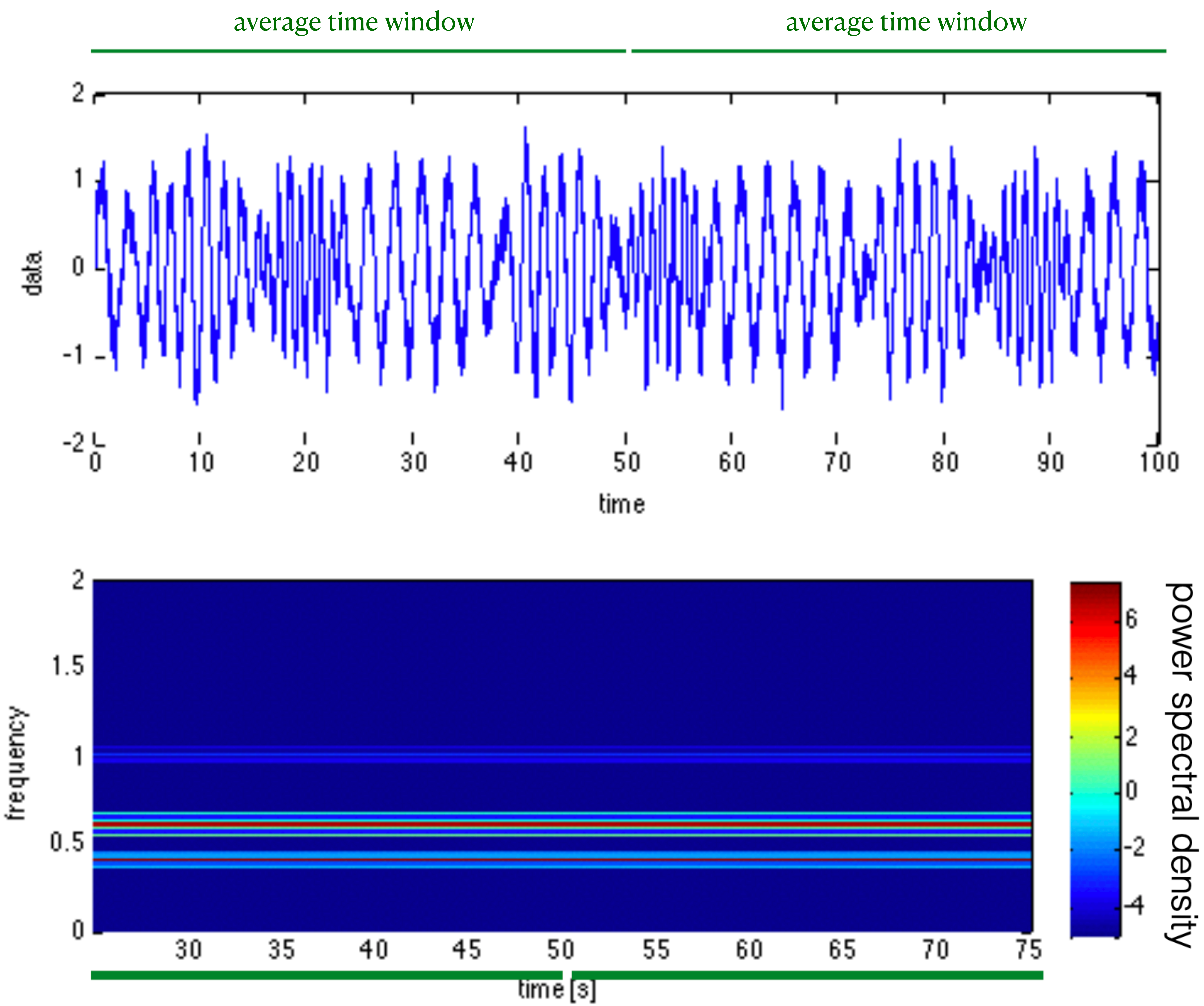
data sampling

Fourier analysis

errors in analysis

linear filters

time-frequency analysis



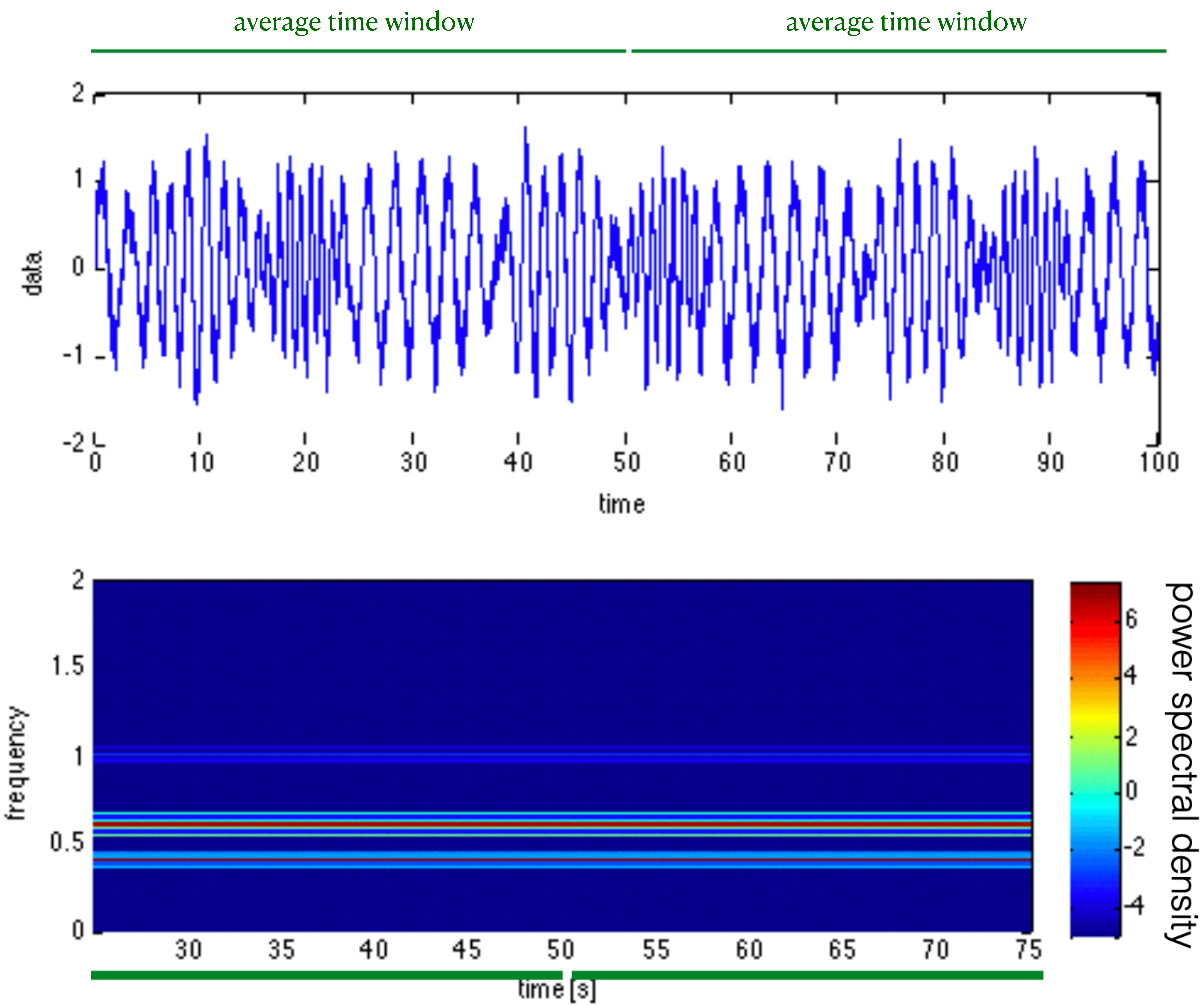
time window:

$\Delta T = 50s$

frequency resolution:

$\Delta f = 0.02\text{Hz}$

very good frequency resolution, very bad time resolution



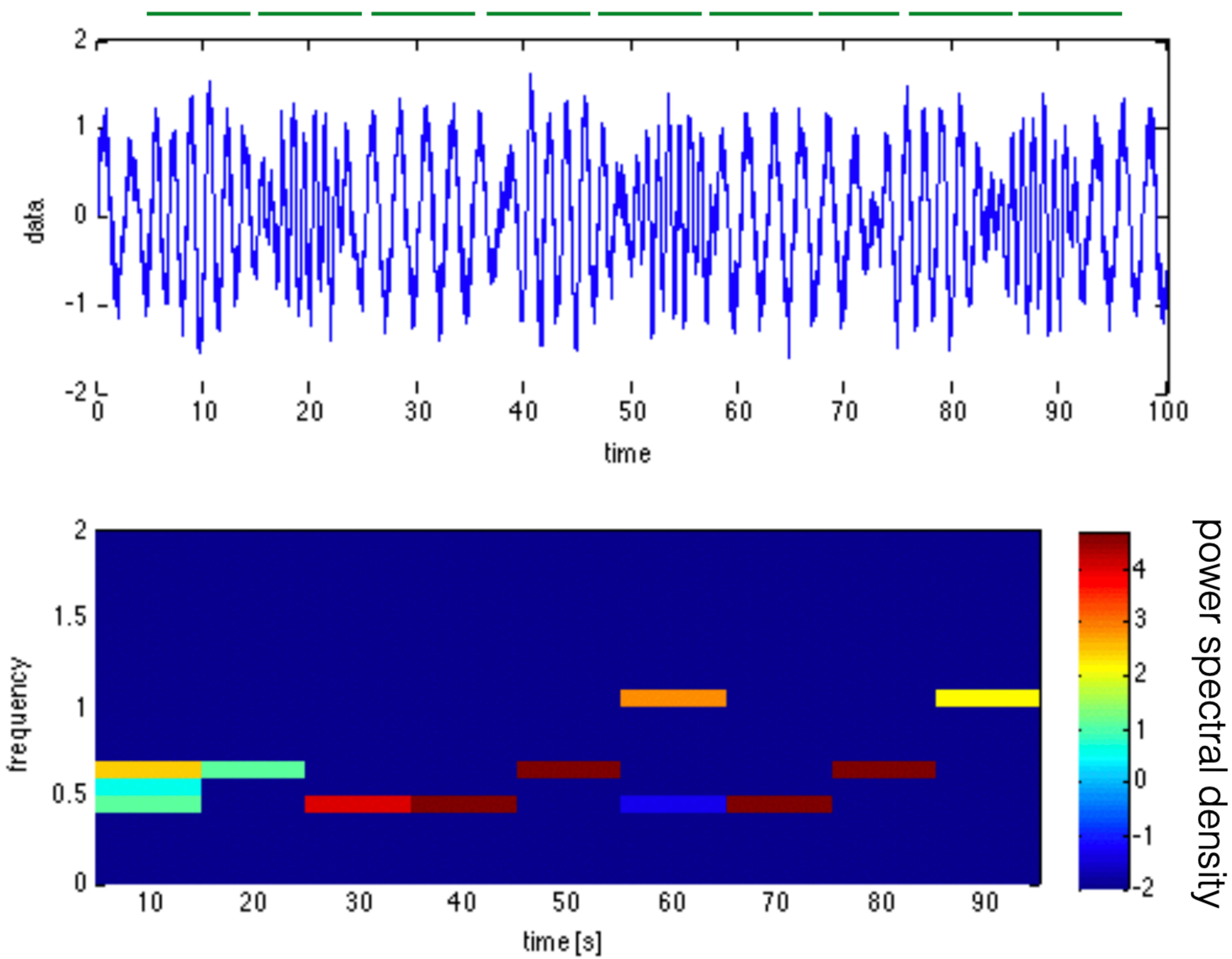
time window:

$\Delta T = 50\mathrm{s}$

frequency resolution:

$\Delta f = 0.02\mathrm{Hz}$

time-independent frequency on large time scale



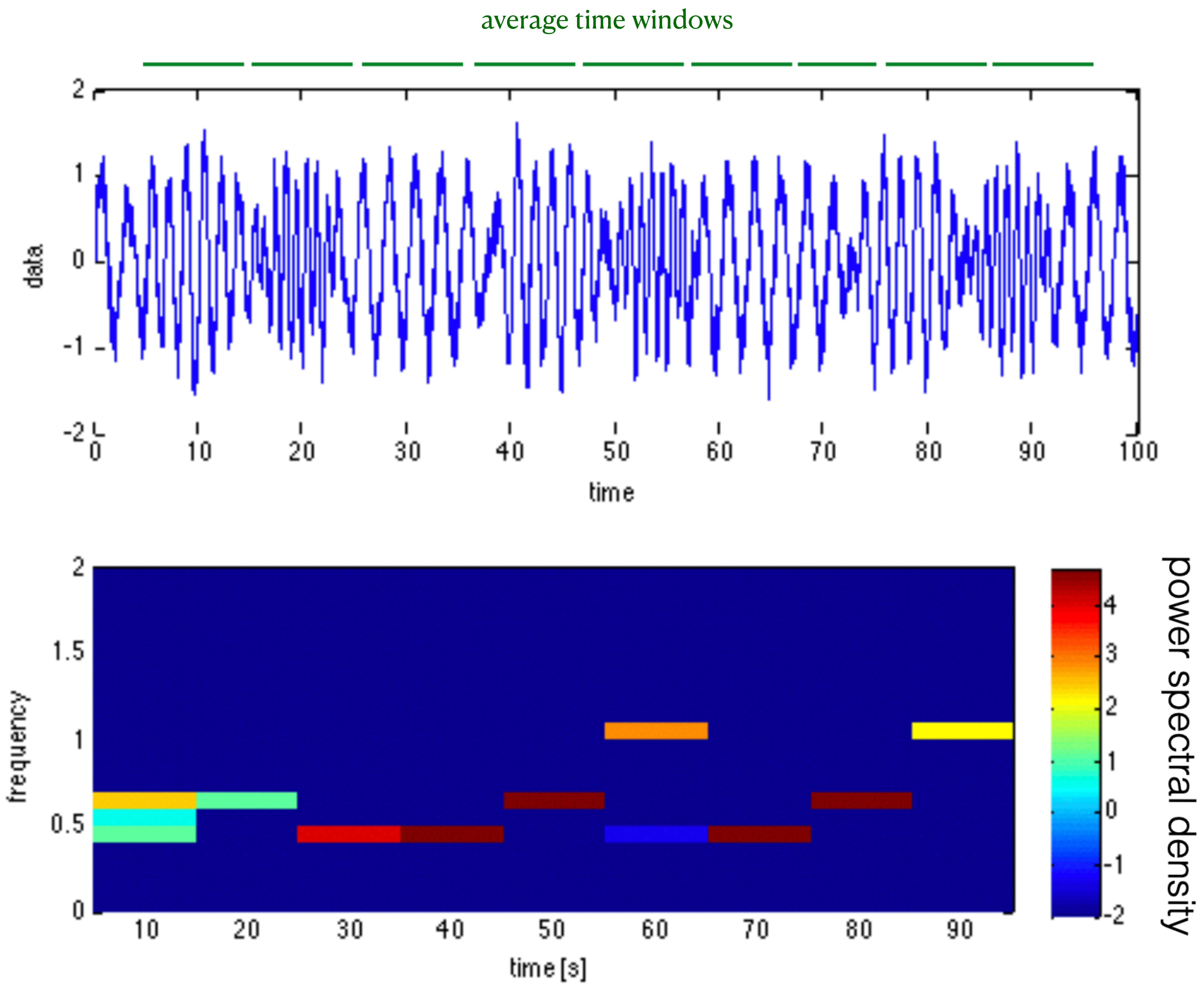
time window:

$\Delta T = 10s$

frequency resolution:

$\Delta f = 0.1\text{Hz}$

good frequency resolution — bad time resolution



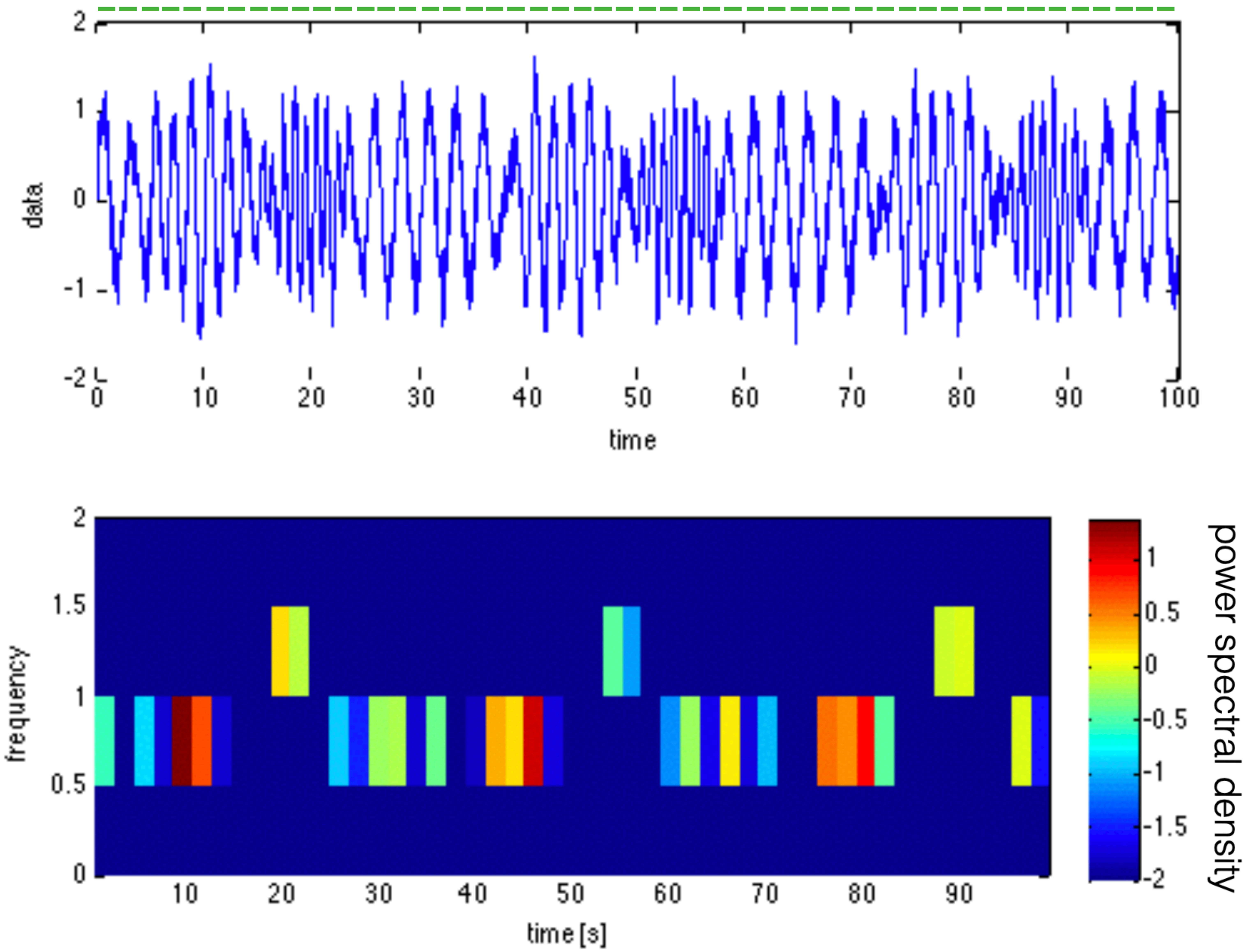
time window:

$\Delta T = 10s$

frequency resolution:

$\Delta f = 0.1\text{Hz}$

time-dependent frequency on shorter time scale



time window:

Δ**T=2s**

frequency resolution:

$$\Delta f = 0.5\text{Hz}$$

bad frequency resolution — good time resolution

it is necessary to balance time and frequency resolution since

$$\Delta T \sim \frac{1}{\Delta f}$$

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$$\longrightarrow \quad \boxed{\Delta T \Delta f = \text{const}}$$

Heisenberg uncertainty relation

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$$\Delta T \sim \frac{1}{\Delta f}$$

$$\longrightarrow \quad \boxed{\Delta T \Delta f = \mathrm{const}}$$

Heisenberg uncertainty relation

Question: **is there an instantaneous frequency ?**

it is necessary to balance time and frequency resolution since

$$\Delta T \sim \frac{1}{\Delta f}$$

$$\Longrightarrow \quad \boxed{\Delta T \Delta f = \mathrm{const}}$$ Heisenberg uncertainty relation

Question: **is there an instantaneous frequency ?**

we come back later to this question.......

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non-Fourier analysis

Short-time Fourier Transform (STFT)

$$s(t,f) = \int_{-\infty}^{\infty} w(t-t')x(t')e^{-i2\pi f t'}dt'$$

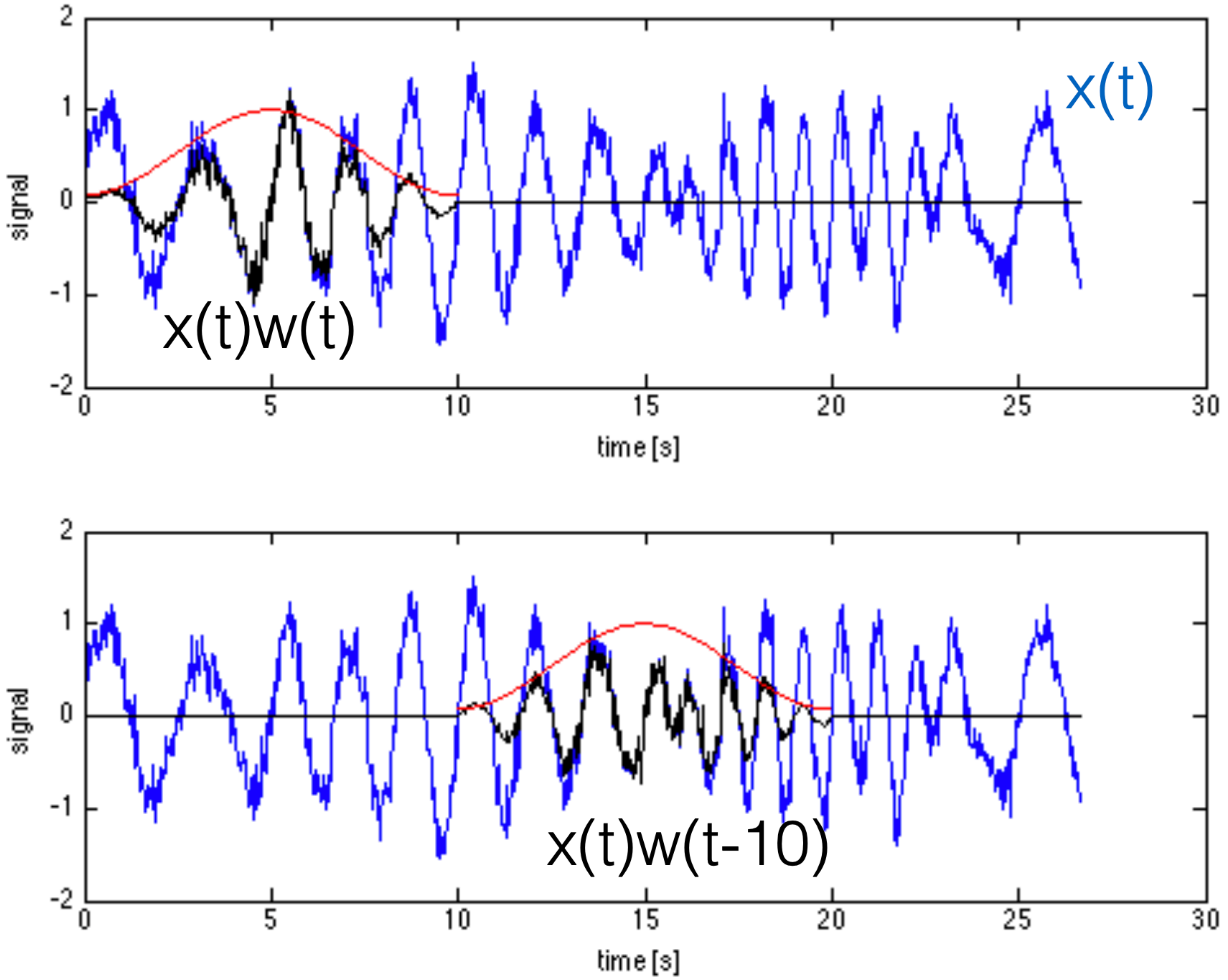
filter window

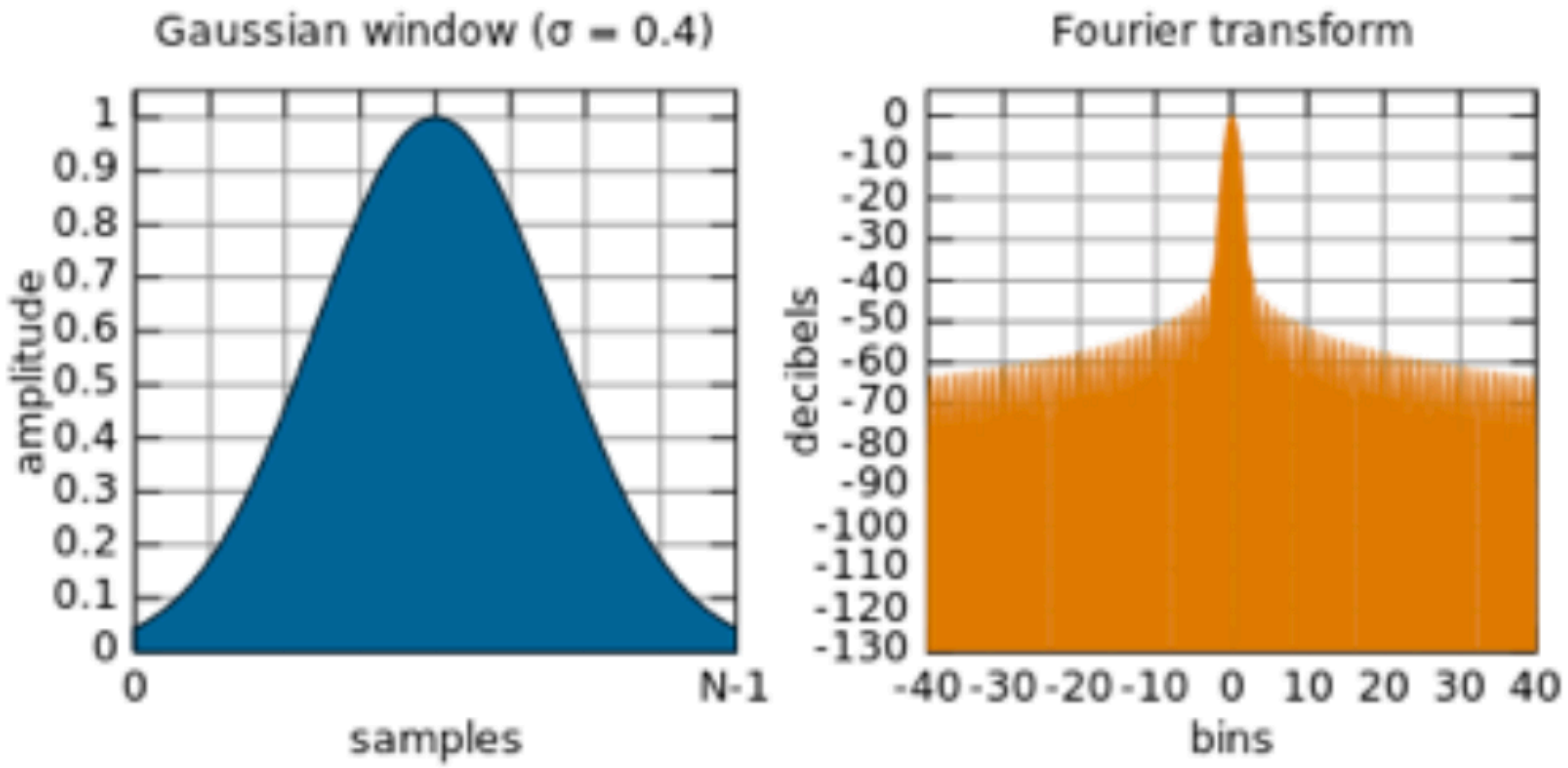
input to filter

finite size !!

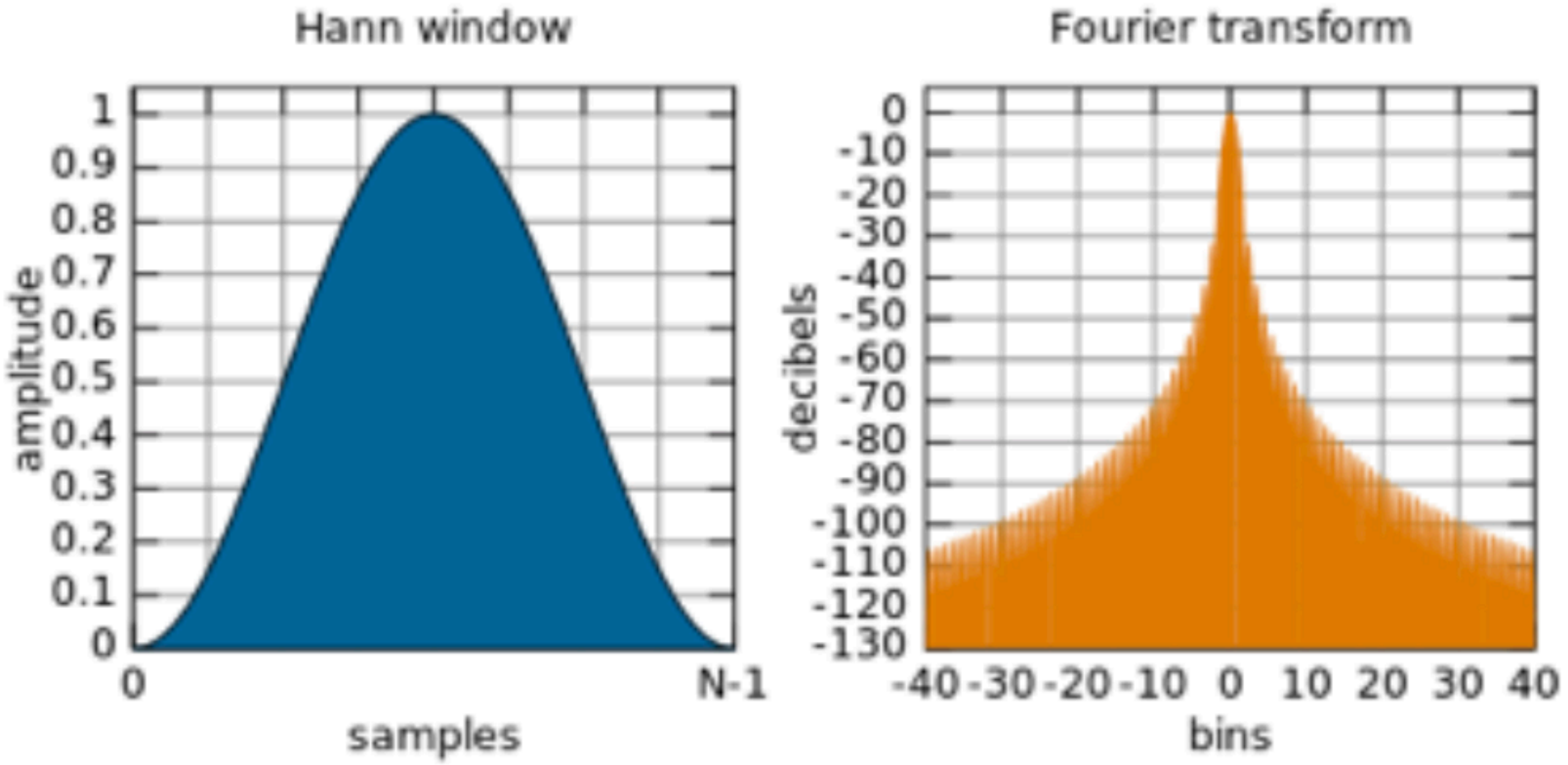
Short-time Fourier Transform (STFT)

$$s(t,f) = \int_{-\infty}^{\infty} w(t-t')x(t')e^{-i2\pi f t'}dt'$$

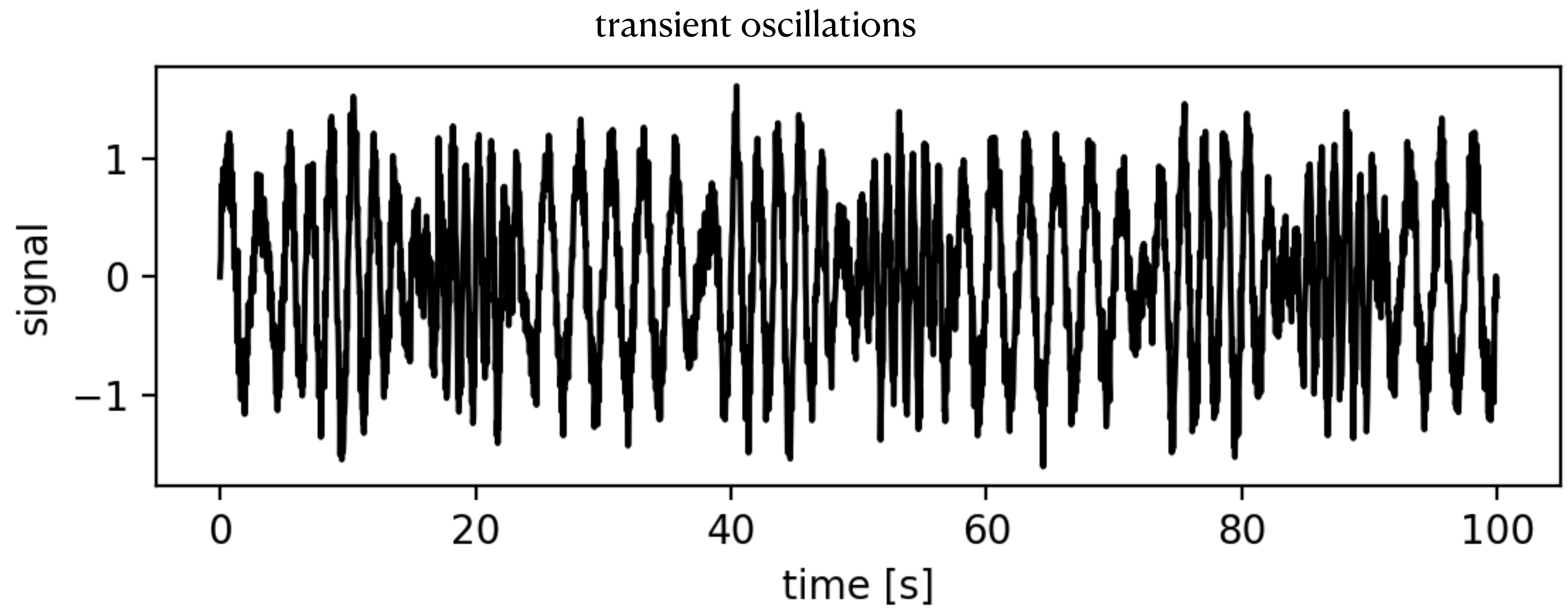




typical **finite size** windows w(t)



choose Hanning window in STFT

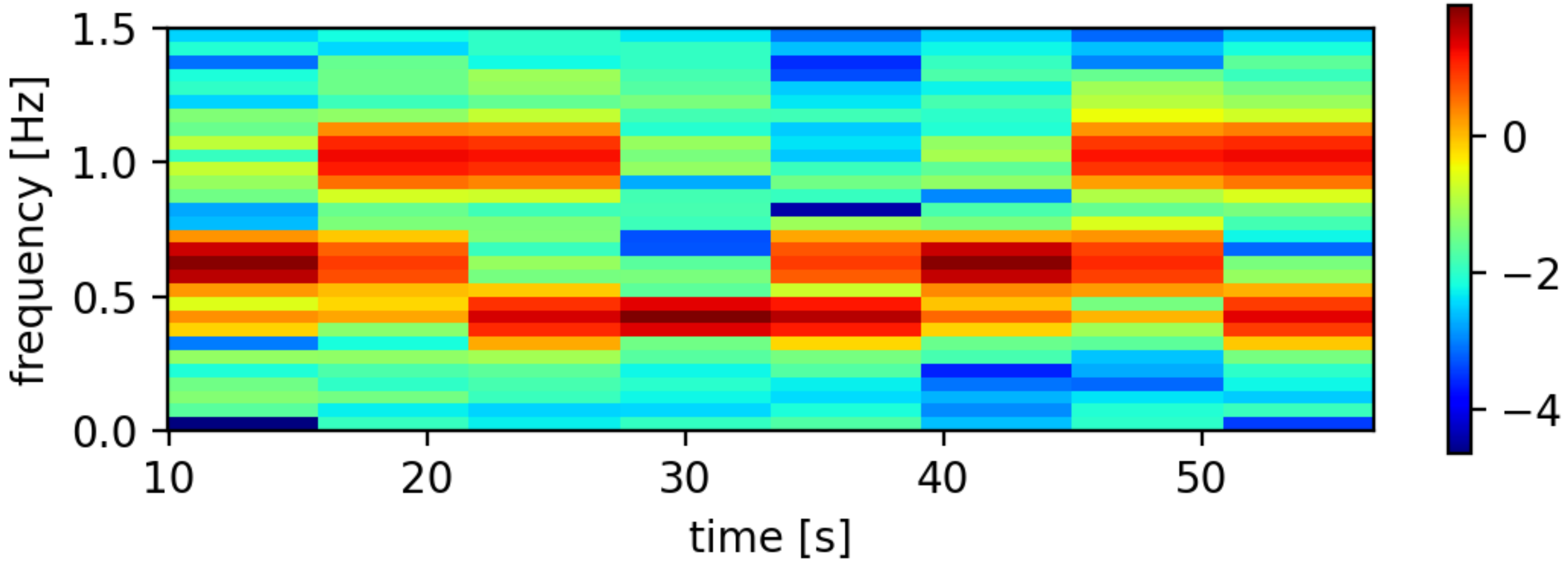


time window:

Δ**T=20s**

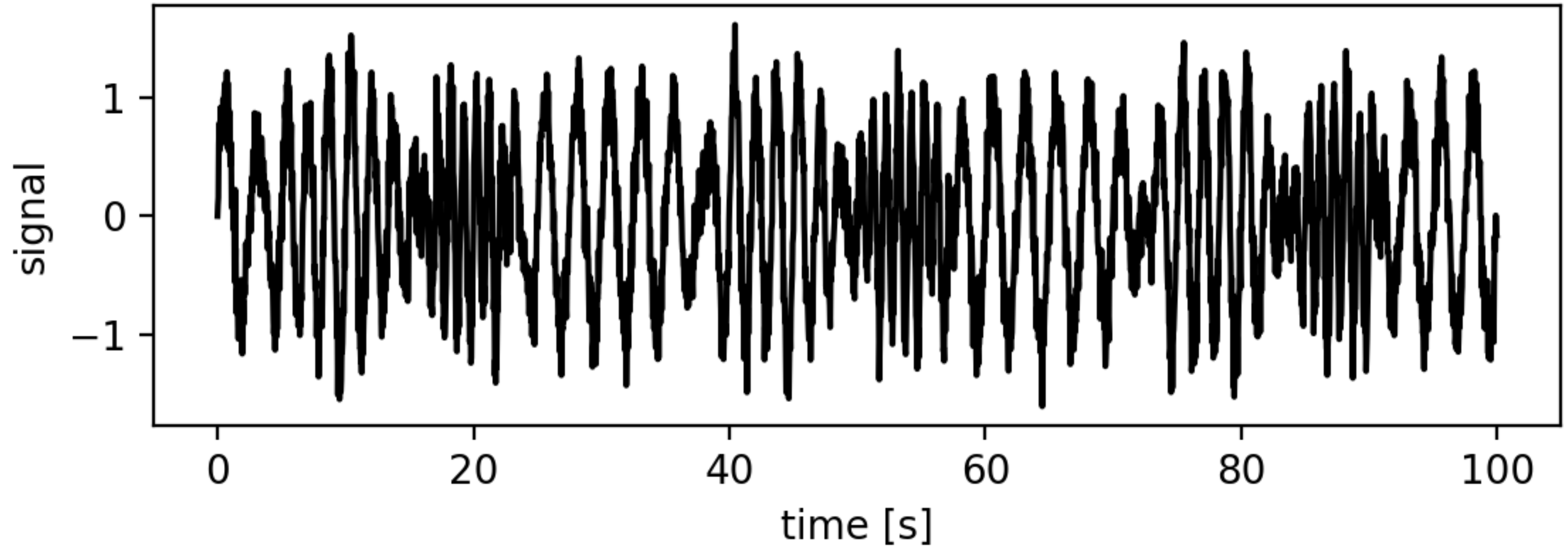
frequency resolution:

$\Delta f = 0.05\text{Hz}$



overlap of 13.4s

—> time shift by **6.6s**



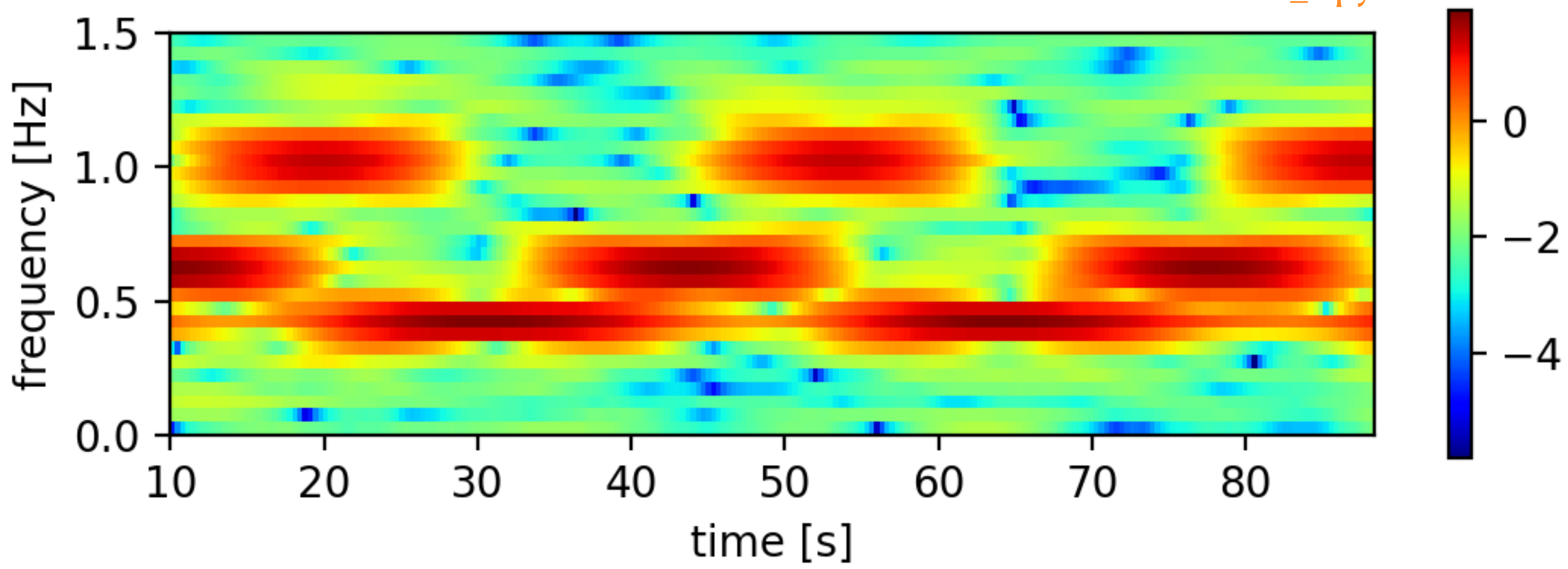
time window:

ΔT=20s

frequency resolution:

$\Delta f = 0.05$Hz

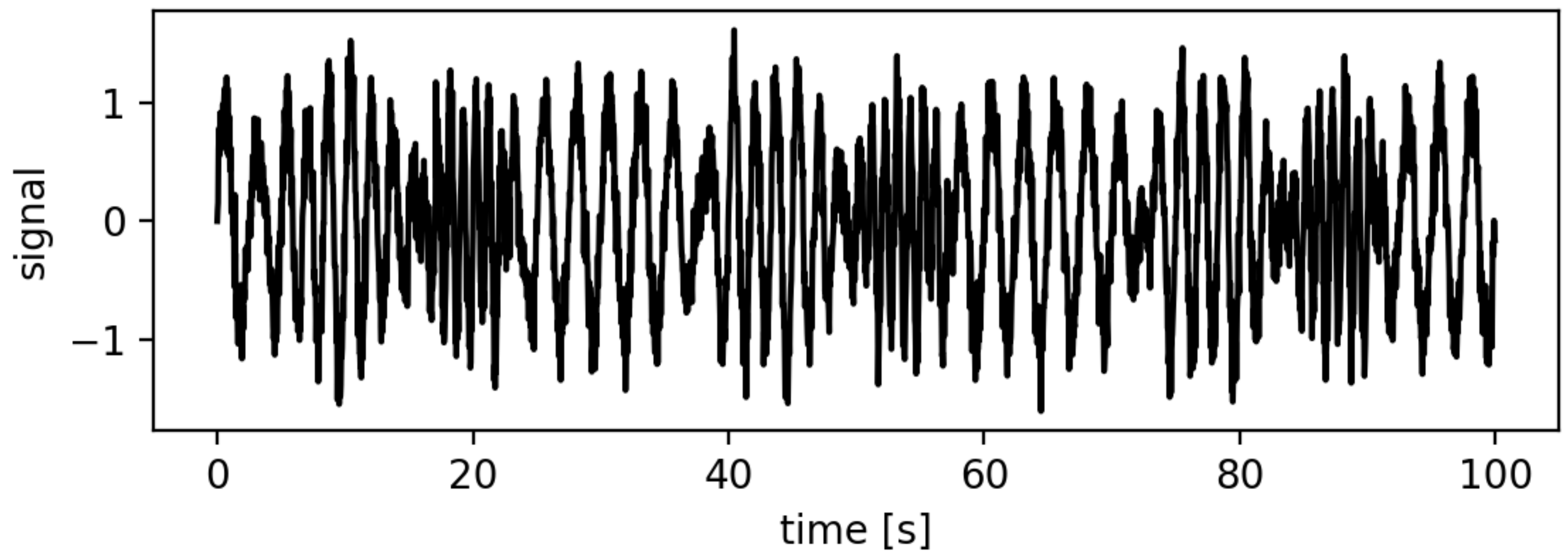
Filter_2.py



increase overlap:

overlap of 19.7s

—> time shift by **0.3s**



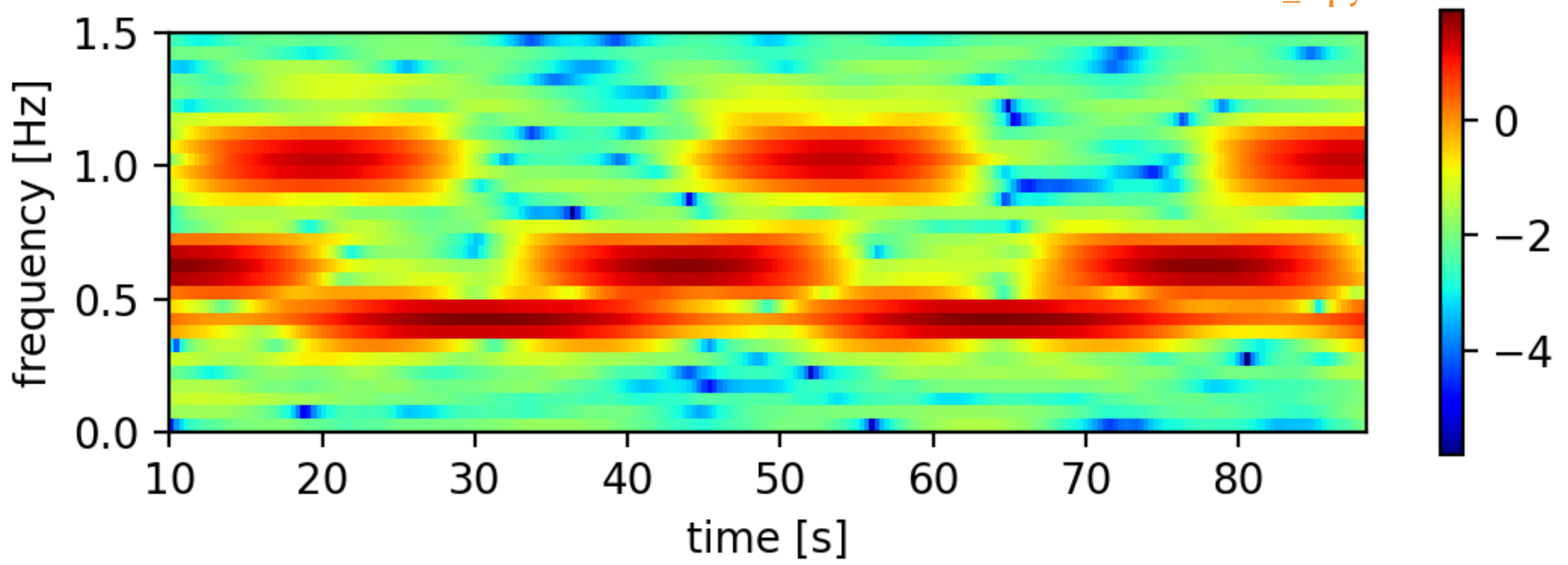
time window:

ΔT=20s

frequency resolution:

$\Delta f = 0.05$Hz

Filter_2.py

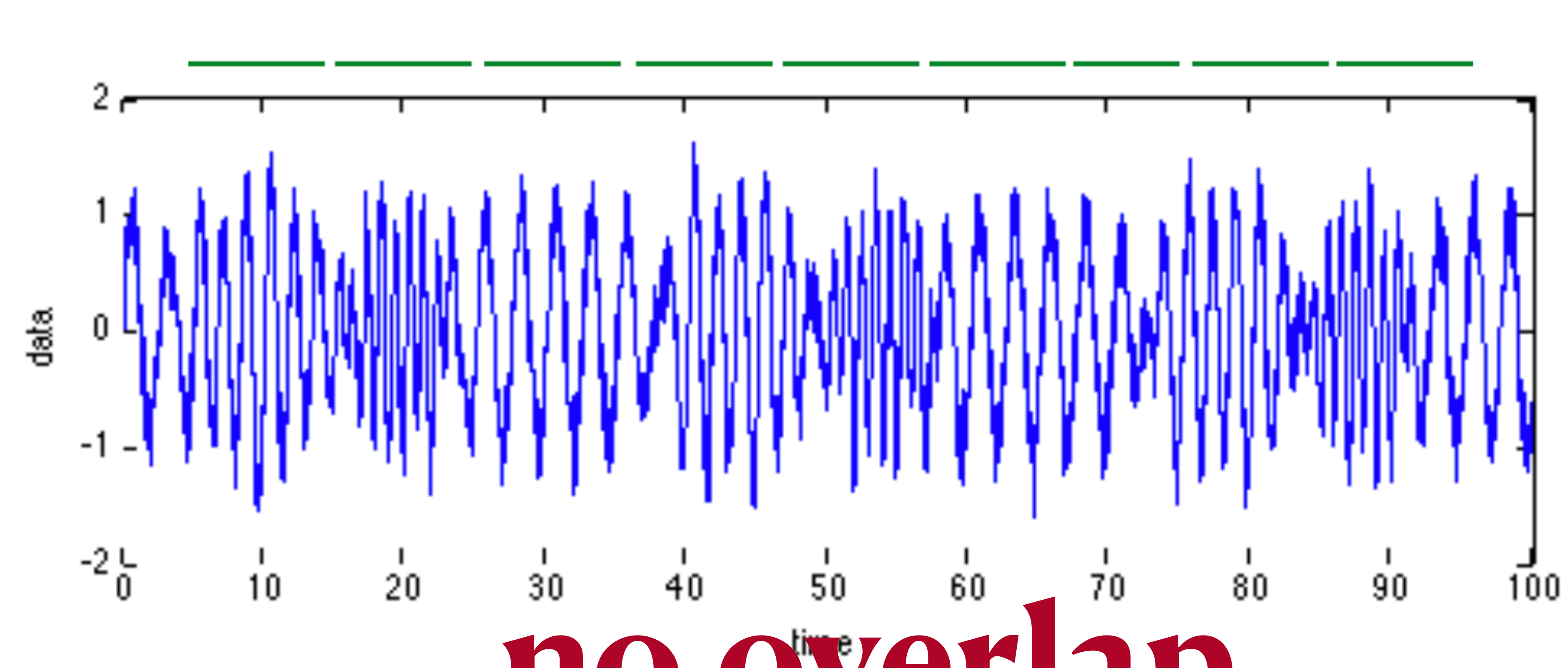


increase overlap:

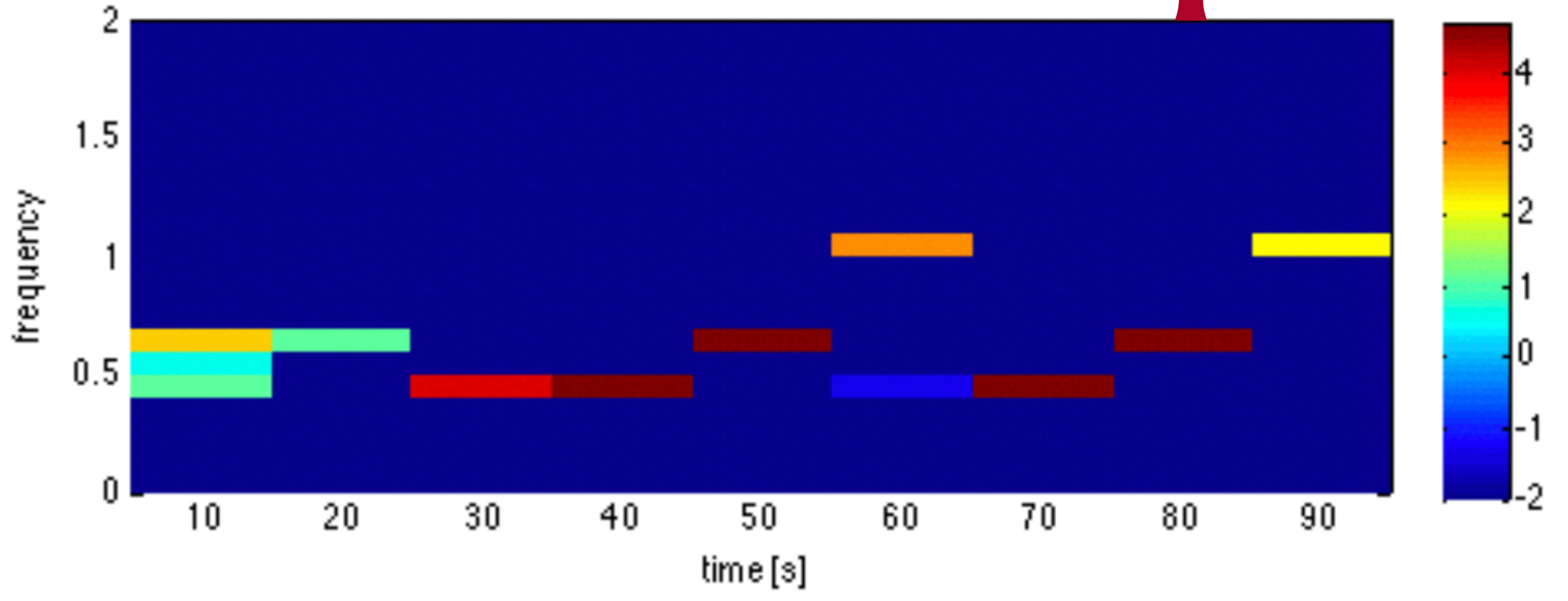
overlap of 19.7s

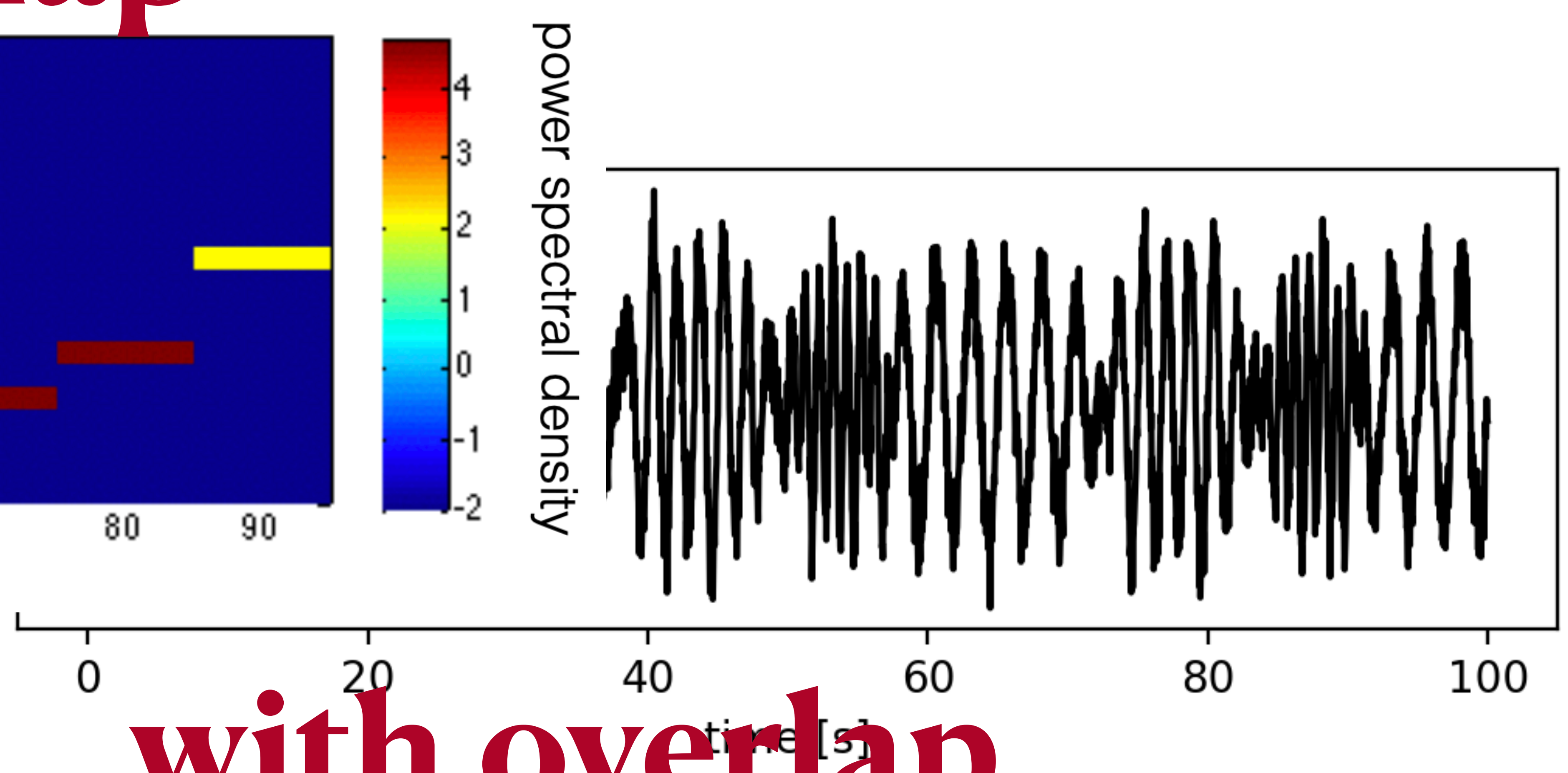
—> time shift by **0.3s**

better visualisation of transient dynamics

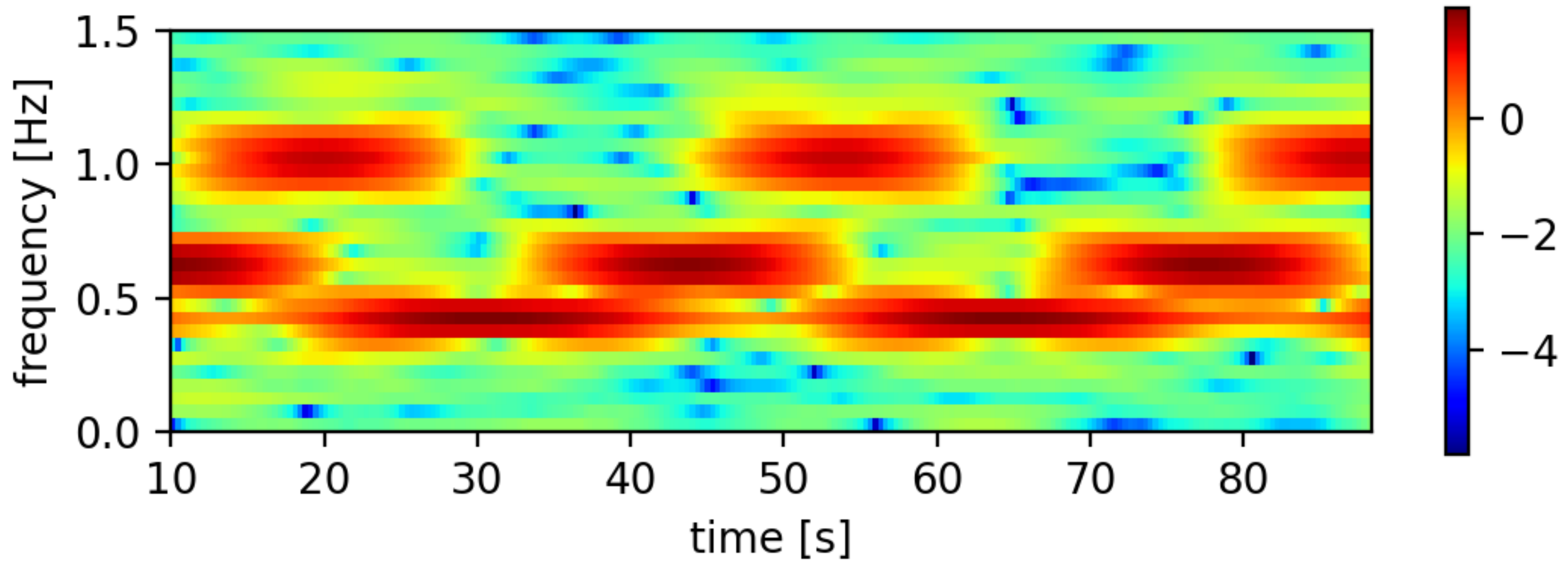


no overlap





with overlap



time window:

Δ**T=20s**

frequency resolution:

