The JPEG Standard

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1. Introduction

- JPEG standard is a collaboration among :
  - International Telecommunication Union (ITU)
  - International Organization for Standardization (ISO)
  - International Electrotechnical Commission (IEC)
- The official names of JPEG :
  - Joint Photographic Experts Group
  - ISO/IEC 10918-1 Digital compression and coding of continuous-tone still image
  - ITU-T Recommendation T.81

1. Introduction

- JPEG have the following mods :
  - Lossless mode, predictive coding
  - Sequential mode, DCT-based coding
  - Progressive mode, DCT-based coding
  - Hierarchical mode
- Baseline system
  - Sequential mode, DCT-based coding, Huffman coding for entropy encoding
  - The most widely used mode in practice

1. Introduction

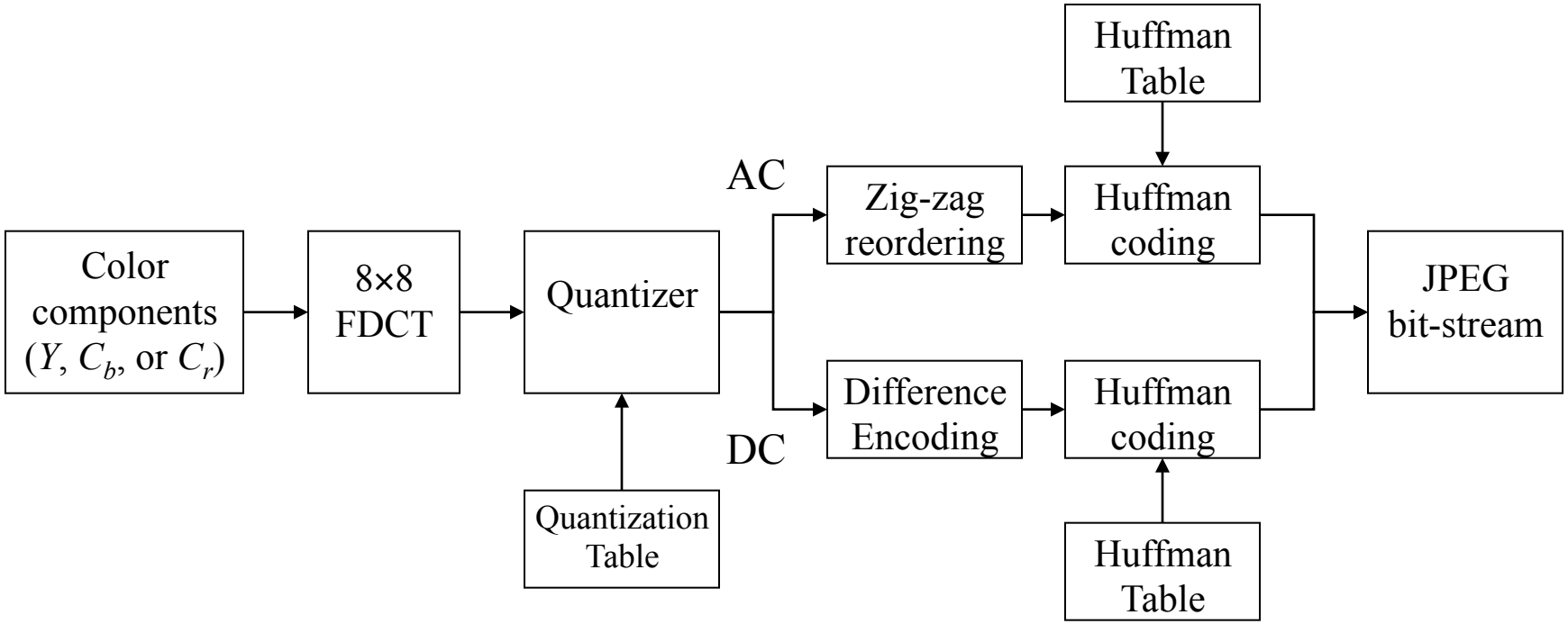


Figure 1. Baseline JPEG encoder

2. Color Space Conversion

$$\begin{bmatrix} Y \\ C_b \\ C_r \end{bmatrix} = \begin{bmatrix} 0.299000 & 0.587000 & 0.114000 \\ -0.168736 & -0.331264 & 0.500002 \\ 0.500000 & -0.418688 & -0.081312 \end{bmatrix} \begin{bmatrix} R \\ G \\ B \end{bmatrix} + \begin{bmatrix} 0 \\ 128 \\ 128 \end{bmatrix}$$

