Spectral analysis

Fundamental elements

Axel Hutt

data sampling

Fourier analysis

errors in analysis

linear filters

time-frequency analysis

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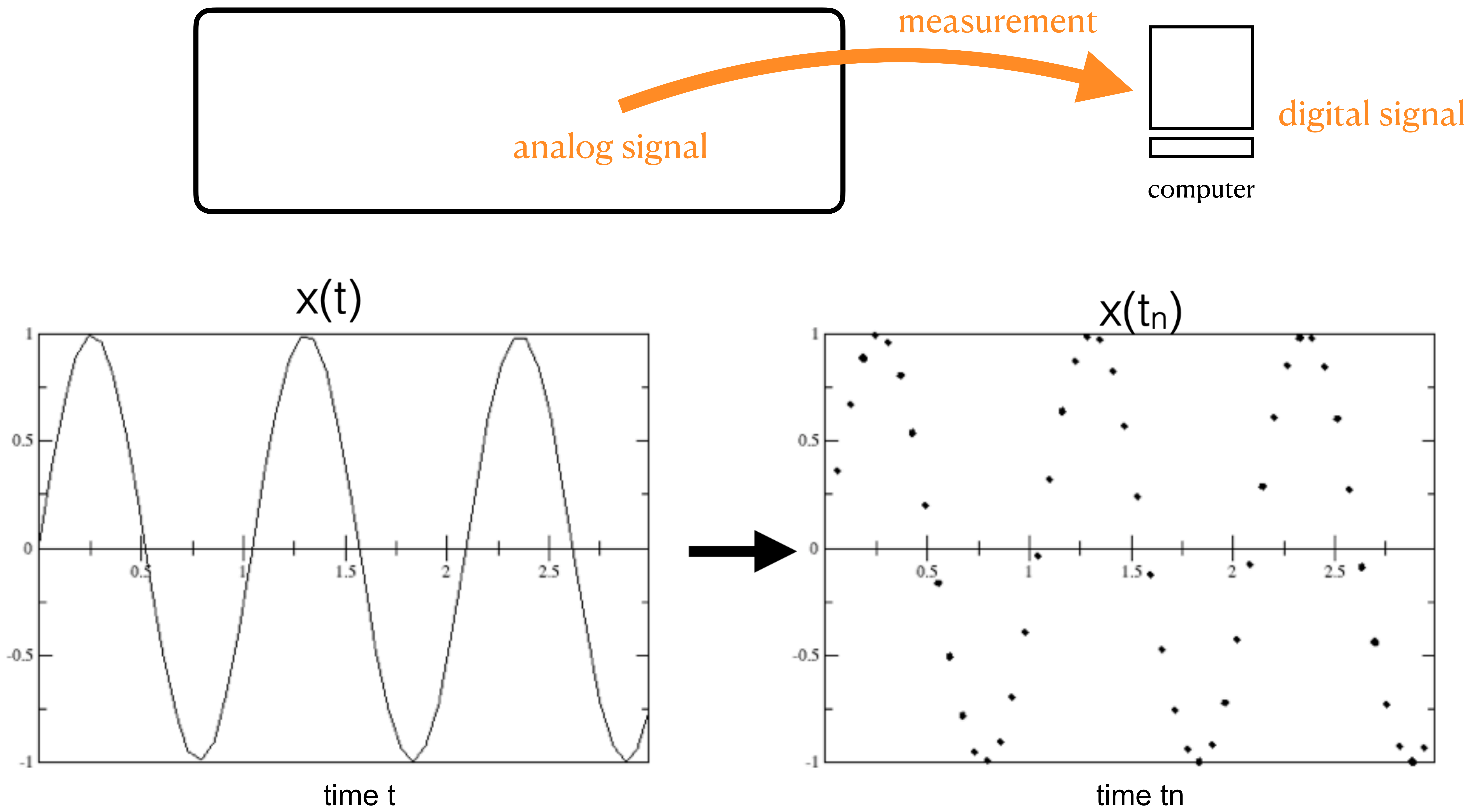
Fourier analysis