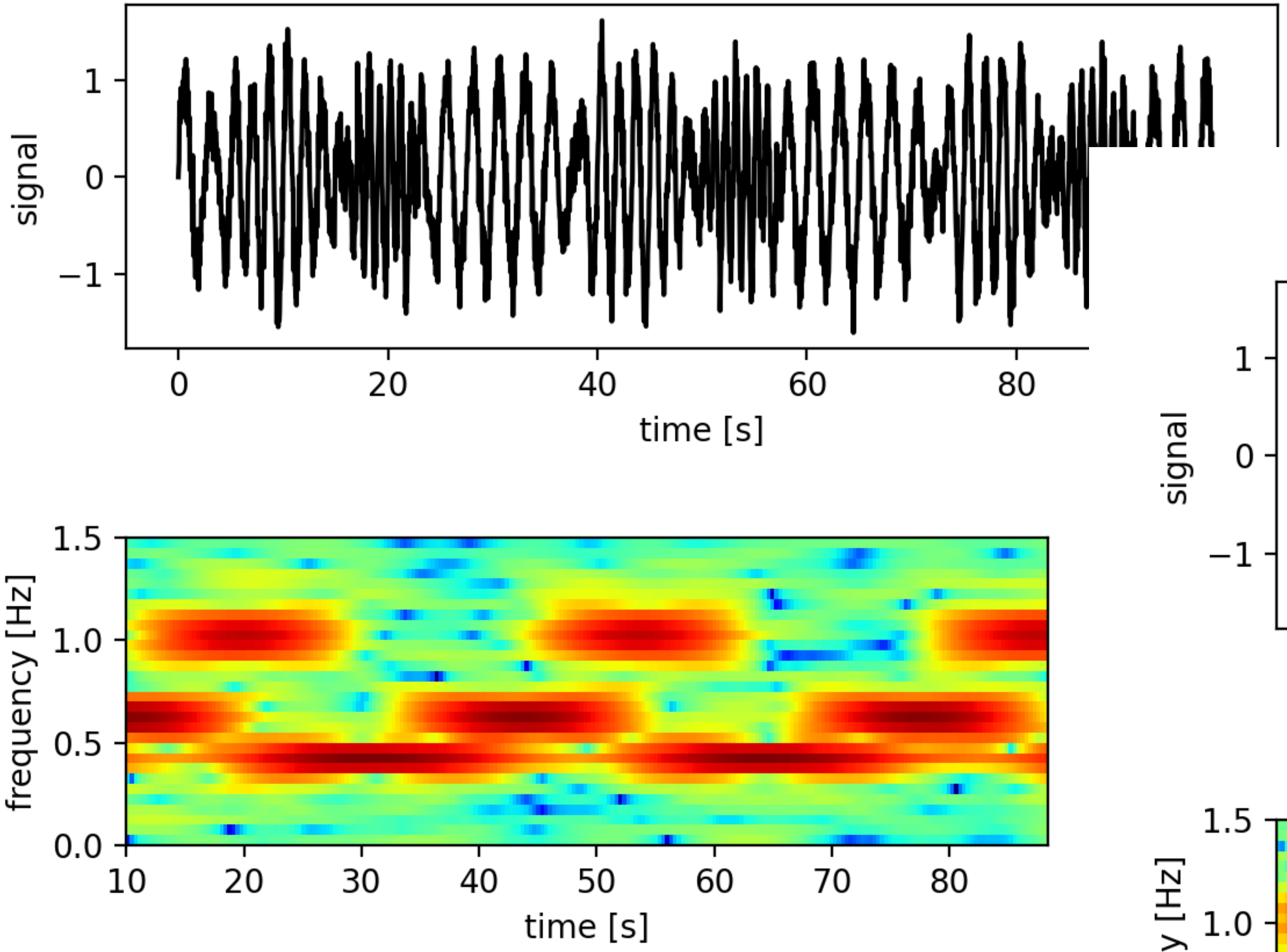
$\Delta f = 0.05\text{Hz}$

overlapping sliding windows

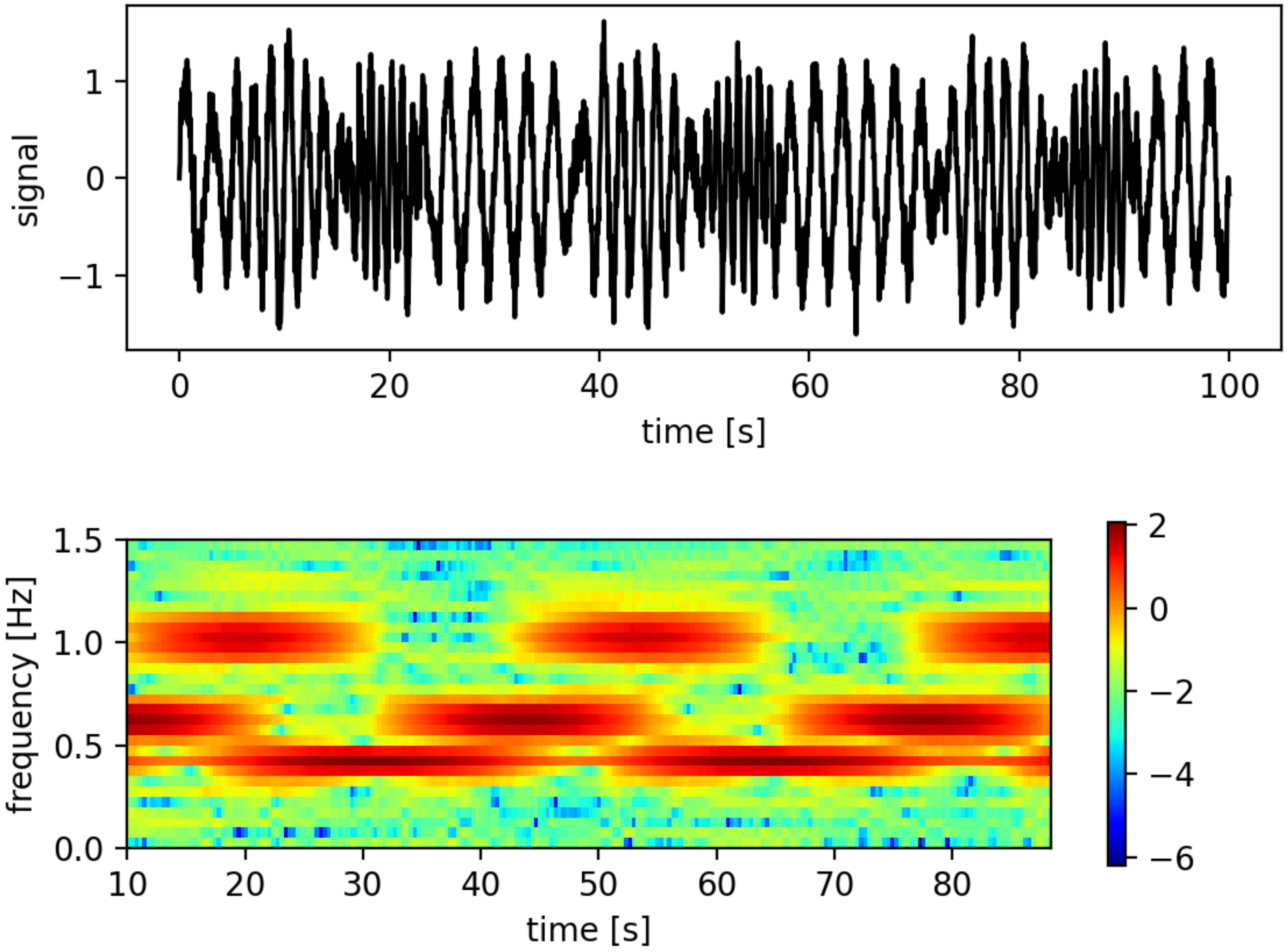
improve

time and frequency resolution

Hanning window



Gaussian window



Hanning and Gaussian results are similar

comment: if

$$(\mu_2)_{|f|^2} = \sigma^2_{|f|^2} = \int_{\mathbb{R}} (x - x_m)^2 \left|f(x)\right|^2 dx$$ **temporal variance**

$$(\mu_2)_{\left|\hat{f}\right|^2} = \sigma^2_{\left|\hat{f}\right|^2} = \int_{\mathbb{R}} (\xi - \xi_m)^2 \left|\hat{f}(\xi)\right|^2 d\xi$$ **frequency variance**

and

$$\|f\|_2^2 = \int_{\mathbb{R}} \left|f(x)\right|^2 dx$$

comment: if

$$(\mu_2)_{|f|^2} = \sigma^2_{|f|^2} = \int_{\mathbb{R}} (x - x_m)^2 \, |f(x)|^2 \, dx$$ **temporal variance**

$$(\mu_2)_{|\hat{f}|^2} = \sigma^2_{|\hat{f}|^2} = \int_{\mathbb{R}} (\xi - \xi_m)^2 \left|\hat{f}(\xi)\right|^2 d\xi$$ **frequency variance**

and

$$\|f\|_2^2 = \int_{\mathbb{R}} |f(x)|^2 \, dx$$

then

$$\boxed{\sigma^2_{|f|^2} \cdot \sigma^2_{|\hat{f}|^2} \geq \frac{\|f\|_2^4}{16\pi^2}}$$

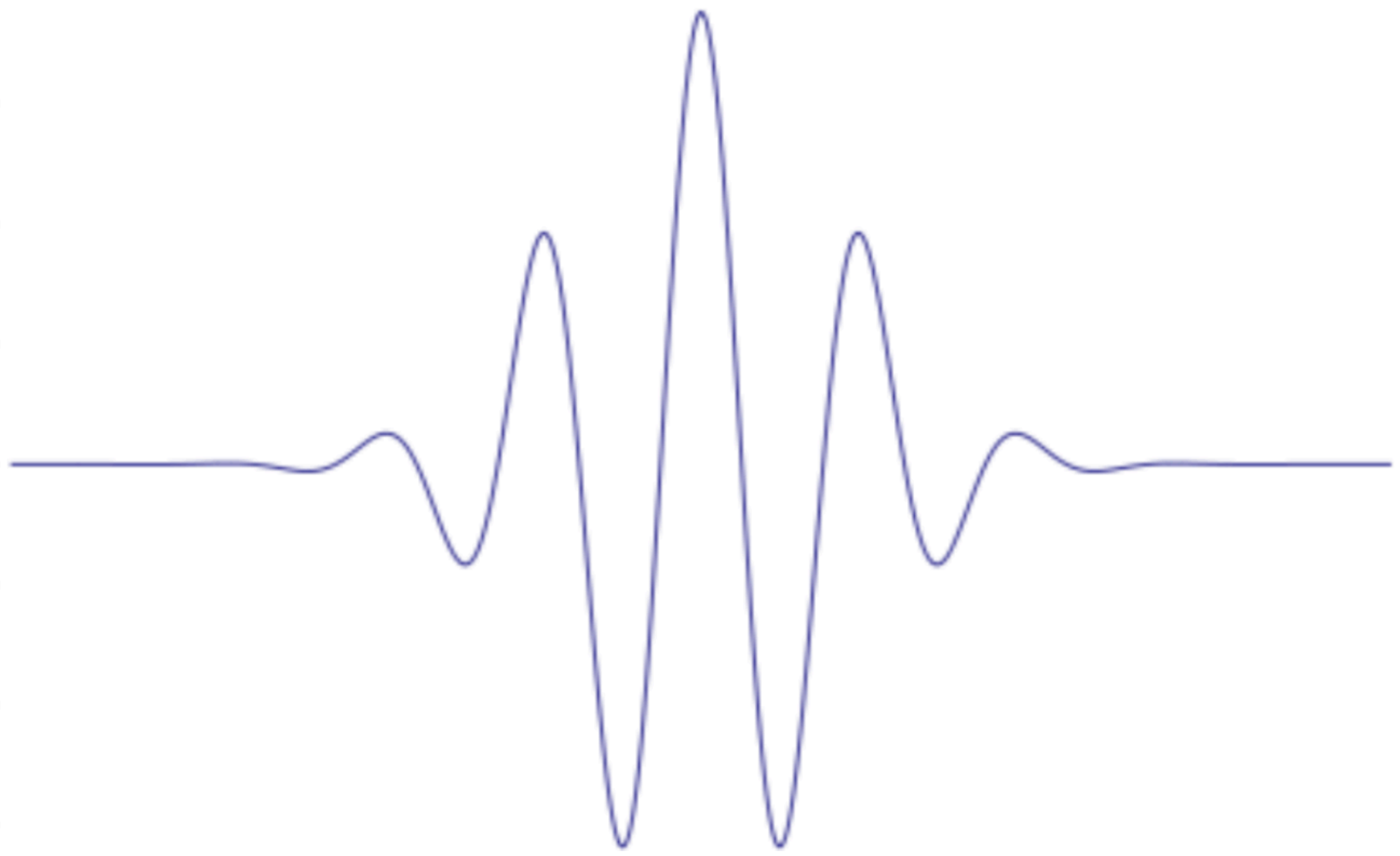
Heisenberg-Weyl inequality

minimum of uncertainty

$$\sigma^2_{|f|^2}\sigma^2_{|\hat{f}|^2} = \frac{||f||_2^4}{16\pi^2}$$

if

$$f(t) = c_0 e^{i2\pi f t} e^{-c_1(x-x_m)^2}$$



minimum uncertainty

$$\sigma^2_{|f|^2}\sigma^2_{|\hat{f}|^2} = \frac{||f||_2^4}{16\pi^2}$$

if

$$f(t) = c_0 e^{i2\pi f t} e^{-c_1(x-x_m)^2}$$

→ best time-frequency resolution if window is of Gaussian shape

→ **Gabor transformation**

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Short-time Fourier Tranform:

$$X(\tau, f) = \int_{-\infty}^{\infty} x(t) w(t-\tau) e^{-i2\pi f t} dt$$

window function

Short-time Fourier Tranform:

$$X(\tau, f) = \int_{-\infty}^{\infty} x(t)w(t-\tau)e^{-i2\pi ft}dt$$

window function

Linear frequency filter:

$$X(\tau, f) = \int_{-\infty}^{\infty} x(t)w(t-\tau)e^{-i2\pi f(t-\tau)}dt$$

$$= \int_{-\infty}^{\infty} x(t)h(t-\tau)dt$$

impulse response function

Short-time Fourier Tranform:

$$X(\tau, f) = \int_{-\infty}^{\infty} x(t)w(t-\tau)e^{-i2\pi ft}dt$$

window function

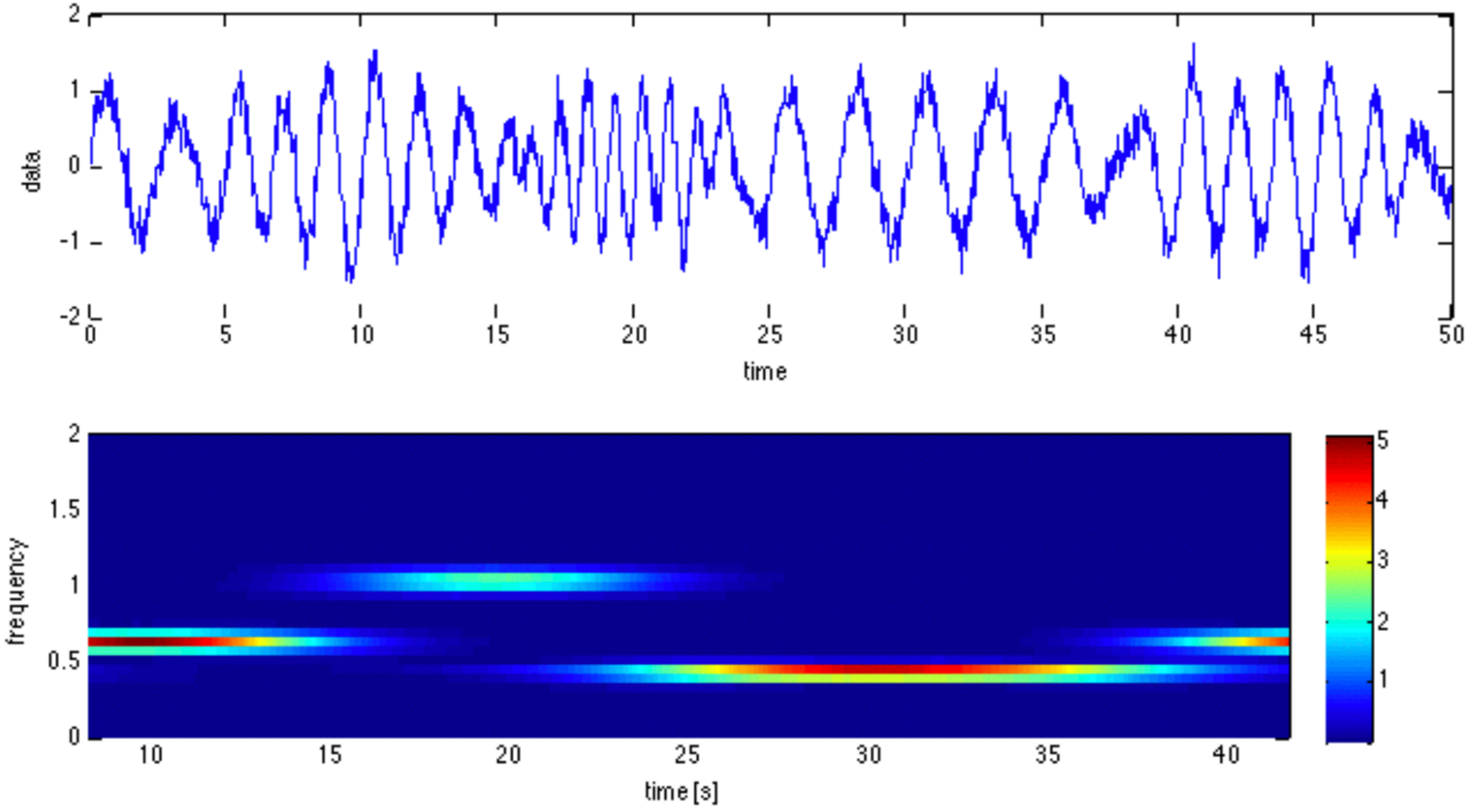
Linear frequency filter:

$$X(\tau, f) = \int_{-\infty}^{\infty} x(t)w(t-\tau)e^{-i2\pi f(t-\tau)}dt$$

$$= \int_{-\infty}^{\infty} x(t)h(t-\tau)dt$$

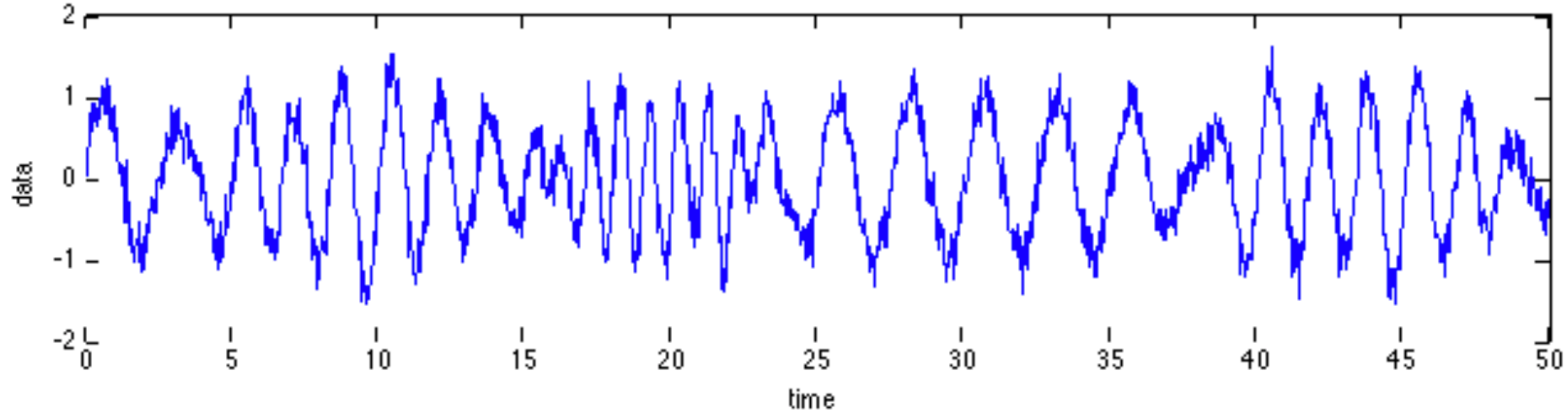
impulse response function

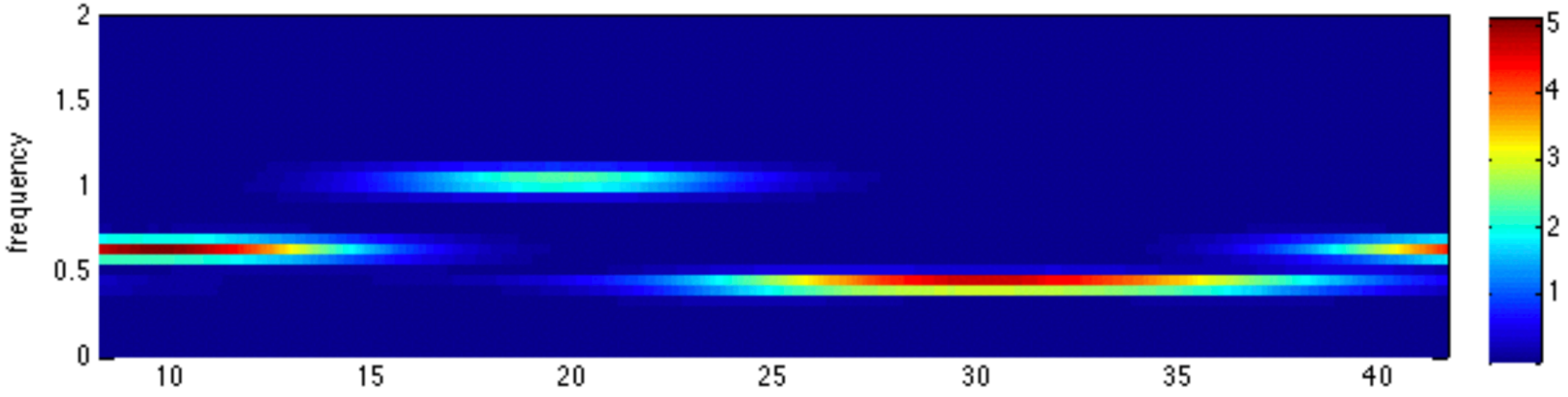
correlation between signal x and impulse response function h



STFT has constant time-frequency resolution,

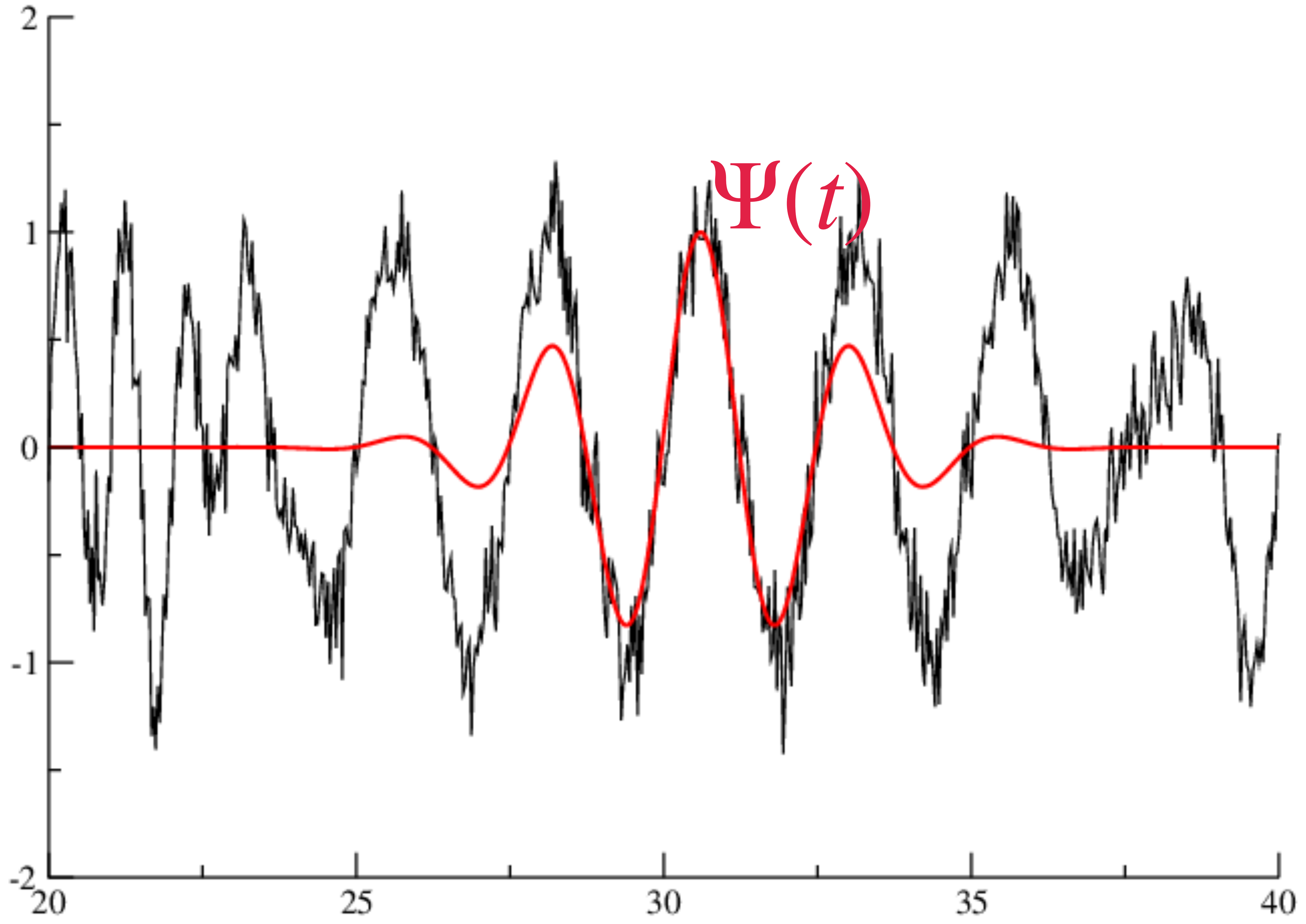
but transient signals need different time resolutions





$$\boxed{X(\tau, a) \sim \int_{-\infty}^{\infty} x(t)\bar{\Psi}\left(\frac{t-\tau}{a}\right)dt}$$

temporal scale factor a

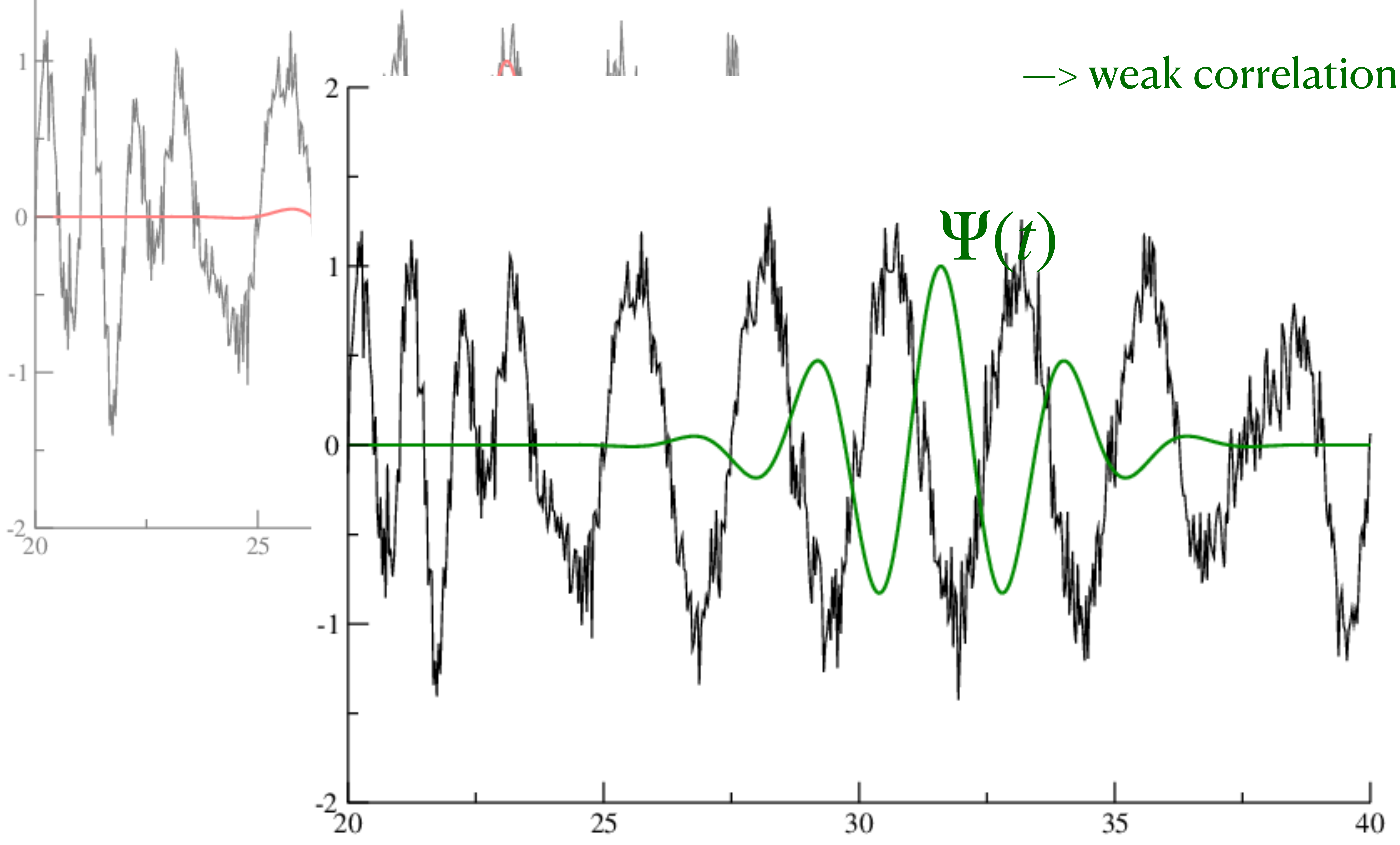


impulse response function fits to signal at that time instance

—> strong correlation X

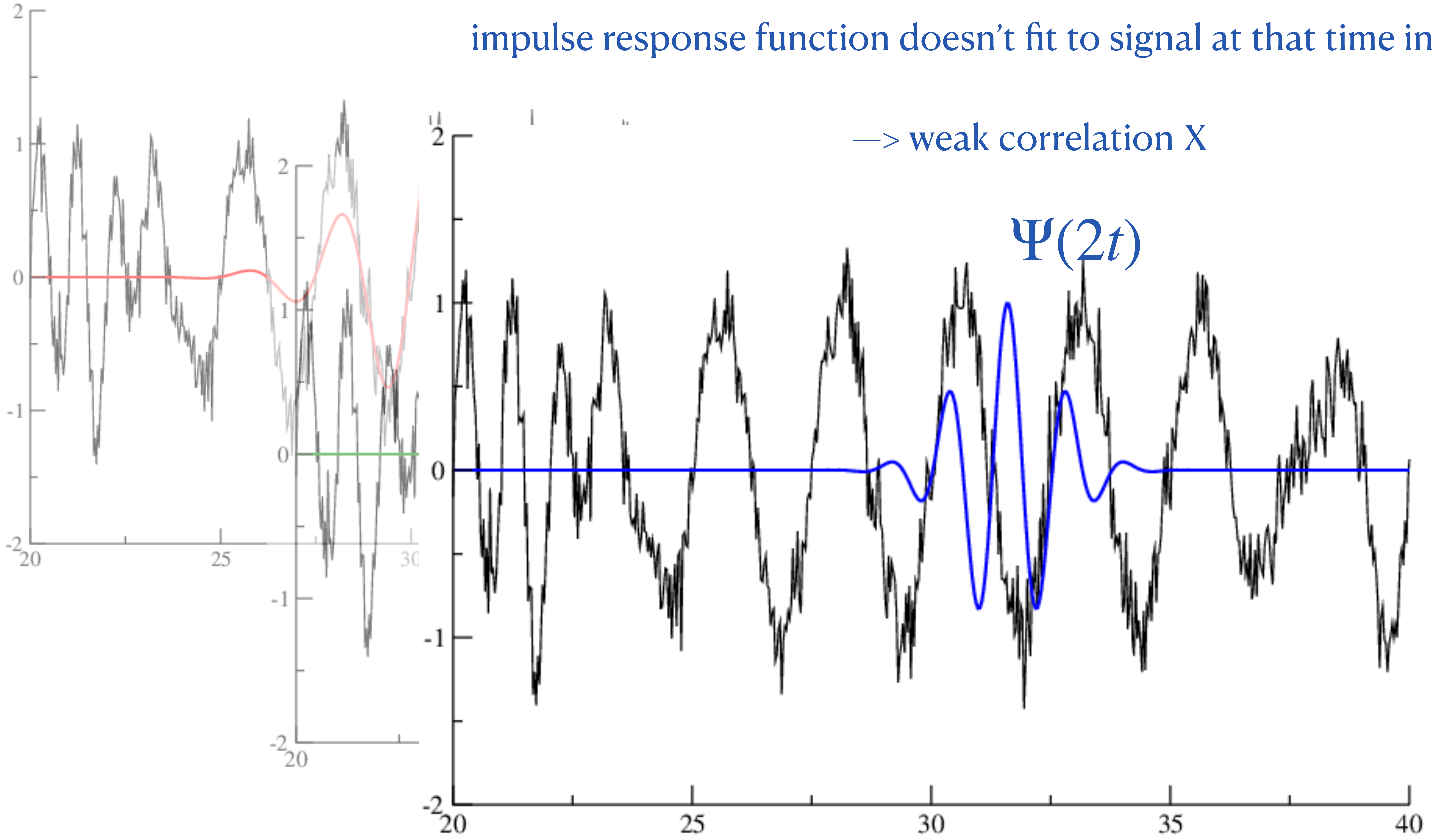
impulse response function does not fit to signal at that time instance

—> weak correlation X



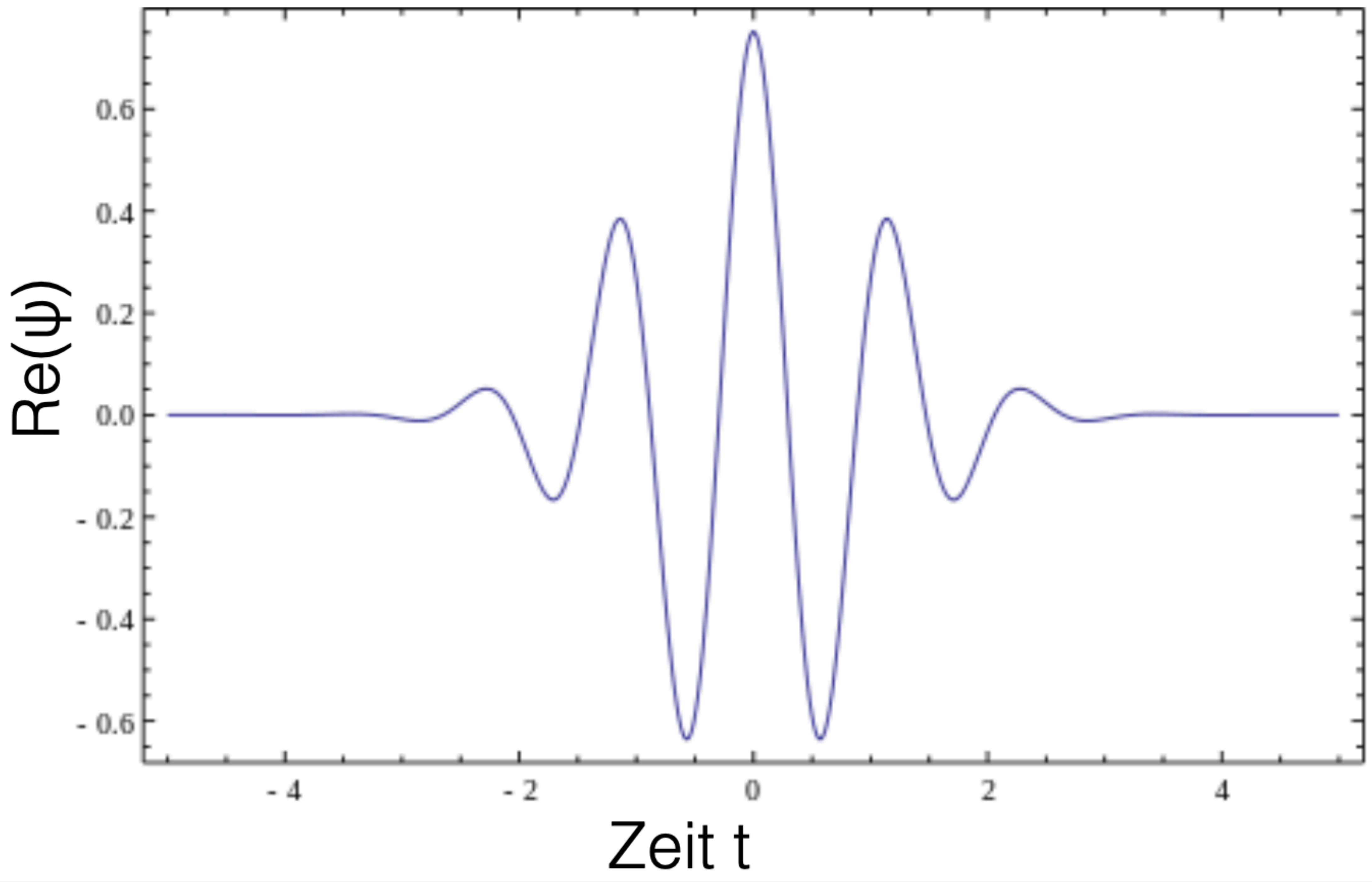
impulse response function doesn't fit to signal at that time instance

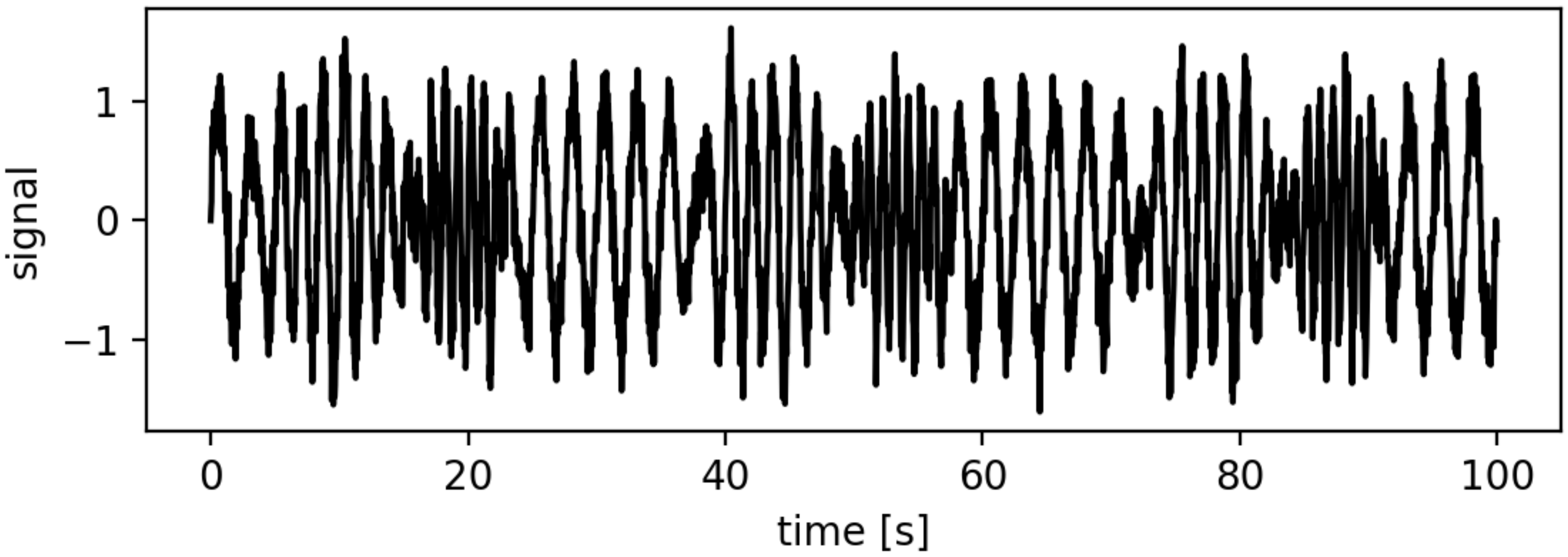
—> weak correlation X



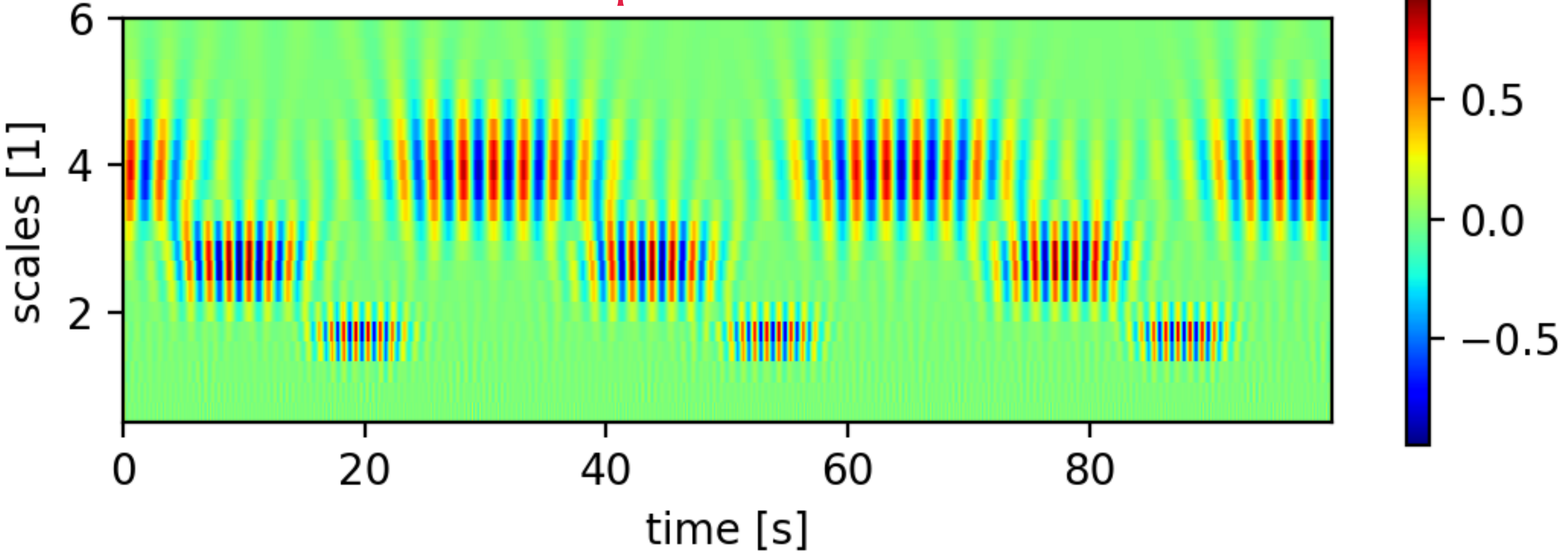
e.g.

$$\Psi_a(t) = k_1 e^{-t^2}(e^{-i\frac{t}{a}} - k_2)$$



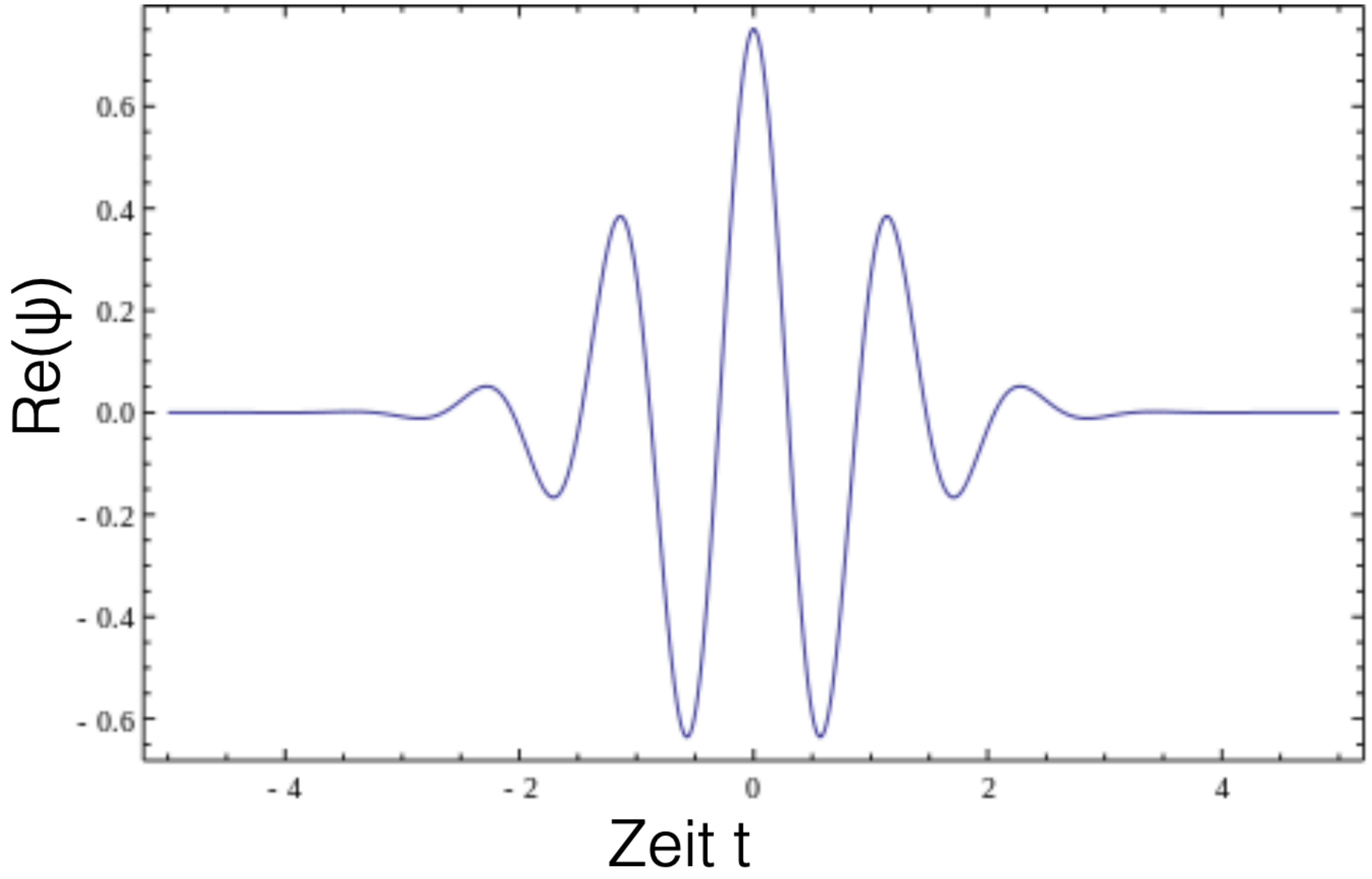


real part of transform X



e.g.

$$\Psi_a(t) = k_1 e^{-t^2}(e^{-i\frac{t}{a}} - k_2)$$



$$\boxed{X(\tau, a) = \frac{1}{\sqrt{a}} \int_{-\infty}^{\infty} x(t) \bar{\Psi}\left(\frac{t-\tau}{a}\right) dt}$$

continuous wavelet transform

$\Psi(t)$: mother wavelet

properties:

administibility $$\int_{-\infty}^{\infty} \Psi\left(t\right) dt = 0$$ mother wavelet has to be oscillatory

properties:

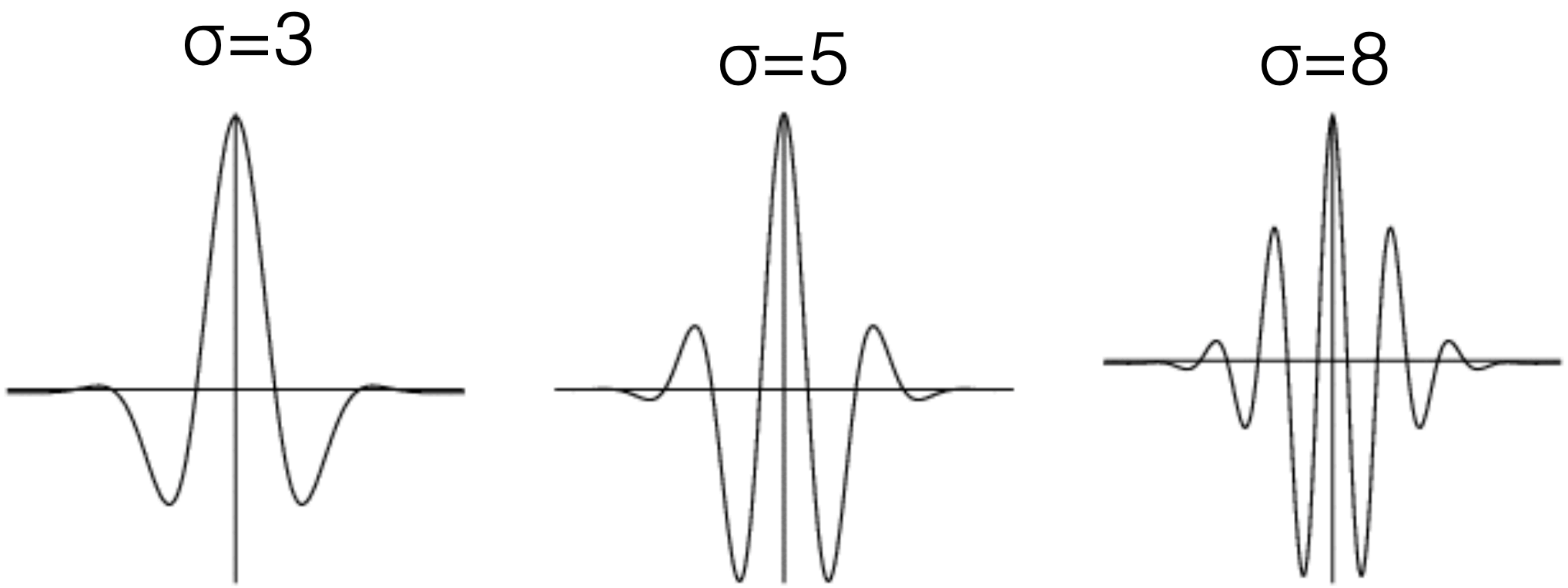
administibility

$$\int_{-\infty}^{\infty} \Psi\,(t)\,dt = 0$$

mother wavelet has to be oscillatory

<u>example</u>: **complex Morlet wavelet**

$$\Psi(t) = k_1 e^{-t^2}(e^{-i\sigma t} - k_2)$$



in neuroscience: $5 \leq \sigma \leq 8$ recommended

$$X(\tau, a) = \frac{1}{\sqrt{a}} \int_{-\infty}^{\infty} x(t) \bar{\Psi}\left(\frac{t-\tau}{a}\right) dt$$

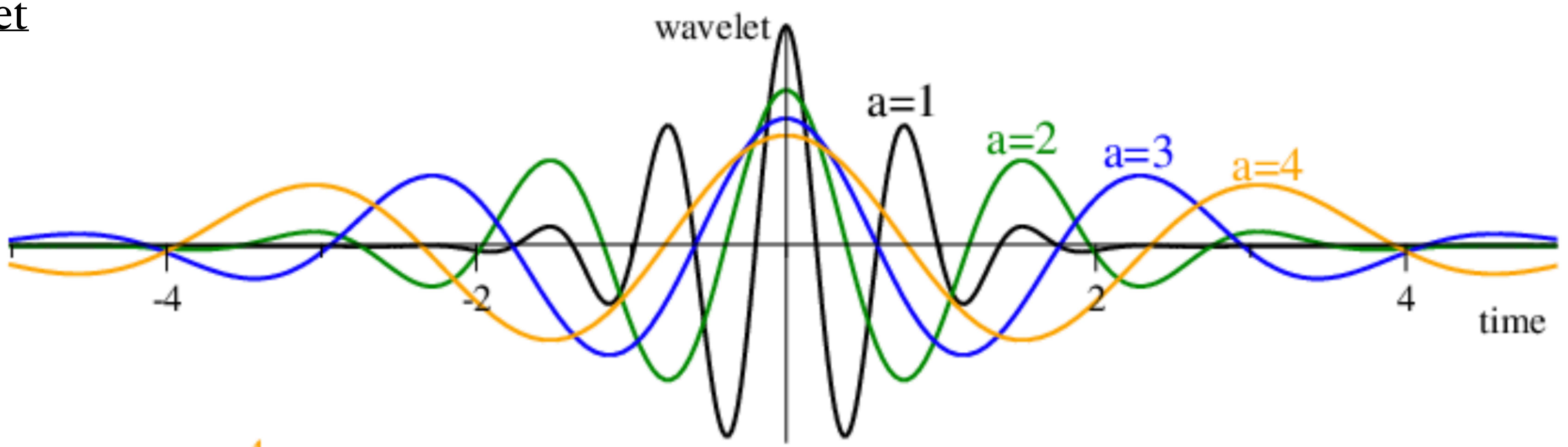
$$X(\tau, a) = \mathrm{IFT}\left[\ \tilde{x}(f) \ \cdot \ \tilde{\bar{\bar{\Psi}}}(af)\sqrt{a}\ \right](\tau)$$

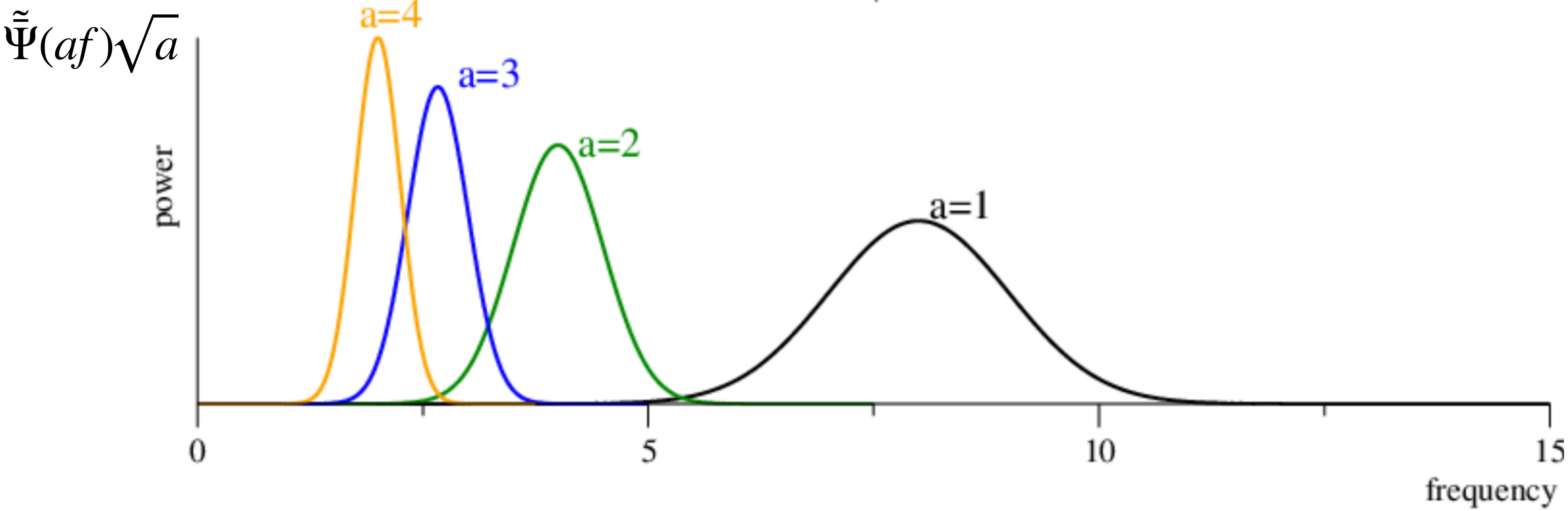
$$X(\tau, a) = \frac{1}{\sqrt{a}} \int_{-\infty}^{\infty} x(t) \bar{\Psi} \left(\frac{t-\tau}{a} \right) dt$$

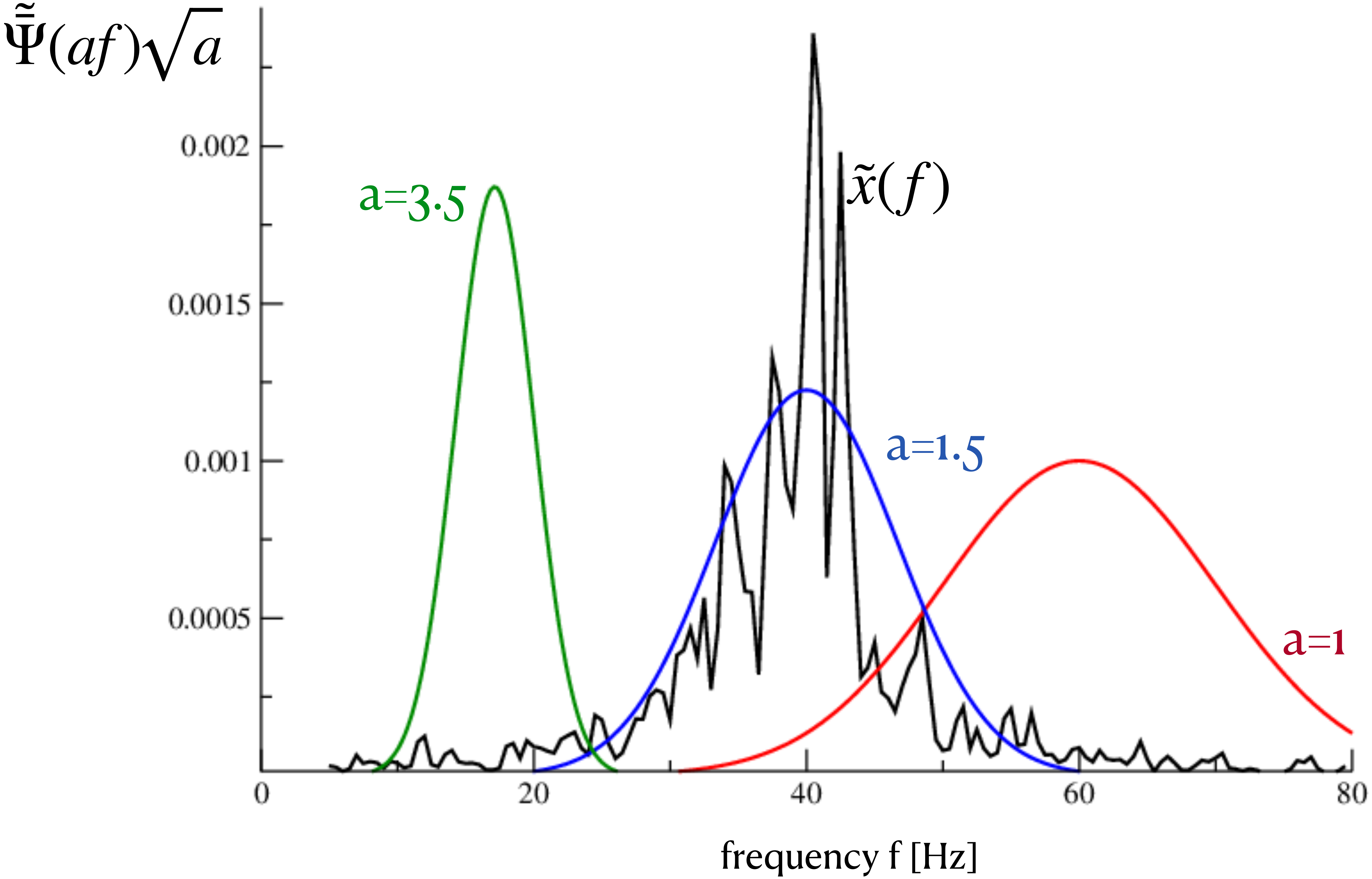
$$X(\tau, a) = \mathrm{IFT} \left[\tilde{x}(f) \cdot \boxed{\bar{\tilde{\Psi}}(af)\sqrt{a}} \right] (\tau)$$

$$X(\tau, a) = \frac{1}{\sqrt{a}} \int_{-\infty}^{\infty} x(t)\bar{\Psi}\left(\frac{t-\tau}{a}\right) dt \qquad X(\tau, a) = \mathrm{IFT}\left[\, \tilde{x}(f) \;\cdot\; \bar{\tilde{\Psi}}(af)\sqrt{a} \,\right](\tau)$$

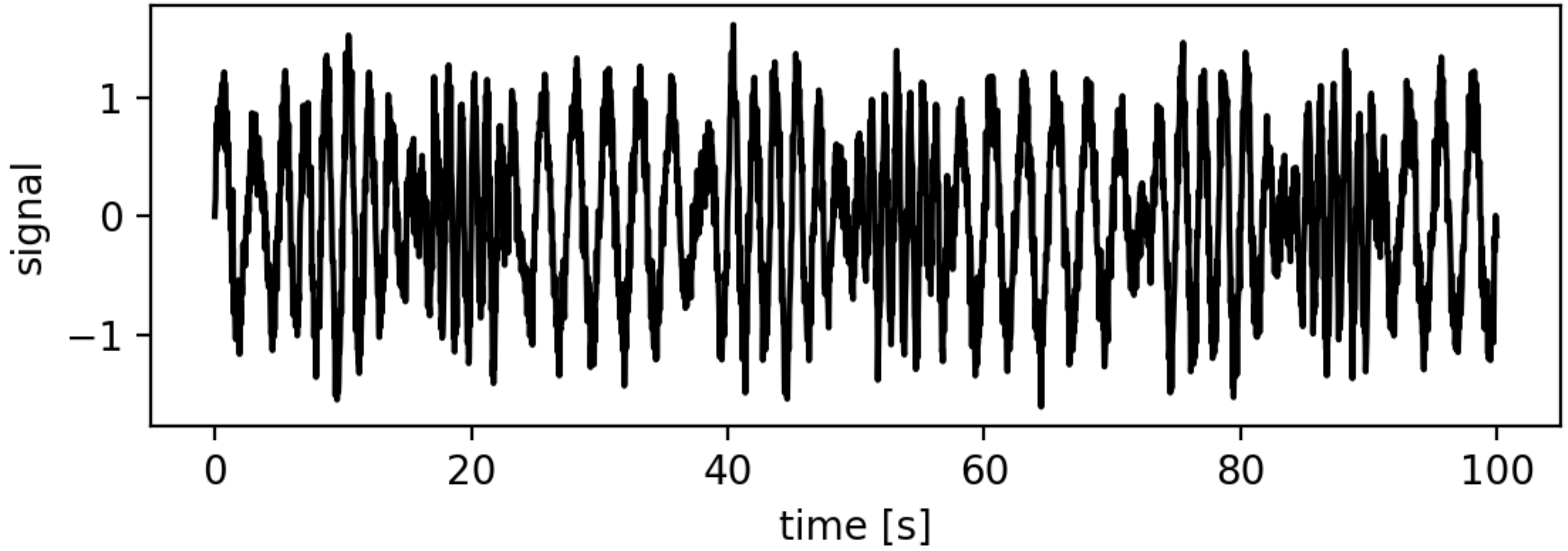
<u>Morlet wavelet</u>



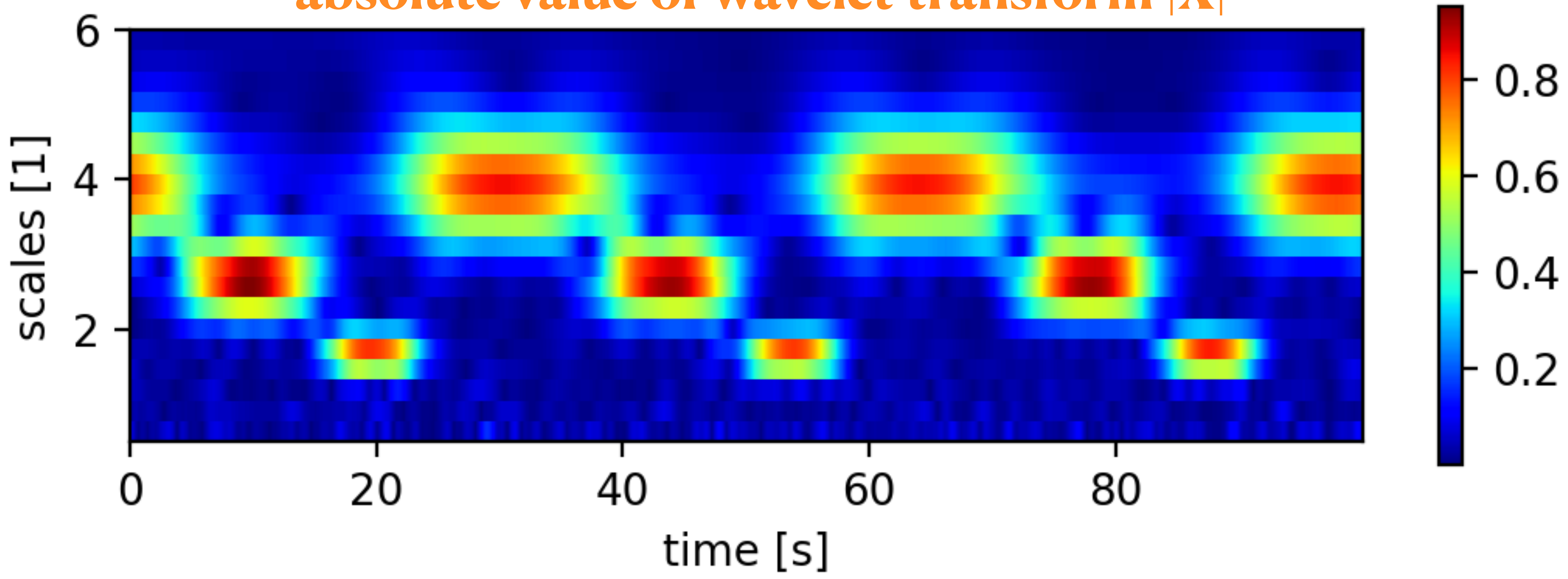




$\tilde{\tilde{\Psi}}(af)\sqrt{a}$
0.002
0.0015
0.001
0.0005
a=3.5
$\tilde{x}(f)$
a=1.5
a=1
0
20
40
60
80
frequency f [Hz]

