01: a C library for empirical testing of random number generators*. ACM Trans. on Mathematical Software 33(4), 22.

See Also

```
other tests of this package freq.test, serial.test, gap.test, order.test and coll.test ks.test for the Kolmogorov Smirnov test and acf for the autocorrelation function.
```

```
# (1) hands of 5 'cards'
#
poker.test(runif(50000))
# (2) hands of 4 'cards'
#
poker.test(runif(40000), 4)
# (3) hands of 42 'cards'
#
poker.test(runif(420000), 42)
```

Description

General linear congruential generators such as Park Miller sequence, generalized feedback shift register such as SF-Mersenne Twister algorithm and WELL generator.

The list of supported generators consists of generators available via direct functions and generators available via set.generator() and runif() interface. Most of the generators belong to both these groups, but some generators are only available directly (SFMT) and some only via runif() interface (Mersenne Twister 2002). This help page describes the list of all the supported generators and the functions for the direct access to those, which are available in this way. See set.generator() for the generators available via runif() interface.

Usage

```
congruRand(n, dim = 1, mod = 2^31-1, mult = 16807, incr = 0, echo)
SFMT(n, dim = 1, mexp = 19937, usepset = TRUE, withtorus = FALSE,
usetime = FALSE)
WELL(n, dim = 1, order = 512, temper = FALSE, version = "a")
knuthTAOCP(n, dim = 1)
setSeed(seed)
```

Arguments

n	number of observations. If $length(n) > 1$, the length is taken to be the required number.
dim	dimension of observations (must be <=100 000, default 1).
seed	a single value, interpreted as a positive integer for the seed. e.g. append your day, your month and your year of birth.
mod	an integer defining the modulus of the linear congruential generator.
mult	an integer defining the multiplier of the linear congruential generator.
incr	an integer defining the increment of the linear congruential generator.
echo	a logical to plot the seed while computing the sequence.
mexp	an integer for the Mersenne exponent of SFMT algorithm. See details
withtorus	a numeric in]0,1] defining the proportion of the torus sequence appended to the SFMT sequence; or a logical equals to FALSE (default).
usepset	a logical to use a set of 12 parameters set for SFMT. default TRUE.
usetime	a logical to use the machine time to start the Torus sequence, default TRUE. if FALSE, the Torus sequence start from the first term.
order	a positive integer for the order of the characteristic polynomial. see details
temper	a logical if you want to do a tempering stage. see details
version	a character either 'a' or 'b'. see details

Details

The currently available generator are given below.

Linear congruential generators: The kth term of a linear congruential generator is defined as

$$u_k = \frac{(a * u_{k-1} + c) \bmod m}{m}$$

where a denotes the multiplier, c the increment and m the modulus, with the constraint 0 <= a < m and 0 <= c < m. The default setting is the Park Miller sequence with a = 16807, $m = 2^31 - 1$ and c = 0.

Knuth TAOCP 2002 (double version): The Knuth-TACOP-2002 is a Fibonnaci-lagged generator invented by Knuth(2002), based on the following recurrence.

$$x_n = (x_{n-37} + x_{n-100}) \bmod 2^{30},$$

In R, there is the integer version of this generator.

All the C code for this generator called RAN_ARRAY by Knuth is the code of D. Knuth (cf. http://www-cs-faculty.stanford.edu/~uno/programs.html) except some C code, we add, to *interface* with R.

Mersenne Twister 2002 generator: The generator suggested by Makoto Matsumoto and Takuji Nishimura with the improved initialization from 2002. See httml for more information on the generator itself. This generator is available only via set.generator() and runif() interface. Mersenne Twister generator used in base R is the same generator (the recurrence), but with a different initialization and the output transformation. The implementation included in randtoolbox allows to generate the same random numbers as in Matlab, see examples in set.generator().

SF Mersenne-Twister algorithm: SFMT function implements the SIMD-oriented Fast Mersenne Twister algorithm (cf. http://www.math.sci.hiroshima-u.ac.jp/~m-mat/MT/SFMT/index. html). The SFMT generator has a period of length 2^m-1 where m is a Mersenne exponent. In the function SFMT, m is given through mexp argument. By default it is 19937 like the "old" MT algorithm. The possible values for the Mersenne exponent are 607, 1279, 2281, 4253, 11213, 19937, 44497, 86243, 132049, 216091.