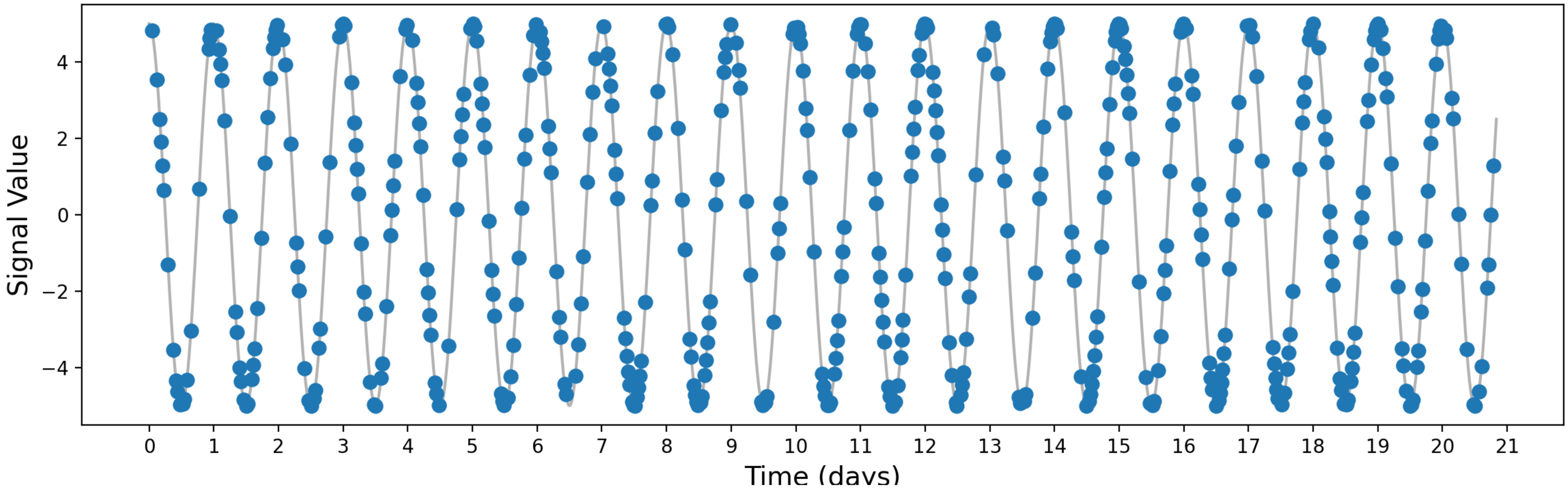
errors in analysis

linear filters

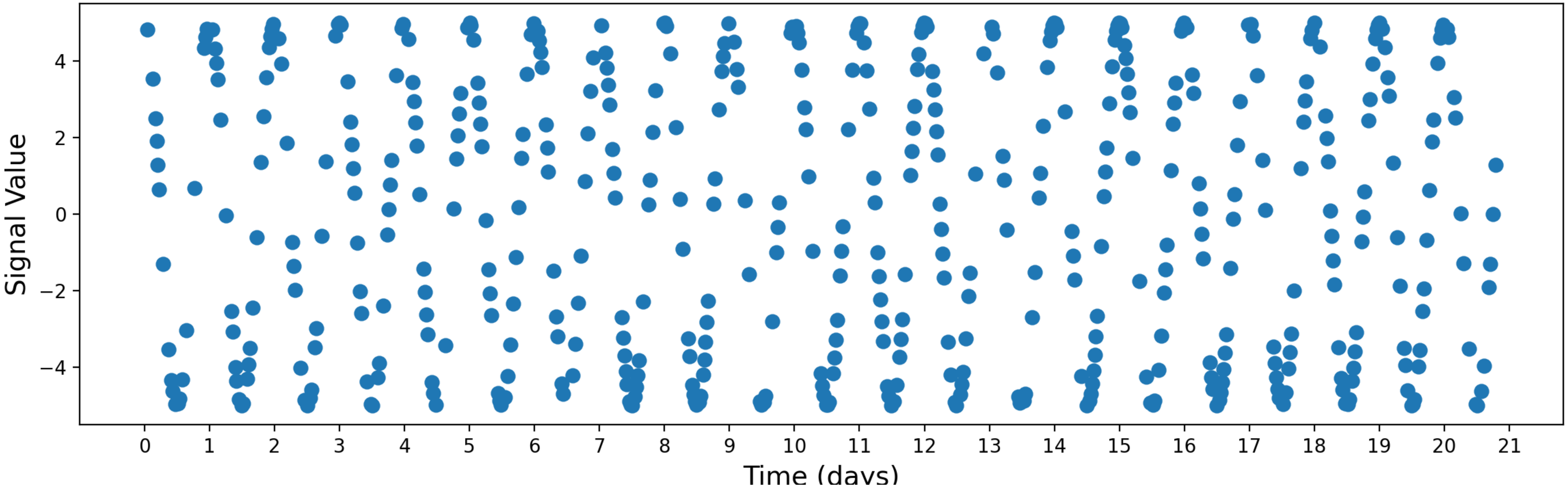
time-frequency analysis


measurement
analog signal
digital signal
computer
x(t)
time t
x(tn)
time tn

unevenly sampled data

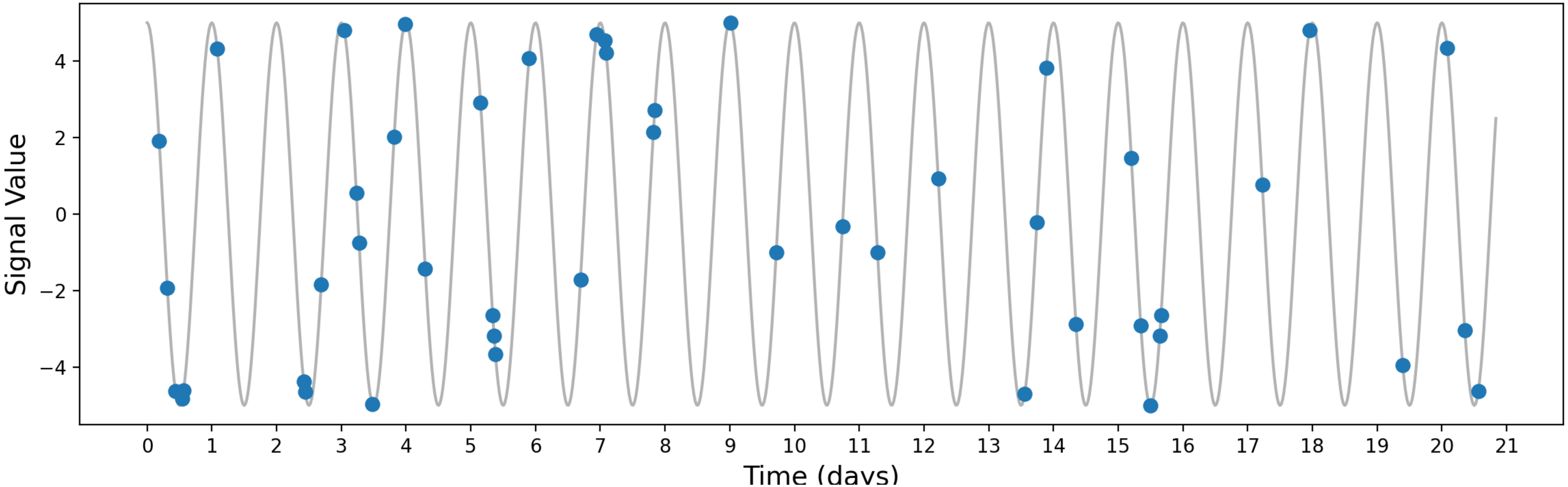