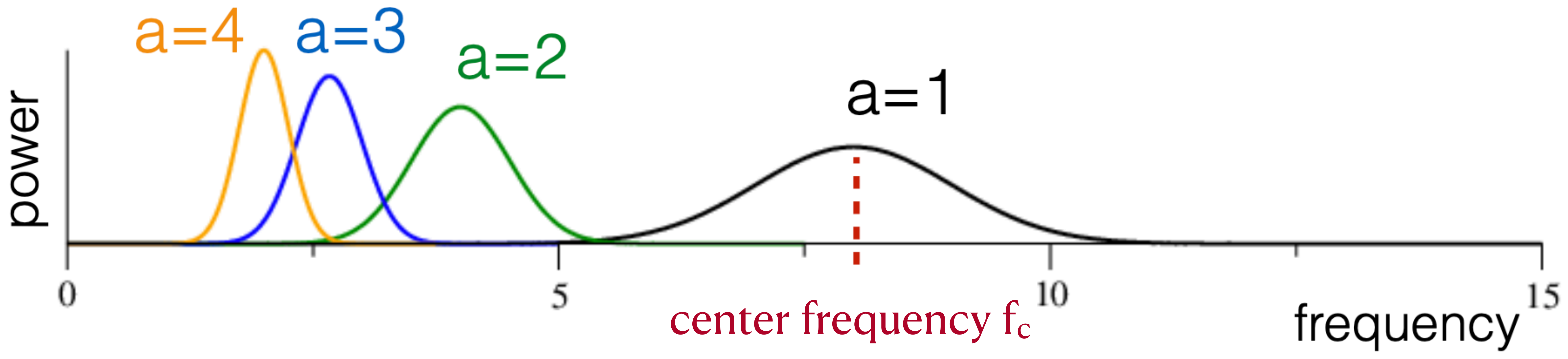


absolute value of wavelet transform |X|

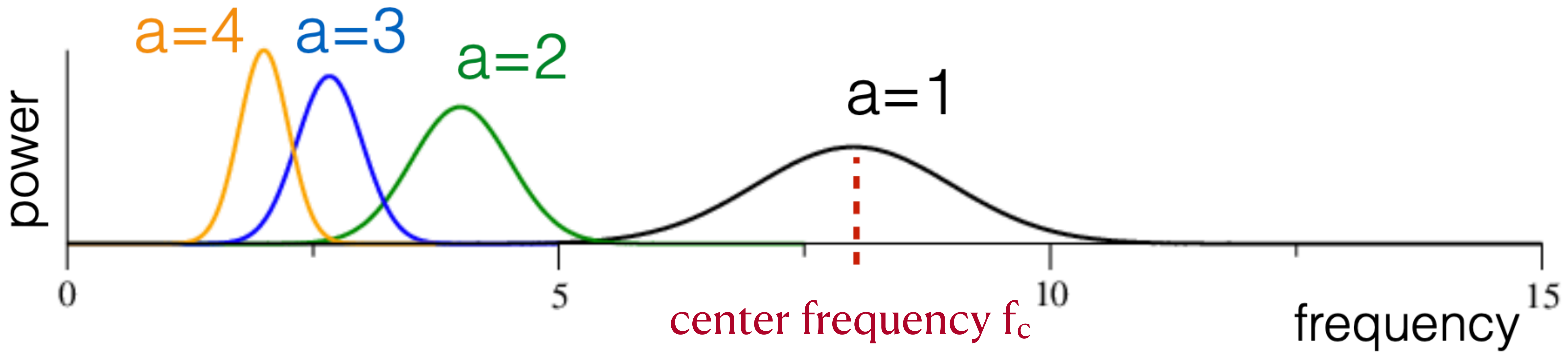


scalogram with Morlet wavelet

relation between scales and frequency



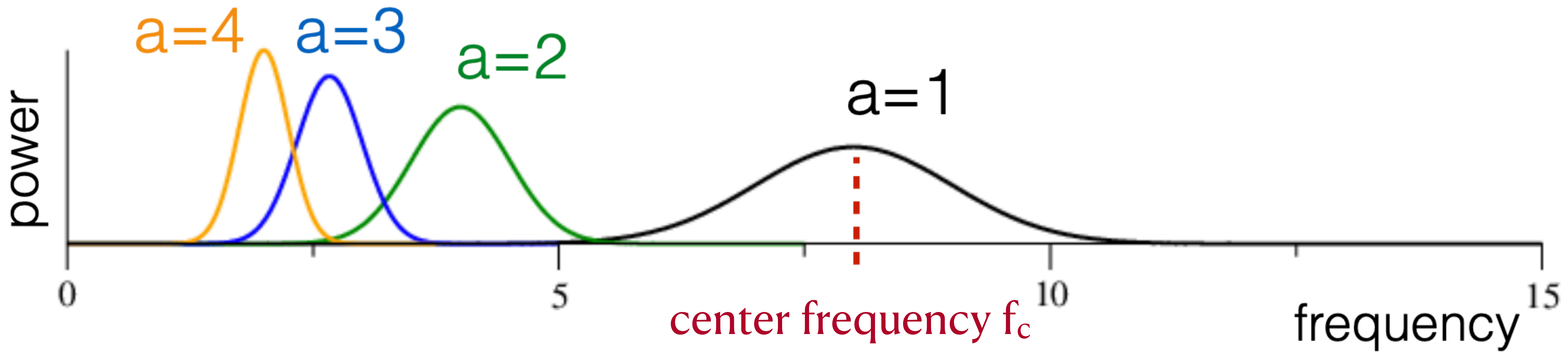
relation between scales and frequency



pseudo frequency $f_p = \frac{f_c}{a}$

pseudo frequency is the frequency of maximum mother wavelet power

relation between scales and frequency

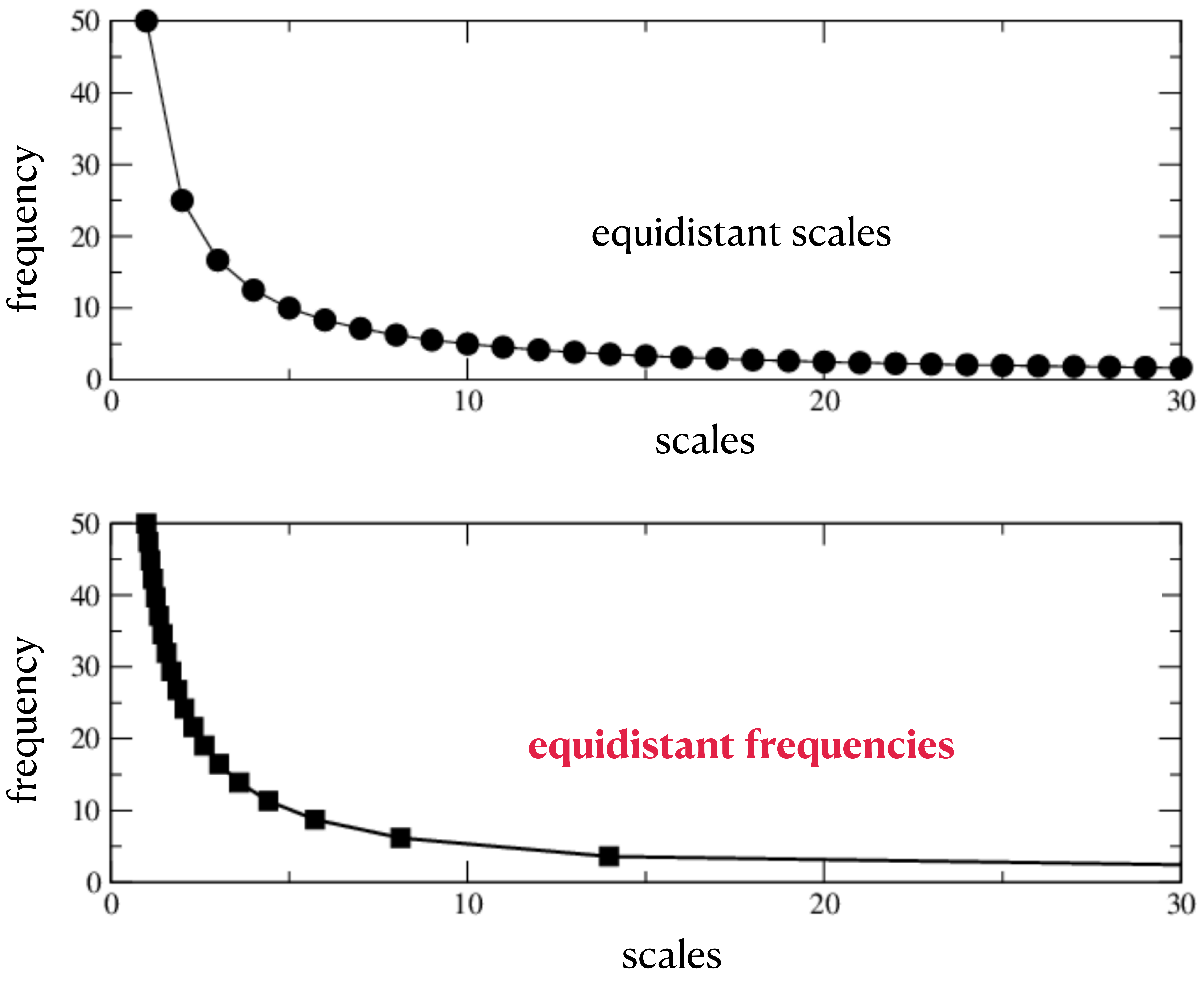


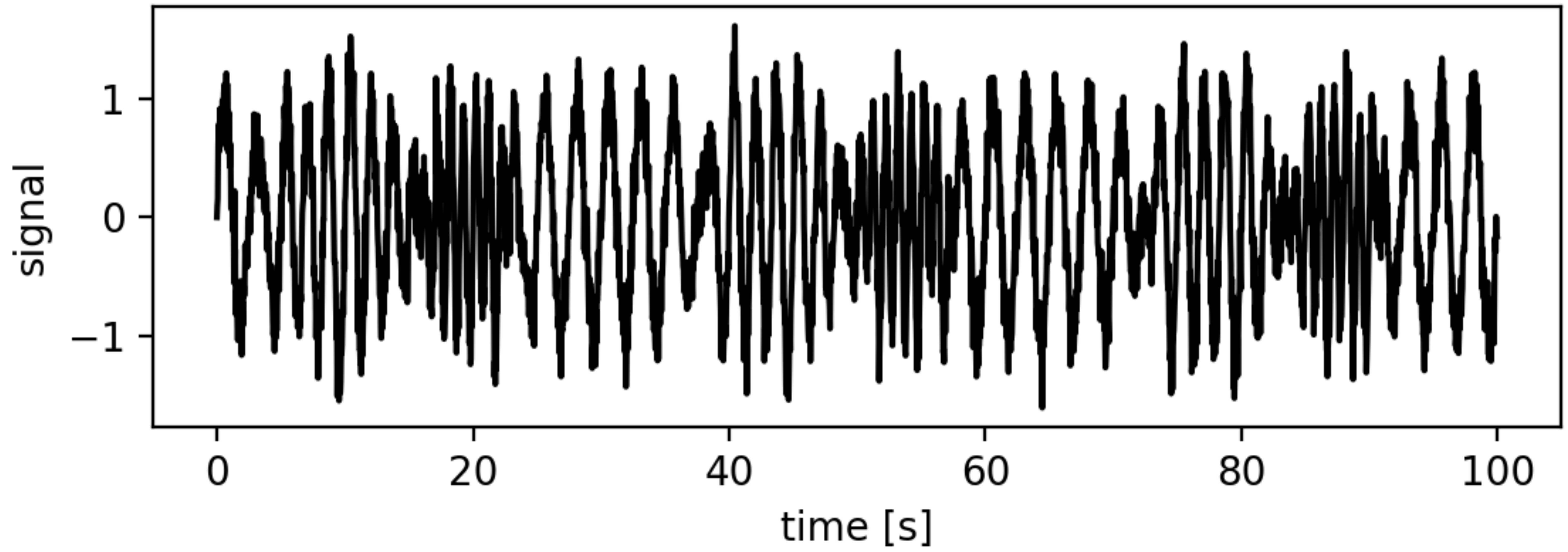
pseudo frequency $f_p = \dfrac{f_c}{a}$

pseudo frequency is the frequency of maximum mother wavelet power

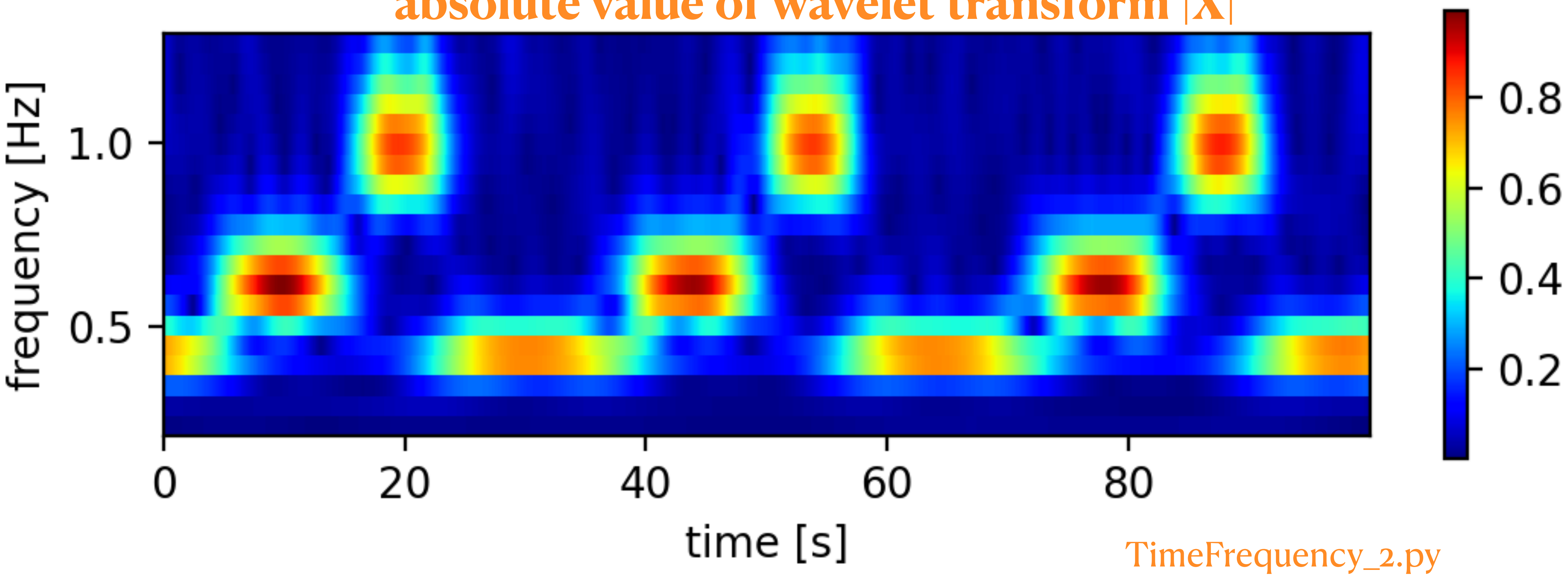
“pseudo” : not a unique frequency, but represents a distribution

relation between scales and frequency





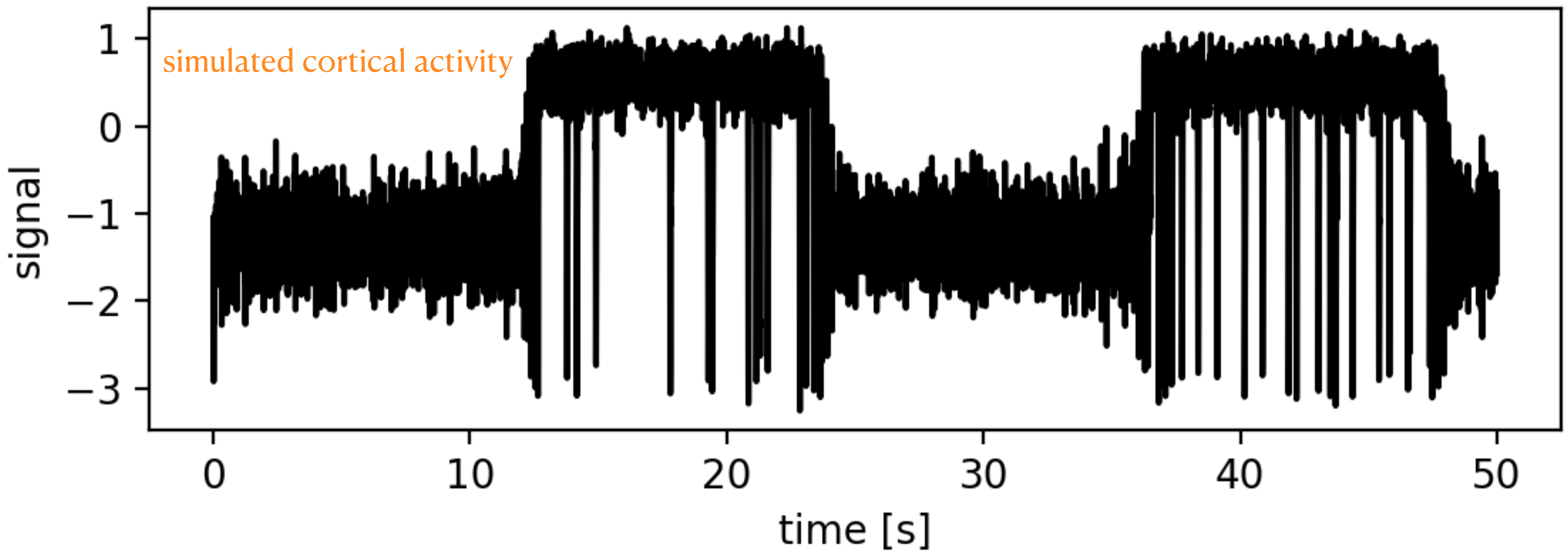
absolute value of wavelet transform |X|

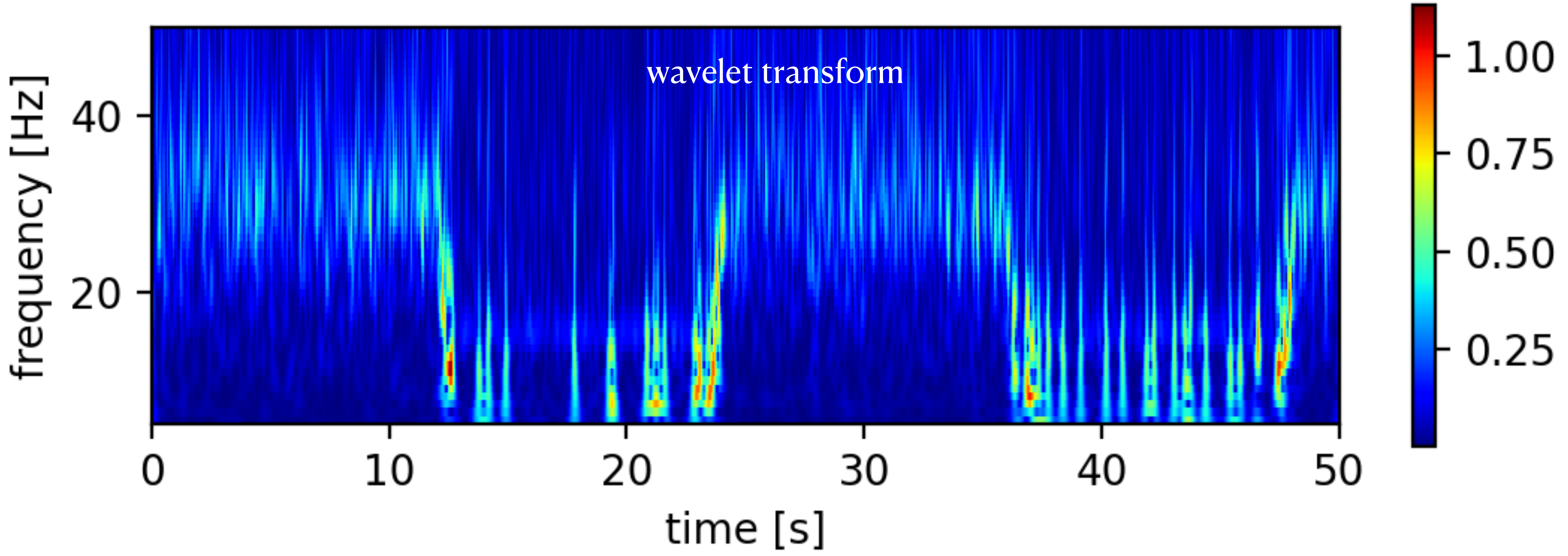


time-frequency plot

TimeFrequency_2.py

Example:

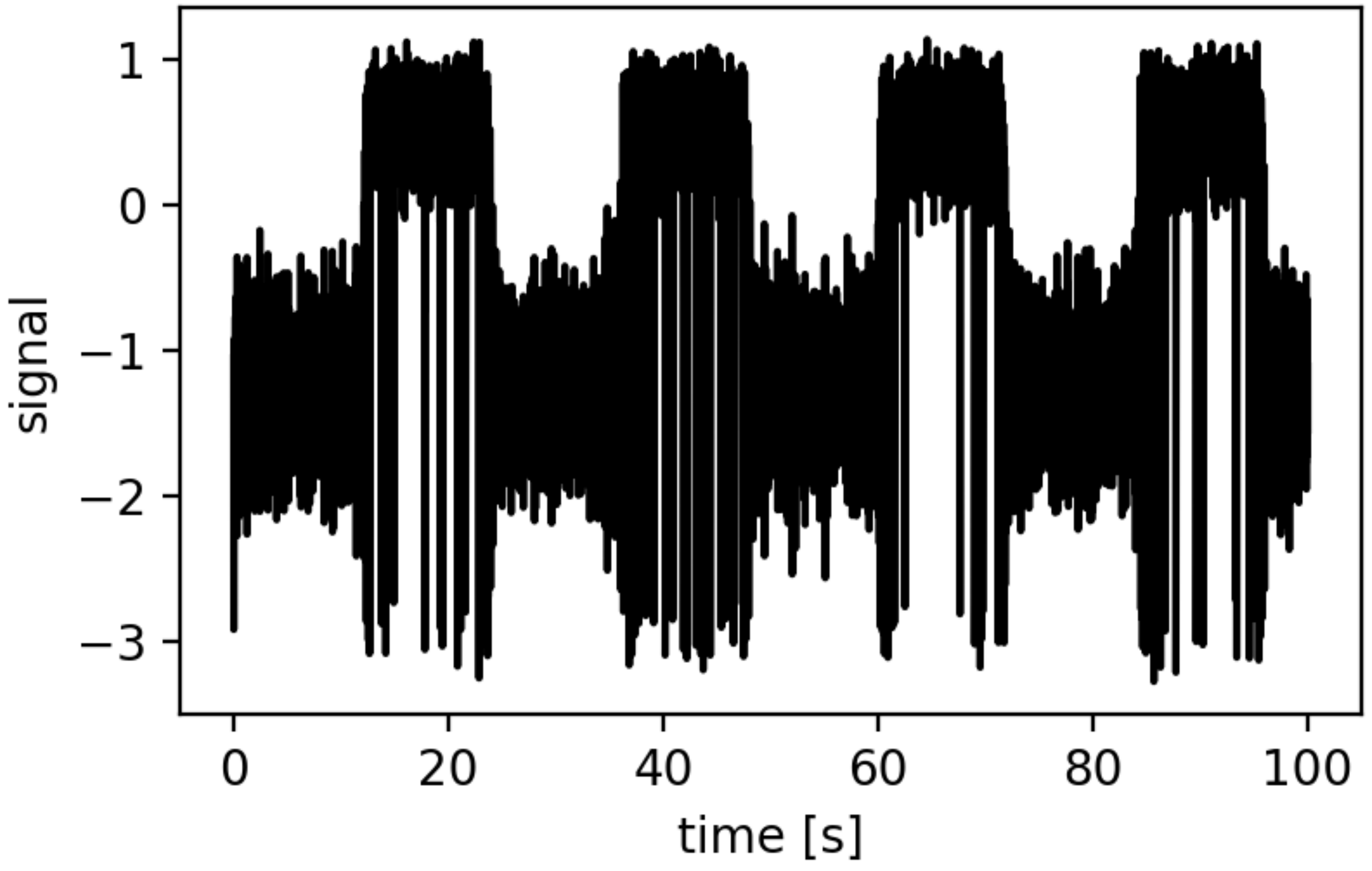


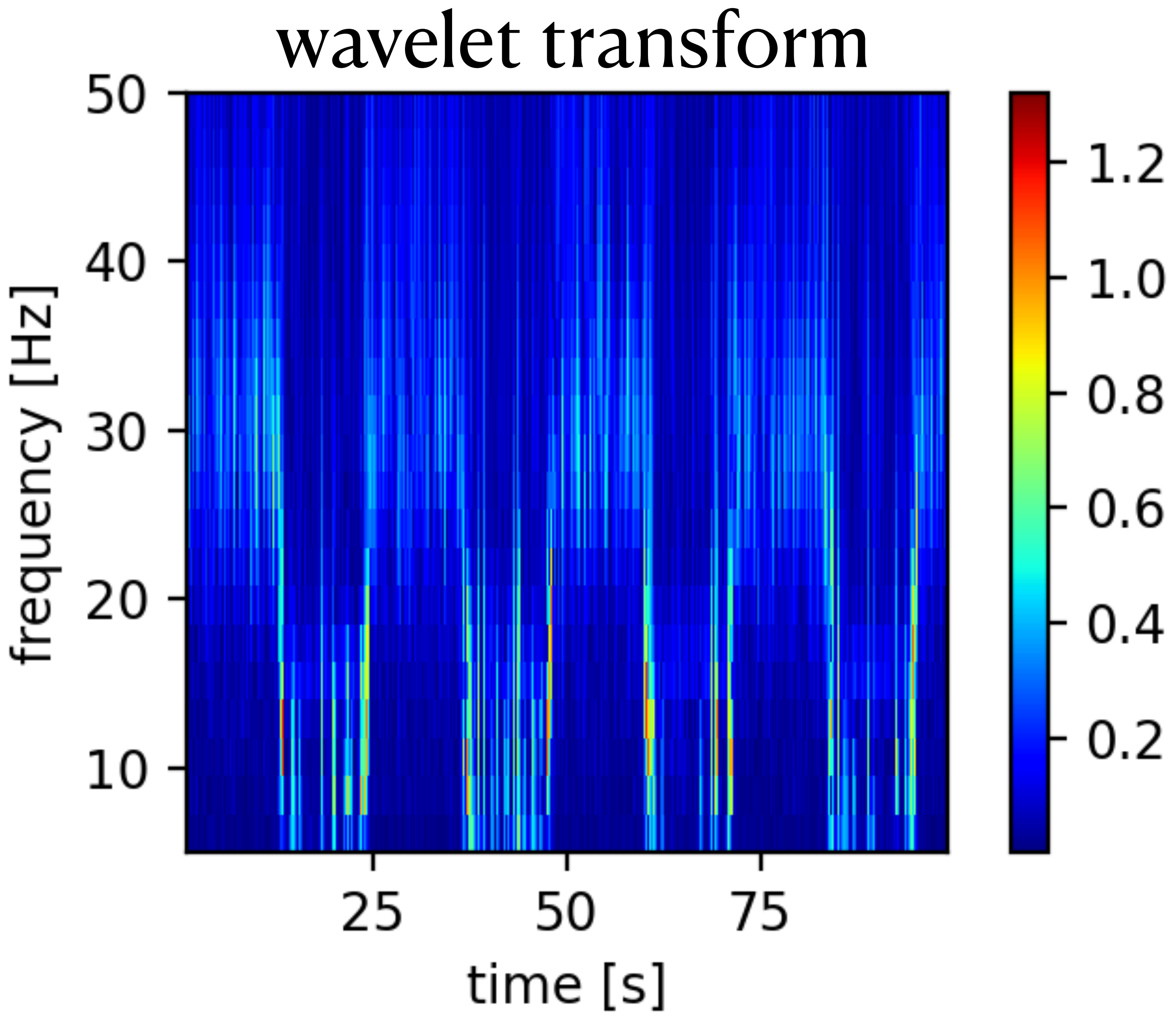


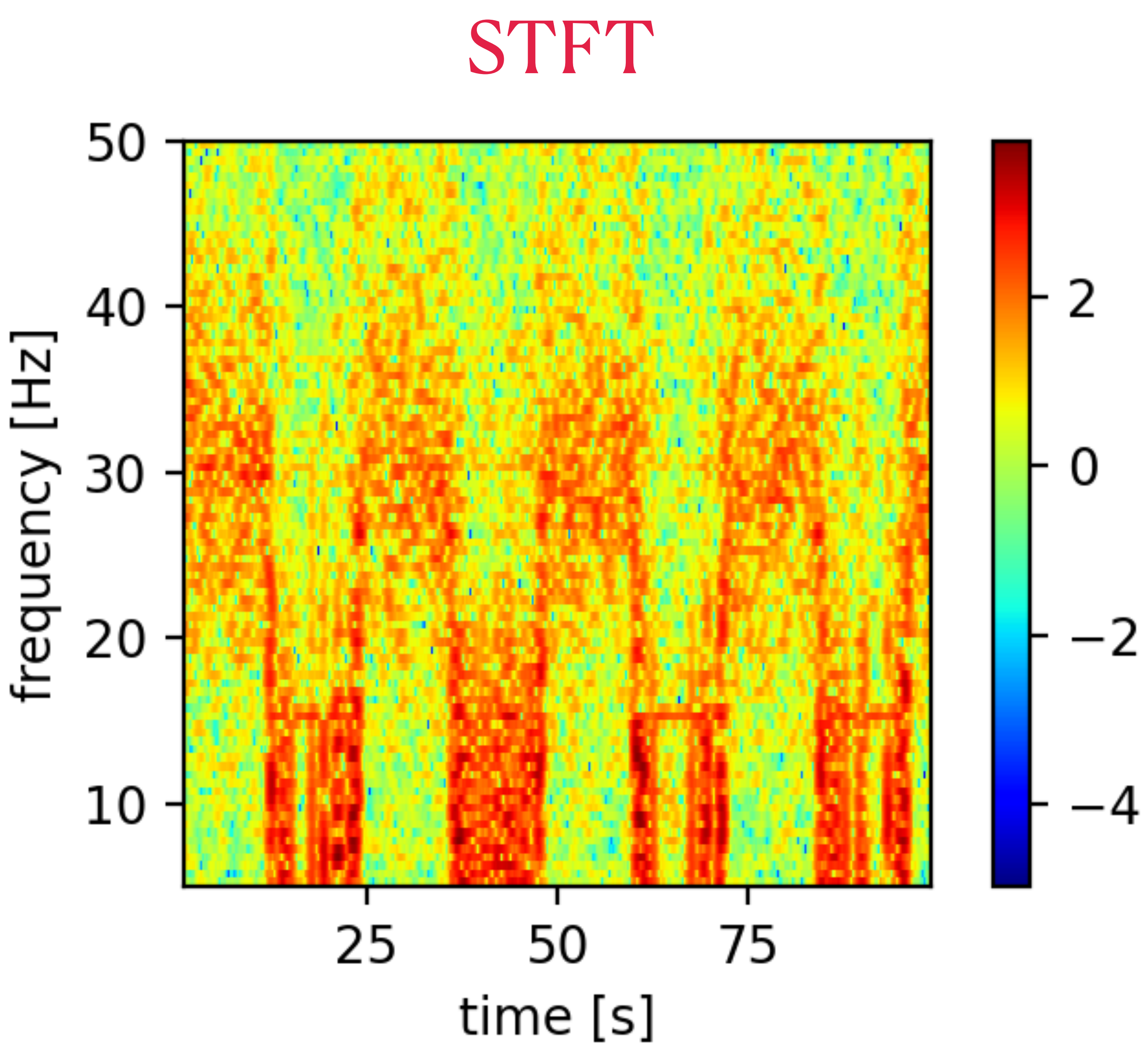
(A. Hutt and J. Lefebvre, Brain Top. (2021))

Example:

simulated cortical activity



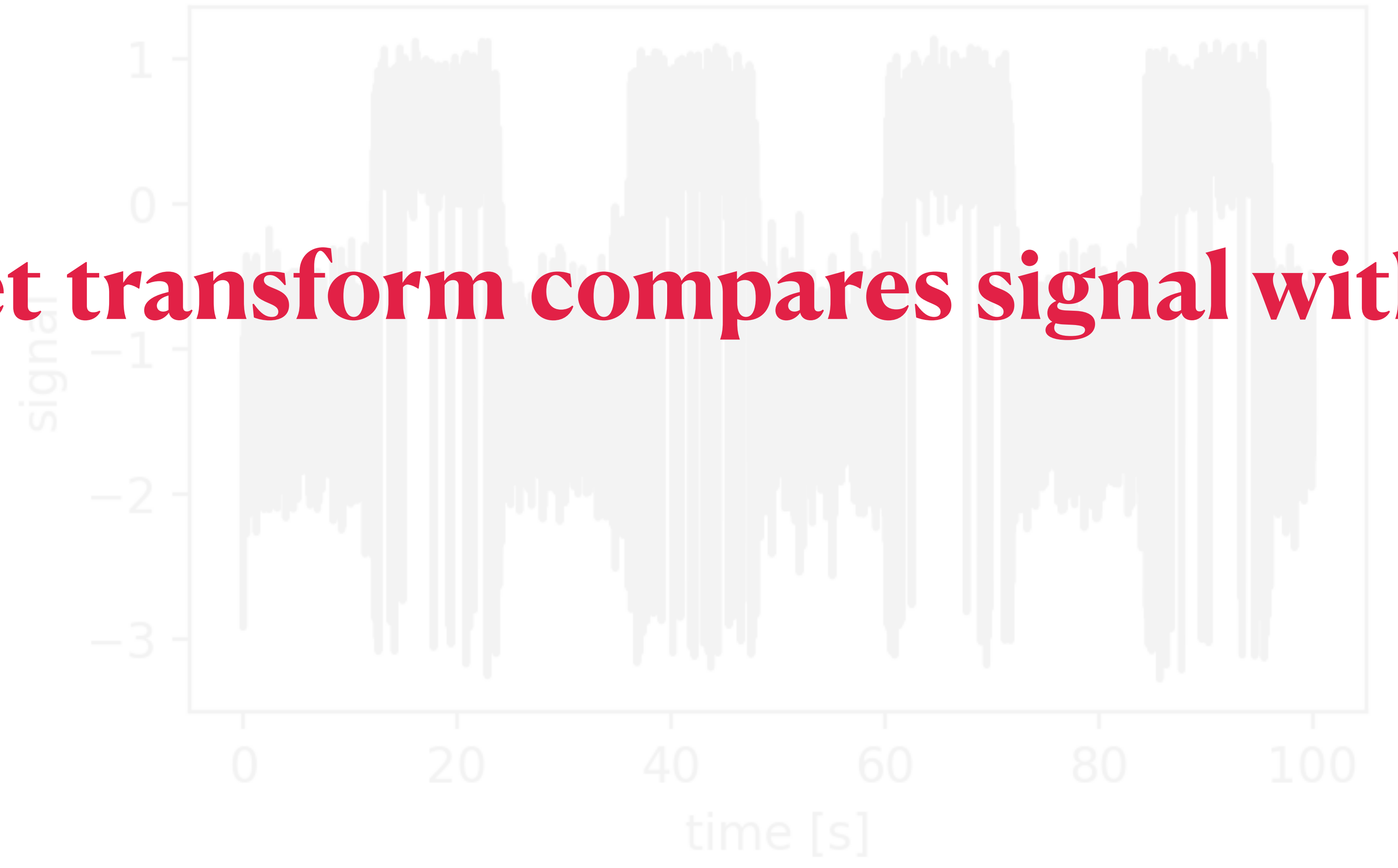




data taken from: A. Hutt and J. Lefebvre, Brain Top. (2021); doi:10.1007/s10548-021-00855-z

Example:

simulated cortical activity



wavelet transform compares signal with mother wavelet

wavelet transform

STFT

data taken from: A. Hutt and J. Lefebvre, Brain Top. (2021); doi:10.1007/s10548-021-00855-z

Example:

simulated cortical activity

wavelet transform compares signal with mother wavelet

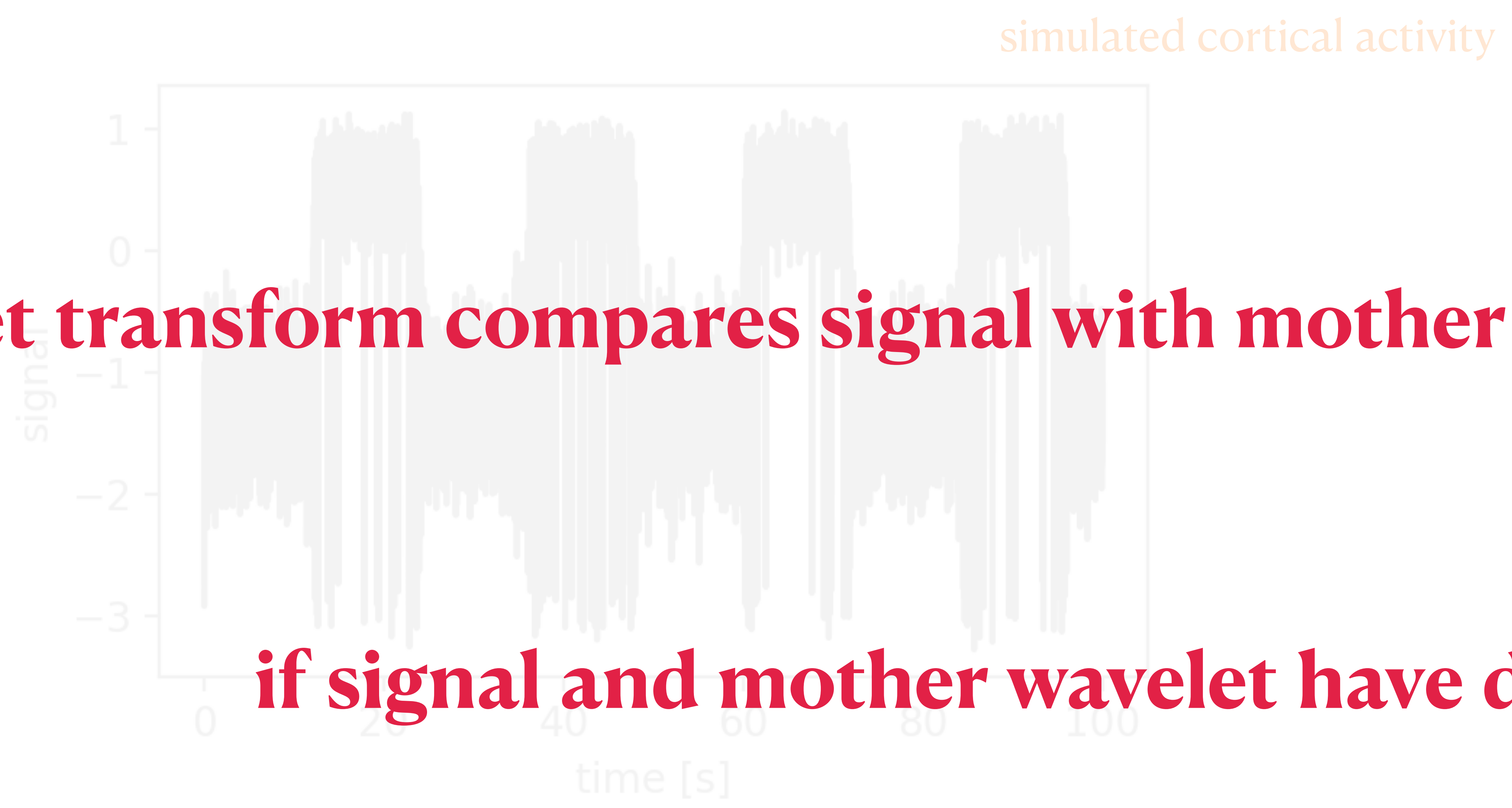
if signal and mother wavelet have different shapes: low value

wavelet transform

STFT

data taken from: A. Hutt and J. Lefebvre, Brain Top. (2021); doi:10.1007/s10548-021-00855-z

Example:



wavelet transform compares signal with mother wavelet

if signal and mother wavelet have different shapes: low value

take home message: try STFT and wavelet transform .. or others

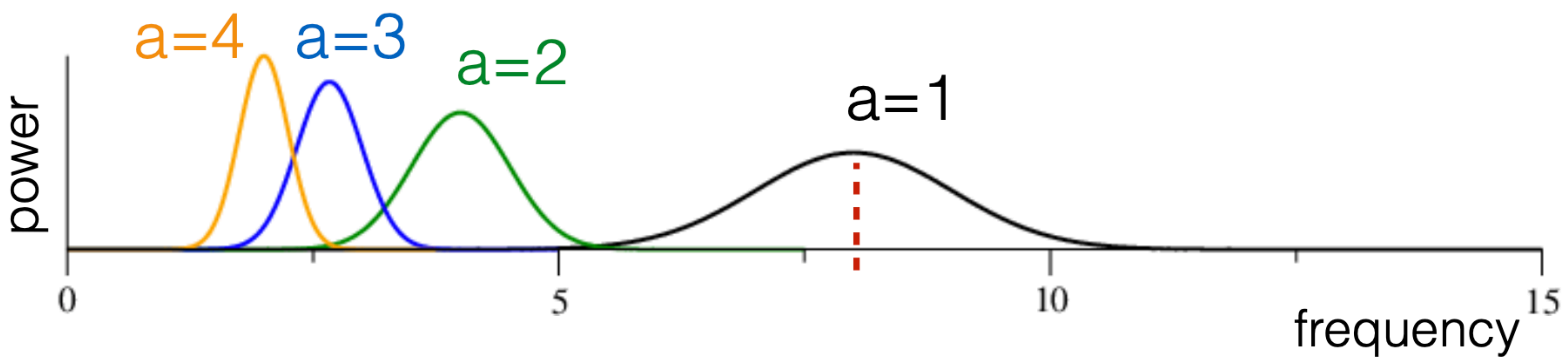
data taken from: A. Hutt and J. Lefebvre, Brain Top. (2021); doi:10.1007/s10548-021-00855-z

related Python libraries

PyWavelets

https://github.com/PyWavelets/pywt

comment: what is the Discrete Wavelet Transform ?

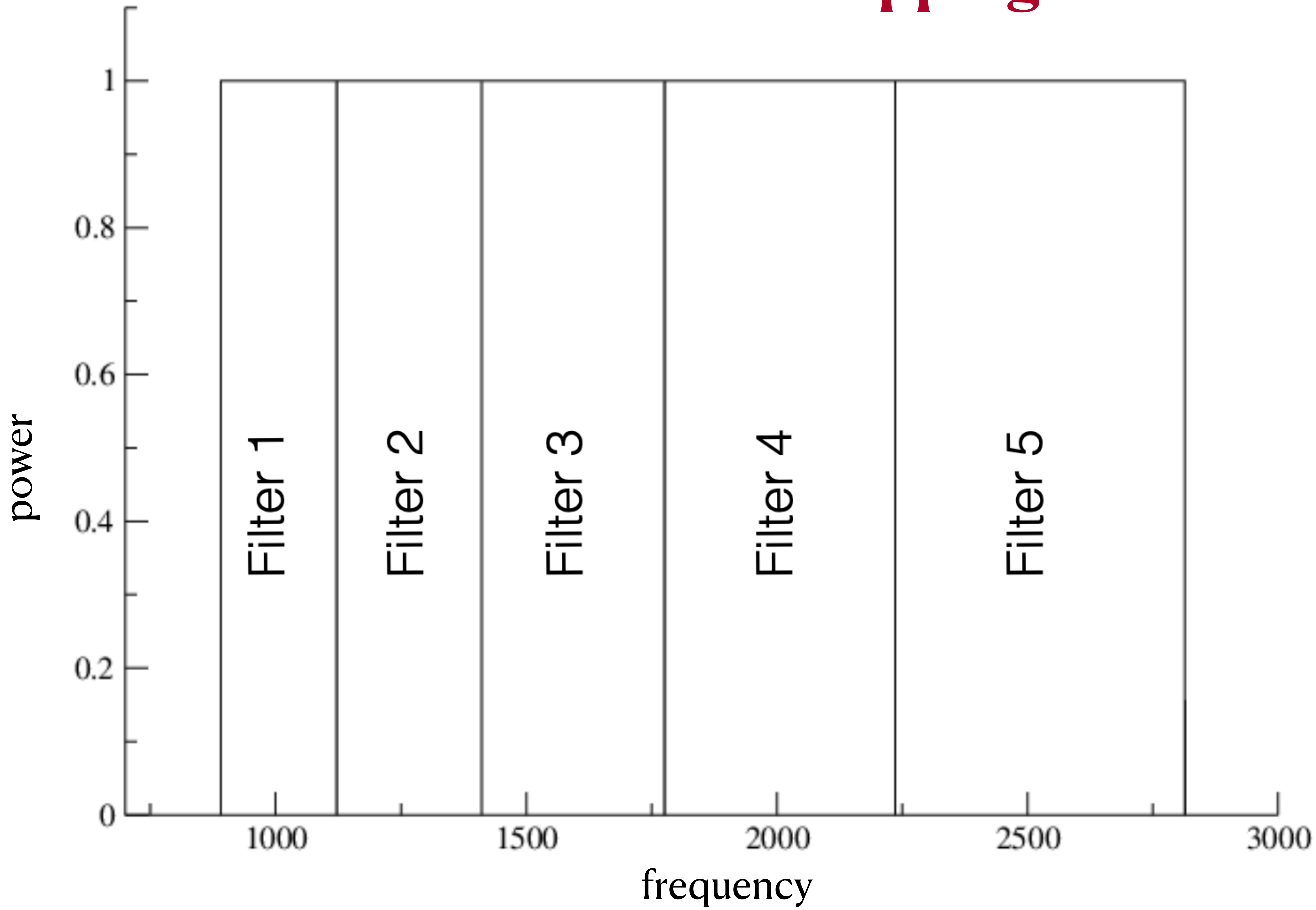


overlapping frequency bands for different a !!

comment: what is the Discrete Wavelet Transform ?

Discrete Wavelet Transform

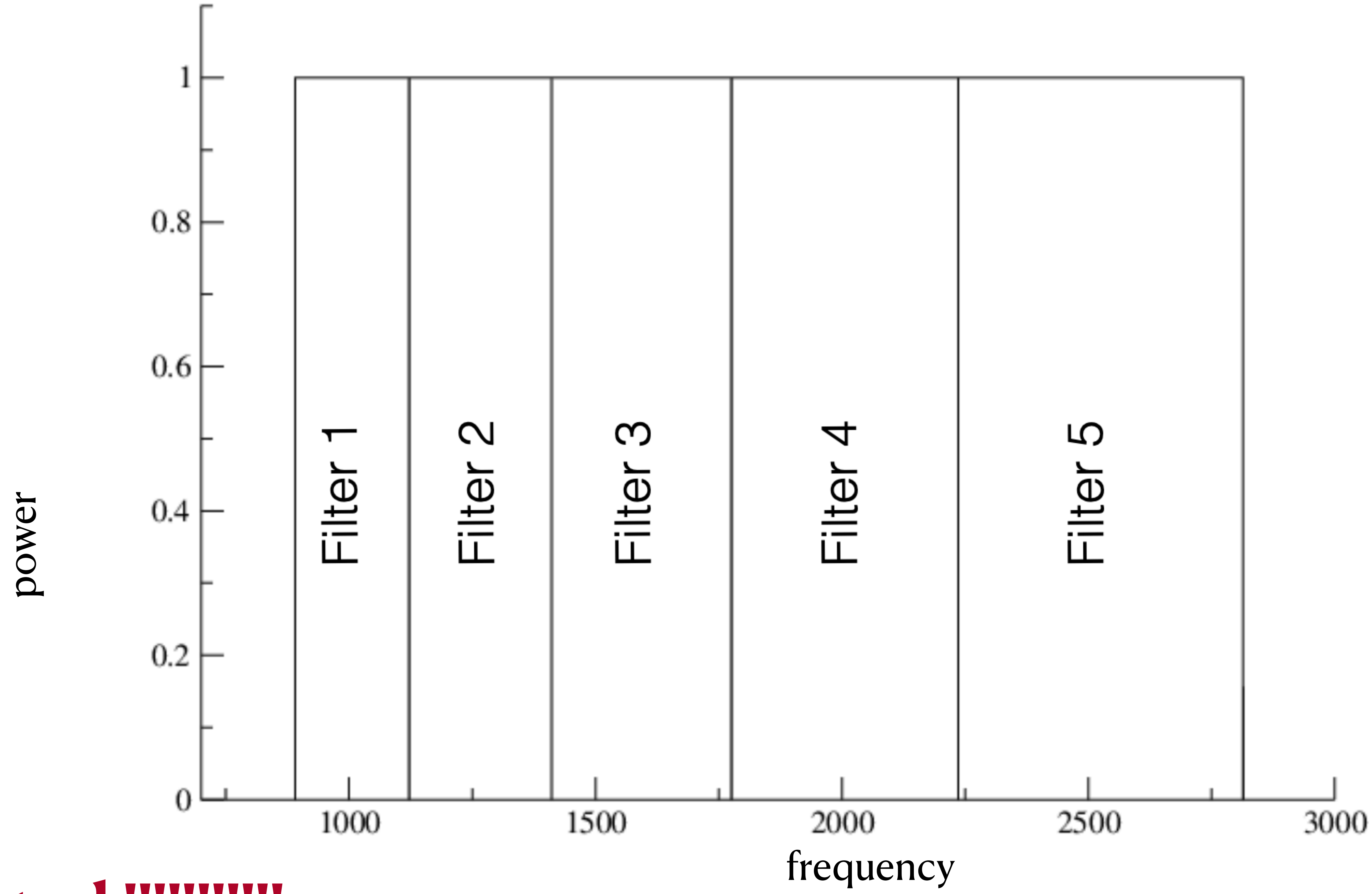
filter bank with **non-overlapping** bands



comment: what is the Discrete Wavelet Transform ?

Discrete Wavelet Transform

filter bank with **non-overlapping** bands



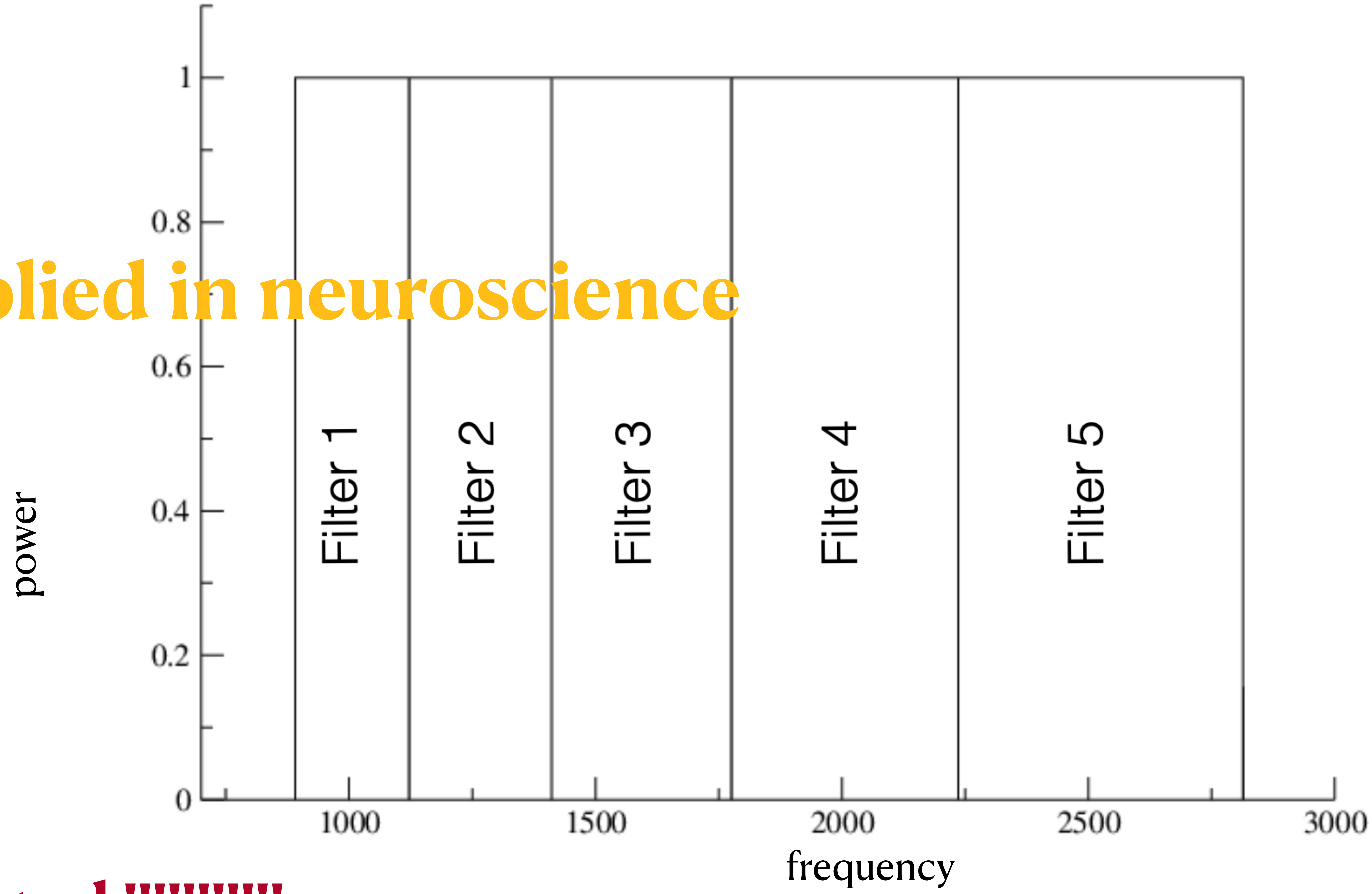
optimal decomposition of signal !!!!!!!!!!!

comment: what is the Discrete Wavelet Transform ?

filter bank with **non-overlapping** bands

but rarely applied in neuroscience

Discrete Wavelet Transform



optimal decomposition of signal !!!!!!!!!!!

data sampling

Fourier analysis

errors in analysis

linear filters

time-frequency analysis

uni-resolution analysis

multi-resolution analysis

non-Fourier analysis

data sampling

Fourier analysis

errors in analysis

linear filters

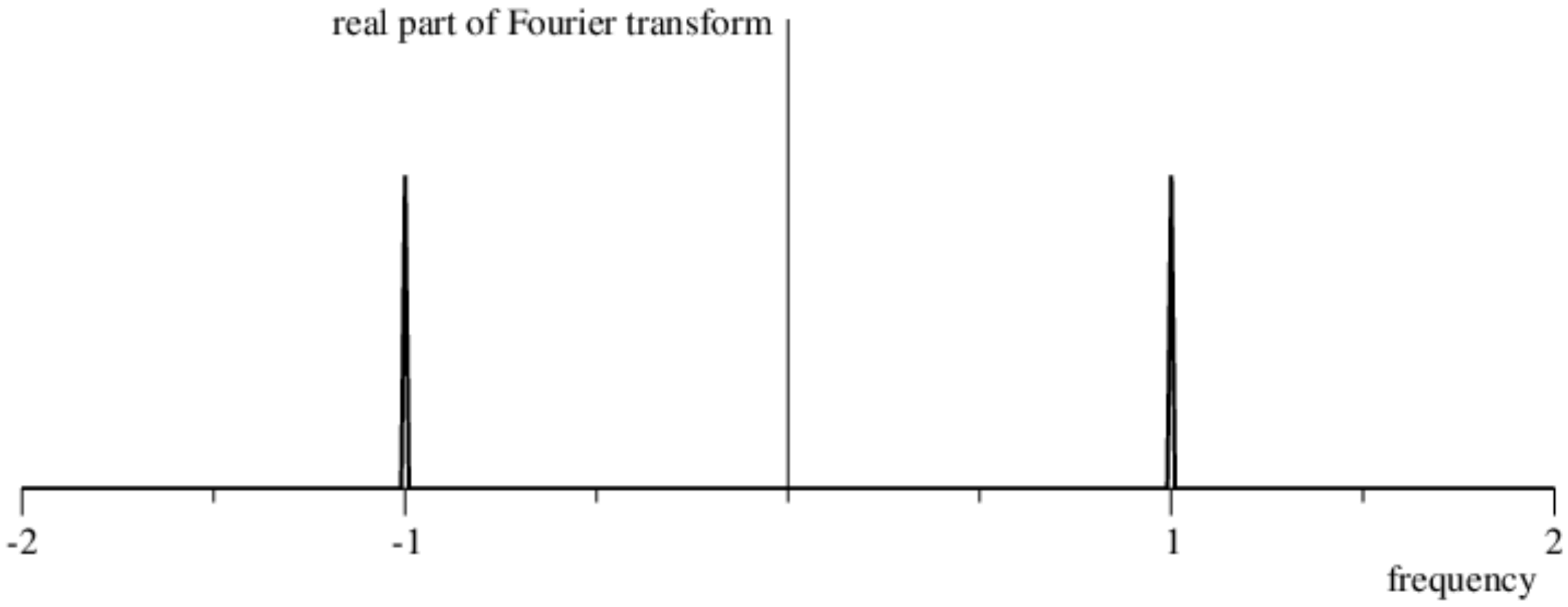
time-frequency analysis

non-Fourier analysis

Hilbert Transform

Hilbert Transform

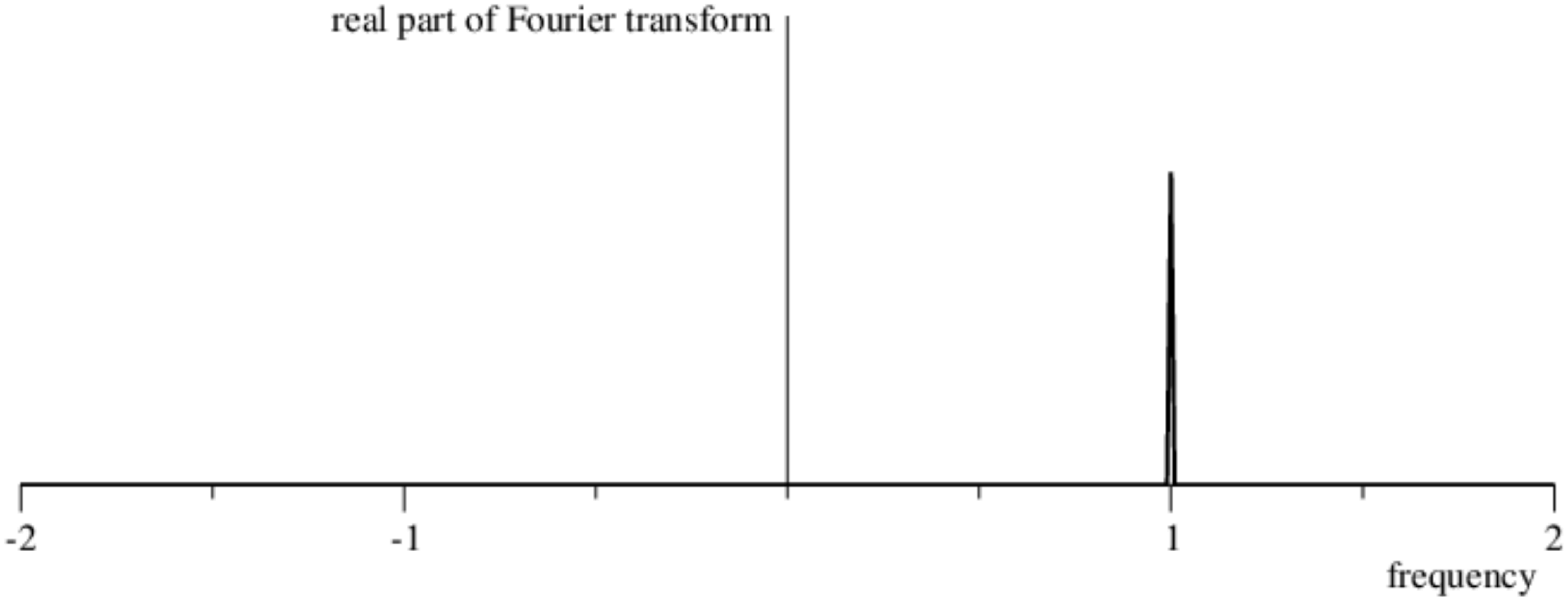
$$s(t) = \cos(2\pi f_0 t)$$



an oscillation with a single frequency has a power spectrum with negative frequency

Hilbert Transform

$$s_a(t) = \cos(2\pi ft) + i\sin(2\pi ft) = e^{i2\pi ft}$$



analytical signal $s_a(t)$ contains a single positive frequency

Hilbert Transform

in general: $$\boxed{s_a(t) = s(t) + i\mathcal{H}[s](t)}$$

$$\mathcal{H}[s](t) = \frac{1}{\pi} P.V. \int_{-\infty}^{\infty} \frac{s(\tau)}{t-\tau}\, d\tau$$

Hilbert Transform

in general: $$\boxed{s_a(t) = s(t) + i\mathcal{H}[s](t)}$$

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$$\boxed{s_a(t) = R(t)e^{i\phi(t)}}$$