(a) translate from RGB to YC_bC_r

$$\begin{bmatrix} R \\ G \\ B \end{bmatrix} = \begin{bmatrix} 1.0 & 0.0 & 1.40210 \\ 1.0 & -0.34414 & -0.71414 \\ 1.0 & 1.77180 & 0.0 \end{bmatrix} \begin{bmatrix} Y \\ C_b - 128 \\ C_r - 128 \end{bmatrix}$$

(b) translate from YC_bC_r to RGB

3. Downsampling

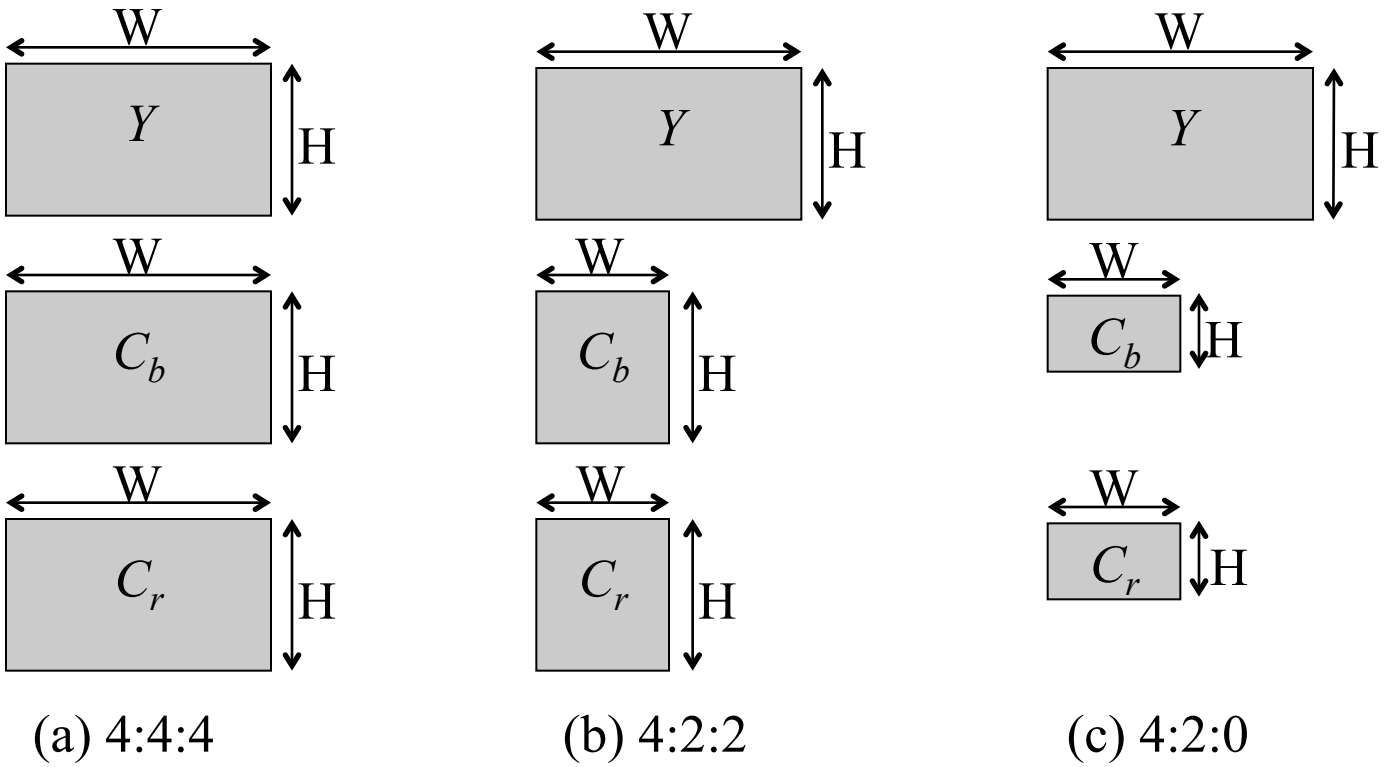


Figure 2. Three color format in the baseline system