unevenly sampled data

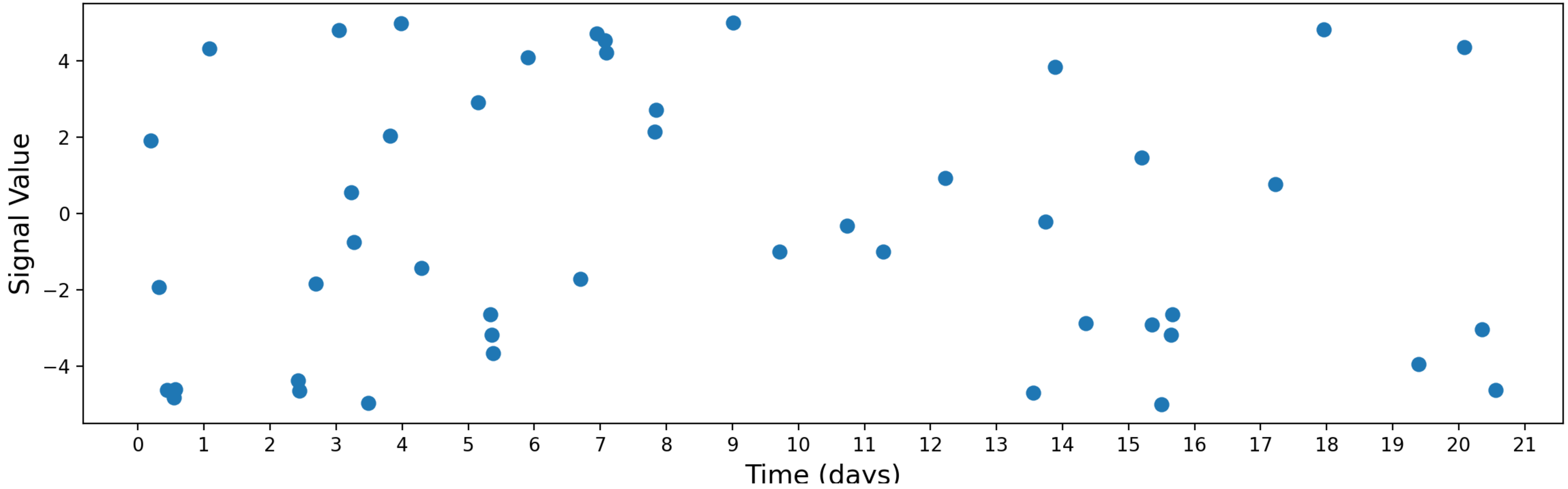


oscillations may be visible

unevenly sampled data - fewer data

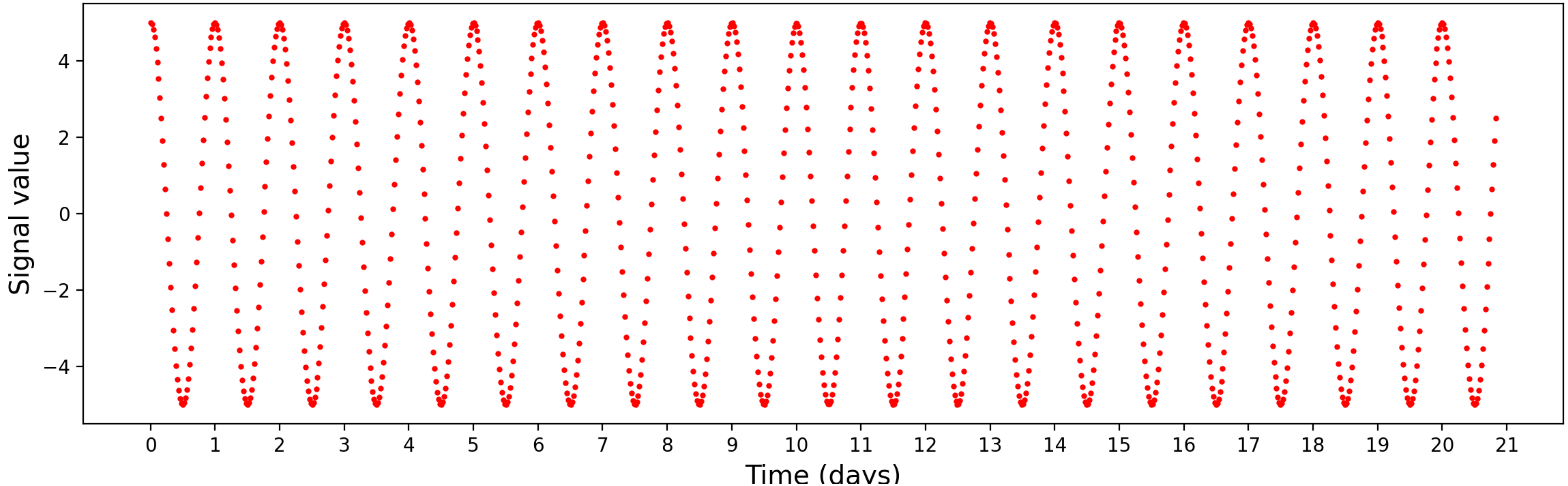


unevenly sampled data - fewer data

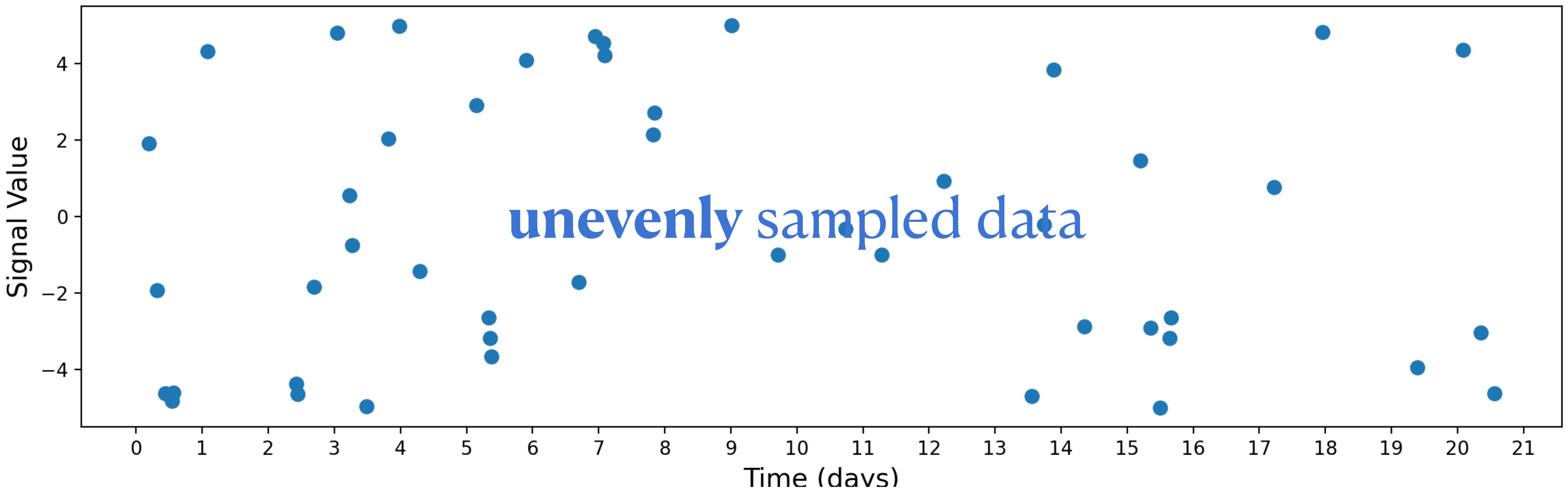