R: instantaneous amplitude

ϕ: instantaneous phase

Hilbert Transform

in general: $$\boxed{s_a(t) = s(t) + i\mathcal{H}[s](t)}$$

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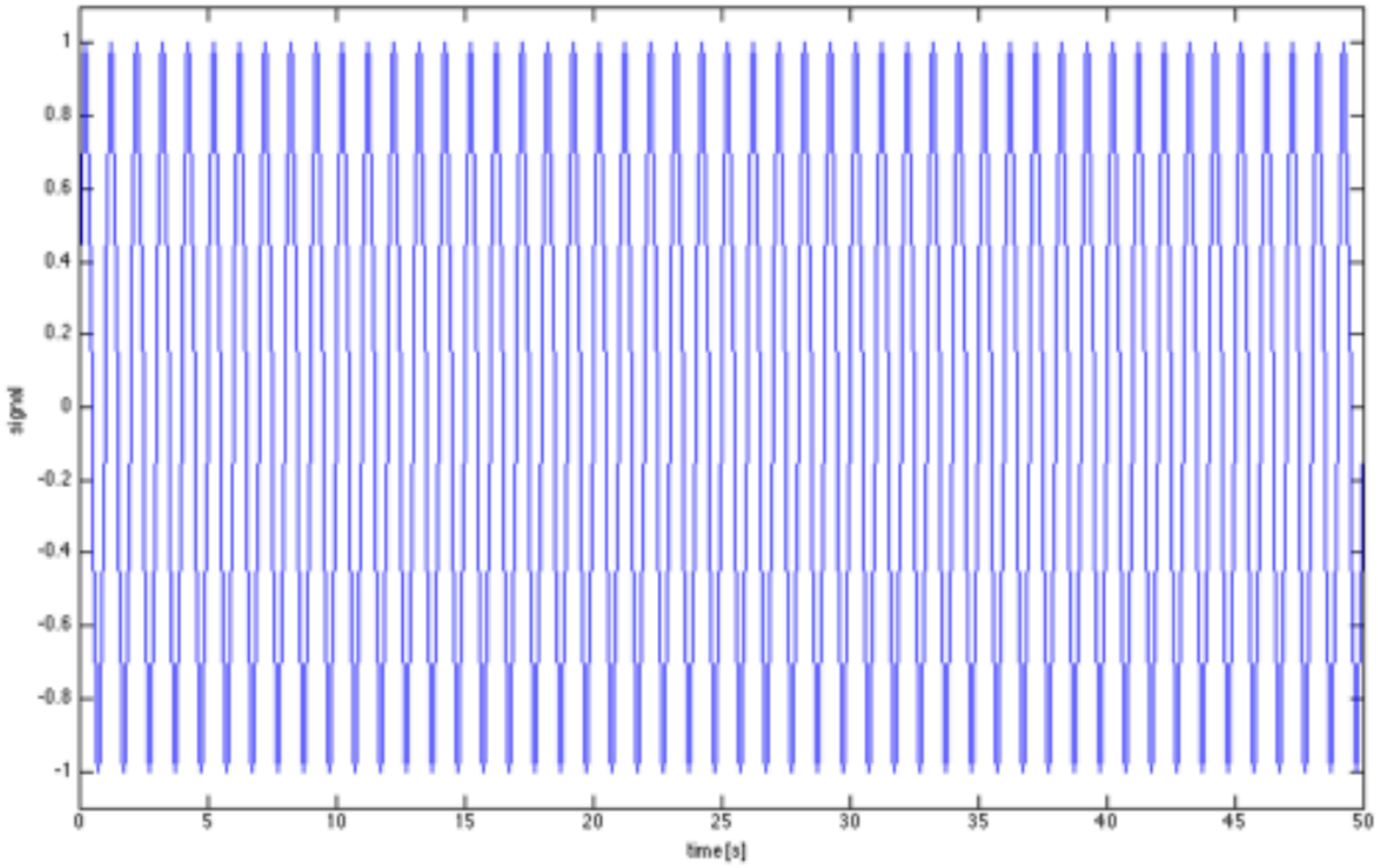
$$\boxed{s_a(t) = R(t)e^{i\phi(t)}}$$

R: instantaneous amplitude

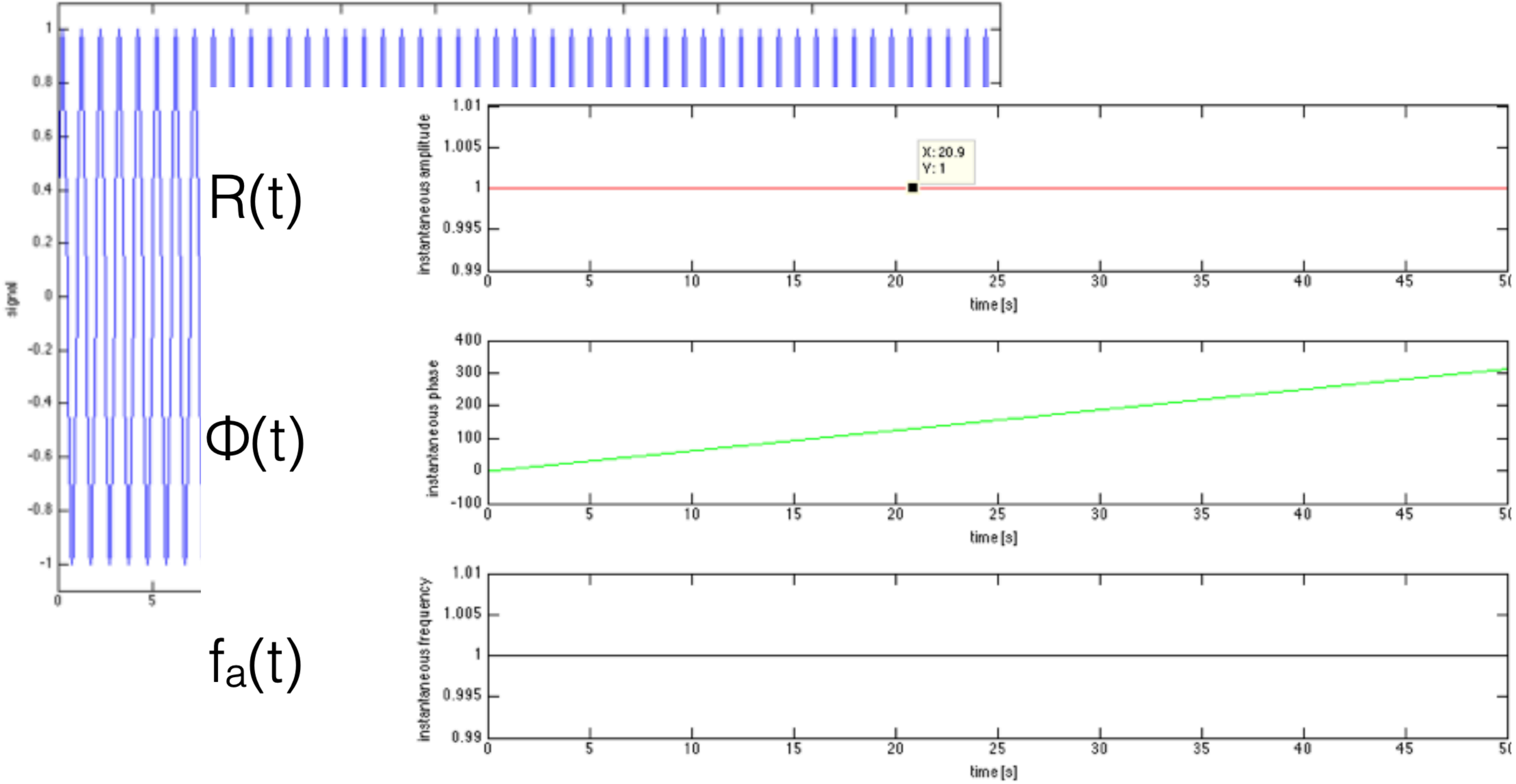
ϕ: instantaneous phase

$\rightarrow$ $\boxed{\text{instantaneous frequency} \quad f_a(t) = \frac{1}{2\pi}\frac{d\phi(t)}{dt}}$

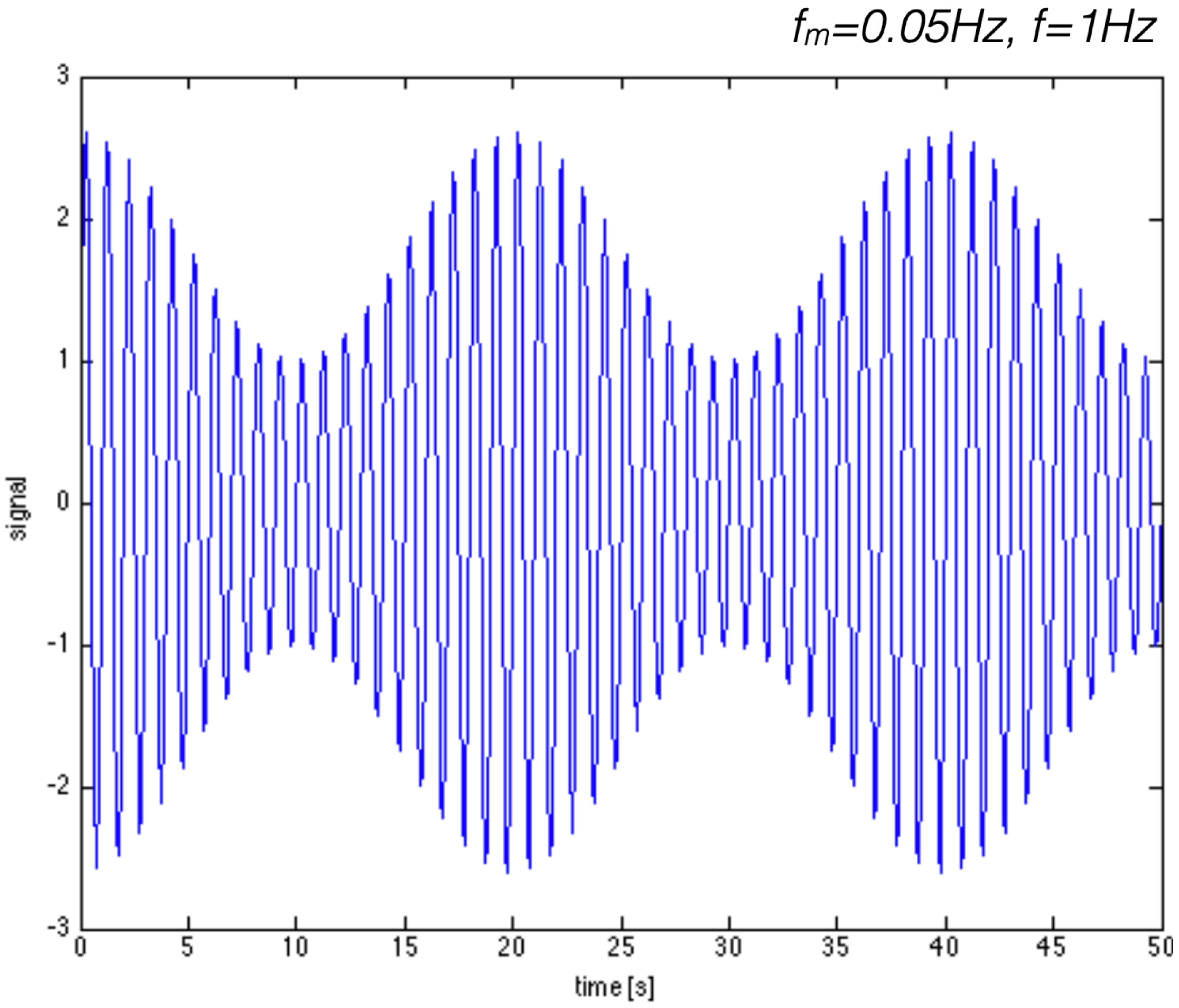
example signal: single frequency oscillation s(t)=sin(2πt)



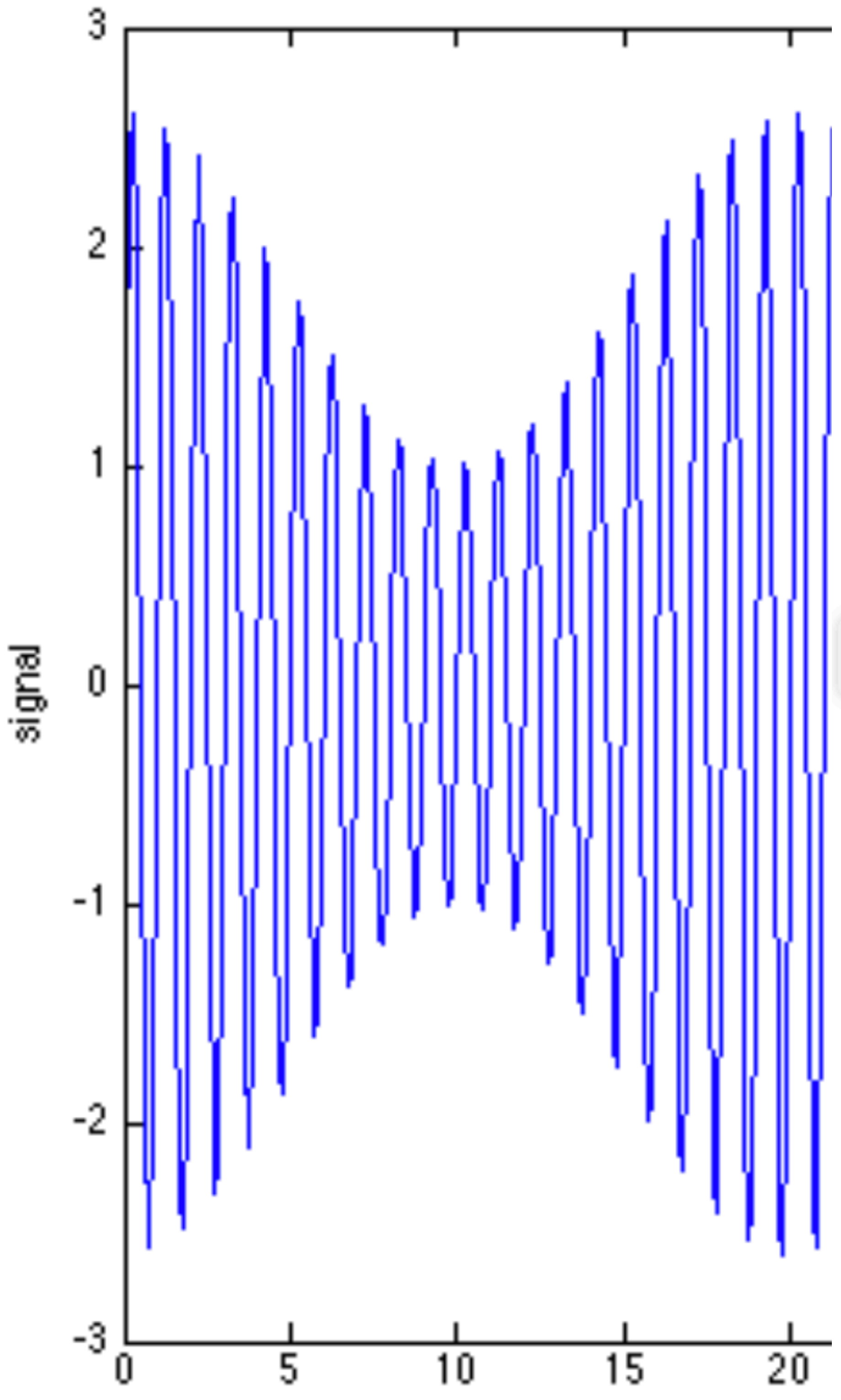
example signal: single frequency oscillation $s(t)=\sin(2\pi t)$



example: amplitude-modulated oscillation $s(t) = [1 + 0.8\cos(2\pi f_m t)]\sin(2\pi f t)$

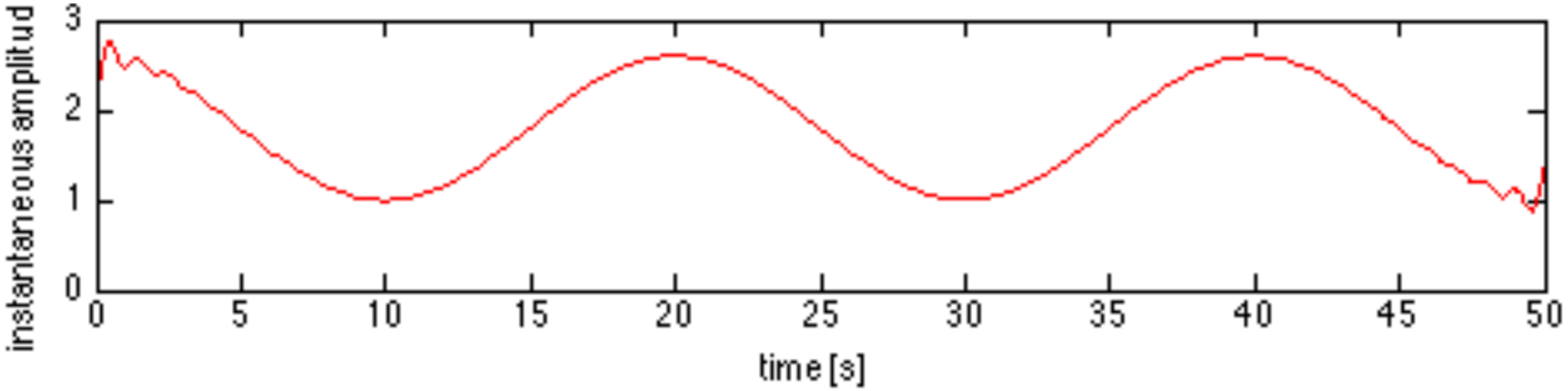


$$s(t) = [1 + 0.8\cos(2\pi f_m t)]\sin(2\pi f t)$$

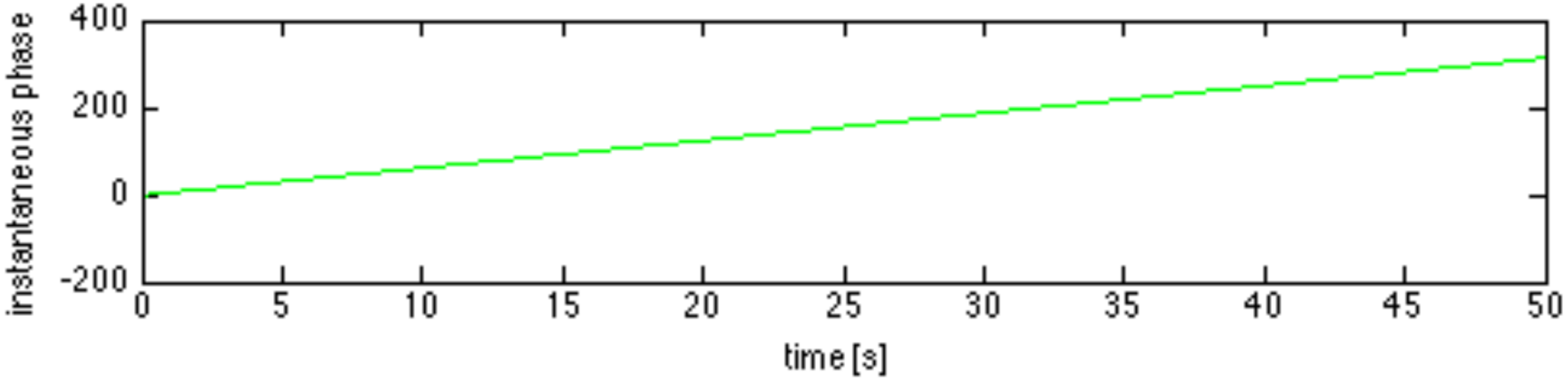


Screenshot

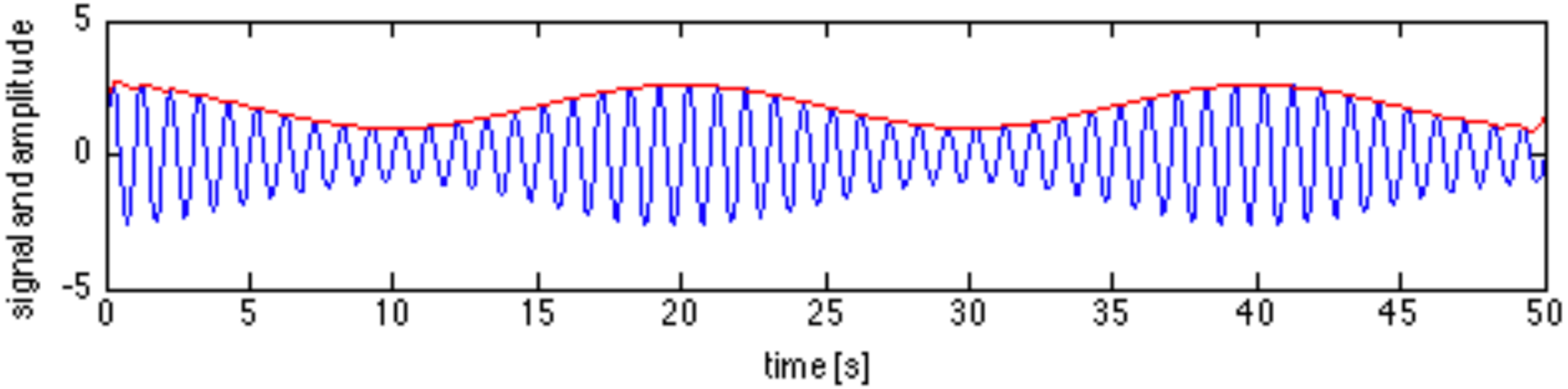
R(t)



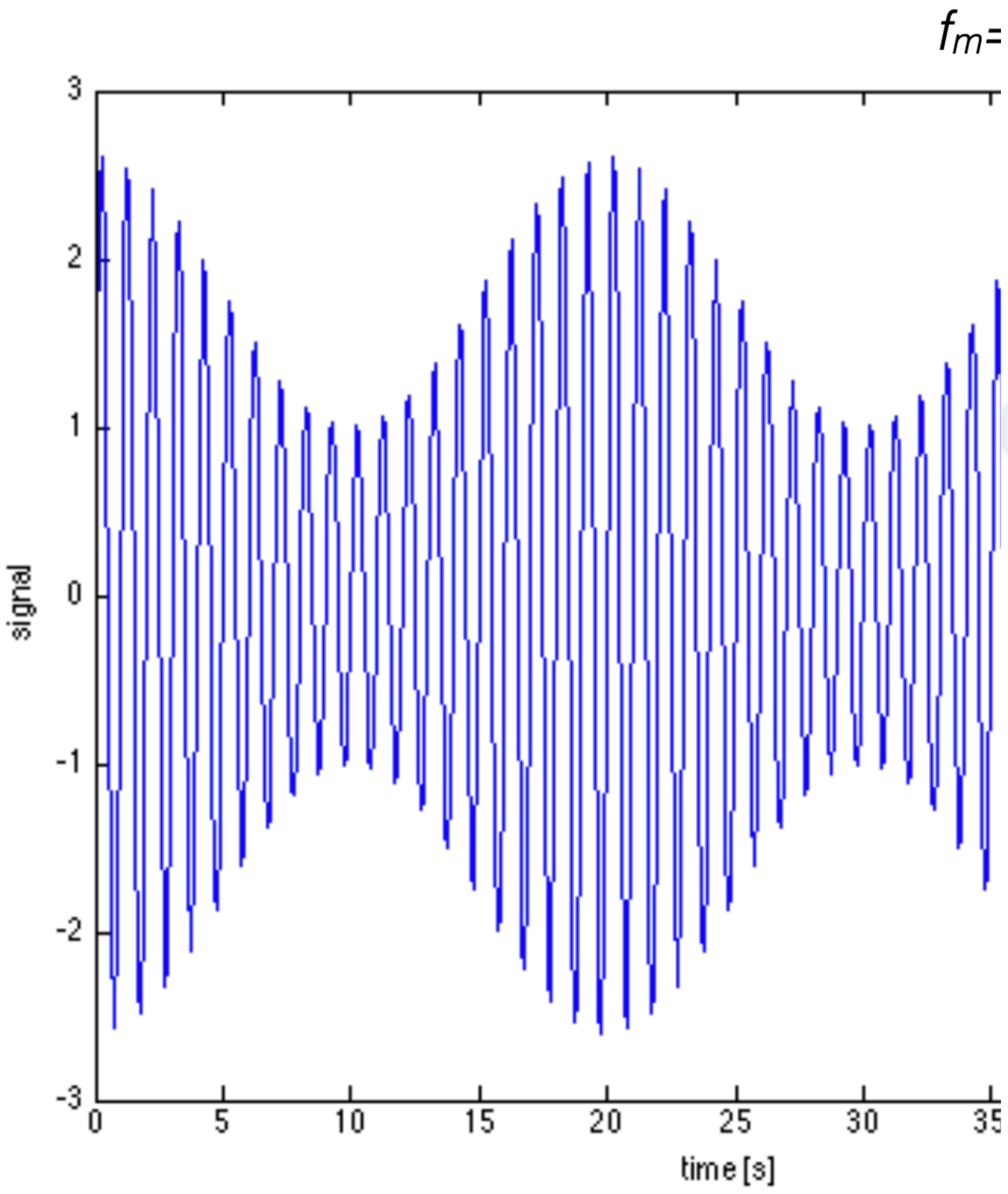
Φ(t)



R(t)cos(Φ(t))



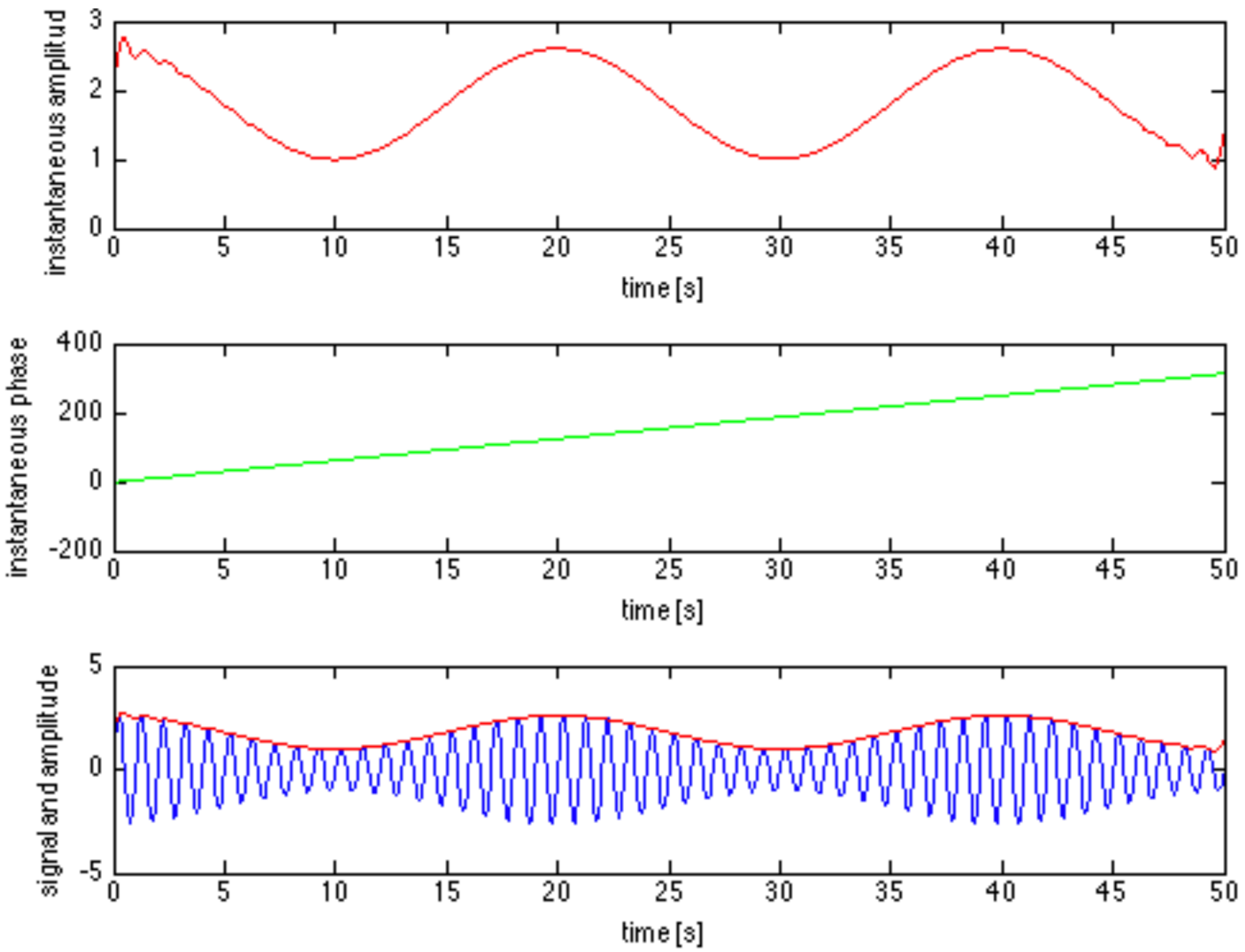
$$s(t) = [1 + 0.8\cos(2\pi f_m t)]\sin(2\pi f t)$$



R(t)

Φ(t)

R(t)cos(Φ(t))



Hilbert transform allows to determine the amplitude modulation

$$
\begin{aligned}
s(t) &= [1 + 0.8\cos(2\pi f_m t)]\sin(2\pi f t) \\
&= \sin(2\pi f t) + 0.4\sin(2\pi(f + f_m)t) + 0.4\sin(2\pi(f - f_m)t)
\end{aligned}
$$

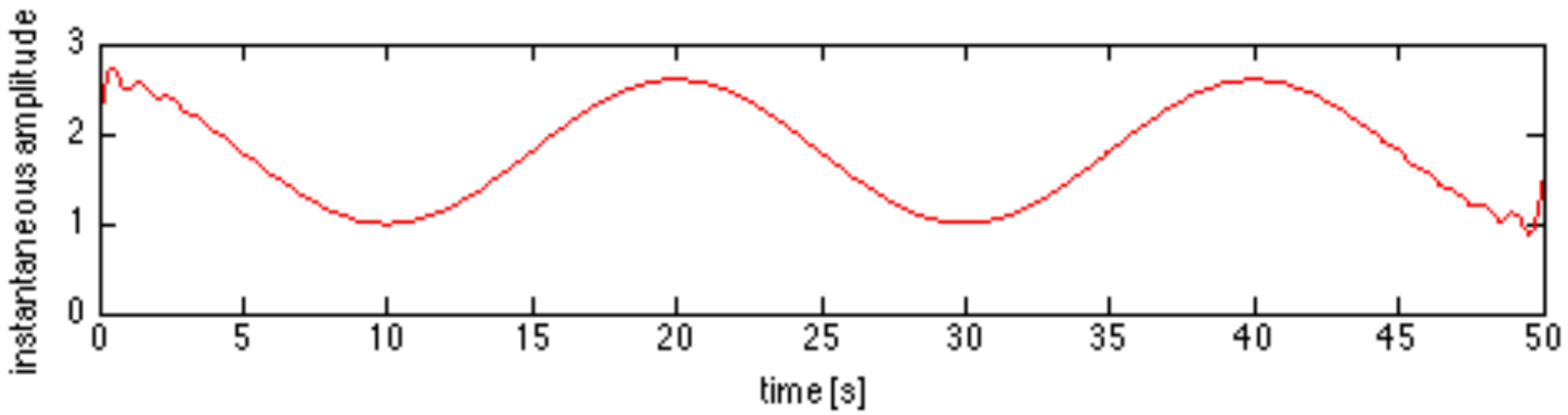
$$
\begin{aligned}
s(t) &= [1 + 0.8\cos(2\pi f_m t)]\sin(2\pi f t) \\
&= \sin(2\pi f t) + 0.4\sin(2\pi(f + f_m)t) + 0.4\sin(2\pi(f - f_m)t)
\end{aligned}
$$

3 frequencies: $f\ ,\ f + f_m\ ,\ f - f_m$

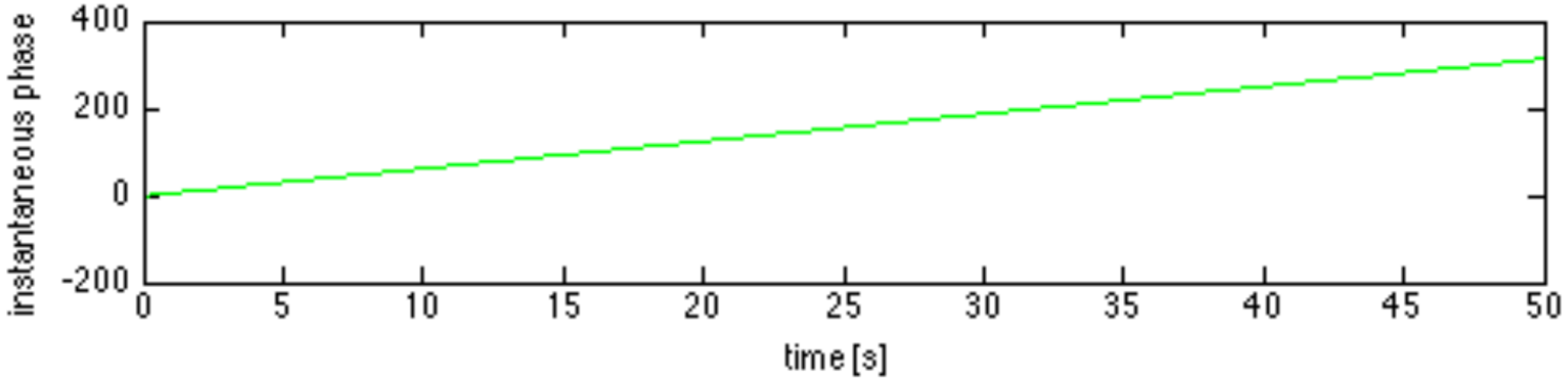
$$s(t) = [1 + 0.8\cos(2\pi f_m t)]\sin(2\pi f t)$$
$$= \sin(2\pi f t) + 0.4\sin(2\pi(f + f_m)t) + 0.4\sin(2\pi(f - f_m)t)$$

3 frequencies: $f\ ,\ f + f_m\ ,\ f - f_m$

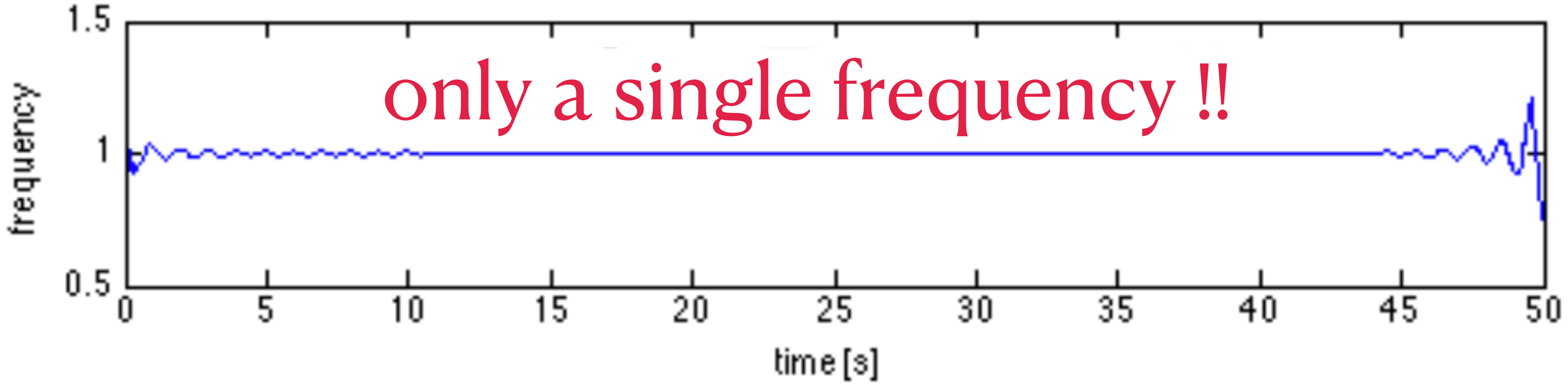
R(t)



Φ(t)



$f_a(t)$



analytical signal allows to describe amplitude modulation,

but poorly instantaneous frequency !!

data sampling

Fourier analysis

errors in analysis

linear filters

time-frequency analysis

non-Fourier analysis

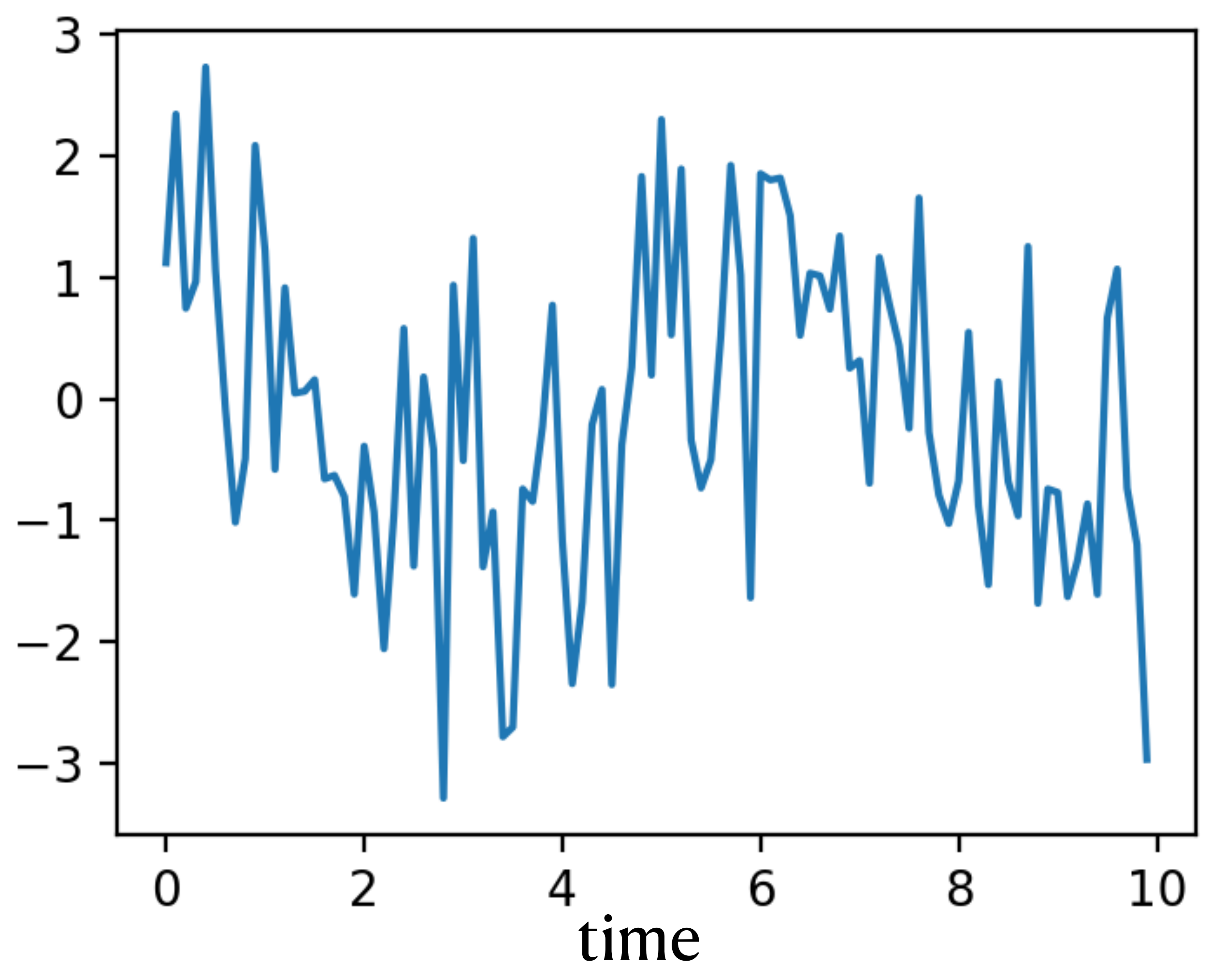
Hilbert Transform

synchronisation

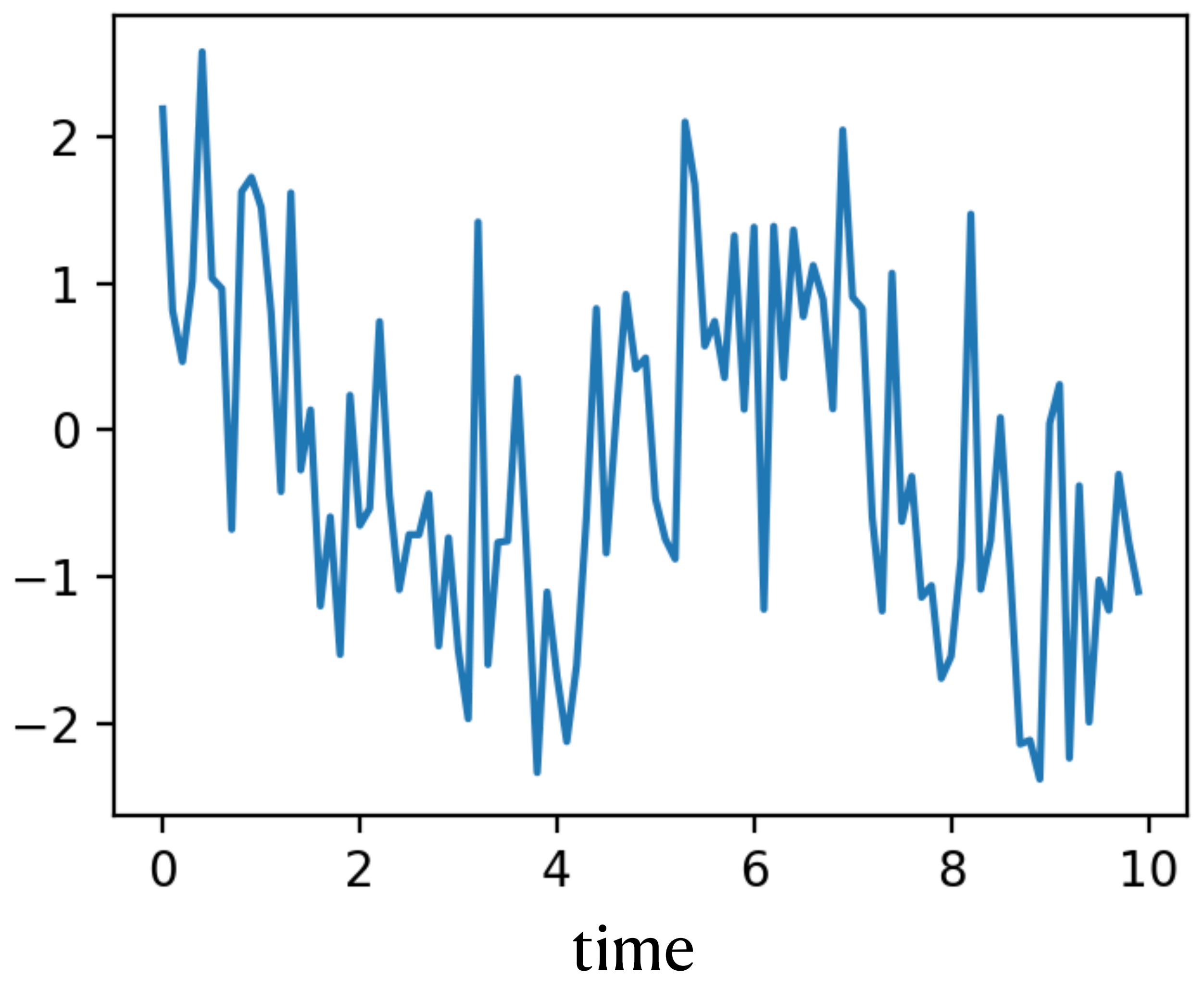
phase synchronisation

spectral coherence

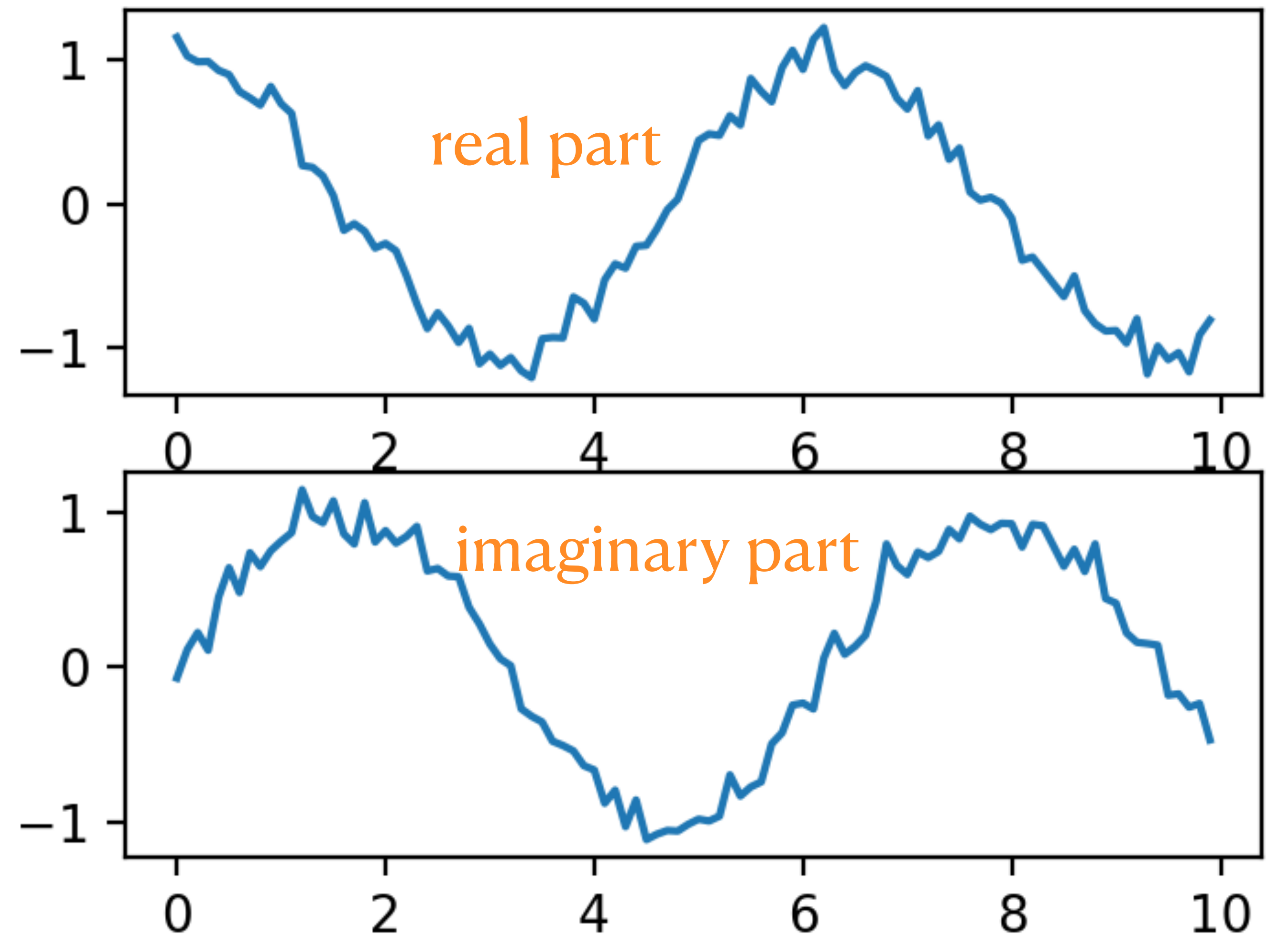
phase-amplitude coupling



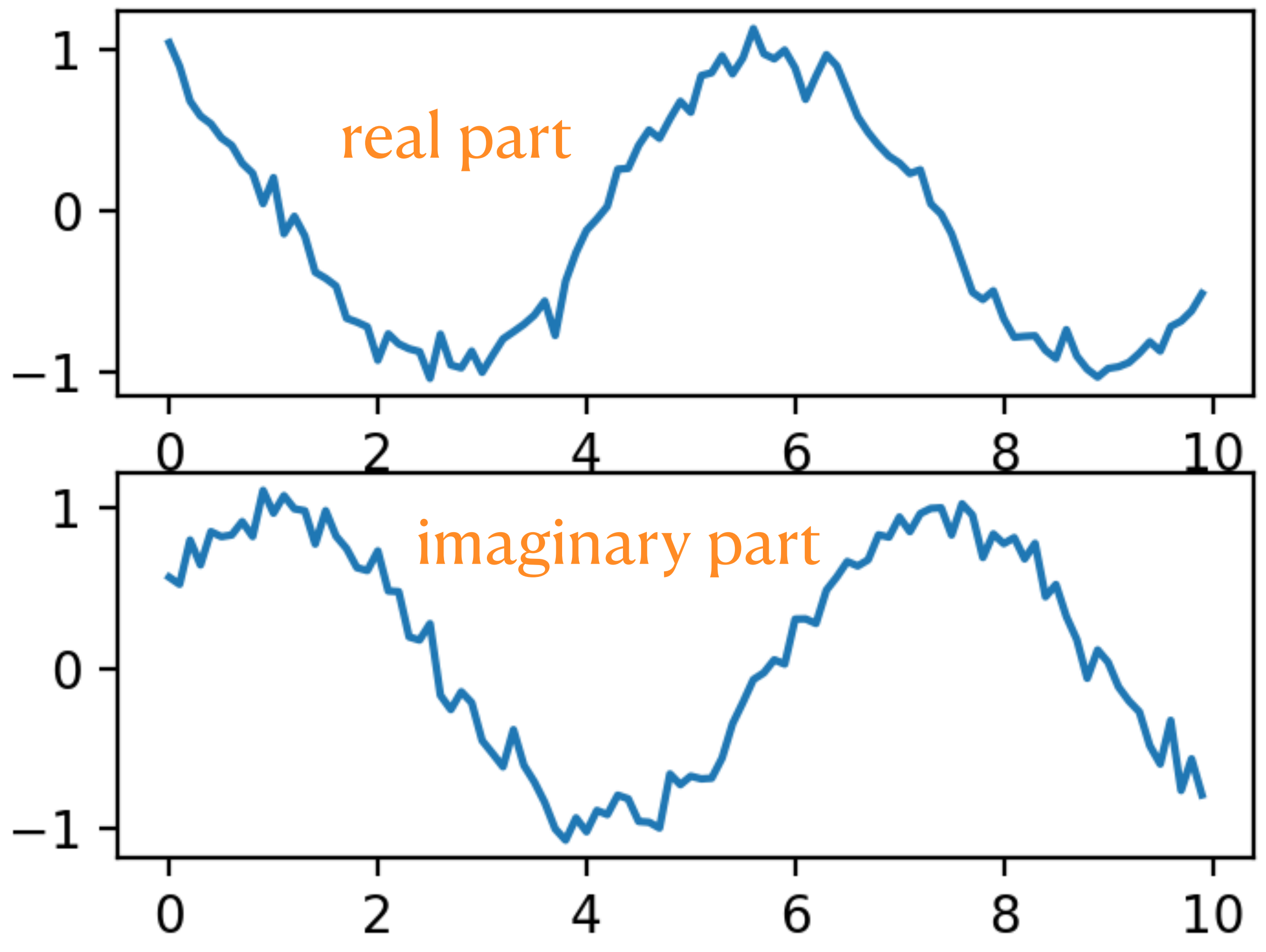
time series 1



time series 2



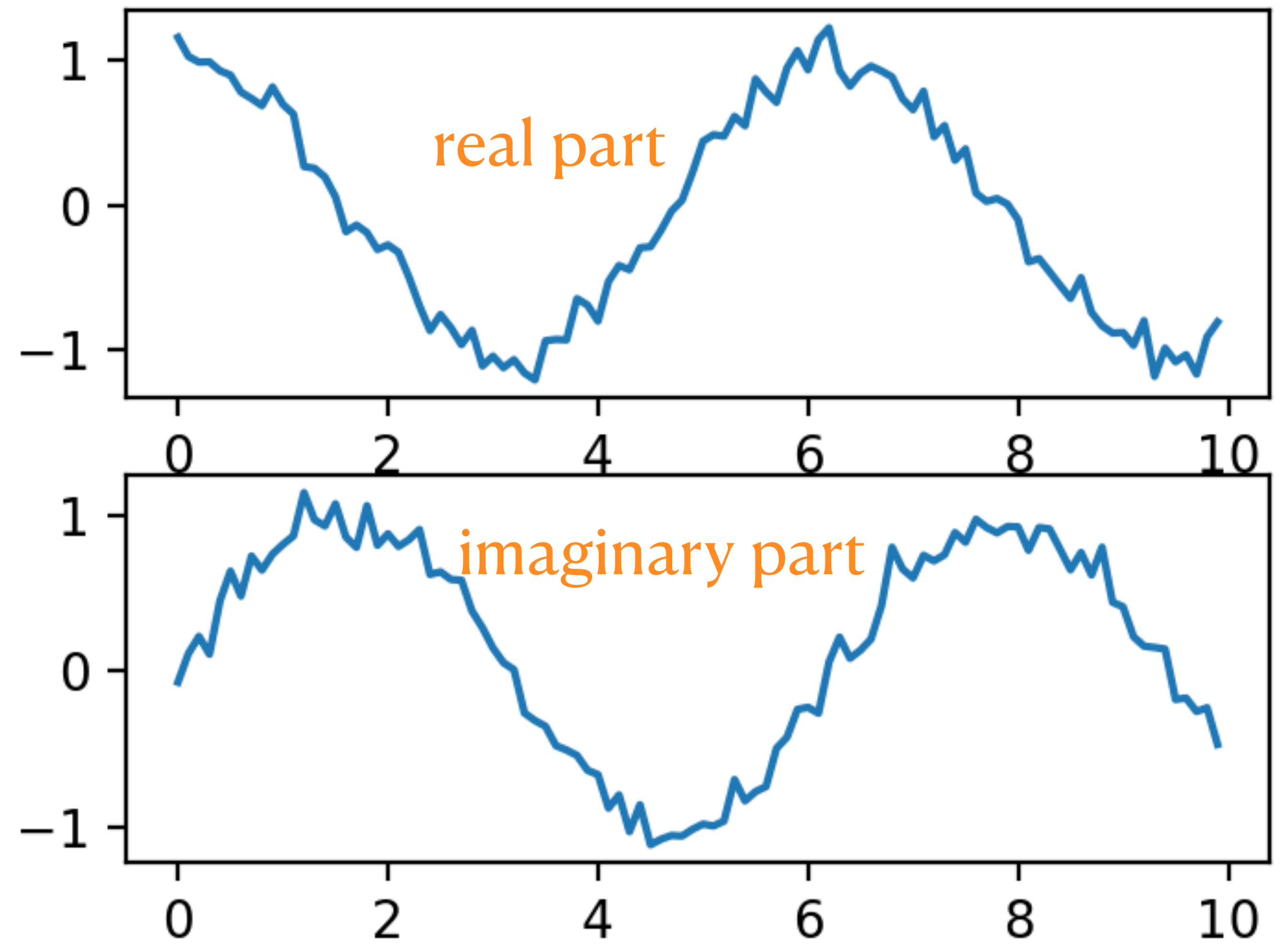
signal 1



signal 2

signal computed from time series by, e.g.