4. Discrete Cosine Transform

Forward DCT:

$$F(u,v) = \frac{1}{4} C(u)C(v) \sum_{x=0}^{7} \sum_{y=0}^{7} f(x,y) \cos\left[\frac{\pi(2x+1)u}{16}\right] \cos\left[\frac{\pi(2y+1)v}{16}\right]$$

for $u = 0,...,7$ and $v = 0,...,7$

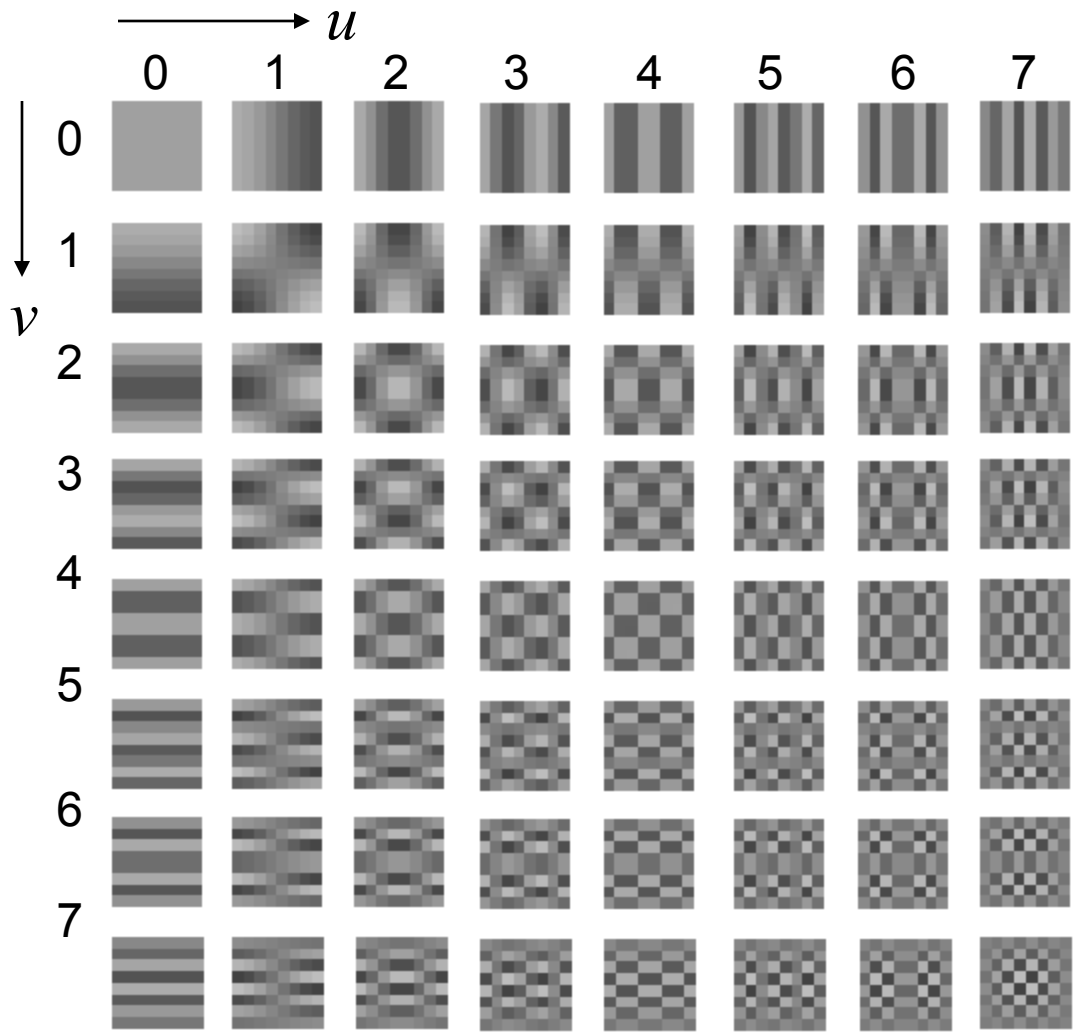
where $C(k) = \begin{cases} 1/\sqrt{2} & \text{for } k = 0 \\ 1 & \text{otherwise} \end{cases}$