oscillations not visible anymore

evenly sampled data

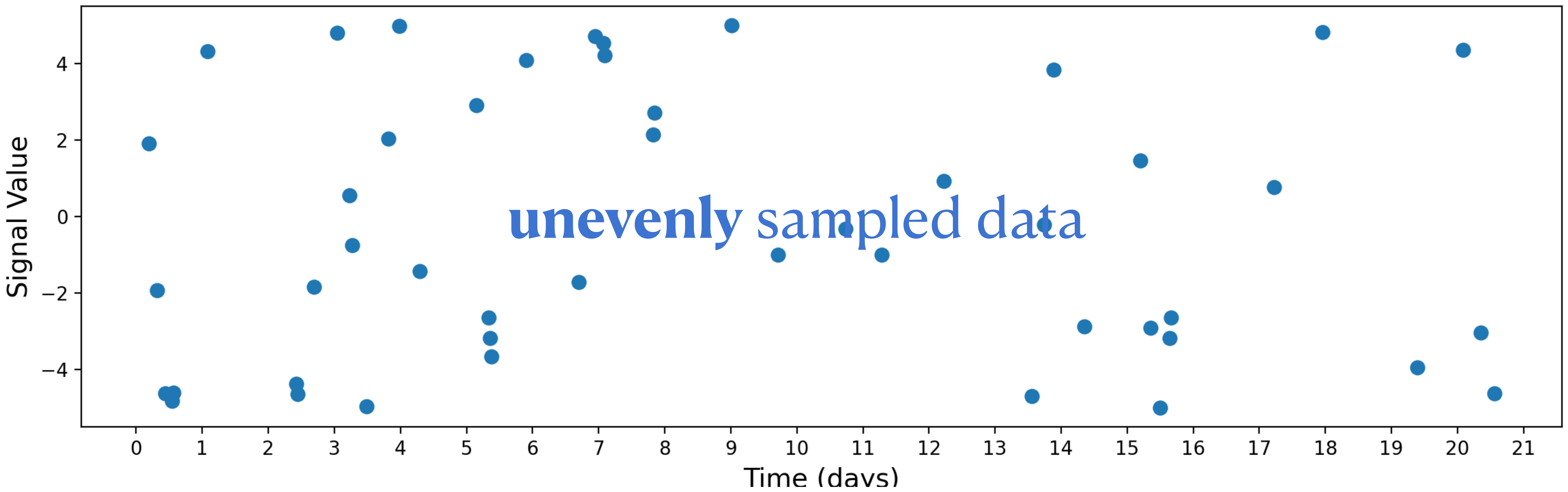


oscillations clearly visible



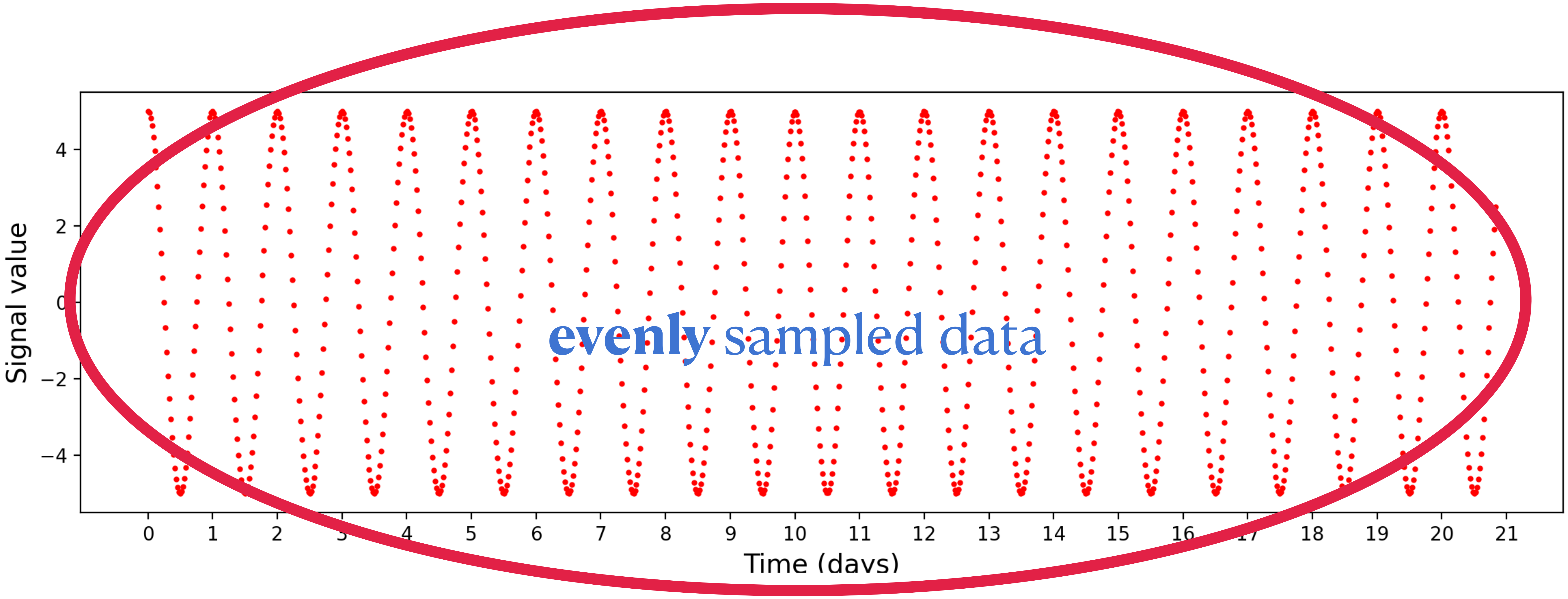
unevenly sampled data

see later slides

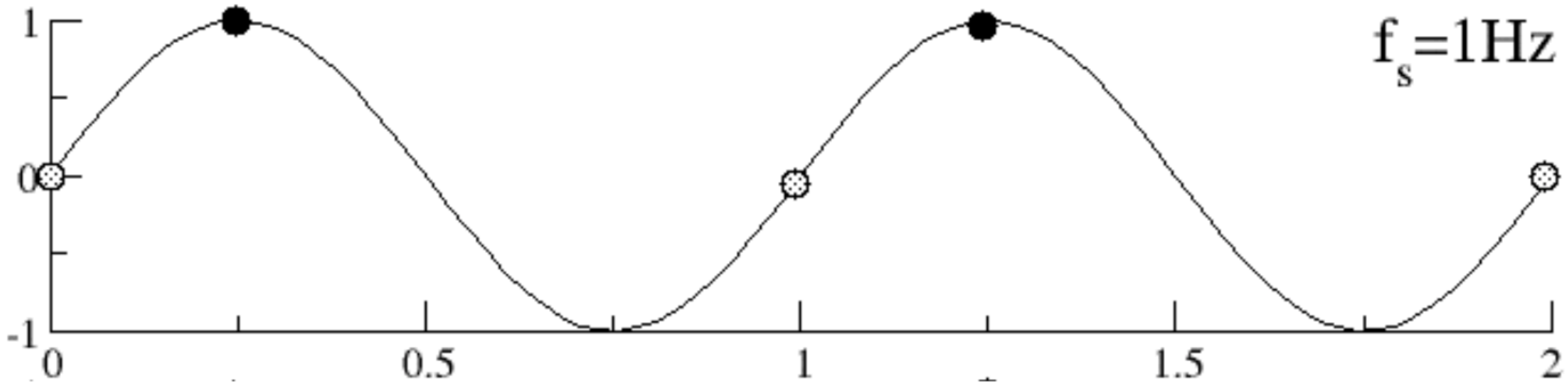


see later slides