- wavelet transform
- Hilbert transform in a bandpass-filtered signal
- Short-time Fourier Transform (STFT)

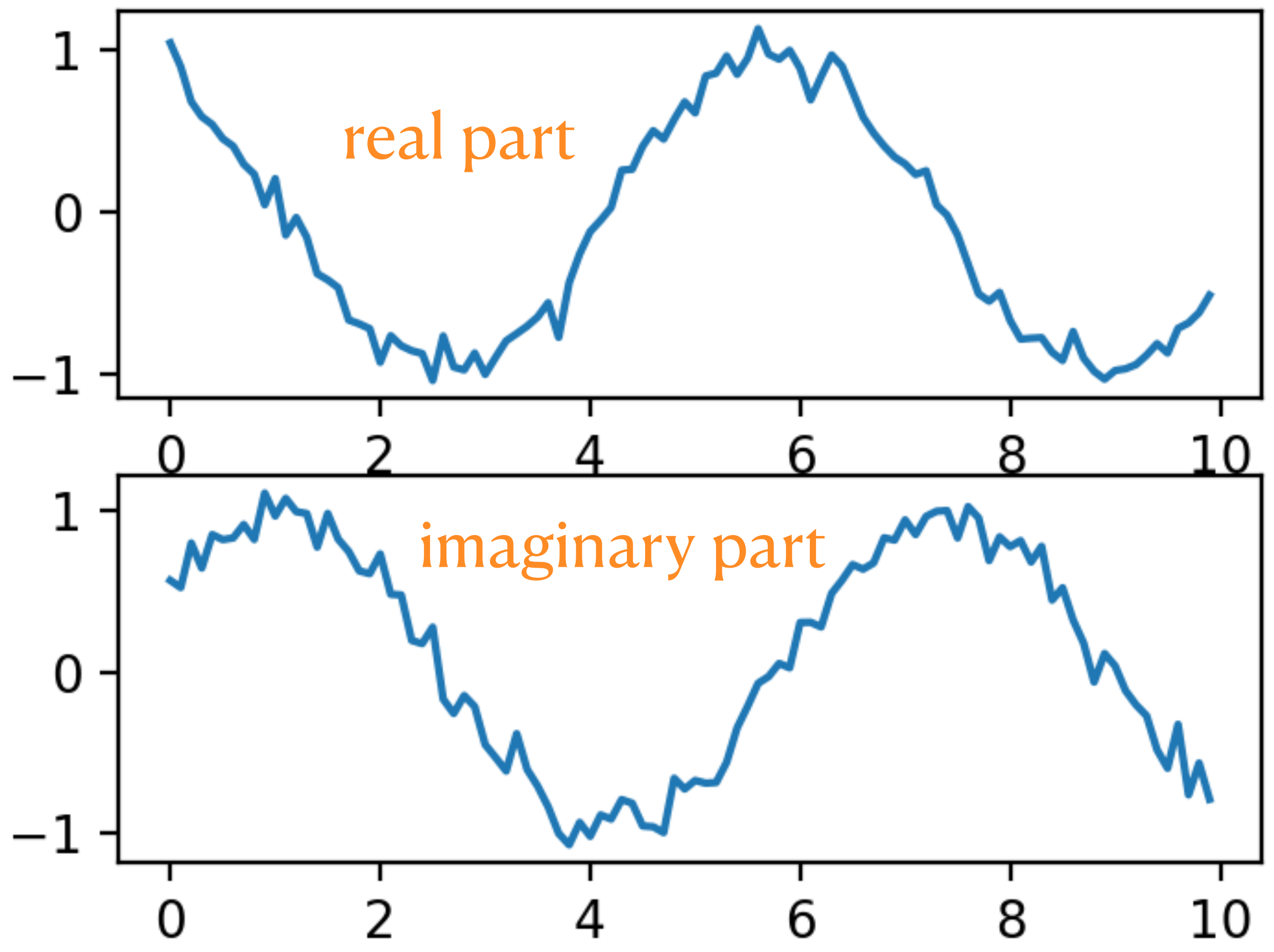


signal 1

$$z_1(t) = R_1(t)e^{i\phi_1(t)} = \underbrace{R_1(t)\cos(\phi_1(t))}_{\text{real part}} + \underbrace{iR_1(t)\sin(\phi_1(t))}_{\text{imaginary part}}$$

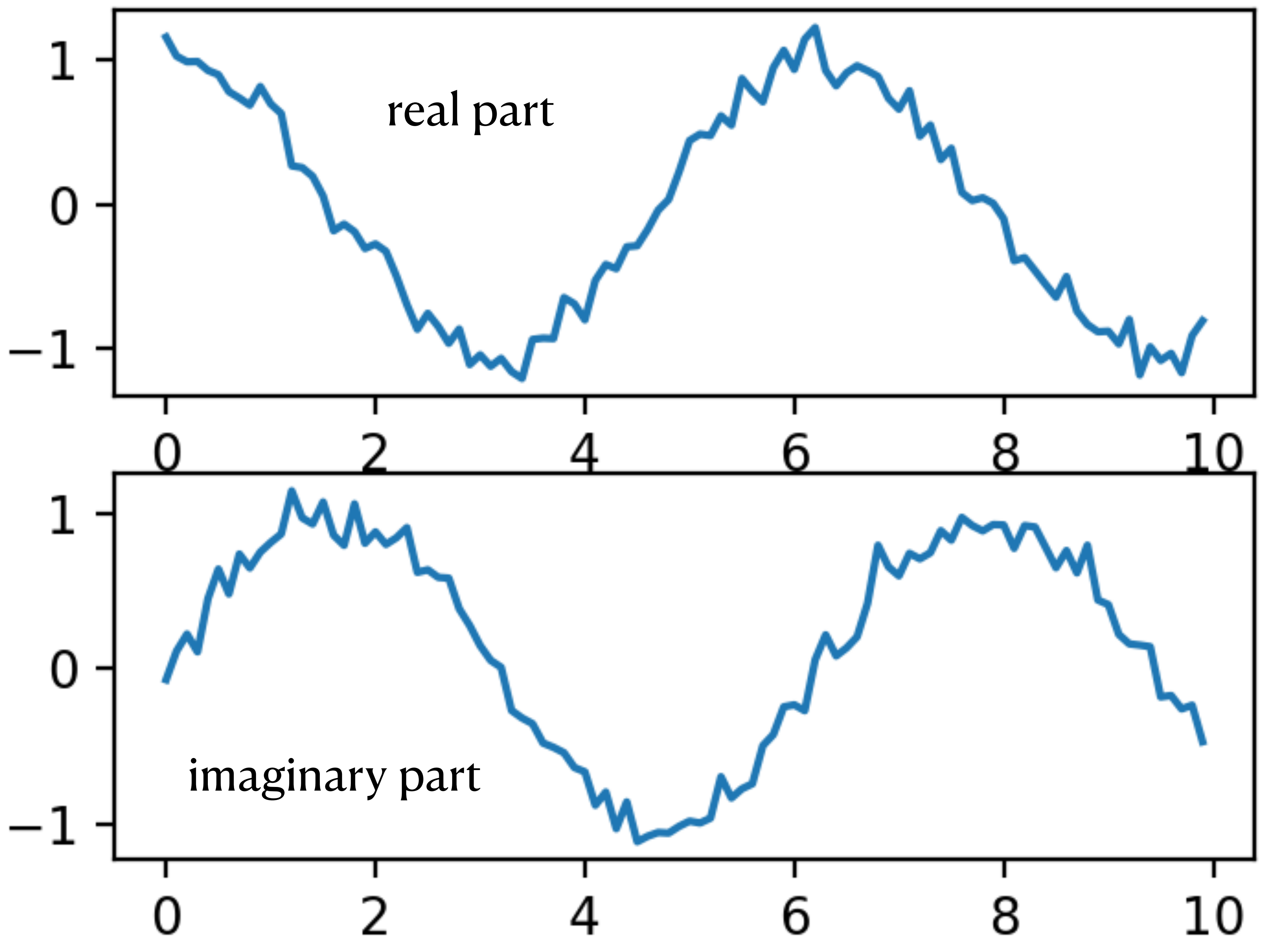
$R_1(t),\ R_2(t)$: magnitude of time series

$\phi_1(t),\ \phi_2(t)$: phase of time series



signal 2

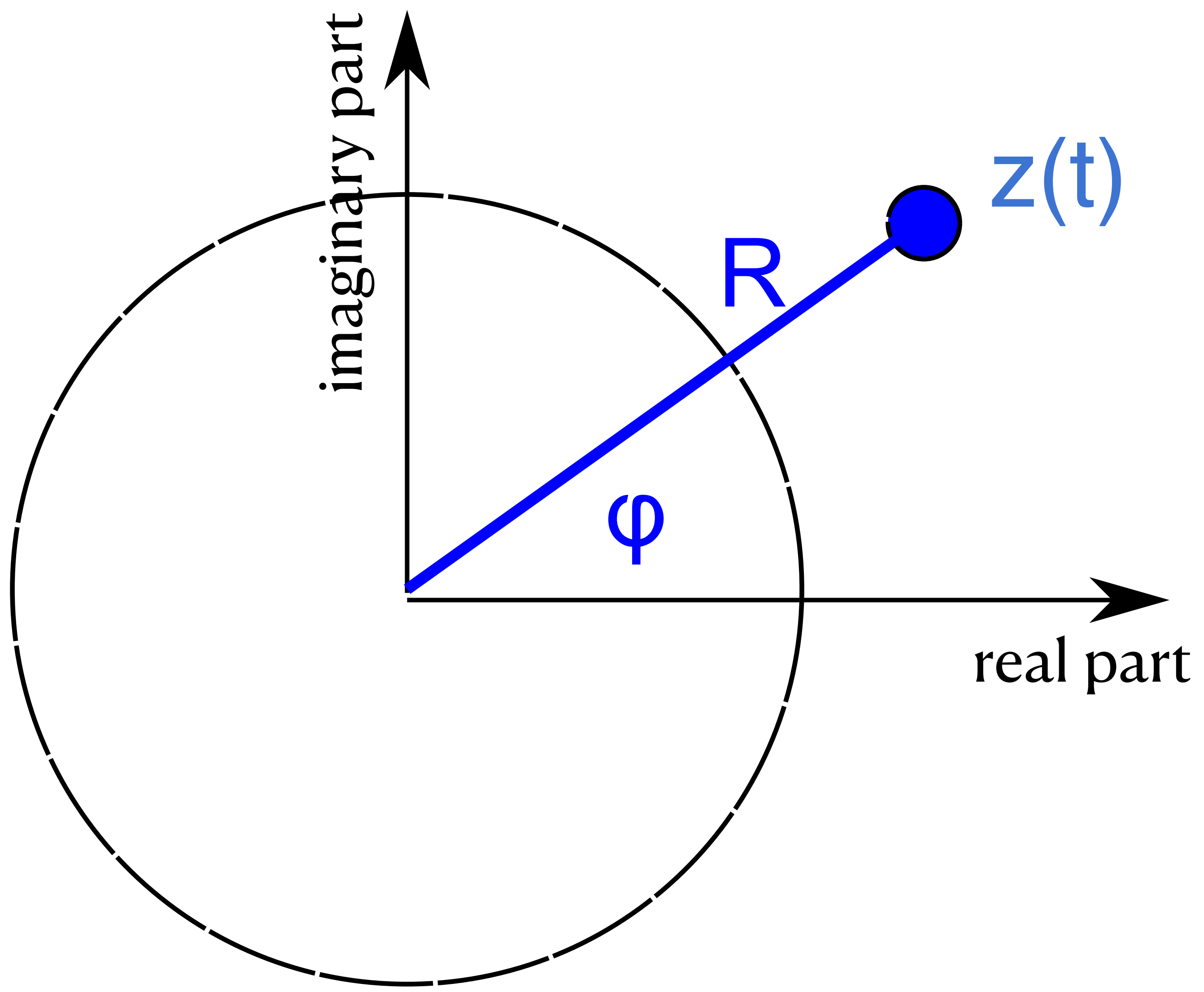
$$z_2(t) = R_2(t)e^{i\phi_2(t)} = \underbrace{R_2(t)\cos(\phi_2(t))}_{\text{real part}} + \underbrace{iR_2(t)\sin(\phi_2(t))}_{\text{imaginary part}}$$

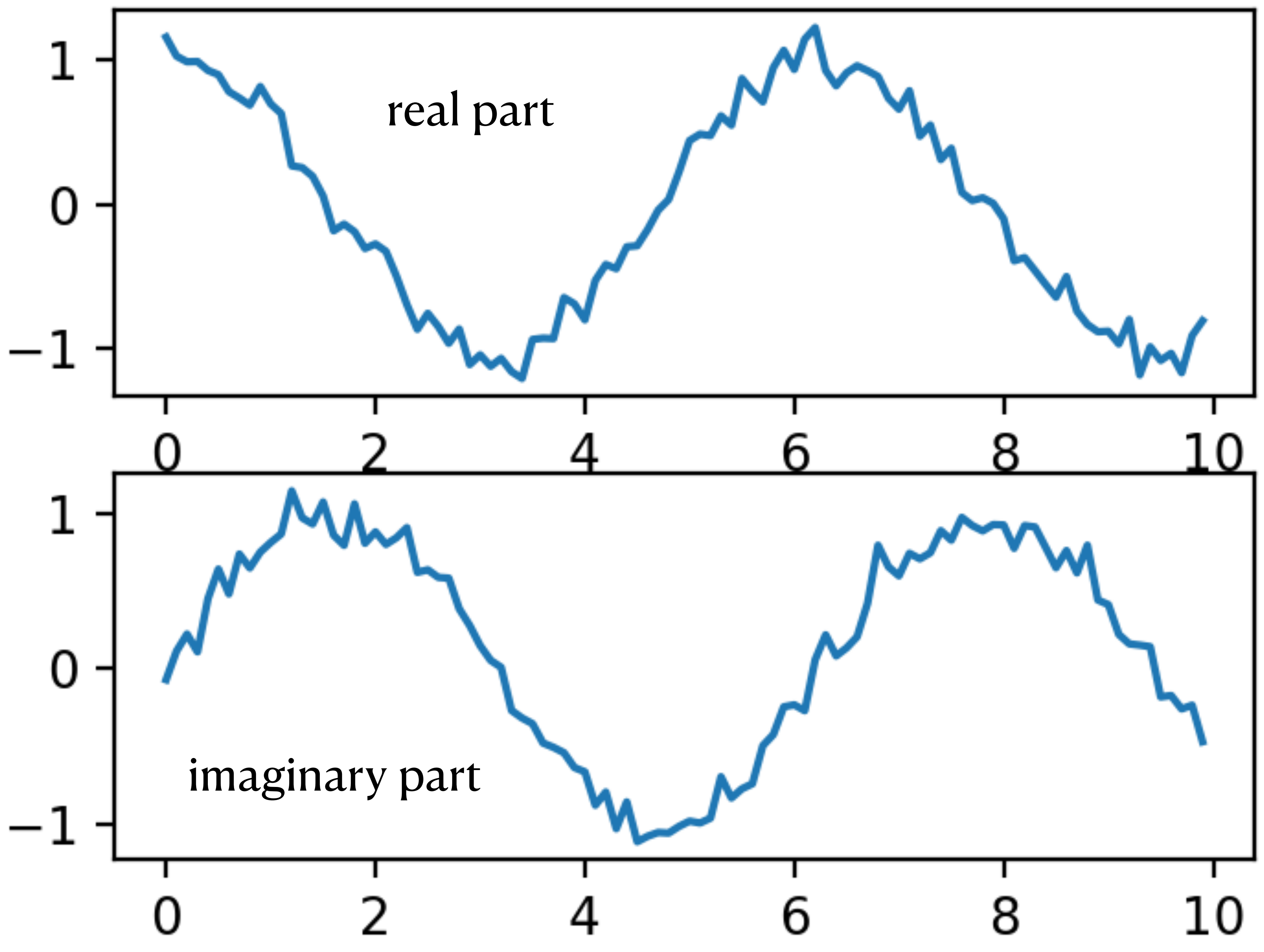


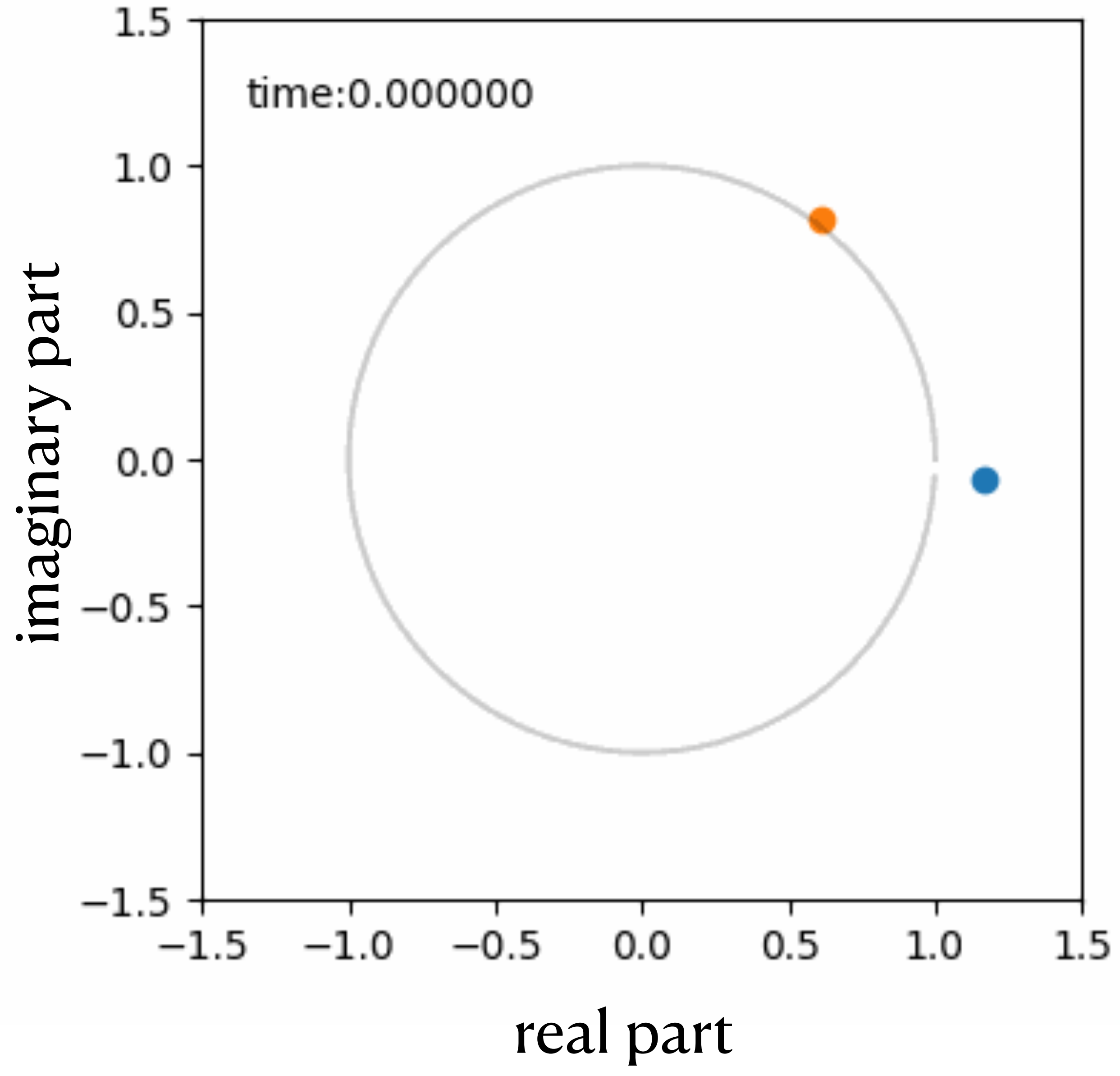
We extract a complex-valued time series z(t)=$Re^{i\phi}$

real part: $R\cos(\phi)$

imaginary part: $R\sin(\phi)$







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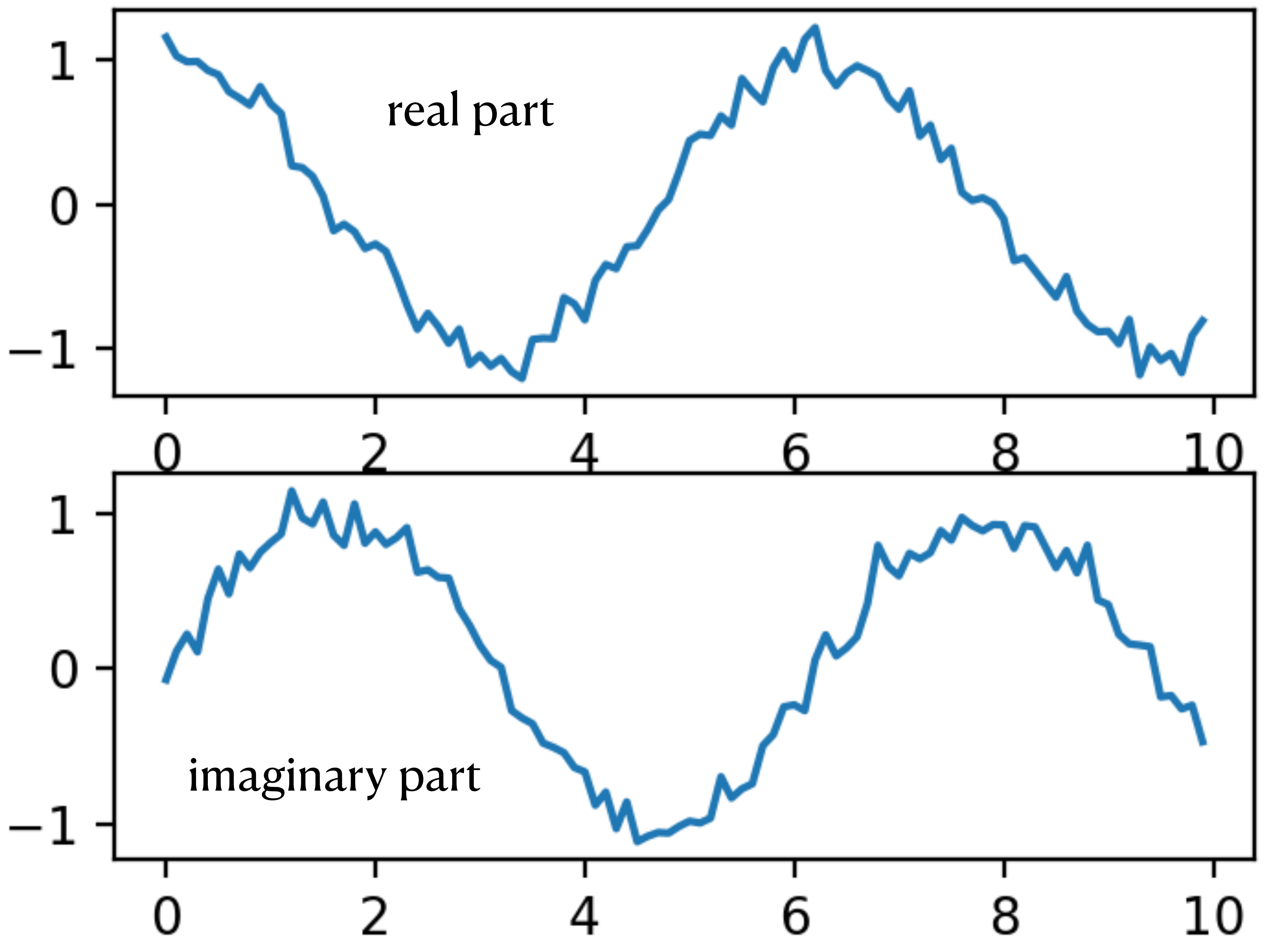
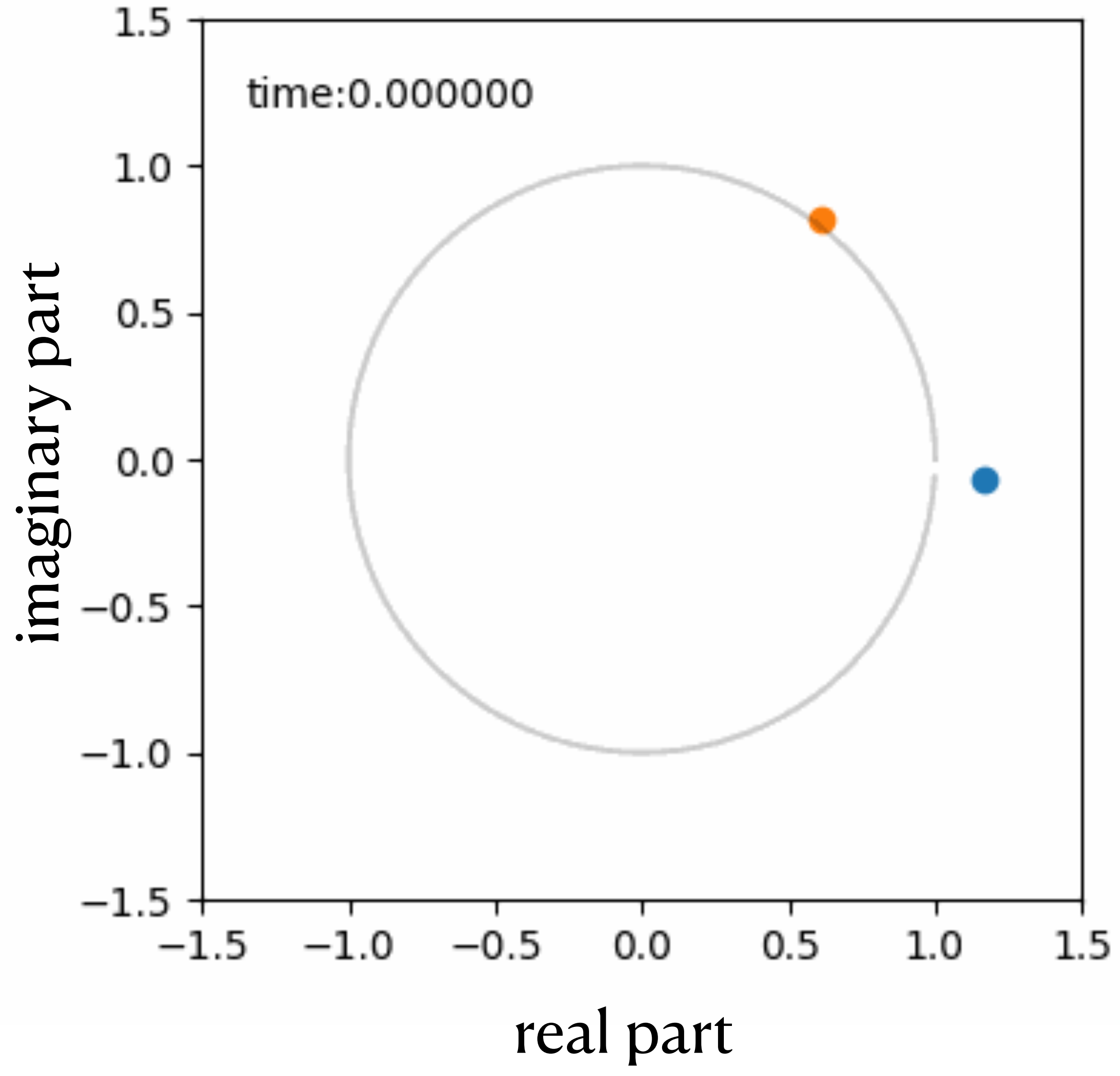




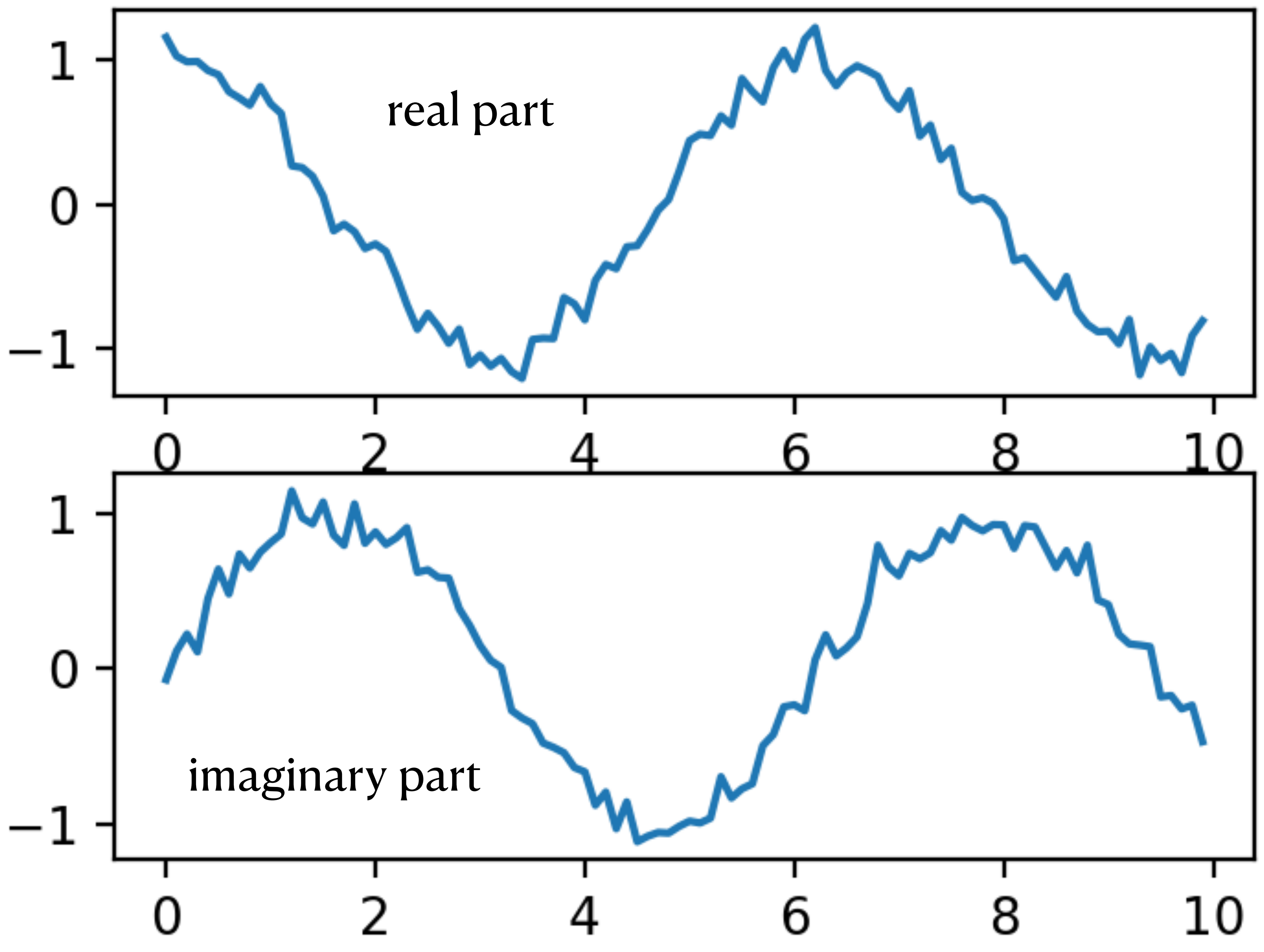
We extract a complex-valued time series z(t)=$Re^{i\phi}$

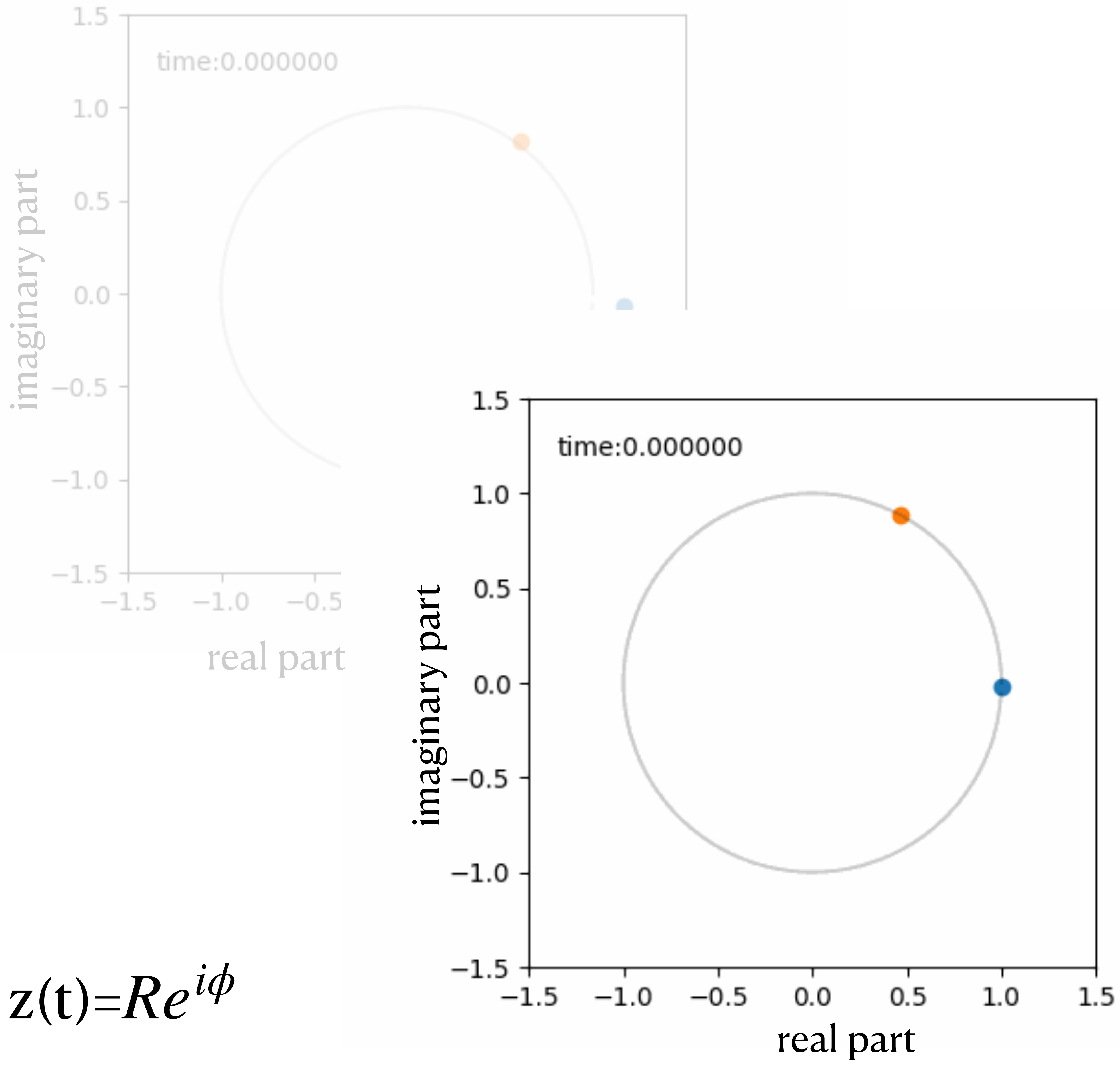
real part: $R\cos(\phi)$

imaginary part: $R\sin(\phi)$

$\phi(t) = f \cdot t$ phase = *frequency* f $\cdot$ *time* t

$R(t)$: magnitude independent of phase



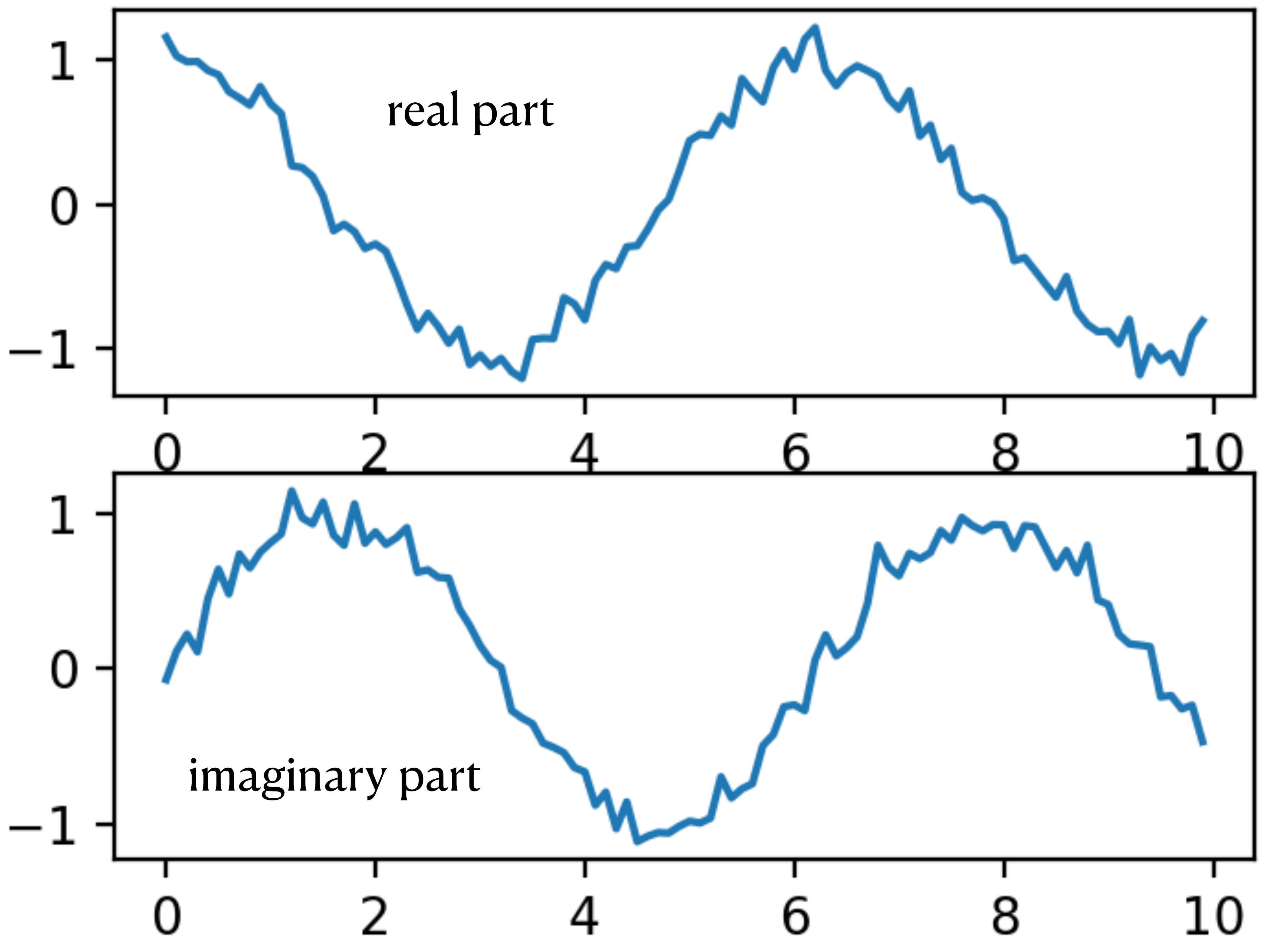


reduction to phase

We extract a complex-valued time series z(t)=$Re^{i\phi}$

real part: $R\cos(\phi)$

imaginary part: $R\sin(\phi)$



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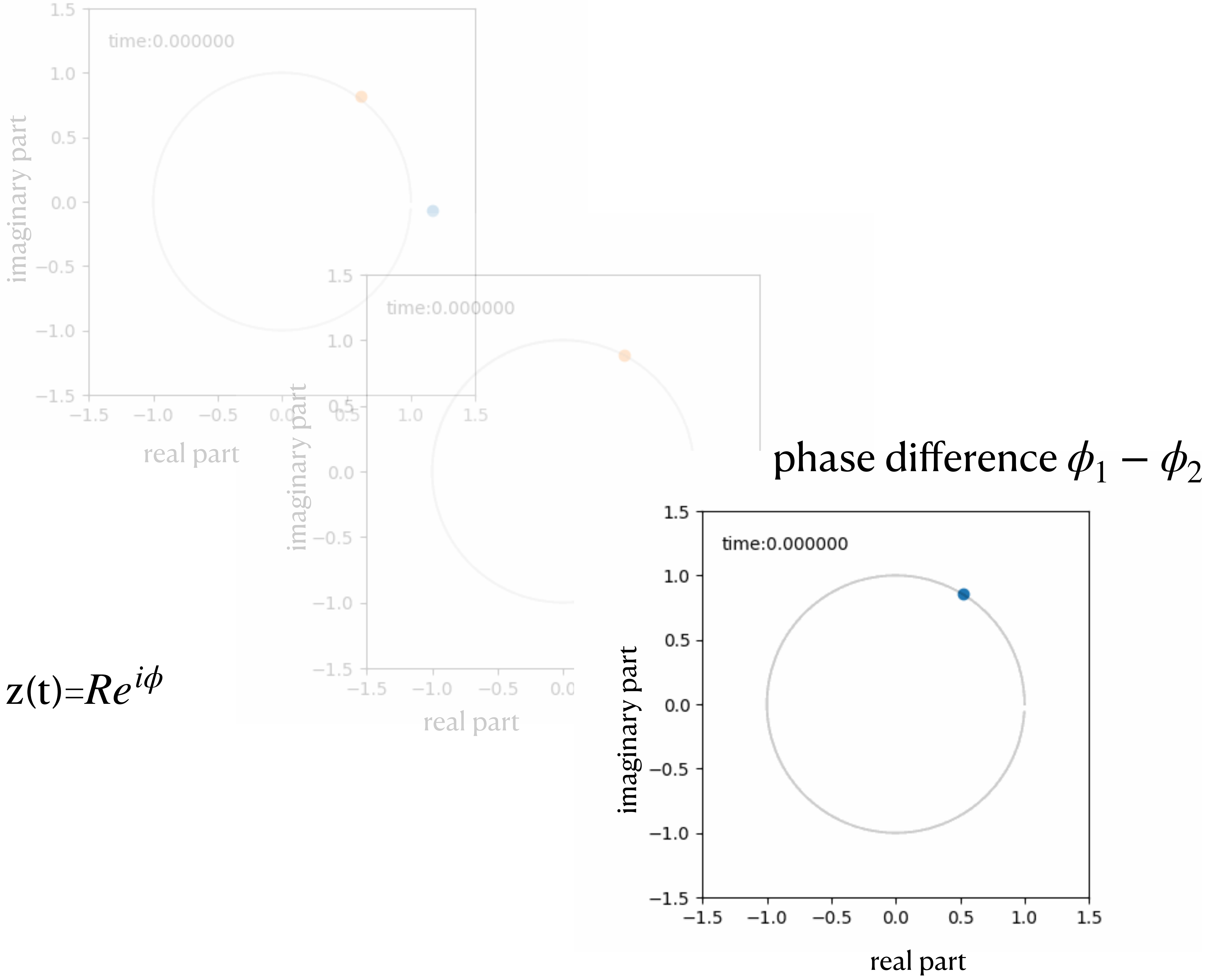
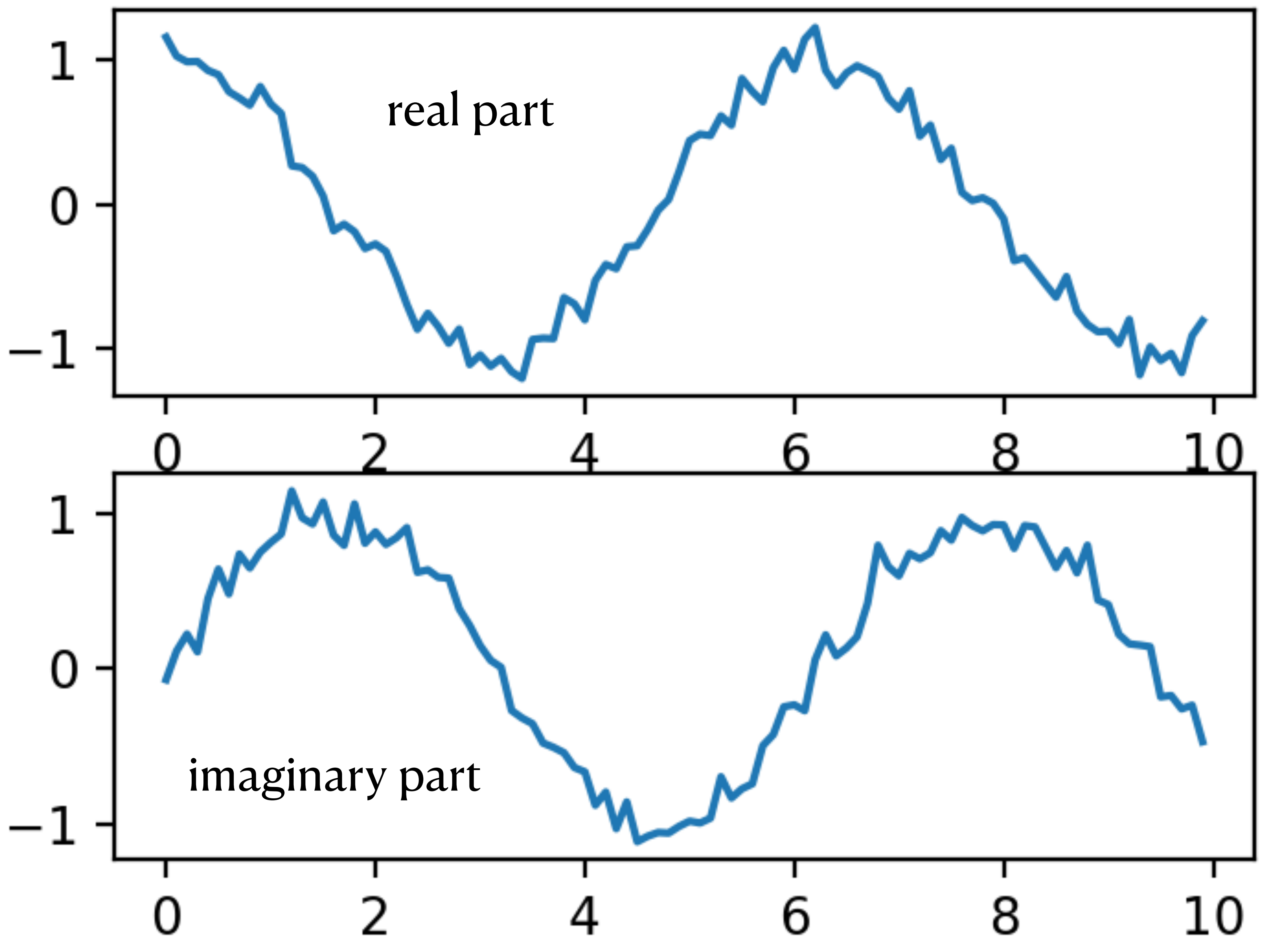
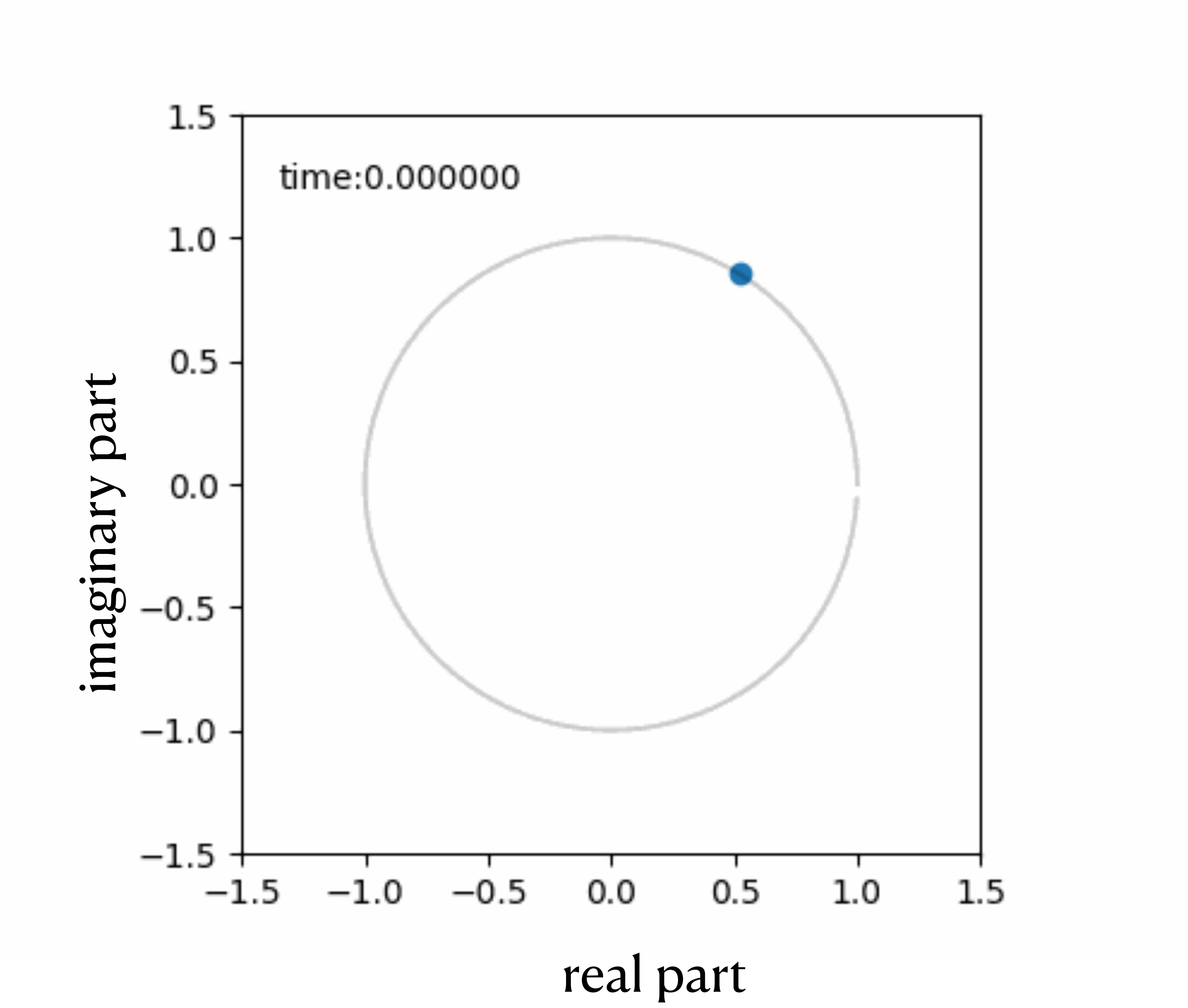




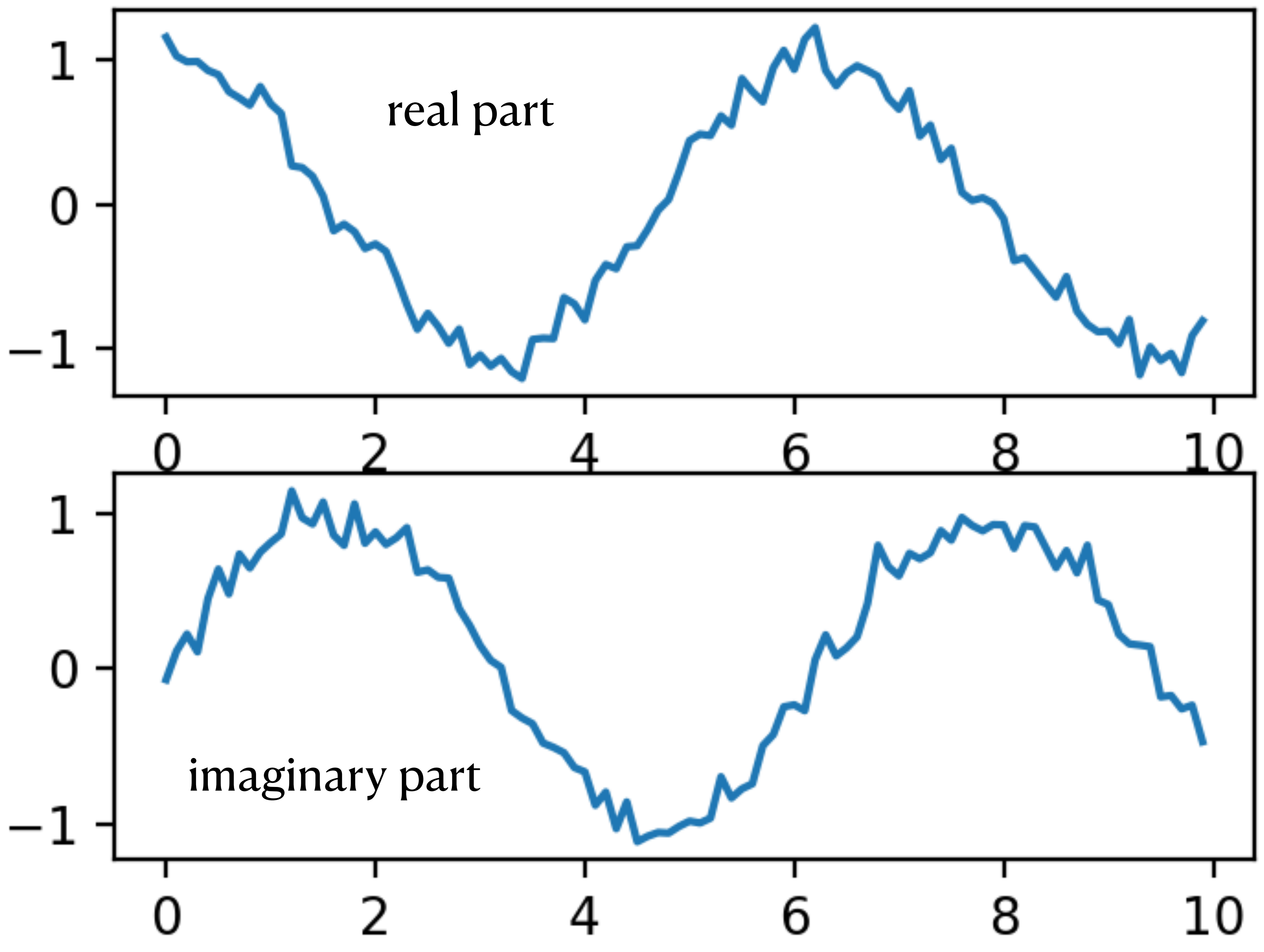


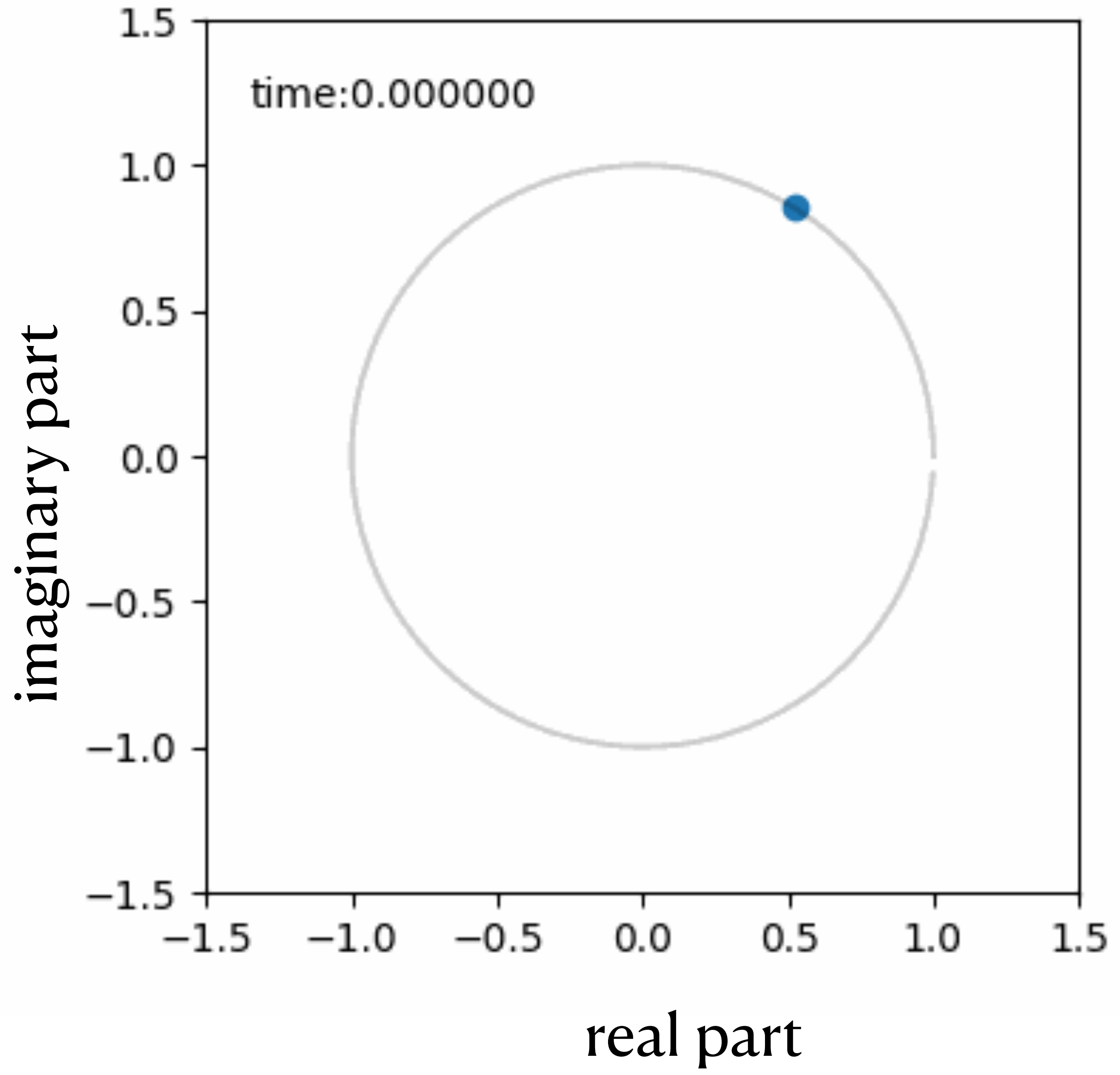
We extract a complex-valued time series z(t)=$Re^{i\phi}$

real part: $R\cos(\phi)$

imaginary part: $R\sin(\phi)$

phase difference does not move anymore,





We extract a complex-valued time series z(t)=$Re^{i\phi}$

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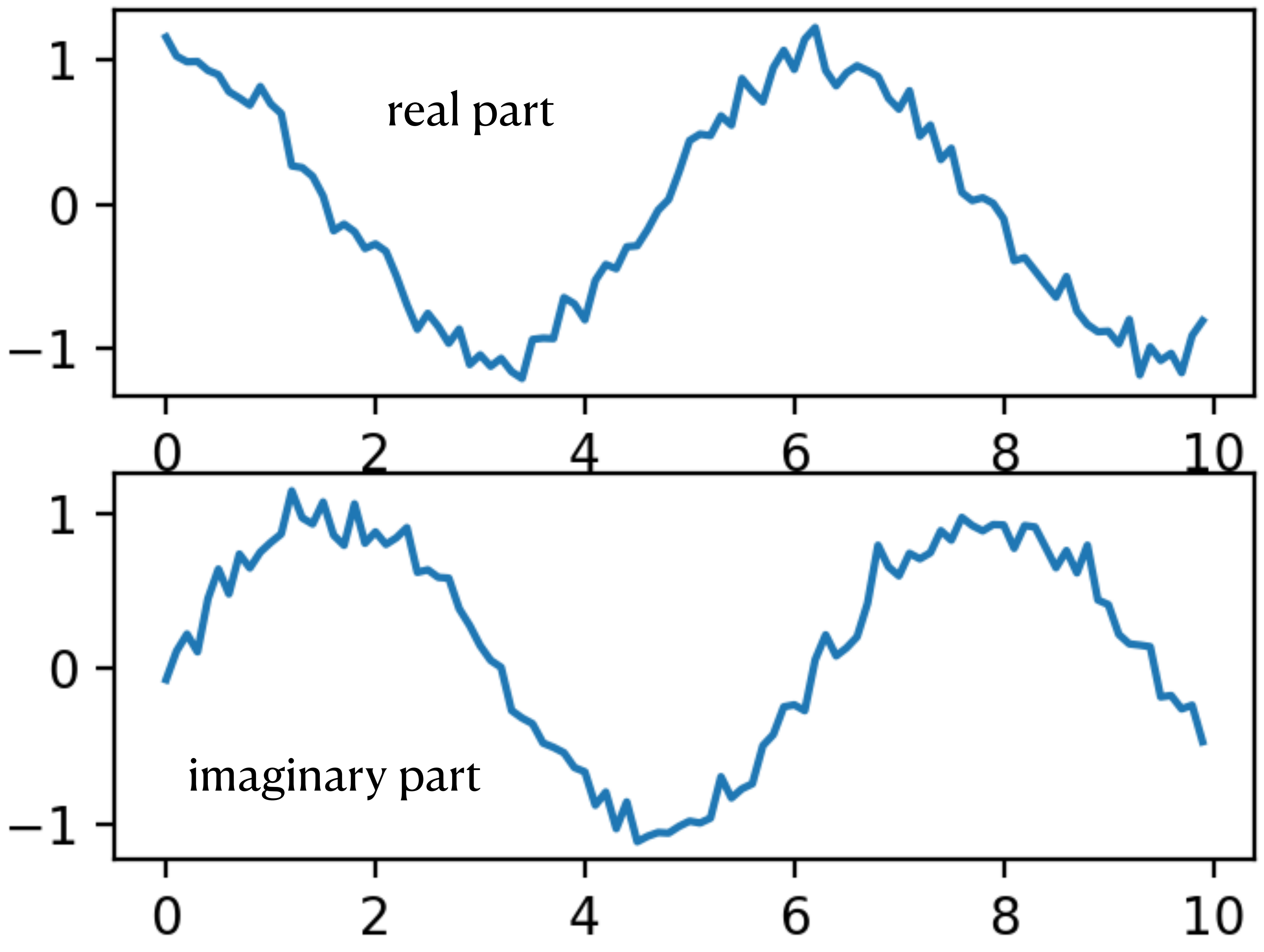
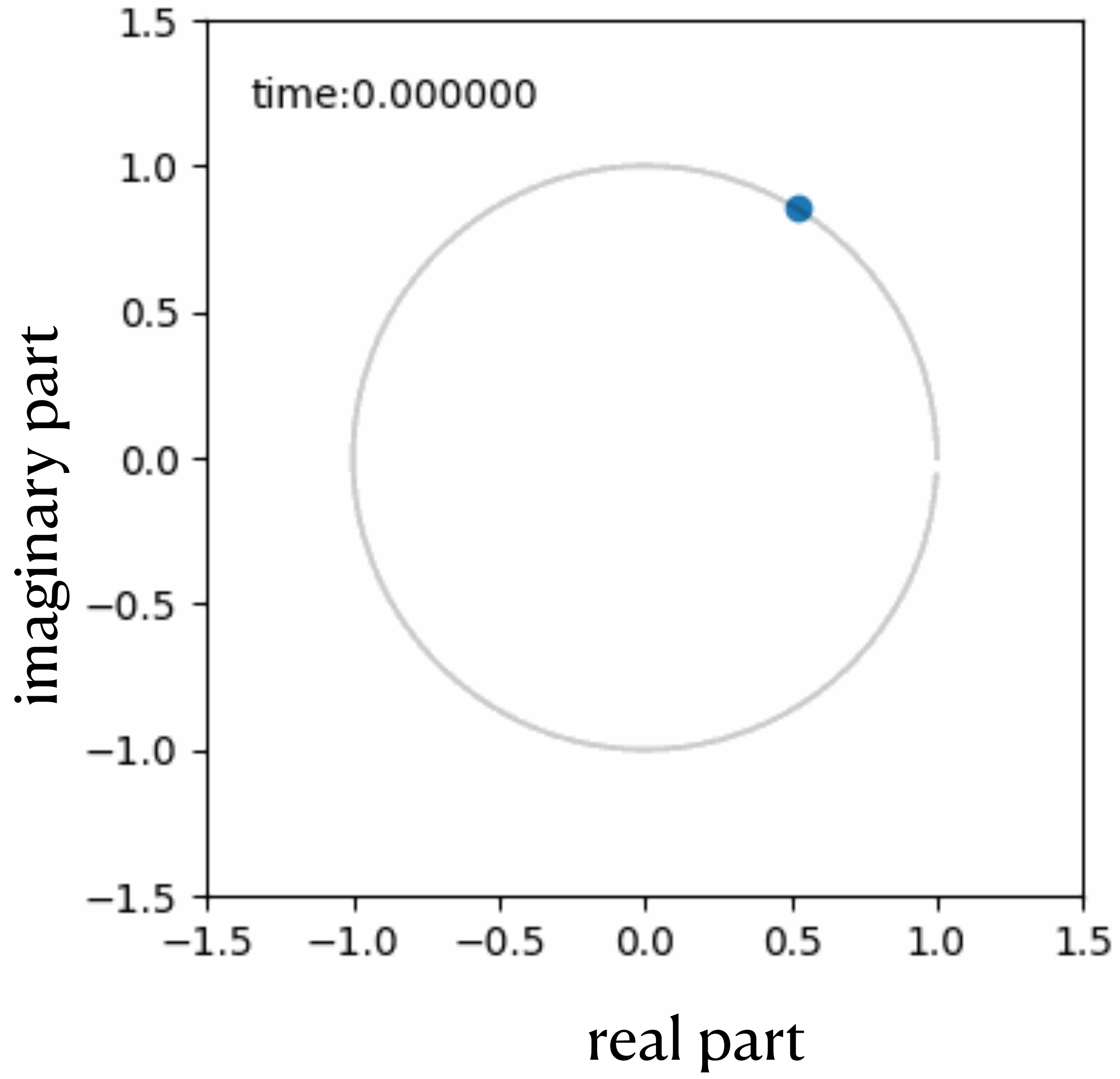
phase difference does not move anymore,

—> phase difference is almost constant





We extract a complex-valued time series z(t)=$Re^{i\phi}$

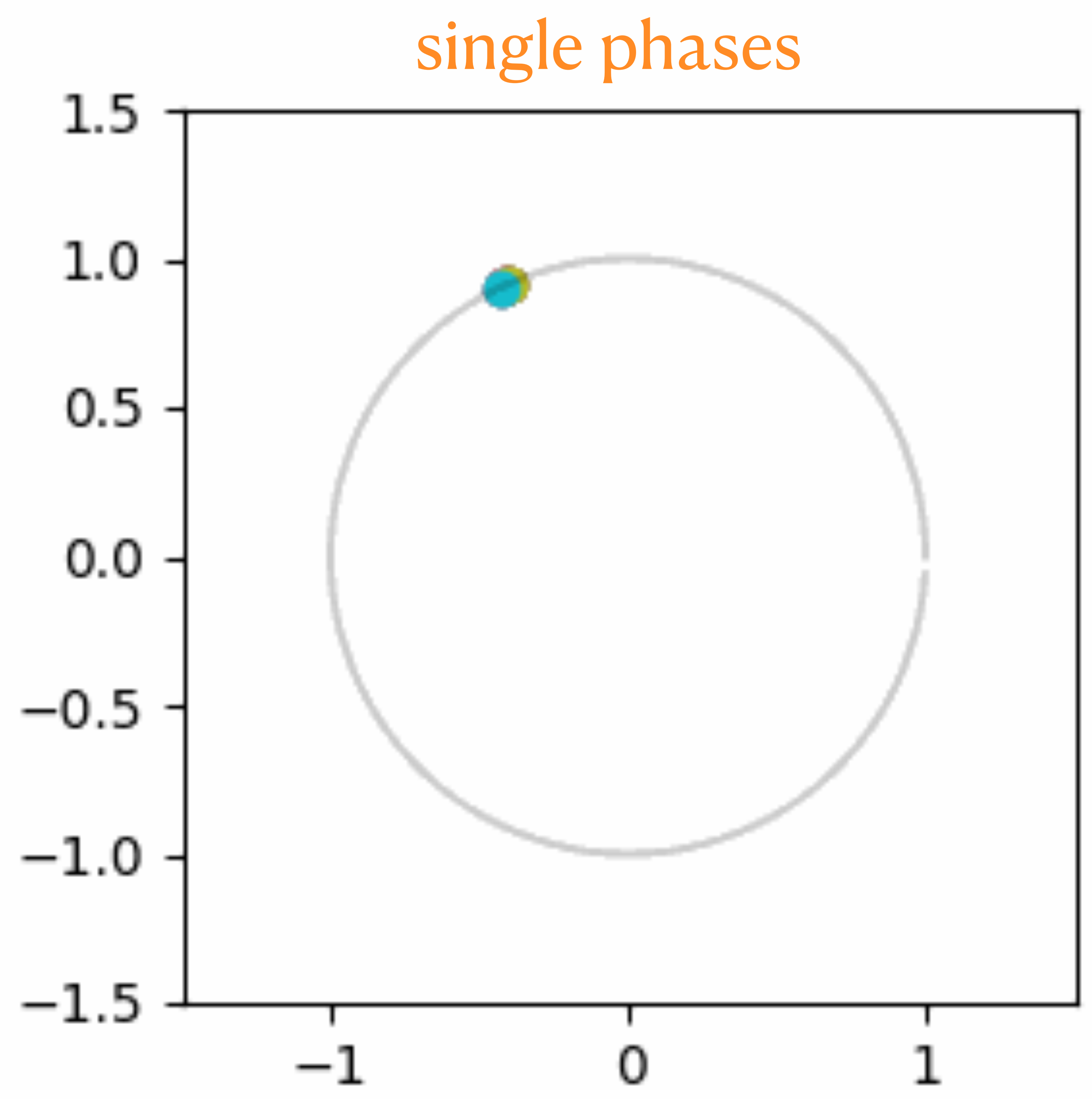
real part: $R\cos(\phi)$

imaginary part: $R\sin(\phi)$

phase difference does not move anymore,

—> phase difference is almost constant

—> **phase synchronization**



number of phases = 10

frequency f of phases are random (Gaussian distribution)

mean frequency f=1Hz
frequency variance 0.03Hz²

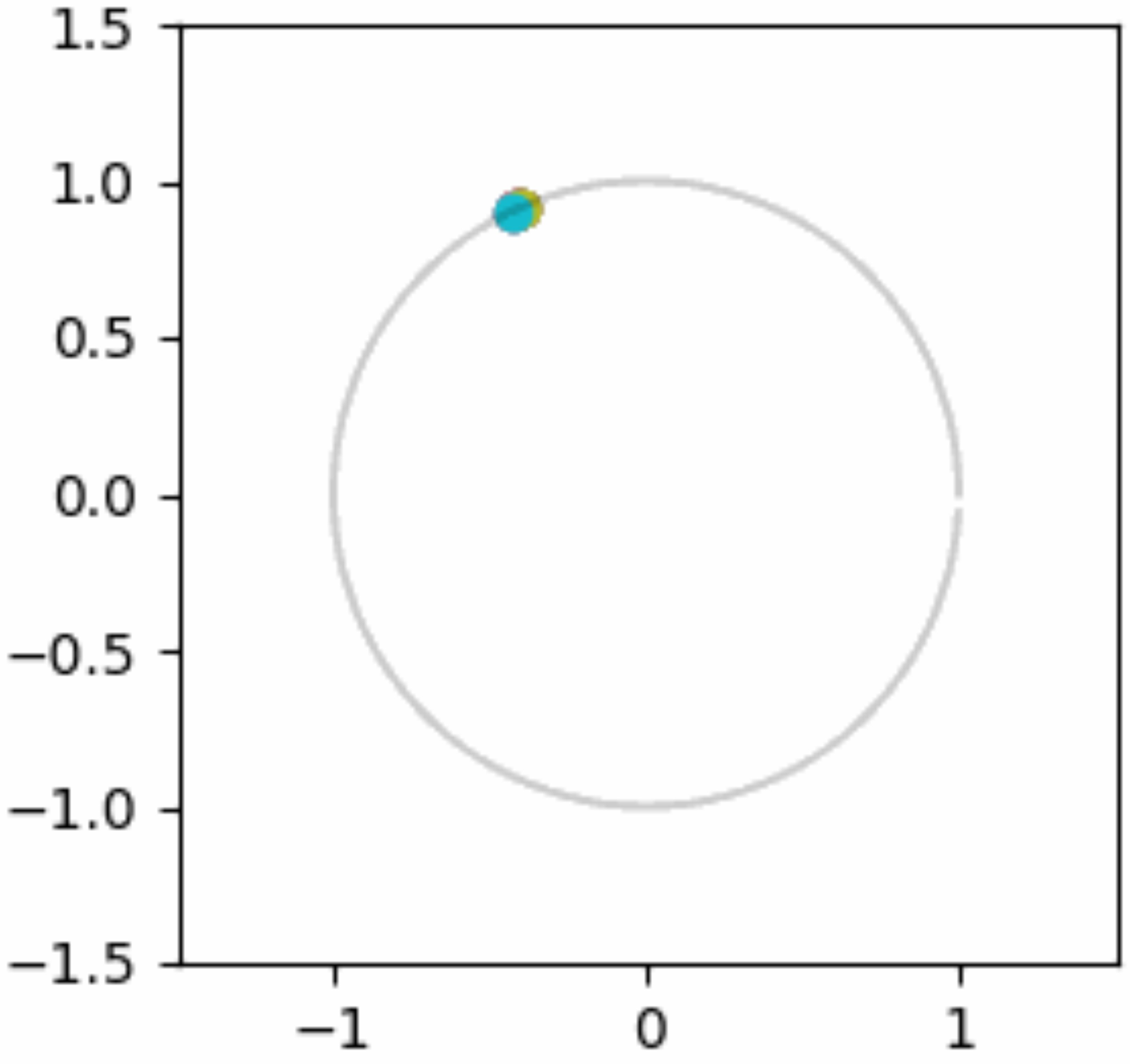
divergence over time due to different frequencies

number of phases = 10

frequency f of phases are random (Gaussian distribution)