There are numerous parameters for the SFMT algorithm (see the article for details). By default, we use a different set of parameters (among 32 sets) at *each call* of SFMT (usepset=TRUE). The user can use a fixed set of parameters with usepset=FALSE. Let us note there is for the moment just *one* set of parameters for 44497, 86243, 132049, 216091 mersenne exponent. Sets of parameters can be found in appendix of the vignette.

The use of different parameter sets is motivated by the following citation of Matsumoto and Saito on this topic :

"Using one same pesudorandom number generator for generating multiple independent streams by changing the initial values may cause a problem (with negligibly small probability). To avoid the problem, using diffrent parameters for each generation is prefered. See Matsumoto M. and Nishimura T. (1998) for detailed information."

All the C code for SFMT algorithm used in this package is the code of M. Matsumoto and M. Saito (cf. http://www.math.sci.hiroshima-u.ac.jp/~m-mat/MT/emt.html), except we add some C code to *interface* with R. Streaming SIMD Extensions 2 (SSE2) operations are not yet supported.

WELL generator: The WELL (which stands for Well Equidistributed Long-period Linear) is in a sentence a generator with better equidistribution than Mersenne Twister algorithm but this gain of quality has to be paid by a slight higher cost of time. See Panneton et al. (2006) for details.

The order argument of WELL generator is the order of the characteristic polynomial, which is denoted by k in Paneton F., L'Ecuyer P. and Matsumoto M. (2006). Possible values for order are 512, 521, 607, 1024 where no tempering are needed (thus possible). Order can also be 800, 19937, 21071, 23209, 44497 where a tempering stage is possible through the temper argument. Furthermore a possible 'b' version of WELL RNGs are possible for the following order 521, 607, 1024, 800, 19937, 23209 with the version argument.

All the C code for WELL generator used in this package is the code of P. L'Ecuyer (cf. http://www.iro.umontreal.ca/~lecuyer/), except some C code, we add, to *interface* with R.

Set the seed: The function setSeed is similar to the function set.seed in R. It sets the seed to the one given by the user. Do not use a seed with too few ones in its binary representation. Generally, we append our day, our month and our year of birth or append a day, a month and a year. We recall by default with use the machine time to set the seed except for quasi random number generation.

Set the generator: Some of the generators are available using runif() interface. See set.generator() for more information.

See the pdf vignette for details.

Value

SFMT, WELL, congruRand and knuthTAOCP generate random variables in]0,1[, [0,1[and [0,1[respectively. It returns a nxdim matrix, when dim>1 otherwise a vector of length n.

congruRand may raise an error code if parameters are not correctly specified: -1 if the multiplier is zero; -2 if the multiplier is greater or equal than the modulus; -3 if the increment is greater or equal than the modulus; -4 if the multiplier times the modulus minus 1 is greater than 2^64-1 minus the increment; -5 if the seed is greater or equal than the modulus.

setSeed sets the seed of the randtoolbox package (i.e. both for the knuthTAOCP, SFMT, WELL and congruRand functions).

Author(s)

Christophe Dutang and Petr Savicky

References

Knuth D. (1997), *The Art of Computer Programming V2 Seminumerical Algorithms*, Third Edition, Massachusetts: Addison-Wesley.

Matsumoto M. and Nishimura T. (1998), *Dynamic Creation of Pseudorandom Number Generators*, Monte Carlo and Quasi-Monte Carlo Methods, Springer, pp 56–69. (available online)

Matsumoto M., Saito M. (2008), SIMD-oriented Fast Mersenne Twister: a 128-bit Pseudorandom Number Generator. (available online)

Paneton F., L'Ecuyer P. and Matsumoto M. (2006), *Improved Long-Period Generators Based on Linear Recurrences Modulo 2*, ACM Transactions on Mathematical Software. (preprint available online)

Park S. K., Miller K. W. (1988), *Random number generators: good ones are hard to find*. Association for Computing Machinery, vol. 31, 10, pp 1192-2001. (available online)

Wikipedia (2008), a linear congruential generator.