see next slides

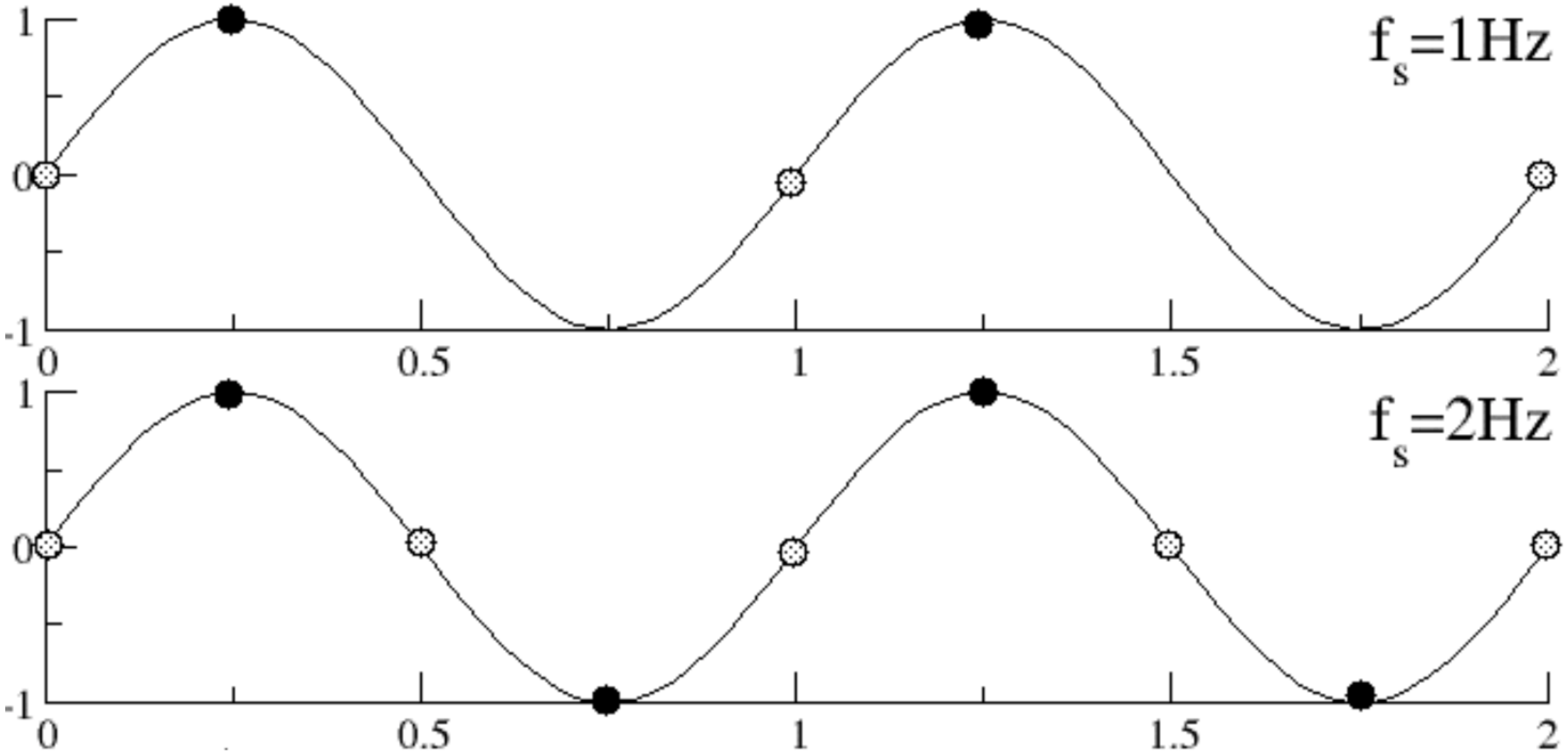


$T_m = 1s$



$T_s = 1s$

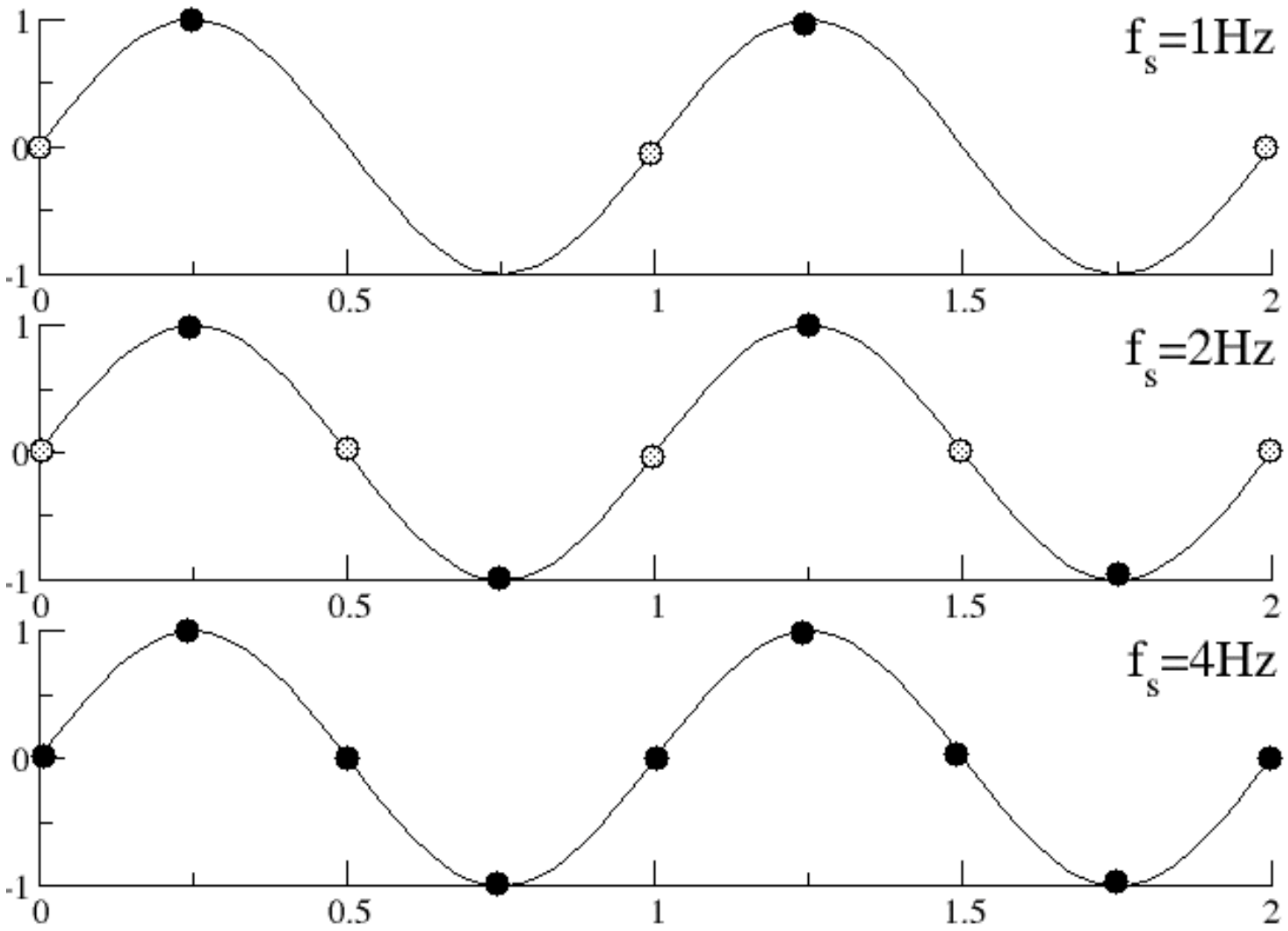
$T_m = 1s$



$T_s = 1s$

$T_s = 0.5s$

$T_m = 1s$

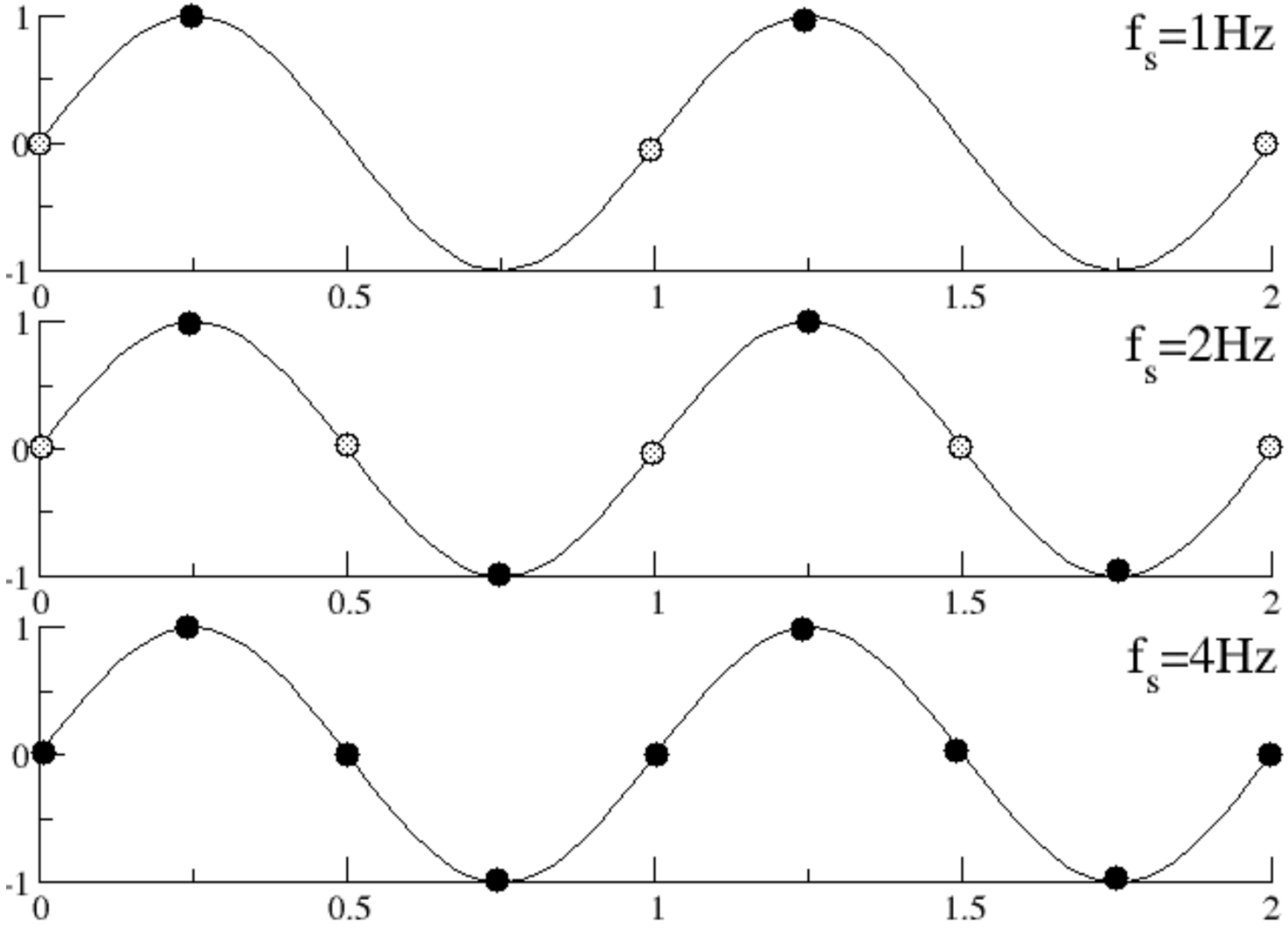


$T_s = 1s$

$T_s = 0.5s$

$T_s = 0.25s$

$$T_m = 1s$$