Inverse DCT:

$$f(x,y) = \frac{1}{4} \sum_{u=0}^{7} \sum_{v=0}^{7} C(u)C(v)F(u,v) \cos\left[\frac{\pi(2x+1)u}{16}\right] \cos\left[\frac{\pi(2y+1)v}{16}\right]$$

for $x = 0,...,7$ and $y = 0,...,7$

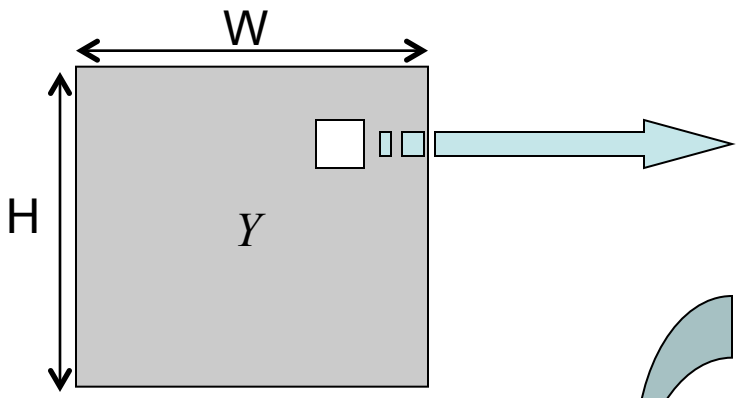
4. Discrete Cosine Transform



The 8x8 DCT basis

4. Discrete Cosine Transform

Example :



the luminance of an image

48	39	40	68	60	38	50	121
149	82	79	101	113	106	27	62
58	63	77	69	124	107	74	125
80	97	74	54	59	71	91	66
18	34	33	46	64	61	32	37
149	108	80	106	116	61	73	92
211	233	159	88	107	158	161	109
212	104	40	44	71	136	113	66

8x8 values of luminance

DCT

699.25	43.18	55.25	72.11	24.00	-25.51	11.21	-4.14
-129.78	-71.50	-70.26	-73.35	59.43	-24.02	22.61	-2.05
85.71	30.32	61.78	44.87	14.84	17.35	15.51	-13.19
-40.81	10.17	-17.53	-55.81	30.50	-2.28	-21.00	-1.26
-157.50	-49.39	13.27	-1.78	-8.75	22.47	-8.47	-9.23
92.49	-9.03	45.72	-48.13	-58.51	-9.01	-28.54	10.38
-53.09	-62.97	-3.49	-19.62	56.09	-2.25	-3.28	11.91
-20.54	-55.90	-20.59	-18.19	-26.58	-27.07	8.47	0.31

8x8 DCT coefficiences

5. Quantization

$$F_q(u,v) = Round\left(\frac{F(u,v)}{Q(u,v)}\right)$$

16	11	10	16	24	40	51	61
12	12	14	19	26	58	60	55
14	13	16	24	40	57	69	56
14	17	22	29	51	87	80	62
18	22	37	56	68	109	103	77
24	35	55	64	81	104	113	92
49	64	78	87	103	121	120	101
72	92	95	98	112	100	103	99