See Also

.Random.seed for what is done in R about random number generation and runifInterface for the runif interface.

```
require(rngWELL)
# (1) the Park Miller sequence
# Park Miller sequence, i.e. mod = 2^31-1, mult = 16807, incr=0
# the first 10 seeds used in Park Miller sequence
# 16807
         1
# 282475249
                    2
# 1622650073
                     3
# 984943658
# 1144108930
# 470211272
                    6
# 101027544
                  8
# 1457850878
# 1458777923
                    9
# 2007237709
setSeed(1)
congruRand(10, echo=TRUE)
# the 9998+ th terms
# 925166085 9998
# 1484786315
                 9999
             10000
10001
10002
# 1043618065
# 1589873406
# 2010798668
setSeed(1614852353) #seed for the 9997th term
congruRand(5, echo=TRUE)
# (2) the SF Mersenne Twister algorithm
SFMT(1000)
#Kolmogorov Smirnov test
#KS statistic should be around 0.037
ks.test(SFMT(1000), punif)
#KS statistic should be around 0.0076
ks.test(SFMT(10000), punif)
#different mersenne exponent with a fixed parameter set
#
```

```
SFMT(10, mexp = 607, usepset = FALSE)
SFMT(10, mexp = 1279, usepset = FALSE)
SFMT(10, mexp = 2281, usepset = FALSE)
SFMT(10, mexp = 4253, usepset = FALSE)
SFMT(10, mexp = 11213, usepset = FALSE)
SFMT(10, mexp = 19937, usepset = FALSE)
SFMT(10, mexp = 44497, usepset = FALSE)
SFMT(10, mexp = 86243, usepset = FALSE)
SFMT(10, mexp = 132049, usepset = FALSE)
SFMT(10, mexp = 216091, usepset = FALSE)
#use different sets of parameters [default when possible]
#
for(i in 1:7) print(SFMT(1, mexp = 607))
for(i in 1:7) print(SFMT(1, mexp = 2281))
for(i in 1:7) print(SFMT(1, mexp = 4253))
for(i in 1:7) print(SFMT(1, mexp = 11213))
for(i in 1:7) print(SFMT(1, mexp = 19937))
#use a fixed set and a fixed seed
#should be the same output
setSeed(08082008)
SFMT(1, usepset = FALSE)
setSeed(08082008)
SFMT(1, usepset = FALSE)
# (3) withtorus argument
# one third of outputs comes from Torus algorithm
u <- SFMT(1000, with=1/3)
# the third term of the following code is the first term of torus sequence
print(u[666:670] )
# (4) WELL generator
# 'basic' calls
# WELL512
WELL(10, order = 512)
# WELL1024
WELL(10, order = 1024)
# WELL19937
WELL(10, order = 19937)
# WELL44497
WELL(10, order = 44497)
# WELL19937 with tempering
WELL(10, order = 19937, temper = TRUE)
# WELL44497 with tempering
WELL(10, order = 44497, temper = TRUE)
# tempering vs no tempering
```

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```
setSeed4WELL(08082008)
WELL(10, order =19937)
setSeed4WELL(08082008)
WELL(10, order =19937, temper=TRUE)
# (5) Knuth TAOCP generator
knuthTAOCP(10)
knuthTAOCP(10, 2)
# (6) How to set the seed?
# all example is duplicated to ensure setSeed works
# congruRand
setSeed(1302)
congruRand(1)
setSeed(1302)
congruRand(1)
# SFMT
setSeed(1302)
SFMT(1, usepset=FALSE)
setSeed(1302)
SFMT(1, usepset=FALSE)
# BEWARE if you do not set usepset to FALSE
setSeed(1302)
SFMT(1)
setSeed(1302)
SFMT(1)
# WELL
setSeed(1302)
WELL(1)
setSeed(1302)
WELL(1)
# Knuth TAOCP
setSeed(1302)
knuthTAOCP(1)
setSeed(1302)
knuthTAOCP(1)
\# (7) computation times on my macbook, mean of 1000 runs
## Not run:
# algorithm time in seconds for n=10^6
# classical Mersenne Twister 0.066
# SF Mersenne Twister
                                0.044
# WELL generator 0.065
# Knuth TAOCP 0.046
# Park Miller
                       0.108
n <- 1e+06
```