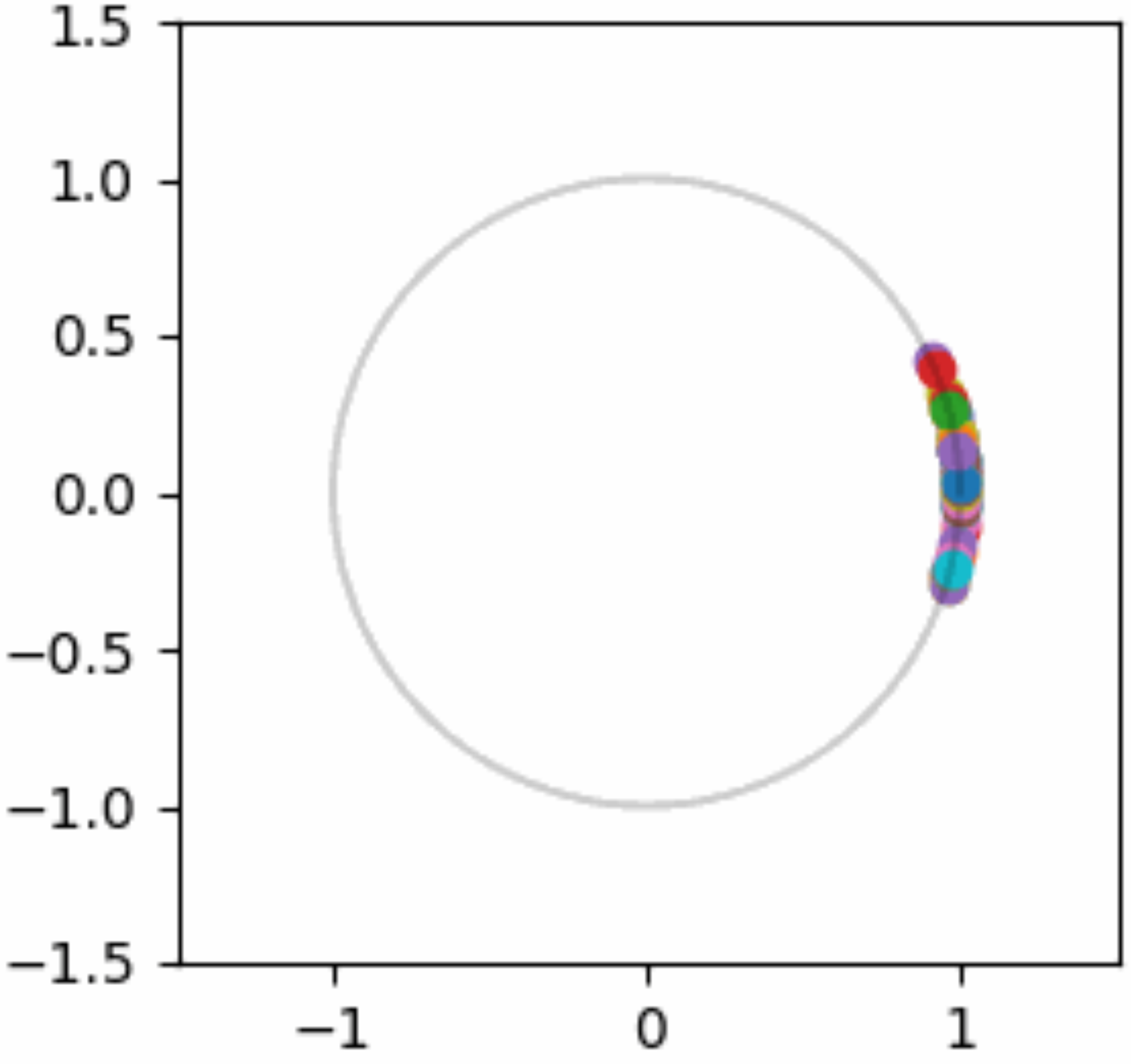
mean frequency f=1Hz

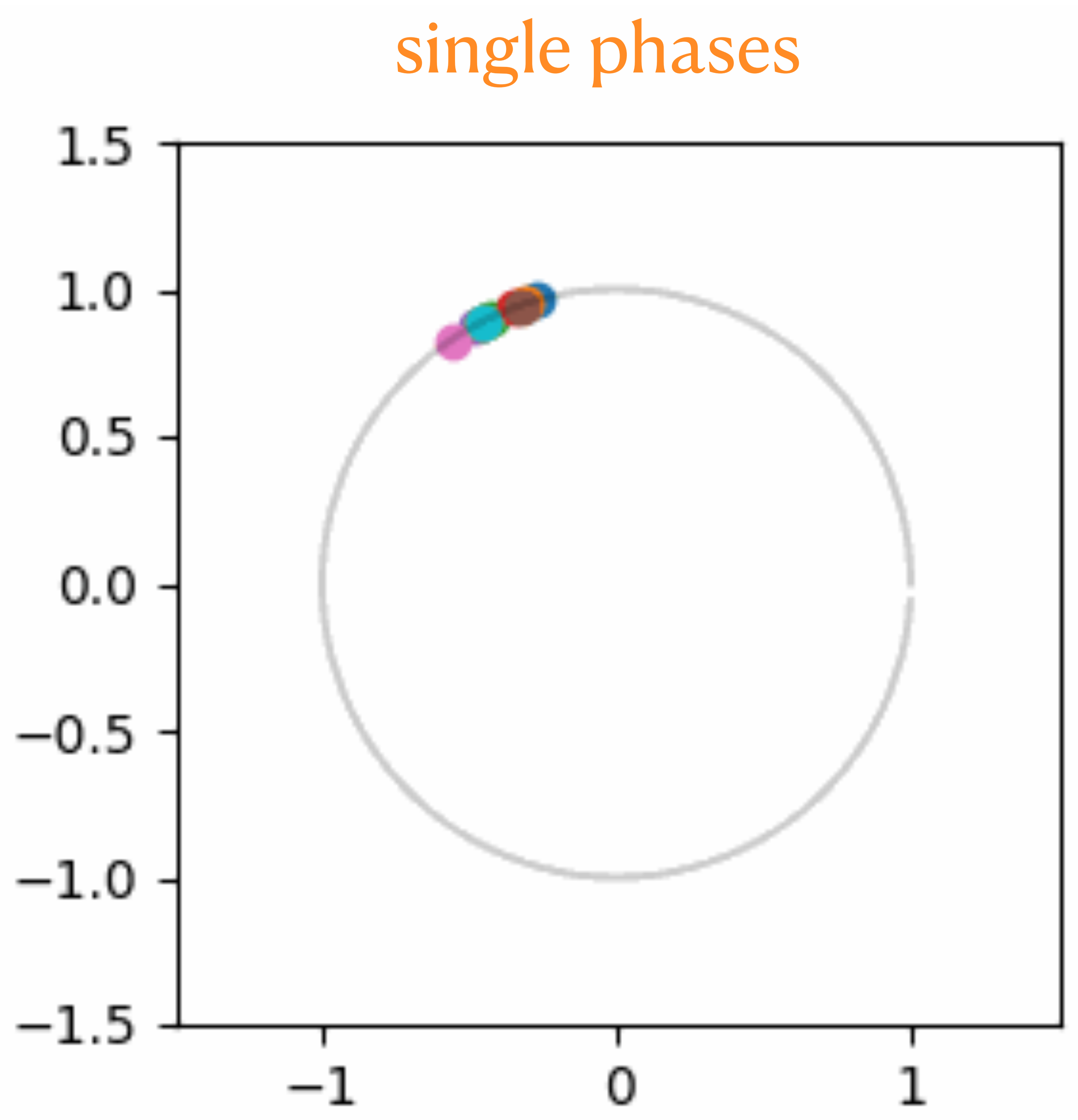
frequency variance 0.03Hz²



no synchronisation

all phase differences





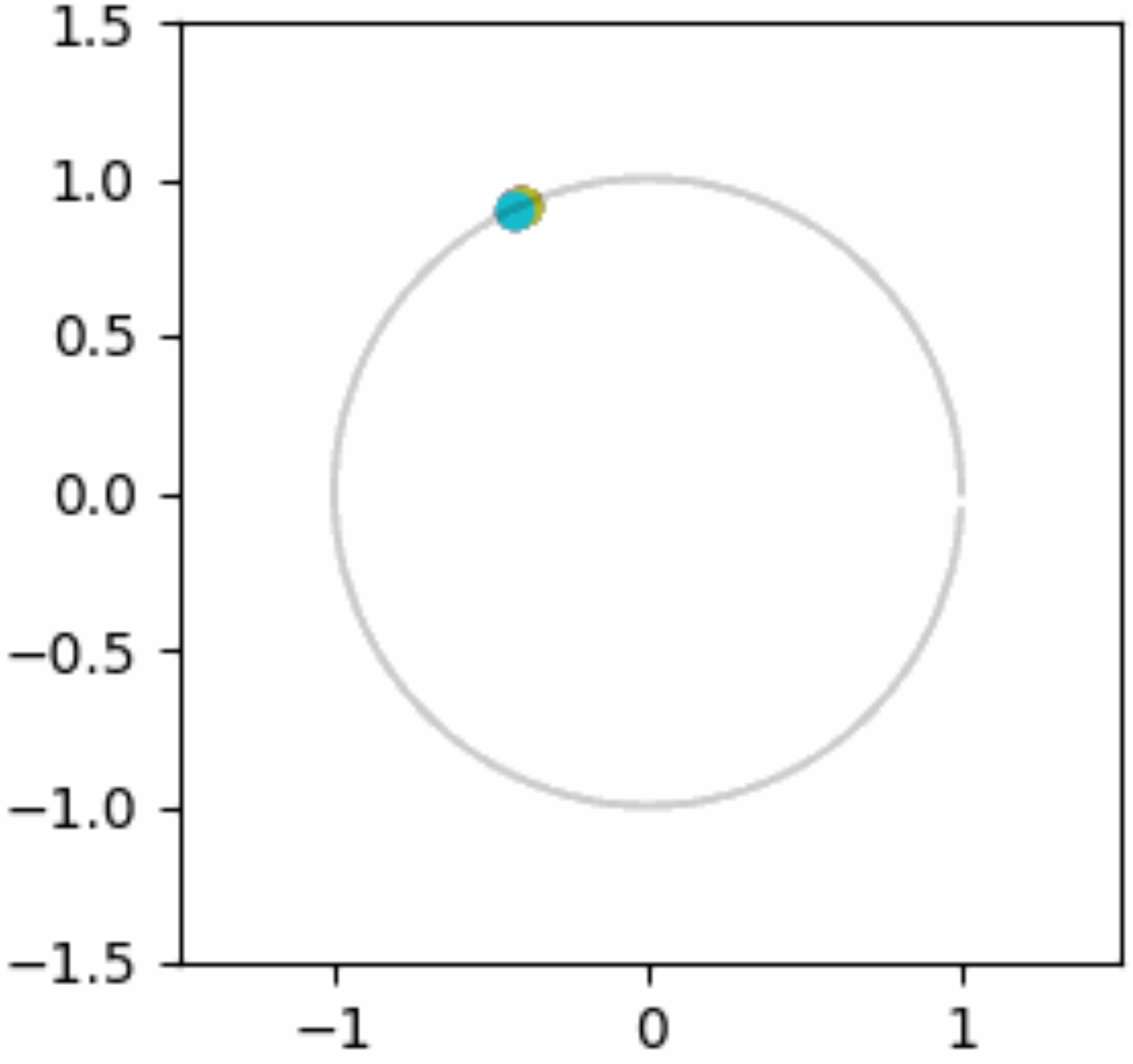
number of phases = 10

frequency f of phases are **identical** f=1Hz

<—> 1 rhythm

stable over time due to **identical** frequencies

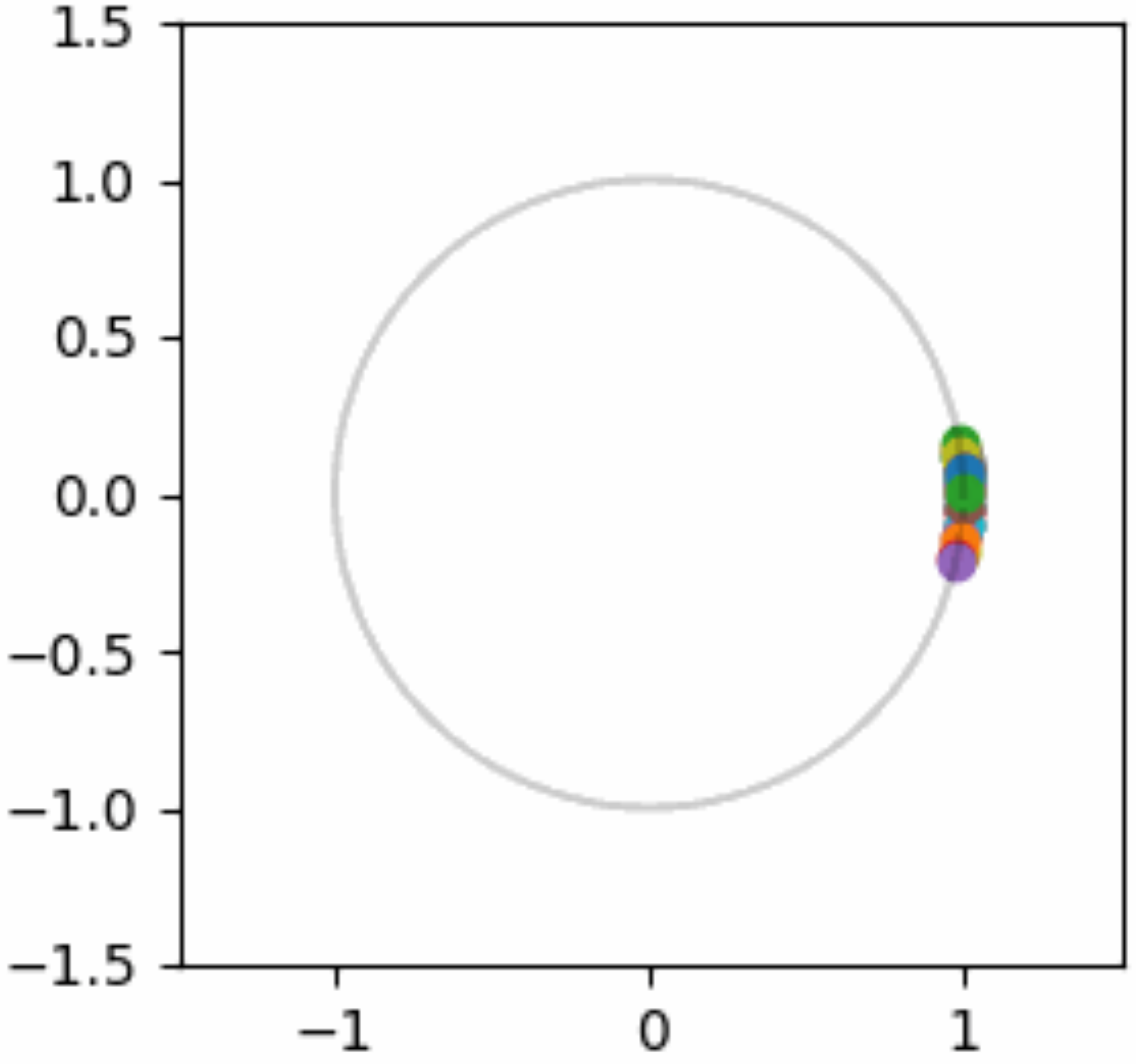
number of phases = 10



frequency f of phases are **identical** f=1Hz

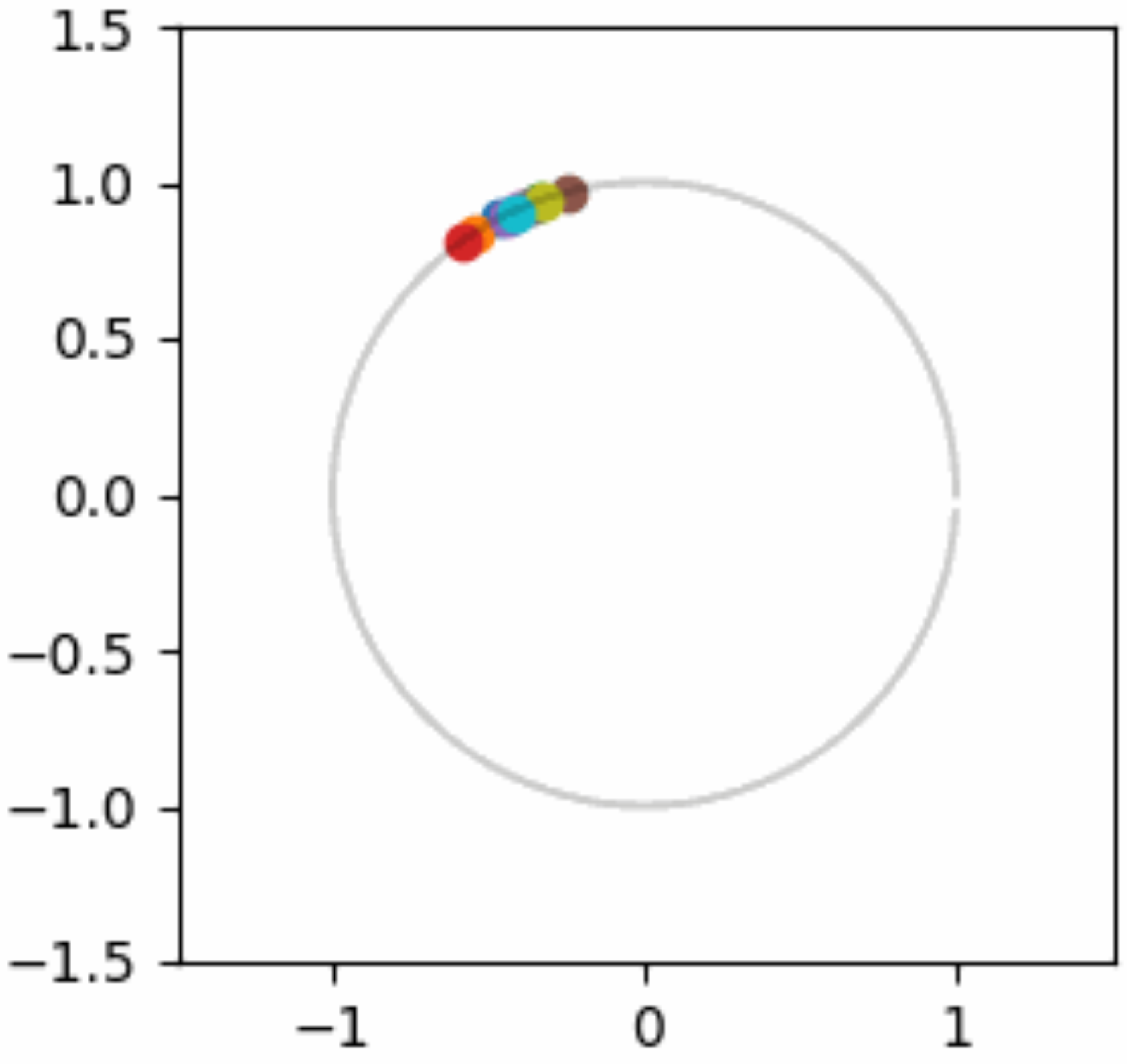
<—> 1 rhythm

all phase differences



synchronisation

single phases



number of phases = 10

5 phases of f=1.5Hz , 5 phases of f=1.0Hz

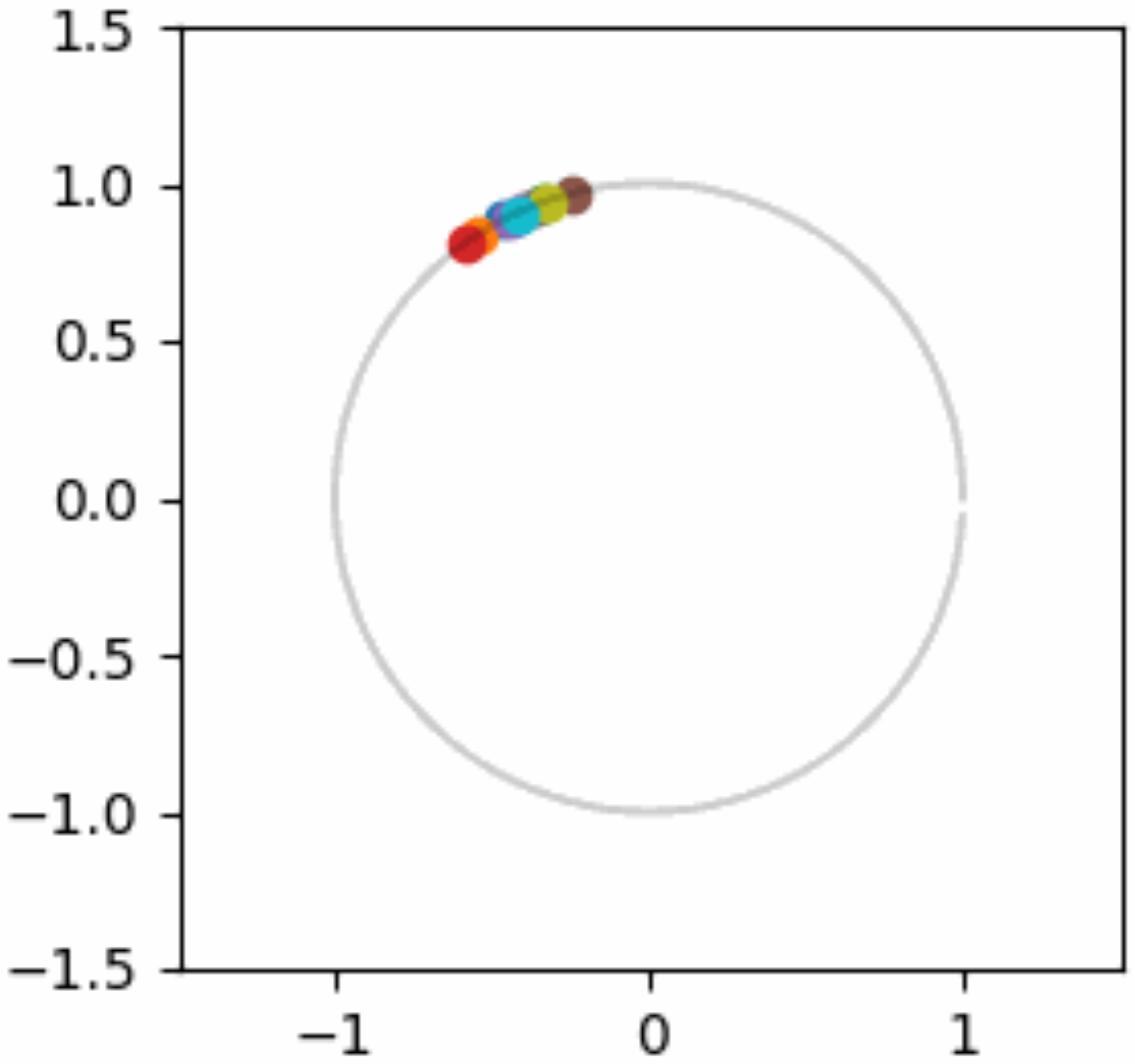
<—> two rhythms

phases cluster and clusters diverge over time

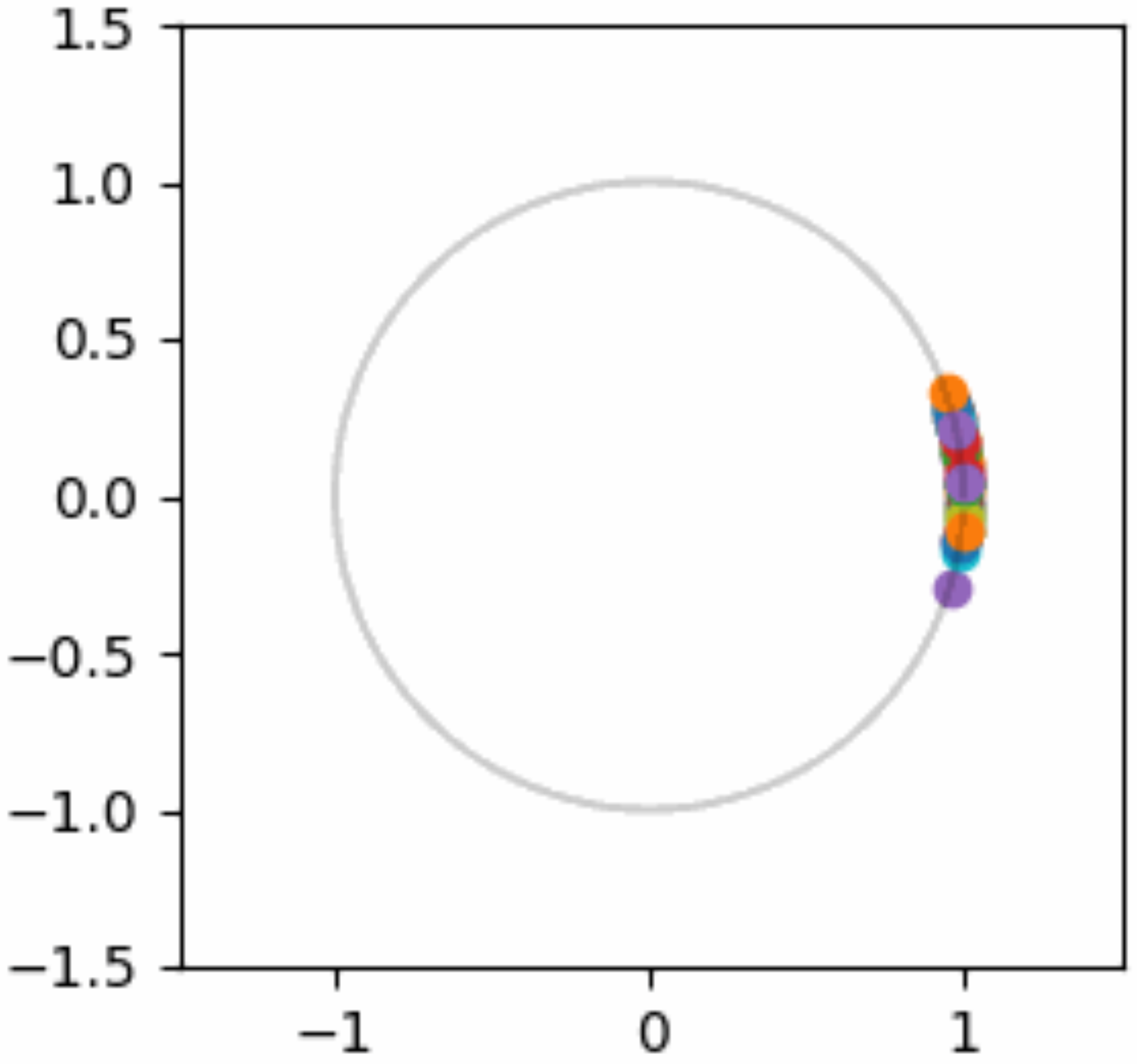
number of phases = 10

5 phases of f=1.5Hz , 5 phases of f=1.0Hz

<—> two rhythms



all phase differences



all synchronised phases build stable cluster, the others move

partial synchronisation

Idea to quantify phase synchronisation:

- analytical signal or complex-valued time-frequency transform leads to complex-valued instantaneous signal

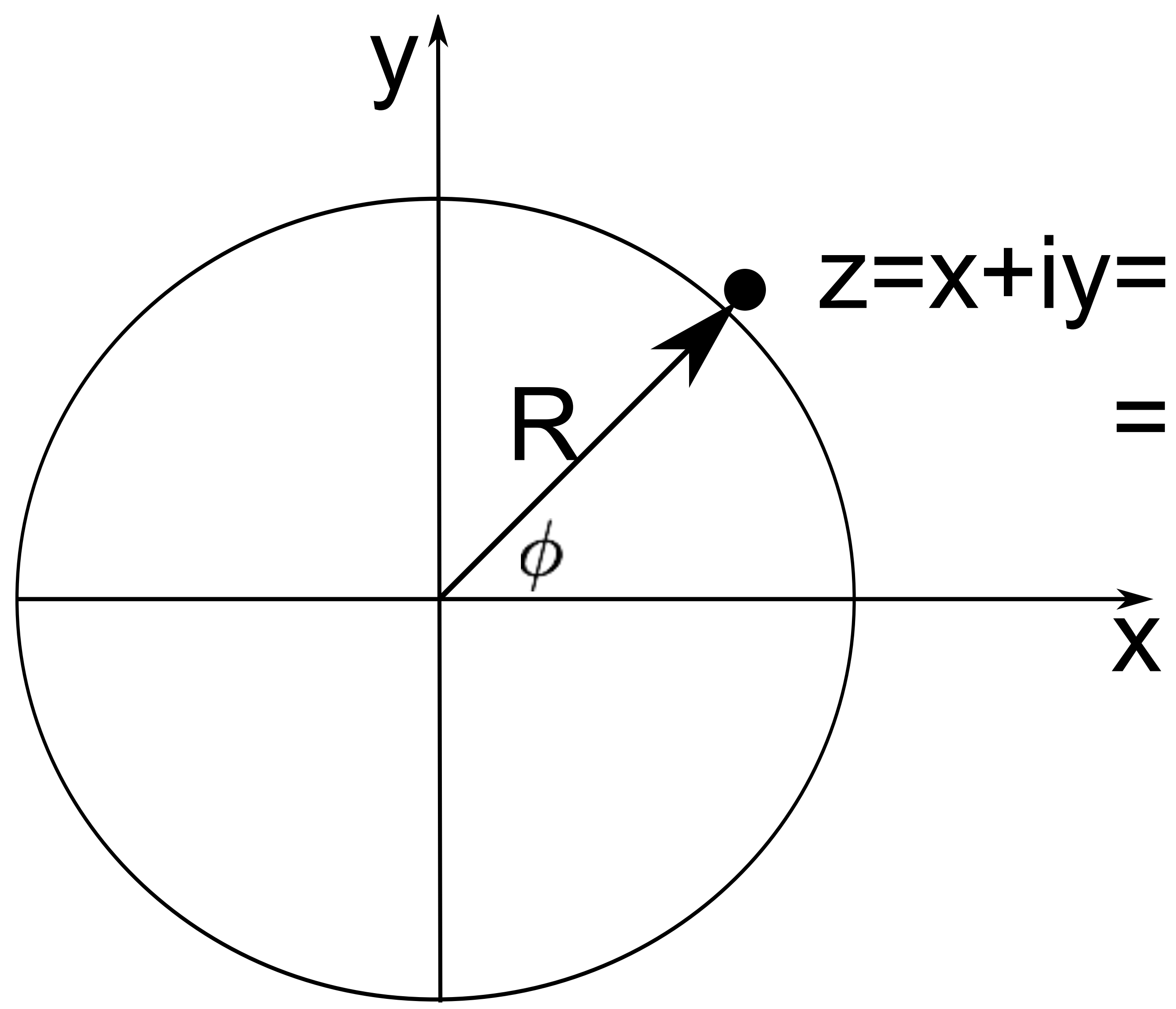
$$z(t) = R(t)e^{i\phi(t)}$$

Idea to quantify phase synchronisation:

- analytical signal or complex-valued time-frequency transform leads to complex-valued instantaneous signal

$$z(t) = R(t)e^{i\phi(t)}$$

in a time instant:



z=x+iy=Re(z)+iIm(z)

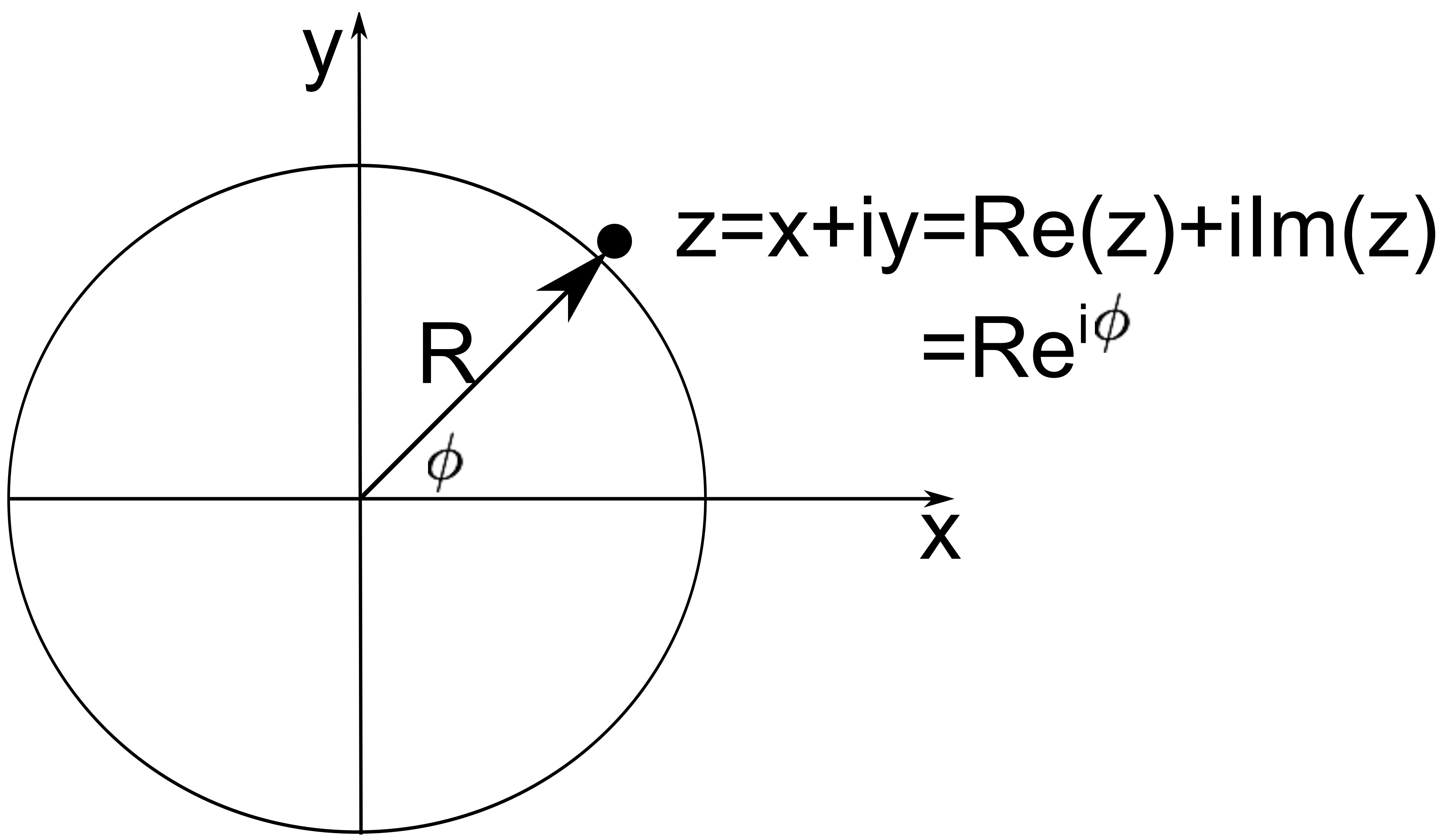
$=Re^{i\phi}$

Idea to quantify phase synchronisation:

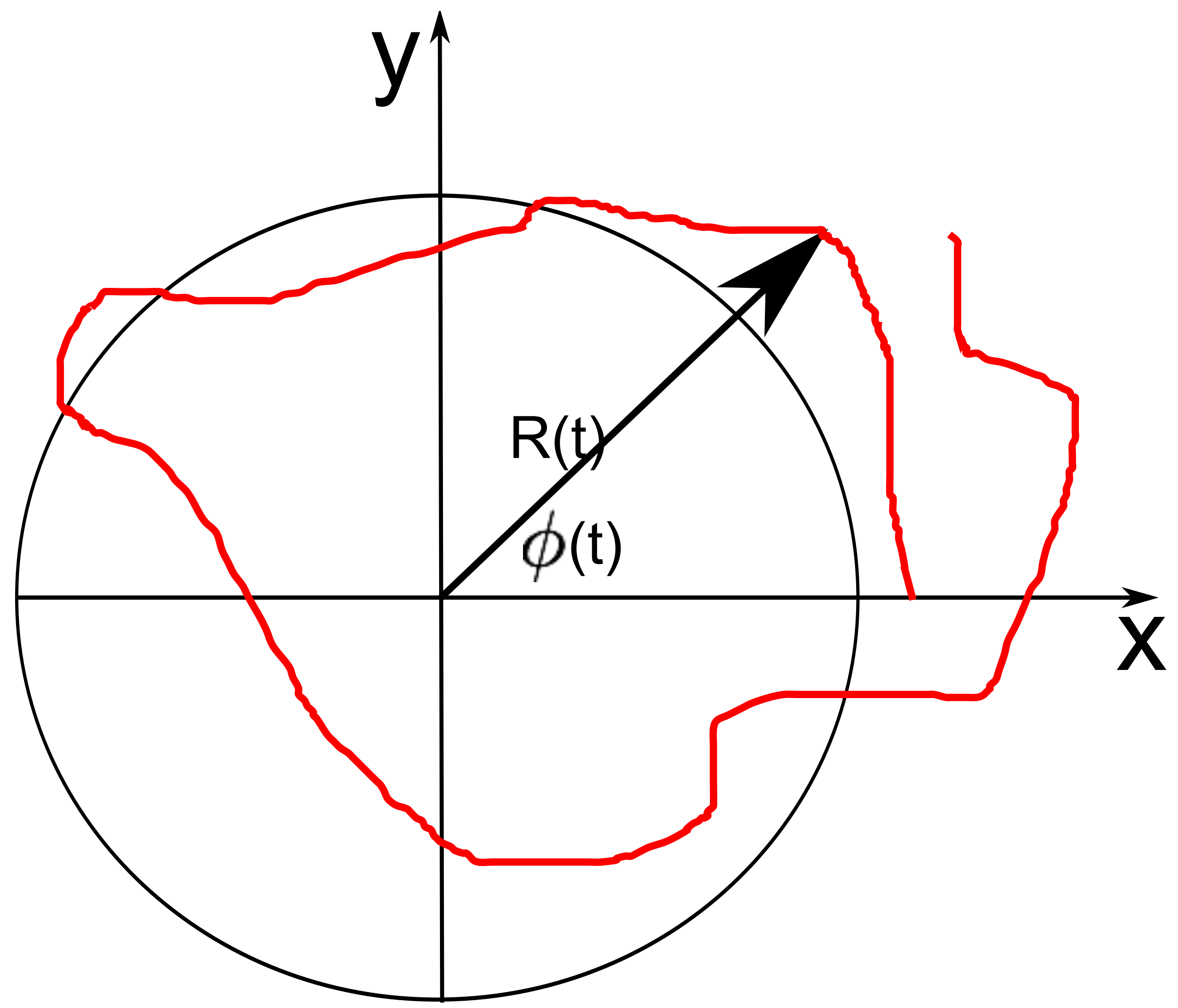
- analytical signal or complex-valued time-frequency transform leads to zu complex-valued instantaneous signal

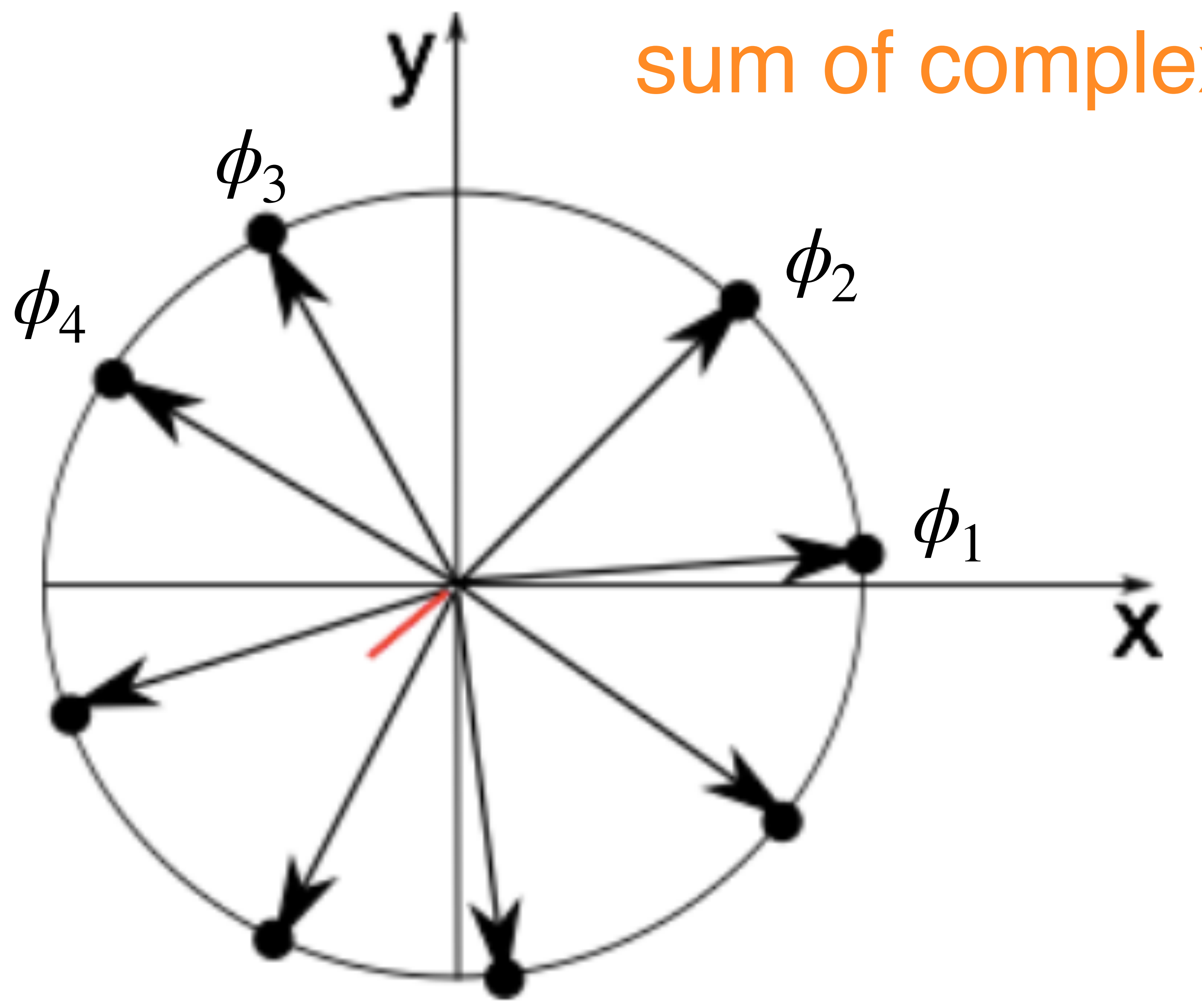
$$z(t) = R(t)e^{i\phi(t)}$$

in a time instant:



complex-valued time series





sum of complex values

ϕ_n : phases of signals

$$\Psi = \frac{1}{N}\sum_{n=1}^{N} e^{i\phi_n}$$

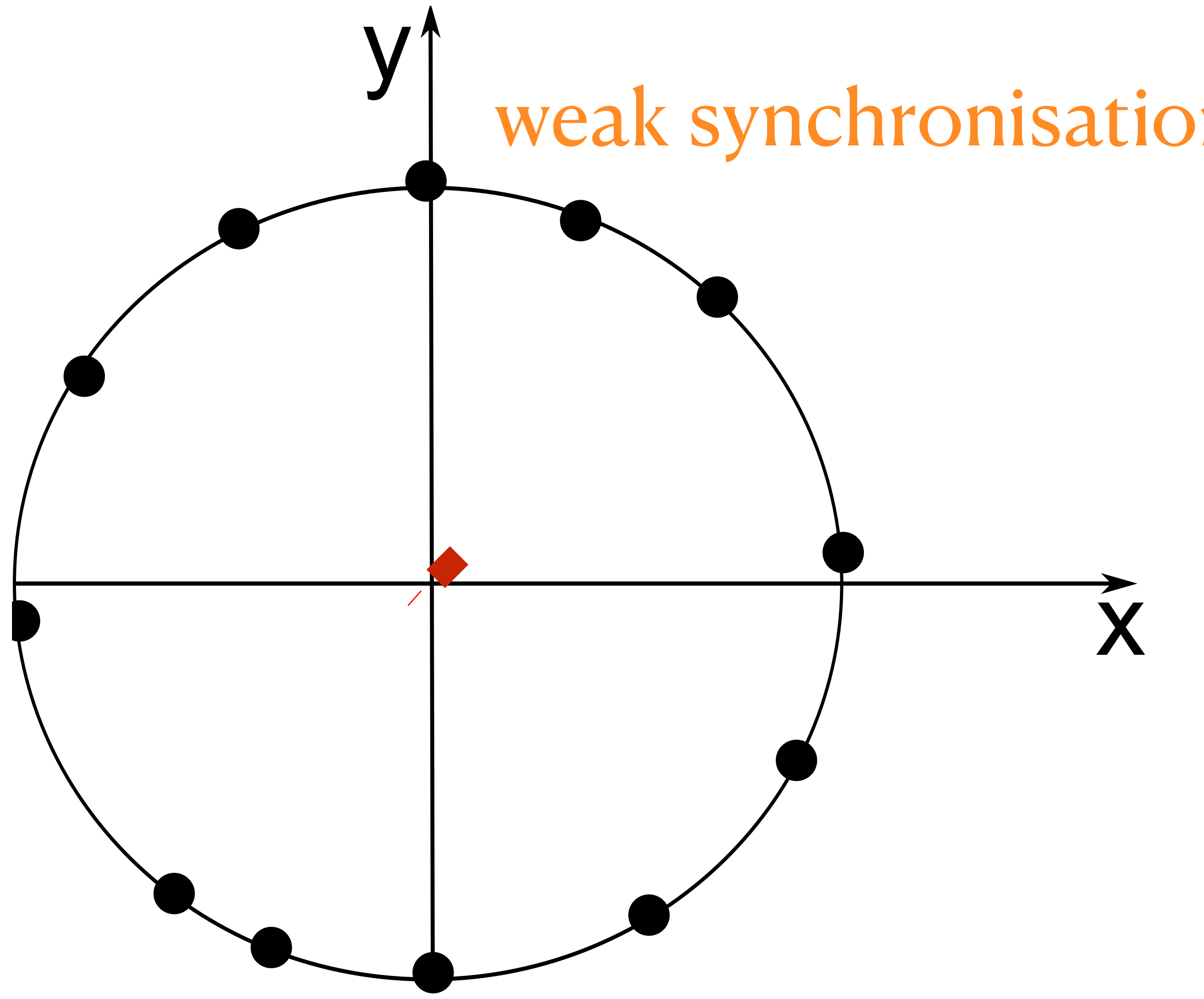
$$= \frac{1}{N}\sum_{n=1}^{N}\left(\underbrace{\cos(\phi_n)}_{x_n} + i\,\underbrace{\sin(\phi_n)}_{y_n}\right)$$

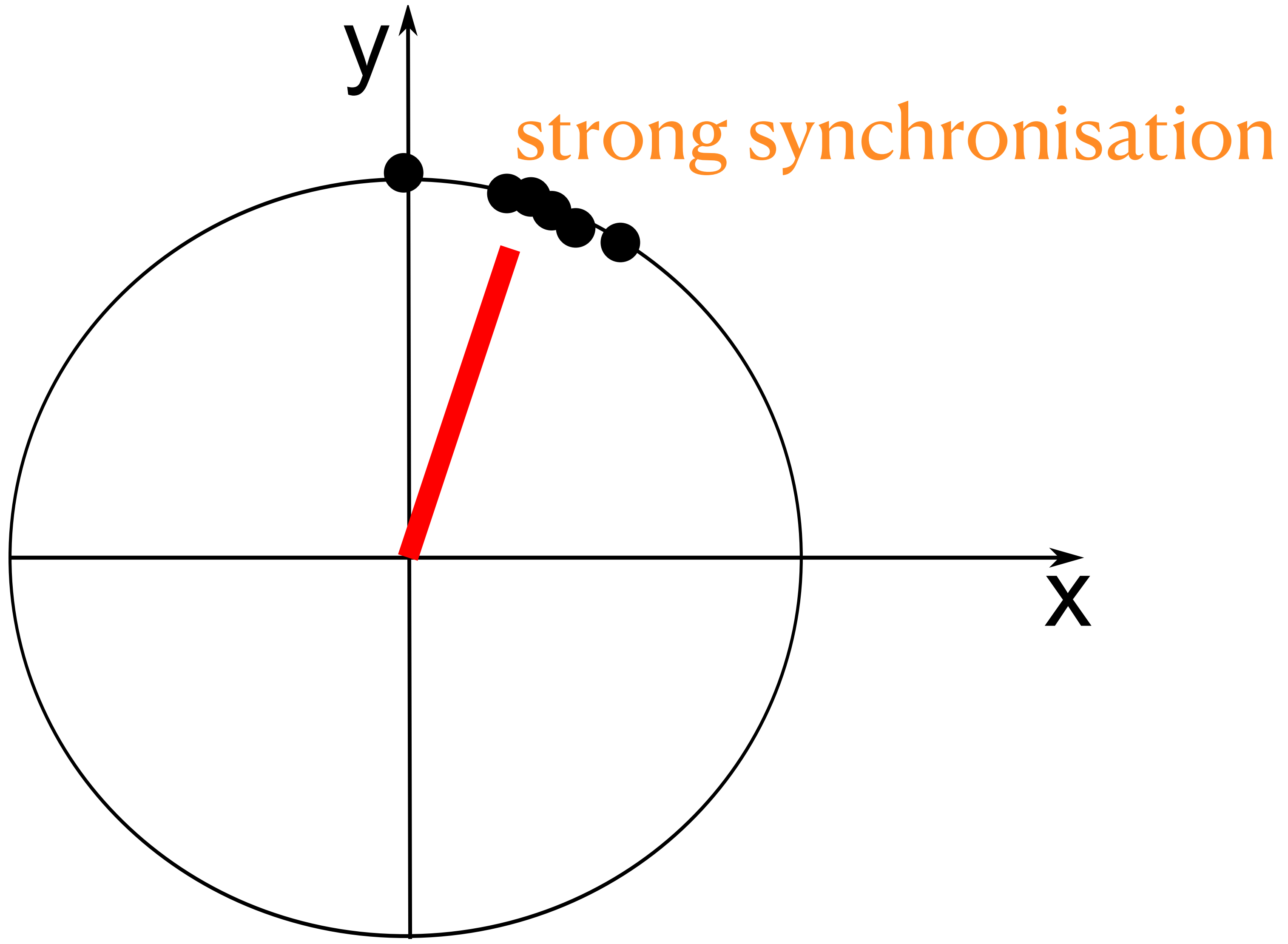
$$= \bar{x} + i\bar{y}$$

if the vektor Ψ

- is short, i.e. $|\,\Psi\,| \approx 0$, then $\bar{x}$ and $\bar{y}$ are small and the phases are different to each other —> **no synchronisation**

- is long, i.e. $|\,\Psi\,| \approx 1$, then $\bar{x}$ and $\bar{y}$ are large and the phases are similar to each other —> **synchronisation**





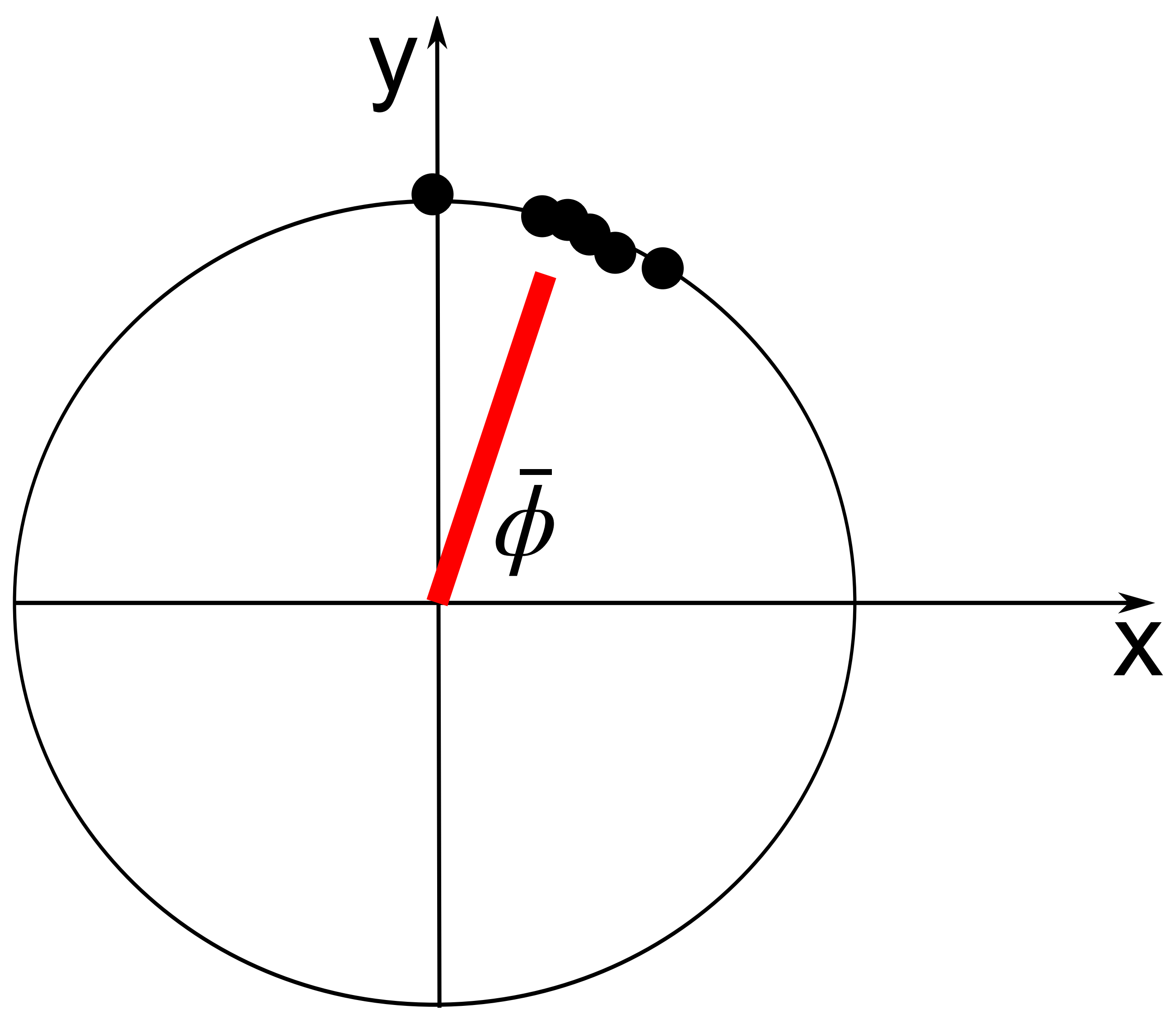
mean phase $\bar{\phi} = \arctan\left(\frac{\bar{y}}{\bar{x}}\right)$

synchronisation index

$$\boxed{\begin{aligned} R &= |\Psi| = \sqrt{\bar{x}^2 + \bar{y}^2} \\ &= \frac{1}{N}\sqrt{\left(\sum_{n=1}^{N}\cos\phi_n\right)^2 + \left(\sum_{n=1}^{N}\sin\phi_n\right)^2} \end{aligned}}$$

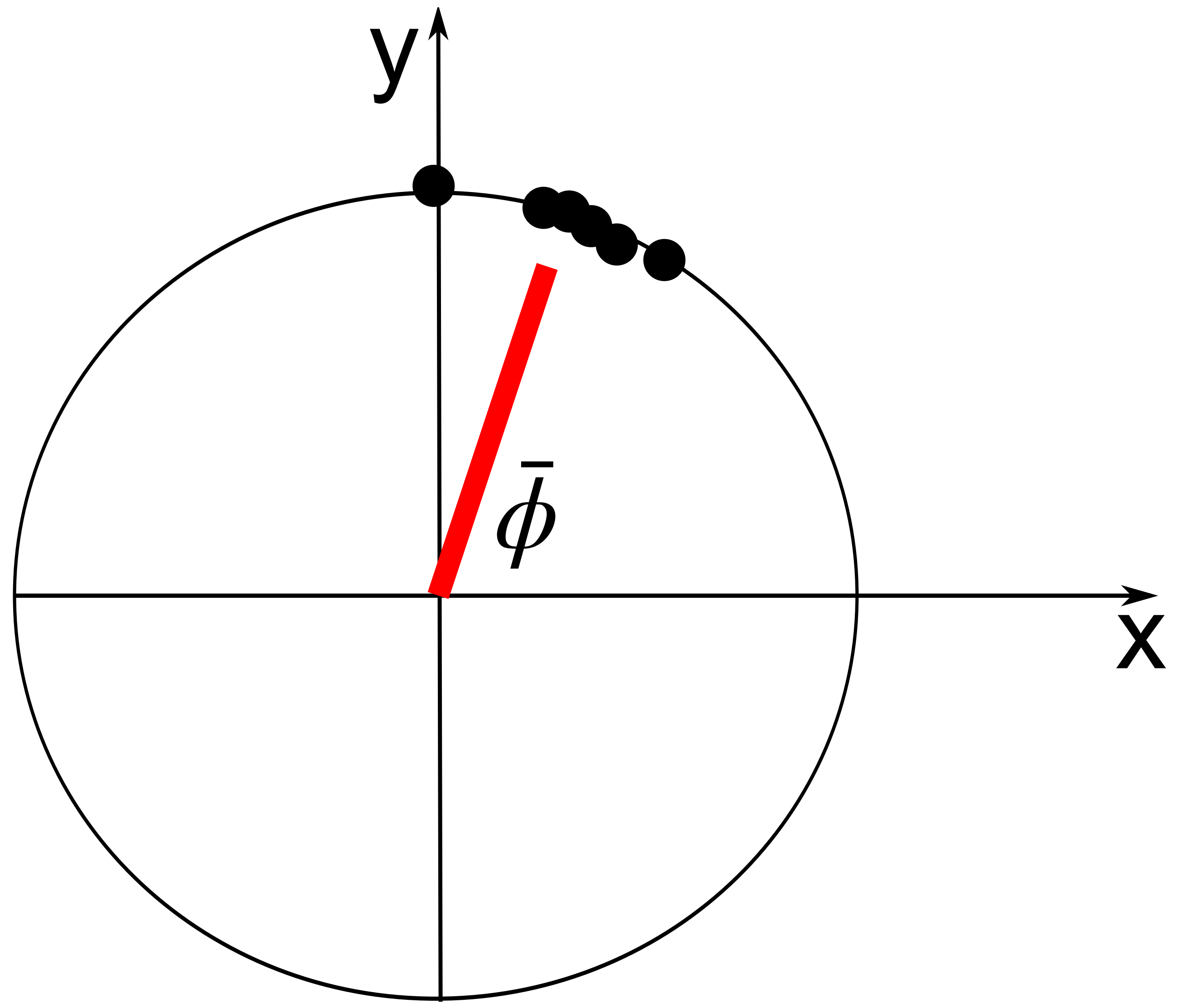
remarks: $\sigma_s^2 = 1 - R$ is also called the *circular variance*

R is also called **Phase Locking Value** (**PLV**, Lachaux et al. (1998))



possible problems:

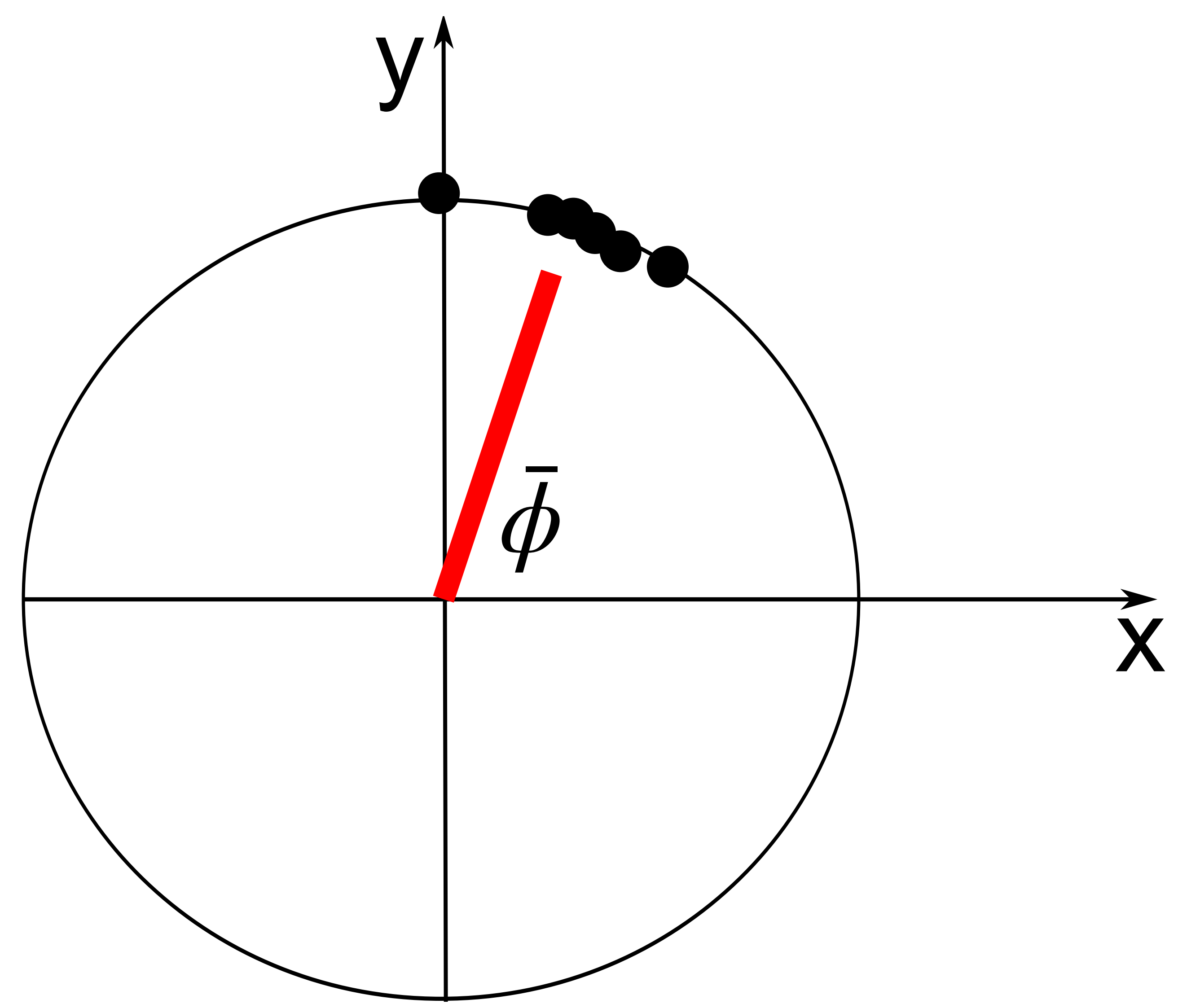
PLV is independent of the mean phase angle $\bar{\phi}$



possible problems:

PLV is independent of the mean phase angle $\bar{\phi}$

in EEG, zero-phase synchronization (R large, $\bar{\phi} \approx 0$)
may reflect volume conduction



<u>possible problems:</u>

PLV is independent of the mean phase angle $\bar{\phi}$

in EEG, zero-phase synchronization (R large, $\bar{\phi} \approx 0$)
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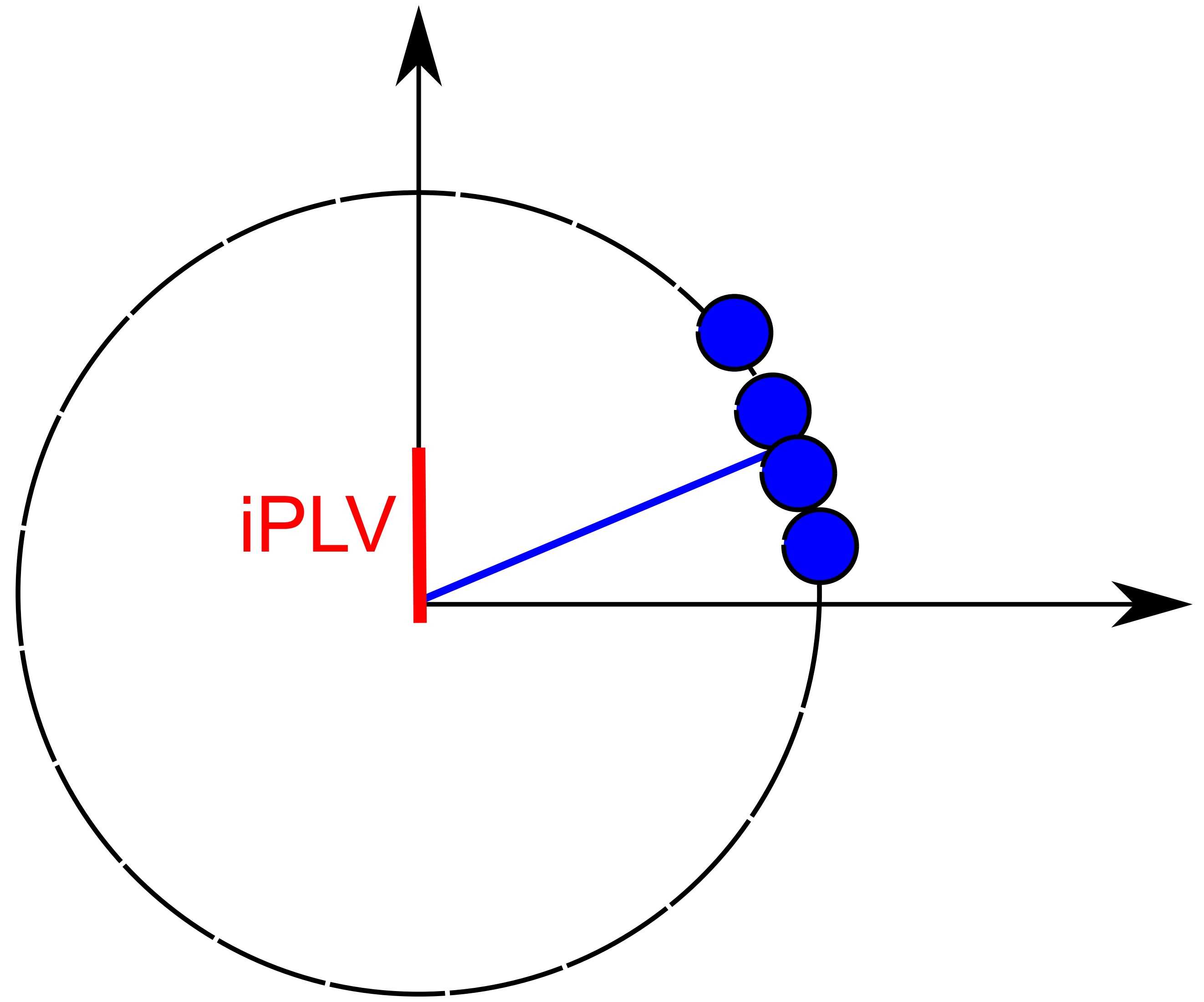
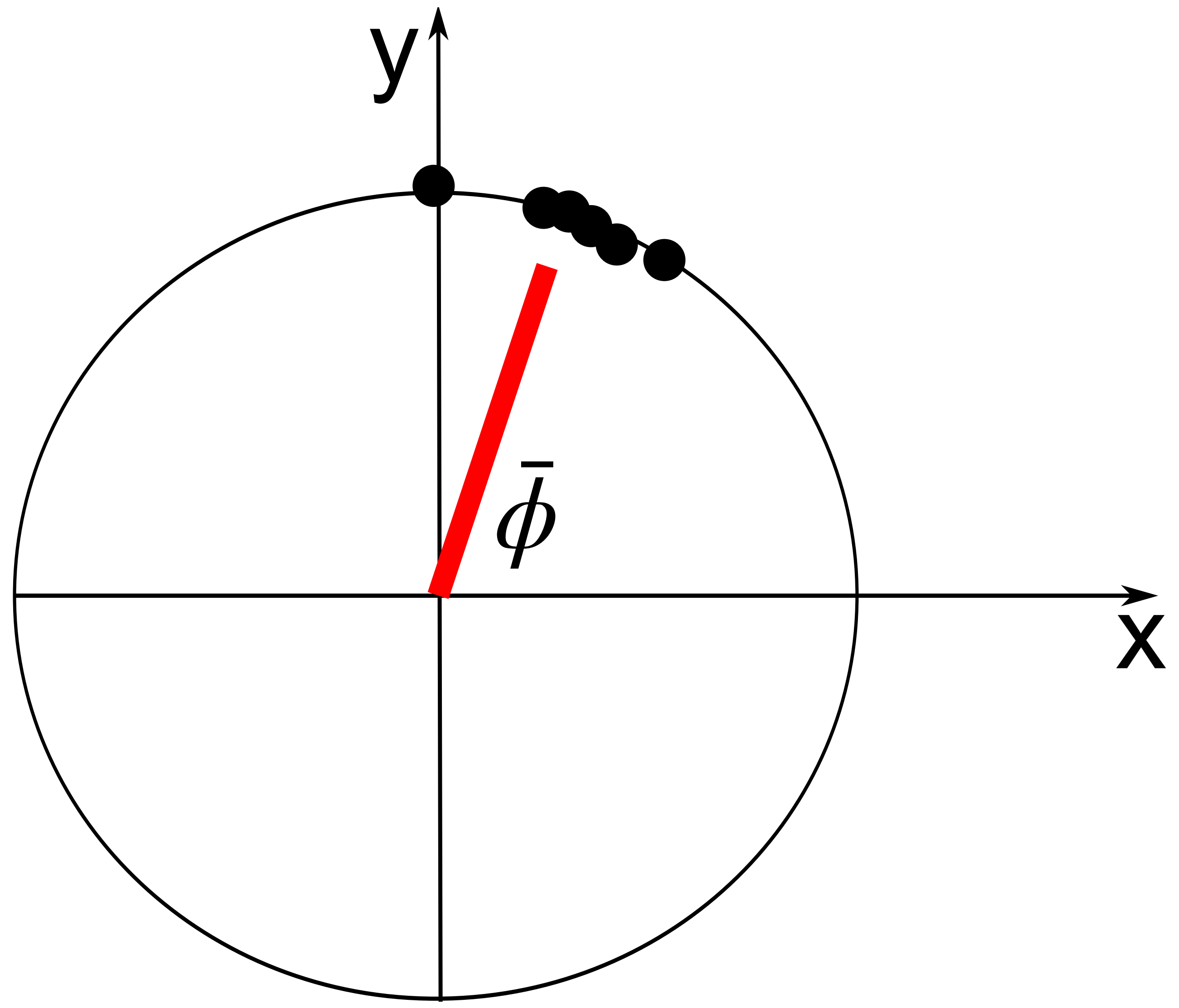
to avoid R large, $\bar{\phi} \approx 0$, one can down-weight R with $\bar{\phi} \approx 0$ by

taking the imaginary part of R:





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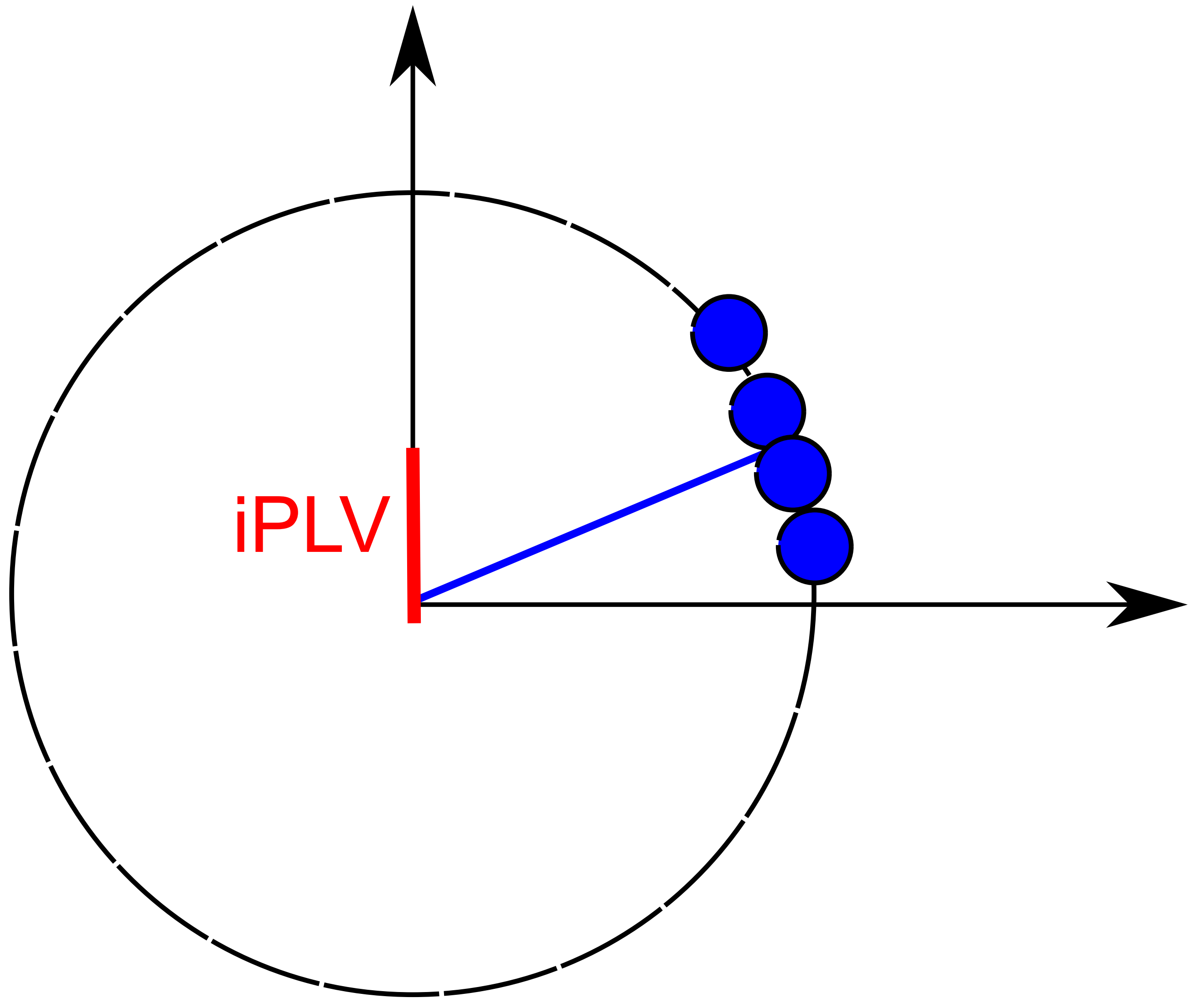
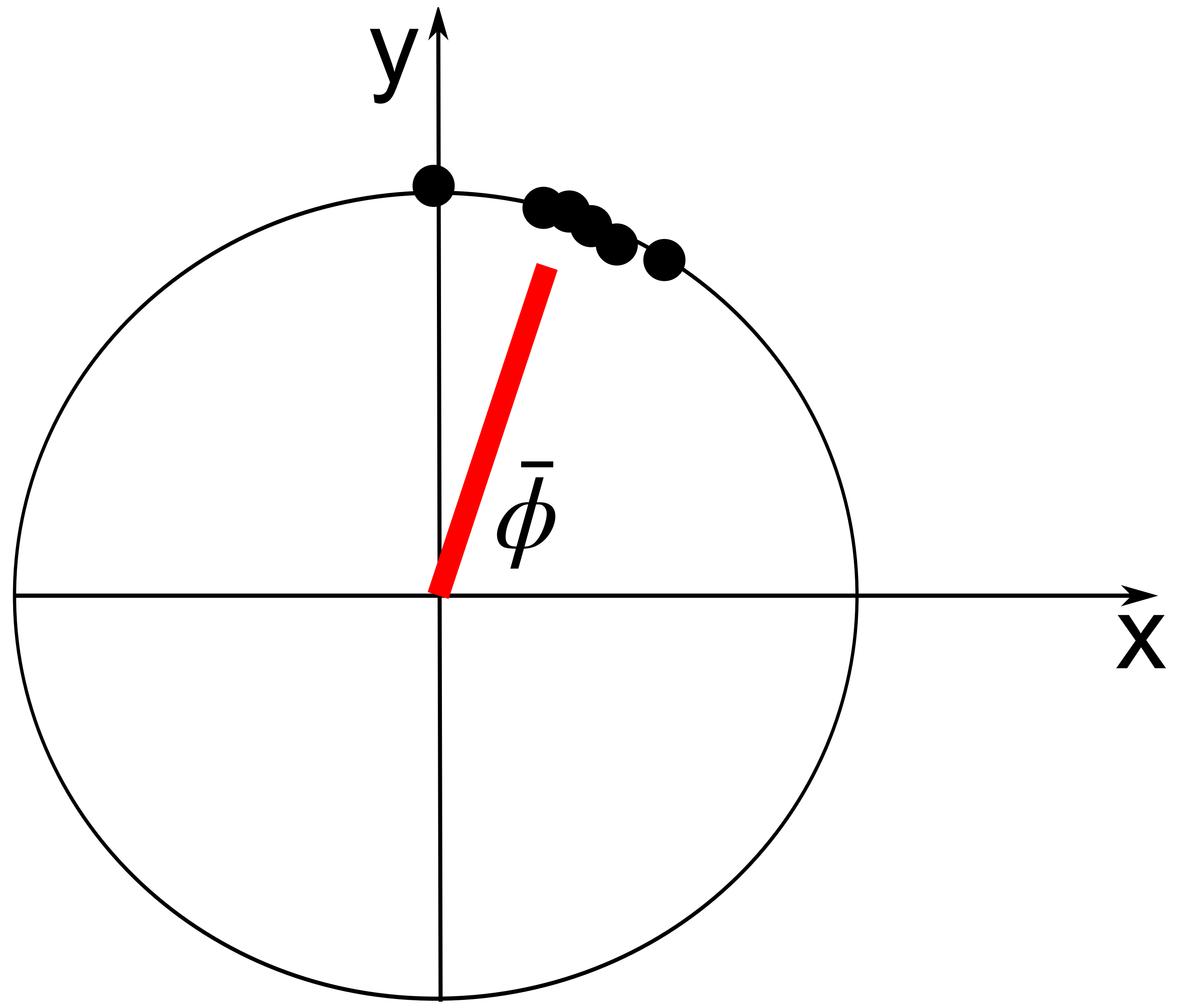
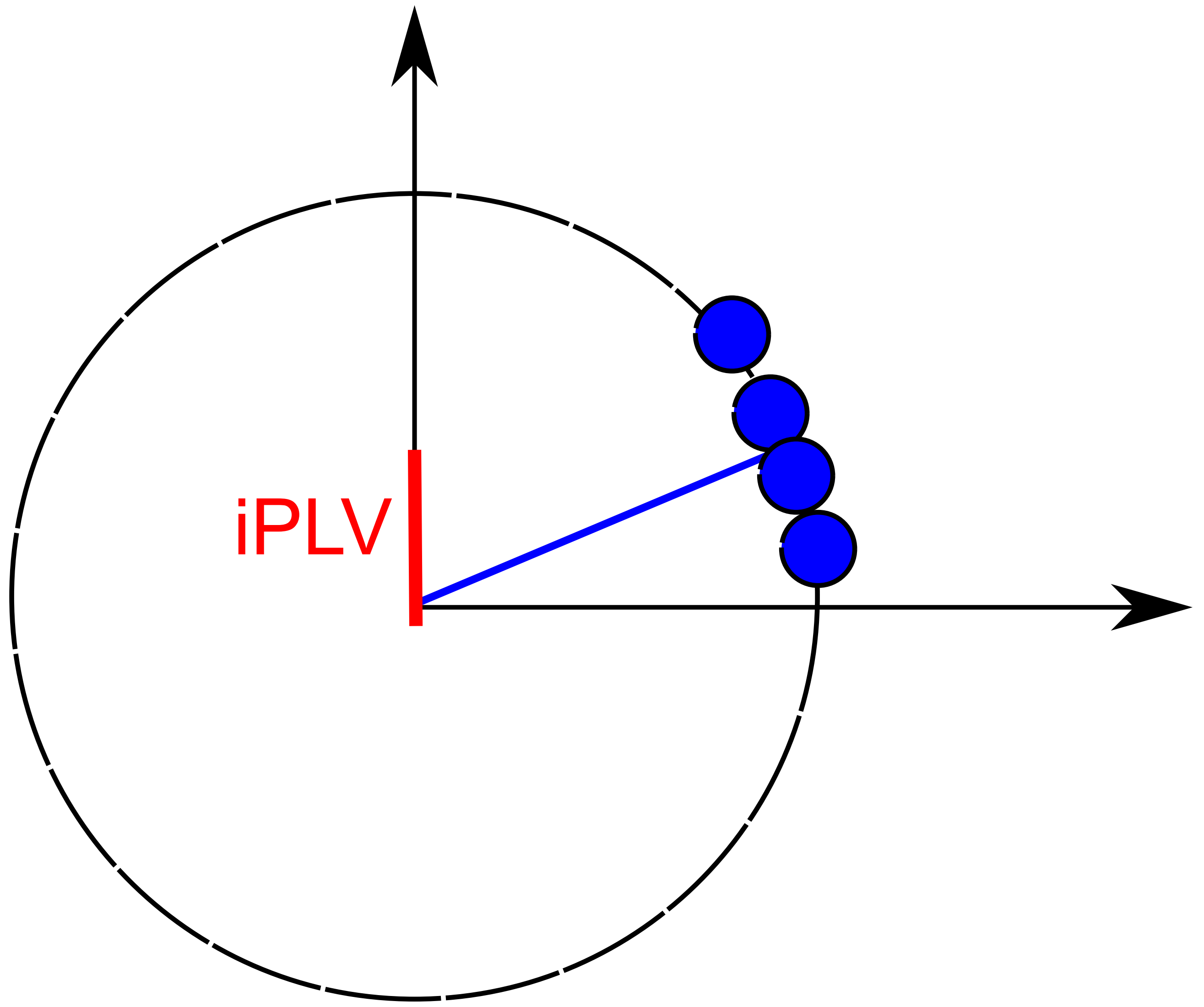


$$R_i = \frac{1}{N} \left| \sum_{n=1}^{N} \sin(\phi_n) \right| \qquad \textbf{(iPLV)}$$

(see Bruna et al., J. Neural Eng. 15:056011 (2018))





weak synchronisation according to iPLV

possible problems:

PLV is independent of the mean phase angle $\bar\phi$

in EEG, zero-phase synchronization (R large, $\bar\phi \approx 0$)
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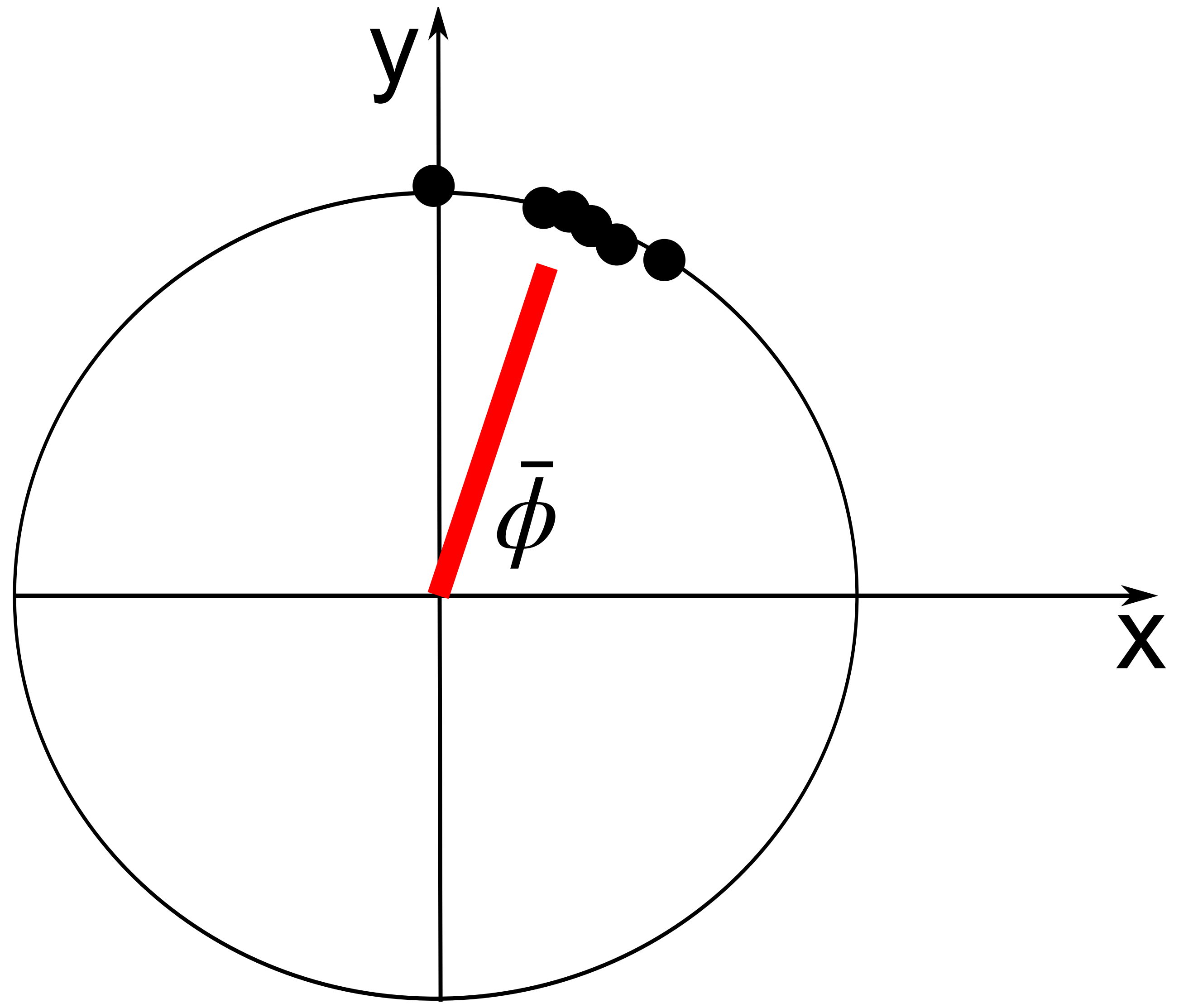

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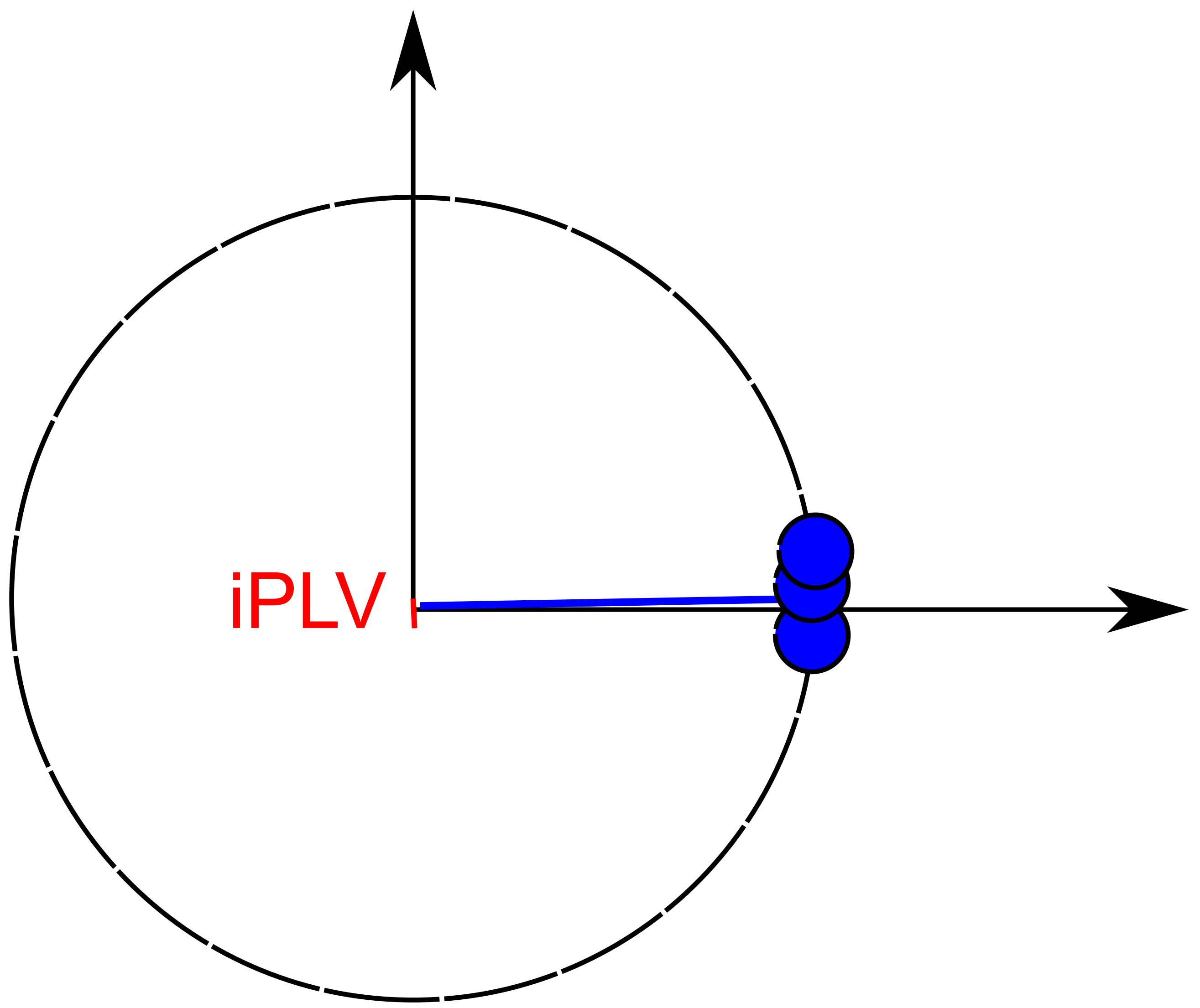


no synchronisation according to iPLV

Kuramoto Phasenmodell:

example: artificial dataset

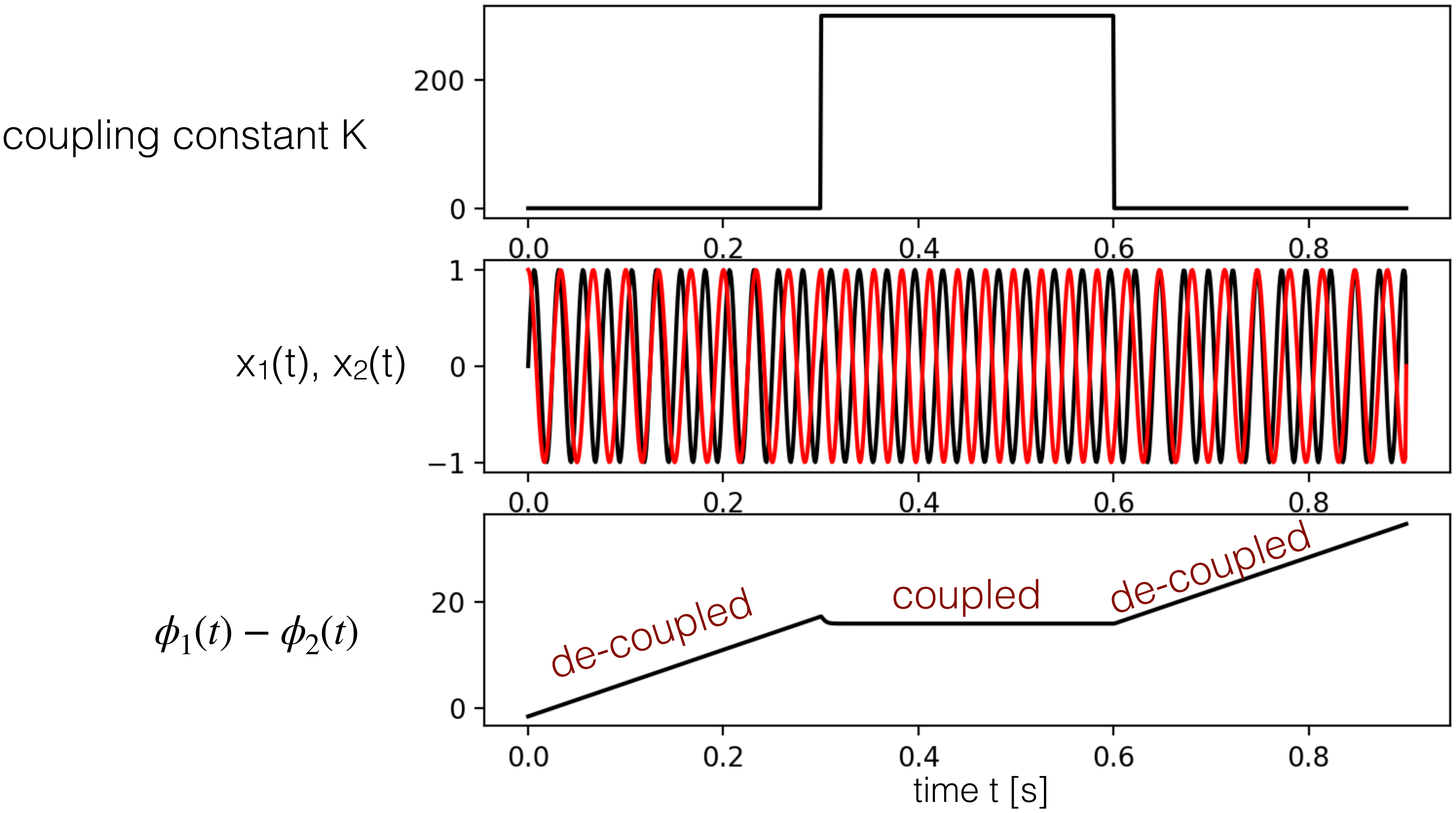
$$\dot{\phi}_1(t) = 2\pi f_1 + \frac{K}{2}\sin(\phi_1 - \phi_2)$$

$$\dot{\phi}_2(t) = 2\pi f_2 - \frac{K}{2}\sin(\phi_1 - \phi_2)$$

$$s_1(t) = \sin\phi_1(t)$$

$$s_2(t) = \sin\phi_2(t)$$

- two non-linearly coupled oscillators (Kuramoto model with K=300.0)
- frequency of oscillators: 40.0Hz and 30.0Hz
- coupling sets in at t=0.3s und is removed at t=0.6s



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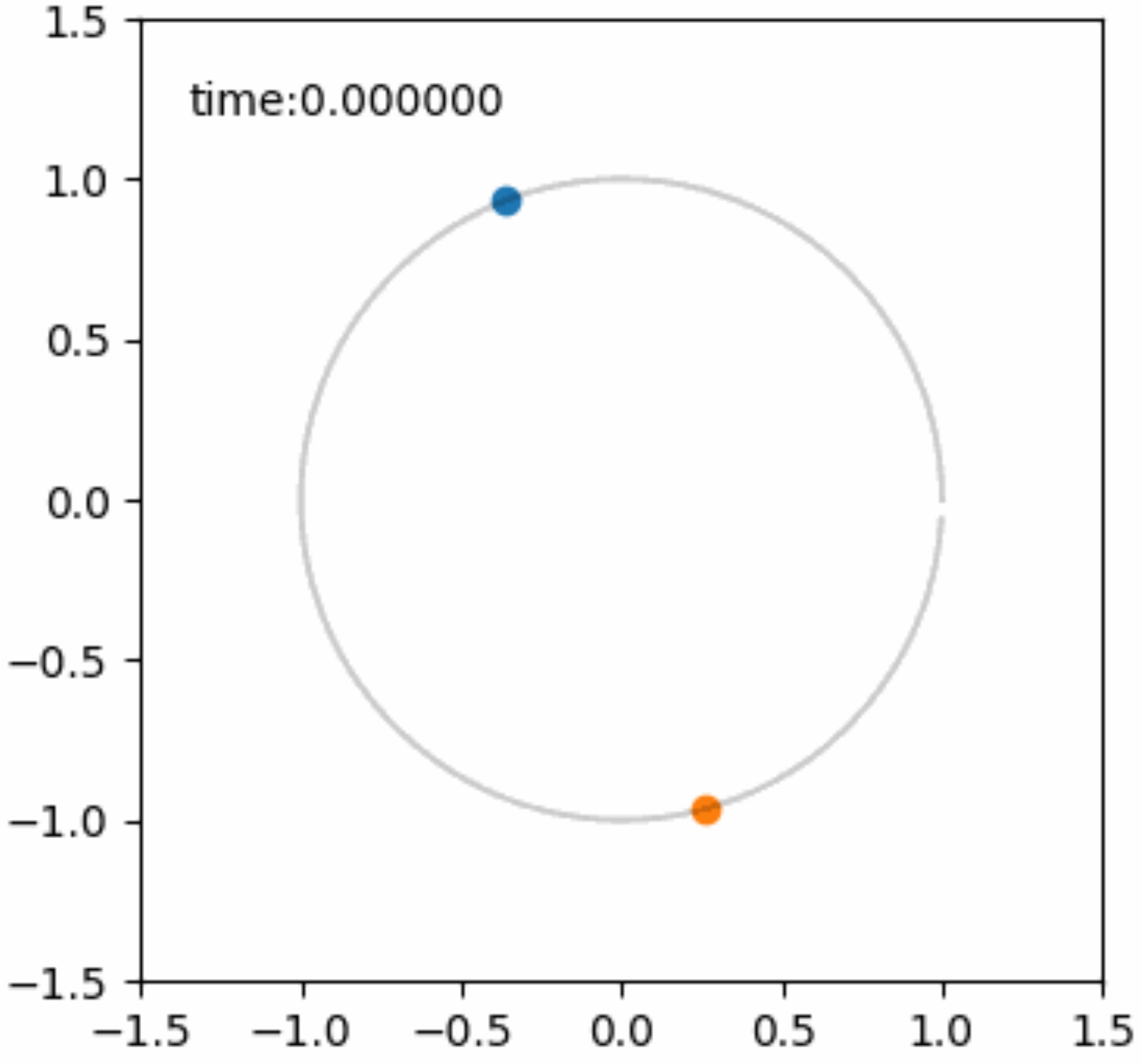
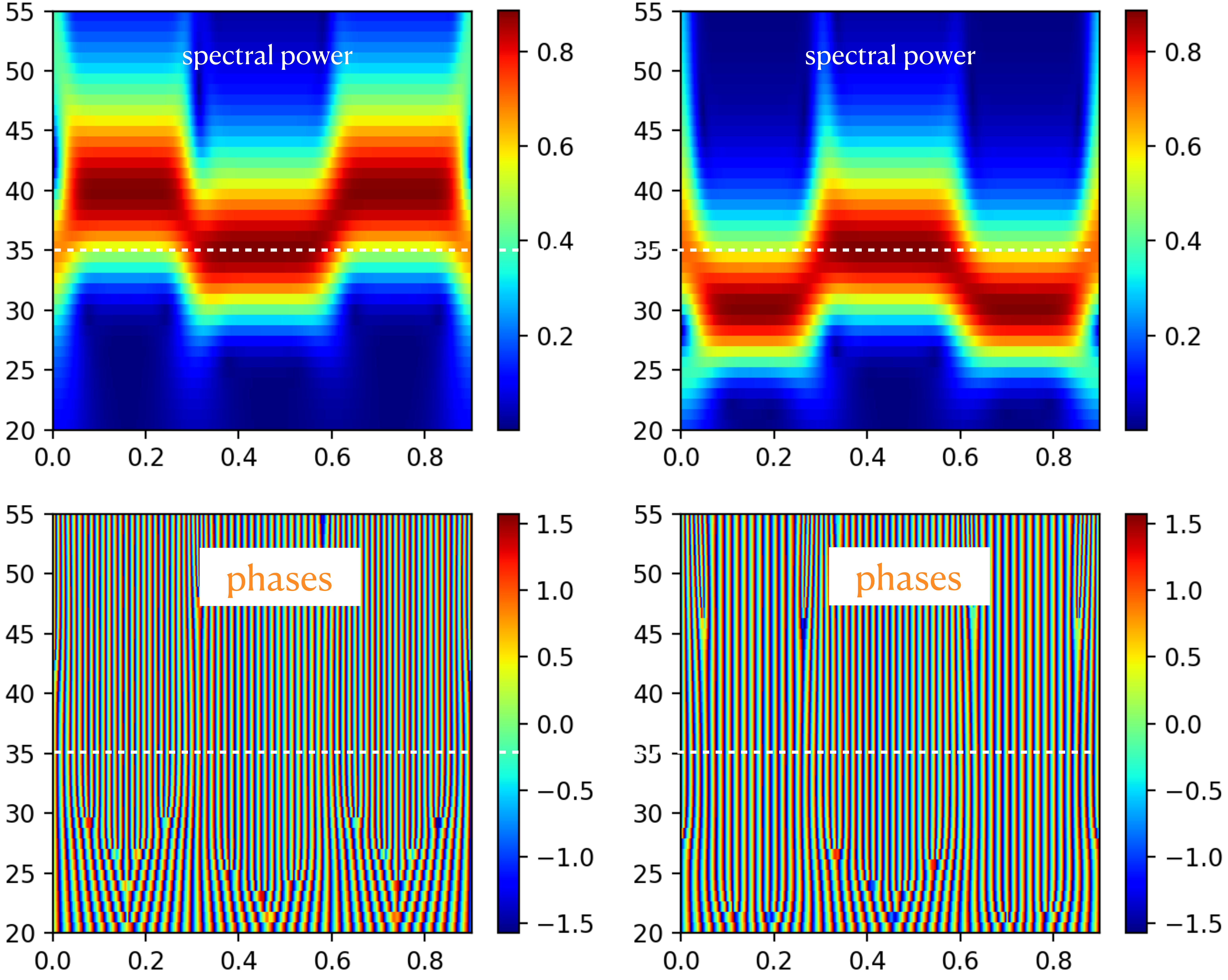
f1=40Hz f1=30Hz

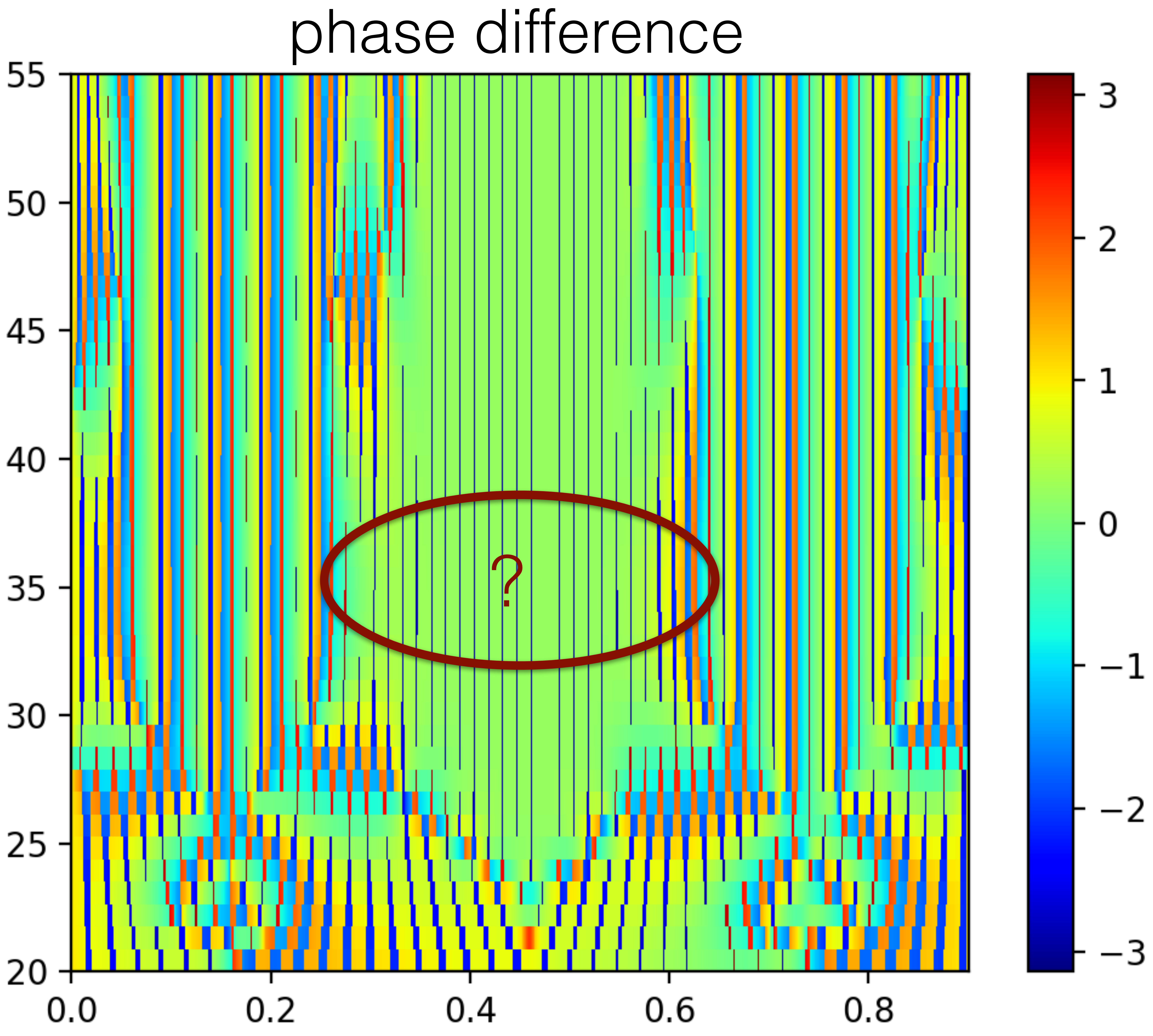


time-frequency description

frequency synchronisation

by the phase coupling

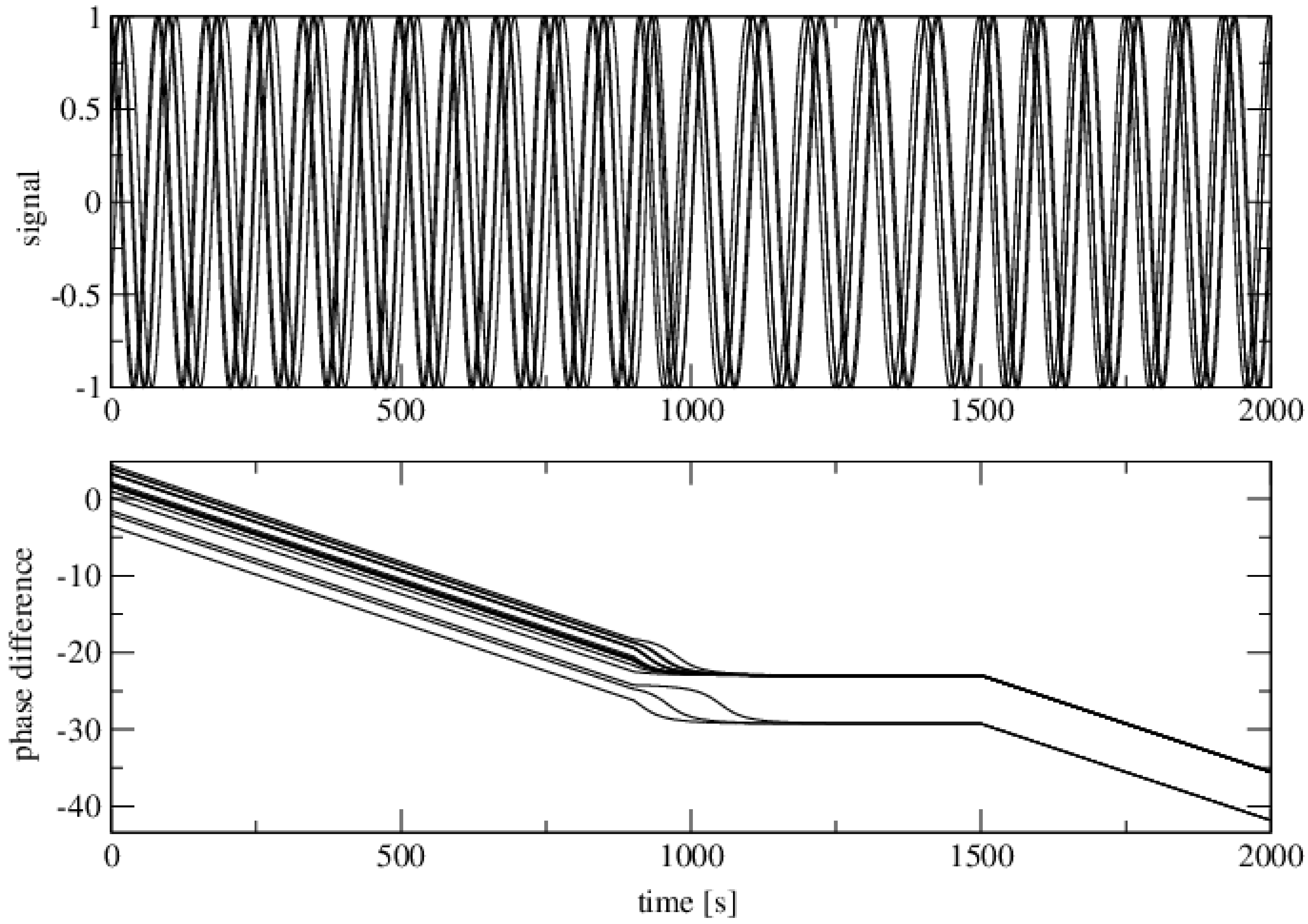




interpretation is difficult

statistical estimation may be necessary

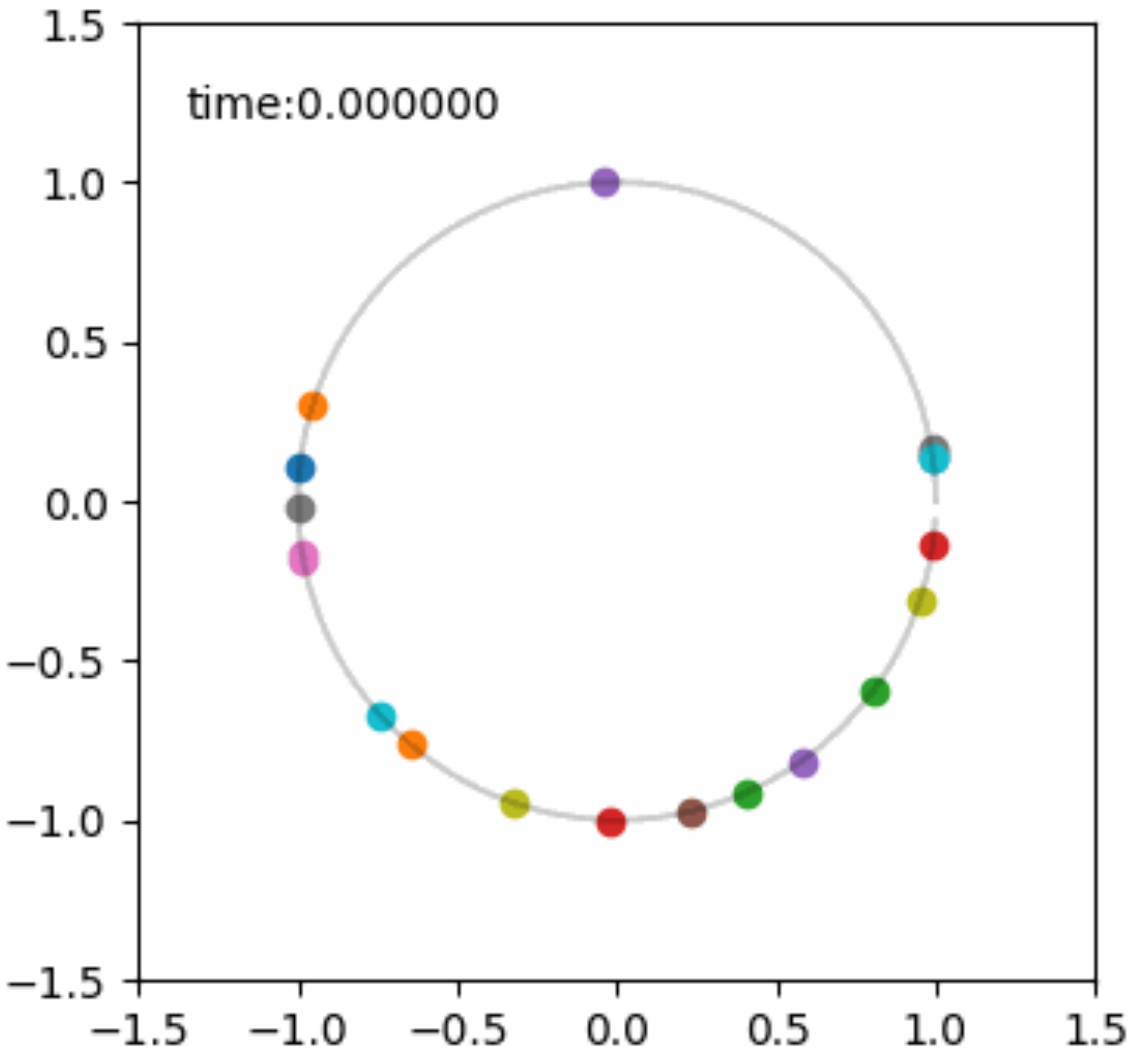
new simulation



with 40 trials with different initial conditions

phase synchronisation exhibits constant phase difference if coupling is present

phase differences of 40 trials:



data sampling

Fourier analysis

errors in analysis

linear filters

time-frequency analysis

non-Fourier analysis

synchronisation

Hilbert Transform **Empirical Mode Decomposition**

phase synchronisation **spectral coherence**

phase-amplitude coupling

linear measure of correlation: **coherence**

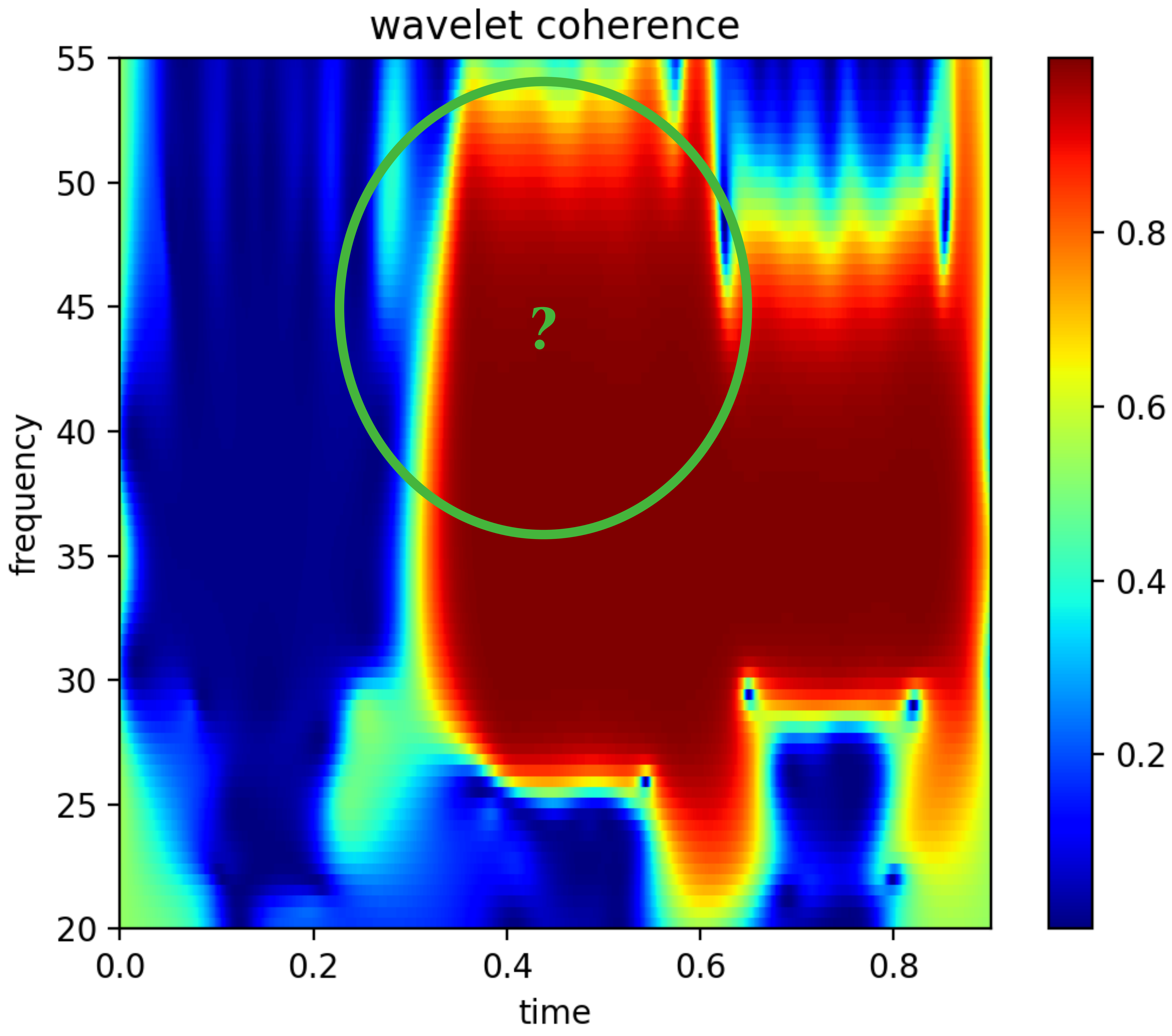
spectral coherence for stationary signals

$$G_{xy}(f) = \frac{|S_{xy}(f)|^2}{S_{xx}(f)S_{yy}(f)}$$

wavelet coherence for non-stationary signals

$$G_{xy}(t,f) = \frac{\hat{S}[|W_{xy}(t,f)|^2]}{\hat{S}[|W_{xx}(t,f)|]\hat{S}[|W_{yy}(t,f)|]}$$

$\hat{S}$: average over trials


wavelet coherence
55
50
45
40
35
30
25
20
frequency
0.0
0.2
0.4
0.6
0.8
time
?
0.8
0.6
0.4
0.2

problem with *spectral coherence*:

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- *coherence* is interpretable for linear interactions only

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suggestion of different nonlinear measure:

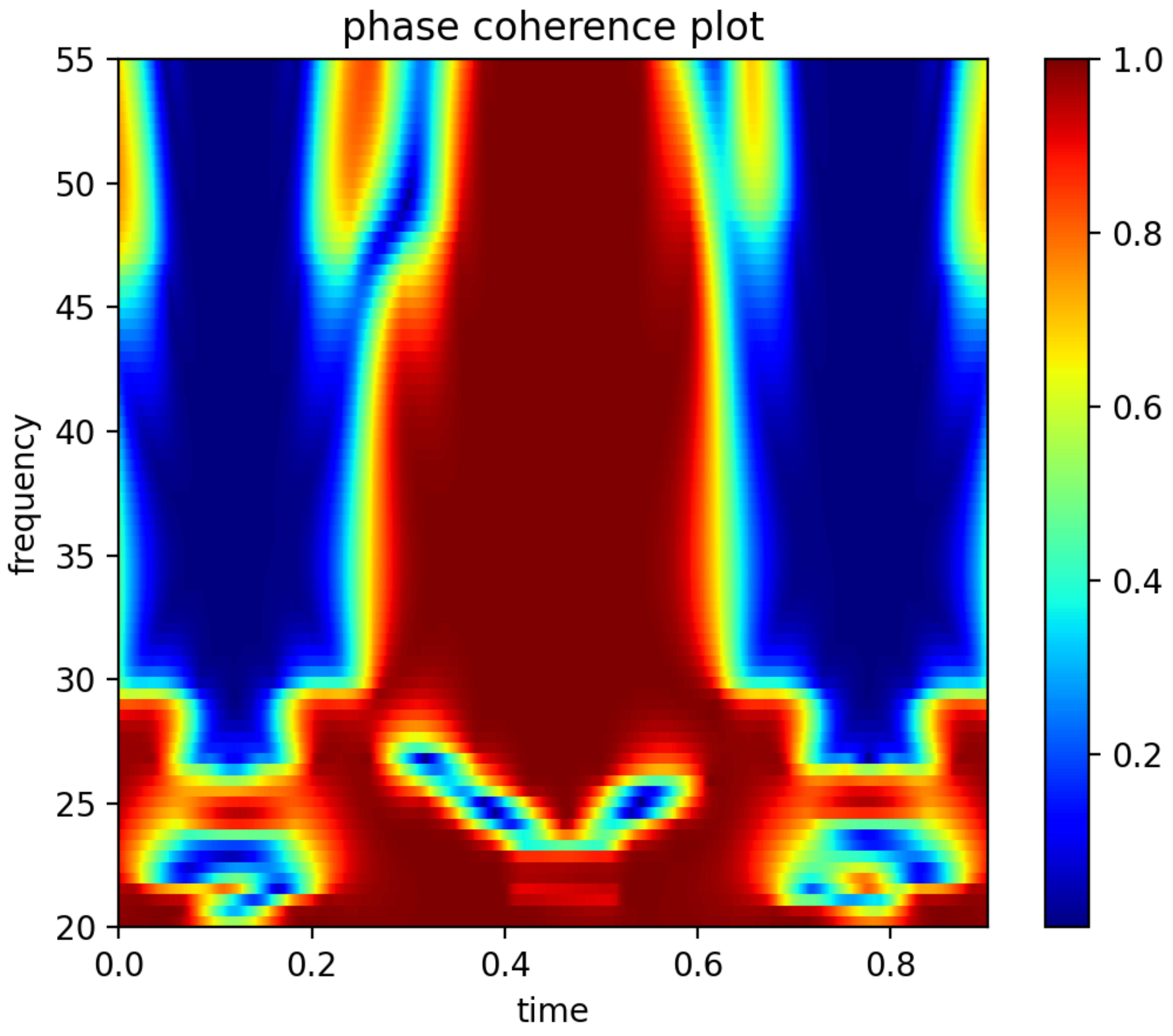
phase locking value (PLV) or **mean phase coherence**