17	18	24	47	99	99	99	99
18	21	26	66	99	99	99	99
24	26	56	99	99	99	99	99
47	66	99	99	99	99	99	99
99	99	99	99	99	99	99	99
99	99	99	99	99	99	99	99
99	99	99	99	99	99	99	99
99	99	99	99	99	99	99	99

Figure 3. Luminance and Chrominance quantization matrix

5. Quantization

Example :

699.25	43.18	55.25	72.11	24.00	-25.51	11.21	-4.14
-129.78	-71.50	-70.26	-73.35	59.43	-24.02	22.61	-2.05
85.71	30.32	61.78	44.87	14.84	17.35	15.51	-13.19
-40.81	10.17	-17.53	-55.81	30.50	-2.28	-21.00	-1.26
-157.50	-49.39	13.27	-1.78	-8.75	22.47	-8.47	-9.23
92.49	-9.03	45.72	-48.13	-58.51	-9.01	-28.54	10.38
-53.09	-62.97	-3.49	-19.62	56.09	-2.25	-3.28	11.91
-20.54	-55.90	-20.59	-18.19	-26.58	-27.07	8.47	0.31

$F(u,v)$
8x8 DCT coefficiences

16	11	10	16	24	40	51	61
12	12	14	19	26	58	60	55
14	13	16	24	40	57	69	56
14	17	22	29	51	87	80	62
18	22	37	56	68	109	103	77
24	35	55	64	81	104	113	92
49	64	78	87	103	121	120	101
72	92	95	98	112	100	103	99

$Q(u,v)$
Quantization matrix

5. Quantization

Example :

43.70	3.93	5.52	4.51	1.00	-0.64	0.22	-0.07
-10.82	-5.96	-5.02	-3.86	2.29	-0.41	0.38	-0.04
6.12	2.33	3.86	1.87	0.37	0.30	0.22	-0.24
-2.91	0.60	-0.80	-1.92	0.60	-0.03	-0.26	-0.02
-8.75	-2.25	0.36	-0.03	-0.13	0.21	-0.08	-0.12
3.85	-0.26	0.83	-0.75	-0.72	-0.09	-0.25	0.11
-1.08	-0.98	-0.04	-0.23	0.54	-0.02	-0.03	0.12
-0.29	-0.61	-0.22	-0.19	-0.24	-0.27	0.08	0.00

$$\frac{F(u,v)}{Q(u,v)}$$

44	4	6	5	1	-1	0	0
-11	-6	-5	-4	2	0	0	0
6	2	4	2	0	0	0	0
-3	1	-1	-2	1	0	0	0
-9	-2	0	0	0	0	0	0
4	0	1	-1	-1	0	0	0
-1	-1	0	0	1	0	0	0
0	-1	0	0	0	0	0	0

$$F_q(u,v) =$$

$$Round\left(\frac{F(u,v)}{Q(u,v)}\right)$$

6. Zig-Zag Reordering

0	1	5	6	14	15	27	28
2	4	7	13	16	26	29	42
3	8	12	17	25	30	41	43
9	11	18	24	31	40	44	53
10	19	23	32	39	45	52	54
20	22	33	38	46	51	55	60
21	34	37	47	50	56	59	61
35	36	48	49	57	58	62	63

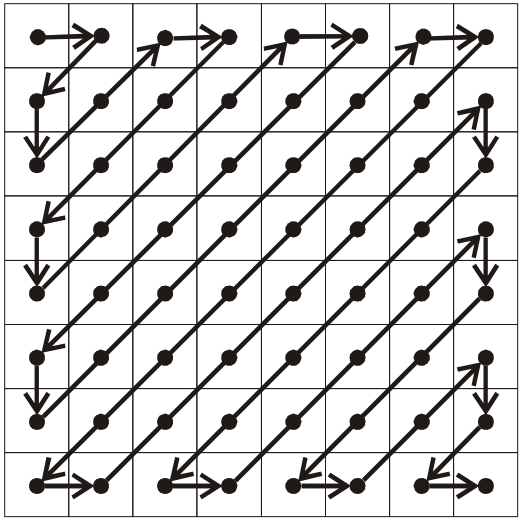


Figure 4. Zig-Zag reordering matrix

6. Zig-Zag Reordering

Example :