```
mean( replicate( 1000, system.time( runif(n), gcFirst=TRUE)[3]) )
mean( replicate( 1000, system.time( SFMT(n), gcFirst=TRUE)[3]) )
mean( replicate( 1000, system.time( WELL(n), gcFirst=TRUE)[3]) )
mean( replicate( 1000, system.time( knuthTAOCP(n), gcFirst=TRUE)[3]) )
mean( replicate( 1000, system.time( congruRand(n), gcFirst=TRUE)[3]) )
## End(Not run)
```

quasiRNG

Toolbox for quasi random number generation

Description

the Torus algorithm, the Sobol and Halton sequences.

Usage

Arguments

n	number of observations. If $length(n) > 1$, the length is taken to be the required number.
dim	dimension of observations default 1.
init	a logical, if TRUE the sequence is initialized and restarts, otherwise not. By default TRUE.
normal	a logical if normal deviates are needed, default FALSE.
scrambling	an integer value, if 1, 2 or 3 the sequence is scrambled otherwise not. If scrambling=1, Owen type type of scrambling is applied, if scrambling=2, Faure-Tezuka type of scrambling, is applied, and if scrambling=3, both Owen+Faure-Tezuka type of scrambling is applied. By default 0.
seed	an integer value, the random seed for initialization of the scrambling process. By default 4711. On effective if scrambling>0.
prime	a single prime number or a vector of prime numbers to be used in the Torus sequence. (optional argument).
mixed	a logical to combine the Torus algorithm with SFMT algorithm, default FALSE.
usetime	a logical to use the machine time to start the Torus sequence, default FALSE, i.e. when usetime=FALSE the Torus sequence start from the first term. usetime is only used when mixed=FALSE.

method a character string either "C" or "Fortran". Note that mixed=TRUE is only avail-

able when method="C".

mexp an integer for the Mersenne exponent of SFMT algorithm, only used when

mixed=TRUE.

Details

The currently available generator are given below.

Torus algorithm: The kth term of the Torus algorithm in d dimension is given by

$$u_k = (frac(k\sqrt{p_1}), ..., frac(k\sqrt{p_d}))$$

where p_i denotes the ith prime number, frac the fractional part (i.e. frac(x) = x - floor(x)). We use the 100 000 first prime numbers from http://primes.utm.edu/, thus the dimension is limited to 100 000. If the user supplys prime numbers through the argument prime, we do NOT check for primality and we cast numerics to integers, (i.e. prime=7.1234 will be cast to prime=7 before computing Torus sequence). The Torus sequence starts from k=1 when initialized with init = TRUE and no not depending on machine time usetime = FALSE. This is the default. When init = FALSE, the sequence is not initialized (to 1) and starts from the last term. We can also use the machine time to start the sequence with usetime = TRUE, which overrides init.

scrambled Sobol sequences Calculates a matrix of uniform and normal deviated Sobol low discrepancy numbers. Optional scrambling of the sequence can be selected. The Sobol sequence restarts and is initialized when init = TRUE and otherwise not.

Halton sequences Calculates a matrix of uniform or normal deviated halton low discrepancy numbers. Let us note that Halton sequence in dimension is the Van Der Corput sequence. The Halton sequence starts from k=1 when initialized with init = TRUE and no not depending on machine time usetime = FALSE. This is the default. When init = FALSE, the sequence is not initialized (to 1) and starts from the last term. We can also use the machine time to start the sequence with usetime = TRUE, which overrides init.

See the pdf vignette for details.

Value

torus, halton and sobol generates random variables in]0,1[. It returns a nxdim matrix, when dim>1 otherwise a vector of length n.

Author(s)

Christophe Dutang and Diethelm Wuertz

References

Bratley P., Fox B.L. (1988); *Algorithm 659: Implementing Sobol's Quasirandom Sequence Generator*, ACM Transactions on Mathematical Software 14, 88–100.

Joe S., Kuo F.Y. (1998); Remark on Algorithm 659: Implementing Sobol's Quaisrandom Sequence Generator.

Planchet F., Jacquemin J. (2003), *L'utilisation de methodes de simulation en assurance*. Bulletin Français d'Actuariat, vol. 6, 11, 3-69. (available online)

See Also

 $\sf pseudoRNG$ for pseudo random number generation, .Random.seed for what is done in R about random number generation.