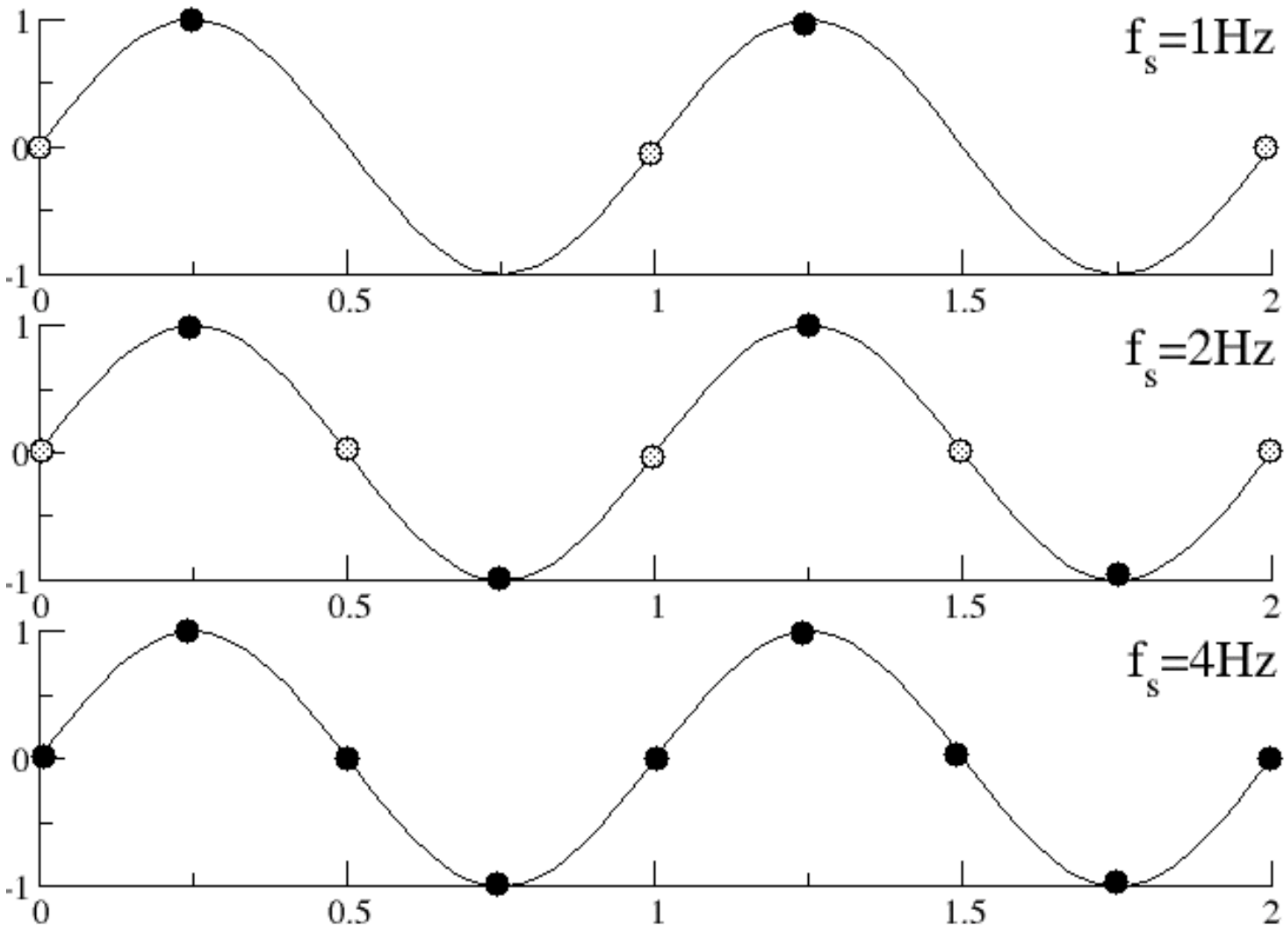
$$T_s = 1s$$

$$T_s = 0.5s$$

$$T_s = 0.25s$$

good sampling: $T_s \leq T_m/2$

$$T_m = 1s$$



$$T_s = 1s$$

$$T_s = 0.5s$$

$$T_s = 0.25s$$

Nyquist law : $\boxed{f_s \geq 2f_m}$

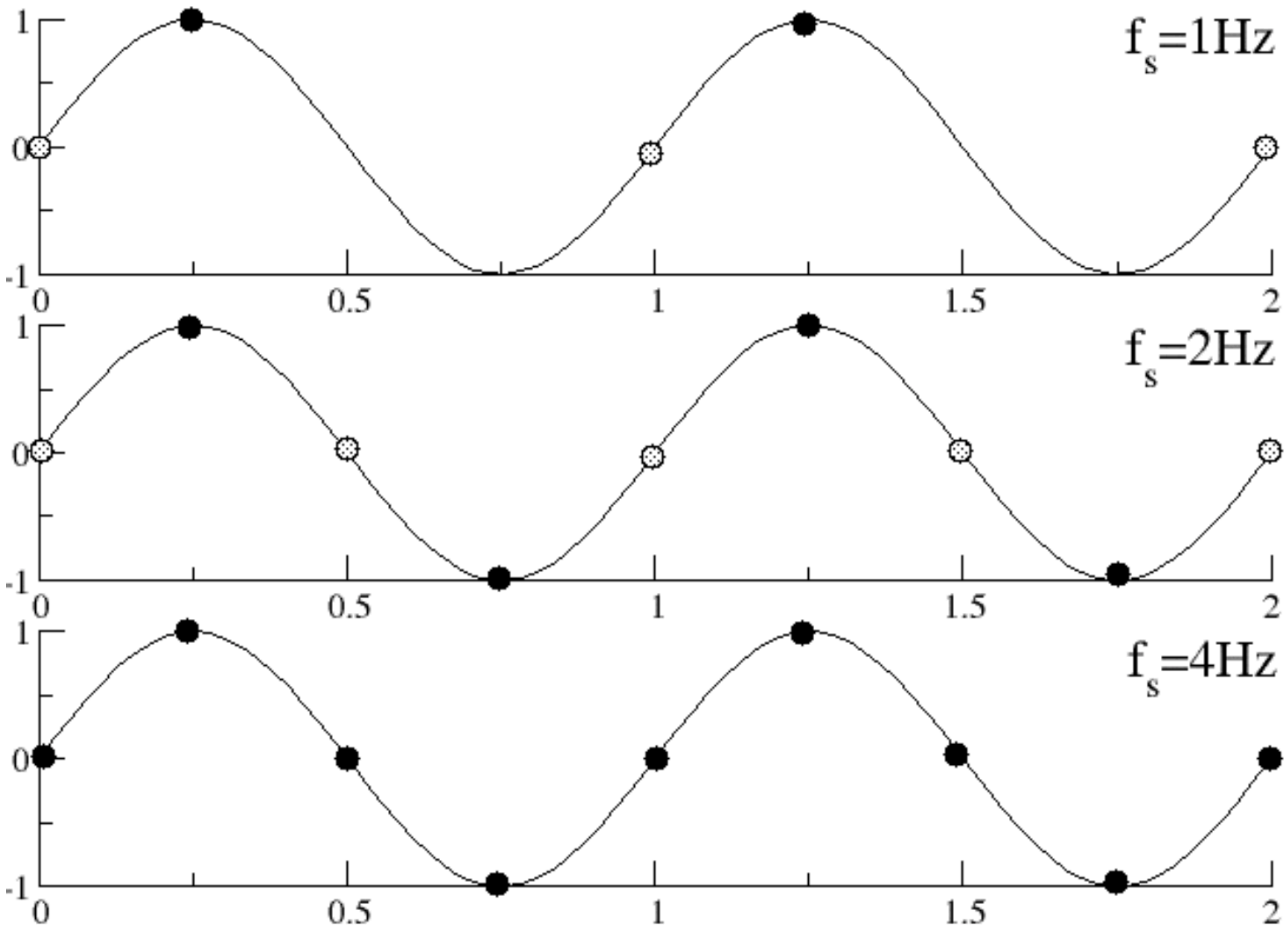
Sampling theorem
(Shannon/Nyquist/Whitaker/Kotelnikov)

given:
- continuous function s(t)
- s(t) has maximum frequency f_m
- s(t) is sampled with frequency f_s , i.e. $s(t) \to s(t_n)\ ,\ t_n = n\Delta t = n/f_s$

hypothesis: information loss by sampling continuous function s(t)