44	4	6	5	1	-1	0	0
-11	-6	-5	-4	2	0	0	0
6	2	4	2	0	0	0	0
-3	1	-1	-2	1	0	0	0
-9	-2	0	0	0	0	0	0
4	0	1	-1	-1	0	0	0
-1	-1	0	0	1	0	0	0
0	-1	0	0	0	0	0	0

$F_q(u,v)$

Zig-Zag Reordering :

44,
4,-11,
6,-6,6,
5,-5,2,-3,
-9,1,4,-4,1,
-1,2,2,-1,-2,4,
-1,0,0,-2,0,0,0,
0,0,0,1,0,1,-1,0,
-1,0,-1,0,0,0,0,
0,0,0,-1,0,0,
0,1,0,0,0,
0,0,0,0,
0,0,0,
0,0,
0

7. Zero Run Length Coding

- Example 1

63 AC coefficience:
57, 45, 0, 0, 0, 0, 23, 0, -30, -16, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0,..., 0

50 zeros

Run Length Coding :
(0,57) ; (0,45) ; (4,23) ; (1,-30) ; (0,-16) ; (2,1) ; EOB → (0,0)

- Example 2

63 AC coefficience:
57, 0, 0, ... , 0, 3, 0, 0, 0, 0, 2, 0, 0, ... , 0, 895, EOB

18 zeros　　33 zeros

Run Length Coding :
(0,57) ; (15,0) ; (2,3) ; (4,2) ; (15,0) ; (15,0) ; (1,895) ; (0,0)

8. Difference Coding

Encode :

$Diff_i = DC_i - DC_{i-1}$

Decode :