```
# (1) the Torus algorithm
torus(100)
# example of setting the seed
setSeed(1)
torus(5)
setSeed(6)
torus(5)
#the same
setSeed(1)
torus(10)
#no use of the machine time
torus(10, use=FALSE)
#Kolmogorov Smirnov test
#KS statistic should be around 0.0019
ks.test(torus(1000), punif)
#KS statistic should be around 0.0003
ks.test(torus(10000), punif)
#the mixed Torus sequence
torus(10, mix=TRUE)
par(mfrow = c(1,2))
acf(torus(10<sup>6</sup>))
acf(torus(10^6, mix=TRUE))
#usage of the init argument
torus(5)
torus(5, init=FALSE)
#should be equal to the combination of the two
#previous call
torus(10)
# (2) Halton sequences
# uniform variate
halton(n = 10, dim = 5)
# normal variate
halton(n = 10, dim = 5, normal = TRUE)
```

```
#usage of the init argument
halton(5)
halton(5, init=FALSE)
#should be equal to the combination of the two
#previous call
halton(10)
# some plots
par(mfrow = c(2, 2), cex = 0.75)
hist(halton(n = 5000, dim = 1), main = "Uniform Halton",
  xlab = "x", col = "steelblue3", border = "white")
hist(halton(n = 5000, dim = 1, norm = TRUE), main = "Normal Halton",
  xlab = "x", col = "steelblue3", border = "white")
# (3) Sobol sequences
# uniform variate
sobol(n = 10, dim = 5, scrambling = 3)
# normal variate
sobol(n = 10, dim = 5, scrambling = 3, normal = TRUE)
# some plots
hist(sobol(5000, 1, scrambling = 2), main = "Uniform Sobol",
  xlab = "x", col = "steelblue3", border = "white")
hist(sobol(5000, 1, scrambling = 2, normal = TRUE), main = "Normal Sobol",
  xlab = "x", col = "steelblue3", border = "white")
#usage of the init argument
sobol(5)
sobol(5, init=FALSE)
#should be equal to the combination of the two
#previous call
sobol(10)
# (4) computation times on my macbook, mean of 1000 runs
## Not run:
# algorithm time in seconds for n=10^6
# Torus algo 0.058
# mixed Torus algo
                            0.087
# Halton sequence 0.878
# Sobol sequence 0.214
n <- 1e+06
mean( replicate( 1000, system.time( torus(n), gcFirst=TRUE)[3]) )
mean( replicate( 1000, system.time( torus(n, mixed=TRUE), gcFirst=T)[3]) )
mean( replicate( 1000, system.time( halton(n), gcFirst=TRUE)[3]) )
```

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```
mean( replicate( 1000, system.time( sobol(n), gcFirst=TRUE)[3]) )
## End(Not run)
```

rngWELLScriptR An implementation of the recurrence of WELL generators in R language

Description

An implementation of the recurrence of WELL generators in R language for testing purposes. It is too slow to be used for random number generation.

Usage

```
rngWELLScriptR(n, s, generator, includeState=FALSE)
```

Arguments

n	Integer. The length of the output sequence.	

An 0,1-matrix representing the state of the required WELL generator as obtained

by getWELLState().

generator Character string. Name of the generator from the list "512a", "521a", "521b", "607a", "607b", "800a",

includeState Logical. Controls, whether the output should contain the final internal state

additionally to the numerical output or not.

Value

If includeState=FALSE, a numeric vector of length n containing the numerical output of the generator. If includeState=TRUE, a list with components x (the numerical output) and state (the final internal state of the generator).

See Also

```
getWELLState
```

```
set.generator("WELL", order=512, version="a", seed=123456)
s <- getWELLState()
x <- runif(500)
y <- rngWELLScriptR(500, s, "512a")
all(x == y)
# [1] TRUE</pre>
```

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runifInterface	Functions for using runif() and rnorm() with randtoolbox generators

Description

These functions allow to set some of the random number generators from randtoolbox package to be used instead of the standard generator in the functions, which use random numbers, for example runif(), rnorm(), sample() and set.seed().