objective: find sampling frequency f_s, for which no information loss occurs

solution: $f_s \geq 2f_m$



comment: in practice it is a good idea to choose $f_s \geq 4 f_m$

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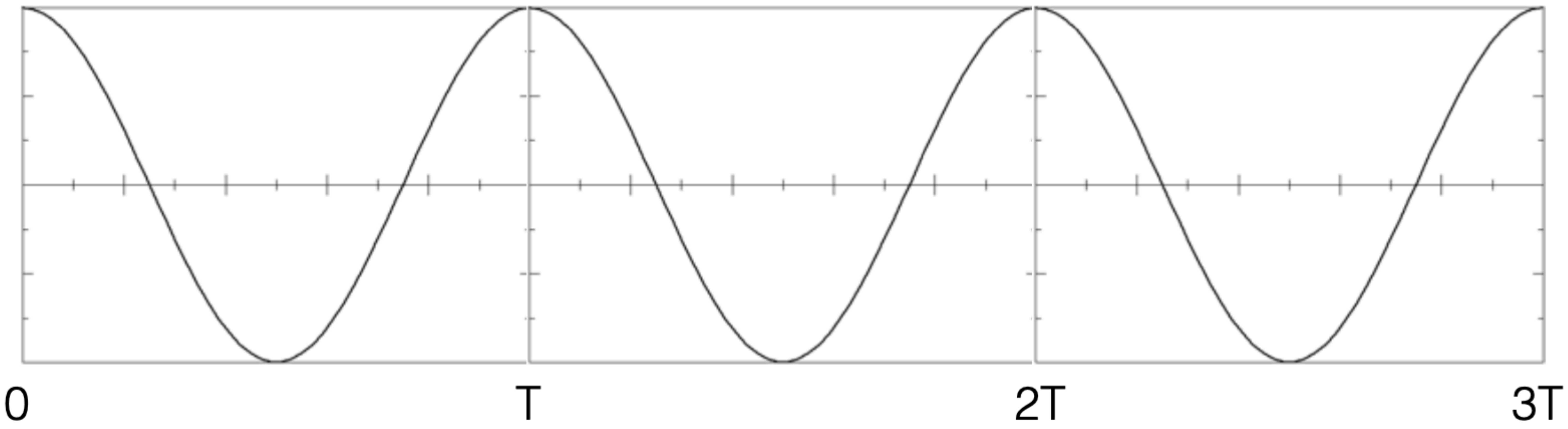
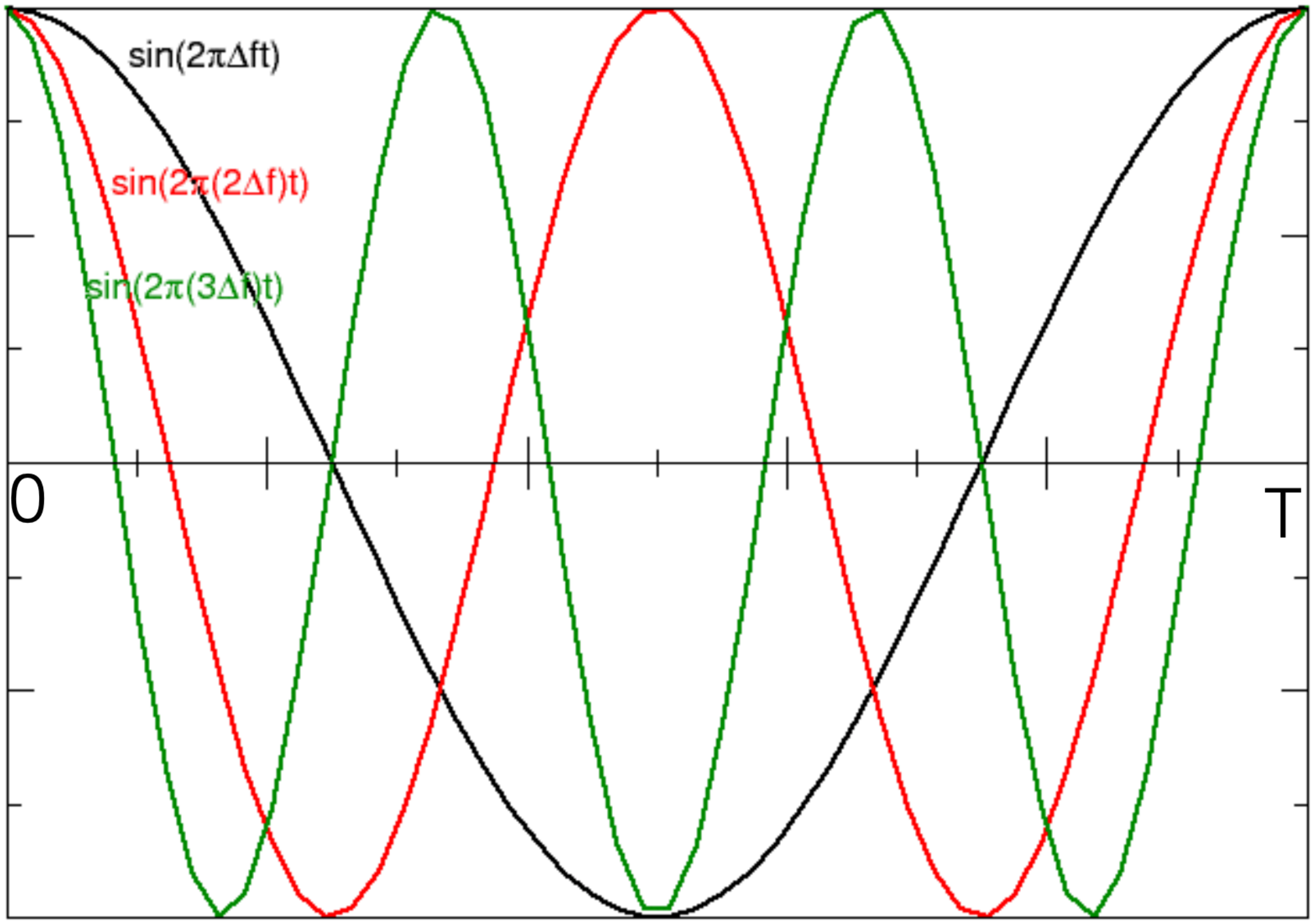
all-time assumption: the signal under study is periodic



the signal under study is periodic

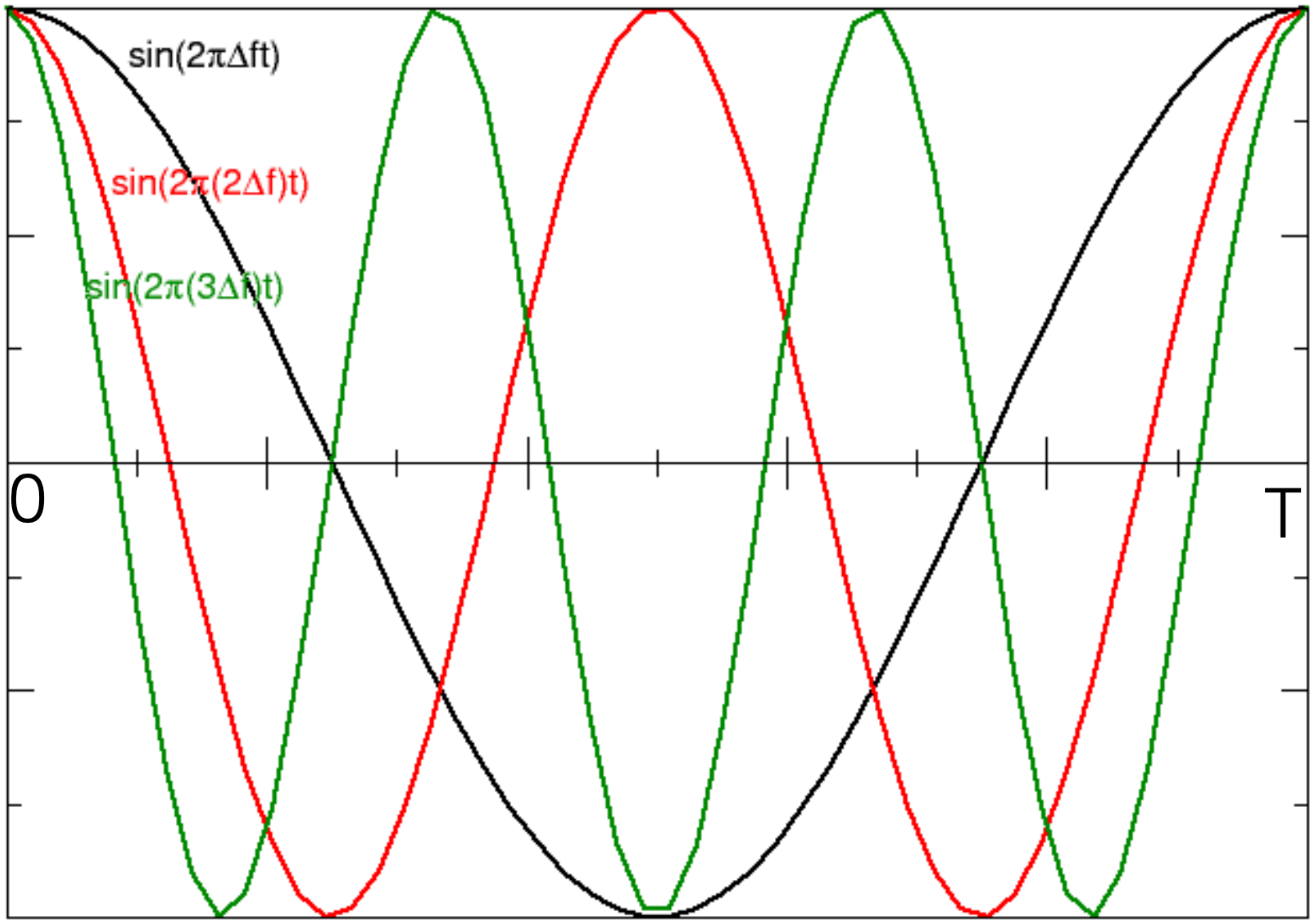


one period in interval

two periods in interval

three periods in interval

the signal under study is periodic



one period in interval

two periods in interval

three periods in interval

smallest frequency: $\boxed{\Delta f = \frac{1}{T}}$

$\boxed{f_n = n\Delta f}$