$DC_i = DC_{i-1} + Diff_i$

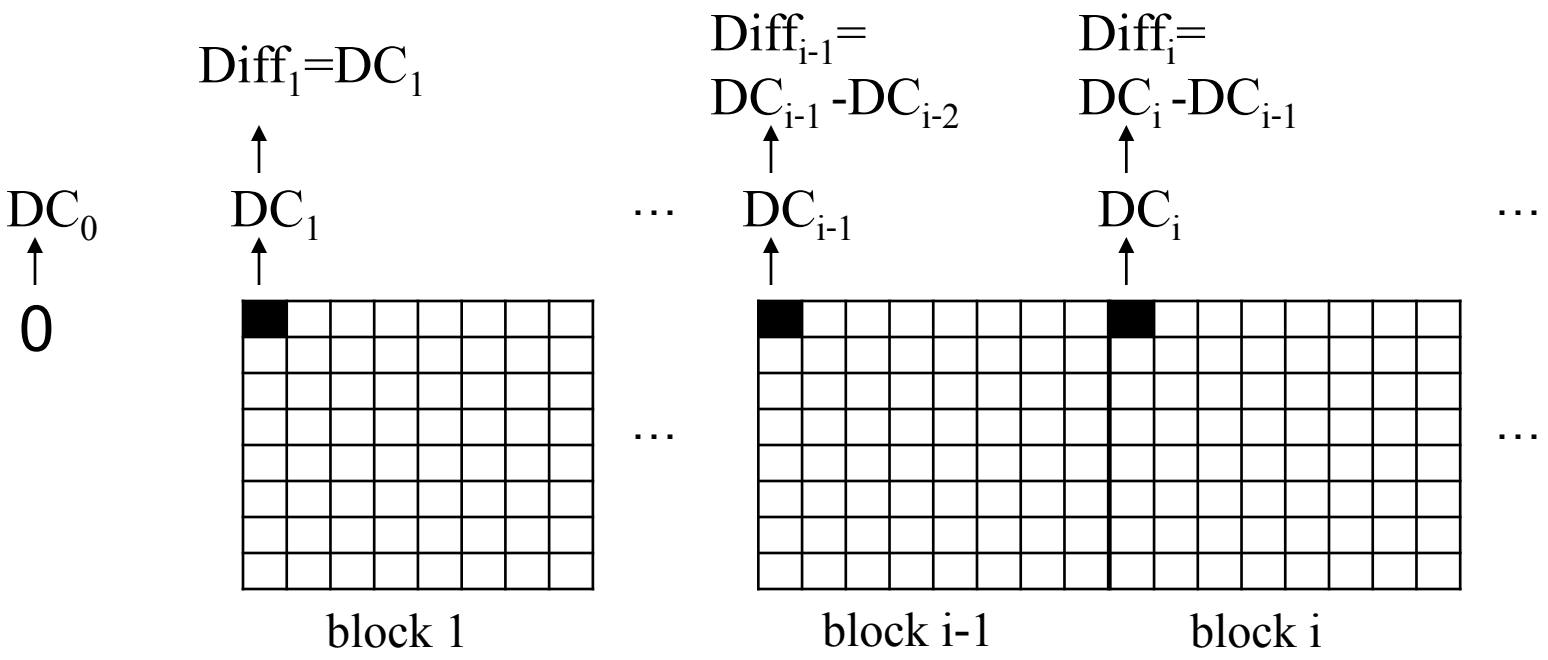


Figure 5. DCs of 8×8 blocks

9. Huffman Coding

Category	Values	Bits for the value
1	-1,1	0,1
2	-3,-2,2,3	00,01,10,11
3	-7,-6,-5,-4,4,5,6,7	000,001,010,011,100,101,110,111
4	-15,...,-8,8,...,15	0000,...,0111,1000,...,1111
5	-31,...,-16,16,...31	00000,...,01111,10000,...,11111
6	-63,...,-32,32,...63	000000,...,011111,100000,...,111111
7	-127,...,-64,64,...,127	0000000,...,0111111,1000000,...,1111111
8	-255,..,-128,128,..,255	...
9	-511,..,-256,256,..,511	...
10	-1023,..,-512,512,..,1023	...
11	-2047,..,-1024,1024,..,2047	...

Figure 6. Table of values and bits for the value

9. Huffman Coding

- Example

Run lenth coding of 63 AC coefficiences :
(0,57) ; (0,45) ; (4,23) ; (1,-30) ; (0,-8) ; (2,1) ; (0,0)

Encode the right value of these pair as category and bits for the value, except the special markers like (0,0) or (15,0) :
(0,6,111001) ; (0,6,101101) ; (4,5,10111);
(1,5,00001) ; (0,4,0111) ; (2,1,1) ; (0,0)

The difference of DC coefficience :
-511

Encode the value as category and bits for the value :
9, 000000000

9. Huffman Coding

run/category	code length	code word
0/0	4	1010
...		
0/6	7	1111000
...		
0/10	16	1111111110000011
1/1	4	1100
...		
1/5	11	11111110110
...		
4/5	16	1111111110011000
...		
15/10	16	1111111111111110

Figure 7. Huffman table of luminance AC coefficience

9. Huffman Coding

category	code length	code word
0	2	00
1	3	010
2	3	011
3	3	100
4	3	101
5	3	110
6	4	1110
7	5	11110
8	6	111110
9	7	1111110
10	8	11111110
11	9	111111110

Figure 8. Huffman table of luminance DC coefficience

9. Huffman Coding

- Example

The AC coefficiences :
(0,6,111001) ; (0,6,101101) ; (4,5,10111);
(1,5,00001) ; (0,4,0111) ; (2,1,1) ; (0,0)

Encode the left two value in () using Huffman encoding :
1111000 111001 , 1111000 101101 , 1111111110011000 10111 ,
11111110110 00001 , 1011 0111 , 11100 1 , 1010

The DC coefficience :
9, 000000000

Encode the category using Huffman encoding :
1111110 000000000

End