Usage

Arguments

•	
name	A character string for the RNG name.
parameters	A numeric or character vector describing a specific RNG from the family specified by the name.
seed	A number, whose value is an integer between 0 and 2^32-1 , to be used as a seed.
•••	Arguments describing named components of the vector of parameters, if argument parameters is missing or NULL.
only.dsc	Logical. If TRUE, a description of the specified RNG is returned and the generator is not initialized.
description	A list describing a specific RNG as created by $set.generator(,only.dsc=TRUE)$ or $get.description()$.

Details

Random number generators provided by R extension packages are set using RNGkind("user-supplied"). The package **randtoolbox** assumes that this function is not called by the user directly and is called from the functions set.generator() and put.description().

Random number generators in **randtoolbox** are represented at the R level by a list containing mandatory components name,parameters,state and possibly an optional component authors. The function set.generator() internally creates this list from the user supplied information and then runs put.description() on this list, which initializes the generator. If the generator is initialized, then the function get.description() may be used to get the actual state of the generator, which may be stored in an R object and used later in put.description() to continue the sequence of the random numbers from the point, where get.description() was called. This may be used to generate several independent streams of random numbers generated by different generators.

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The component parameters is a character or a numeric vector, whose structure is different for different types of the generators. This vector may be passed to set.generator(), if it is prepared before its call, however, it is also possible to pass its named components via the ... parametr of set.generator() and the vector parameters is created internally. If the vector parameters is not supplied and the arguments in ... are not sufficient to create it, an error message is produced.

Linear congruential generators: Parameters for the linear congruential generators (name="congruRand") are integers represented either as a character or a numeric vector. The components are

mod The modulus.

mult The multiplier.

incr The increment.

WELL generators: Parameters for the WELL generators is a character vector with components

order The number of bits in the internal state. Possible values are 512, 521, 607, 800, 1024, 19937, 21701, 23209, 44497.

version The version letter "a", "b", or "c" to be appended to the order.

The concatenation of order and version should belong to "512a", "521a", "521b", "607a", "607b", "800a", "800b", "1 When order and version are specified in . . . parametr of set.generator(), then the parameter order is optional and if missing, it is assumed that the parameter version contains also the number of bits in the internal state and the combination belongs to the list above.

Mersenne Twister 2002 generator: Parameters for the Mersenne Twister 2002 is a character vector with components

 $\textbf{initialization} \ \ Either \ "init 2002" \ or \ "array 2002". \ The \ initialization \ to \ be \ used.$

resolution Either 53 or 32. The number of random bits in each number.

Value

set.generator() with the parameter only.dsc=TRUE and get.description() return the list describing a generator. put.description() with the parameter only.dsc=TRUE (the default) and put.description() return NULL.

Author(s)

Petr Savicky and Christophe Dutang

See Also

RNGkind and .Random. seed for random number generation in R.

```
#set WELL19937a
set.generator("WELL", version="19937a", seed=12345)
runif(5)

#Store the current state and generate 10 random numbers
storedState <- get.description()
x <- runif(10)</pre>
```

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```
## Not run:
#Park Miller congruential generator
set.generator(name="congruRand", mod=2^31-1, mult=16807, incr=0, seed=12345)
runif(5)
setSeed(12345)
congruRand(5, dim=1, mod=2^31-1, mult=16807, incr=0)
# the Knuth Lewis RNG
set.generator(name="congruRand", mod="4294967296", mult="1664525", incr="1013904223", seed=1)
runif(5)
setSeed(1)
congruRand(5, dim=1, mod=4294967296, mult=1664525, incr=1013904223)
## End(Not run)
#Restore the generator from storedState and regenerate the same numbers
put.description(storedState)
x == runif(10)
# generate the same random numbers as in Matlab
set.generator("MersenneTwister", initialization="init2002", resolution=53, seed=12345)
runif(5)
# [1] 0.9296161 0.3163756 0.1839188 0.2045603 0.5677250
# Matlab commands rand('twister', 12345); rand(1, 5) generate the same numbers,
# which in short format are 0.9296
                                                 0.1839
                                      0.3164
                                                            0.2046
                                                                      0.5677
#Restore the original R setting
set.generator("default")
RNGkind()
```

serial.test

the Serial test

Description

The Serial test for testing random number generators.

Usage