Lachaux et al. (1999): average of time-dependent PLV over trials

$$\bar{R}(t,f) = \frac{1}{N}\sum_{n=1}^{N} R_n(t,f)$$

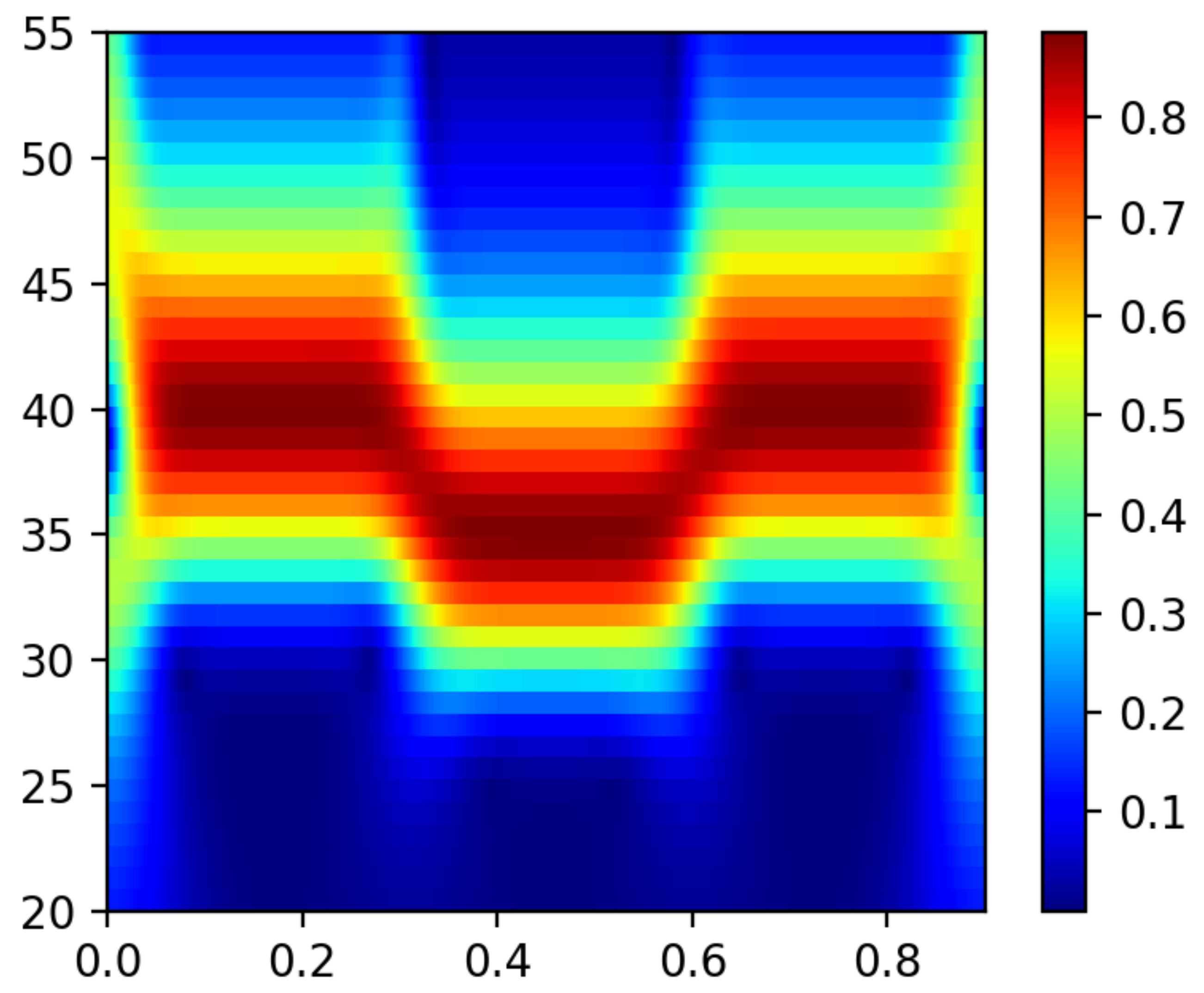
N: number of trials

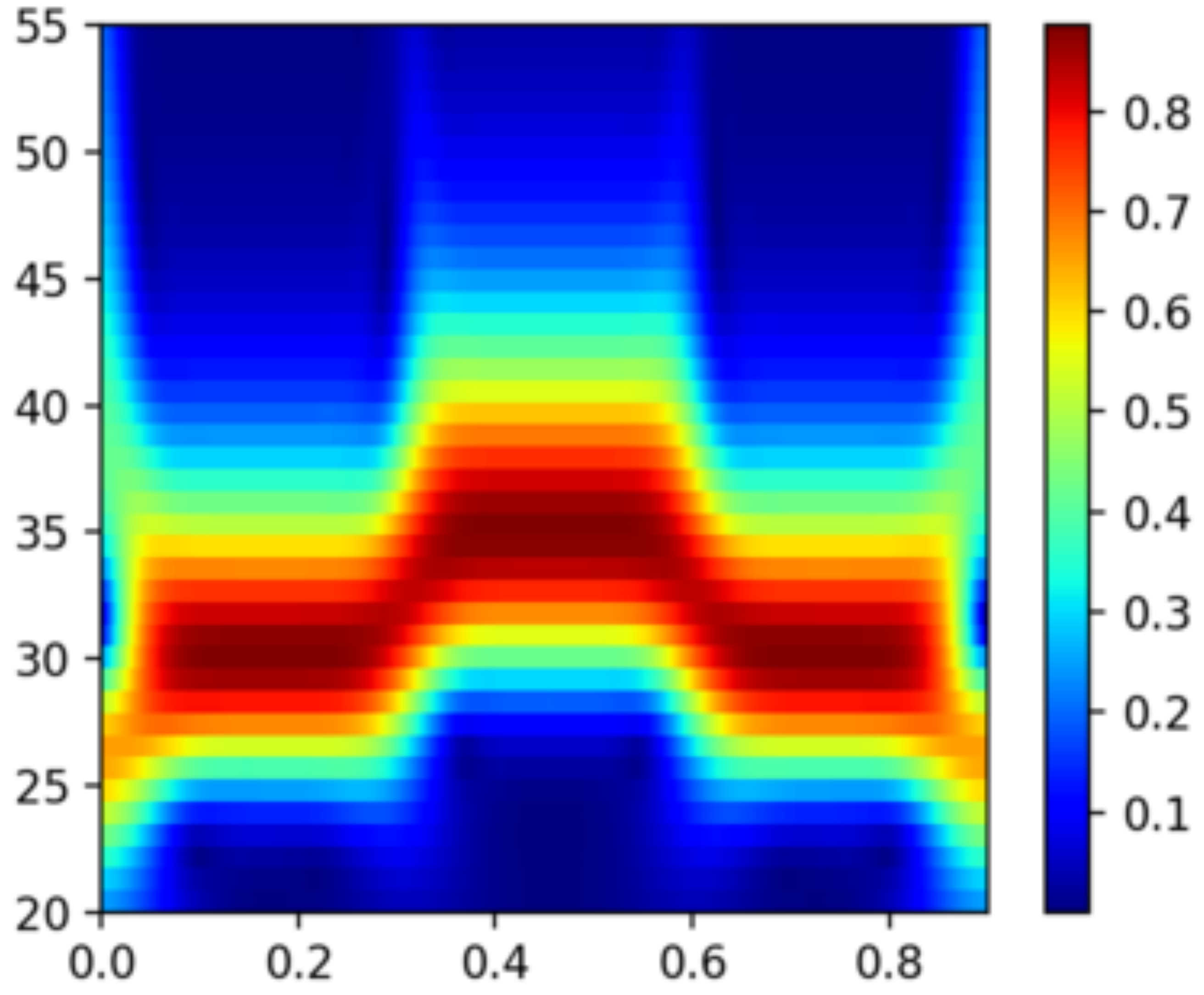
$R_n(t,f)$: PLV of trial n at time t and frequency f



problem: errorneous *coherence* and *PLV*

error in *coherence*:



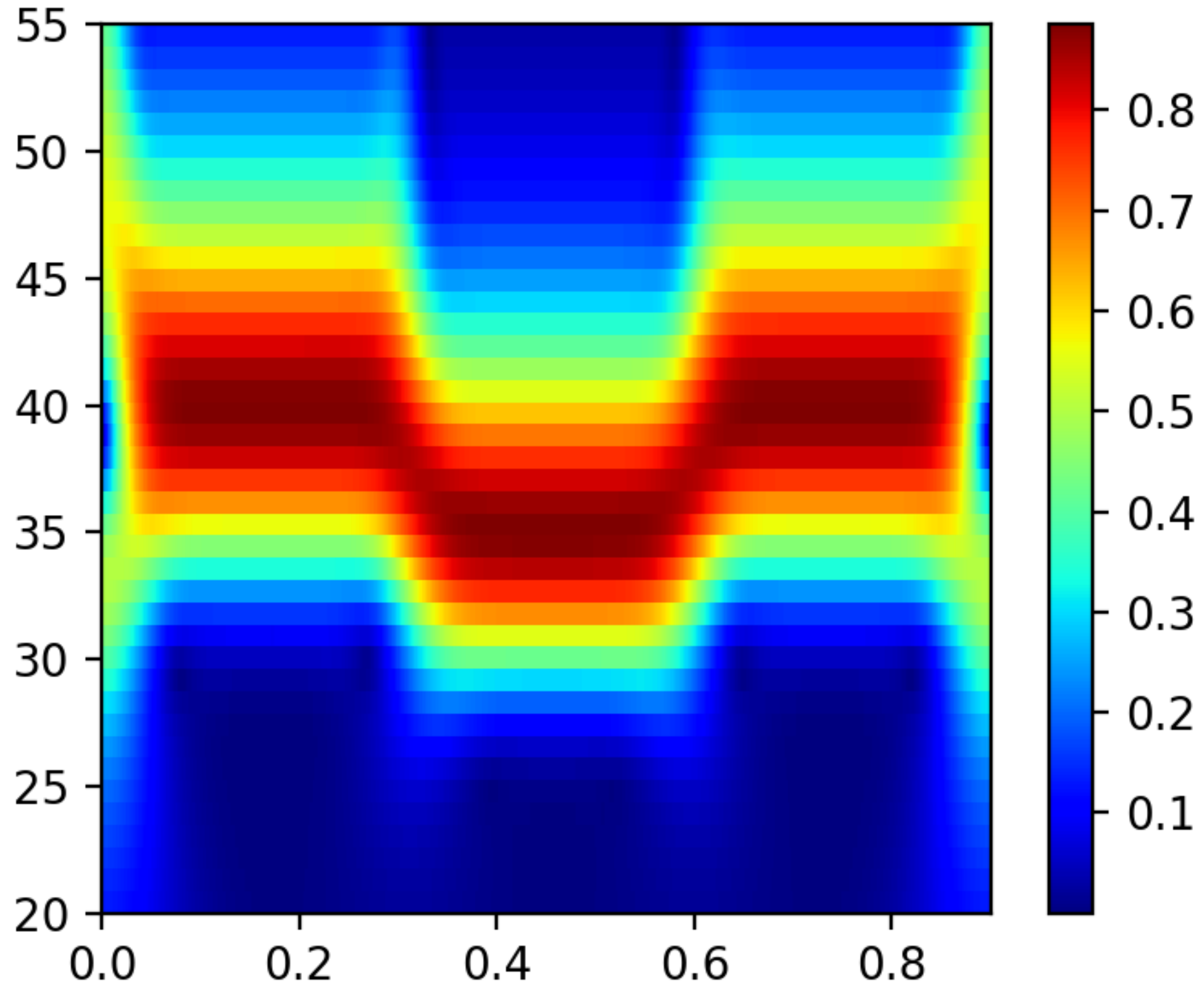


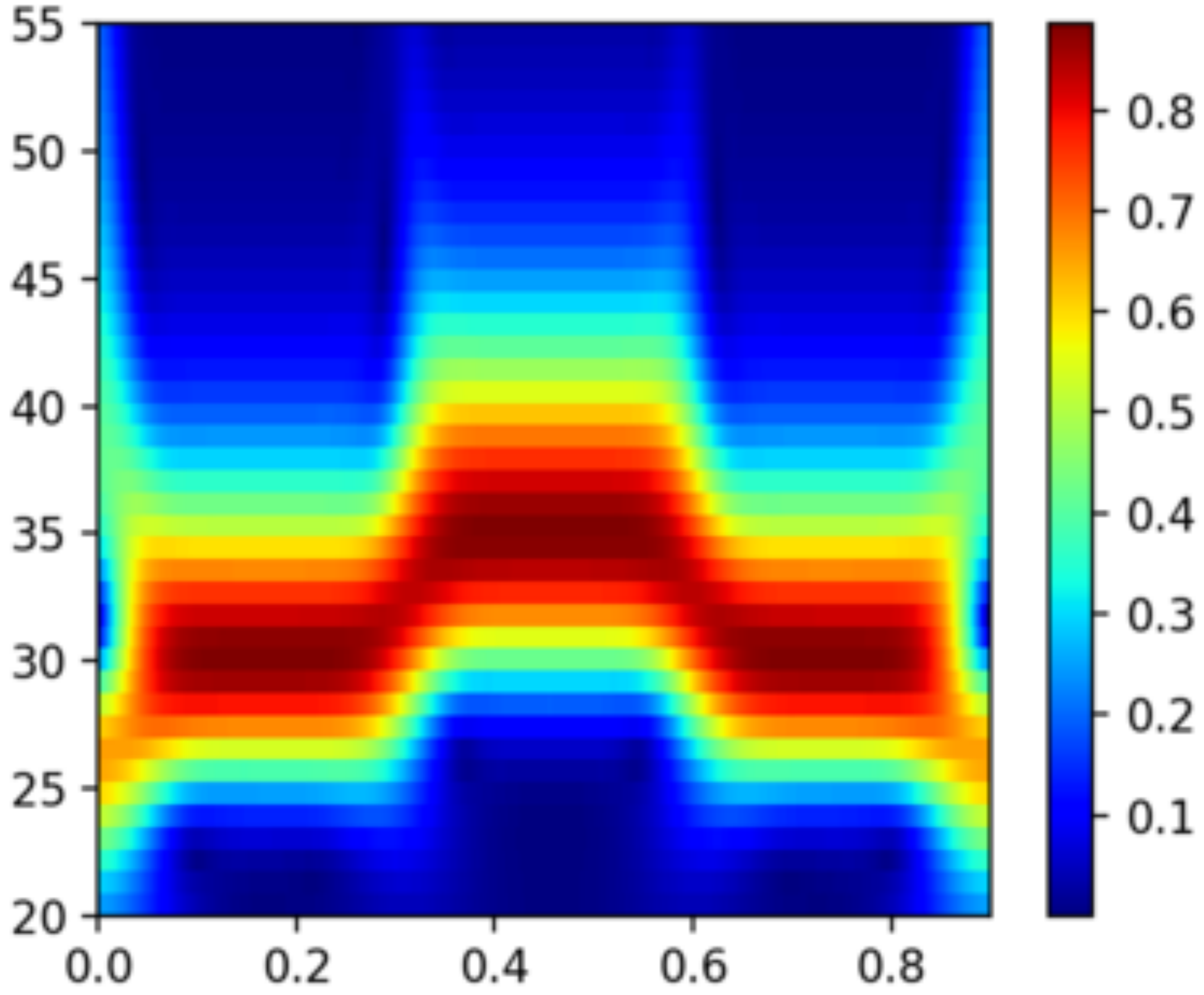
error in PLV:

definition of phase is independent of amplitude

if power (activity) is very low, no coherence should be computed

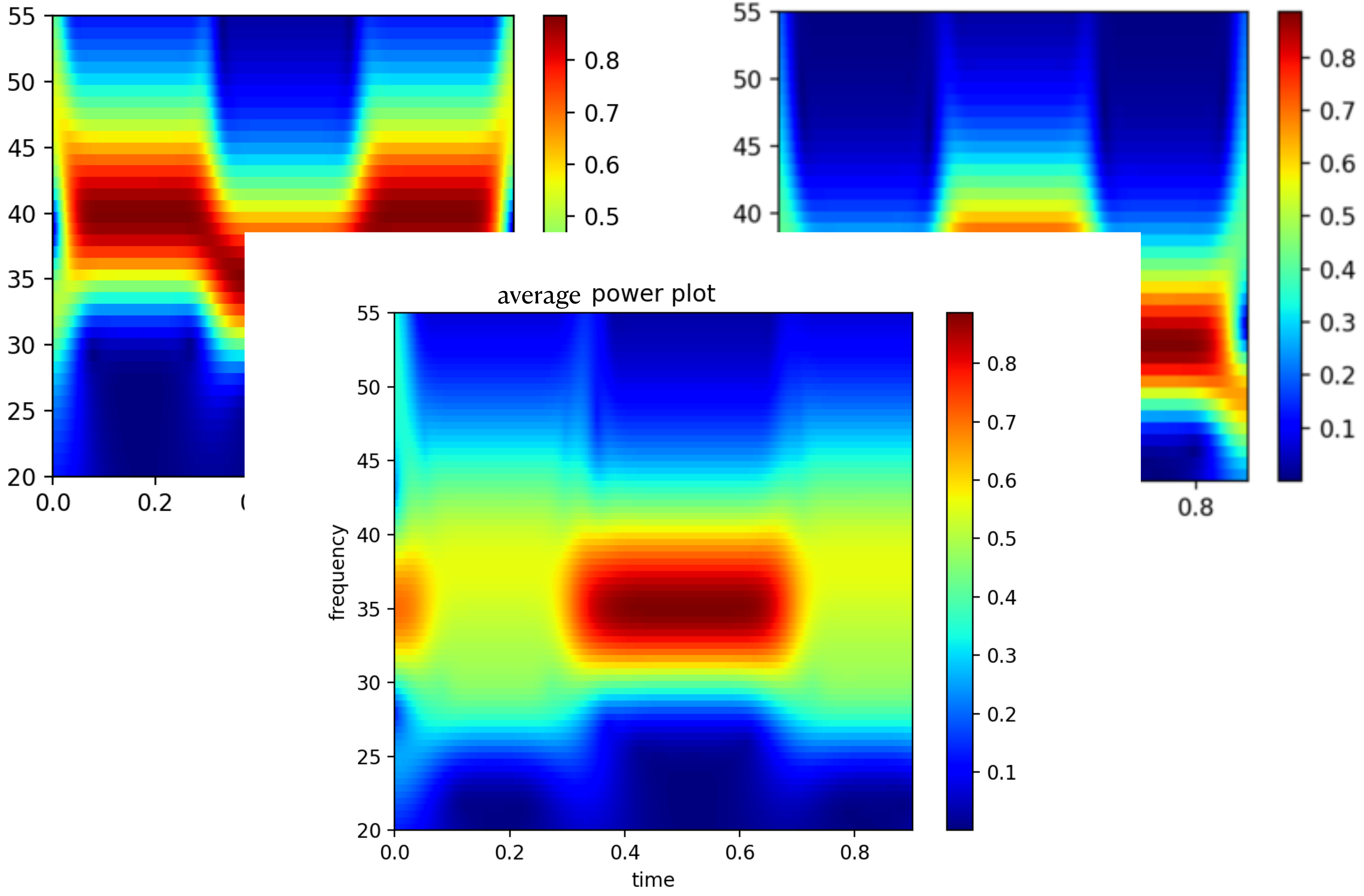
idea (Hutt et al., Neuroimage 197:414 (2018)):





consider these time-frequency pairs,
where **PSD** is **large in both channels**.

idea (Hutt et al., Neuroimage 197:414 (2018)):



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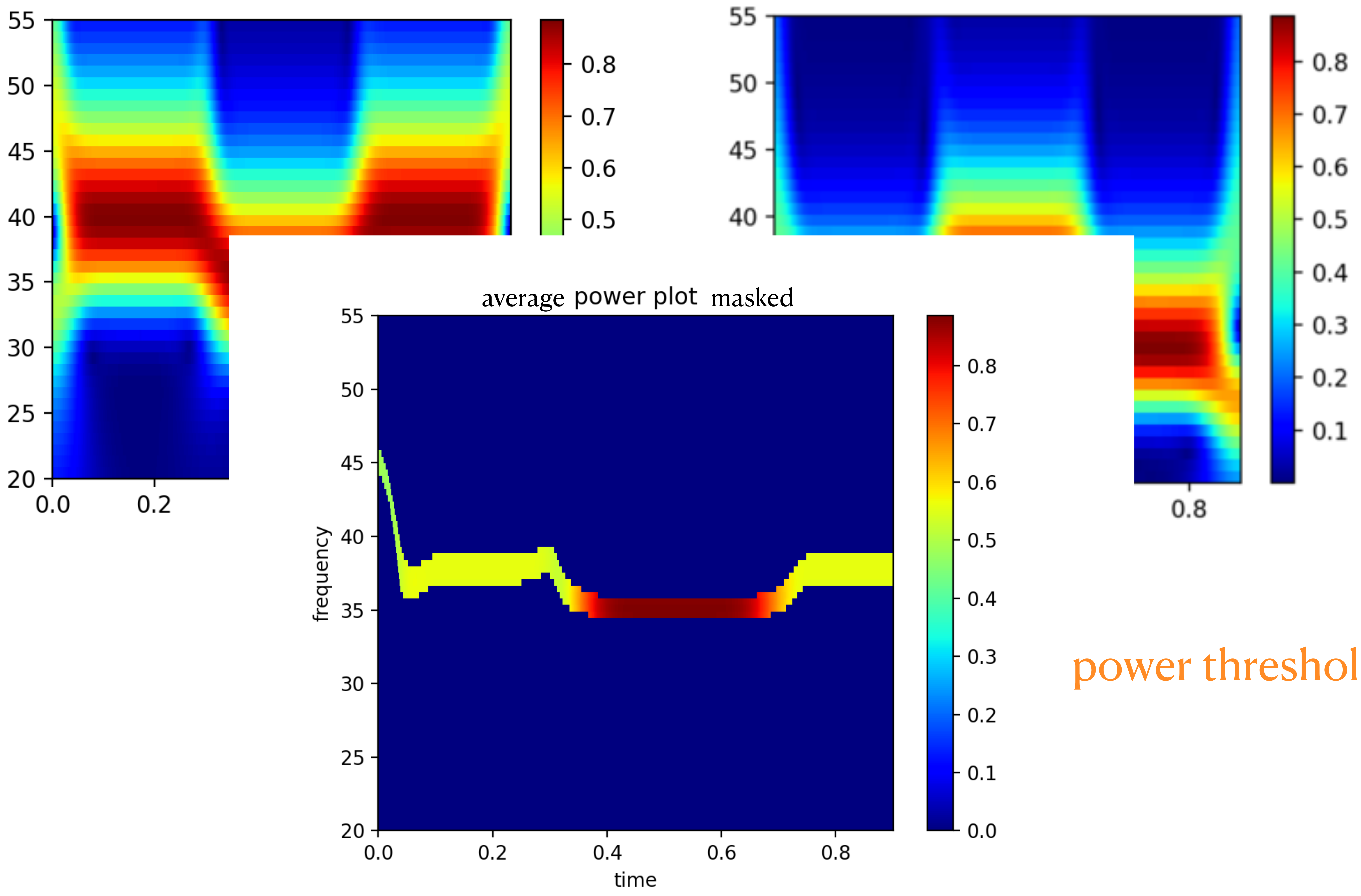
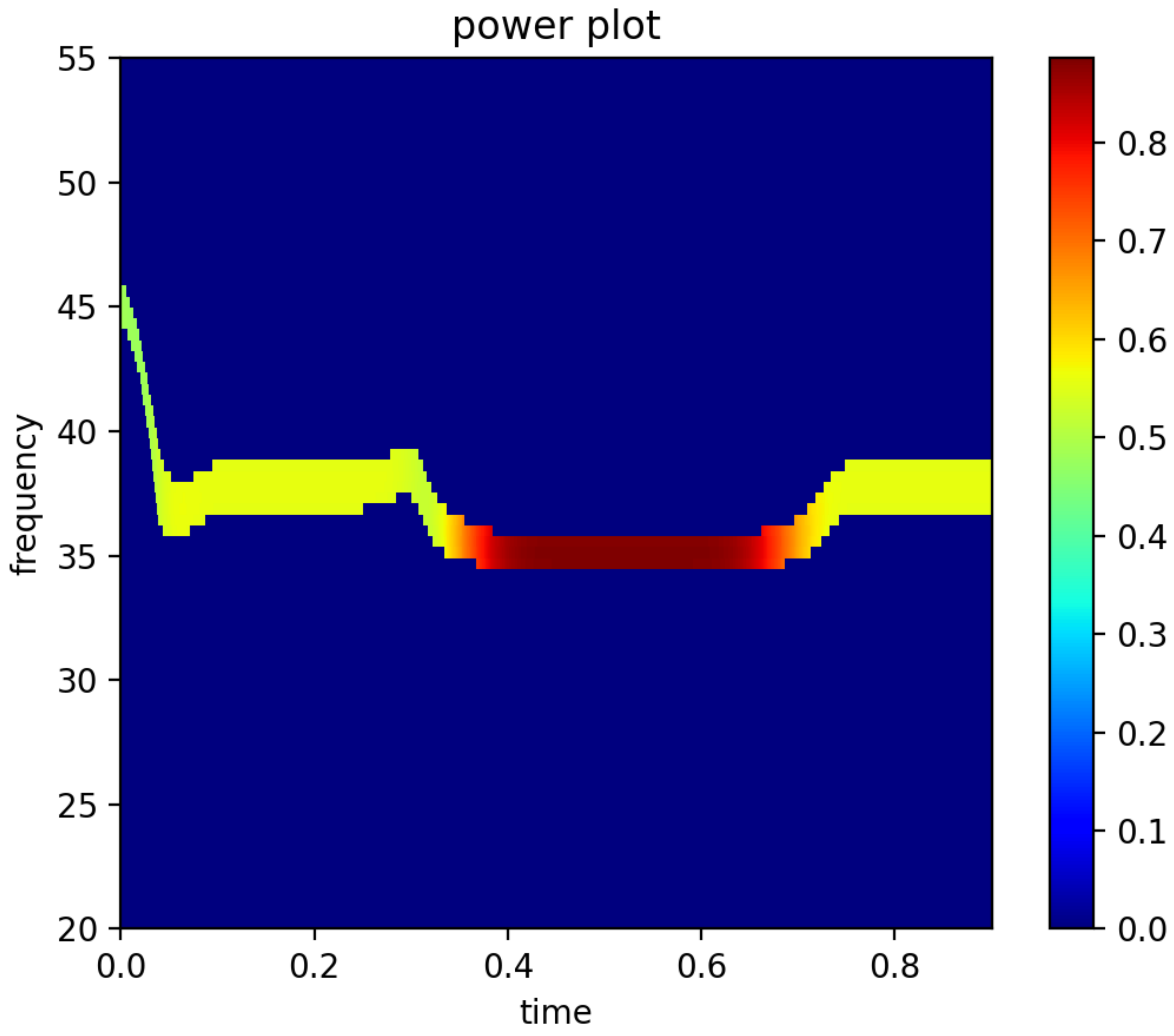
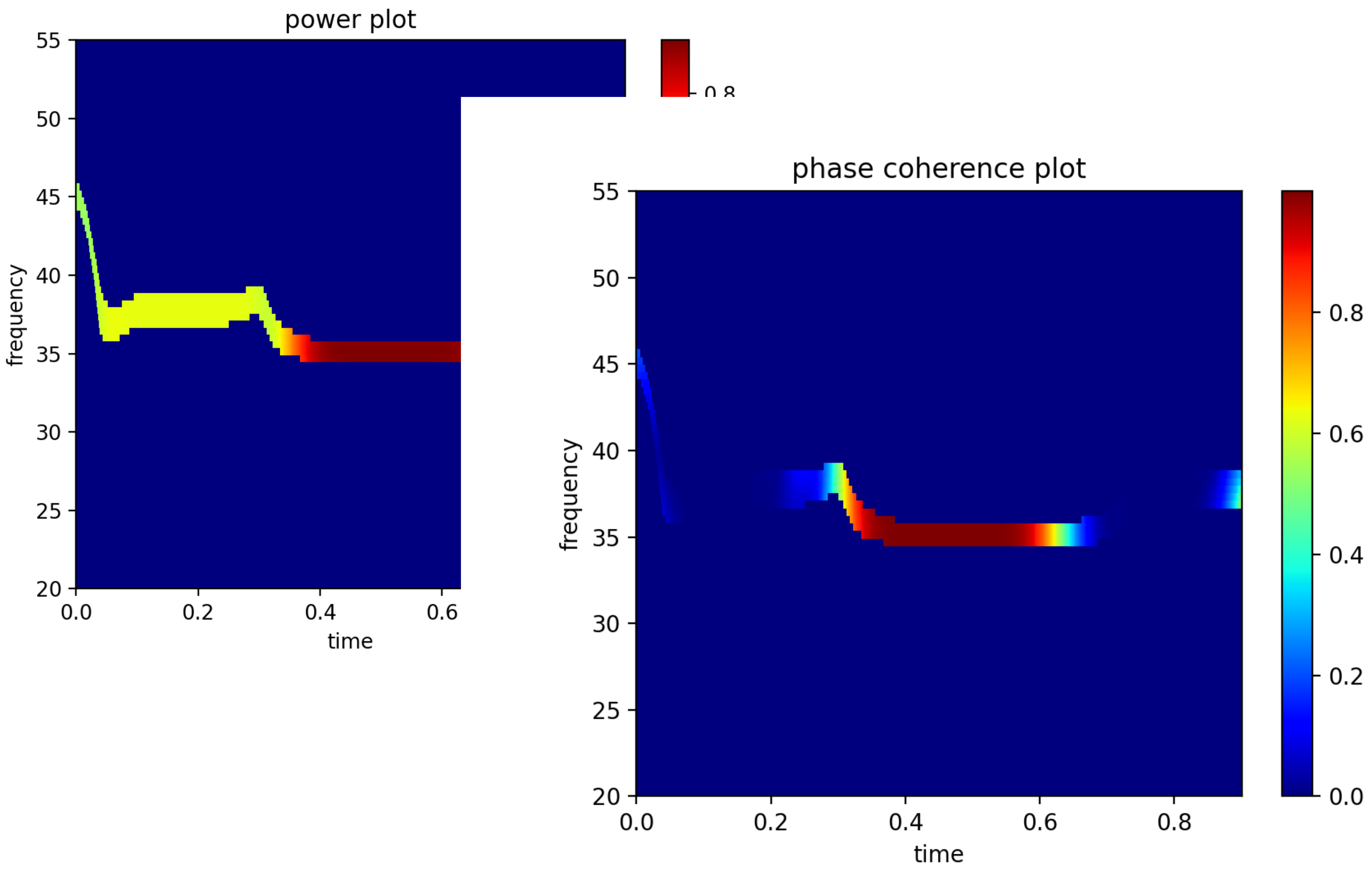


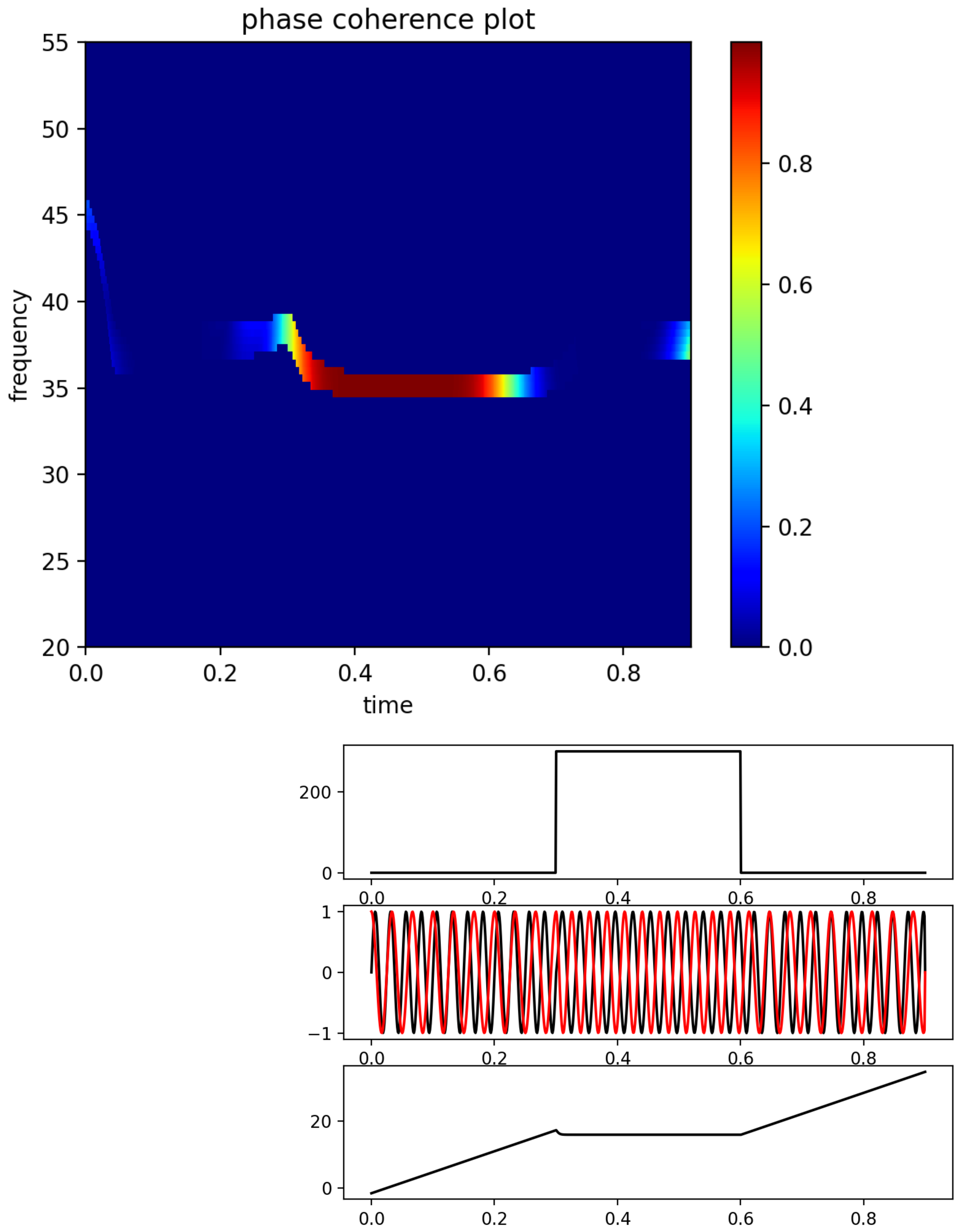
power threshold: 0.992



“template” for R



result is good !!



improved phase coherence detection

data sampling

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phase synchronisation **spectral coherence**

phase-amplitude coupling

simple model for phase-amplitude coupling

recall:

$$s(t) = [1 + 0.8\cos(2\pi f_m t)]\sin(2\pi f t)$$
$$= \sin(2\pi f t) + 0.4\sin(2\pi(f + f_m)t) + 0.4\sin(2\pi(f - f_m)t)$$

3 frequencies: $f \ , \ f + f_m \ , \ f - f_m$

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3 frequencies: $f \;,\; f + f_m \;,\; f - f_m$

according to the general rule

$$\sin(\alpha)\cos(\beta) = (\sin(\alpha+\beta) + \sin(\alpha-\beta))/2$$

simple model for phase-amplitude coupling

recall:

$$s(t) = [1 + 0.8\cos(2\pi f_m t)]\sin(2\pi f t)$$
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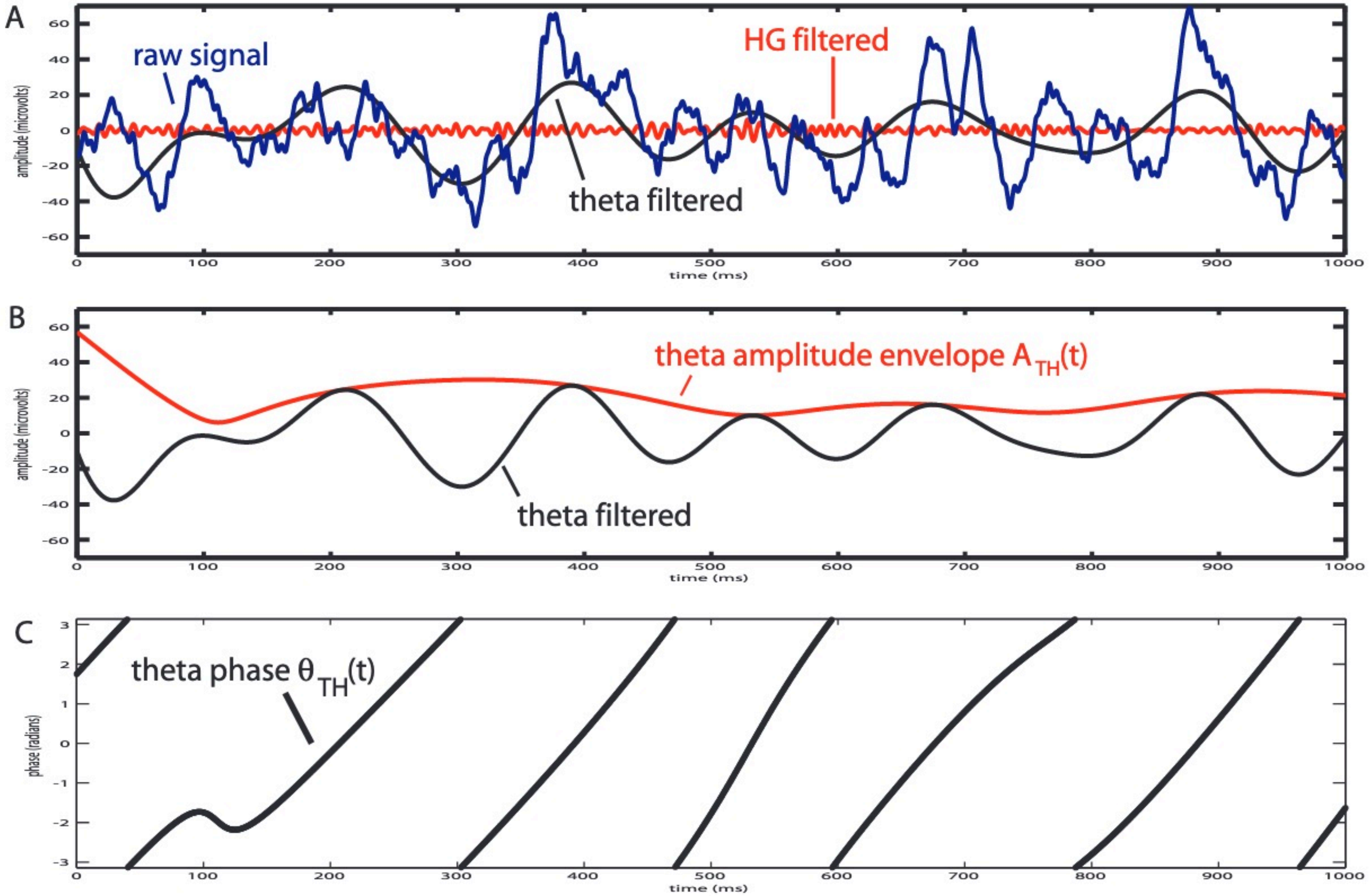
3 frequencies: f , $f + f_m$, $f - f_m$

each **periodically amplitude-modulated** oscillation
is identical to the **sum of 2 two oscillations** with constant amplitude

- different definitions for correlation between phase and amplitude time series

Modulation index (Canolty et al., Science 313:1626 (2006))



θ-oscillation determines amplitude of high-γ oscillations

- different definitions for correlation

Modulation index (Canolty et al., Science 313:1626 (2006))

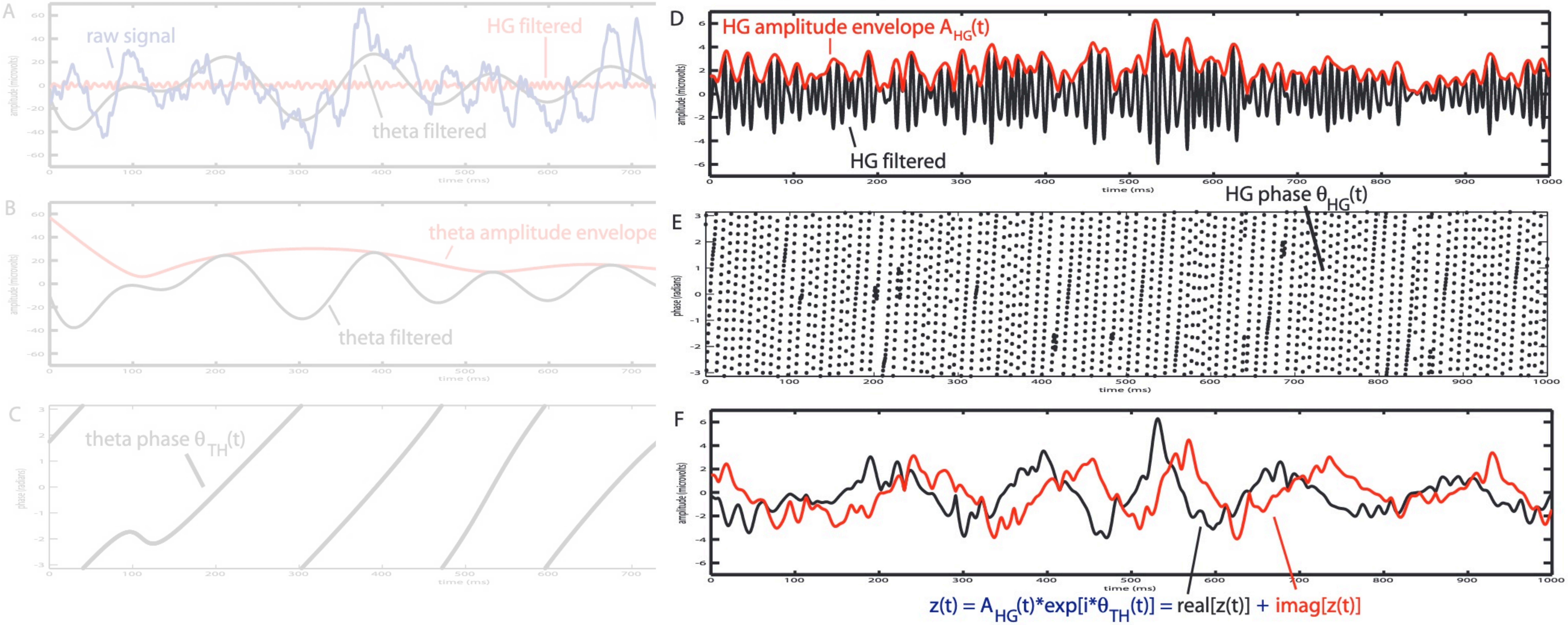
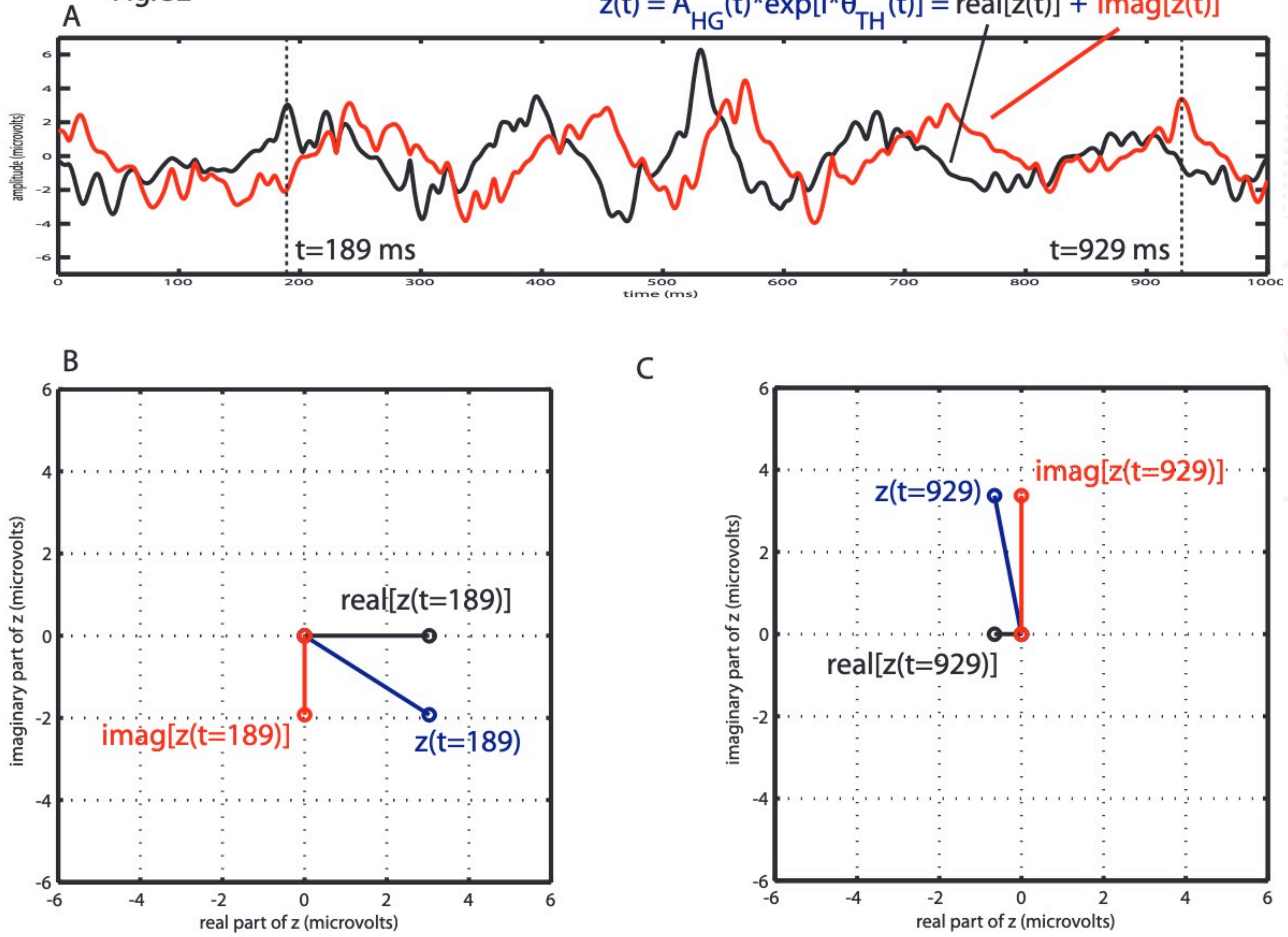
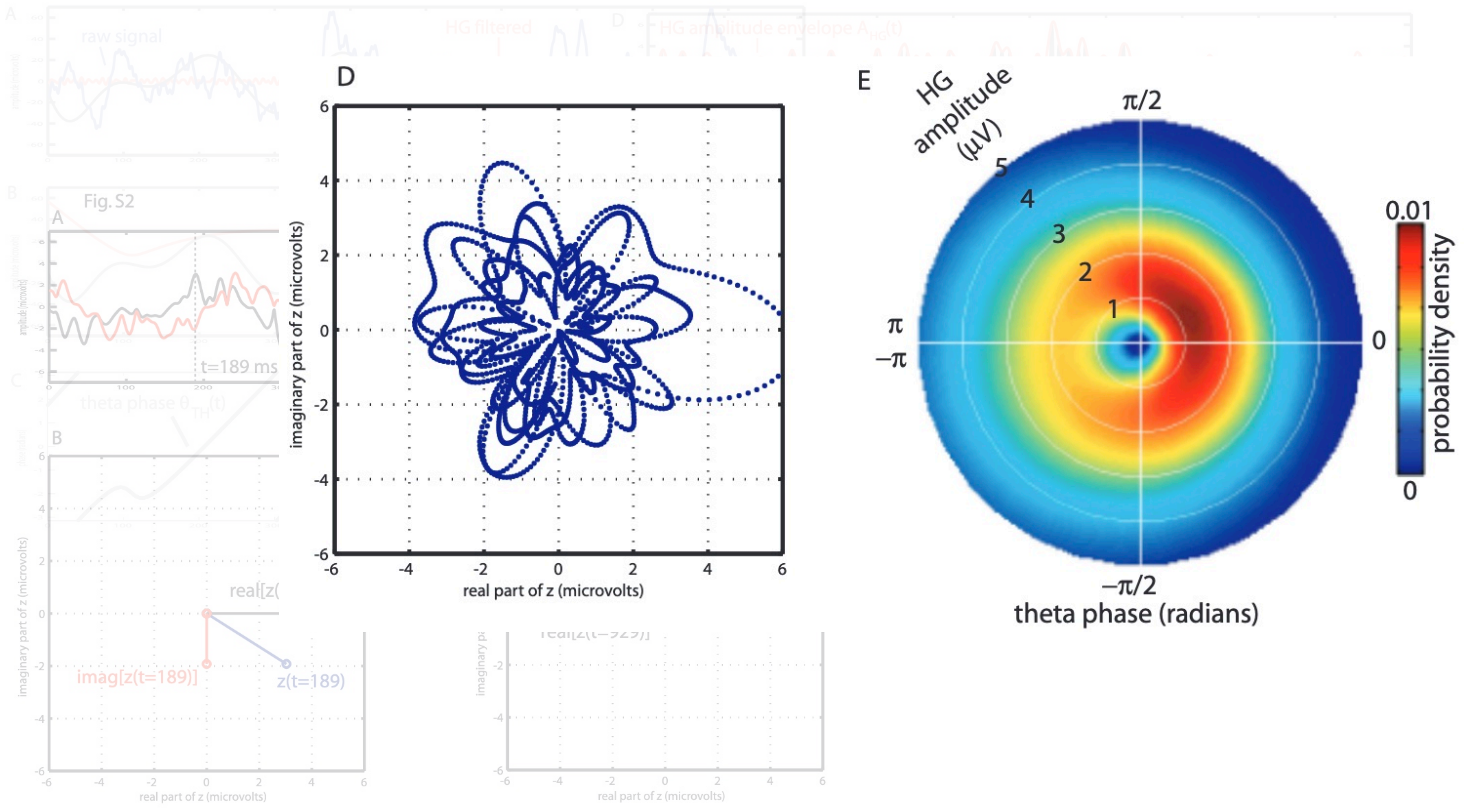


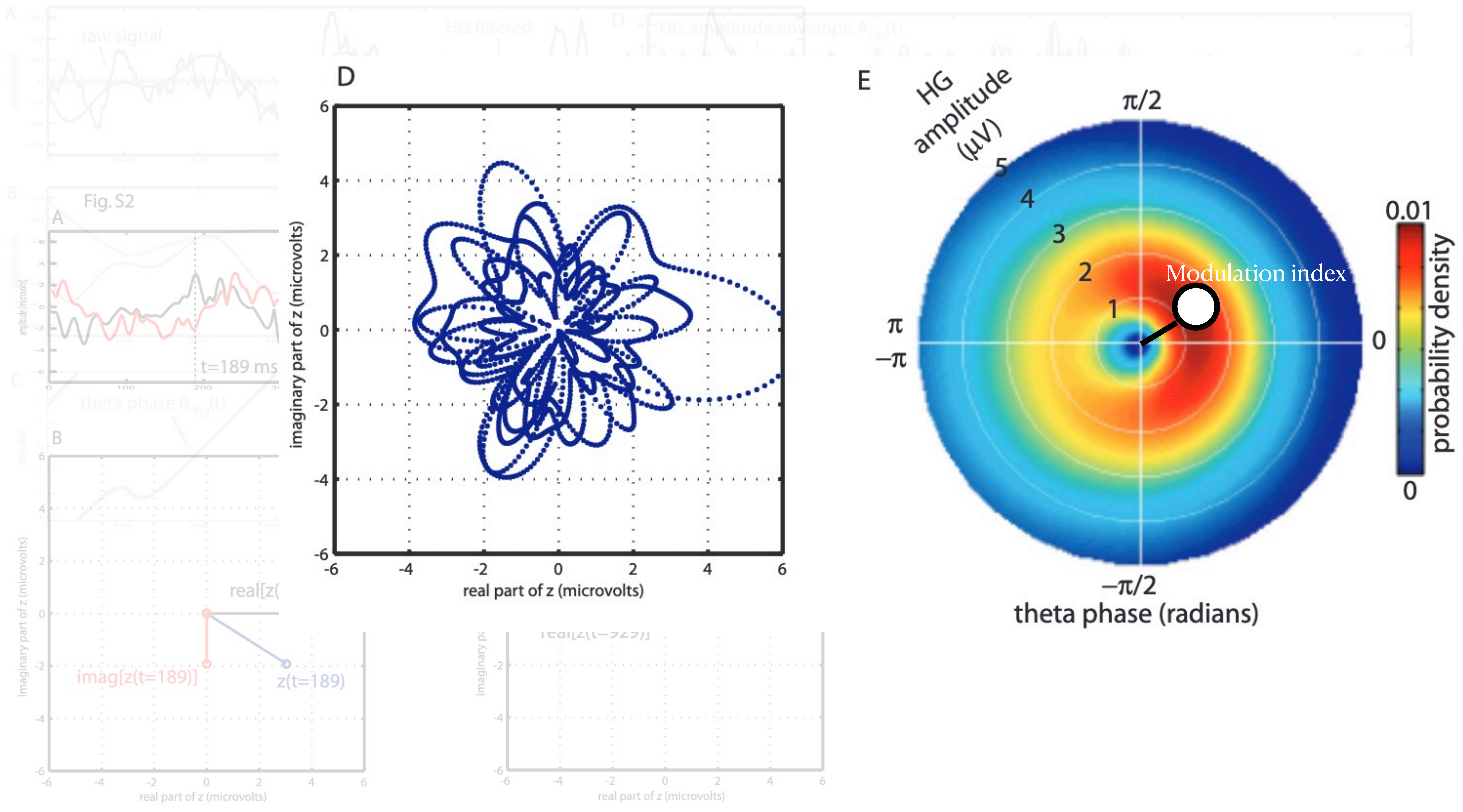
Fig. S2



A
$z(t) = A_{HG}(t)*\exp[i*\theta_{TH}(t)] = \text{real}[z(t)] + \text{imag}[z(t)]$
amplitude (microvolts)
t=189 ms
t=929 ms
time (ms)
B
real[z(t=189)]
imag[z(t=189)]
z(t=189)
imaginary part of z (microvolts)
real part of z (microvolts)
C
z(t=929)
imag[z(t=929)]
real[z(t=929)]
imaginary part of z (microvolts)
real part of z (microvolts)



raw signal
HG filtered
HG amplitude envelope
Fig. S2
t=189 ms
imag[z(t=189)]
z(t=189)
real part of z (microvolts)
imaginary part of z (microvolts)
D
imaginary part of z (microvolts)
real part of z (microvolts)
E
HG amplitude (μV)
1
2
3
4
5
π/2
−π/2
π
−π
0
theta phase (radians)
0.01
0
probability density


D
imaginary part of z (microvolts)
real part of z (microvolts)
E
HG amplitude (μV)
π/2
−π/2
π
−π
0
1
2
3
4
5
Modulation index
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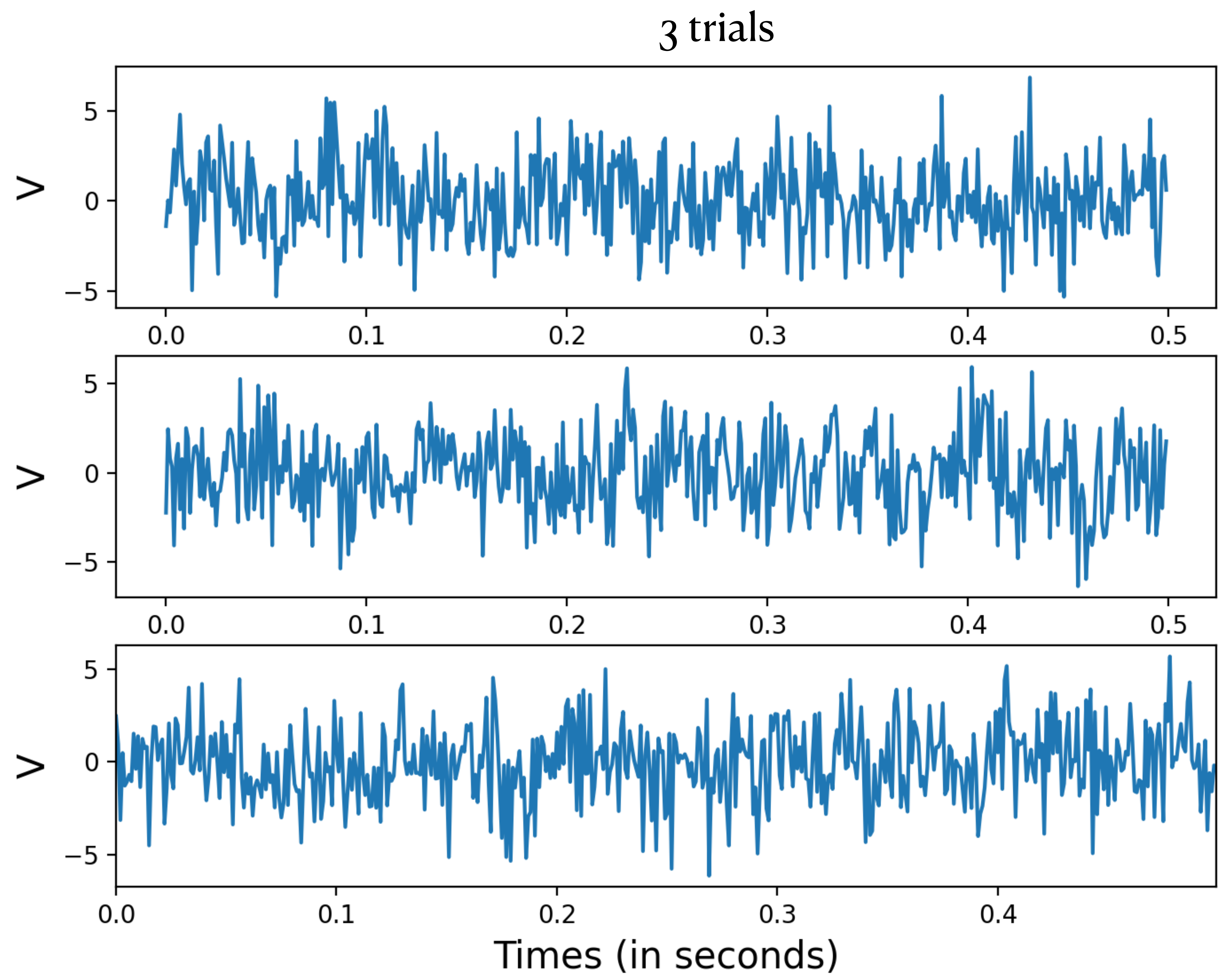
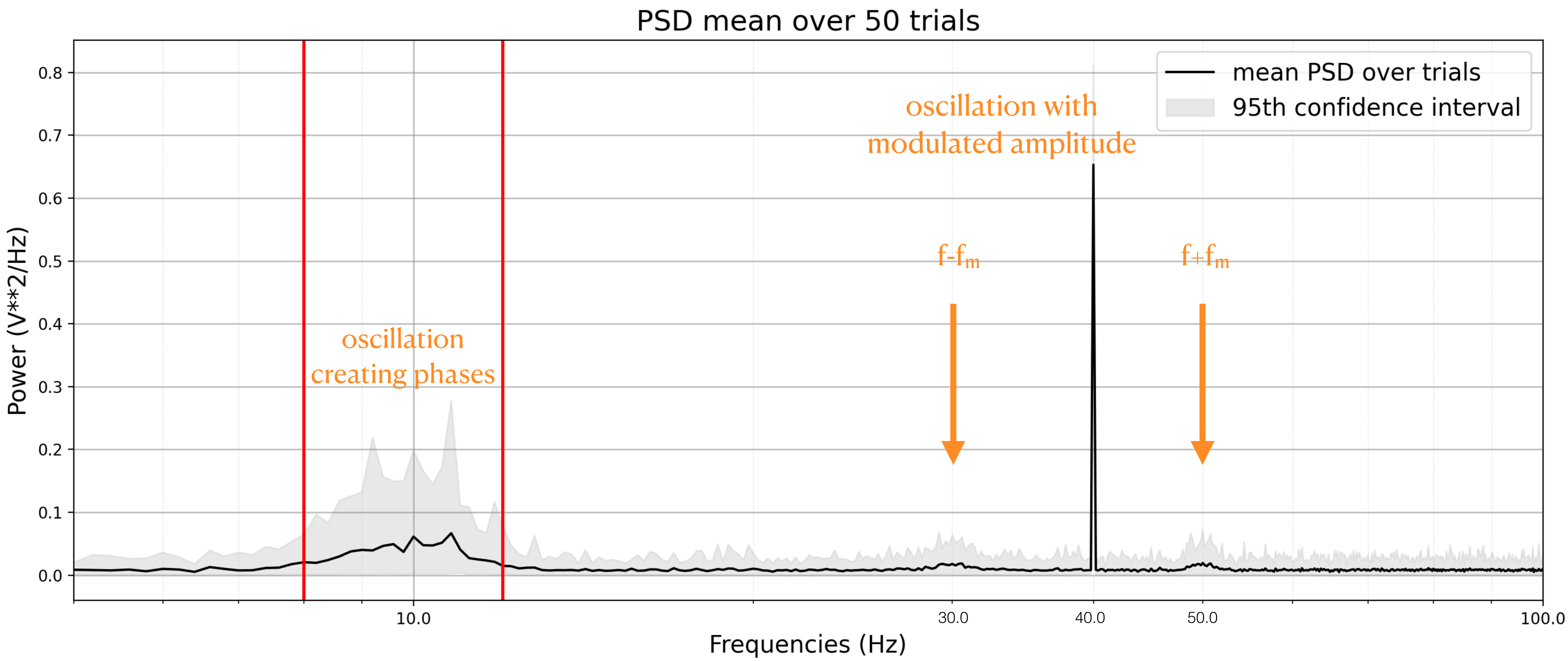
raw signal
HG filtered
HG amplitude envelope
Fig. S2
t=189 ms
theta phase
imag[z(t=189)]
z(t=189)



simulated example signal:

- 10Hz rhythm (α)
- 40Hz rhythm (γ)
- α-oscillation determines amplitude of γ-oscillation
- additive and multiplicative noise
- 50 trials

Python toolbox: TensorPac


PSD mean over 50 trials
mean PSD over trials
95th confidence interval
oscillation with
modulated amplitude
f-fm
f+fm
oscillation
creating phases
Power (V**2/Hz)
Frequencies (Hz)
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
10.0
30.0
40.0
50.0
100.0

To compute Phase-Amplitude Coupling (PAC) between phases and amplitudes:

- filter signal in frequency band at *low frequency (phase)* and *high frequency (amplitude)*

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high frequency band width $\Delta f_{\mathrm{high}} > f_{\mathrm{low}}$

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high frequency band width $\Delta f_{\text{high}} > \text{f}_{\text{low}}$

$f \;,\; f + f_m \;,\; f - f_m$

To compute Phase-Amplitude Coupling (PAC) between phases and amplitudes:

- filter signal in frequency band at *low frequency (phase)* and *high frequency (amplitude)*

    - compute *instantaneous phases (low frequencies)* and *instantaneous amplitudes (high frequencies)*
      by Hilbert transform or Wavelet transform

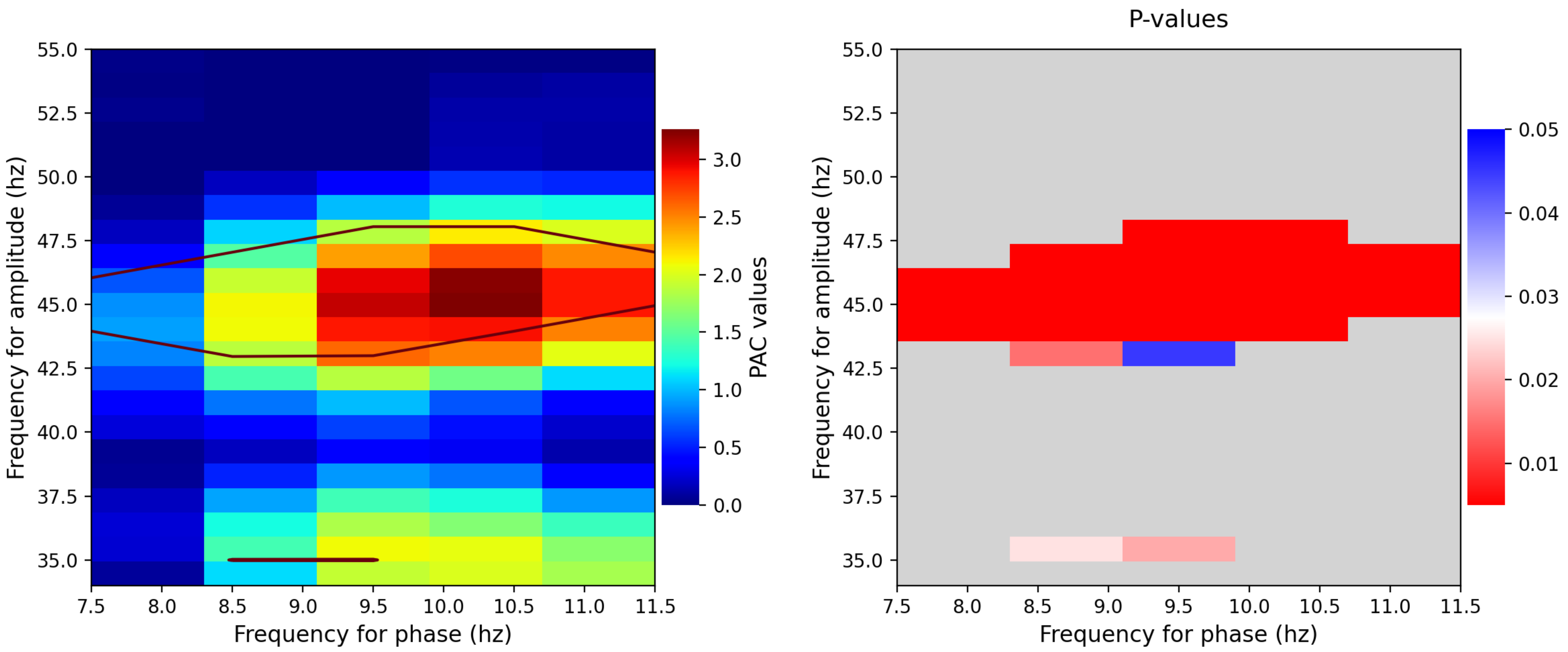
To compute Phase-Amplitude Coupling (PAC) between phases and amplitudes:

- filter signal in frequency band at *low frequency (phase)* and *high frequency (amplitude)*

- compute *instantaneous phases (low frequencies)* and *instantaneous amplitudes (high frequencies)*
  by Hilbert transform or Wavelet transform

- compute correlation between amplitudes and phases
  —> *comodulogram*

To compute Phase-Amplitude Coupling (PAC):



Thank you for your attention

any later questions to me: write to *axel.hutt@inria.fr*