```
serial.test(u , d = 8, echo = TRUE)
```

Arguments

u sample of random numbers in]0,1[.
echo logical to plot detailed results, default TRUE

d a numeric for the dimension, see details. When necessary we assume that d is a multiple of the length of u.

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Details

We consider a vector u, realisation of i.i.d. uniform random variables U_1, \ldots, U_n .

The serial test computes a serie of integer pairs (p_i, p_{i+1}) from the sample u with $p_i = \lfloor u_i d \rfloor$ (u must have an even length). Let n_j be the number of pairs such that $j = p_i \times d + p_{i+1}$. If d=2, we count the number of pairs equals to 00, 01, 10 and 11. Since all the combination of two elements in $\{0, \ldots, d-1\}$ are equiprobable, the chi-squared statistic is

$$S = \sum_{j=0}^{d-1} \frac{n_j - n/(2d^2)^2}{n/(2d^2)}.$$

Value

a list with the following components:

statistic the value of the chi-squared statistic.

p.value the p-value of the test.

observed the observed counts.

expected the expected counts under the null hypothesis.

residuals the Pearson residuals, (observed - expected) / sqrt(expected).

Author(s)

Christophe Dutang.

References

Planchet F., Jacquemin J. (2003), *L'utilisation de methodes de simulation en assurance*. Bulletin Français d'Actuariat, vol. 6, 11, 3-69. (available online)

L'Ecuyer P. (2001), Software for uniform random number generation distinguishing the good and the bad. Proceedings of the 2001 Winter Simulation Conference. (available online)

L'Ecuyer P. (2007), *Test U01: a C library for empirical testing of random number generators*. ACM Trans. on Mathematical Software 33(4), 22.

See Also

```
other tests of this package freq.test, gap.test, poker.test, order.test and coll.test ks.test for the Kolmogorov Smirnov test and acf for the autocorrelation function.
```

```
# (1)
#
serial.test(runif(1000))
print( serial.test( runif(1000000), d=2, e=FALSE) )
# (2)
#
```

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```
serial.test(runif(5000), 5)
```

soboltestfunctions

Some test functions for Sobol sequences

Description

Some test functions for Sobol sequences

Usage

```
sobol.directions(polycoef, deg, prevmj, nbpoint, echo=FALSE)
sobol.basic(n, polycoef, deg, prevmj, echo=FALSE, output=c("real", "integer"))
sobol.R(n, d, echo=FALSE)
int2bit(x)
bit2int(x)
bit2unitreal(x)
```

Arguments

n number of observations.d dimension of observations.

polycoef binary coefficients of the primitive polynomial.

degree of the primitive polynomial.

prevmj matrix where each column is the binary representation of integers m_i

nbpoint number of additionnal integers m_j . echo a logical to show some traces.

output a character string either "real" or "integer" to specify whether the radical

inverse function is computed output="real" or not output="integer".

x an integer to convert in base 2 or its binary representation

Details

sobol.directions computes the direction numbers used when computing Sobol sequences.

sobol.basic compute the Sobol sequence in one dimension according to a primitive polynomial and specified integers m_i .

int2bit computes the binary representation of the integer part of a real, bit2int computes an integer from its binary representation. bit2unitreal computes the radical inverse function in base 2.

Value

a vector of length n for sobol.basic or a matrix for sobol.directions.

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Author(s)

Christophe Dutang

References

Glasserman P., (2003); Monte Carlo Methods in Financial Engineering, Springer.

See Also

quasiRNG for quasi random number generation.

```
#page 306 of Glassermann
p13 <- int2bit(13)
prevterm <- sapply(c(1,3,3), int2bit)
colnames(prevterm) <- apply(prevterm, 2, bit2int)
sobol.directions(p13, 3, prevterm, 2, echo=FALSE)

sobol.basic(15, p13, 3, c(1,3,3), echo=FALSE, output="i")

#page 307 of Glassermann
trueval <- c(16, 24, 8, 12, 28, 20, 4, 30, 14,
6, 22, 18, 2, 10, 26, 5, 21, 29, 13, 9, 25, 17,
1, 27, 11, 3, 19, 23, 7, 15, 31)/32
cbind(sobol.basic(31, p13, 3, prevterm, echo=FALSE, output="r"), trueval)</pre>
```

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